Effect of the Dilaton Field on the Entropic Force

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ABSTRACT

According to the Eric Verlinde's arguments on the gravity, we study the entropic force of two spacetimes without and with dilaton field, which are the Schwarzschild black hole and the charged dilaton black hole respectively. Generally, the existence of the dilaton field makes over the spacetime to have unusual asymptotic structure. During the calculations of the entropic force, the key point is to describe the holographic screen of the associated spacetime. In this thesis, we mainly consider three surfaces as being candidates for the holographic screen. These surfaces are called as the static holographic screen, the accelerating surface and the stretched horizon. Thus, by comparing the results of the entropic force of the associated spacetimes, we want to stress the effect of the dilaton field on the entropic force.

Keywords: Entropic force, holographic screen, dilaton field, emergent gravity.

ÖZ

Eric Verlinde’nin yerçekimi üzerindeki argümanlarına dayanarak dilaton alanına sahip olmayan ve sahip olan iki uzay-zamanın, ki sırasıyla bunlar Schwarzschild ve yüklü dilaton kara delikleridir, entropik kuvvetlerini hesaplıyoruz. Genellikle dilaton alanının varlığı uzay-zamanı asimtotik olarak düzgün olmayan bir yapıya dönüştürür. Entropik kuvvet hesaplamaları sırasında kilit nokta ilgili uzay-zamanın holografik ekranını tanımlamaktır. Bu tezde, holografik ekran adayı olarak başlıca üç yüzeyi dikkate alıyoruz. Bu yüzeyler statik holografik ekran, ivmelenen yüzey ve gergin yüzey olarak adlandırılırlar. Böylece, ilgili uzay-zamanların entropik kuvvetlerinin sonuçlarını karşılaştırarak dilaton alanının entropik kuvvet üzerindeki etkisini ön plana çıkarmak istiyoruz.

Anahtar Kelimeler: Entropik kuvvet, holografik ekran, dilaton alanı, beliren yerçekimi.

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LIST OF SYMBOLS/ABBREVIATIONS

a	Dilaton Parameter
$\hat{a}$	Acceleration
c	Speed of Light
C	Constant
AF	Asymptotically Flat
AS	Accelerating Surface
BH	Black Hole
CDBH	Charged Dilaton Black Hole
DoF	Degrees of Freedom
E	Energy
F	Force
G	Gravitational Constant
HS	Holographic Surface
Q	Heat
L	Local
m	Mass of the Object
M	Mass of the BH
NAF	Non-Asymptotically Flat
Φ	Flux
QL	Quasilocal
SH	Stretched Horizon
$\hbar$	Planck's Constant

k_B .... Boltzmann's Constant

κ .... Surface Gravity

K .... Kretchmann Scalar

r .... Distance From the Center of the Black Hole

R .... Radius of the Sphere

$\mathcal{R}$ .... Ricci Scalar

S .... Entropy

T .... Temperature

T_H .... Hawking Temperature

U .... Unruh

Δx .... Displacement

Chapter 1

INTRODUCTION

1.1 Review of the Gravity and Holography

Gravity is one of the four fundamental interactions of nature. Because of its existence, objects with mass attract each other. Since1916 in which Einstein published his famous studies [1-2] on general relativity, it has been discussed that gravity is different from the other fundamental forces. In point of view of the general relativity we consider the gravity as gravitational fields arising due to the deformation of spacetime. Although the Einstein's general relativity theory has a Newtonian limit, until today there is no confirmed signal proving the existence of gravitational waves. So one may deduce that the only explanation for the absence of these fields is, maybe we were thinking in the wrong way. That is why nowadays the researchers look for a new approach to describe the gravity in the framework of general relativity and quantum mechanics – the so-called quantum gravity theory.

Bekenstein [3,4] from Gedanken experiments found out that black hole (BH) could have intrinsic entropy when the thermodynamics laws applied for the BH physics. Hawking [5] in continue found out that BH can radiate its energy in the form of Planckian black body radiation. Throughout this period, 't Hooft [6,7] and Susskind [8,9] showed that the basic principles of quantum mechanics and statistical mechanics have to be made to co-exist with BH evaporation. Later on, Jacobson [10] demonstrated that Einstein equations could be derived from combining the thermodynamics with equivalence principle. Padmanabhan [11,12] observed that equipartition law for horizon degrees of freedom combined with the Smarr's formula leads to Newton's law of gravity. Recently, Verlinde [13] has proposed the Newtonian law as an entropic force by using holographic principle and equipartition rule.

1.2 Holographic Principle

t' Hooft [6-7] found that the maximum information storage capacity of a BH depends on its area, not on its volume. Holographic principle says that the entire phenomena occurring in a three-dimensional region can be described by only on the two-dimensional boundary of the region. In other words, we can describe the whole world with a two-dimensional lattice of spins. A two-dimensional surface as a screen separates sites as pixels [14]. Of more relevance to a theory of quantum gravity, BH will be assumed as entropy storage found on the horizon, no more than one bit per Planck area. Namely, the entropy can be considered as a measure of information. We can map the horizon on the screen as follows, see figure 1 [15].

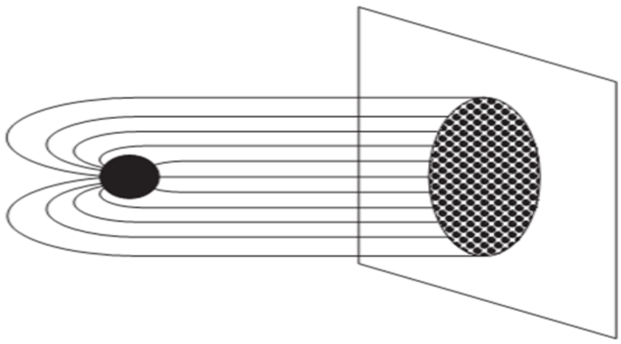

Figure 1: A BH projected to the screen

We have another image of slowly passing first BH in front of the second in an attempt to eclipse it, see figure 2.

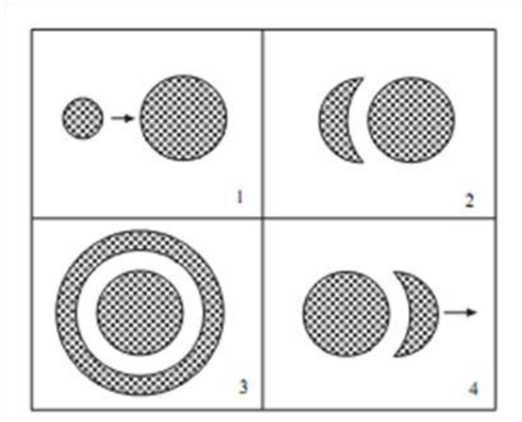

Figure 2

In summary, the holographic principle shows that the world is two-dimensional, but two-dimensional surface contains information that is encoded in such a way that we can recover our three-dimensional world. Hence, one can also make another assumption: "We are the holograms, living on the boundary surface of a five-dimension spacetime". On the other hand, the holographic principle yields also another consequence: gravity is not a fundamental interaction any more, it is just an emergent phenomenon that appears from the statistical behavior of microscopic degrees of freedom encoded on holographic screen (HS). This suggests that gravity originates from changing of mass, time and space (information). By summarizing some well-known physical principles and laws we make the relation between information and gravity more clear. [16]

1) Landauer's principle: To erase information dS, at least energy described by $dE = TdS$ should be consumed. (Information is related to the heat).

2) $E = mc^2$: Energy is related to the mass (matter)

3) Einstein's equations: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ (Matter generates gravity)

4) Unruh effect: Quantum fluctuation looks thermal to some observers

By combining **1** and **2,** we see that "matter is related to information", **1+2+3** means "Gravity is related to information" and **1+4** shows that " Quantum mechanics is related to information". This configuration can be seen best in figure 3.

Therefore, one can make the following remarks:

i- Information is fundamental→ Physical laws should be such that they respect observers' information about given matter and spacetime

ii- Holographic principle → Amount of information in a region bounded by causal horizon is finite in bits and proportional to the area of the horizon

iii- Landauer's principle, $dE = TdS$ or second law of thermodynamics → information- energy relation

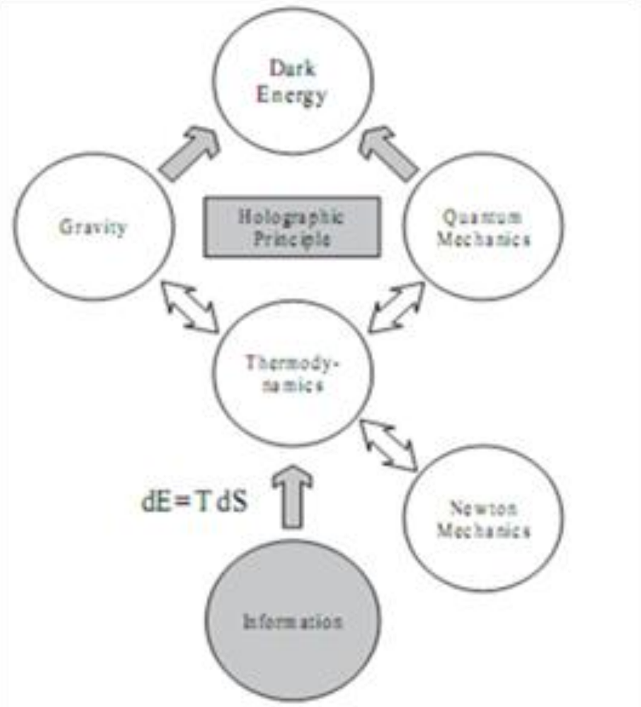

Figure 3: Relation between various physical concepts.

1.3 Entropic Force

Thermodynamics and relativity has been connected over 35 years ago, with pioneering work of Bekenstein [3,4], Hawking [5], and Unruh [17,18]. Based on Bekenstein and Hawking's works, Jacobson [10] discovered that Einstein's equations [1-2] can be viewed as thermodynamic equation of state under a set of minimal assumptions, the equivalence principle and the identification of the area of a causal horizon with its entropy. Recently, Verlinde [13] has shown that Newton's law of gravitation can be understood as an entropic force.

The idea of BH entropy is rather enigmatic. The first glimmerings of the notion began in the early days of thermodynamics, the study of the movement of heat. In 1850, Rudolf Clausius [19] generalized Carnot's finding who analyzed the working of a steam engine in detail. Clausius called the unavailable energy of a system the entropy of that system. According to Clausius, the entropy of any closed system always increases with time. This statement which is a version of the second law of thermodynamics holds for all closed systems.

On the other hand, Boltzmann [20] said, entropy is the number of distinct arrangements of microstates in a system that gives rise to the same energy of the system. The entropy of the volume of a gas, for example, is the number of different arrangements of molecules that give rise to the same energy for the gas. The unit of Boltzmann's entropy is energy divided by temperature [14].

There is another definition for entropy. It was developed by Claude Shannon [21], who introduced the concept of entropy as a branch of applied mathematics called information theory in 1948. He showed that maximum amount of information we can pack into a system depends upon the surface area of the system, not the volume. We can think of the entropy of a BH as being information written on its event horizon.

Each bit of information corresponds to four Planck areas. Bekenstein [3-4] stated that when object falls into a BH the mass and the area of the horizon and the entropy of the BH increases (general form of the second law of thermodynamics). In 1986, Rafael Sorkin [22] showed that the entropy we calculate for a BH must involve the most fundamental degrees of freedom (DoF). So even if the structure of matter is unknown, we can still find the true entropy of a BH [14].

If we return to the concept of the entropic force, it can be understood as an effective macroscopic force that starts in a system with many DoF. The equation of this macroscopic force can be formulated in terms of the entropy differences without considering the microscopic dynamics.

In order to visualize this macroscopic force, we can consider the elastic force in a polymer. By using a mechanism, we can pull the endpoints of the polymer by an applied force F. Thus we bring it out of its equilibrium. Furthermore, if we assign its one end as the origin, and shift its other endpoint along the x –axis, we get the entropy as

$$S\,(E,x) = k_B \, ln\Omega(E,x), \tag{1.3.1}$$

where k_B is the Boltzmann's constant and the volume $\Omega\,(E,x)$, which is a function of the total energy E of the system and the position x belongs to the configuration space of the system. Therefore, the applied force F at a temperature T can be obtained by the following saddle point equations

$$\frac{1}{T} = \frac{\partial S}{\partial E} \quad \text{and} \quad \frac{F}{T} = \frac{\partial S}{\partial x} \ , \tag{1.3.2}$$

According to the Newton’s third law, the applied force F should be equal to the entropic force. Namely, the polymer is restored to its equilibrium point by the entropic force. Thus, we can deduce that the magnitude of an entropic force is directly proportional to the temperature, and it is aligned with the direction of increasing entropy. Actually, Verlinde has extended the latter idea to the gravitational force between test masses using a set of assumptions loosely motivated by the concepts emerged from the study of the relation between thermodynamics and relativity, [13,23].

In order to understand Verlinde's entropic force, we imagine first a patch of a HS [24], and an object of mass m that penetrates it from the side at which spacetime has already emerged. While the object with its microscopic DoF attaches to the screen, it also influences the amount of information stored on the screen, see figure 4.

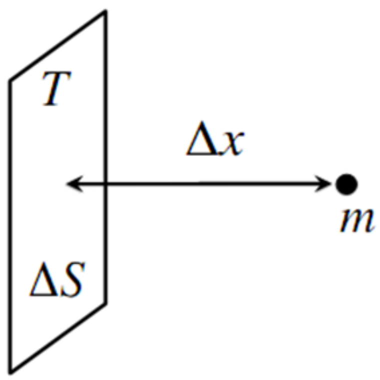

Figure 4

According to the Bekenstein's arguments [25], the change in entropy (ΔS) which is correlated with the information on a boundary (screen) is given by

$$\Delta S = 2\pi k_B \quad \text{when} \quad \Delta x = \frac{\hbar}{mc}, \tag{1.3.3}$$

Soon after, we will understand the reason of the factor 2π appearing in the above equation.

Let us rewrite this formula by assuming that ΔS is linear to the displacement, Δx.

$$\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x, \tag{1.3.4}$$

For reaching the entropic force then we gain inspiration from the osmosis, which crosses a semi-permeable membrane. When an object transfers through a membrane with a temperature T, it experiences an effective force F, which satisfies the following equality

$$F\Delta x = T\Delta S, \tag{1.3.5}$$

F is called the entropic force. Remarkably, in order to have a $F \neq 0$, a non-zero temperature is needed. On the other hand, we know from the Newton's second law that whenever $F \neq 0$ yields a non-zero acceleration, $\hat{a} \neq 0$. . Unruh [17,18] proved that acceleration and temperature are related. Referring to his studies, we get the following expressions

$$k_B T = \frac{1}{2\pi}\frac{\hbar\hat{a}}{\mathrm{c}}, \qquad T = \frac{1}{2\pi k_B}\frac{\hbar\hat{a}}{\mathrm{c}}, \tag{1.3.6}$$

where $\hat{a}$ is the acceleration. This equation shows that temperature caused by the acceleration. From now on, we can call T as the temperature associated with the bits on the HS. Now we understand why the equation (1.3.3) contains the factor 2π.

$$F\Delta x = \frac{1}{2\pi k_B}\frac{\hbar\hat{a}}{\mathrm{c}}\Delta S, \tag{1.3.7}$$

and

$$\Delta S = F\Delta x 2\pi k_B \frac{c}{\hbar\hat{a}}, \tag{1.3.8}$$

Thus, we reach to the second law of Newton by using the factor 2π,

$$F = ma \ \rightarrow \Delta S = 2\pi k_B \frac{mc}{\hbar}\Delta x, \tag{1.3.9}$$

The above arguments indicate that in order to gain insight into the origin of the entropic force we need to explore where the temperature comes from. For this purpose let us imagine that our boundary is finite and closed surface. One of the ways is to consider the boundary as an information storage device. Each bit occupies let us say one unit cell and N is the number of used bits. N will be proportional to the area of the boundary. Then one has

$$N = \frac{Ac^3}{G\hbar}\,, \tag{1.3.10}$$

Whit a new constant G known as Newton's constant. Suppose that there is a total energy E which is divided over the bits N. It can be determined by the equipartition rule

$$E = \frac{1}{2} N k_B T, \tag{1.3.11}$$

Since

$$E = Mc^2, \tag{1.3.12}$$

from Eq. (1.3.11), one obtains

$$\frac{1}{T} = \frac{1}{2Mc^2} N k_B, \tag{1.3.13}$$

where M is the mass that comes out from the part of space surrounded by the screen, see figure 5.

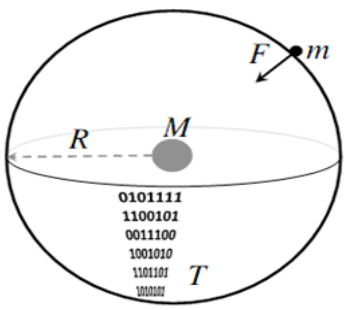

Figure 5: An object with mass m locating near a spherical HS. The energy is uniformly distributed over the unit cells, and is equivalent to the mass M that would emerge in the part of space enclosed by the HS.

By using the following equations, we can find the entropic force as follows.

$$A = 4\pi R^2, \tag{1.3.14}$$

$$F\Delta x = T\Delta S \rightarrow F = T\frac{\Delta S}{\Delta x} = 2\pi m T\frac{k_B c}{\hbar}, \tag{1.3.15}$$

$$F = \frac{4\pi m M c^2}{N\hbar}, \tag{1.3.16}$$

$$F = G\frac{mM}{R^2}. \tag{1.3.17}$$

The latter result is nothing but the Newton's law of gravitation, which is practically derived by virtue of the holographic principle. The number of DoF is linear with the area of the HS and, the energy is evenly distributed over takes the form of the Newton's law. Therefore, we conclude that in order to determine the gravitational force, one needs only the amount of information determined by the entropy and the energy, which is associated with.

In this thesis, our main aim is to derive Verlinde's entropic force in a spacetime with dilaton field. Today, the studies on the dilaton field shows that it can be the dominant messenger between standard model fields and dark matter [26]. In this way, we want to find the form of the dilatonic entropic force. To this end, we choose the metric of charged dilaton black hole (CDBH) [27] as our spacetime, which admits unusual asymptotics. They do not only exhibit asymptotic flatness (AF), but also they have non-asymptotically flat (NAF) structure depending on their dilaton parameter, a. One of the important properties of the CDBHs is that they have Schwarzschild limit when the dilaton field vanishes, $a \rightarrow \infty$. Here, we basically follow one of the recent studies on the entropic force whose authors are Myung and Kim [28]. They have introduced some HS candidates. We basically consider three of them, which are accelerating surface (AS), static HS and stretched horizon (SH). Our strategy is the following: we first calculate the temperature of each HS, and insert those obtained temperatures separately into the entropic force formula (1.3.15). Thus, we also aim to verify that the obtained entropic forces reduce to the Schwarzschild results, and thus to the Newton's force law [28]. Furthermore, we want to find out the effect of the dilaton field on the entropic force by some graphics.

The thesis is organized as follows. In chapter 2, we consider the Schwarzschild BH as a test spacetime to calculate the entropic force on the three different HSs; static HS, AS, and SH. In chapter 3, we extend our calculations made in chapter 2 to the CDBHs. The obtained results of chapter 2 and 3 are also compared graphically. Chapter 4 is devoted to the conclusion. We discuss about the obtained results, and highlight the possible detection of the dilaton fields.

Chapter 2

ENTROPIC FORCE OF THE SCHWARZSCHILD BLACK HOLE

2.1 Features of the Schwarzschild Black Hole

According to the Newton's theory in order to escape from gravity of a heavy object with mass M and radius R, the escape velocity should be $v = \sqrt{\frac{2Gm}{R}}$. Now think what will happen if mass M is compressed into small volume that escape velocity become larger than the speed of light? First in 1783, this question was raised by John Mitchell. Later, Pierre Simon de Laplace investigated further and asked, does the light fall back toward the surface of that object? Today, we can answer this question according to the Einstein's both special and general relativity theories. Later on, Karl Schwarzschild discovered that Einstein's equations have a solution in closed form. Singularity appeared in his solution is such a point that the physics laws break down there and a traveler could go through it but never comes back. Indeed, also light could not emerge out of central region of this solution. John Archibald Wheeler called these strange objects, BHs. Schwarzschild and many other researchers thought that his theoretical solution has no physical meaning and they had doubt on existence of BHs. But nowadays we have evidence that BHs exist in nature. Astrophysicists have detected super massive BHs at the center of many galaxies, and our galaxy has a BH at its core. When one gets close to the BH, spacetime becomes rarely curved.

And when one passes the event horizon, which is a surface enclosing the singularity, the velocity required to escape from the gravitational clutches of the BH becomes larger than the speed of light, so all hopes to escape from the BH is lost. Although astrophysical BHs are massive, from Einstein's theory BH with a much smaller mass is also possible. If we can find a way to crush mass into a small volume, then we can create a BH. Another important issue about a BH is its entropy, which we shall discuss about it in next chapters, [14, 15]

2.1.1 Schwarzschild Coordinates and Its Geometrical Properties

Schwarzschild coordinates, or the related tortoise coordinates, cover only exterior of the horizon. Schwarzschild geometry is simplest spherically symmetric static uncharged BHs geometry. Its metric is described by

$$ds^2 = -\left(1 - \frac{2MG}{r}\right)dt^2 + \frac{1}{\left(1 - \frac{2MG}{r}\right)}\,dr^2 + r^2 d\Omega^2, \qquad (2.1.1)$$

Where the spherical line-element is $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$. Throughout the thesis, without loss of generality, we use $c = \hbar = k_B = 1$. M and G are the mass of the BH and universal gravitational constant. The coordinate t is called Schwarzschild time, and it shows the time recorded by a standard clock at rest at spatial infinity. The coordinate r is known as Schwarzschild radial coordinate. It is the distance from origin, and it shows that the area of two-sphere at r is $4\pi r^2$. The symbols θ and ϕ are polar and azimuthal angles. The horizon is the place where g_{00} vanishes, and defined by the coordinate $r_{EH} = 2MG$. At the horizon g_{rr} becomes singular. No local invariant properties of the geometry are singular at $r_{EH} = 2MG$. So, in simple free-falling in laboratory at $r = 2MG$ we would record nothing unusual.

We can easily read the metric tensor $g_{\mu\nu}$ from the Schwarzschild metric (2.1.1).

Table 1: Metric components of the Schwarzschild spacetime

$g_{\mu\nu}$	$g_{tt} = -f$	$g_{rr} = 1/f$	$g_{\theta\theta} = r^2$	$g_{\phi\phi} = r^2 sin^2\theta$
$g^{\mu\nu}$	$g^{tt} = -1/f$	$g^{rr} = f$	$g^{\theta\theta} = 1/r^2$	$g^{\phi\phi} = 1/r^2 sin^2\theta$

where $f = 1 - \frac{2MG}{r}$.

The Christoffel symbols are found via the components of metric tensor

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\lambda}(g_{\lambda\beta,\gamma} + g_{\lambda\gamma,\beta} - g_{\beta\gamma,\lambda}), \qquad (2.1.2)$$

We just write the nonzero component of Christoffel symbols

$$\Gamma^{r}_{rr} = \frac{-GM}{r^2} \frac{1}{\left(1 - \frac{2GM}{r}\right)} = -f' \frac{1}{f}, \qquad (2.1.3)$$

$$\Gamma^{r}_{\theta\theta} = -r\left(1 - \frac{2GM}{r}\right) = -rf, \qquad (2.1.4)$$

$$\Gamma^{r}_{tt} = \frac{-GM}{r^2}\left(1 - \frac{2GM}{r^2}\right) = \frac{1}{2} f' f, \qquad (2.1.5)$$

$$\Gamma^{r}_{\phi\phi} = -f(r sin^2\theta), \qquad (2.1.6)$$

where a prime denotes derivative with respect to r.

The Riemann curvature tensor formula is defined as

$$R^{\alpha}_{\beta\gamma\delta} = \Gamma^{\alpha}_{\beta\delta,\gamma} - \Gamma^{\alpha}_{\beta\gamma,\delta} + \Gamma^{\alpha}_{\lambda\gamma}\Gamma^{\lambda}_{\beta\delta} - \Gamma^{\alpha}_{\lambda\delta}\Gamma^{\lambda}_{\beta\gamma}, \qquad (2.1.7)$$

And the Kretschmann scalar [29] which represents singularity at $r = 0$ is given by

$$\mathrm{K} = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = 48G^2\frac{M^2}{r^6}, \qquad (2.1.8)$$

2.1.2 Hawking Temperature and Entropy of Schwarzschild Black Hole

Stephen Hawking found that BHs actually emit radiation like a black body spectrum when he was studying the quantum theory of electromagnetism near BHs. He showed that vacuum differences between spatial infinity and near horizon regions produce thermal effects, which occur near the horizon of the BH. This new phenomenon can be considered as the evaporation of the BH. The particles of thermal atmosphere gradually leak through the barrier and carry off energy in the form of thermal radiation. We can understand this process by Rindler quantum field theory by observing this fact: When a particle approaches the event horizon of the BH it is broken into matter and anti-matter, and matter can run away from the BH and we detect it as Hawking radiation, and conversely the anti-matter one decreases the mass M of the BH. Temperature of the Hawking radiation is given by

$$T_H = \frac{\kappa}{2\pi}, \qquad (2.1.9)$$

where κ is an acceleration due to gravity the so-called surface gravity and can be found by the following formula

$$\kappa = \left[lim_{r\to r_{EH}}\left(\frac{-1}{4}g^{tt}g^{ij}g_{tt,i}g_{tt,j}\right)\right]^{\frac{1}{2}}; \;\; i,j \equiv t,r,\theta,\phi, \qquad (2.1.10)$$

By using $g_{\mu\nu}$ from table 1, we find that

$$\kappa = \left[lim_{r\to r_{EH}=2GM}\left(\frac{GM}{r^2}\right)^2\right]^{\frac{1}{2}} = \frac{GM}{(2GM)^2} = \frac{1}{4GM}\,, \qquad (2.1.11)$$

Thus the Hawking temperature (2.1.9) becomes

$$T = \frac{\kappa}{2\pi} = \frac{1}{8\pi GM}\,, \qquad (2.1.12)$$

So, for a distant observer, BH is a body with that T temperature and energy M. It means that there is entropy for the BH. For finding the entropy, we use the first law of thermodynamics

$$dE = TdS\,, \tag{2.1.13}$$

E can be replaced with mass M so that

$$dM = \frac{1}{8\pi MG} dS, \tag{2.1.14}$$

which can be integrated to give

$$S = 4\pi M^2 G\,, \tag{2.1.15}$$

Since the radius of the Schwarzschild BH is $2MG$ and consequently the area of the horizon is $A_h = 4\pi(4M^2G^2)$, one can see that

$$S_{BH} = \frac{A_h}{4G}, \tag{2.1.16}$$

This is the famous Bekenstein–Hawking entropy.

The entropy shows the number of microscopically distinct quantum states, which is called coarse graining, into the single macroscopic state that we know it as a BH. The number of states is of order

$$\frac{dN}{dM} \sim exp[4\pi M^2 G] = e^{S_{BH}}\,. \tag{2.1.17}$$

Here dN is the number of distinct quantum states with mass M in the interval dM. It means that we can study entropy in terms of local properties of the BH.

2.2 Entropic Force of the Schwarzschild Black Hole on the Isothermal Cavity

It is clear from (1.3.15) that to obtain the entropic force on any screen of BH; the first step is finding the temperature on that screen. The Schwarzschild BH could be in thermal equilibrium with a finite size heat tank in asymptotically flat spacetimes. It can be made by enclosing BH into a cavity. So the temperature that we should find for entropic force corresponds to the Tolman temperature and the energy is the quasilocal energy on the isothermal cavity. By using the Tolman red-shift transformation on the BH system [29], the local temperature observed by an observer located on $r > r_{EH}$ is

$$T_L(r) = \frac{T_\infty}{\sqrt{-g_{tt}}} = \frac{1}{8\pi GM}\frac{1}{\sqrt{1-\frac{2GM}{r}}}, \qquad (2.2.1)$$

where

$$T_\infty = \frac{1}{8\pi GM} = \frac{1}{4\pi r_{EH}} = T_H, \qquad (2.2.2)$$

and similarly we have local energy

$$E_L = \frac{E_\infty}{\sqrt{-g_{tt}}}, \qquad (2.2.3)$$

which shows that UV/IR scaling transformation (the Tolman redshift transformation) of the energy between the bulk and the HS? It is important to notice that there is no difference between the local BH entropy near the horizon S_L and the entropy at infinity S_∞ ;

$$S_L = \pi r_{EH}^2 = \mathrm{S}_\infty, \qquad (2.2.4)$$

This is the Bekenstein-Hawking entropy for the Schwarzschild BH, and it is invariant under UV/IR transformation, [30].

Now one can introduce the energy of the HS as E_{HS}

$$E_{HS}(r) = \frac{E_\infty}{\sqrt{1-\frac{2GM}{r}}}, \qquad \text{where} \quad E_\infty = M, \qquad (2.2.5)$$

and

$$E_{HS} = \frac{M}{\sqrt{1-\frac{2GM}{r}}}, \qquad (2.2.6)$$

In order to find the holographic temperature T_{HS} we use (2.2.5) and (2.2.8)

$$E_{HS}(r) = 2S_{HS}(r)T_{HS}(r), \tag{2.2.7}$$

where

$$S_{HS}(r) = \frac{\pi r^2}{G}\,, \tag{2.2.8}$$

From here we can easily see that

$$T_{HS}(r) = \frac{GM}{2\pi r^2\sqrt{1-\frac{2GM}{r}}}\,. \tag{2.2.9}$$

We are very close to our aim, which is finding the entropic force on the HS by using (1.3.15).

$$F_{HS} = \frac{GmM}{r^2}\frac{1}{\sqrt{1-\frac{r_{EH}}{r}}}\,, \tag{2.2.10}$$

We know that Newtonian force work in very long distance, so when

$$r \gg r_{EH} \;\rightarrow\; \sqrt{1-\frac{r_{EH}}{r}} = 1 \;\rightarrow\; F_{HS} = \frac{GmM}{r^2}\,. \tag{2.2.11}$$

Thus, we reach the Newtonian force that is expected from Verlinde's assumption [28]. Namely, the Newtonian force is obtained by using entropy and thermodynamic equations.

Similarly, the quasilocal energy is derived by using the first law of thermodynamics $dQ = TdS$ and assuming that the Bekenstein-Hawking entropy S_{BH} is not changed on the cavity.

$$S_{BH} = \frac{A_{HS}}{4G} = \frac{4\pi r_{EH}^2}{4G} = 4\pi GM^2, \tag{2.2.12}$$

Since

$$dE_{QL} = T_L dS_{BH}, \tag{2.2.13}$$

By putting (2.2.1) into (2.2.13)

$$dE_{QL} = \frac{dM}{\sqrt{1 - \frac{2GM}{r}}} \, , \tag{2.2.14}$$

We integrate it to find energy

$$E_{QL} = \int dM \frac{1}{\sqrt{1 - \frac{2GM}{r}}} = \frac{-r}{G}\sqrt{1 - \frac{2GM}{r}} + C \ \ , \tag{2.2.15}$$

For finding the constant, we use the asymptotic behavior of the quasilocal energy, i.e, when $r \to \infty$, one must get $E \to M = \frac{r_{EH}}{2G}$. Thus

$$lim_{r\to\infty}\left[\frac{-r}{G}\sqrt{1 - \frac{2GM}{r}} + C\right] = lim_{r\to\infty}\left[\frac{-r}{G} + M + \cdots + C\right] = M, \tag{2.2.16}$$

and it can be easily seen that

$$C = \frac{r}{G} \, , \tag{2.2.17}$$

Finally, the quasilocal energy can be rewritten as

$$E_{QL} = \frac{r}{G}\left[1 - \sqrt{1 - \frac{2GM}{r}}\right]. \tag{2.2.18}$$

In this sense, the isothermal cavity, which was an artificial device to make a phase transition from a hot gas to a BH, is different from HS. It seems unlikely to define the entropic force on the isothermal cavity except the case that it is located near the event horizon, [28].

2.3 Entropic Force of the Schwarzschild Black Hole on the Static Holographic Screen

In the previous section, we have reached to the entropic force by finding the local temperature of surface. We want to find entropic force by using proper acceleration.

Our aim is to show that there is no special way to reach the entropic force i.e., we can find it from different ways.

For the first step, we introduce the component form of the proper acceleration [23, 28]

$$\hat{a}^{\mu} = \frac{1}{f}\xi^{\alpha}\xi^{\mu}_{;\alpha}\,, \qquad (2.3.1)$$

ξ^{α} is a timelike Killing vector of spacetime, and semicolon means covariant derivative. In general, a Killing vector is governed by the following equation

$$\xi_{\mu;\nu} + \xi_{\nu;\mu} = 0, \qquad (2.3.2)$$

In terms of the proper acceleration, the energy of the static HS is found by

$$E(V)_{HS} = \frac{1}{4\pi G}\oint_{\partial V} \hat{a}^{\mu} n_{\mu}\, dA\,, \qquad (2.3.3)$$

where V is the volume of the bulk, which is enclosed by a spacelike hypersurface i.e., $\partial V = S^2$, and n_{μ} is a spacelike unit normal vector of S^2. Alternatively, we can express the energy of the HS in the Komar integral form

$$E(V)_{HS} = \frac{1}{8\pi G}\oint_{\partial V} \xi^{\mu;\nu}\, d\Sigma_{\mu\nu}, \qquad (2.3.4)$$

in which

$$d\Sigma_{\mu\nu} = \left(n_{\mu}\xi_{\nu} - n_{\nu}\xi_{\mu}\right)dA. \qquad (2.3.5)$$

Meanwhile, Eq. (2.3.4) is defined only for the stationary spacetimes. Now, for outside region of the BH, one of the non-zero components of the Killing vector which satisfies Eq. (2.3.2) is

$$\xi^{t} \equiv 1. \qquad (2.3.6)$$

The non-zero component of n_r on the S^2 hypersurface is

$$n_r = \frac{1}{\sqrt{f}}\,, \qquad (2.3.7)$$

For solving (2.3.3), we need to find out the proper acceleration $\hat{a}$. We have

$$\hat{a} = \hat{a}^{\mu} n_{\mu} = \hat{a}^{r} n_{r}, \qquad (2.3.8)$$

And from (2.3.1)

$$\hat{a}^{r} = \frac{1}{f} \xi^{t} \xi^{r}_{;t} = \frac{1}{f} \xi^{t} \left[\xi^{r}_{,t} + \Gamma^{r}_{\alpha t} \xi^{\alpha} \right] = \frac{1}{f} \left[\Gamma^{r}_{tt} \xi^{t} \right] = \frac{1}{f} \Gamma^{r}_{tt} \,, \qquad (2.3.9)$$

which is equivalent to

$$= \frac{1}{f} \left[\frac{1}{2} g^{rk} \left(g_{tk,r} + g_{tk,r} - g_{tt,k} \right) \right],$$

$$= \frac{1}{2f} g^{rr} \left(g_{tr,r} + g_{tr,r} - g_{tt,r} \right) =$$

$$= \frac{f'}{2}. \qquad (2.3.10)$$

Therefore, the proper acceleration (2.3.8) is obtained as

$$\hat{a} = \frac{f'}{2\sqrt{f}} \,, \qquad (2.3.11)$$

So the energy on the static HS (2.3.3) with $dA = r^{2} sin\theta d\theta d\phi$ becomes

$$E(V)_{HS} = \frac{1}{4\pi G} \oint_{\partial V} \frac{f'}{2} \frac{1}{\sqrt{f}} r^{2} sin\theta d\theta d\phi,$$

$$= \frac{1}{8\pi G} \oint_{\partial V} \frac{2GM}{r^{2}} \frac{1}{\sqrt{1 - \frac{2GM}{r}}} r^{2} sin\theta d\theta d\phi,$$

$$= \frac{M}{\sqrt{1 - \frac{2GM}{r}}} = E_{HS}, \qquad E_{\infty} = M, \qquad (2.3.12)$$

This is the local energy from viewpoint of an observer located at rest with respect to the Schwarzschild coordinate r. On the other hand, the entropy on the HS, S_{HS}, for $r > r_{EH}$ is

$$S_{HS}(r) = \frac{\pi r^{2}}{G}, \qquad (2.3.13)$$

Since we know now the energy on HS, by using the equipartition rule we can find the temperature of the HS. The procedure is as follows

$$E_{HS}(r) = 2S_{HS}(r)T_{HS}(r),$$

$$\frac{M}{\sqrt{1-\frac{2GM}{r}}} = \frac{2\pi r^2}{G} T_{HS}(r),$$

$$\rightarrow\ T_{HS}(r) = \frac{GM}{2\pi r^2\sqrt{1-\frac{2GM}{r}}}\,, \tag{2.3.14}$$

Finally by using (1.3.15) the entropic force of the Schwarzschild BH yields

$$F_{HS} = 2\pi m T_{HS}(r) = \frac{1}{\sqrt{1-\frac{2GM}{r}}}\frac{GmM}{r^2}, \tag{2.3.15}$$

We can clearly see that when $r \rightarrow r_{EH}$ the object having mass m feels an infinitely strong attraction force, while at large distances $r \gg r_{EH}$ this force reduces to the well-known Newtonian force. We also remark that for obtaining the entropic force of a BH the most important step is to make use of the equipartition rule to gain the temperature on the static screen.

2.4 Entropic Force of the Schwarzschild Black Hole on the Accelerating Surface

In the previous sections, we studied the entropic force on the static HS and isothermal cavity, now we want to extend our entropic force calculations to the AS, which is also considered as a HS. This surface is a spacelike two-surface, and it does not belong to the part of any horizon of spacetime. Its dynamical properties satisfy the Einstein's field equations [1,2]. The AS is a smooth, orientable, simply connected, spacelike two-surface of the spacetime. Every point on the AS accelerates with a constant proper acceleration $\hat{a}$ with direction of unit vector n^{μ}. One can learn more things about the AS by referring to [31,32]. Here we have another definition for the component form of the proper acceleration unlike the definition given in (2.3.1)

$$\hat{a}^{\mu} = u^{\alpha} u^{\mu}_{;\alpha}, \tag{2.4.1}$$

where u^{μ} denotes a future pointing unit tangent vector field of the congruence of the timelike world lines of the points on an arbitrary spacelike two-surface of spacetime [31]. Let us recall that the proper acceleration is defined as

$$\hat{a}^{\mu} n_{\mu} \equiv constant = \hat{a}\,, \tag{2.4.2}$$

It shows that all the points on the surface have the same acceleration equal to amount $\hat{a}$ in direction of unit vector n_{μ}. We can easily verify that unit vector field u_{μ} is orthogonal to the acceleration $\hat{a}^{\mu}$ and the unit vector n_{μ}.

$$\hat{a}^{\mu} u_{\mu} = 0, \tag{2.4.3}$$

$$u^{\mu} n_{\mu} = 0, \tag{2.4.4}$$

One of the properties of the AS [31] is

$$\sqrt{\hat{a}^{\mu} \hat{a}_{\mu}} = \hat{a} = constant = \hat{a}^{\mu} n_{\mu}, \tag{2.4.5}$$

At every point of two-surface. Essentially, the constancy of the proper acceleration explains why we choose the AS. The main reason is that this surface is very similar to event horizon of the BH. As if the surface gravity $\kappa = \frac{1}{4GM}$ is constant everywhere on the event horizon, the proper acceleration $\hat{a}$ is constant through the AS. Since the BH is associated with the concept of heat, entropy and temperature, we also hope to do the same thing for the AS.

The flux of the proper acceleration vector field through the AS is defined as

$$\Phi_{AS} = \hat{a} A_{AS}, \tag{2.4.6}$$

$$\Phi_{AS} = \int_{S} \hat{a}^{\mu} n_{\mu} dA, \tag{2.4.7}$$

Now it is time to find entropic force with our knowledge about the AS. In previous sections, we obtained the entropic force by finding the acceleration and its associated temperature. Here we choose a different way to obtain the temperature. To this end, we use the relation between a flux of the proper acceleration and energy (heat) which is given by an equation of the form

$$\delta Q = \frac{1}{4\pi G} d\Phi_{As} \,, \tag{2.4.8}$$

For the Schwarzschild BH (2.1.1), the only non-zero component for the future pointing unit vector is

$$u^t = \frac{1}{\sqrt{1 - \frac{2GM}{r}}} \,, \tag{2.4.9}$$

So we can find the non-zero component of the proper acceleration vector a^μ

$$\hat{a}^r = u^t u^r_{;t} = u^t \left[u^r_{,t} + \Gamma^r_{tt} u^t \right] = \frac{1}{\sqrt{f}} \frac{f f'}{2} \frac{1}{\sqrt{f}} = \frac{f'}{2} \,, \tag{2.4.10}$$

Thus the proper acceleration (2.4.5) becomes

$$\hat{a} = \hat{a}^\mu n_\mu = \frac{f'}{2} \frac{1}{\sqrt{f}} = \frac{GM}{r^2} \frac{1}{\sqrt{1 - \frac{2GM}{r}}} \,, \tag{2.4.11}$$

Therefore, we obtain the Unruh temperature as

$$T_U(r) = \frac{\hat{a}}{2\pi} = \frac{GM}{2\pi r^2} \frac{1}{\sqrt{1 - \frac{2GM}{r}}} \,, \tag{2.4.12}$$

By comparing the above equation with (2.3.14), we find out that

$$T_U = T_{HS}, \tag{2.4.13}$$

This equality is well known in the local quantum field theory. It is called the Unruh effect, which tells that an accelerating observer detects thermal particles even in the case of no particles. If we go to the Newtonian limit for the acceleration and the flux, one can see that

$$\hat{a} = \frac{GM}{r^2}, \qquad \text{and} \qquad \Phi_{AS} = \frac{4\pi GM}{\sqrt{1 - \frac{2GM}{r}}} . \qquad (2.4.14)$$

This is the acceleration of the free falling particle in the gravitational field due to the mass M.

The derivative of the flux can be written as

$$d\Phi_{AS} = \frac{\partial \Phi_{AS}}{\partial M} dM + \frac{\partial \Phi_{AS}}{\partial r} dr,$$

$$= \frac{4\pi G(r - GM)}{r\left(\frac{r - 2GM}{r}\right)^{\frac{3}{2}}} dM + \frac{-4\pi G^2 M^2}{r^2}\left(1 - \frac{2GM}{r}\right)^{\frac{-3}{2}} dr , \qquad (2.4.15)$$

According to the constancy of the proper acceleration on the AS; $d\hat{a}(r, M) = 0$. This condition leads to

$$d\hat{a} = \frac{\partial \hat{a}}{\partial r} dr + \frac{\partial \hat{a}}{\partial M} dM = 0 , \qquad (2.4.16)$$

which corresponds to

$$dM = \frac{-\frac{\partial \hat{a}}{\partial r}}{\frac{\partial \hat{a}}{\partial M}} dr , \qquad (2.4.17)$$

From Eq. (2.4.11)

$$\frac{\partial \hat{a}}{\partial r} = \frac{GM}{r^3}\left(1 - \frac{2GM}{r}\right)^{\frac{-3}{2}} \left[-2 + \frac{GM}{r}\right] , \qquad (2.4.18)$$

$$\frac{\partial \hat{a}}{\partial M} = Gr^{-2}\left(1 - \frac{2GM}{r}\right)^{\frac{-3}{2}} \left[1 - \frac{GM}{r}\right] , \qquad (2.4.19)$$

After substituting the above results into (2.4.17), we obtain

$$dM = \frac{2Mr - 3GM^2}{r^2 - GMr} dr \ , \tag{2.4.20}$$

Therefore the change of the heat (2.4.8) becomes

$$\delta Q \ = \frac{2M}{r} \frac{1}{\sqrt{1 - \frac{2GM}{r}}} dr. \tag{2.4.21}$$

We want to use the above equation like the equipartition law of energy to determine the temperature on the AS. In this case, it is seen that the entropy takes the following form

$$S_{AS} = \frac{A}{2G} = \frac{2\pi r^2}{G} \ , \tag{2.4.22}$$

Then we differentiate it to find dr in terms of S_{AS}

$$dS_{AS} = 4\pi r \frac{dr}{G} \ , \tag{2.4.23}$$

Inserting it into Eq. (2.4.20), we get

$$\delta Q \ = \frac{GM}{2\pi r^2} \frac{1}{\sqrt{1 - \frac{2GM}{r}}} dS_{AS} = T_U(r) dS_{AS} = \frac{\hat{a}}{2\pi} dS_{AS} \, , \tag{2.4.24}$$

It shows that change of the heat is balanced by the change of the entropy when fixing the acceleration on the AS. Thus, by using the temperature of the AS, $T_U(r)$, we prove that the entropic force obtained for the static HS matches with the entropic force of the AS:

$$F_{HS} = 2\pi m T_U = m\hat{a} = \frac{1}{\sqrt{1 - \frac{2GM}{r}}} \frac{GmM}{r^2} \ . \tag{2.4.25}$$

This result approves that the AS can be used as a HS.

2.5 Entropic Force of the Schwarzschild Black Hole on the Stretched Horizon

In this section, we shall consider the SH as a HS, and attempt to derive the entropic force on it. In fact, SH is such a place that all the thermodynamic quantities, which are measured by an observer located at the proper distance l_{pl} away from the horizon. The location of the SH is chosen at $r = r_{EH} + {l_{pl}^2}/{r_{EH}}$, [28]. This umlaut length (${l_{pl}^2}/{r_{EH}} = \sqrt{-g_{tt}}\, l_{pl}$) is due to the redshift transformation of $\sqrt{-g_{tt}} \cong {l_{pl}}/{r_{EH}}$ near the event horizon.

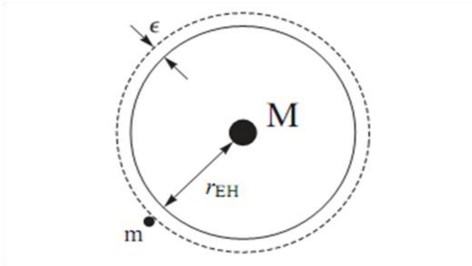

Figure 6: Solid circle here represents the event horizon r_{EH} of the Schwarzschild BH, and the dotted circle, at the position $r = r_{EH} + {l_{pl}^2}/{r_{EH}}$ near the event horizon, shows the entropic force equality of isothermal cavity, static HS and AS.

In order to find the local temperature on the SH, we put $r = r_{EH} + {l_{pl}^2}/{r_{EH}}$ instead of r in the (2.2.1) then we have

$$T_L^{SH} = \lim_{r \to r_{EH} + {l_{pl}^2}/{r_{EH}}} T_L,$$

$$\cong \frac{1}{4\pi r_{EH}} \frac{r_{EH}}{l_{pl}} \sqrt{1 + {l_{pl}^2}/{r_{EH}^2}},$$

$$\cong \frac{1}{4\pi l_{pl}} \left(1 + \frac{1}{8 l_{pl}^2 M^2}\right) , \qquad (2.5.1)$$

in which $G = l_{pl}^2$, $r_{EH} = 2GM$ and the series of $(1+x)^n \cong 1 + nx$ is used. In the leading order, we can see that local temperature of SH is independent of the BH mass M [30]. We repeat the same processes for the static HS temperature $T_{HS}(r)$ or the Unruh temperature $T_U(r)$, (2.4.12) as

$$T_{HS}^{SH} = \lim_{r \to r_{EH} + {l_{pl}^2}/{r_{EH}}} T_{HS}(r) = \lim_{r \to r_{EH} + {l_{pl}^2}/{r_{EH}}} T_U(r),$$

$$\cong \frac{1}{4\pi l_{pl}} \frac{1}{\left(1 + {l_{pl}^2}/{{r_{EH}}^2}\right)^2} \left(1 + {l_{pl}^2}/{{r_{EH}}^2}\right)^{\frac{1}{2}},$$

$$\cong \frac{1}{4\pi l_{pl}} \left(1 - \frac{3}{2M^2 l_{pl}^2}\right) , \qquad (2.5.2)$$

It can be easily seen that three temperatures are the same in leading order:

$$T_L^{SH} = T_{HS}^{SH} = T_U^{SH} = \frac{1}{4\pi l_{pl}} , \qquad (2.5.3)$$

Similarly for the local energy on the SH(2.1.12),

$$E_{HS}^{SH} = \lim_{r \to r_{EH} + {l_{pl}^2}/{r_{EH}}} E_L ,$$

$$\cong \frac{M r_{EH}}{l_{pl}} \left(1 + {l_{pl}^2}/{{r_{EH}}^2}\right)^{\frac{1}{2}},$$

$$\cong \frac{{r_{EH}}^2}{2 l_{pl}^3} \sqrt{1 + {l_{pl}^2}/{{r_{EH}}^2}} = \frac{A_{EH}}{8\pi l_{pl}^3} \left(1 + \frac{1}{2} {l_{pl}^2}/{{r_{EH}}^2}\right),$$

$$\cong \frac{A_{EH}}{8\pi l_{pl}^3} \left(1 + \frac{1}{8} \frac{1}{M^2 l_{pl}^2}\right) \cong \frac{A_{EH}}{8\pi l_{pl}^3},$$

$$\cong \frac{N^{SH} T_{HS}^{SH}}{2} . \qquad (2.5.4)$$

where $A_{EH} = 4\pi r_{EH}^2$ and $N^{SH} = \frac{A_{SH}}{l_{pl}^2 = G}$. Finally we can see that the energy and entropy on the SH take different form for the HS for the non-relativistic case.

$$E_{HS}^{SH} = 2l_{pl}M^2 \quad \text{and} \quad T_{HS}^{SH} = \frac{1}{4\pi l_{pl}} \ , \tag{2.5.5}$$

This result shows that energy is proportional to mass square M^2, and the temperature is independent of the mass. This is a feature of thermodynamic quantities of a BH on the SH. For this reason, E_{HS}^{SH} can be called as the Rindler energy observed from near the horizon, and M (energy) is the Schwarzschild mass defined by an observer at infinity. Now, for finding the entropic force, we use the first law of thermodynamics on the SH.

$$dE_{HS}^{SH} = dT_{HS}^{SH}\, dS^{SH} \ , \tag{2.5.6}$$

where the entropy on the SH is introduced as

$$S^{SH} = \frac{A_{EH}}{2l_{pl}^2} \ , \tag{2.5.7}$$

It is obvious that the latter result is different from the original Bekenstein-Hawking entropy $S_{BH} = \frac{A_{EH}}{4l_{pl}^2}$ [3, 4]. Finally, by combining Eqs. (1.3.15) and (2.5.3), we can easily calculate the entropic force on the SH, which yields

$$F_{SH} = 2\pi m T_L^{SH} = 2\pi m T_{HS}^{SH} = 2\pi m T_U^{SH} = \frac{m}{2l_{pl}} \ , \tag{2.5.8}$$

According to the Newton's second law, $F = m\hat{a}$, one can read the acceleration on the SH as

$$\hat{a}_{SH} = \frac{1}{2l_{pl}} \ . \tag{2.5.9}$$

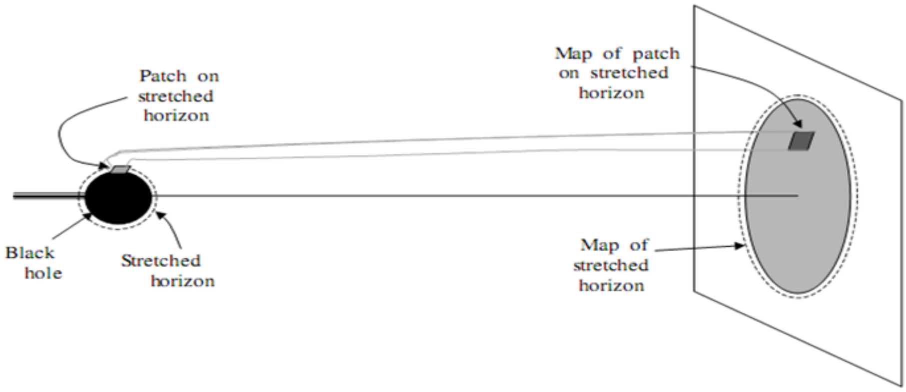

Figure 7: Image of the SH on an asymptotic screen.

Chapter 3

ENTROPIC FORCE OF THE CHARGED DILATON BLAK HOLE

3.1 Features of the Charged Dilaton Black Hole

CDBHs are the static and spherically symmetric solutions to the Einstein-Maxwell-Dilaton theory in the low energy limit of the string theory [33]. These solutions have a regular horizon and a curvature singularity at the origin. The metrics of the CDBHs are neither AF nor NAF. The charge feature of the CDBH is transformable from magnetic to electric one by simply replacing the sign of the dilaton parameter"a".

First, Gibbons and Maeda [34] found the CDBH solutions, however they were in rather general form. Their solutions were admitting the higher dimensional CDBHs. Later on, the four dimensional version of the CDBHs were re-obtained by Garfinkle, Horowitz and Strominger [35]. Today one can see various studies in the literature which are related with the CDBHs, see for instance [36-40].

In this section, we will focus only on the four dimensional electrically charged dilaton BHs. These CDBHs can be best seen in the paper of Chan et al. [27]. The beauty of their solution is that they have Schwarzschild limit. Their CDBH solution is obtained from the following four dimensional action

$$S = \int d^4x \sqrt{-\mathrm{g}}\left(\mathcal{R} - 2(\nabla\phi)^2 - e^{-2a\phi}F^2\right), \qquad (3.1.1)$$

where the above action is expressed in the Einstein frame. Here $\mathcal{R}$ is the scalar curvature, $F^2 = F^{\mu\nu}F_{\mu\nu}$ is the Maxwell invariant and the dilaton parameter a governs the dilaton field ϕ. $F^{\mu\nu}$ is known as the Maxwell field, which is related to $U(1)$ subgroup of $E_8 \times E_8$ or spin(32)/Z_2. After varying the action (3.1.1) and solving the required field equations, the electrically charged dilaton BH solutions are given by the following metric [27] as

$$ds^2 = -fdt^2 + \frac{dr^2}{f} + \bar{R}^2(d\theta^2 + sin^2\theta d\varphi^2)\,, \qquad (3.1.2)$$

where f and $\bar{R}$ are

$$f = \frac{1}{\gamma^2}\left(1 - \frac{r_+}{r}\right) r^{\frac{2}{1+a^2}}\,, \qquad (3.1.3)$$

$$\bar{R} = \gamma r^{\bar{N}}, \qquad (3.1.4)$$

and

$$r_+ = \frac{2M}{\bar{N}}\,, \qquad (3.1.5)$$

Here r_+ shows the event horizon of the CDBH. Eq. (3.1.5) can be obtained by the virtue of quasilocal mass definition of the Brown and York [41] which is designed for the NAF BHs. Meanwhile, γ is an arbitrary real constant and another parameter $\bar{N}$ is expressed by

$$\bar{N} = \frac{a^2}{1 + a^2}\,, \qquad (3.1.6)$$

The solution of the dilaton field is found in the following form

$$\phi(r) = \phi_0 + \phi_1 lnr\,, \qquad (3.1.7)$$

where

$$\phi_0 = -\frac{ln\left[Q^2\frac{(1+a^2)}{\gamma^2}\right]}{2a} \qquad \text{and} \qquad \phi_1 = \frac{\bar{N}}{a}\,, \qquad (3.1.8)$$

in which Q corresponds to the electric charge, and hence the solution for electromagnetic field is found as

$$F_{tr} = Q\frac{e^{2a\phi}}{\bar{R}^2}\ , \tag{3.1.9}$$

The present form of the dilaton field (3.1.7) is common in two and three dimensional solutions, but it is used also as a sign of pathology in dimensions higher than three. Due to the existence of the dilaton field in the action (3.1.1), the CDBHs are not vacuum solutions (energy-momentum tensor never vanishes). Furthermore, if we terminate the dilaton field ($a^2 \to \infty$) with $\gamma = 1$, the CDBH solution reduces to the Schwarzschild solution. Besides, in the particular case of $a = 1$, the CDBH is known as the linear dilaton BH [42]. Without loss of generality, from now on we will use $\gamma = 1$.

One of the interesting features of the CDBHs is that even when $r_+ = 0$ the metric (3.1.2) still represents the features of being a BH. This is because of the singularity at $r = 0$ is null and marginally trapped, so that it prevents the outgoing signals to reach the external observer. Another intriguing issue of the CDBH is about its charge Q: there is no $Q = 0$ case and no extremal limit on Q.

Table2: Metric components of CDBH

g_{tt}	$-f = -r^{2/_{1+a^2}}\left(1-\frac{r_+}{r}\right)$	g^{tt}	$-1/f = -r^{-2/_{1+a^2}}\,1/\left(1-\frac{r_+}{r}\right)$
g_{rr}	$1/f = r^{-2/_{1+a^2}}\,1/\left(1-\frac{r_+}{r}\right)$	g^{rr}	$f = r^{2/_{1+a^2}}\left(1-\frac{r_+}{r}\right)$
$g_{\theta\theta}$	$\bar{R}^2$	$g^{\theta\theta}$	$\bar{R}^{-2}$
$g_{\phi\phi}$	$\bar{R}^2 sin^2\theta$	$g^{\phi\phi}$	$\bar{R}^{-2} sin^{-2}\theta$

By using the above table, we can see that the Ricci scalar of the CDBH takes the form

$$\mathcal{R} = g^{il}g^{jm}R_{ijlm},$$

$$= \frac{2a^2}{(1+a^2)^2} r^{-2\bar{N}}\left(1-\frac{2M}{\bar{N}r}\right), \quad (3.1.10)$$

It can be easily seen that it is finite everywhere except at $r = 0$, which is the location of the singularity hidden by the event horizon, r_+. On the other hand, the Hawking temperature of the CDBH according to the definition (2.1.9) is

$$T_H^{CDBH} = \frac{1}{4\pi} r_+^{\frac{1-a^2}{1+a^2}}. \quad (3.1.11)$$

Thus, one can easily observe that the BH temperature increases as the mass of the CDBH decreases for $a^2 > 1$. if $a \to \infty$ we can see the Schwarzschild limit:

$$lim_{a\to\infty} T_H^{CDBH} = \frac{1}{8\pi GM} = T_H^{SHBH} \quad (3.1.12)$$

Now we can easily find local temperature T_L^{CDBH}

$$T_L^{CDBH} = \frac{T_H^{CDBH}}{\sqrt{-g_{tt}}} = \frac{T_H^{CDBH}}{\sqrt{f}} = \frac{{r_+}^{-\frac{a^2}{a^2+1}}}{4\pi\sqrt{1-\frac{r_+}{r}}} \qquad (3.1.13)$$

$$lim_{\substack{a\to\infty \\ \gamma\to 1}} T_L^{CDBH} = \frac{{r_{EH}}^{-1}}{4\pi\sqrt{1-\frac{r_{EH}}{r}}} = \frac{1}{8\pi GM}\frac{1}{\sqrt{1-\frac{2GM}{r}}} = T_L^{SHBH} \qquad (3.1.14)$$

In the following sections, we aim to find the entropic force of the CDBHs by following the calculations that we made in chapter 2. Again, we shall consider three HS candidates, which are the static HS, AS and SH. After finding the temperature of each associated surface, and we will simply read the entropic force (1.3.15). Finally, we will analyze the effect of dilaton parameter on the entropic force. To this end, we shall plot some graphs and by using these graphs we will opine about the entropic force with dilaton field.

3.2 Entropic Force of the Charged Dilaton Black Hole via the Static Holographic Screen

In this section we briefly follow the section (2.3) to derive the entropic force on the static HS of the CDBH. As it was found in Eq. (2.3.10), the r-component of the proper acceleration $\hat{a}^r$ of the static HS of the CDBH becomes

$$\hat{\bar{a}}^r = \frac{1}{2}f' = \frac{1}{2}\, r^{\frac{1-a^2}{1+a^2}}\left[\frac{2}{1+a^2}\left(1-\frac{r_+}{r}\right)+\frac{r_+}{r}\right] , \qquad (3.2.1)$$

According to the energy $E\ (V)$ definition of the static HS (2.3.3), one can generalize this concept of energy for the CDBH as

$$E_{HS}^{CDBH}(V) = \frac{1}{4\pi}\oint \frac{f'}{2}\frac{1}{\sqrt{f}}\bar{R}^2 sin\theta d\theta d\varphi \ , \qquad (3.2.2)$$

and it can be integrated to give

$$E_{HS}^{CDBH}(V) = \frac{1}{1+a^2} r^{\bar{N}} \frac{\left[1 + \frac{r_+(a^2-1)}{2r}\right]}{\sqrt{1 - \frac{r_+}{r}}}, \tag{3.2.3}$$

We can easily verify that in the vanishing dilaton case ($a \to \infty$), the energy reduces to its Schwarzschild form (2.3.12), i.e.,

$$E_{HS}^{CDBH} \to \frac{M}{\sqrt{1 - \frac{r_+}{r}}} = E_{HS}, \tag{3.2.4}$$

Now, in order to obtain the holographic temperature of the CDBH on the HS, we use the local equipartition rule

$$E_{HS}^{CDBH} = 2 S_{HS}^{CDBH} T_{HS}^{CDBH}, \tag{3.2.5}$$

where S_{HS}^{CDBH} is the entropy on the HS located at r, outside the horizon of the CDBH, and it is defined as

$$S_{HS}^{CDBH} = \pi \bar{R}^2, \tag{3.2.6}$$

By using the Eqs. (3.2.3-6), one can read the temperature on the HS for CDBH as

$$T_{HS}^{CDBH} = \frac{1}{2\pi(1+a^2)} r^{-\bar{N}} \frac{\left[1 + \frac{r_+(a^2-1)}{2r}\right]}{\sqrt{1 - \frac{r_+}{r}}}, \tag{3.2.7}$$

As happened in the energy case, T_{HS}^{CDBH} reduces to its Schwarzschild form (2.3.14) with vanishing dilaton field

$$T_{HS} = \lim_{a \to \infty} T_{HS}^{CDBH} = \frac{M}{2\pi r^2} \frac{1}{\sqrt{1 - \frac{r_+}{r}}}, \tag{3.2.8}$$

Since we have the temperature of the static HS, with the aid of Eq. (1.3.15), we can calculate the entropic force for the CDBH on the static HS

$$F_{HS}^{CDBH} = 2\pi m T_{HS}^{CDBH} = \frac{m}{(1+a^2)} r^{-\bar{N}} \frac{\left[1 + \frac{r_+(a^2-1)}{2r}\right]}{\sqrt{1 - \frac{r_+}{r}}}, \tag{3.2.9}$$

which is the force exerting on the object having mass m when it is on the static HS of the CDBH with mass M. As expected, in the case of no dilaton field, F_{HS}^{CDBH} is nothing but the F_{HS} (2.3.15):

$$F_{HS} = \lim_{a\to\infty} F_{HS}^{CDBH} = \frac{Mm}{r^2\sqrt{1-\frac{r_+}{r}}}, \tag{3.2.10}$$

On the other hand, at large distances $r \gg r_+$, F_{HS}^{CDBH} takes the following form

$$F_{HS}^{CDBH} = F_{MN} \to mr^{-\bar{N}}\left(\frac{M}{r} + \frac{1}{1+a^2}\right). \tag{3.2.11}$$

We can see that by eliminating the dilaton field in the above equation, we recover the Newtonian force. For this reason, we have preferred to symbolize the large distance limit of F_{HS}^{CDBH} as F_{MN} – Modified Newtonian Force. In section (3.5), we will discuss about the effect of the dilaton field on the gravitational force by sketching some proper plots for F_{MN}.

3.3 Entropic Force of the Charged Dilaton Black Hole on the Accelerating Surface

In this section our purpose is to define the entropic force on the AS of the CDBH. Since we have already recognized the AS in the section (2.4), here we shall simply extend the obtained results of the Schwarzschild BH to the CDBH.

From Eq. (2.4.11), the proper acceleration on the AS of the CDBH is found as

$$\hat{a} = \frac{f'}{2\sqrt{f}} = \frac{r^{-\bar{N}}}{(1+a^2)} \frac{\left[1 + \frac{r_+(a^2-1)}{2r}\right]}{\sqrt{1-\frac{r_+}{r}}}, \tag{3.3.1}$$

Then according to the definition of Unruh temperature [17, 18], we obtain the temperature of the AS on the CDBH, which is the so-called CDBH Unruh temperature

$$T_U^{CDBH} = \frac{\hat{\bar{a}}}{2\pi} = \frac{r^{-\bar{N}}}{2\pi(1+a^2)} \frac{\left[1 + \frac{r_+(a^2-1)}{2r}\right]}{\sqrt{1 - \frac{r_+}{r}}} , \qquad (3.3.2)$$

This result means that an accelerating observer located on AS with r distance far away from the center of the CDBH detects thermal radiation as T_U^{CDBH}. Another interpretation of it is that when the accelerating observer looks at the space around the CDBH, which is filled with the quantized field, he will observe the CDBH as a state containing many particles in thermal equilibrium with T_U^{CDBH} [42]. Comparing Eq. (3.3.2) with Eq. (3.2.7), one can see that $T_U^{CDBH} = T_{HS}^{CDBH}$.

Now, if we follow the procedure about the flux of heat through the AS that we have done in chapter (2.4), the definition of Φ_{AS} of the CDBH should be modified to

$$\Phi_{AS}^{CDBH} = \hat{\bar{a}} A_{AS}^{CDBH} = 4\pi \hat{\bar{a}} \bar{R}^2 , \qquad (3.3.3)$$

where A_{AS}^{CDBH} is the surface area of AS of the CDBH. Thus, the flux becomes

$$\Phi_{AS}^{CDBH} = \frac{4\pi r^{-2\bar{N}}}{(1+a^2)} \frac{\left[1 + \frac{r_+(a^2-1)}{2r}\right]}{\sqrt{1 - \frac{r_+}{r}}} , \qquad (3.3.4)$$

Since the change of the heat is defined as

$$\delta Q = \frac{1}{4\pi} d\Phi_{AS}^{CDBH} , \qquad (3.3.5)$$

where

$$d\Phi_{AS}^{CDBH} = \frac{\partial \Phi_{AS}^{CDBH}}{\partial M} dM + \frac{\partial \Phi_{AS}^{CDBH}}{\partial r} dr , \qquad (3.3.6)$$

and we know also that the proper acceleration on the AS is constant such that $d\hat{\bar{a}}(r, M) = 0$, so that we can express dM in terms of dr.After a tedious calculation, we find dM as

$$dM = \frac{\{M(a^4-1)[(1+a^2)(3a^2+1)M - 2a^4 r] - a^6 r^2\}}{[(a^4-1)M - a^4 r] r (1+a^2)^2} \, dr , \qquad (3.3.7)$$

Substituting this into Eqs. (3.3.5) and (3.3.6), we get the change in the heat of the AS

$$\delta Q = \frac{2\bar{N}}{1+a^2} r^{-\frac{1}{1+a^2}} \frac{\left[1 + \frac{r_+(a^2-1)}{2r}\right]}{\sqrt{1-\frac{r_+}{r}}} \, dr \,, \tag{3.3.8}$$

The foregoing equation can also be expressed in terms of the differential entropy of the CDBH, which is equal to

$$dS_{AS}^{CDBH} = 4\pi\bar{N} r^{\frac{a^2-1}{a^2+1}} \, dr \,, \tag{3.3.9}$$

and thus we have

$$\begin{aligned} \delta Q &= \frac{r^{-\bar{N}}}{2\pi(1+a^2)} \frac{\left[1 + \frac{r_+(a^2-1)}{2r}\right]}{\sqrt{1-\frac{r_+}{r}}} \, dS_{AS}^{CDBH} \,, \\ &= T_U^{CDBH} dS_{AS}^{CDBH}, \\ &= T_{AS}^{CDBH} \, dS_{AS}^{CDBH} \,, \end{aligned} \tag{3.3.10}$$

Therefore, we deduce that if we keep the temperature on the AS as constant, then the change of the heat is balanced by the change of the entropy. Now, since we have obtained the temperature on AS, the entropic force can readily be obtained from Eq. (1.3.15) as

$$F_{AS}^{CDBH} = 2\pi m T_U^{CDBH} = \frac{m r^{-\bar{N}}}{(1+a^2)} \frac{\left[1 + \frac{r_+(a^2-1)}{2r}\right]}{\sqrt{1-\frac{r_+}{r}}} \,. \tag{3.3.11}$$

which is exactly same with the F_{HS}^{CDBH}(3.2.9).

3.4 Entropic Force of the Charged Dilaton Black Hole on the Stretched Horizon

In this section we will explore the form of entropic force on the SH for the CDBH. As mentioned before, this surface is a particular surface in which all the temperatures on it become equivalent. We shall simply follow the procedure given in section (2.5).

The local temperature (3.1.14) can be modified to the CDBH by shifting r to $r = r_+ + {}^1\!/_{r_+}$ where the Planck constant l_p can be fixed to 1 without losing the generality. Thus it takes the following form

$$T_L^{SH-CDBH} = \frac{1}{2\pi(1+a^2)}(r_+)^{\frac{1}{1+a^2}}\left[\frac{1}{2}(1+a^2)+\frac{1}{r_+^2}\right],$$

$$\cong \frac{1}{4\pi}(r_+)^{\frac{1}{1+a^2}} \ , \qquad (3.4.1)$$

One can easily check that with vanishing dilaton field it reduces to the local temperature of the Schwarzschild BH (2.5.3), namely

$$\lim_{a\to\infty} T_L^{SH-CDBH} = \frac{1}{4\pi} \ , \qquad (3.4.2)$$

If we apply the same shifting in r to the Unruh temperature and/or to the static HS temperature described by Eqs. (3.2.7) and (3.3.2) respectively, we see that

$$T_U^{SH-CDBH} = T_{HS}^{SH-CDBH} = \frac{1}{4\pi}(r_+)^{\frac{1}{1+a^2}}\left[1+\frac{2}{r_+^2(1+a^2)}\right]\left(1+\frac{1}{r_+^2}\right)^{-\frac{3a^2+1}{2(1+a^2)}},$$

$$\cong \frac{1}{4\pi}\left[1-\frac{3(a^2-1)}{2r_+^2(1+a^2)}\right](r_+)^{\frac{1}{1+a^2}}, \qquad (3.4.3)$$

As stated before, all the temperature on the SH with different approaches should have the same form. This can be realized by checking the leading order of the temperatures (3.4.1) and (3.4.3):

$$T_U^{SH-CDBH} = T_{HS}^{SH-CDBH} = T_L^{SH-CDBH} = \frac{1}{4\pi}(r_+)^{\frac{1}{1+a^2}} \ , \qquad (3.4.4)$$

Once a time, we observe that limit of $a \to \infty$ reduces the above equation to Eq. (3.4.2). We can also read the local energy (3.2.3) of the CDBH on the SH as

$$E_L^{SH-CDBH} = E_{Hs}^{SH-CDBH} \cong {r_+}^{\bar{N}} M,$$

$$= \bar{N}\frac{A_{EH}^{CDBH}}{8\pi} r_+^{\frac{1}{a^2+1}}, \qquad (3.4.5)$$

Remarkably, one can crosscheck from Eqs. (3.4.4-5) that the equipartition rule is satisfied: $E_{HS}^{SH-CDBH} = \frac{1}{2}\bar{N}^{SH}T_{HS}^{SH-CDBH}$ where $\bar{N}^{SH} = \bar{N}A_{EH}^{CDBH}$ is the number of bits stored on the HS. Therefore, we deduce that the dilaton parameter a scales the total information stored on the screen. When $a \to \infty$, in the zero dilaton field, the total information on the HS takes its maximum, and conversely increasing dilaton field decreases the amount of information stored on the screen [42].

Since the Bekenstein-Hawking entropy is the quarter of the horizon area, one concludes that

$$\bar{N}^{SH} = 4\bar{N}S_{BH}^{CDBH}, \tag{3.4.6}$$

Recalling the first law of thermodynamics

$$dE_{HS}^{SH-CDBH} = T_{HS}^{SH-CDBH}d\,S_{CDBH}^{SH}\ , \tag{3.4.7}$$

the entropy of the CDBH on the SH should be

$$S_{CDBH}^{SH} = \frac{1+2a^2}{1+a^2}\,S_{BH}^{CDBH}\ , \tag{3.4.8}$$

which corresponds to

$$1 < \frac{S_{CDBH}^{SH}}{S_{BH}^{CDBH}} < 2\ , \tag{3.4.9}$$

Above results show that the information stored on the SH or the entropy of the SH obviously depends on the dilaton parameter a. The $S_{CDBH}^{SH} \cong S_{BH}^{CDBH}$ while $a \to 0$ (maximal dilaton field case, see the reference [42]) and it could be $S_{CDBH}^{SH} \cong 2S_{BH}^{CDBH}$ in the absence of the dilaton field ($a \to \infty$) [28].

Finally, by using (3.4.3) we can find the entropic force of the CDBH on the SH

$$F_{SH}^{CDBH} = 2\pi m T_{HS}^{SH-CDBH} = \frac{1}{2}m(r_+)^{\frac{1}{1+a^2}} = m\hat{a}_{SH}^{CDBH} \tag{3.4.10}$$

where $\hat{a}_{SH}^{CDBH} = \frac{1}{2}(r_+)^{\frac{1}{1+a^2}}$ is called the proper acceleration defined on the SH [42].

3.5 Graphics of the Entropic Force With and Without Dilaton Field

In the previous chapters, we have defined the entropic force basically on the three different HS surfaces both for the Schwarzschild BH and the CDBH. Now we especially want to compare the entropic forces of these two BHs by sketching the plots. Through the comparison we mainly focus on the entropic forces of the static HS and the AS.

Let us first revisit the equations that we need. If one goes back to Eq. (2.3.15), we see that the entropic force of the Schwarzschild BH with $G = 1$ is defined as

$$F_{HS} = \frac{mM}{r^2}\frac{1}{\sqrt{1-\frac{2M}{r}}} \tag{3.5.1}$$

such that in the large distance limit $r \gg r_{EH}$ it is the conventional Newtonian force

$$F_N = F_{HS} = \frac{mM}{r^2} \tag{3.5.2}$$

However for the CDBH, its entropic force is given by Eq. (3.2.9)

$$F_{HS}^{CDBH} = \frac{m}{(1+a^2)} r^{-\bar{N}} \frac{[1 + r_+(a^2 - 1)]}{\sqrt{1-\frac{r_+}{r}}} \tag{3.5.3}$$

and its large distance limit $r \gg r_+$ is called the Modified Newtonian force.

$$F_{MN} = mr^{-N}\left(\frac{M}{r} + \frac{1}{1+a^2}\right) \tag{3.5.4}$$

Firstly, we plot the F_{HS} versus r graphs by using the Eqs. (3.5.1) and (3.5.3). The physical parameters are chosen as $M = 1$ and $m = 10^{-9}$. For the plots of F_{HS}^{CDBH}, different values of a are chosen in order to see the effect of dilaton field on the entropic force of the CDBH.

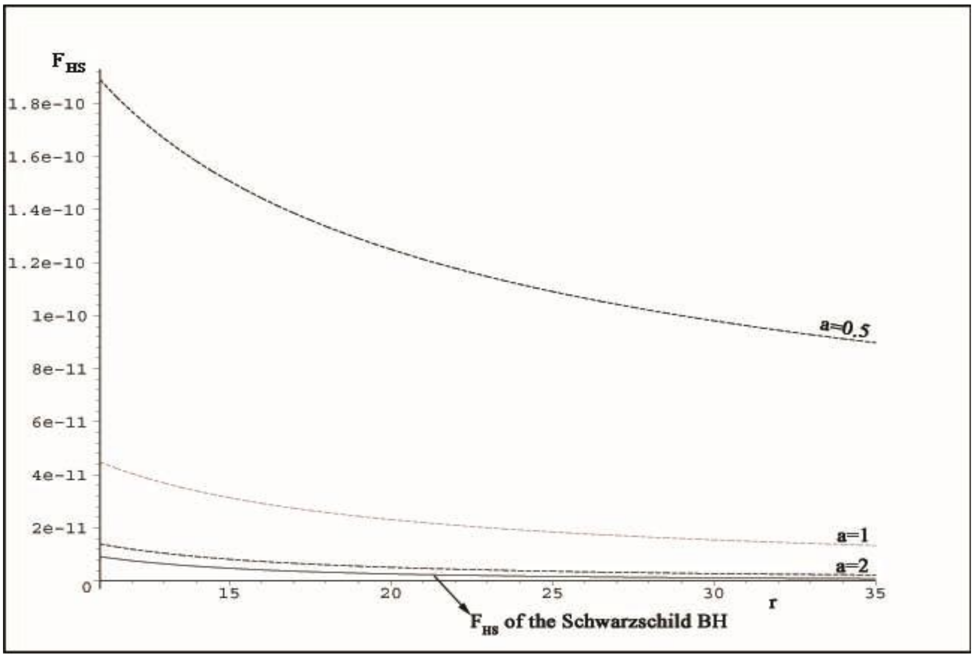

Figure 8. The solid line belongs to F_{HS} of the Schwarzschild BH, however the dotted lines represent F_{HS} of the CDBH with different dilaton parameters.

Secondly, we plot the F versus r graphs which are governed by Eqs. (3.5.2) and (3.5.4). Again the physical parameters are chosen as $M = 1$ and $m = 10^{-9}$. For the plots of F_{MN}, different values of a are chosen in order to see the effect of dilaton field on the large distance limit of the entropic force.

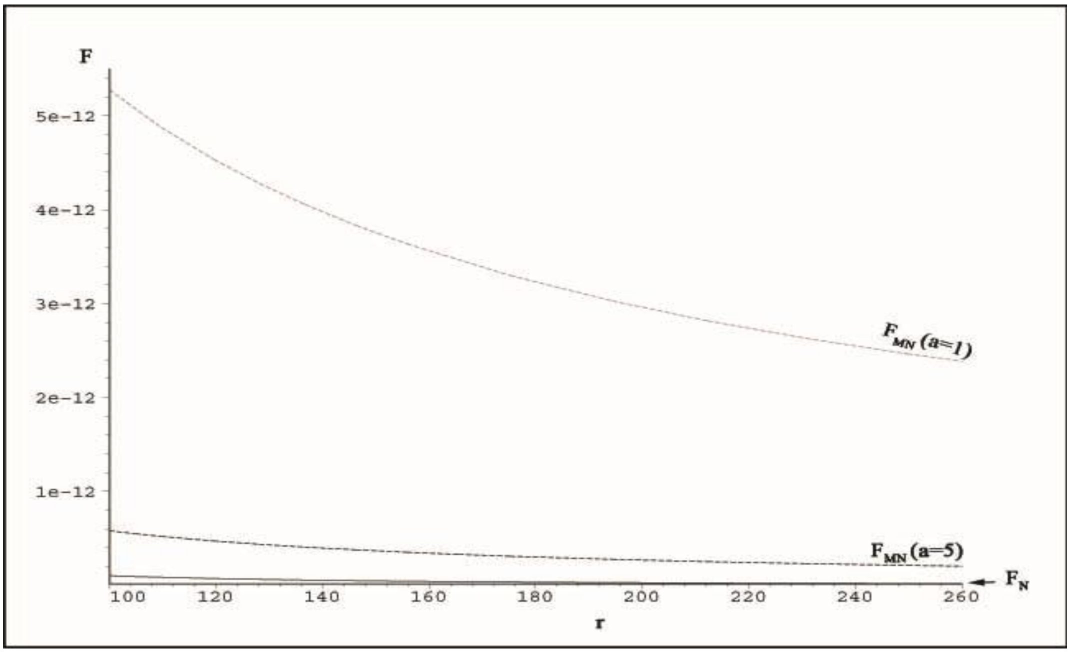

Figure 9: The solid line belongs to F_N, however, the dotted lines represent F_{MN} with different dilaton parameters.

It is obvious from figure (8) that in the high dilaton field regime the distinct behaviors of the entropic forces of both the Schwarzschild BH and the CDBH become more apparent. On the other hand, all types of entropic forces tend to vanish at spatial infinity like the Newtonian force. Especially, in figure (9), which pictures the entropic forces at large distances we realize that the effect of the dilaton field on the gravitational force of an object and the CDBH can be a measurable value. In other words, there could be an indirect way to detect the dilaton fields, and thus the existence of the CDBHs, in the universe, experimentally.

Chapter 4

CONCLUSION

In this thesis we have considered two different BHs, which are the Schwarzschild BH and the CDBH. After gaining an experience about the entropic force calculations on the Schwarzschild BH, we have extended these calculations to the CDBH.

In chapter 1, we have made a revision about the entropic force in order to clarify how one could define the entropic force on the HS by using the equipartition rule and the laws of thermodynamics, as proposed by Verlinde [13, 43]. This review tells us why the gravity is not the fundamental force anymore; instead it emerges due to the change in the entropy. In our entropic force calculations, three different surfaces are taken into account in place of the HS. These surfaces are the static HS, the AS, and the SH. Then in the following chapters using Eq. (1.3.15) we have calculated the entropic forces of the Schwarzschild BH and the CDBH, which are different from the conventional Newtonian force at any distance r. However, all the obtained entropic forces with no additional field like dilaton (except the gravitational field) match with the Newtonian force at large distances.

The most remarkable points that we have obtained in this thesis are due to the dilaton parameter a. It is shown that this parameter does not only modify the entropic force of gravity but it scales the number of bits of information stored on the HS. Particularly, in section (3.4) we have seen that an increase in the dilaton field causes a decrease in amount of the information on the SH. Besides, after plotting the entropic forces of the Schwarzschild BH and the CDBH against the distance r, we make the effect of dilaton field on the entropic force more apparent. It is seen that a distant observer feels the gravitational force of the CDBH stronger than the Schwarzschild and the Newtonian ones. If the Verlinde's arguments about the gravity are verified in the near future, our latter result may also provide us a chance to discover the dilaton fields in the universe. Because some researchers have reported that the dilaton fields could be responsible for the dark energy in the cosmos, see for instance [26].

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