Predicting Influential Parameters and Suggesting Empirical Equation for Sediment Transport Capacity of Rills

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ABSTRACT

The lack of well defined, strong correlation between sediment transport capacity and most influential variables selected for the amendment of sediment transport capacity is one of the crucial reasons of deviation from measured result. Although several scholars have suggested different parameters to represent sediment transport capacity unfortunately the suggested parameters could not yet compensate the problem.

In this study, the input dataset were grouped in three distinct categories. The most influential simple parameters by providing insight into importance of other parameters were detailed. Nine different equations were suggested to obtain the most representative equation so as to define the sediment transport capacity of the rills base on the most three influential parameters, one from each distinct category.

The validation and the accuracy of the selected parameters were computed by the sensitivity analysis (SA) through multiple linear regression (MLR) method. The preciseness of the suggested equations was compered through statistical error estimation measures.

It is found that, the most influential parameter for sediment transport capacity of the rills is the slope from the channel morphology. The most representative equation to define sediment transport capacity of the rills is the combination of slope of the channel, the Reynolds number and the soil particle size d_{65}.

Keywords: $d_{65,}$ Reynolds number, rill erosion, sediment transport capacity, slope.

ÖZ

Sediman taşıma kapasitesi ile onu en çok etkileyen değişkenler arasındaki kuvvetli korilasyon eksikliğinin varlığı, elde edilen ampirik veya yarı-ampirik denklemlerin ölçüm değerlerindeki sapmaların oluşturmasında en önemli sebebtir. Her nekadar da, bazı araştırmacılar farklı değişkenlerin kullanımını önermişlerse de maalasef bu parametrelerin hiçbiri bu sorunu halen çözememiştir.

Bu çalışmada, giriş doneleri üç farklı sınıflamaya gruplanmıştır. En çok etki eden basit değişkenler, diğer önemli değişkenlerin göreceli etkileri detalı incelenmiştir. Her grubun bir değişkeni kullanılarak , üç farklı değişkenle, dereciklerin oluşmasında etkin olan sediman taşıma kapasitesi bulmak için dokuz farklı denklem sunulmuştur.

Etkin değişkenlerin muteberliği ve kesinliği, çoklu-regrasyon metodunun duyarlılık analizi ile hesaplanmıştır. Denklemlerin doğruluğu ıstatistiksel hata ölçüm yaklaşımları ile karşılaştırılmıştır.

Kanal morfolojisinden eğim, dereciklerdeki sediman taşınım kapasitesini en çok etkileyen değişken olduğu belirlenmiştir. Dereciklerin en temsili sediman taşınım kapasite denklemi eğim, Reynolds sayısı, ve toprak parçacığı çapı d65'den oluşmuştur.

Anahtar kelimeler: d65, derecik erezyonu, eğim, Reynolds sayısı, sediman taşınım kapasitesi.

To My Beloved Ones

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A	Wetted cross sectional area of the rill [L^2]
a, b, c, h	Empirically calculated coefficient [-]
C_c	Coefficient of curvature [-]
C_t	Total sediment concentration [$MT^{-1}L^{-1}$]
C_u	Coefficient of uniformity
D	Hydraulic depth of the channel [L]
d	Sediment particle size [L]
D_f	Rate of erosion in rill [$ML^{-2}T^{-1}$]
D_c	Detachment capacity by the flow [$ML^{-2}T^{-1}$]
d_{10}, d_{16}, d_{25}, d_{30}, d_{35}, d_{50}, d_{60}, d_{65}, d_{75}, d_{84}, and d_{90}	The subscript (numbers) represents percentage finer of the soil sample obtained from the sieve analysis [L]
f	Darcy-Weisbach Friction Factor [-]
Fr	Froude number [-]
f_s	Darcy-Weisbach hydraulic roughness coefficient for smooth, bare soil [-]
G	Rate of sediment load per unit length [$ML^{-1}T^{-1}$]
g	Gravitational acceleration [LT^{-2}]
K_t	Transport coefficient [-]
MNE	Mean Normalized Error [-]
N	Cumulative number of the common datasets [-]
q	Volumetric discharge per unit width [L^2T^{-1}]
Q_{av}	Volumetric average discharge [L^3T^{-1}]

Q_{cr}	Volumetric critical discharge [L^3T^{-1}]
q_s	Sediment discharge [MT^{-1}]
$q_{\bar{c}}$	The calculated average value
q_{cGM}	The geometric mean
$q_{\bar{m}}$	The measured average value
R_d	The discrepancy ratio [-]
Re	Reynolds number [-]
Re^*	Reynolds number based on Shear stress [-]
Re_{d65}	Reynolds number based on d_{65} [-]
Rh	Hydraulic Radius [L]
RMSE	Root Mean Squared Error
S	Slope of channel bed [-]
S_x	The Sample standard deviation
SG	Particle specific gravity [-]
t	Time [T]
T_c	Sediment transport capacity [$ML^{-1}T^{-1}$]
U	Thiel's Inequality Coefficient
U^*	Average shear velocity [LT^{-1}]
V	Error analysis [-]
v_{av}	Average velocity of the flow passing within that specific rill cross-section [LT^{-1}]
v_{cr}	Critical velocity of the flow passing within that specific rill cross-section [LT^{-1}]
v_{max}	Maximum velocity of the flow passing within that specific rill cross-section [LT^{-1}]

W	Bottom width of the rill channel [L]
$X_{i,j}$	i^{th} input data point for the j^{th} input
x	Distance in longitudinal flow direction [L]
Y	Elevation above datum line [L]
Y_i	i^{th}output data point for the i^{th}input data point
y_s	the portion of total hydraulic depth acting on detached soil particle [L]
y_t	Total hydraulic depth [L]
Y''_{cr}	Dimensionless critical shear stress [-]
Y''	Dimensionless shear stress [-]

Greek Symbols

β	Standardized coefficients [-]
β′	Coefficient in Yang's equation [-]
β''	Parameters in Yalin's equation [-]
ɣ	Unit weight of water $[ML^{-2}T^{-2}]$
ɣ′	Coefficient in yang equation [-]
δ''	Parameters in Yalin's equation [-]

ε_i	Error for the i^{th} data point [-]
ν	Kinematic viscosity of water $[L^2T^{-1}]$
ρ_w	Mass density of water $[ML^{-3}]$
τ_{av}	Shear stress on the detached soil $[ML^{-1}T^{-2}]$
Ω	Unit stream power $[MLT^{-3}]$
Ω_{cr}	Critical unit stream power $[MLT^{-3}]$
ω	Fall velocity of sediment particle $[LT^{-1}]$

Chapter 1

INTRODUCTION

1.1 General

Placidly altering the earth's surface by soil erosion can be seen all over the world. That soil erosion and deposition cause asperities in all parts of the world is undeniable. Soil erosion is an underestimated subject which affects physically well-being and economy growth all over the world. It has a limiting impact on the reservoirs capacity due sedimentation where the geometry of the channel is affected negatively by increased sediments. It is the most significant factor for long-term decay of fertile soil and the most important factor in non-point water pollution (Lei et al., 2008). By taking all above mentioned argument into consideration modeling the soil erosion to diminish the associated consequences is logical.

Nonetheless soil erosion would not abolish, it can be controlled. The significant role of human in boosting the soil erosion is undeniable by ruining the vegetation cover of the soil since the vegetated soil or soil which holds more residues will have more resistance against the eroding factors. Hence the runoff consequently diminishes the shear stress of the flow and as a result the transport capacity of the flow will decline. Rill erosion is the main source of erosion in hill slope profile; many scholars have tried to model it.

1.2 Overview

The overview of the general contents of the chapters contained in this thesis:

- Chapter 2: Watershed and Sediment Transport Mechanism

This chapter deals with the definition of the most important terms in sediment transport science. Also the parameters which have been studied are summarized with their definitions.

- Chapter 3: Theory of Rill Erosion

Classification of sediment transport due to stage and the factors contributing in the rill erosion discussed in early stages of this chapter. The models which have been used to describe soil erosion especially in the rill are elaborated. Equations available in literature to define sediment transport capacity are detailed in this chapter.

- Chapter 4: Statistics and Data-Fitting Measures

The basic statistical measures like mean, standard deviation, etc. are detailed. The measures of well- fitting which are the most important to determine the preciseness of the model is elaborated. The definition and the significance of sensitivity analysis and approach to run sensitivity analysis are as well detailed.

- Chapter 5: Methodology

The characteristics of data are detailed and the procedure of preparation of data, running sensitivity analysis and deriving of the most representative empirical equation for sediment transport capacity of the rill is explained.

- Chapter 6: Result and Discussion

The most influential parameters that directly affect the sediment transport capacity in rills are determined and tabulated. Also wherever necessary, the findings are presented graphically. Comments on these results are as well indicated. Several recommendations are given for those researchers and/or practical engineers whom are studying on this topic.

1.3 Aim of This Study

The scope of this study is to determine the key parameters that directly or indirectly affect the sediment transport capacity in rill erosion. Mathematical method, Multiple Linear Regression, for Sensitivity Analysis was applied to establish new formulas to determine sediment transport capacity for rills. The findings of this study will be expected to lead the researchers to improve existing models and formulas or establish better predictive models. Simplification of the present formulas by taking into consideration the decoupled detachment and transport processes.

Chapter 2

WATERSHED AND SEDIMENT TRANSPORT MECHANISM

2.1 General

Hill slopes are the main coverage of landscape which can be categorized into two main groups with respect to slope as follow:

1. steep hill slopes (Angle $\geq 15°$)and

2. gentle hill slopes ($0° \leq$ Angle $< 15°$).

Inasmuch as water, sediments, and rocks tend to release their potential and stay in the lower potential, they try to move downslope. By taking the present soil thickness topography into consideration, the sensitive balance between erosion, deposition and weathering would be realized. Human activities have significant role to accelerate soil loss which can result in:

i. losing fertile soil from cultivated lands,

ii. generating excessive erosion which results in decreasing sediment in main stream,

iii. causing catastrophic events like landslide.

2.2 Sediment Transport Terminology

There are some extremely significant terms in sediment science which has to be express to have better understanding before going through the concept.

2.2.1 Watershed

Particularly watersheds consist of several hill slope which empty them to several streams or impoundments.

2.2.2 Hill slope

The whole of the landscape is particularly covered by varying hill slopes.

2.2.3 Sediment

Sediment can be identified as a fragmentary material which is commonly formed by dissolution of rocks through physical or chemical processes from the Earth's surface and then transported and deposited in other places.

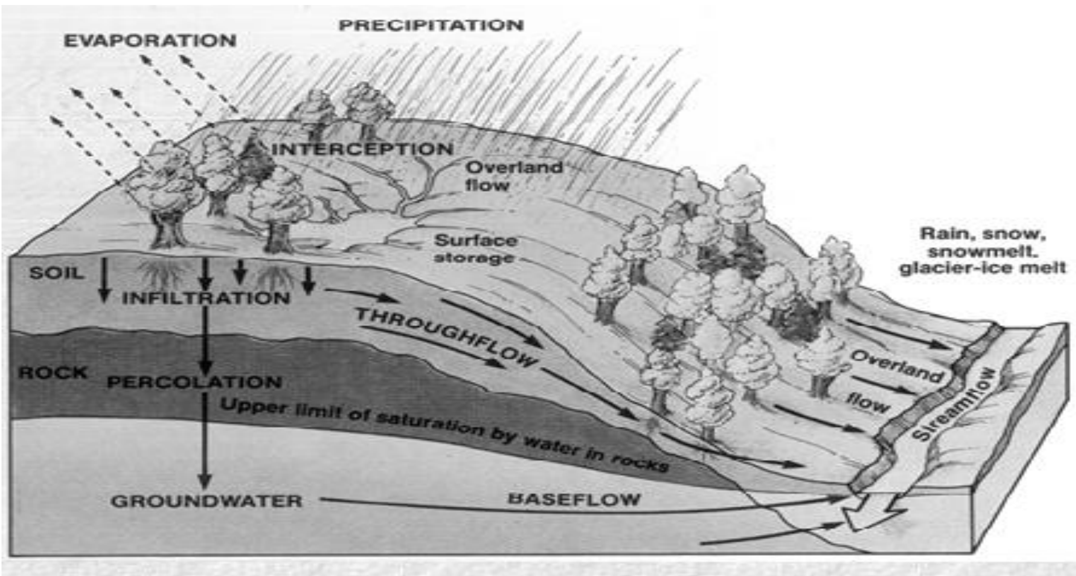

Figure 2.1: Hill slope profile (University of Colorado, Boulder Geography Department)

2.2.4 Erosion

The mechanism of eliminating soil and rock from Earth's surface by natural processes such as wind or water flow, and then conveying and depositing in other locations.

2.2.4.1 Wind Erosion

In the arid and semi-arid areas the most significant force to cause erosion is wind. In addition wind erosion can cause land degradation, evaporation, desertification, harmful airborne dust, and crop damage. Human activities (i.e. deforestation, urbanization, and agriculture) have a significant boost far above natural rates. It is worth full to mention that, mainly silt particle are vulnerable to this kind of erosion.

2.2.4.2 Water Erosion

Engendering soil erosion due to water take place in different ways:

2.2.4.2.1 Inter-rill erosion

Theoretically inter-rill erosion consists of two major parts:

2.2.4.2.1.1 Splash Erosion

It is the first step of water erosion procedure. When raindrops strike the soil surface the kinematic energy separate the soil particle from the surface and splash. This phenomenon depends on the size and the velocity of colliding raindrop.

In an average, the distance travelled by soil particles towards downslope is greater than the opposite side. Considering the bulk amount of raindrops, the initiation of erosion is significant and can contribute to the next steps of erosion by increasing the flowing water turbulence. As the drops strike the flowing water they diminish the soil resistance against the erosion (Wainwright et al., 2000).

2.2.4.2.1.2 Sheet Erosion

The overland flow causes the sheet flow which transport the sediment detached by splash erosion. Its velocity and depth change downslope by variation of the precipitation characteristics both in time, duration and intensity. It is believe that, keeping the strength of the soil against erosion as constant; the sheet erosion will be a function of hydromechanics of the flow and geomorphology of the slope. The origin of the term “sheet”, in this type of erosion, comes from roughly uniform the shape of the eroded land after the erosion. Since sheet erosion and sheet splash erosion take place simultaneously; it is difficult to distinguish them. Therefore this term is interchangeably used for both cases (Whiting et al., 2001).

Figure 2.3: Splash erosion (NRCS)

Figure 2.2: Sheet erosion (M. Mamo, Labels added by UNL)

2.2.4.2.2 Rill Erosion

Once the travel distance of the sheet flow is fair enough (critical) the formation of micro-channels along the soil on the hill slope due to runoff referred as rill commences. In fluvial geomorphology, rill defined as a narrow and shallow incision into topsoil layers, resulting from erosion by overland flow or surface runoff on topsoil layer which is subjected to several storm events (Brayan, 2000).

2.2.4.2.3 Gully Erosion

Basically, if rill does not expose to the tillage it will turn to gully since it is deepening and widening its channel. This transformation is not completely quantified. The transformation of rill to gully is playing a crucial role in producing sediment from hill slopes. Although temporarily tillage can annihilate rills, it makes soil more vulnerable to rill erosion. Loosening soil consequently increases the chance of eroding more soil from terrain by the successive storms.

Figure 2.4: Gully erosion in a pasture (NRCS)

Figure 2.5: Rill Erosion (Wirtz et al. 2013)

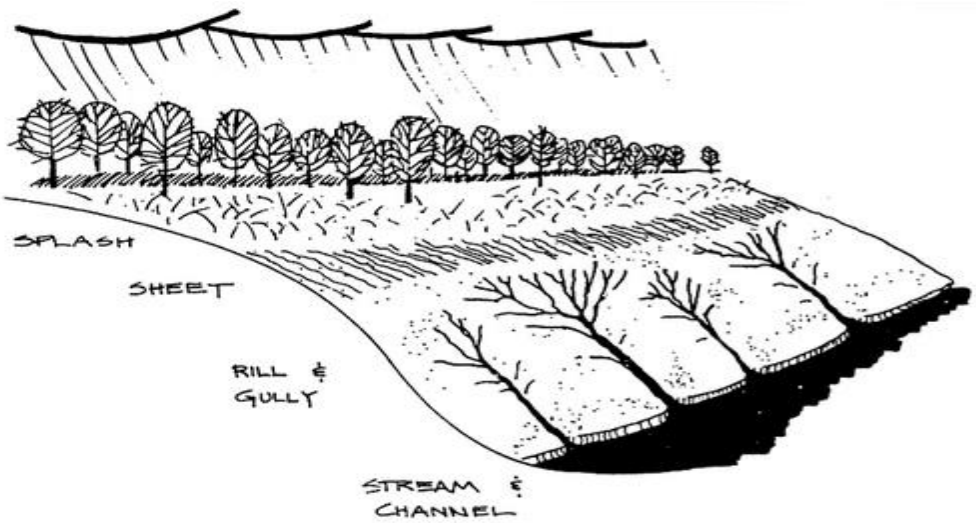

Figure 2.6: Schematic hill slope erosion (http://www.cep.unep.org/pubs/Techreports/tr32en/fig1.gif)

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2.3 Important parameters

In this study, input dataset has been divided into three groups based on previous theories so as to correlate the rill erosion:

i- Soil characteristics, ii- flowing water properties, and iii- channel morphology.

2.3.1 Soil Characteristics

By reviewing the related sediment transport literature it is observed that, different scholars used different particle sizes in their approaches from sieve analysis. So in this study all those suggested eleven different particle sizes were taken into consideration: d_{10}, d_{16}, d_{25}, d_{30}, d_{35}, d_{50}, d_{60}, d_{65}, d_{75}, d_{84}, and d_{90}. The numbers represents percentage finer of the soil sample obtained from the sieve analysis. Additionally two widely used dimensionless soil mechanics parameters based on soil gradation, were selected:

i-Coefficient of Curvature (C_c) and ii- Coefficient of Uniformity C_u.

Sandy soil is thought to be well graded if $1< C_c < 3$ and , $C_u \geq 6$.

2.3.2 Flowing Water Properties

Several researchers use different parameters in their studies to correlate the effect of flowing water by considering fluid (water) and flow characteristics separately with the sediment transport equations. In this study the combination of above mentioned two characteristics were investigated through seven main parameters.

2.3.2.1 Maximum Velocity (v_{max})

It is the expected maximum velocity of the flow passing within that specific rill cross-section due to the unsteadiness of the flow [m/s].

2.3.2.2 Average Velocity (v_{av})

It is the average velocity of the flow passing within that specific rill cross-section [m/s].

2.3.2.3 Average Discharge (Q_{av})

It is the total average volumetric amount of water passing within that specific cross-section of the rill for fix duration [m^3/s].

2.3.2.4 Kinematic Viscosity (ν)

It is the viscosity of a fluid that varies with respect to temperature [m^2/s].

2.3.2.5 Reynolds Based on Hydraulic Radius (Re)

Is dimensionless parameter obtained from a ratio of the inertial forces to the viscous forces within the fluid based on flow characteristics [-].

2.3.2.6 Reynolds Number Based on d_{65} (Re_{d65})

Is dimensionless parameter obtained from a ratio of the inertial forces to the viscous forces by considering the effective particle size diameter (d_{65}) [-].

2.3.2.7 Darcy-Weisbach Friction Factor (*f*)

It is the friction factor occurred against the flow direction due to viscosity of the fluid and the surface roughness of the wetted channel perimeter [-].

2.3.2.8 Froude Number (Fr)

It is a dimensionless number defined as the ratio of a characteristic velocity to a gravitational wave velocity [-].

2.3.3 Channel Morphology

The characteristics of the eroding rill channel is playing significant role to determine eroding rill properties. The mainly effective four properties are:

2.3.3.1 Wetted Cross Section (A)

It is the ratio of the flowing average discharge to the average velocity [m^2].

2.3.3.2 Width of the Channel (W)

It is the width of the channel at any specific cross section [m].

2.3.3.3 Slope of Channel (S)

It is the ratio of the elevation difference between any two points on rill to its longitudinal distance measured along the stream [-].

2.3.3.4 Hydraulic Radius (R_h)

It is the wetted cross-sectional area divided by the wetted perimeter for that specific cross section [m].

2.3.4 Transport Capacity (T_c)

The maximum sediment load that can be conveyed by the flowing fluid is called sediment transport capacity. It is the main implication of identification of detachment and deposition procedure in process-base erosion studies. Transport capacity is the prominent factor for determination of whether flow is depositing or detaching sediment Huang et al. (1999). In SI units the transport capacity per unit of length is expressed as [kg/ (ms)].

Chapter 3

THEORY OF RILL EROSION

3.1 General

Generally all over the world, when a rainstorm falls on the steep slope, rill erosion can be seen. Most of sediments which are produced by sheet erosion move short distances, then concentrated into micro-scale channels (rill) and then transported by the rill flow (Liu, et al., 2006).

Rill erosion is a major problem especially in the cultivated hill slope since it erodes the fertile soil (Lei, et al., 2001).

Two main processes in hill slope soil erosion due flowing water are inter-rill, rill erosion; however their mechanisms are totally distinct (Wirtz, et al. 2012). In the inter-rill erosion, detachment engenders and improves by drop-impact as well as soil characteristics and it is believed to be chiefly dependent to rainfall intensity (Bordie et al., 2007; Beuselinck et al., 2002). On the other hand, rill erosion is engendered by the water flow concentration and it can be considered as the most significant process through sediment production in hill slopes. It is significant to mention that, the slope and the surface roughness play dominant roles in delivery of eroded sediment from inter-rill to rill (Brodie, I., Rosewell,C, 2007; Bryan, 2000).

Generally, detachment in rill occurs when both:

i- flowing water shear stress exceeds a specified threshold of soil resistance capacity (τ_w>τ soil)and

ii- the amount of conveying sediment by the flow is less than the sediment transport capacity of that flow (Knapen et al., 2007).

3.2 Soil Erosion Models

Numerous empirical methods have been developed with respect to field observations and laboratory experiments since the modeling of rill erosion is labyrinthine type. There are a lot of factors contributing the rill erosion, like the characteristics of the soil, slope surface condition, and flow dynamics features.

Soil erosion models are instruments used to correlate the source of erosional variance with the measurable quantities and they play crucial role in soil and water conservation management (Nearing, 1998).

It should be remembered that the soil erosion by water plays a significant role for the geomorphological studies hence having a vital importance.

Models based on their concept can be categorized in three groups:

i- Empirical

ii- Physical process or process-base

iii- Stochastic approaches

Nevertheless, up to date, there is not any model based on the stochastic approach. In most of the models the general belief is, there is a linear relation between shear stress of flowing water and soil detachment (Wirtz et al. 2013).

WEPP (Nearing et al., 1989) soil erosion model which is the most famous process-base model is led from Foster-Meyer (Meyer et al., 1969) model. These two process-based models differ from Rose (1985) dynamic model, which takes into account a balance between three simultaneous and continuous procedures: i-rainfall detachment, ii-runoff detachment and iii-sediment deposition. For sediment delivery this model considers taking the Sedimnet transport capacity 'T_c' and volumetric sediment discharge 'q_s', values as a prominont factor for detemining sediment deposition and detachment amount.

Beside, WEPP model needs to be divided into rill and inter-rill areas, however, in Rose model there is no need of such a division.

Through the last decades a number of approaches have been conducted to understand soil detachment and transport mechanism in rills and a huge amount of time has been consumed to evaluate their appropriateness, but there are a lot of discrepancies or even contradictories among them. The first and the most significant reason of variation from each other is methodology in monitoring and the experiments set-ups (Govers et al., 2007; Giménez et al., 2002; Hessel et al., 1993). The second fundamental reasons of these discrepancies are due to the random component of measured data. But the models due to their deterministic quiddity do not take this major component into consideration (Nearing, 1998)**.** The third significant reason is a limited data is accessible in scales accordant to those used in these equation formulations of the models (Hung et al., 1996).

Through past decades, lots of scholars have been trying to amend and investigate the validation of WEPP erosion model. Such as Huang et al. (1996), were the one who suggested the concept of detachment-transport coupling in WEPP erosion model for rill erosion equations. According to them, sediment transport capacity is the governing parameter and in the field conditions where the slope-length is longer, the sediment load supply from inter-rill exists.

Based on WEPP for calculating rill detachment the following function can be applied:

$$D_f = D_c \left[1 - \frac{G}{T_c}\right] \qquad (3.1)$$

where;

D_f: rate of erosion in rill per unit area [kg/(m^{2}s)]

D_c: detachment capacity by the flow [kg/(m^{2}s)]

G: sediment load per unit length [kg/(ms)].

3.3 Pervious equations

Since it is believed that, the transport capacity is one of the dominant parameter used for defining sediment transport in inter-rill, rill, gully, and streams equations, vast number of scholars attempted to express the transport capacity through soil or flowing water or geomorphological characteristics.

Williams, 1975; Foster and Meyer, 1972; McWhorter et al., 1979; and Foster, 1982 have chosen single parameter such as average discharge (Q_{av}), effective discharge (Q_{av}-Q_{cr}), effective shear stress (tractive force) (τ_{av}-τ_{cr}), to define T_c theoretically. But these equations once applied to the alluvial channel in practice they were deviating from the actual values and the unique result was either over estimate or under estimate implying that the selected parameters were not enough and appropriate.

Through the years Yang and his colleagues determine that unit stream power is the prominent agent in the final sediment concentration of stream with alluvial and gravely beds. They have described an empirical solution through the unit steam power Ω as a rate of potential per time per unit of weight.

$$\log C_t = ɣ' + \beta' \log\left[\frac{\Omega - \Omega_{cr}}{\omega}\right] \quad (3.2)$$

in which

$$ɣ' = 5.435 - 0.286 \log\left(\frac{\omega d}{\nu}\right) - 0.457 \log\left(\frac{U^*}{\omega}\right) \quad (3.3)$$

$$\beta' = 1.799 - 0.40 - \log\left(\frac{\omega d}{\nu}\right) - 0.314\left(\frac{U^*}{\omega}\right) \quad (3.4)$$

$$\Omega = \frac{dY}{dt} = \frac{dx}{dt}\frac{dY}{dx} = v_{av}S \quad (3.5)$$

$$\Omega_{cr} = v_{cr}S$$

where;

C_t: the total sediment concentration or transport capacity [ppm];

Ω_{cr}: critical unit stream power at incipient motion [kgm/s^3];

$\Omega - \Omega_{cr}$: effective stream power [kgm/s^3];

ω: sediment fall velocity in water [m/s];

d_{50}: the median particle size [mm];

ν: kinematic viscosity of flowing water [m^2/s];

U^* : the average shear velocity (= $(gDS)^{1/2}$ in which D is the flowing water depth [m]) [m/s];

Y: the elevation above datum line [m],

t : time [s];

Yang also found that, V_{cr} is in inverse relation with shear velocity Reynolds number Re^*(=$U^* d/\nu$) as follow:

$$\frac{v_{cr}}{\omega} = \frac{2.5}{\log Re^* - 0.06} + 0.66 \qquad 0 < Re^* < 70 \qquad (3.6)$$

$$\frac{v_{cr}}{\omega} = 2.05 \qquad Re^* > 70 \qquad (3.7)$$

where;

v_{av}: average velocity in longitude direction [m/s];

x: distance in longitudinal direction [m];

S: the energy gradient of the flow and roughly can be replaced by the slope of the channel bed (Yang et al., 1982).

Moore and Burch (1986), after running analyses on three independent sets of data which represent fine and coarse texture non-aggregated soils on rills, sheet erosion and the combination of these two determined that although unit stream power theory is not complicated to estimate sediment transport capacity of flow, it is vigorous for both rill and sheet flow cases.

Bennett (1974) mentioned vast majority of dynamic erosion models for equate the water erosion of upland areas by solving continuity equation for sediment transport where the numerical approaches were applied to hydrological models for obtaining the hydrological inputs. Since the equations utilize in these models based on sediment transport capacity, determination of deposition or detachment occurrence would be accomplished by comparison of sediment transport capacity to the sediment load. Beside, putting discussion into further context, Foster (1982) claimed that, the rate of deposition and detachment is a function of subtraction of sediment transport capacity and sediment load.

Based on the suggestion by Alonso et al. (1981) who examined nine different transport capacity equations, among them Yalin equation (Yalin, 1963) is the most reliable one for prediction of shallow overland flow.

Yalin, 1963, equation can be defined as:

$$\frac{T_c}{(SG)\ d_{50}\ \ \rho_w^{0.5}\ \ \tau_{av}^{0.5}} = 0.635\delta''\left[1 - \frac{1}{\beta''}\ \ln(1+\beta'')\right] \qquad (3.8)$$

$$\beta'' = 2.45\ (SG)^{-0.04}(Y_{cr})^{\ 05} \qquad (3.9)$$

$$\delta'' = (Y/Y_{cr})\ -1\ (\text{when } Y< Y_{cr},\ \delta''=0) \qquad (3.10)$$

$$Y'' = \frac{\tau_s/\rho_w}{(SG-1)g\ d_{50}} \qquad (3.11)$$

in which;

T_c: sediment transport capacity [kg/(ms)];

ρ_w : density of water [kg/m^3] ;

g: acceleration due gravity [m/s^2];

d_{50}: the median sediment diameter [mm];

Y" : dimensionless shear stress [-];

Y''_{cr} : dimensionless critical shear stress to be obtained from Shields Diagram [-];

τ_{av} : average shear stress acting on the detached soil [Pa].

As average shear stress 'τ_{av}' is much greater than the critical shear stress 'τ_{cr}' threshold, Yalin equation will be reduce to:

$T_c = K_t \tau_{av}^{1.5}$ (Foster and Meyer, 1972) (3.12)

where;

K_t is defined as transport coefficient of the cropland soil based on sand, very fine sand and organic matters characteristics.

i- soils sand content $\geq$ 30% :

$K_{t=}$ $0.00197 + 0.00030 * \mathrm{VFS} + 0.03863 * \mathrm{EXP}\,(-1.84 * \mathrm{ORGMAT})$ (3.13)

where;

VFS: percent of very fine sand ($62.5\ \mu m < d < 125\ \mu m$);

ORGMAT: percent organic matter in the surface soil (based on WEPP assumption, organic matter equals 1.724 times organic carbon content).

Range of applicability of equation (3.13):

- the value for VFS used must be less than or equal to 40% (if the value for VFS is greater than 40%, use 40%),
- the value for ORGMAT must be greater than 0.35% (if the value for ORGMAT is less than 0.35%, use 0.35%).

ii − soils sand content < 30% :

$K_{t=}0.0069 + 0.134 * \mathrm{EXP}\,(-0.20 * \mathrm{CLAY})$ (3.14)

where;

CLAY : percent of clay

Range of validity of the equation (3.14):

- the value of the CLAY must be 10% or greater (in a case value for CLAY is less than 10%, use 10% in the equations).

τ_{av} would be calculated as the |Eq.(3.15).

$$\tau_{av} = \gamma\, y_s\, S \qquad (3.15)$$

where;

y_s: the portion of total hydraulic depth channel cross-section acting on detached soil particle as [m]:

$$y_s = y_t \frac{f_s}{f} = \left[\frac{f_t q_{av}^2}{8gS}\right]^{1/3} \frac{f_s}{f} \quad \text{(Foster, 1982)} \qquad (3.16)$$

where;

y_t : total hydraulic depth of the channel cross section ;

f_s: Darcy-Weisbach hydraulic roughness coefficient for smooth, bare soil;

f: hydraulic roughness coefficient;

q_{av}: average discharge per unit width [$m^3/s/m$].

Nearing et al. (1997), propose an equation to drive sediment transport capacity by considering it as a function of stream power.

$$\log_{10} T_c = c + \frac{h.\, e^{[a+b\log(\Omega)]}}{1 + e^{[a+b\log(\Omega)]}} \qquad (3.17)$$

Where a, b, c, h are empirically calculated coefficient.

Lei et al. (2001), run an experiment to define sediment transport capacity using a laboratory flume experimental method consequently and proved that T_c has a linear relationship with the average slope of the channel (S) and the average flow rate (Q_{av}) as:

$$T_c = a + bS + c\,Q_{av} \qquad (3.18)$$

where a, b, c are the regression coefficient.

It is important to note that although T_c is theoretically defined and widely used in modeling the soil erosion, it cannot be measured directly (HUANG et al., 1999).

Chapter 4

STATISTICS AND DATA-FITTING MEASURES

4.1 General

The characteristic of collected dataset plays a significant role in finding the best fitted analyzing approach. Inasmuch as little information about the specification of data will be obtained through the analysis, which may be based on the inappropriate assumptions the collected dataset commentary would not be true or decisive. So the prominent specification of collected data to determine the best analyzing approach of water resource data will be detailed.

The target population mainly defines the dataset which has to be analyzed. Scarcely the whole population data is available for scientist due to two main reasons:

i- commonly it is impossible to collect all the dataset,

ii- it is not economical to collect them all.

Hence rather than the population data, statisticians suggest to select a subdivision among the population data and defining it as sample. Sample data has to be collected aligned with the concept that the conclusion derived from sample can expand through whole of the population. Since sample size is small in comparison with the number of population data, the calculated statistics are just an approximation about the population properties (e.g. location, spread, and skewness). Utilization of the term "sample" used for each statistics measures implies that these measures are just approximation of population values.

For instant, the common location measures for sample are the sample mean or the sample median; the common spread measures are the sample standard deviation and the sample interquartile range and the sample skewness implies how these sample data deviates from the average value.

In order to indicate the goodness of fit between the computed and measured results, the statistical measures that have been commonly used in the past by many researchers and are detailed below were used in this study as well.

4.2 Basic Statistics

4.2.1 Average

$$q_{\bar{c}} = \frac{\sum_{i=1}^{N} q_{ci}}{N} \qquad (4.1)$$

in which;

$q_{\bar{c}}$ is the calculated average value,

q_{ci} is the computed value of the i^{th} dataset of the common datasets,

i is the dataset number, and

N is the cumulative number of the common datasets used in that analysis.

4.2.2 Geometric Mean

The geometric mean of non-negative values can be obtained by multiplying them all together and then taking the n^{th} root of them, sometimes it is called antilog mean, which can be define as:

$$q_{cGM} = \sqrt[N]{\prod_{i=1}^{N} q_{ci}} \qquad (4.2)$$

Where;

q_{cGM} is the calculated geometric mean.

4.2.3 Sample Standard Deviation

$$S_x = \sqrt{\frac{\sum_{i=1}^{N}(q_{ci} - q_{\bar{c}})^2}{(N-1)}} \qquad (4.3)$$

Where;

S_x is the calculated standard deviation value of the data.

4.3 Statistical Error Estimation

4.3.1 Discrepancy Ratio

There is a consensus, among the researchers that using the discrepancy ratio (R_d) the calculated and measured data can be correlated. In this study R_d is defined as ratio of calculated average value ($q_{\bar{c}}$) to measured average value ($q_{\bar{m}}$).

$$R_d = \frac{q_{\bar{c}}}{q_{\bar{m}}} \qquad (4.4)$$

The closest R_d value to 1 the best representative model fitting to the measured values. As R_d deviates from unity, to above or below 1, the model overestimates or underestimates the measured datasets respectively.

4.3.2 Error Analysis

The error analysis (V) is the ratio between the standard deviation of the dataset to its related average value:

$$V = \frac{S_x}{q_{\bar{m}}} \qquad (4.5)$$

Values less than 1 are acceptable for V and values more than 10 should be rejected.

4.3.3 Sample Standard Error

$$s_{\bar{x}} = \sqrt{\frac{\sum_{i=1}^{N}(q_{ci} - q_{\bar{c}})^2}{N(N-1)}} \quad (4.6)$$

where;

S_x is the calculated standard deviation value of the data.

4.3.4 Root Mean Squared Error (RMSE)

One of the most common and appropriate method of examining the simulation models is Root Mean Squared Error (RMSE). This term implies the deviation between the computed and measured values.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(q_{ci} - q_{mi})^2}{N}} \quad (4.7)$$

The closer RMSE to zero, the best fit between the models, computed dataset, and measured data. This method was applied to determine the well fitness of the suggested equations.

4.3.5 Thiel's Inequality Coefficient

Thiel's inequality coefficient (U) is a simulation statistics related to RMSE,

$$U = \frac{RMSE}{[\frac{1}{N}\sum_{i=1}^{N} {q_{ci}}^2]^{1/2} + [\frac{1}{N}\sum_{i=1}^{N} {q_{mi}}^2]^{1/2}} \quad (4.8)$$

The range of U is between 1 and zero. As it approaches to zero it implies q_{ci} approaches to q_{mi}.

4.3.6 Mean Normalized Error

Mean Normalized Error (MNE) is one of the statistical mean to quantify the goodness of fit.

$$MNE = \frac{100}{N}\sum_{i=1}^{N}\left|\frac{q_{ci} - q_{mi}}{q_{mi}}\right| \quad (4.9)$$

When the value of MNE approaches to zero implies q_{ci} approaches toq_{mi}.

4.4 Approaches of Sensitivity Analysis

4.4.1 Introduction

Sensitivity Analysis (SA) is an investigation which tries to correlate qualitatively or quantitatively the result variation of mathematical model to distinct sources of input change. In a simple way, SA is defined as technique to determine the sensitivity of a mathematical model to changes either in input parameter or model structure. Both of the approaches try to collect information from system with minimum number of physical or numerical experiments.

In more general terms uncertainty and sensitivity analysis play a significant role to make models more confident by investigating uncertainties commonly accompany the parameters.

There is a distinct difference between uncertainty analysis and sensitivity analysis. Although both of the analyses investigate the overall uncertainty in the model result, sensitivity analysis put an effort to probe the source of uncertainty which plays more significant role on studied result.

Since sensitivity analysis depict the respond of model to variation of model input, it is convenient for constructing or evaluating the model and is a tool to guarantee either the quality of modeling or assessment.

Although there is a vast classification of sensitivity analysis through the literature, in this research categorization is: 1- mathematical, 2- statistical, and 3- graphical due to the assertion of Devore and Peck (1996). Note that, other classifications mostly are based on capability, instead of methodology, of a particular technique. Categorizing would help analysts to determine whether a particular method is applicable on specific model or aligned with the analysis objective or not.

4.4.2 Sensitivity Analysis by Mathematical Methods

Mathematical method by selecting a sample from the range of an input which can be extended to total range of an input and computing the result of the model, examine sensitivity of model output to the range of variation of that specific input. Although these techniques quantify the effect of variation of input on the output, they are not capable of measuring the change rate in the output due to the change in the input.

Occasionally these methods have been applied to point out the most significant inputs (Brun et al. 2001); verification, validation, and recognizing the inputs which need further data attainment or research.

4.4.3 Multiple-Linear Regression

In statistics linear regression is a technique to find a model which can apportion one or more explanatory variables 'X' with a dependent variable 'Y'. Since linear regression is fitting a function, generally linear, between several input and an output it can be noted as Multiple- Linear Regression (MLR) and elaborate as:

$$Y_i = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + \cdots + \beta_j X_{j,i} + \varepsilon_i \qquad (4.10)$$

Y_i = i^{th} output data point for the i^{th} input data point;

$X_{j,i}$ = i^{th} input data point for the j^{th} input;

β_j = coefficient of regression for the j^{th} input; and

ε_i = error for the i^{th} data point.

Based on Neter et al. (1996) Regression analysis by applying probabilistic sensitivity analysis will be beneficial for three significant aims: 1-Explaining the relationship between input parameter if exists, 2-identifying the significance of predictor input for a calculated result, and 3- estimating output based on the significant predictors.

Hereby, it is significant to mention that, such analysis most carry out appropriately on an independent random sample data. To determine the importance of input on output, several statistics measure can be investigated, regression coefficient, standard errors of regression coefficients, and level of significance of regression coefficients.

To determine the goodness of fitted model on the actual data, coefficient of determination 'R^2' can be applied; where R^2 is indicator of deviation of calculated dependent variable by model.

In statistics standardized coefficients are measures, to specify independent parameter importance in multiple-regression particularly when they hold different units or dimensions, some statistical software package noted the standardized regression coefficient as Betas 'β'. They obtained from analysis on independent variables which are standardize (i.e. variance is equal to 1). Since in this study the input variables are independent and the dimensions are not unit, these coefficients are applied to determine the significance of input variables. Beside, to prevent the different dimensions of input variables to affect the βs' coefficient; before running the analysis all the input variables were normalize (or standardize).

Inasmuch as the input parameters hold different units and dimensions, standardize coefficient (Beta 'β') which make the input variable more comparable. For utilizing standardized coefficient, the independent input variables have to normalize and standardize. Standardization was done using the equation 4.11 to remove the influence of different units and dimensions from the regression analysis.

$$\text{Standardized value} = \frac{\text{Value} - \text{Mean}}{\text{Standard Deviation}} \qquad (4.11)$$

4.5 Interpolation and Extrapolation

Approach of derive new data points within the range of a discrete set of known data points in the mathematical field of numerical analysis noted by interpolation. If the required data lies outside the two known point, determination of the required data called extrapolation. The technique depends on the characteristics of the data whether the data obey linear or nonlinear function (i.e. quadratic, cubic, logarithmic, etc.).

4.5.1 Linear Interpolation

One of the simplest approaches in interpolation technique is assuming a linear relationship between the points, which is denoted by linear Interpolation. The expression of such a regression for point (x,y) is as follows:

$$y = y_a + (y_b - y_a)\ \frac{x - x_a}{x_b - x_a} \qquad (4.12)$$

in which x_a, x_b, y_a, and y_b are the coordinates of the two given points.

Chapter 5

METHODOLOGY

5.1 Introduction

The input parameters which have an influence on the sediment transportation, erosion, and deposition within the rill or the overland flows can be categorized into three distinct sciences. In this study from soil mechanics, particle diameters, from hydromechanics, the flow velocity and discharge were selected, and from geomorphology, slope of the channel, cross-sectional width and area were chosen. The presence of these sciences simultaneously, as mentioned in the previous chapters, is one of the main reasons which make the understanding of sediment transport and deposition labyrinthine.

Base on Nearing et al. (1990) amendment of physically based model consist of two major stages: 1- amending model algorithms, equation, and structure based on available knowledge (i.e. theories and basic principles), 2- evaluation of the model. The second step basically consists of three sequences at least. The first and the most significant step is probing the validation of model by comparing the measured data to calculated model results; in the second priority carry out a sensitivity analysis to investigate the sensitivity of model output to the change of input parameter by probing the relative magnitude of deviation in the model output as a function of variation in model input; and lastly probing the applicability range of the model.

5.2 Data

The dataset which is used in this study is the compendium of Elliot et. al., 1989 used to depict approaches so as to measure the soil erodibility and the characteristics of the soil in field and reported the calculated data of the soil.

The datasets consist of thirty three locations in USA with different soil and geomorphologic conditions. This dataset composed of: specific weight, the kinematic viscosity of the fluid, flow average discharge and velocity (both max and average), channel cross-sectional area, width, hydraulic radius, friction factor, Reynolds number, and flow sediment transport capacity. As it tabulated in Table 5.1 the data set cover most of available values for each parameter. Hence the result study would be applicable for wide range of cases.

For each location, six distinct rills were investigated through average of 22 runs. The location information is tabulated in the table A.1 and the map of each location as well detailed in Appendix A.

The dataset consist of three stages:

i-stage one: considers the effect of the rainfall only (initiation of erosion due to soil loosening),

ii- stage two: considers the erosion within the rill while the rainfall continues (erosion within the rill due to the rain fall and added flow), and

iii- stage three: considers the erosion with in the rill after the rain fall (erosion within the rill after the end of the rainfall). In this study the effect of rain fall over the rill only has been studied (inter-rill and gullies were not considered).The data of stages ii and iii was combined and utilized in this study to get more general equation.

Table 5.1: Range of input parameters

Variable	Range	
	Max	Min
V_{max}	0.338889	0.192333
V_{av}.	0.001882	0.000891
A	0.012833	0.006427
Rh	0.1559	0.078292
W	44.21857	1.45
Tc	51.599	1.211806
f	6395.794	1743.319
Re	0.049022	0.03257
Q_{av}	8.95	3.865
S	8.78E-06	7.58E-07
ν	44.12742	0.504819
Re_{d65}	0.687	0.551
d_{10}	0.02684	0.000378
d_{16}	0.036211	0.000591
d_{25}	0.050043	0.00071
d_{30}	0.054845	0.001247
d_{35}	0.098599	0.006463
d_{50}	0.135575	0.009779
d_{60}	0.154063	0.011437
d_{65}	0.201381	0.021693
d_{75}	0.341703	0.039552
d_{84}	0.908169	0.052934
d_{90}	19.675	0.555
Fr	1.348645	0.570398
C_c	8.407694	0.142816
C_u	184.3972	2.839037

5.3 Developing a general equation for Sediment Transport Capacity in rill

5.3.1 General

To establish a relationship between sediment transport capacity and influential parameters, the following steps have been followed to prevent bias in analysis and having more representative and simple equation.

5.3.2 Preparation of the Existing Data

Since there are several dimensional and non-dimensional parameters of varying ranges, and sample sizes, the dataset was reorganized:

i- to eliminate the partially available data groups that causes bias in the sensitivity analysis appropriate averaging techniques were adopted for each data group. In a case of sieve analysis by reviewing the literature the geometric mean approach was applied since the sieve analysis results is believed to obey the logarithmic character. To obtain required the effective grain size, while obeying logarithmic rules, logarithmic interpolation was applied to acquire them. For the remaining parameter which required averaging, the simple average method was applied through ‘Excel’ program.

ii- to correlate data appropriately through the statistical measures by using SPSS for Sensitivity Analysis, the datasets were standardized or normalized through Excel beforehand.

5.3.3 Running Sensitivity Analysis (SA)

Running sensitivity analysis was applied by carrying out Multiple Linear Regression (MLR) through SPSS (formerly known as Statistical Package for the Social Sciences). Standardized coefficient was used to determine the sensitivity of the sediment transport capacity to different input parameters in this study. The parameters holding higher values of β imply higher significance. Appropriate table was organized to summarize coefficients resulted from Multiple Linear Regression based on the Beta values. Here 95% of confidence interval was used to determine the most significant parameters. In order to select the most sensitive or significant parameter of the Sediment Transport Capacity for each effective group (i.e. soil characteristics, flowing water property, and channel morphology) a parameter holding higher value of β value has been selected. These parameters are believed to be statistically the most appropriate parameters that may represent the Sediment Transport Capacity of rill flow during and after rainfall.

5.3.4 Derive a Representative Equation

Derivation of a representative equation has been done by the Wolfram Mathematica® program. In this study, specifically written codes try to establish the proposed model (linear, logarithmic, etc.) coefficient by fitting the three input data (the three most influential parameters obtained from the SA through MLR of the distinct mentioned groups) with the model.

The measured T_c was compered by the calculated T_c from the model to determine preciseness (goodness of fit) of the model. This is carried out through below mentioned statistical measures:

i- MNE

ii- RMSE

iii- R^2

iv- U

v- Rd

vi-

Chapter 6

RESULT AND DISCUSSION

6.1 General

In this study, to obtain a relationship between sediment transport capacity for the rill with the help of the SA through MLR, the soil characteristic (13 distinct) parameters, the flowing water properties (8 distinct) parameters, and the channel morphology (4 distinct) parameters were studied. Table 6.1, details the parameters and their significance through β values evaluation based on MLR.

6.2 MLR Results

The most influential soil characteristics parameter out of eleven is 'd_{65}' of β= 0.477, the most significance flowing water properties out of seven is 'Re' of β = 0.151 whereas the most important channel morphology parameter out of four is 'S' of β = 0.720. It is significant to mention that the absolute value of β is important for determination of the most influential parameter.

As tabulated below, since from the soil characteristics parameters d_{65} is the most influential parameter, a Reynolds number based on the mentioned parameter was proposed. But it reveals that, the Reynolds number based on hydraulic radius plays more significant role to determine sediment transport capacity than the proposed one.

Table 6.1: β Coefficients of the studied parameters

Model	Standardized Coefficients β
S	0.681722061
d_{65}	0.450855874
Rh	0.33406474
W	0.319166501
d_{84}	0.27470146
d_{60}	0.265773
Re	0.23395
d_{16}	0.200883
d_{75}	0.198172
Fr	0.191264
V_{av}	0.170729
d_{25}	0.157632
d_{35}	0.105694
d_{10}	0.102327
f	0.100984
Re_{d65}	0.094255
ν	0.091307
A	0.089201
C_u	0.059974
d_{90}	0.057547
V_{max}	0.039639

Table 6. 2: (Continiued)

d_{50}	0.039125
C_c	0.018031
d_{30}	0.016212
Q_{av}	0.004319

6.3 Representative Equations for T_c

The aim of this study is to generate equation(s) so as to represent T_c in the simplest manner. This is done by combining the most influential parameters of different characteristics that affect T_c of rills, and attempting to find empirical relationships among these parameters.

Since the slope of the channel (S) has much more significant role to determine the sediment transport capacity (T_c) as it can be realized from the result of MLR through β (= 0.72) in Table 6.1, initially one parametric empirical equation was applied between S and T_c separately. By taking above mentioned reason into consideration, keeping the most sensitive parameter (S) in all of the proposed empirical equation is undisputable; hence the other two influential parameters were added sequentially based on β values.

The coefficients of proposed models have been calculated by using WolfarmMathematica®, by means of written codes to find the best fit which represent the suggested models and given in Table 6.2.

Table 6.3: Statistical detail of the proposed models

No.		Proposed Model and Derived Equations	R^2	RMSE	Rd	V	U	MNE
1	Prop. Model	$a_0 S^{b_1} + a_4$	0.861	28.5	1	0.03	0.078	14.071
	Derived Eq.	$T_c = 55.487 + 0.712 S^{2.66}$						
2	Prop. Model	$a_0 {d_{65}}^{b_0} + a_5$	0.0173	75.753	1	0.08	0.216	46.825
	Derived Eq.	$-8654.526 + 8867.951 {d_{65}}^{0.002}$						
3	Prop. Model	$a_0 \mathrm{Re}^{b_2} + a_5$	0.245	66.389	0.998	0.07	0.187	41.012
	Derived Eq.	$-700.147 + 309.047 x_4^{0.126}$						
4	Prop. Model	$a_0 S^{b_0} + a_1 {d_{65}}^{b_1} + a_4 \mathrm{Re}^{b_3} + a_5$	0.332	62.45	0.999	0.066	0.176	37.785
	Derived Eq.	$T_c = -40.624 - \frac{88.646}{S^{0.603}} - 2207.872 {d_{65}}^{1011.376} + 5.528 \mathrm{Re}^{0.495}$						
5	Prop. Model	$a_0 S^{b_0} {d_{65}}^{b_1} + a_4$	0.866	27.95	1	0.0296	0.0766	13.658

Table 6.2: (continued)

	Derived Eq.	$T_c = 47.959 + \frac{0.877S^{2.471}}{{d_{65}}^{0.078}}$						
6	Prop. Model	$a_0 S^{b_0} {d_{65}}^{b_1} Re^{b_2} + a_4$	0.917	21.987	1	0.0233	0.0601	11.426
	Derived Eq.	$T_c = 46.532 + \frac{0.0209S^{2.126}Re^{0.553}}{{d_{65}}^{0.077}}$						
7	Prop. Model	$a_0 S^{b_0} {d_{65}}^{b_1} Re^{b_2} + a_4 S^{b_3} + a_5$	0.486	54.763	0.999	0.0580	0.1529	28.852
	Derived Eq.	$T_c = 112.629 - \frac{54.623}{S^{0.577}} + 0.038S^{1.101}{d_{65}}^{0.754}Re^{0.935}$						
8	Prop. Model	$a_0 S^{b_0} {d_{65}}^{b_1} Re^{b_2} + a_4 {d_{65}}^{b_3} + a_5$	0.486	21.987	1	0.0233	0.0601	11.426
	Derived Eq.	$T_c = 46.532 + 6188.435{d_{65}}^{2794.248} + \frac{0.0209S^{2.126}Re^{0.535}}{{d_{65}}^{0.077}}$						
9	Prop. Model	$a_0 S^{b_0} {x_3}^{b_1} Re^{b_2} + a_4 {d_{65}}^{b_3} S^{b_4} + a_5$	0.554	51.031	0.999	0.0540	0.1422	28.221
	Derived Eq.	$T_c = 272.237 + 715.01S^{4.115}{d_{65}}^{320.671} - \frac{69.926Re^{0.524}}{S^{1.989}}$						

The Coefficient of Determination 'R^2' has been calculated by taking y=x equation and the calculated and measured datasets into consideration and given in Figures 6.1- 6.9.

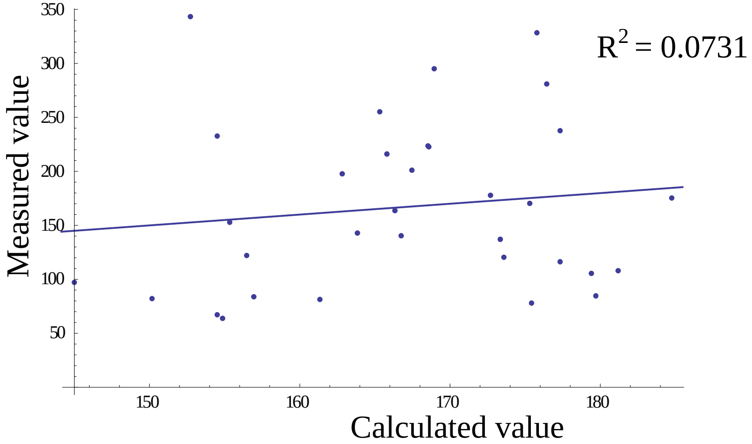

Figure 6.2: Measured versus calculated value of Model 2

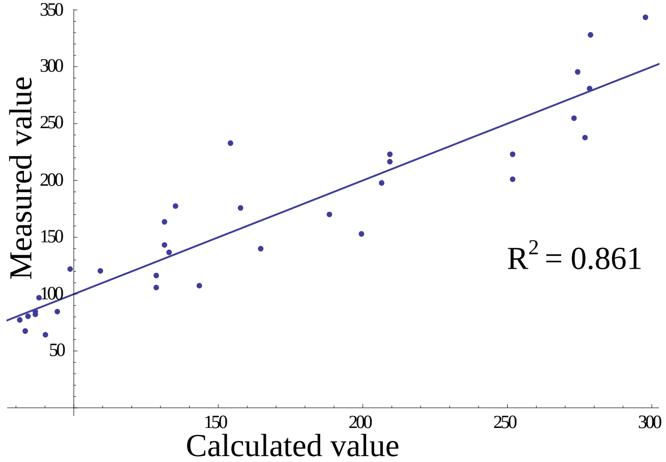

Figure 6.1: Measured versus calculated value of Model 1

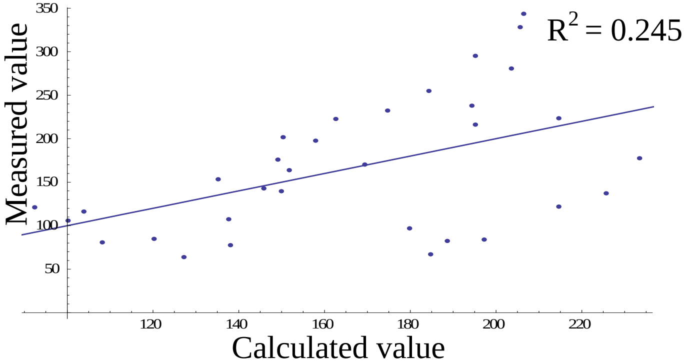

Figure 6.3: Measured versus calculated value of Model 3

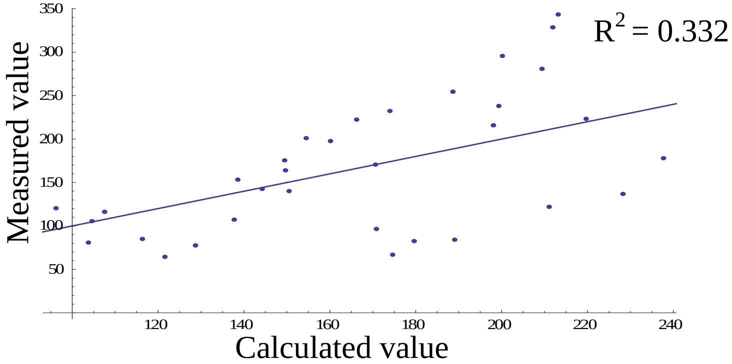

Figure 6.4: Measured versus calculated value of Model 4

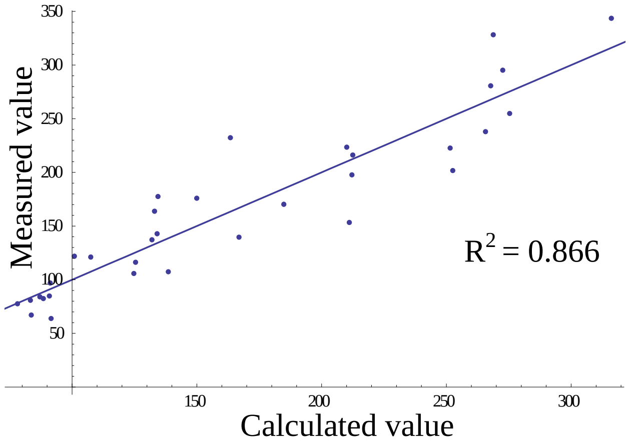

Figure 6.6: Measured versus calculated value of Model 5

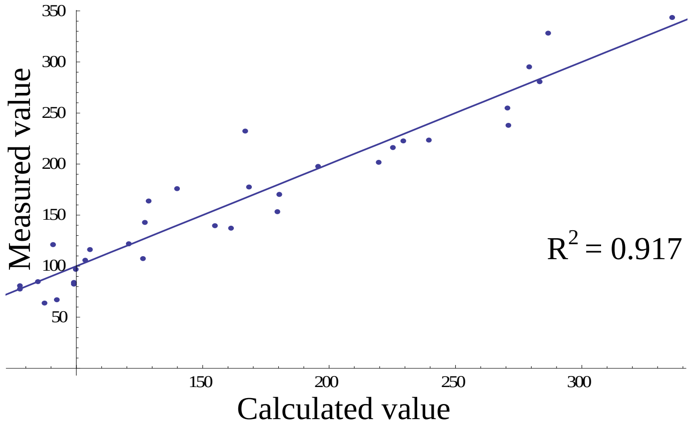

Figure 6.5: Measured versus calculated value of Model 6

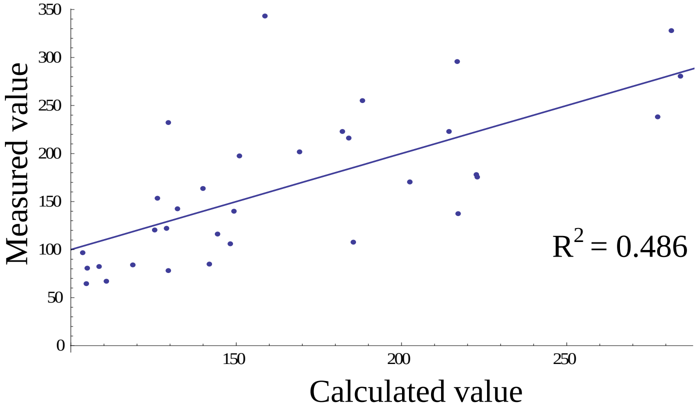

Figure 6.8: Measured versus calculated value of Model 7

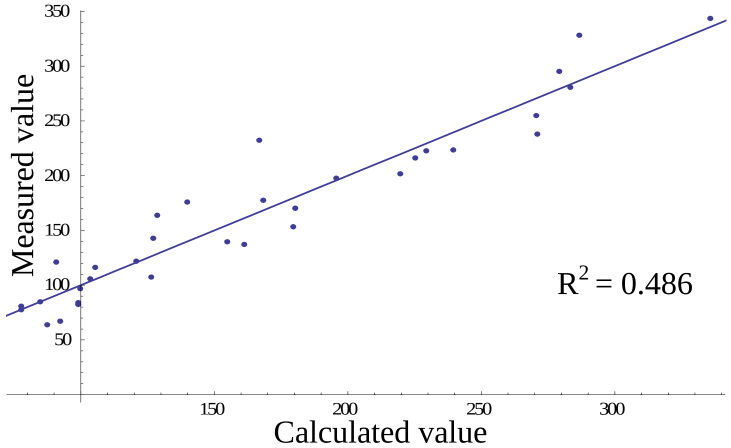

Figure 6.7: Measured versus calculated value of Model 8

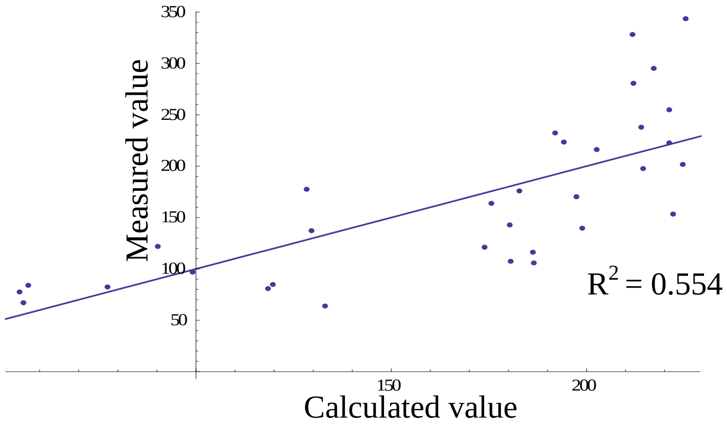

Figure 6.9: Measured versus calculated value of Model 9

6.4 Discussion

From the outcome of MLR the suggested most influential 3 distinct parameters were used in different combinations by several proposed models.

6.4.1 Notion1

By considering the above mentioned results in Table 6.1 through β coefficients and Table 6.2, it is clearly observed that, the sediment transport capacity (Tc) is significantly sensitive to the slope (S) of the rill channel where a single parametric equation based on S (given in Eqn. no. 1) would be a representative empirical equation for Tc with a reasonable R^2 (= 0.861).

6.4.2 Notion 2

The attempt was continued to verify whether other parameters would represent a single empirical equation through Eqn.2 and 3, but the result of R^2 decline dramatically.

6.4.3 Notion 3

As Table 6.1 suggests d_{65} as an influential parameter of $\beta = -0.4774$, does not play a significant role in the improvement of R^2 when it was coupled empirically with Eqn.1 as given in Eqn. 4, hence, adding the third distinct parameter to Eqn. 5, the best possible result was obtained as given in Eqn. 6.

6.4.4 Notion 4

An attempt was carried out from Eqn. 7 to 9 so as to increase R^2 by adding empirical terms to the sixth equation but none of those trials reveled better results.

6.4.5 Notion 5

Although from the figures 6.1-6.9 the well fitness of the proposed models could be comprehended, the relationship of Tc and the most influential parameter are undefined. Therefore figuratively strive was carried out to see the correlations one by one.

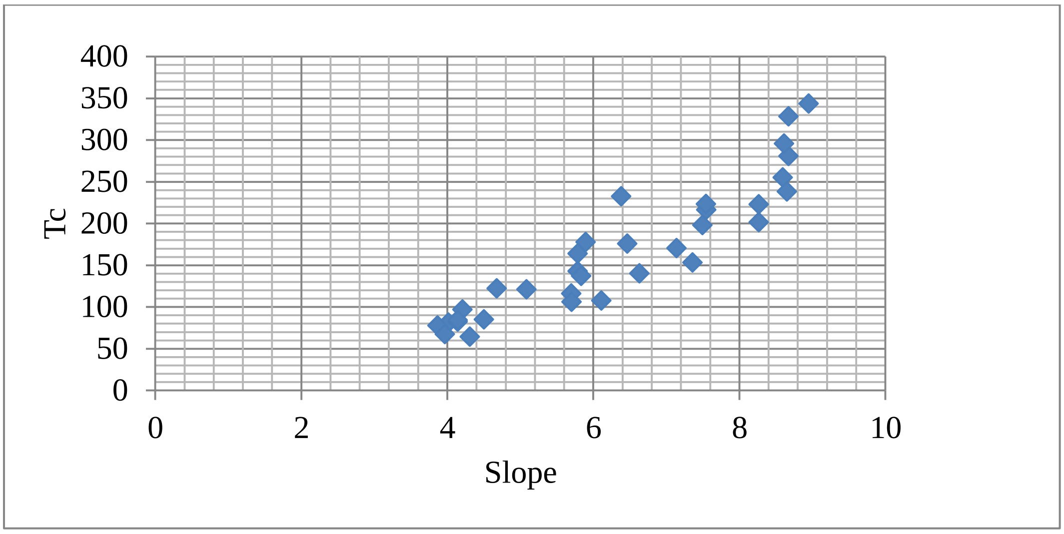

Figure 6.10: Correlation of Tc and channel slope

As illustrated in Figure 6.10, between sediment transport capacity of the rill and slope as discussed already, there is a linear correlation.

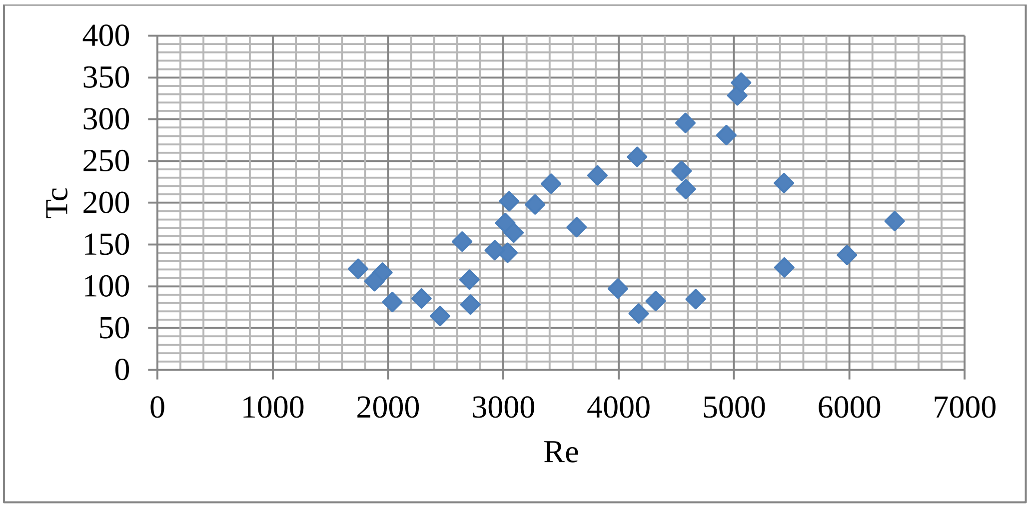

Figure 6.11: Correlation of Tc and Reynolds number

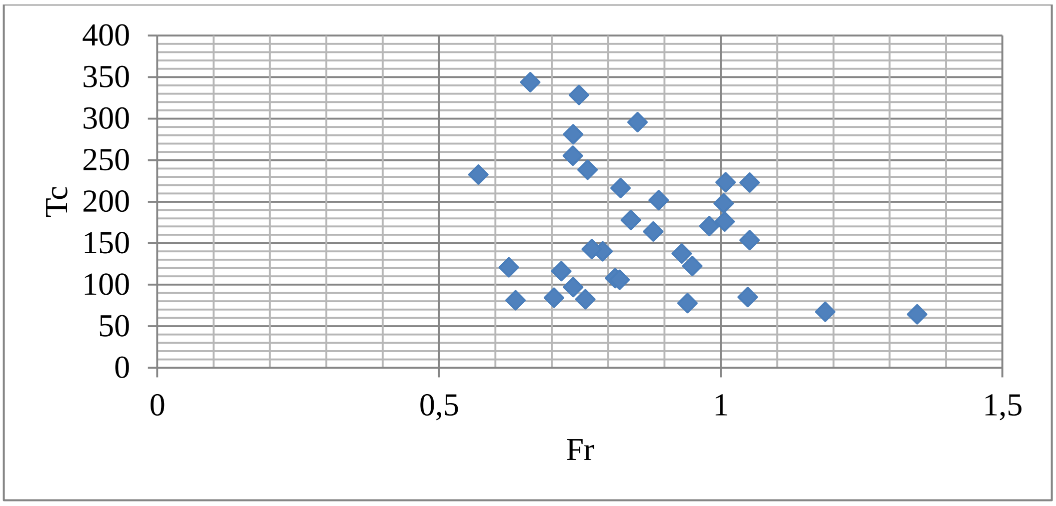

Figure 6.12: Correlation of Tc and Froude number

As it is depicted in Figure 6.11 and Figure 6.12 Reynolds number shows better correlation with Tc in comparison with Froude number. Although in open channel hydromechanics, Froude plays a crucial role, in this case it felt into contradictory due to the critical role of shear stress in water erosion mechanism. Since the shear stress play a significant role in soil erosion and deposition, the viscous forces and consequently Reynolds number seems to have more influential role in determination of sediment transport capacity.

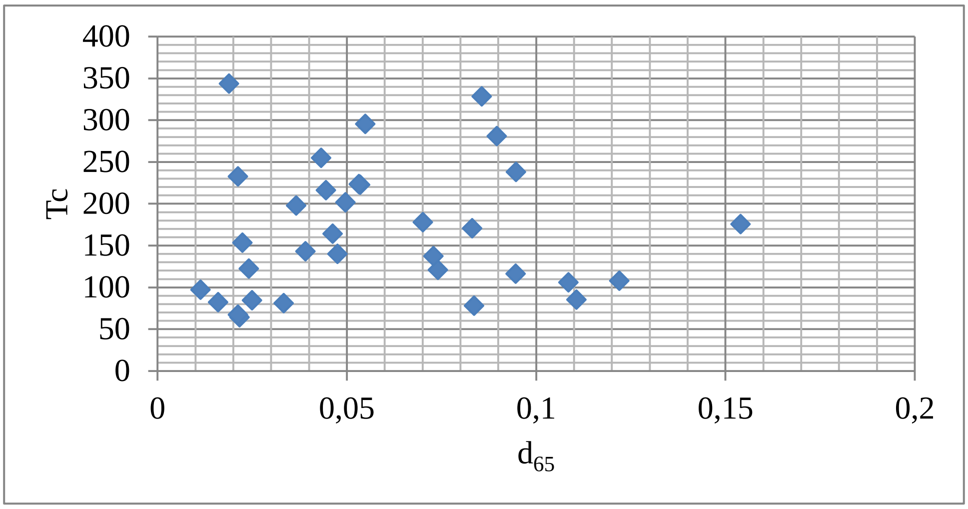

Figure 6.13: Correlation of Tc and d_{65}

As it is illustrated in Figure 6.13, the distribution of d_{65} with respect to sediment transport capacity (Tc) is highly scattered and not reviling any correlation.

6.5 Conclusion

Among 13 distinct soil parameters, d_{65}, 8 distinct flowing water properties, Re, and 4 distinct channel morphology, S, were determined to be the most influential parameters by applying Sensitivity Analysis (SA) through Multiple Linear Regression (MLR) so as to represent sediment transport capacity (Tc) for rills during and after rainfall effects. The generated empirical equation based on compendium of Elliot et. al., 1989 dataset, with $R^2 = 0.917$ is:

$$T_c = 46.532 + \frac{0.0209S^{2.126}Re^{0.553}}{{d_{65}}^{0.077}}$$

6.6 Further Studies

It is suggested that, other than sediment transport capacity T_c there are some other significant parameters which can be investigated such as detachment rate, deposition rate, inter-rill-rill coupled erosion effect and etc.

In order to generalize the selected equation, more dataset from the field observation and/or laboratory experiments from different locations (rather than selected locations) would be selected so as to have more governing equation for all locations. It is significant to mention that, the more general equation may deviate from the measured values, to obviate the mentioned problems, applicability limitations to the equations could be applied i.e. Froude number, sediment particle size, Reynolds number, slope and etc.

It is worth to mention that, the applied methodology in this study is applicable in other fields of science.

The approach which has been applied in this study would be applicable in other study in any other science to run sensitivity analysis.

Since there is no available relevant dataset for TRNC, field observation and laboratory test can be conducted in different studies to measure and formulate sediment transport capacity of rill for TRNC case is required.

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APPENDIX

Appendix A: Location information of 32 fields (WEPP)

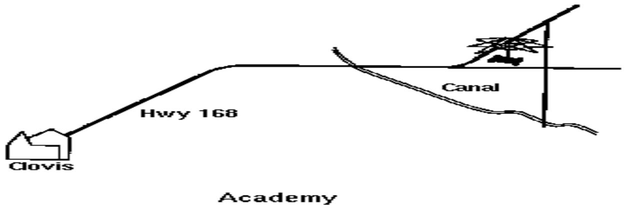

Figure A. 1: Academy map

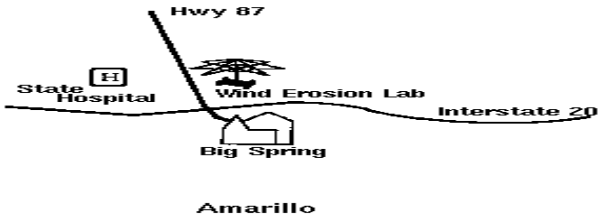

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Figure A. 2: Amarillo map

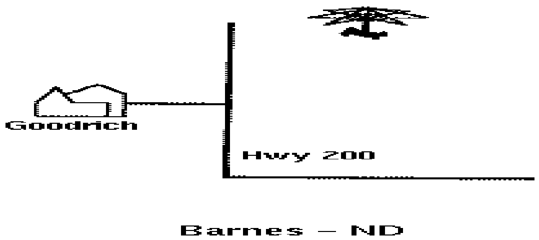

Figure A. 4: Barnes-ND map

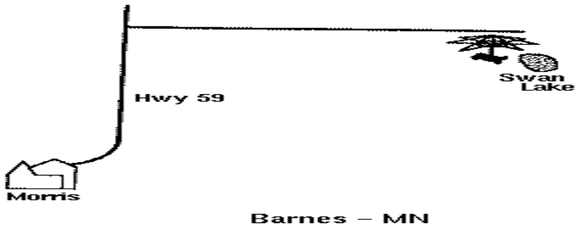

Figure A. 3: Barnes- MN map

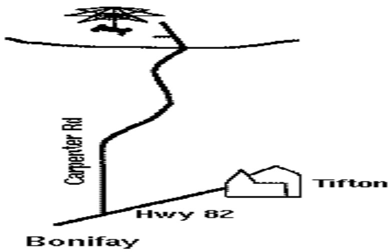

Figure A. 6: Bonifay map

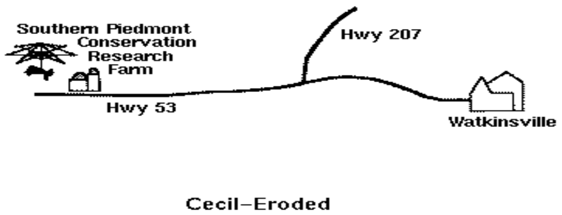

Figure A. 8: Cecil- Eroded map

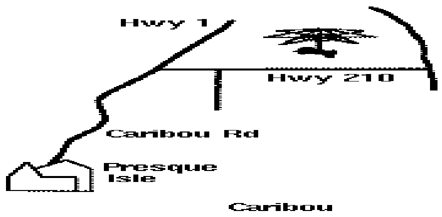

Figure A. 5 Caribou map

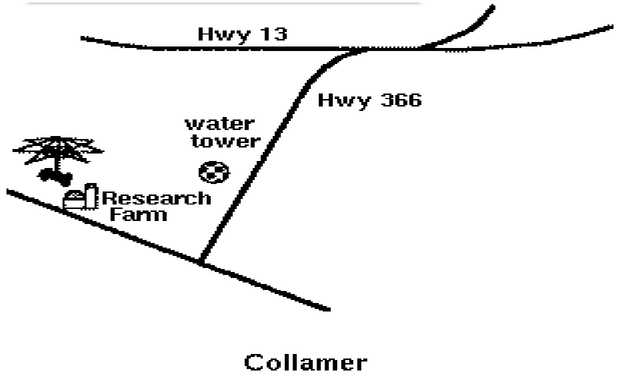

Figure A. 7: Collamer map

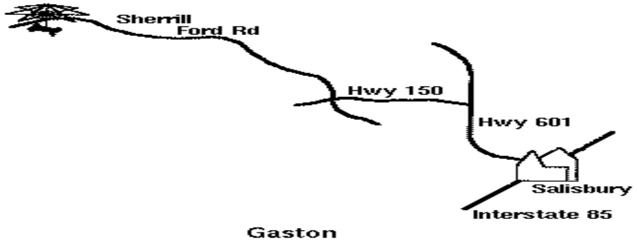

Figure A. 12: Gaston map

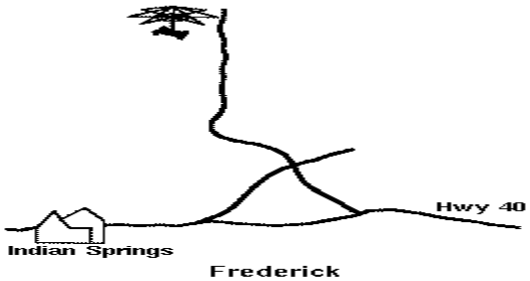

Figure A. 10: Frederick map

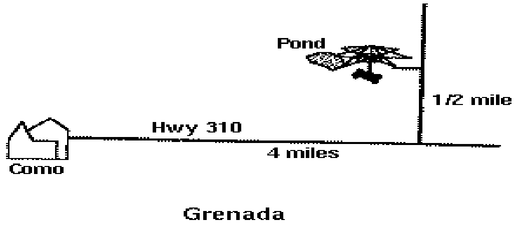

Figure A. 11: Grenada map

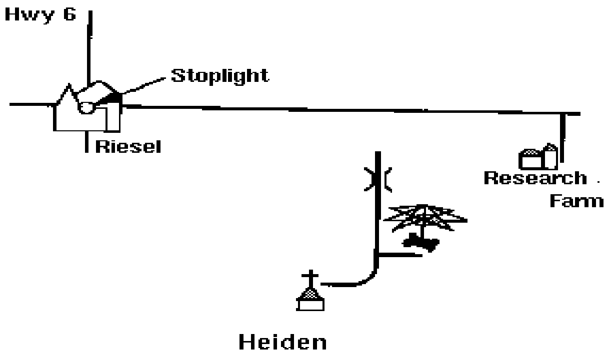

Figure A. 9: Heiden map

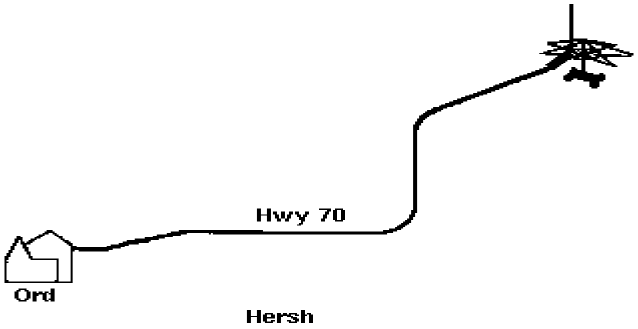

Figure A. 13: Hersh map

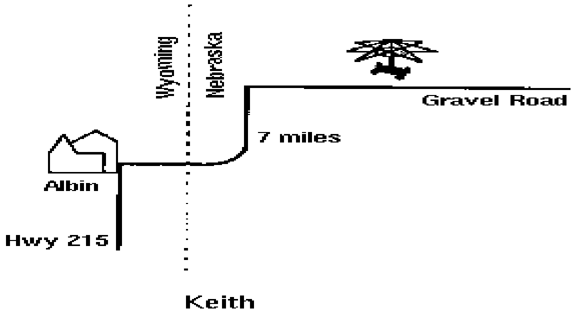

Figure A. 14 Keith map

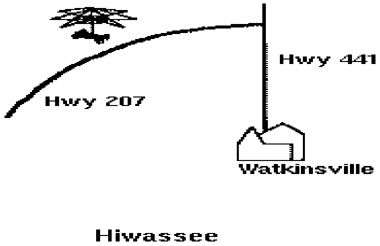

Figure A. 15: Hiwassee map

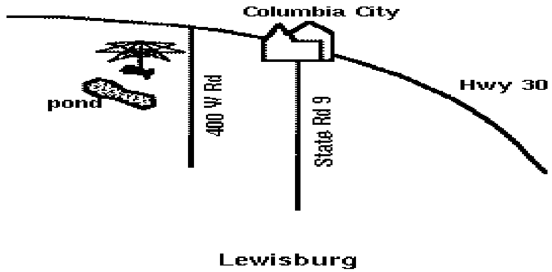

Figure A. 16 Lewisburg map

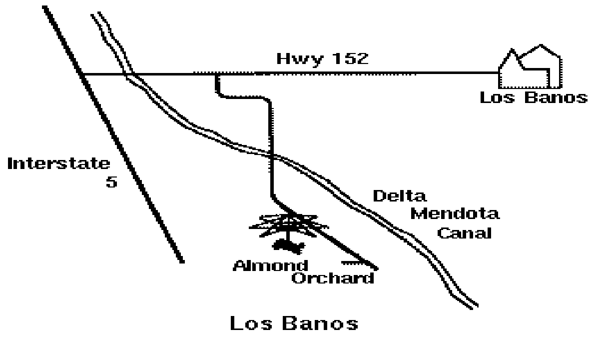

Figure A. 20: Los Banos map

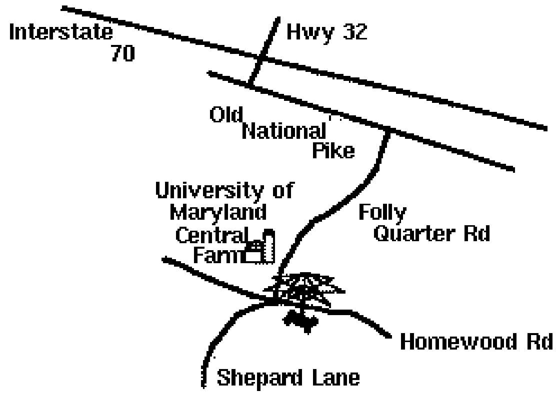

Figure A. 18 : Manor map

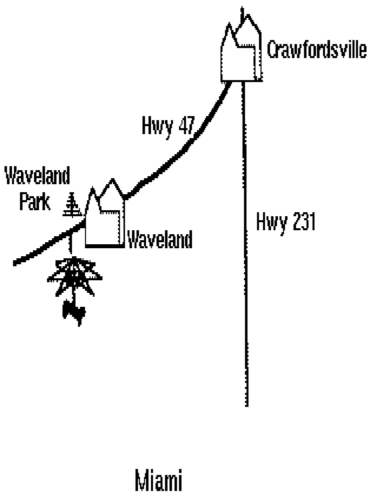

Figure A. 19: Miami map

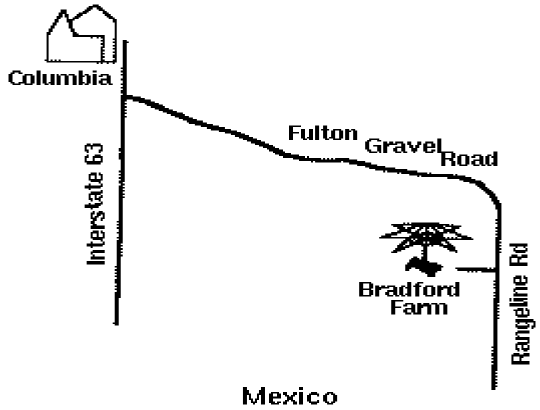

Figure A. 17: Mexico map

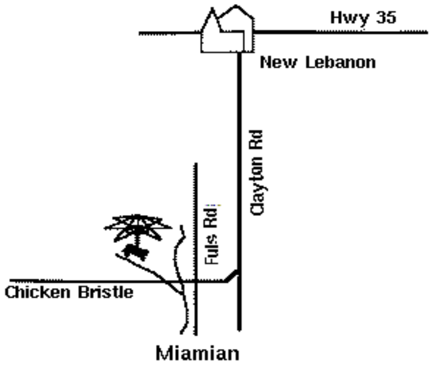

Figure A. 23: Miamian map

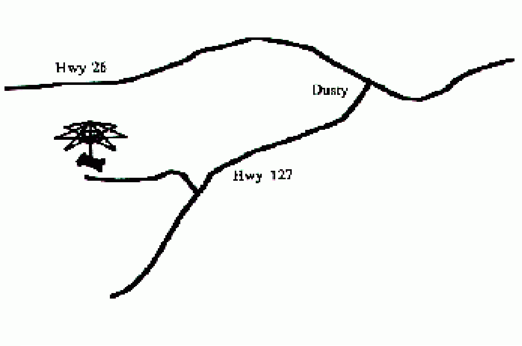

Figure A. 22 : Nansene map

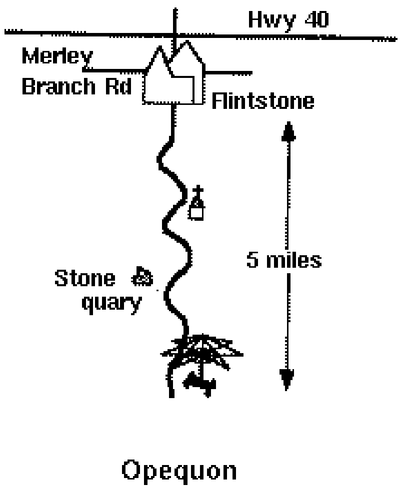

Figure A. 24: Opequon map

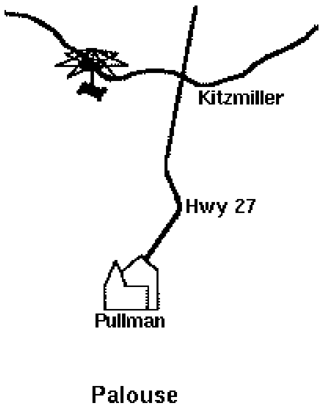

Figure A. 21: Palouse map

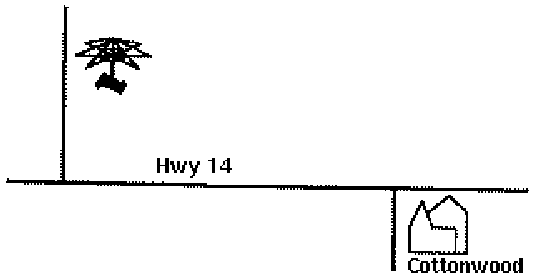

Figure A. 27: Pierre map

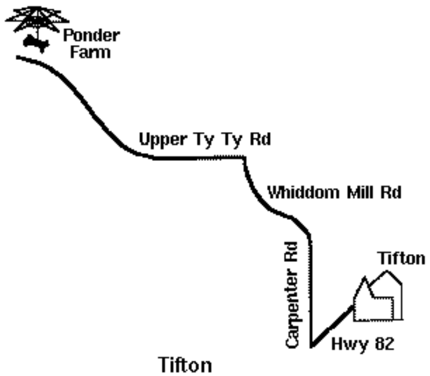

Figure A. 26: Tifton map

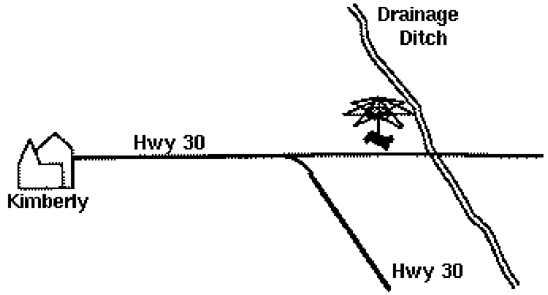

Figure A. 25: Portneuf map

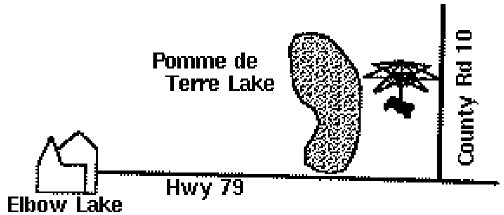

Figure A. 28: Sverdrup map

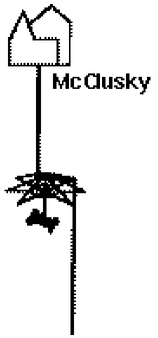

Figure A. 31: Williams map

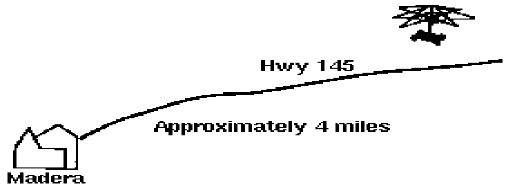

Figure A. 29: Whitney map

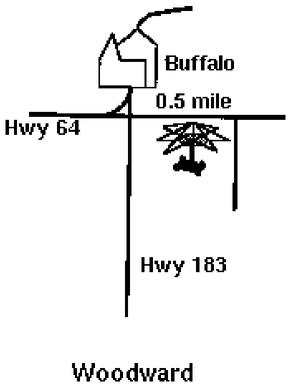

Figure A. 30: Woodward map

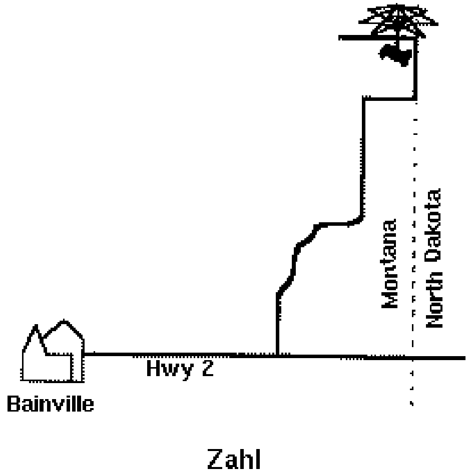

Figure A. 32: Zahl map

Table A. 1: Location details of 32 fields information

Soils	County	State	Latitude	Longitude	Elevation	Direction
Sharpsburg	Lancaster	NE	40-51-20-	096-28- -	374 m MSL	2160' W and 420' S of the NEcorner of SEC.12 T.10N, R.8W
Hersh	Valley	NE	41-42-30-N	098-48-45-W		75' W and .4 mi W of the NE corner of SEC.16 T.20N, R.13W
Keith	Banner	NE	41-49-04-N	103-58-01-W	1586 m MSL	1320' N and 500' W of SE cornerof SEC.35 T.18N, R.58W
Amarillo	Howard	TX	32-16- -N	101-30- -W	686 m MSL	Located o the ARS Research Station at Big Spring, TX
Woodward	Harper	OK	36-49-24-N	099-25-12-W	572 m MSL	84 m W and 152 m S of NE corner of SEC.18 T.27N, R.22W
Heiden	Falls	TX	31-27-16-N	096-52-52-W	160 m MSL	From Riesel: 2.0 mi E on FR 1860, 0.6 mi S 0.6 mi W 1.5 mi S onCo. Rd. 400' E in field
Whitney	Madera	CA	37-00-31-N	120-57-05-E	104 m MSL	1500' W and 600' S of NE corner of SEC.1 T.11S, R.18E
Academy	Fresno	CA	36-52-01-N	119-33-46-E	146 m MSL	700' E and 200' N of SW corner of SEC.22 T.12S, R.22E
Los Banos	Merced	CA	39-25-00-N	120-57-00-E	76 m MSL	1650' W and 600' S of NE corner of SEC.26 T.10S, R.9E
Portneuf	Twin Falls	ID				on ARS Research Station: Kimberely, ID
Nansene	Whitman	WA				
Palouse	Witman	WA				on ARS Research Station: Pullman, WA

Table A. 2: Continued

Zahl	Roosevelt	MT				
Pierre	Pennington	SD				
Williams	Sheridan	ND	47-25-00-N	100-25-00-W	614 m MSL	2585' N and 1915' W of SE corner of SEC.1 T.145N, R.77W
Barnes - ND	Sheridan	ND	47-34- -N	100-06- -W	584 m MSL	2340' S and 2515' D of NW corner of SEC.9 T.147N, R.74W
Sverdrup	Grant	MN				NW 1/4 of the SW 1/4 of SEC.8 T.129N, R.41W
Barnes - MN	Stevens	MN				SW 1/4 of the NW 1/4 of SEC.35 T.126N, R.41W
Mexico	Boone	MO				on U. of MO. Agronomy Farm
Grenada	Panola	MS	34-30-57-S	089-05-04-W		0.45 mi 325' W of SE corner of SEC.32 T.6S, R.6W
Tifton	Worth	GA	31-30-29-N	083-39-23-W	107 m MSL	0.2 mi N of upper Tyty, 1 mi W of intersec. w Tyty Whiddom Mill Rd
Bonifay	Tift	GA	32-29-30-N	083-32-34-W	104 m MSL	
Cecil (Eroded)	Oconee	GA	30-52-07-N	083-27-07-W	232 m MSL	1.3 mi SW of HWY 207 on 53, 100' NW of HWY 53
Hiwassee	Oconee	GA	33-53-07-N	083-25-51-W	233 m MSL	1.2 mi W of HWY 441 on HWY 207 N side of HWY
Gaston	Rown	NC	35-41-45-N	080-37-10-E	213 m MSL	
Opequon	Allegany	MD	39-01-54-N	078-37-30-W	351 m MSL	5 mi S of Flintstone, Evitts Creek Quadrangle; 2 mi E of Twigstown, 800's of Murleys Branch Rd.

Table A. 3: Continued

Frederick	Washington	MD	41-43-59-N	078-01-57-W	201 m MSL	1.25 mi S MD/PA state line 500' NW of Little Cove Indian Springs Rd.
Manor	Howard	MD	39-14-50-N	076-55-35-W	145 m MSL	Clarksville Quad, 2000' E of intersection of Folly Quarter and Homewood Rds; 100' S of Homewood Rd.
Caribou	Aroostook	ME	46-00-55-N	068-01-11-E	190 m MSL	
Collamer	Tompkins	NY				Located on the Cornell University campus
Miamian	Montgomery	OH				
Lewisburg	Whitley	IN	41-09-42-N	084-34-05-W	275 m MSL	528' S and 596' W of NE 1/4 of SEC.12 T.13N, R.8E 1/2 mi SEC.12 T.13N, R.8E 1/2 mi S of USSO SHEET 18. 400W
Miami	Montgomery	IN	39-52-42-N	087-05-35-W	220 m MSL	1452' W and 1056' S of NE 1/4 of SEC.34 T.17N, R.6W SHEET 67 1/4 mi S of HWY 47