Physics on the Rotating Earth

Najm Abdola Saleh

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Prof. Dr. Elvan Yılmaz
Director

I certify that this thesis satisfies the requirements as a thesis for the degree of Master of Science in Physics.

Prof. Dr. Mustafa Halilsoy
Chair, Department of Physics

We certify that we have read this thesis and that in our opinion it is fully adequate in scope and quality as a thesis for the degree of Master of Science in Physics.

Prof. Dr. Mustafa Halilsoy
Supervisor

Examining Committee

1. Prof. Dr. Özay Gürtuğ
2. Prof. Dr. Mustafa Halilsoy
3. Assoc. Prof. Dr. Izzet Sakalli

ABSTRACT

In this thesis we study physics on the rotating Earth by studying the moving coordinate systems and rotating coordinate systems. First, we illustrate briefly some kind of translations like Galilean transformation which consists of two inertial frames, one of them moving with respect the other stationary We show how to transform between the two reference frames. Then we give the Lorentz transformations in which time is no more absolute when the speed approaches to the speed of light. We review Abelian and Non-Abelian groups. But then we will focus on the Newton's equations of motion on the Earth and we will explain in details the derivation of these equations. We derive both the Coriolis and centrifugal forces.

Later on we explain some applications about rotating Earth. The most important example is a projectile motion. We illustrate by derivation how it is the best way to show the reason of the deflections of missiles in long range distances. Another famous example is the Foucault pendulum, which is an important example to prove that the Earth is rotating about its axis. And finally, we give some applications to show the effect of Coriolis and centrifugal forces in our daily life.

Keywords: Coriolis and Centrifugal Force, Foucault Pendulum.

ÖZ

Hareketli ve dönen koordinat sistemlerinin dünya üzerindeki fiziğe etkileri incelenmiştir.Önce birbirine göre hareketli Galile koordinat sistemleri göz önüne alınmıştır.İki koordinat sistemi arasındaki dönüşüm verilmiştir.Işık hızına yakın durumlarda**,** ki zamanın mutlak özelliği geçersiz olur Lorentz dönüşümleri ele alınmıştır. Abel/ Abel olmayan gruplar gözden geçirilmiştir.Dönen dünya üzerindeki Newton hareket denklemleri ile Coriolis ve merkezkaç kuvetler türetilmiştir. Fırlatılmış cisimlerde dönmenin etkileri incelenmiştir. Uzun menzilli roket hareketindeki sapmalar iyi bir örnek olarak ele alınmıştır .Foucauft sarkacı dünyanin dönme etkisine başka bir önemli örnek teşkil etmekte olup dünyanın dönüşünü de kanıtlamaktadır.Coriolis ve merkezkaç kuvetlerinin günlük hayatımızdaki örnekleri irdelenmiştir.

Anahtar Kelimeler: Coriolis ve Merkezkaç Kuvetleri, Foucault Sarkacı .

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LIST OF SYMBOL/ABBREVIATIONS

SO3	special orthogonal in 3-dimentions
I	identity
d	distance
$\vec{v}$	velocity
X	distance
t	time
c	speed of light
ω	angular velocity
$\ddot{r}$	acceleeration
$F_{cor.}$	Coriolis force
$F_{cen.}$	centrifugal force
$\ddot{r}_{fix.}$	acceleration of fixed coordinate system
$\ddot{r}_{mov.}$	acceleration of moving coordinate system
F	force
g	acceleration due to the gravity
$\ddot{x}$	acceleration in x-direction
$\ddot{y}$	acceleration in y-direction
$\ddot{z}$	acceleration in z-direction
$\dot{x}$	velocity in x-direction
$\dot{y}$	velocity in y-direction

$\dot{z}$	velocity in z-direction
W`	apparent weight
m	mass
T	tension
IF	inertial frame
NIF	non inertial frame

Chapter 1

INTRODUCTION

We live on a rotating Earth which at the same time rotates in an elliptical orbit around the Sun. Our solar system also moves both translational and rotationally in the Milky way galaxy. Ultimately everything is in motion relative to others in our universe and our universe undergoes an accelerated expansion for the last 5 billion years. The reason of such accelerated expansion is due to the dark energy which may be attributed to the repulsive pressure existing in the universe. The fact that such a running away may not last for ever is due to the super massive black holes or wormholes lying at the heart of galaxies in our universe. Doıng physıcs ın such an evolutıonary envıronment becomes a state of art and thanks to the covarıant approach of general relativity proposed by Einstein ın 1916.This naturally modified Newtonian mechanics which used to be valid in an Inertial frame. Being universal the physical laws must be valid for all coordinate frames equally well. This is precisely what is meant by covariance of the physical laws[1]. Such an approach becomes physically feasible provided the laws of physics can be cast in to a tensor formalism [2,3]. Tensors are mathematically 'good 'objects that once they satisfy a relation/ equation it becomes satisfied in any other frames which are related to the original frame by a coordinate transformation. Physically, change of a coordinate frame amounts to a coordinate transformation from one frame to the other. Such transformations must obey certain basic requirements in order to be physically admissible. For instance non-vanishing Jacobian, existence of inverse and the related properties cast the transformations into a canonical from which is said to form a particular mathematical class, named Group. Since every object moves translational and rotationally the Group that is to be taken into consideration is known as the Poincare group. This consists of an Abelian and Non-Abelian parts, so that over all the group that confront us is Non-Abelian. It is well-known that two successive rotations around different axes do not commute which is meant by Non-Abelian. However, if we restrict our operations into the common axis of rotation then we reduce to an Abelian subgroup of the overall larger group which is Non-Abelian. The concept of symmetry in physics relates to the transformation properties and among all these mathematical processes determination of invariants becomes essential. That is, the things that do not change while everything else is changing are the things that we label as invariants of the motion. To recall an analogy in the electromagnetic theory the combinations $\sim (B^2 - E^2)$ and $\sim (E \cdot B)$, where E and B are the electric and magnetic fields, are frame independent and they are said to be the invariants of the electromagnetic field[3,4]. Similarly in Newton's laws for instance, we have conservation of motion preserve their identity under certain classes of transformation of linear momentum under translational motion. This means from Newton's second law $\overrightarrow{F} = \frac{\overrightarrow{dp}}{dt}$ that under translation no new forces arise to distort a given object. As a result a cube/ sphere or anything doesn't become distorted under translation. Abelian character of successive translations is the reason why the object preserves its shape under translation and it relates to the conservation of linear momentum from Newton's second law. When we come to rotations things change completely [5]. Although certain things do not change under rotation the physical laws of Newton require modifications. The square of angular momentum, for instance, is an invariant under rotation whereas the angular momentum as a vector transforms in this process. As a matter of fact every vector change, in particular the velocity and acceleration vectors also do change and as a result the Newton's laws change accordingly. The problem becomes therefore how to modify the Newton's laws so that they become still applicable in a rotating frame. Since the time is considered 'absolute' it doesn't change from one frame to the other because the associated rule of transformation is the Galilean transformation. But once the speed among frames surpasses the classical limit and approach the speed of light then automatically the Galilean transformation is replaced by the Lorentz transformation in which time is no more absolute. In this project, however, we shall confine ourselves with the classical limit in which $v \ll c$ so that Lorentz transformation will be out of questions.

The rotation/ motion of our Earth around Sun and the motion of Sun /Solar system in the Galaxy will be ignored in this study. We shall consider only the rotation of Earth around its axis which is about $\boldsymbol{\Omega} = 7.29 \times 10^{-5}$ rad/sec . As a result, since all vectors change under rotation the Newton's law of motion will change accordingly. The new version of the equation of motion will be derived and its consequences will be discussed. We shall give many examples from our daily life which change accordingly due to the rotation of Earth. To mention only one at this stage let us refer to rocket launching sites in our world. Cape Canaveral (Florida USA), French Guiana (in South America, for European spaces agency (ESA) also) and Baikonur (Kazakhistan) are all located nearer to the equators. The reason is to get extra advantage from the rotational velocity of the Earth. The choice of site causes an extra boost in velocity of the rocket up to 500 m/s, which amounts to saving fuel and money in the rocket launch process.

A simple pendulum processes on the rotating Earth, this may be used to prove, as it was done first by Foucault, that our Earth is rotating. The period of precession gives information about the location on the Earth. Depending on the parallel and meridian lines, i.e. latitude (colatitudes) angles, the precession period of a long pendulum changes and this may be used to identify any point on the Earth.

The missiles or long ranged artilleries can't reach their destination without taking into account the rotational effect of Earth. Global positioning system (GPS) also works feasible provided Earth's rotation is taken into consideration, computed and loaded to the data. For further accuracy let us remark that the curvature of Earth due to Einstein's general relativity must be added. It has been realized that the general relativistic effects even dominates over the special relativistic ones in the long range missile projectiles. In this project general relativistic contributions will be out of our scope but local effects of rotation are to be computed exactly. The true vertical/ weight of a projectile/ mass will be compared with the apparent one. Much of the physics that we are familiar in an inertial frame becomes modified in a non-inertial frame. Since we live on the surface of the Earth which rotates our frame automatically becomes a non-inertial frame. Fortunately Newton's laws of motion can easily be formulated in a rotating/inertial frame [6]. In our calculation we shall use the rotational effects to the first order only that is, $\omega^2 \approx 0$ will be adopted for the square of the angular velocity to keep the ω only to the first order. Two famous non-inertial forces, namely the Coriolis and centrifugal forces will be investigated. It should be added that these two forces are not real, they arise only in non-inertial frame. In physics a force is real if it has a physical source, such as mass, energy, charge, pressure etc. which are non-zero even in an inertial frame.

As we move away from the source all physically real sources have vanishing effect, whereas non-inertial forces increase unbounded. This is precisely the case for Coriolis and centrifugal forces. In a car turning around a corner the outward force we experience is the centrifugal force which arises due to the rotation of the car. As long as the car moves on a straight line no such force shows itself, for this reason this frame in a straight line is called an inertial frame in which the Newton’s law of inertia is trivially satisfied.

Similarly, the Coriolis force/ acceleration shows itself in many real life processes. Deflection of flying rockets, winds, ocean water and many other cases involve the imprints of this effect.

In this review project we shall investigate all these problems in a simple language/formalism that will provide a simple guide to those who want to know about the rotational effect of our Earth.

Chapter 2

INERTIAL AND NON INERTIOAL FRAMES

2.1 Introduction

In classical mechanics inertial frame is defined to be the frame in which Newton's laws take the simplest form. That is $\vec{F}$= m $\vec{a}$, or in Cartesian components $\vec{F}_x$= m$\vec{a}_x$, $\vec{F}_y$= m $\vec{a}_y$ and $\vec{F}_z$= m $\vec{a}_z$. When the frame is not inertial then Newton's laws will naturally be modified accordingly to take into account the effect of rotation. That means, new fictitious forces emerge [7-10]. Motion in physics is associated with the group structures of mathematics. The group of classical mechanics is known to be the Galilean group. Special relativistic group is the Lorentz/Poincare group. Poincare group is the translational addition of the Lorentz group. The number of independent parameters of the group indicate the physical degree of motion. For the Galilean group the parameter is the velocity vector $\vec{v}$ in which the time is absolute. In the Lorentz group the parameters are 6 namely $\vec{v}$ (the translational velocity) and $\vec{\omega}$ (the rotational velocity). Addition of the 4-translational degree of freedom $x^a \rightarrow x^a + \tau^a$ where (a= 1,2,3,4, 4 for the time component) and τ^a = constant yields the Poincare group with 10 parameters of degrees of freedom. In the simplest case we consider the Galilean and Lorentz transformation in 1-dimention.

In classical mechanics the most important type of transformation is the canonical transformation. This is a transformation that preserves area in phase space. That is, the area in the flow of the system is conserved. If we label the coordinate by p (the canonical momentum) and q (the canonical coordinate) the area is dq dp. Under a time independent canonical transformation $Q = Q(q,p)$ and $P = P(q,p)$ constancy of the area means that we have :

$$dq\, dp = dQ\, dP.$$

Let us note that the order of product also is important in this expression. It amounts to the fact that:

$$dQ\, dP = |J|\, dq\, dp$$

In which $|J|$ stands for the Jacobian of the transformation so that we must take $|J|=1$. For the details of the subject we refer to the book of Goldstein [8].

In this chapter we shall give a definition of a group, its Abelian/Non-Abelian properties. The full rotation group that our Earth experiences is SO(3), the special orthogonal group in 3-dimentions[5]. This group is Non-Abelian however, when we restrict ourselves entirely to the rotation about a fixed axis, which is a planer rotation, it satisfies an Abelian group of motion, The same property is valid also for the Lorentz group I.e. if we restrict ourselves to the 2 - dimensional (that means 1 space and 1 time) motion it becomes an Abelian group of motion. For completeness we shall review briefly these mathematical concepts.

2.2 Abelian and Non-Abelian Groups

What is a group?

Suppose G is a set of certain objects and a, b, c ϵ G, with a given operation, such as matrix multiplication.

Then if the following conditions are satisfied:

1) Closure condition:

$a, b \,\epsilon\, G \longrightarrow a.b \,\epsilon\, G$

2) associative relation:

(a.b).c = a.(b.c).

3) there exists a unit element in which:

I.a =a. I=a

4) for any a ϵ G, there exist a^{-1} in which

$a \, . \, a^{-1} = a^{-1} . \, a = I$ Then G is a Group.

Example: The set of counter clockwise coordinate relationship:

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}, \; R_1, R_2, R_3 \;\epsilon\; G$$

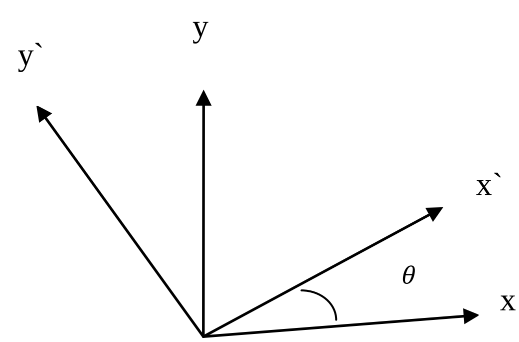

Figure 1: Two Coordinate Systems

$R_1, R_2 \in G$

$$R_1 = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix}, \qquad R_2 = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{bmatrix}$$

1) $R_1.R_2 = \begin{bmatrix} \cos\theta_1\cos\theta_2 - sin\theta_1 sin\theta_2 & \cos\theta_1\cos\theta_2 + sin\theta_1\cos\theta_2 \\ -(sin\theta_1\cos\theta_2 + cos\theta_1 sin\theta_1) & -sin\theta_1 sin\theta_2 + \cos\theta_1\cos\theta \end{bmatrix}$

$$R_1.R_2 = \begin{bmatrix} \cos(\theta_1+\theta_2) & \sin(\theta_1+\theta_2) \\ -(\sin\theta_1+\theta_{2)} & \cos(\theta_1+\theta_2) \end{bmatrix} \in G \text{ if } \theta = \theta_1 + \theta_2$$

2) $(R_1R_2)R_3 = R_1(R_2R_3)$

3) $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}, R = I \text{ if } \theta = 0$

4) $R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}, R^{-1}(\theta) = \begin{bmatrix} \cos(-\theta) & \sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{bmatrix}$

$$R\,R^{-1} = I$$

IF multiplication defined in the Group G is commutative (commute), means a,b $\in$ G, then a b = b a, so this Group is called an Abelian Group, If not, it is said to be Non-Abelian Group.

2.3 Galilean Transformation

If we have two inertial frames, one of them moving relative to the other which is stationary, we can transform between the two reference frames.

As shown in fig.(2):

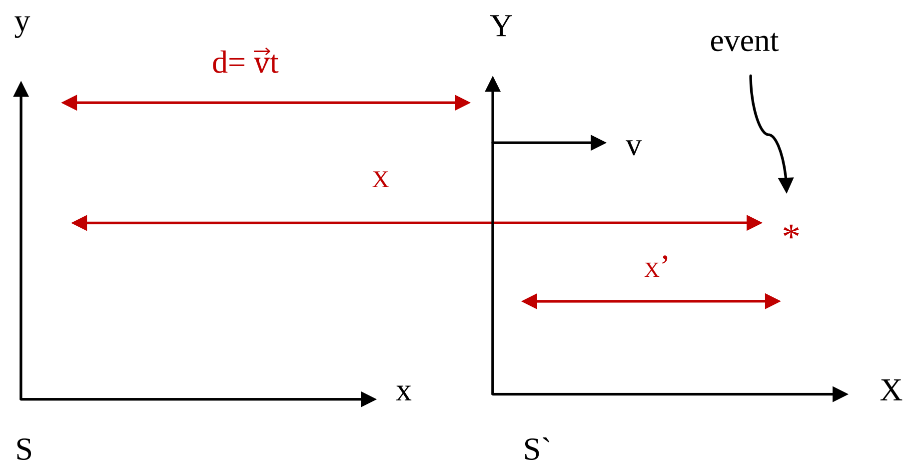

Figure 2: Galilean Transformation

We have two reference frames together at (t = 0), and we have an event as shown, and also we have a distance X according to (S) frame.

What is the position would S` measured from(S`) to event (X`) ?

As the time changes, S` is moving with some velocity, so it moves some distance (d) to right, such that:

$$d = \vec{v}\ t$$

So we say that:

$$X = \vec{v}\ t + X` \tag{2.1}$$

This is Galilean transformation for position,

To find X` :

$$X` = X - \vec{v}\ t \tag{2.2}$$

Since our study is one-dimensional, we have:

$\boxed{y = y`}$ , $\boxed{z = z`}$ $\boxed{t = t`}$ (2.3)

It means that the time that an event happens according to (S) frame, is equal to the time that an event happens according to (S`) frame.

Therefore,

$$X = v\,t + X` \longrightarrow X` = X - vt$$

$$y = y` \longrightarrow y` = y$$

$$z = z` \longrightarrow z` = z$$

$$t = t` \longrightarrow t` = t$$

This is Galilean transformation in the X- direction,

Note that the general Galilean transformation should read:

$$\vec{r'} = \vec{r} - \vec{v}t$$

$$t` = t \qquad (2.5)$$

Example: If an event happens at (X = 100 m), and (s`) travelling at (10 m/s) in (2)s then:

$$d = \vec{v}t$$

= (10 m/s) (2 s) = 20 m the distance between (s) and(s`)

So: $X` = X - \vec{v}t$

= 100-20 = 80 m the distance between (s`) and even.

So we convert between the two reference frames.

Example: Is the wave equation in 1- dimension:

$\frac{d^2\emptyset}{dx^2}$ - $\frac{1}{c^2}\frac{d^2\emptyset}{dt^2} = 0$, invariant under the Galilean transformation? Prove it.

Answer: No.

$$\frac{d^2\emptyset}{dx^2} - \frac{1}{c^2}\frac{d^2\emptyset}{dt^2} = 0$$

$$\frac{d}{dx} = \frac{dx'}{dx}\frac{d}{dx'} =$$

$$\frac{d}{dt} = \frac{dt'}{dt}\frac{d}{dt'} + \frac{dx'}{dt}\frac{d}{dx'}$$

$$\frac{d}{dt} = \frac{d}{dt'} + (-v)\frac{d}{dx'}$$

$$\frac{d^2\emptyset}{dx^2} - \frac{1}{c^2}\left(\frac{d}{dt'} - v\frac{d}{dx'}\right)\left(\frac{d\emptyset}{dt'} - v\frac{d\emptyset}{dx'}\right) = 0$$

$$\frac{d^2\emptyset}{dx'^2} - \frac{1}{c^2}\left(\frac{d^2\emptyset}{dt'^2} - 2v\frac{d^2\emptyset}{dt'dx'} + v^2\frac{d^2\emptyset}{dx'^2}\right) = 0$$

2.4 Galilean Transformation in Matrix Former

$$\begin{bmatrix} t' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} t_0 \\ x_0 \\ y_0 \\ z_0 \end{bmatrix} + L\begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix} \quad (2.6)$$

Where: $t_0, x_0, y_0, z_0 = 0$ at $t = 0$

L is a transformation operator (matrix)

$$L = \begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} \\ L_{21} & L_{22} & L_{23} & L_{24} \\ L_{31} & L_{32} & L_{33} & L_{34} \\ L_{41} & L_{42} & L_{43} & L_{44} \end{bmatrix}$$

There fore,

$$\begin{bmatrix} t' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} \\ L_{21} & L_{22} & L_{23} & L_{24} \\ L_{31} & L_{32} & L_{33} & L_{34} \\ L_{41} & L_{42} & L_{43} & L_{44} \end{bmatrix} \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix} \tag{2.7}$$

$t` = t$ if $L_{11} = 1,$ and L_{12} , L_{13} , $L_{14} = 0$

$X` = X - vt$ if $L_{21} = -u,\ L_{22} = 1,$ and L_{23} , $L_{24} = 0$

$y` = y$ if $L_{33} = 1,$ and L_{31} , L_{32} , $L_{34} = 0$

$z` = z$ if $L_{44} = 1,$ and L_{41} , L_{42} , $L_{43} = 0$

So the transform matrix for Galilean is:

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -v & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So:

$$\begin{bmatrix} t' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -v & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix} \tag{2.8}$$

$$\begin{bmatrix} t' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} t \\ -vt + u \\ y \\ z \end{bmatrix}$$ This is the Galilean transformation matrix. (2.9)

2.5 Lorentz Transformation in (1-1) Dimensions

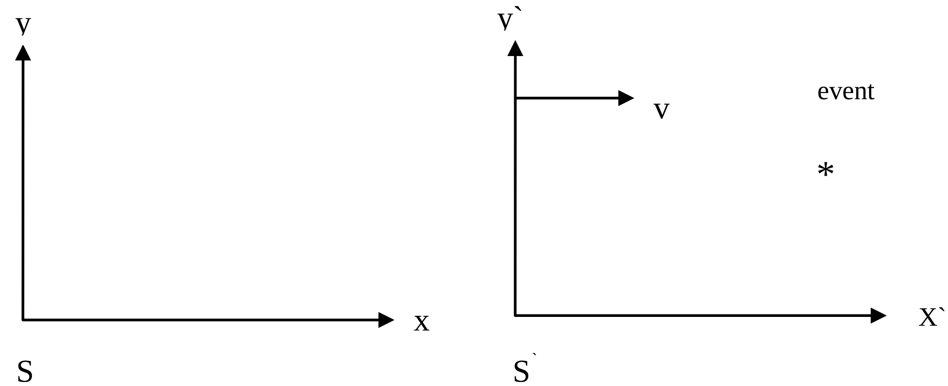

Figure 3: Lorentz Transformation

$$\begin{bmatrix} ct'' \\ x' \\ y' \\ z' \end{bmatrix} = L \begin{bmatrix} ct' \\ x \\ y \\ z \end{bmatrix} \tag{2.10}$$

L is a Lorentz transformation matrix (4×4).

Therefore:

$$\begin{bmatrix} ct'' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} \\ L_{21} & L_{22} & L_{23} & L_{24} \\ L_{31} & L_{32} & L_{33} & L_{34} \\ L_{41} & L_{42} & L_{43} & L_{44} \end{bmatrix} \begin{bmatrix} ct' \\ x \\ y \\ z \end{bmatrix} \tag{2.11}$$

$$ct' = L_{11}\, ct + L_{12} x + L_{13}\, y + L_{14}\, z$$

$$L_{11} = \gamma, \quad L_{12} = -\gamma \frac{v}{c^2}, \quad L_{13}, \ L_{14} = 0$$

$$t` = \gamma \left(t - \frac{v}{c^2} x\right) \tag{2.12}$$

Where { $\gamma = \frac{1}{\sqrt{1-(\frac{v}{C})^2}} = \frac{1}{\sqrt{1-\beta^2}}$ }

$x' = L_{21}\,ct + L_{22}x + L_{23}\,y + L_{24}\,z$

$L_{21} = -\gamma\,\frac{v}{c}, \quad L_{22} = \gamma, \quad L_{23} = L_{24} = 0$

$$x' = \gamma\,(x - vt) \tag{2.13}$$

$y' = L_{31}\,ct + L_{32}x + L_{33}\,y + L_{34}\,z$

$L_{33} = 1 \quad , \quad L_{32} = L_{31} = L_{34} = 0$

$$y' = y \tag{2.14}$$

$z' = L_{41}\,ct + L_{42}\,x + L_{43}\,y + L_{44}\,z$

$L_{44} = 1 \quad , \quad L_{42} = L_{43} = L_{41} = 0$

$$z' = z \tag{2.15}$$

So that Lorentz transformation matrix is:

$$L = \begin{bmatrix} \gamma & -\gamma\,\frac{v}{C} & 0 & 0 \\ -\gamma\,\frac{v}{C} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.16}$$

So:

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\frac{v}{c} & 0 & 0 \\ -\gamma\frac{v}{c} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct' \\ x \\ y \\ z \end{bmatrix} \tag{2.17}$$

Which is Lorentz transformation matrix.

Note that the wave equation,

$$\nabla^2\emptyset - \frac{1}{c^2}\frac{d^2\emptyset}{dt^2} = 0$$

Is invariant under the 1-dimensional Lorentz transformation.

2.6 Coordinate Systems in Rotating Frames

Let (XYZ) and (xyz) be two coordinate systems with the common origins (0),

The system (xyz) rotates w.r.t the system (X.Y.Z)

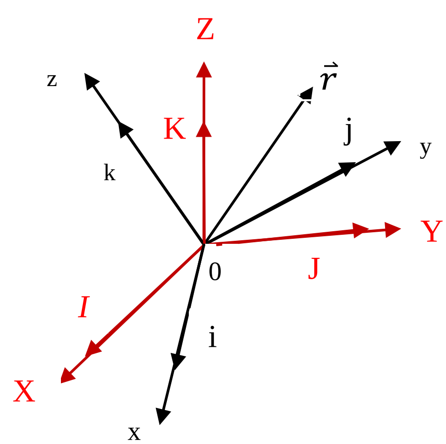

Figure 4: Coordinate System in Rotating Frames

At the same time, a vector ($\vec{r}$) which is changing during the time, to an inertial frame w.r.t rotating (x.y.z).

Now, what is the time rate of change of the vector $\vec{r}$ =x i + y j + z k?

$$\left(\frac{d\vec{r}}{dt}\right)_{mov.} = \frac{dx}{dt}i + \frac{dy}{dt}j + \frac{dz}{dt}k \tag{2.18}$$

And also the time rate of changing ($\vec{r}$) w.r.t the (XYZ) is:

$$\frac{d\vec{r}}{dt} = \frac{dx}{dt}i + \frac{dy}{dt}j + \frac{dz}{dt}k + x\frac{di}{dt} + y\frac{dj}{dt} + z\frac{dk}{dt}$$

This lead to:

$$\left(\frac{d\vec{r}}{dt}\right)_{Fix} = \left(\frac{d\vec{r}}{dt}\right)_{mov.} + x\frac{di}{dt} + y\frac{dj}{dt} + z\frac{dk}{dt} \tag{2.19}$$

i.i = 1,

Take derivative: $\frac{di}{dt}.i = 0$

Therefore: $$\frac{di}{dt} = a_1 j + a_2 k \tag{2.20}$$

j.j = 1,

Take derivative: $\frac{di}{dt}.j = 0$

Therefore: $$\frac{dj}{dt} = a_3 k + a_4 i \tag{2.21}$$

k.k = 1,

Take derivative: $\frac{dk}{dt}.k = 0$

Therefore: $\frac{dk}{dt} = a_5 i + a_6 j$ (2.22)

Which $a_1, a_2, a_3, a_4, a_5, a_6$ are constant, we should find them,

As $i.j = 0$

Then by derivative

$$\frac{di}{dt} .j + i. \frac{dj}{dt} = 0$$

$$a_1 + a_4 = 0$$

$$a_4 = -a_1 \tag{2.23}$$

As $j . k = 0$

Then take derivative,

$$\frac{dj}{dt} . k + j. \frac{dk}{dt} = 0$$

$$a_3 + a_6 = 0$$

$$a_6 = -a_3 \tag{2.24}$$

And finally as, $k . i = 0$

Then take derivative,

$$\frac{dk}{dt} . i + k . \frac{di}{dt} = 0$$

$$a_5 + a_2 = 0$$

$$a_5 = -a_2 \tag{2.25}$$

By substituting eqs.(2.21), (2.22) and (2.23) into eq. (2.19), we get:

$$\left(\frac{d\vec{r}}{dt}\right)_{Fix} = \left(\frac{d\vec{r}}{dt}\right)_{mov.} + x(a_1 j + a_2 k) + y(a_3 k + a_4 i) + z(a_5 i + a_6 j) \qquad (2.26)$$

But we have,

$$\left.\begin{aligned} a_4 &= -a_1 \\ a_5 &= -a_2 \\ a_6 &= -a_3 \end{aligned}\right\} \quad \text{put into eq. (2.26)}$$

$$\left(\frac{d\vec{r}}{dt}\right)_{Fix} = \left(\frac{d\vec{r}}{dt}\right)_{mov.} + x\,(a_1 j + a_2 k) + y\,(a_3 k - a_1 i) + z\,(-a_2 i - a_3 j)$$

$$\left(\frac{d\vec{r}}{dt}\right)_{Fix} = \left(\frac{d\vec{r}}{dt}\right)_{mov.} + a_1\, x\, j + a_2\, x\, k + a_3\, y\, k - a_1\, y\, i + -a_2\, z\, i - a_3\, z\, j$$

$$\left(\frac{d\vec{r}}{dt}\right)_{Fix.} = \left(\frac{d\vec{r}}{dt}\right)_{mov.} + (-a_1 y - a_2 z)i + (a_1 x - a_3 z)j + (a_2 x + a_3 y)k$$

$$\left(\frac{d\vec{r}}{dt}\right)_{Fix.} = \left(\frac{d\vec{r}}{dt}\right)_{mov.} + \begin{vmatrix} i & j & k \\ a_3 & -a_2 & a_1 \\ x & y & z \end{vmatrix}$$

$$\left(\frac{d\vec{r}}{dt}\right)_{Fix.} = \left(\frac{d\vec{r}}{dt}\right)_{mov.} + \vec{\omega} \times \vec{r} \qquad (2.27)$$

OR $\quad (\dot{r})_{Fix} = (\dot{r})_{mov.} + \vec{\omega} \times \vec{r}$

Where, $\quad \vec{\omega} = \omega_1\, i + \omega_2\, j + \omega_3\, k$

$$\vec{r} = x\, i + y\, j + z\, k$$

$\left(\frac{d\vec{r}}{dt}\right)_{Fix.}$ is the velocity of the vector ($\vec{r}$) w. r. t (XYZ), and it is also said to be (True velocity).

$$\left(\frac{d\vec{r}}{dt}\right)_{mov.} = \frac{dx}{dt}i + \frac{dy}{dt}j + \frac{dz}{dt}k \qquad (2.28)$$

Which is the velocity of a vector ($\vec{r}$) w. r. t (xyz),

and it is also said to be (apparent velocity).

From eq. (2.27) we can write this formula:

$$\boxed{\left(\frac{d}{dt}\right)_{Fix} = \left(\frac{d}{dt}\right)_{mov.} + \omega \times} \qquad (2.29)$$

We can also obtain the acceleration of the system:

$$(\ddot{r})_{Fix} = \left(\frac{d^2r}{dt^2}\right)_{Fix} = \left(\frac{d}{dt}\right)_{Fix} \left(\frac{dr}{dt}\right)_{Fix}$$

$$= \left[\left(\frac{d}{dt}\right)_{mov.} + \omega \times \right]\left[\left(\frac{dr}{dt}\right)_{mov.} + \omega \times r\right]$$

$$= \left(\frac{d}{dt}\right)_{mov.}\left[\left(\frac{dr}{dt}\right)_{mov.} + \omega \times r\right] + \omega \times \left[\left(\frac{dr}{dt}\right)_{mov.} + \omega \times r\right]$$

$$= \left(\frac{d^2r}{dt^2}\right) + \frac{d\omega}{dt} \times r + \omega \times \frac{dr}{dt} + \omega \times \frac{dr}{dt} + \omega \times (w \times r)$$

$$= \left(\frac{d^2r}{dt^2}\right) + \frac{d\omega}{dt} \times r + 2\omega \times \frac{dr}{dt} + \omega \times (\omega \times r)$$

$$(\ddot{r})_{Fix.} = (\ddot{r})_{Mov.} + \frac{d\omega}{dt} \times r + 2\omega \times \frac{dr}{dt} + \omega \times (\omega \times r)$$

Since $\omega = 2\pi/24h$ and exactly is (7.27×10^{-5} rad/s), which is a constant, therefore, we can say that :

$\frac{d\omega}{dt} = 0$, so we finally get:

$$(\ddot{r})_{Fix} = (\ddot{r})_{Mov.} + 2\omega \times \dot{r} + \omega \times (\omega \times r) \qquad (2.30)$$

$(\ddot{r})_{Fix}$ is the true acceleration.

$(\ddot{r})_{Mov.} = \ddot{x}\, i + \ddot{y}\, j + \ddot{z}\, k$ which is an apparent acceleration.

$+[\omega \times (\omega \times r)]$ is a centripetal acceleration.

From eq. (2.30) we can get:

$$(\ddot{r})_{mov.} = (\ddot{r})_{fix.} - 2\vec{\omega} \times \dot{r} - \vec{\omega} \times (\vec{\omega} \times \vec{r}) \qquad (2.31)$$

Which:

$-2\vec{\omega} \times \dot{r}$ is a Coriolis acceleration.

$-\vec{\omega} \times (\vec{\omega} \times \vec{r})$is a centrifugal acceleration.

We can multiply eq. (2.31) by the mass (m), we obtain:

$$m\,\ddot{r}_{mov.} = m\,\ddot{r}_{fix} - 2m\,(\vec{\omega} \times \vec{\dot{r}}) - m\,\vec{\omega} \times (\vec{\omega} \times \vec{r}) \qquad (2.32)$$

OR $\quad m\,\ddot{r}_{mov.} = m\,\ddot{r}_{fix} - F_{cor.} - F_{cen.}$

Which :

$F = m\,\ddot{r}_{mov.}$, is a factious force.

$F = m\,\ddot{r}_{Fix}$, is the force of the particle in the fixed system.

$F_{cor.} = -2m\,(\omega \times \dot{r})$, is a Coriolis force.

$F_{cent.} = -\,m\,([\,\omega \times (\omega \times r)\,])$, is a centrifugal force.

The following figures show the direction of a centrifugal force:

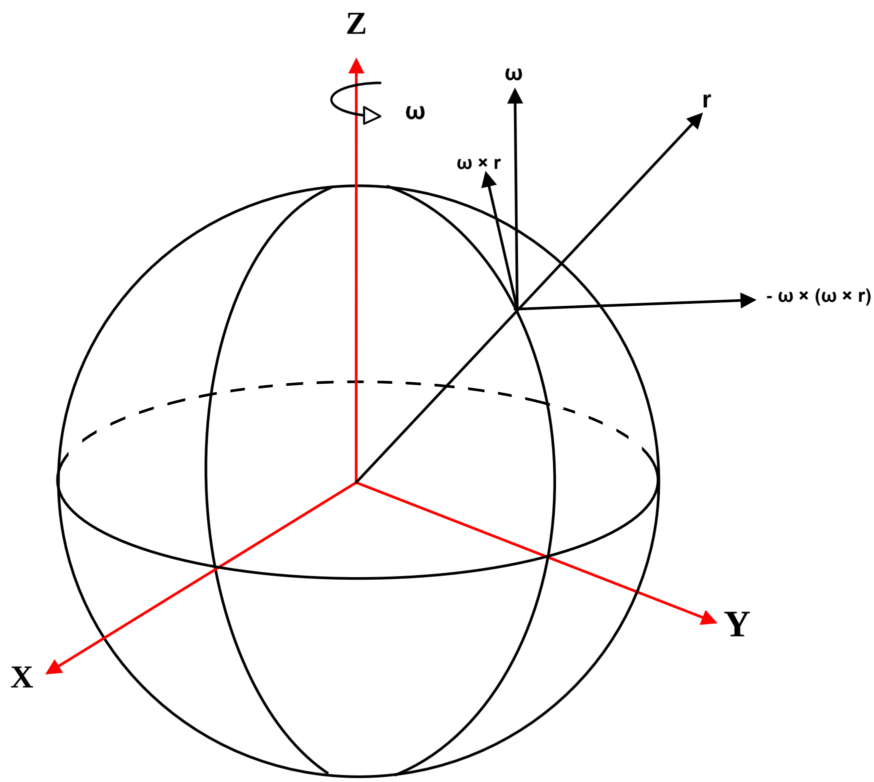

Figure 5: Direction of Centrifugal Force (1)

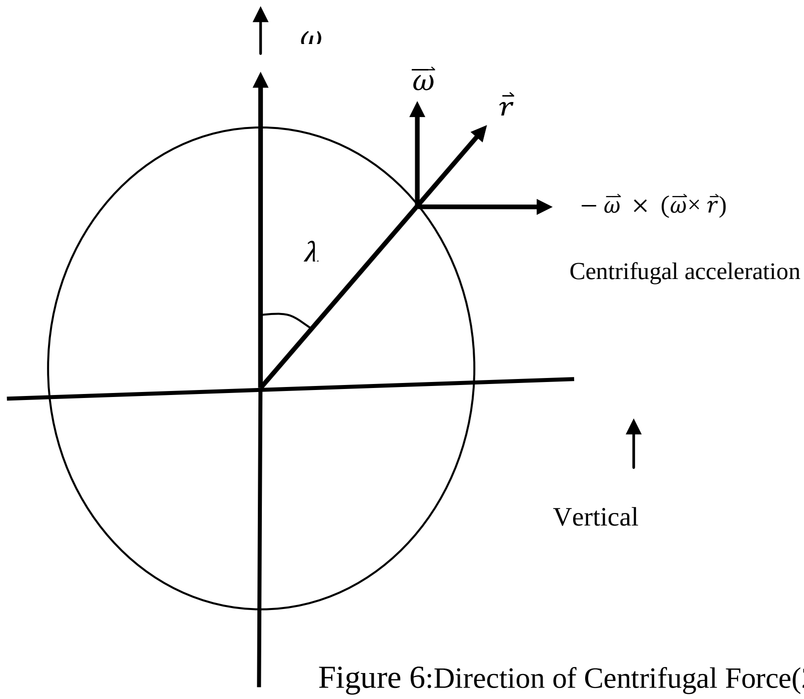

Figure 6:Direction of Centrifugal Force(2).

2.7 Moving Relative to Rotating Earth

Now, if we have two systems (as shows in the fig. (7)) with different origins, one moves w.r.t to another,

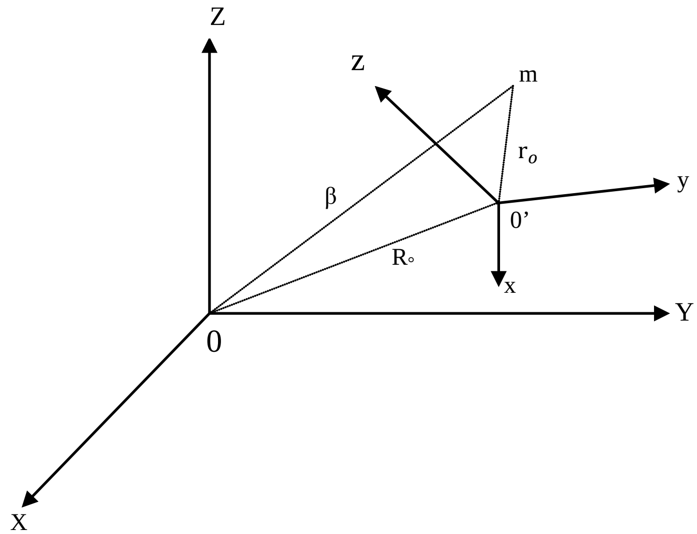

Figure 7: Two Coordinate System in Different Original

Let R be the distance of origin (0) to origin (0\`), The velocity of (m) particle relative to moving system is:

$$(\frac{dr_\circ}{dt})_{mov.} = \dot{r}_{\circ\ mov.} = \dot{x}_\circ\, i + \dot{y}_\circ\, j + \dot{z}_\circ\, k \qquad (2.33)$$

Now if the distance between the particle (m) and the origin (0) is $\beta = R_\circ + r_\circ$, then its velocity w.r.t (XYZ) system will be:

$$(\frac{d\beta}{dt}) = \frac{d}{dt}(R_\circ + r_\circ)_{Fix} \qquad (2.34)$$

$$= (\frac{dR_\circ}{dt})_{Fix} + (\frac{dr_\circ}{dt})_{Fix} \qquad (2.35)$$

$$\frac{d\beta}{dt} = \dot{R}_\circ + \dot{r}_{\circ\ mov}. + \vec{\omega} \times \vec{r}_\circ \qquad (2.36)$$

Which $\dot{R}_\circ$ is the velocity of $(0')$with respect to (0).

If $R_\circ = 0$, this will be the same as eq. (2.27).

$\frac{d\beta}{dt}$ is the particle velocity relative to the Earth's Rotation.

Now let us find the acceleration of the particle (m) with respect to rotating Earth:

The acceleration of the particle (m) relative to (o`) system is:

$$(\frac{d^2r_\circ}{dt^2})_{mov.} = (\ddot{r}_\circ)_{mov.} = \ddot{x}_\circ\, i + \ddot{y}_\circ\, j + \ddot{z}_\circ\, k \qquad (2.37)$$

Since the distance of the particle (m) relative to (0) is $\beta = R_\circ + r_\circ$, then the acceleration of (m)in the fixed system is:

$$(\frac{d^2\beta}{dt^2})_{Fix.} = \frac{d^2}{dt^2}(R_\circ + r_\circ)_{Fix.} = (\frac{d^2R_\circ}{dt^2})_{Fix.} + (\frac{d^2r_\circ}{dt^2})_{Fix.} \qquad (2.38)$$

$$\left(\frac{d^2\beta}{dt^2}\right)_{Fix} = \ddot{R}_\circ + \ddot{r}_{\circ mov} + \dot{\omega} \times r_\circ + 2\,\omega \times \dot{r}_\circ + \omega \times (\omega \times r_\circ) \tag{2.39}$$

2.8 Determine the Equation of Motion of a Particle Moving Near to Earth's Surface

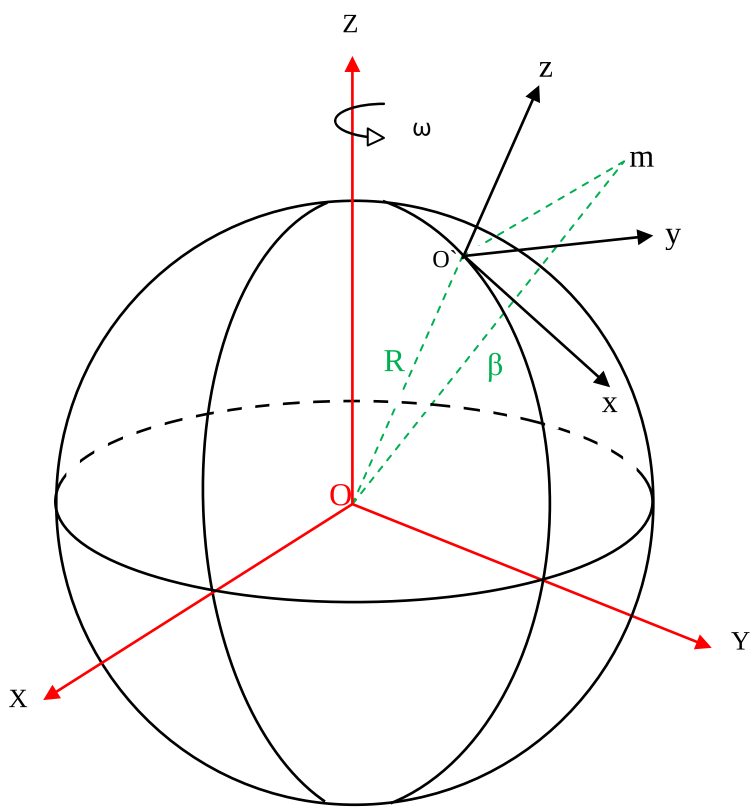

Figure 8: Moving of the Particle w.r.t Two Coordinates

Recall eq. (2.39), under these facts:

$\dot{\omega}=0$, because ω is constant.

$\ddot{R}$, we can neglect it for simplicity.

$a_F = \ddot{\beta} - \omega \times (\omega \times r)$, combine both of them as Gravity of Earth.

Thus $a_F = -g$

Therefore eq. (2.39) becomes:

$$\vec{a}_{(mov)} = -g - 2\,(\vec{\omega} \times \dot{r}) \tag{2.40}$$

$$\vec{\omega} = \omega\hat{k}$$

$$\hat{k} = (\hat{k}\,.\,\hat{\imath})\, i + (\hat{k}\,.\,\hat{\jmath})\, j + (\hat{k}\,.\,\hat{k})\, k$$

$$K\,.\,i = -\sin\lambda$$

$$K\,.\,j = 0$$

$$K\,.\,k = \cos\lambda$$

$$\hat{k} = -\sin\lambda\,\hat{\imath} + \cos\lambda\,\hat{k}$$

$$\omega\hat{k} = -\omega\,\sin\lambda\,\hat{\imath} + \omega\cos\lambda\,\hat{k}$$

$$\vec{\dot{r}}_{(mov.)} = (\dot{x}, \dot{y}, \dot{z})$$

$$\vec{\omega} \times \vec{\dot{r}} = \begin{vmatrix} (+) & (-) & (+) \\ i & j & k \\ -\omega\sin\lambda & 0 & \omega\cos\lambda \\ \dot{x} & \dot{y} & \dot{z} \end{vmatrix} \tag{2.41}$$

$$a_{mov.} = -g - 2\begin{vmatrix} (+) & (-) & (+) \\ i & j & k \\ -\omega\sin\lambda & 0 & \omega\cos\lambda \\ \dot{x} & \dot{y} & \dot{z} \end{vmatrix} \tag{2.42}$$

Put $a_{mov.} = (\ddot{x}, \ddot{y}, \ddot{z})$ and $a_{Fix} = -g$

Therefore

$$\ddot{x} = 2\,\omega\,\dot{y}\,\cos\lambda$$

$$\ddot{y} = 2\,\omega\,(\,\dot{x}\cos\lambda + \dot{z}\sin\lambda) \qquad (2.43)$$

$$\ddot{z} = -\,g + 2\,\omega\,\dot{y}\sin\lambda$$

This net of equations is the summary of physics on the rotating Earth as the example in the next chapter will show.

Chapter 3

APPLICATIONS

3.1 Projectile in General

Example: A projectile is launched at angle λ with arbitrary initial conditions. We wish to determine the rest of the motion in accordance with the equations of motion on the rotating Earth?

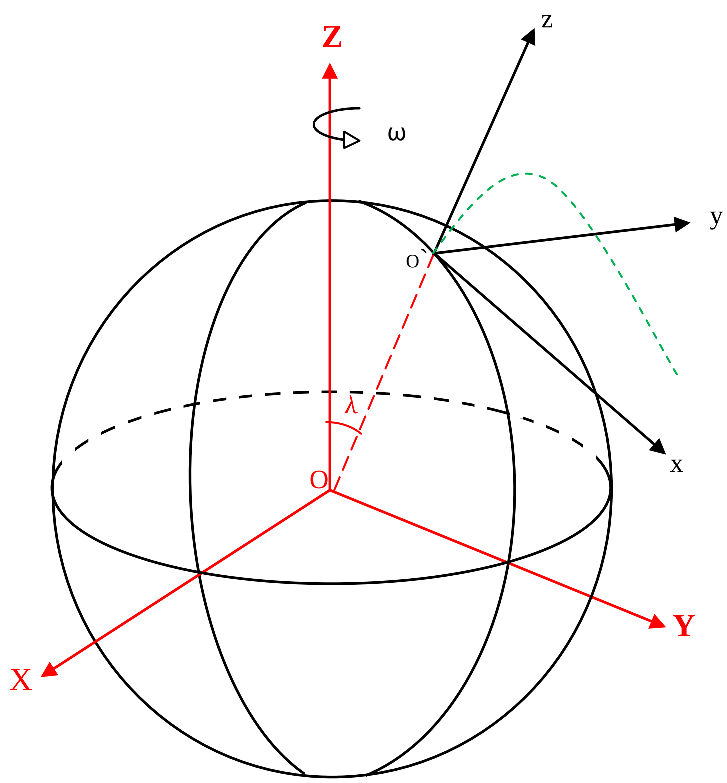

Figure 9: Projectile Motion

At rest we have:

V_{0x} , V_{0y} and V_{0z} ,

Recall eqs. (2.43),

$\ddot{x} = 2\,\omega \cos\lambda\ \dot{y}$

$\ddot{y} = -2\ \omega\,(\dot{x}\cos\lambda + \ \dot{z}\sin\lambda)$

$\ddot{z}\ = -\,g + 2\,\omega\,\dot{y}\sin\lambda$

By integrating both $\ddot{x}$ and $\ddot{z}$, we get:

$$\dot{x} = 2\,\omega \cos\lambda\ y + V_{0x} \qquad (3.1)$$

$$\dot{z}\ = -\,g\,t + 2\,\omega y \sin\lambda + V_{0z} \qquad (3.2)$$

Substitute $\dot{x}$, $\dot{z}$ in $\ddot{y}$, we get:

$$\ddot{y} = -2\,\omega[\ (2\,w \cos\lambda\, y + V_{0x}) \cos\lambda + (-\,g\,t + 2\,\omega y \sin\lambda + V_{0z}) \sin\lambda$$

$$= -2\,\omega[\ (2\,\omega \cos^2\lambda\, y + V_{0x} \cos\lambda) + (-\,g\,t \sin\lambda + 2\,\omega \sin^2\lambda + V_{0z} \sin\lambda)\ ]$$

$$= -4\,\omega^2\cos^2\lambda\, y - 2\,\omega\, V_{0x} \cos\lambda\ + 2\omega\, g\, t \sin\lambda - 4\,\omega^2 \sin^2\lambda - 2\,\omega\, V_{0z} \sin\lambda$$

$$= -2\,\omega\, V_{0x} \cos\lambda\ + 2\omega\, g\, t \sin\lambda - 2\,\omega\, V_{0z} \sin\lambda$$

$$\ddot{y} = -2\,\omega\,(\ V_{0x} \cos\lambda - g\,t \sin\lambda + V_{0z} \sin\lambda) \qquad (3.3)$$

By integrating $\ddot{y}$ we get:

$\dot{y} = -2\,\omega\,(\,V_{0x}\cos\lambda\; t - \frac{1}{2}\,g\,t^2\,\sin\lambda + V_{0z}\sin\lambda\; t) + C_1$

If $t = 0$, then $C_1 = V_{0y}$

By integrating $\dot{y}$ we get,

$$y = -2\,\omega\,(\frac{1}{2}\,g\,t^2\,V_{0x}\cos\lambda - \frac{1}{6}\,g\,t^3\,\sin\lambda + \frac{1}{2}\,t^2\,V_{0z}\sin\lambda\,) + V_{0y}\,t + C_2 \qquad (3.5)$$

If $t = 0$, then $C_2 = 0$

$y = \frac{1}{3}\,\omega\,g\,t^3\,\sin\lambda - \omega\,t^2\,V_{0x}\cos\lambda - \omega\,t^2\,V_{0z}\sin\lambda + V_{0y}\,t$

$$y = t\,[\,V_{0y} + \frac{\omega}{3}\,g\,t^2\,\sin\lambda - t\,\omega\,(\,V_{0x}\cos\lambda + V_{0z}\sin\lambda\,)\,] \qquad (3.6)$$

If $\omega = 0$, then: $\quad y = V_{0y}\,t \qquad (3.9)$

Recall eq. (3.2),

$\dot{z} = -\,g\,t + 2\,\omega\,\sin\lambda\;\; y + V_{0z}$

Put eq. (3.6) in (3.2), we get:

$\dot{z} = -\,g\,t + V_{0z} + 2\,\omega\,\sin\lambda\,.\,t\,[\,V_{0y} + \frac{\omega}{3}\,g\,t^2\,\sin\lambda - t\,\omega\,(V_{0x}\cos\lambda + V_{0z}\sin\lambda\,)\,]$

$\dot{z} = -\,g\,t + V_{0z} + 2\,\omega\,V_{0y}\sin\lambda\,.\,t + \frac{2}{3}\,\omega^2\,g\,\sin^2\lambda\,t^3 - 2\,\omega^2\,\sin\lambda\,t^2\,(V_{0x}\cos\lambda + V_{0z}\,\sin\lambda\,)$

Put $\omega^2 = 0$, we get:

$$\dot{z} = -\,g\,t + V_{0z} + 2\,\omega\,V_{0y}\sin\lambda\,.\,t \qquad (3.7)$$

By integrating $\dot{z}$, we get:

$$Z = V_{0z}\, t - \frac{g}{2} t^2 + \omega\, V_{0y} \sin\lambda\, t^2 + C_3$$

If $t = 0$, then $C_3 = 0$,

$$z = t\,[\, V_{0z} - \frac{g}{2} t^2 + \omega\, V_{0y} \sin\lambda \,.\, t\,] \tag{3.8}$$

If $\omega = 0$, then: $$z = V_{0z} - \frac{g}{2} \,.\, t^2 \tag{3.9}$$

Recall eq. (3.1),

$$\dot{x} = 2\,\omega \cos\lambda\; y + V_{0x}$$

Substitute eq. (3.6) in (3.1), we get:

$$\dot{x} = 2\,\omega \cos\lambda \,.t\, [V_{0y} + \frac{\omega}{3} g\, t^2 \sin\lambda - t\,\omega\, (V_{0x} \cos\lambda + V_{0z} \sin\lambda)\,] + V_{0x}$$

$$\dot{x} = 2\,\omega\, V_{0y} \cos\lambda .t + \frac{2}{3} \omega^2 g\, t^3 \sin\lambda \cos\lambda - 2\,\omega^2 t^2 \cos\lambda\, (V_{0x} \cos\lambda + V_{0z} \sin\lambda\,) + V_{0x}$$

Put $\omega^2 = 0$, we get:

$$\dot{x} = 2\,\omega\, V_{0y} \cos\lambda\; t + V_{0x} \tag{3.10}$$

By integrating $\dot{x}$, we get:

$$x = V_{0x}\, t + \omega\, V_{0y} \cos\lambda\, t^2 + C_4$$

İf $t = 0$, then $C_4 = 0$, then:

$$x = t\,(\, V_{0x} + \omega\, V_{0y} \cos\lambda\; t\,) \tag{3.11}$$

İf $\omega = 0$, then: $$x = V_{0x}\, t \tag{3.12}$$

To find (Z_{max}) let $\frac{dz}{dt} = 0$ OR $\dot{z}=0$

Recall eq. (3.7),

$$\dot{z} = -gt + V_{0z} + 2\omega V_{0y} \sin\lambda \,.\, t$$

Put $\dot{z} = 0$,

$$0 = -gt + V_{0z} + 2\omega V_{0y} \sin\lambda \,.\, t$$

$$t_{max} = \frac{V0z}{g - 2\omega V0y \sin\lambda} \longrightarrow t_{max} = \frac{V0z}{g}[1 - \frac{2\omega V0y}{g} \sin\lambda]^{-1}$$

$$t_{max} \approx \frac{V0z}{g}(1 + \frac{2\omega V0y}{g} \sin\lambda) \tag{3.13}$$

put (3.13) into (3.8),

$$z = t[\ V_{0z} + (\omega V_{0y} \sin\lambda - \frac{g}{2})\, t\ ]$$

$$z = \frac{V0z}{g}(1 + \frac{2\omega V0y}{g} \sin\lambda)[\ V_{0z} + \underbrace{(\omega V_{0y} \sin\lambda - \frac{g}{2})\ (\frac{V0z}{g}\ (1 + \frac{2\omega V0y}{g} \sin\lambda))}_{I_1}\ ] \tag{3.14}$$

$$I_1 = [\ V_{0z} + (\omega V_{0y} \sin\lambda - \frac{g}{2})(\frac{V0z}{g}(1 + \frac{2\omega V0y}{g} \sin\lambda))\ ]$$

$$= V_{0z} + \frac{V0z}{g}(\ \omega V_{0y} \sin\lambda + \frac{2\omega^2 v^2 0y}{g} \sin^2\lambda - \frac{g}{2} - \omega V_{0y} \sin\lambda\ )$$

Put $\omega^2 = 0$,

$$= V_{0z} + \frac{V0z}{g}\ (\ \omega V_{0y} \sin\lambda - \frac{g}{2} - \omega V_{0y} \sin\lambda\ )$$

$$= V_{0z} + \frac{V0z}{g}\left(-\frac{g}{2}\right)$$

$$= V_{0z} - \frac{V0z}{2}$$

$$I_1 = \frac{V0z}{2} \quad \text{put in to (3.4)}$$

$$Z_{max} \approx \frac{V0z}{g}\left(1 + \frac{2\omega\, V0y}{g}\sin\lambda\right)\left[\frac{V0z}{2}\right]$$

$$Z_{max} \approx \frac{v^2 0z}{2g}\left(1 + 2\omega \frac{V0y}{g}\sin\lambda\right) \tag{3.15}$$

From eq. (3.8),

$$Z = t\left[V_{0z} - \frac{g}{2}t + \omega V_{0y}\sin\lambda\ .t\right]$$

For Z=0,

$$0 = t\left[V_{0z} - \frac{g}{2}t + \omega V_{0y}\sin\lambda\ .t\right]$$

$$V_{0z} = t\left[\frac{g}{2} - \omega V_{0y}\sin\lambda\right] \tag{3.16}$$

$$2 V_{0z} = t\left[g - 2\omega V_{0y}\sin\lambda\right]$$

$$t = \frac{2V0z}{g - 2\omega\, V0y\, \sin\lambda}$$

$$t = \frac{2V0z}{g}\left(1 - 2\omega \frac{V0y}{g}\sin\lambda\right)^{-1}$$

$$t_{flight} \approx \frac{2V0z}{g}\left(1 + 2\omega \frac{V0y}{g}\sin\lambda\right) \tag{3.17}$$

Put eq. (3.17) into eq. (3.11) to find X_{max} ,

$$X_{max} = t_{flight} [V_{0x} + \omega V_{0y} \cos\lambda . t_{flight}]$$

$$X_{max} = \frac{2V0z}{g} (1 + 2\omega \frac{V0y}{g} \sin\lambda) [V_{0x} + \omega V_{0y} \cos\lambda . \frac{2V0z}{g} \left(1 + 2\omega \frac{V0y}{g} \sin\lambda \right)]$$

$$X_{max} = \frac{2V0z}{g} (1 + 2\omega \frac{V0y}{g} \sin\lambda) [V_{0x} + 2\omega. \frac{V0y\ V0z}{g} \cos\lambda]+ 4 \omega 2 \frac{v^2 0y\ V0z}{g^2} \sin\lambda \cos\lambda]$$

Put $\omega^2 = 0$,

$$= \frac{2\ V0y}{g} (1 + 2\omega \frac{V0y}{g} \sin\lambda) [V_{0x} +2 \omega \frac{V0y\ V0z}{g} \cos\lambda]$$

$$= \frac{2V0y}{g} [V_{0x} +2\omega \frac{V0y\ V0z}{g} \cos\lambda + 2\omega \frac{V0y\ V0z}{g} \sin\lambda] + 4 \omega^2 \frac{v^2 0y\ Vz}{g^2} \sin\lambda \cos\lambda]$$

Put $\omega^2 = 0$,

$$= \frac{2\ V0y}{g} [V_{0x} + 2\omega \frac{V0y\ V0z}{g} \cos\lambda + 2\omega \frac{V0y\ V0z}{g} \sin\lambda]$$

$$X_{max} = \frac{2\ V0z}{g} [V_{0x} + 2 \omega \frac{V0y}{g} (V_{0x} \sin\lambda + v_{0z} \cos\lambda)] \tag{3.18}$$

To find (y_{max}) , use (t_{flight}) in eq (3.6):

$$y = t [V_{0y} + \frac{\omega}{3} g t^2 \sin\lambda - t \omega (v_{0x} \cos\lambda + V_{0z} \sin\lambda)]$$

$$y = V_{0y} t_f + \frac{\omega}{3} g t^3_{flight} \sin\lambda - t^2_{flight} \omega (v_{0x} \cos\lambda +V_{0z} \sin\lambda)] \tag{3.19}$$

$$t_f = t_{flight} = \frac{2\ V0z}{g} (1+2\omega \frac{V0y}{g} \sin\lambda) \tag{3.20}$$

$$t^2 = t \,.\, t$$

$$=\left(\frac{2V0z}{g}\right)\left(1 + 2\,\omega \frac{V0y}{g}\sin\lambda\right)\left(\frac{2\,V0z}{g}\right)\left(1 + 2\,\omega\frac{V0y}{g}\sin\lambda\right)$$

$$= \frac{4\,V0z^2}{g^2}\left(1 + \frac{2\,\omega\,V0y}{g}\sin\lambda\right)^2$$

$$= \frac{4V^2 0z}{g^2}\left(1 + \frac{4\,\omega\,V0y}{g}\sin\lambda + \frac{4\omega^2 v^2 0y}{g^2}\sin^2\lambda\right)$$

put $\omega^2 = 0$,

$$t^2{}_{flight} = \frac{4\,V0z}{g^2}\left(1 + \frac{4\,\omega\,V0y}{g}\sin\lambda\right) \tag{3.21}$$

$$t^3 = t \,.\, t^2$$

$$=\left(\frac{2V0z}{g}\right)\left(1{+}2\,\omega\frac{V0y}{g}\sin\lambda\right)\left(\frac{4\,v^2 0z}{g^2}\right)\left(1 + \frac{4\omega\,V0y}{g}\sin\lambda\right)$$

$$= \frac{8\,v^3 0z}{g^3}\left(1 + \frac{2\omega\,V0y}{g}\sin\lambda\right)\left(1 + \frac{2\,\omega\,V0y}{g}\sin\lambda\right)$$

$$= \frac{8\,v^3 0z}{g^3}\left(1 + \frac{4\,\omega\,V0y}{g}\sin\lambda + \frac{2\,\omega\,V0y}{g}\sin\lambda + \frac{8\,\omega^2\,v^2 0y}{g^2}\sin^2\lambda\right)$$

Put $\omega^2 = 0$,

$$= \frac{8\,v^3 0z}{g^3}\left(1 + \frac{4\,\omega\,V0y}{g}\sin\lambda + \frac{2\,\omega\,V0y}{g}\sin\lambda\right)$$

$$t^3 = \frac{8\,v^3 0z}{g^3}\left(1 + \frac{6\,\omega\,V0y}{g}\sin\lambda\right) \tag{3.22}$$

Substitute eqs. (3.20), (3.21) and (3.22) into eq. (3.19), we get:

$$y_{max} \approx \frac{2\,V0z\,V0y}{g}\left(1+\frac{2\omega\,V0y}{g}\,\sin\lambda\right)+\frac{\omega}{3}\,g\sin\lambda\,.\frac{8\,v^3 0z}{g^3}\,\left(1+\frac{6\,\omega\,V0y}{g}\,\sin\lambda\right)-\omega\,(V_{0x}\,\cos\lambda +$$

$$V_{0z}\,\sin\lambda).\frac{4\,v^2 0z}{g^2}\left(1+\frac{4\,\omega\,V0y}{g}\,\sin\lambda\right)$$

$$=\frac{2\,V0z\,V0y}{g}\left(1+\frac{2\,\omega\,V0y}{g}\,\sin\lambda\right)+\frac{\omega g}{3}\,\sin\lambda\,.\frac{8\,V0z^3}{g^3}\,+\frac{16\,\omega^2}{g^3}\,\sin^2\lambda\,v^3_{0z}v_{0y}-\omega\,(V_{0x}$$

$$\cos\lambda + V_{0z}\,\sin\lambda).\frac{4\,v^2 0z}{g^2}-\frac{4\,\omega^2\,V0y}{g}\,\sin\lambda\,(V_{0x}\,\cos\lambda + V_{0z}\,\sin\lambda).\frac{4\,v^2 0z}{g^2}$$

Put $\omega^2 = 0$,

$$y_{max} \approx \frac{2\,V0z\,V0y}{g}\left(1+\frac{2\omega\,V0y}{g}\,\sin\lambda\right)+\frac{8}{3}\,v_{0z}^3\,\omega\,\frac{\sin\lambda}{g^2}-\frac{4\,v^2 0z}{g^2}\,\omega\,(V_{0x}\,\cos\lambda + V0z\,\sin\lambda)$$

$$=\frac{2\,V0y\,V0z}{g}+\frac{2\,V^2 0y\,\,V0z}{g^2}\,\omega\,\sin\lambda+\frac{8}{3}\,v_{0z}^3\,\,\frac{w}{g}\,\sin\lambda-\frac{4v^2 0z}{g^2}\,\omega\,(V_{0x}\,\cos\lambda + V_{0z}\,\sin\lambda)$$

$$y_{max} \approx \frac{2V0z\,V0y}{g}+\frac{4\omega\,V0z}{g^2}\left[v_{0y}^2\,\sin\lambda+\frac{2}{3}v_{0z}^2\,\,\sin\lambda - v_{0z}(v_{0x}\,\cos\lambda + v_{0z}\,\sin\lambda)\right.$$

$$y_{max} \approx \frac{2V0z\,V0y}{g}+\frac{4\omega\,V0z}{g^2}\left[\,v_{0y}^2\,\sin\lambda\,-\,\frac{1}{3}\,v_{0z}^2\,\sin\lambda - v_{0x}\,\,v_{0z}\,\cos\lambda\,\right] \qquad (3.23)$$

Special Case:

$V_{0x} = 10$ m/s $= V_0 \cos\alpha$

$V_{0y} = 10$ m/s $= V_0 \sin\alpha$

$V_{0x} = V_{0y} = V_{0z} = 10$ m/s

g =10 m/s2

let $\lambda = 30\,^o$

$\omega = 7.29 \times 10^{-5}$ rad/s

$$X_{max} = \frac{2V0z}{g} \left[V_{0x} + 2\,\omega \frac{V0y}{g} (V_{0x} \sin\lambda + v_{0z} \cos\lambda) \right]$$

$$y_{max} = \frac{2V0z\,V0y}{g} + \frac{4\omega V0z}{g^2} \left[v_{0y}^2 \sin\lambda - \frac{1}{3} v_{0z}^2 \sin\lambda - V_{0x} V_{0z} \cos\lambda \right]$$

$$Z_{max} = \frac{V^2 0z}{2g} \left(1 + 2\omega \frac{V0y}{g} \sin\lambda \right)$$

$$t_{flight} = \frac{2V^2 0z}{2g} \left(1 + 2\,\omega \frac{V0y}{g} \sin\lambda \right)$$

$$t_{max} = \frac{V0z}{g} \left(1 + \frac{2\omega\, V0y}{g} \sin\lambda \right)$$

Let us find the maximum distance in x-direction,

$$X_{max} = \frac{2\,V0z}{g} \left[V_{0x} + 2\,\omega \frac{V0y}{g} \left(V_{0x} \sin\lambda + v_{0z} \cos\lambda \right) \right]$$

$$=2\ \frac{10}{10} \left[10 + 2\,(7.29 \times 10^{-5})\ \frac{10}{10}\ (10 \sin 30 + 10 \cos 30) \right]$$

$$=2\,(10)\,[\,1 + 2\ (7.29 \times 10^{-5})\,(0.5 + .87)\,]$$

$$=20\,[\,1 + 2\ (7.29 \times 10^{-5})\,(1.37)\,]$$

$$=20\,(1 + 20 \times 10^{-5}) \quad = \ 20 + 400 \times 10^{-5}$$

$$=20 + 4 \times 10^{-3}$$

$$X_{max} = 20.004 \text{ m}$$

We show that due to the rotation of the Earth there is a deflection about (0.004) m for each (20) m.

Let us find the maximum distance in y-direction,

$y_{max} = \frac{2V0z\ V0y}{g} + \frac{4\ \omega\ V0z}{g^2}\ [\ v_{0y}^2 \sin\lambda\ -\ \frac{1}{3}\ v_{0z}^2 \sin\lambda - V_{0x}\ V_{0z}\ \cos\lambda\ ]$

$= \frac{2(10^2)}{10} + \frac{4(10)}{10^2}\ (\ 7.29 \times 10^{-5}\ )[\ 10^2 \sin 30\ -\ \frac{1}{3} 10^2 \sin 30 - (10^2)\ \cos 30\ ]$

$= 2(10) + 4\ (10)\ (7.29 \times 10^{-5})\ [\ \sin 30\ -\ \frac{1}{3}\ \sin 30 - \cos 30\ ]$

$= 2(10) + 4\ (10)\ (7.29 \times 10^{-5})[\ 0.5\ -\ \frac{1}{3}\ (0.5) - 0.87\ ]$

$= 2(10) + 4\ (10)\ (7.29 \times 10^{-5})\ [\ 0.5\ -\ 1.037\ ]$

$= 2(10) + 4(10)\ (7.29 \times 10^{-5})[-0.537\ ]$

$= 20 + (-157.6 \times 10^{-5}) = 19.998$ m.

Now, what is the deflection in z-direction?

$Z_{max.} = \frac{V^2 0z}{2g}\ (\ 1 + 2\omega\ \frac{V0y}{g}\ \sin\lambda\ )$

$= \frac{(10^2)}{2(10)}\ (1 + 2\ (\ 7.29 * 10^{-5}\ )\left(\frac{10}{10}\right) (\sin 30)$

$= 5\ [1 + 2\ (7.29 * 10^{-5})(\ 0.5)$

$= 5\ [1 + 7.29 * 10^{-5}]$

$= 5[1 + 0.0000729]$

$= 5 + 0.00036 = 5.00036$ m.

3.2 Apparent Weight (w`)

Example: At Colatitude angle λ, let us find the apparent weight (w`) of an object of mass m?

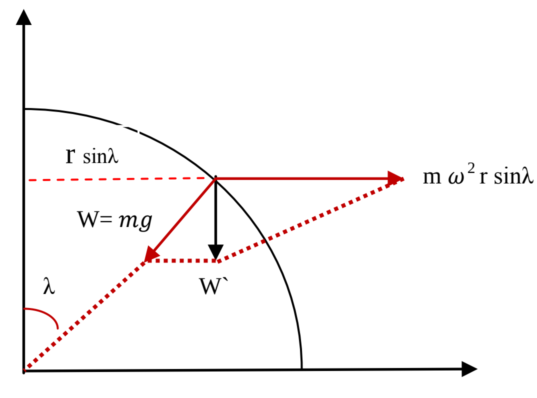

Figure 10: Colatitudes Angle

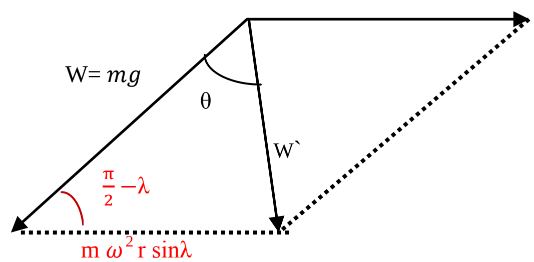

Figure 11: Apparent Weight

By the law of cosines,

$$(w`)^2 = (m\,g)^2 + (m\,\omega^2\, r \sin\lambda)^2 - 2m\,g \sin\lambda \,.\, (m\,\omega^2\, r \sin\lambda) \tag{3.24}$$

$$w` = m\sqrt{g^2 + r\omega^2 \sin^2\lambda\ (\omega^2 r - 2\,g)} \tag{3.25}$$

For $\lambda = 0, \pi$ in North and South poles

$$w` = m\,g = w \tag{3.26}$$

For $\lambda = \frac{\pi}{2}$ in the Equator

$$w` = m\sqrt{g^2 + r\,\omega^2(\omega^2 r - 2\,g)} = m\,(g - r\,\omega^2) \tag{3.27}$$

3.3 True and Apparent Vertical

Example: Find tanθ, if θ Is the angle between the true and apparent vertical?

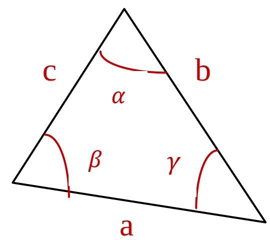

Figure 12: Triangle

From the law of sines:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \tag{3.28}$$

By using figure (11):

$$\frac{w'}{\sin(\frac{\pi}{2}-\lambda)} = \frac{m\,\omega^2\, r\sin\lambda}{\sin\theta} \tag{3.29}$$

But $\sin\left(\frac{\pi}{2}-\lambda\right) = \cos\lambda$

$$\longrightarrow \quad \sin\theta = \frac{m\,\omega^2\, r\sin\lambda\cos\lambda}{w'} \tag{3.30}$$

$$\tan\theta = \frac{\sin\theta}{\sqrt{1-\sin\theta^2}}$$

$$1-\sin^2\theta = \frac{1}{w'^2}\left[\, w'^2 - (m\,\omega^2\, r\sin\lambda\cos\lambda\,)^2\,\right]$$

$$w'^2 = m^2[\,g^2 + r\,\omega^2 \sin^2\lambda\ (\,\omega^2 r - 2\,g)] \tag{3.31}$$

Substitute (3.31) into above implies,

$$\tan\theta = \frac{m\, r\,\omega^2 \sin\lambda\cos\lambda}{g - r\,\omega^2 \sin^2\lambda} \tag{3.32}$$

3.4 Centrifugal Force on Earth

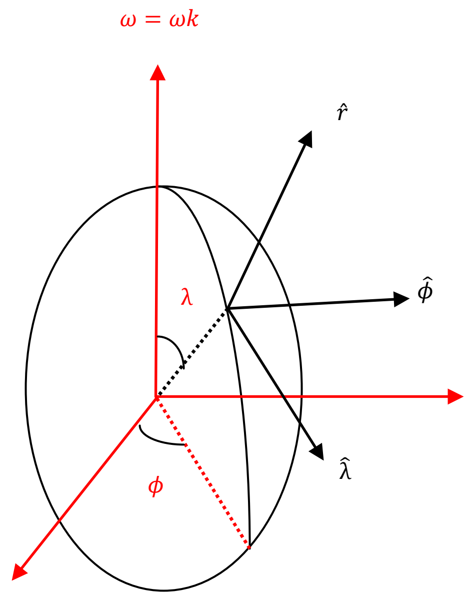

Figure 13: Centrifugal Force on Earth

$$\vec{\omega} = (\omega\,.\,\hat{r})\,\hat{r} + (\omega\,.\,\hat{\lambda})\,\hat{\lambda}$$

$$= \omega(\cos\lambda\,\hat{r} - \sin\hat{\lambda}) \qquad (3.33)$$

$$\vec{w} \times \hat{r} = \omega(\cos\lambda\,\hat{r} - \sin\lambda\hat{\lambda}) \times r\,\hat{r}$$

$$= \omega\, r \sin\lambda\,\hat{\phi} \qquad (3.34)$$

$$\vec{\omega} \times (\vec{\omega} \times \hat{r}) = \omega(\cos\lambda\,\hat{r} - \sin\lambda\,\hat{\lambda}) \times (\omega\, r \sin\lambda\,\hat{\phi})$$

$$= -\,\omega^2 r\sin\lambda\cos\lambda\,.\,\hat{\lambda} - \omega^2 r\,\sin^2\lambda\,\hat{r} \qquad (3.35)$$

$$\vec{F}_{cent.} = -\,m\,\vec{\omega} \times (\vec{\omega} \times \hat{r}) \qquad (3.36)$$

$$= m\,\omega^2\, r\sin\lambda\left(\cos\lambda\,.\,\hat{\lambda} + \sin\lambda\,\hat{r}\right)$$

$$\left|\vec{F}_{cent.}\right| = m\,\omega^2\, r\sin\lambda \qquad (3.37)$$

Example: A car is turning around a circle corner with angular frequency ω at radius R, Find the Period of a simple Pendulum in such a car?

(Hint: If the car is not moving the period is T= 2π $\sqrt{\frac{L}{g}}$, L= Length, g = acceleration Due to the gravity)

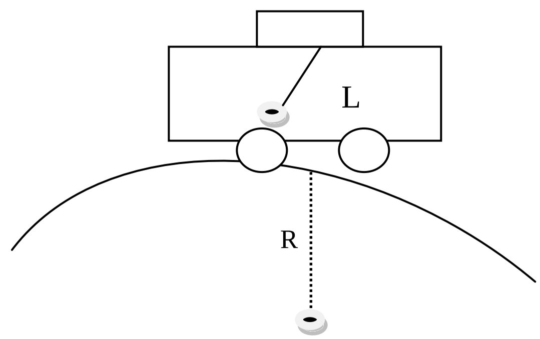

Figure 14: Car in a Curved Line(2)

Answer:

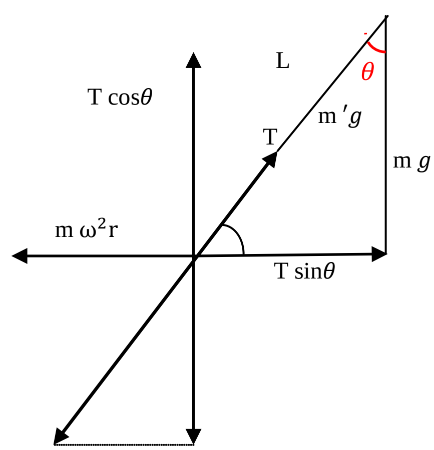

Figure 15:Car in a Curved Line(2)

∑ Fy =0

$$\boxed{T \cos\theta - m\, g = 0} \quad \ldots\ldots\ldots\ldots (1)$$

∑ Fx =0

$$\boxed{T \sin\theta - m\, \omega^2\, r = 0} \quad \ldots\ldots\ldots\ldots (2)$$

From fig.(15):

$$(m\,g')^2 = (m\,\omega^2 r)^2 + (m\,g)^2$$

$$g'^2 = \omega^4 r^2 + g^2$$

$$\boxed{g' = \sqrt{\omega^4 r^2 + g^2}} \quad \ldots\ldots\ldots\ldots (3)$$

From eq. (1) and eq. (2)

$$\frac{T\sin\theta}{T\cos\theta} = \frac{m\,\omega^2\,r}{m\,g'}$$

$$\tan\theta = \frac{r\omega^2}{g'}, \quad \text{And} \quad \tan\theta = \frac{r}{L}$$

$$\longrightarrow \quad \frac{r}{L} = \frac{r\omega^2}{g'}$$

$$\omega^2 = \frac{g'}{L}$$

$$\boxed{\omega = \sqrt{\frac{g'}{L}}} \quad \ldots\ldots\ldots\ldots (4)$$

$$T = \frac{2\pi r}{V} = \boxed{\frac{2\pi}{\omega}} \quad \ldots\ldots\ldots\ldots (5)$$

From (5) (4) and (3), we can get that:

$$\boxed{T = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + r^2\omega^4}}}}$$

If a car is not Rotating $\omega = 0 \longrightarrow \boxed{T = 2\pi\sqrt{\frac{L}{g}}}$

3.6 Foucault Pendulum

Example: Determining the equation of motion of Foucault pendulum?

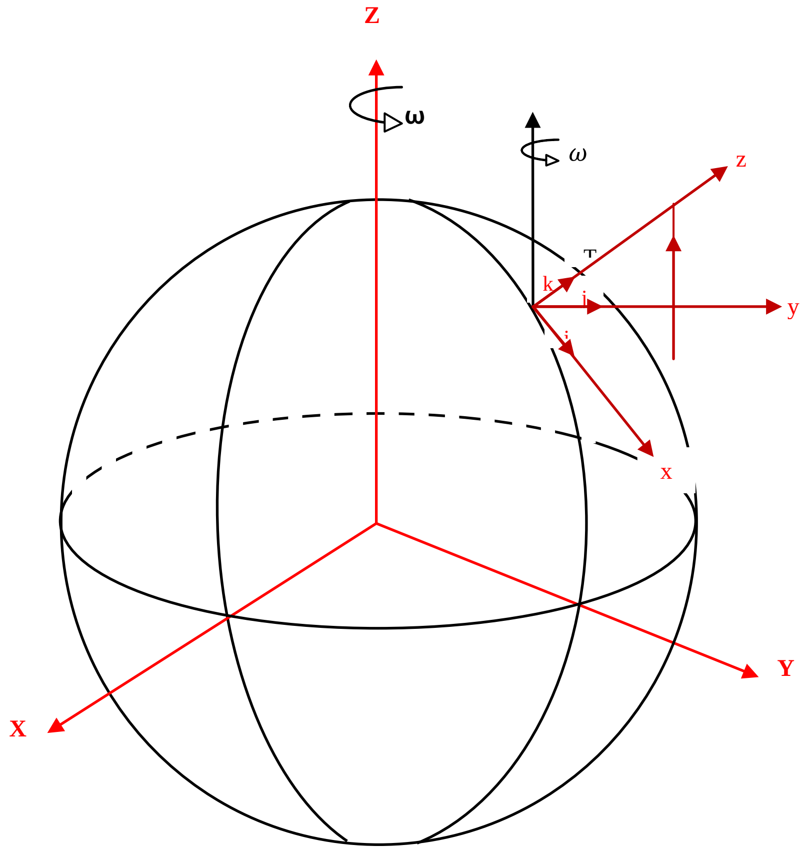

Figure 16: Foucault Pendulum(1)

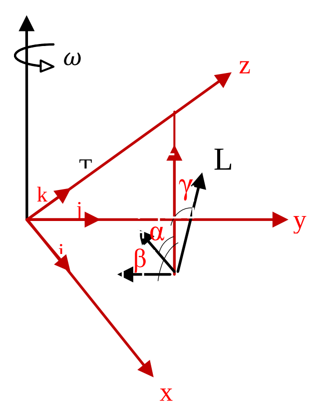

Figure 17: Foucault Pendulum(2)

$$T = T_x\, i + T_y\, j + T_z\, k \tag{3.38}$$

$$T\,.\,i = (T_x\, i + T_y\, j + T_z\, k).i = T_x \tag{3.39}$$

$$T\,.\,j = (T_x\, i + T_y\, j + T_z\, k).j = T_y \tag{3.40}$$

$$T\,.\,k = (T_x\, i + T_y\, j + T_z\, k).k = T_z \tag{3.41}$$

Thus: $T = (T.i)i + (T.j)j + (T.k)k$ (3.42)

But $T.i = |T||i| \cos\alpha = T\cos\alpha = -T\,\frac{x}{L}$ (3.43)

$$T.j = |T|\,|j| \cos\beta = T\cos\beta = -T\,\frac{y}{L} \tag{3.44}$$

$$T.k = |T||k| \cos\gamma = T\cos\gamma = T\,\frac{L-z}{L} \tag{3.45}$$

Put (3.43), (3.44) and (3.45) in (3.42)

$$T = -T\left(\frac{x}{L}\right)i \;-\; T\left(\frac{y}{L}\right)j \;+\; T\left(\frac{L-z}{L}\right)k \tag{3.46}$$

Recall eq.(2.40),

$$m\,(a)_{mov.} = -mg - 2\,(\vec{\omega}\times\vec{r})$$

If we use eq. (2.39) for Foucault pendulum, we can write in this form:

$$m\,(a)_{mov} = T - mg - 2\,(\vec{\omega}\times\vec{r}) \tag{3.47}$$

By using eq. (2.39) and eq. (2.41), we will get:

$$m\,(a)_{\,mov} = -T\left(\frac{x}{L}\right)i - T\left(\frac{y}{L}\right)j + T\left(\frac{L-z}{L}\right)k - mg - 2\begin{vmatrix} i & j & k \\ -\omega\sin\lambda & 0 & \omega\cos\lambda \\ \dot{x} & \dot{y} & \dot{z} \end{vmatrix} \tag{3.48}$$

Put $(a)_{mov} = (\ddot{x}, \ddot{y}, \ddot{z})$

$$\left.\begin{aligned} m\ddot{x} &= -T\left(\frac{x}{L}\right) + 2\,m\,\omega\,\dot{y}\cos\lambda \\ m\ddot{y} &= -T\left(\frac{y}{L}\right) + 2\,m\,(\dot{x}\,\omega\cos\lambda + \dot{z}\,\omega\sin\lambda) \\ m\ddot{z} &= T\left(\frac{L-z}{L}\right) + mg + 2\,m\,\omega\,\dot{y}\sin\lambda \end{aligned}\right\} \tag{3.49}$$

These are equations of motion for Foucault pendulum.

3.7 Coriolis Force on a Merry go Round

Example: A pistol may be considered at the centre of a rotating platform i.e. a Merry go round. The deflection of the bullet is depicted as in the figure and it is due to the Coriolis effect.

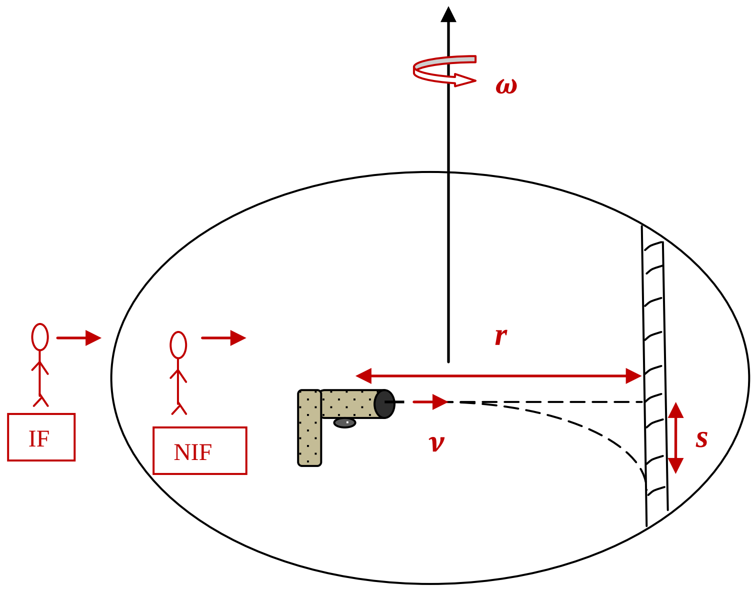

Figure 18: Merry go Round

Answer:

Let: (NIF) is (Non Inertial Frame) and (IF) is (Inertial Frame)

$$s = r(\Delta\theta)$$

$$= r\,\frac{\Delta\theta}{\Delta t}\,\Delta t \quad (3.50)$$

For Δt = small

$$\frac{\Delta\theta}{\Delta t} = \omega \quad (3.51)$$

$$S = r\,\omega\,\Delta t \quad (3.52)$$

$$r = \vec{v}\,t \quad (3.53)$$

$$s = \vec{v}\,\omega\, t^2 \tag{3.54}$$

$$s = \frac{1}{2}\,\vec{a}\,t^2 \tag{3.55}$$

Where $\boxed{a = 2\,\omega\,\vec{v}}$

There is a force acting on the bullet called Coriolis force w.r.t NIF.

The Bullet diverts (shifts) because of the rotating object w.r.t IF.

Chapter 4

CONCLUSION

In this thesis we considered the Earth rotating around its axes. We ignored the motion of the Earth around the sun, and also the motion of the Sun and solar system in the Galaxy.

We have reviewed briefly the Abelian and Non-Abelian groups of mathematics. Galilean transformation equations and Lorentz transformation equations are considered, since they are the basic mathematics of physics. Canonical transformations of classical mechanics is also mentioned.

But the most important point that we have explained in chapter (2) is Newton's equation of motion for the rotating Earth. In particular we have elaborated on Coriolis and centrifugal forces since they are the most important forces. We stressed that the Coriolis and centrifugal forces are not real forces; they are derived forces in non-inertial frames. But we can observe their effect in our daily life. After that we use these equations in some applications to guide us as the effect of rotating Earth in our daily life. We proved this effect for a projectile motion in some details. And also in this analysis we have proved that the missiles can't reach their destinations without taking into account the rotational effect of the Earth.

As shown in a special case, and by using equations (X_{max}., Y_{max}. , Z_{max}.),we considered that the initial velocity in (x, y, z) directions are equal (v_{0x} ,v_{0y} , v_{0z} = 10 m/s

), the angle is 30^0 and the angular velocity of the Earth is taken into account which is (7.29×10^{-5}) rad/s and also the acceleration of Earth is constant (10) m/s^2, We showed that the data will be changed because of the rotation of the Earth. As we obtained the maximum distance in x-direction increased by the amount (0.004) m for each (20) m, while in the y-direction the distance will be decreased by the amount (0.00157) m. In addition, the change in z-direction will be (0.0036) m which is the deflection occurred when z is a maximum. Finally we conclude that, if we want to get the correct data from the GPS system, we should take the rotating Earth in to consideration, because it directly affect in our daily life. Without this information loaded on the computer memory of the GPS system our seeking of direction will be incorrect.

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