Developing Equations for Ideal Gas Air Properties

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ABSTRACT

The equations for property data of air as an ideal gas are developed. The equations are presented as a function of temperature. Although there are several software capable of using property tables for example, EES, VisSim and etc., others require equations for property calculations such as FORTRAN. Equations are developed by using the “CurveFitting Expert” software. Equations are presented for air properties as a function of temperature are; enthalpy, internal energy, entropy, reduced pressure and reduced volume. Many equations were developed but only those which have high correlations are presented (i.e., equations with highest R^2). Moreover, the percent deviations of the calculated properties were studied and the equations which have more than one percent deviation from the tabulated data were neglected. Two equations to calculate temperature as a function of reduced pressure and enthalpy are also presented. The property equations developed in this study were used to simulate a 320 hp actual gas power turbine engine of a small military ship with reheater and recuperator. EES, MATLAB software and the developed equations were used to calculate various properties and the efficiency of the cycle. The difference in the final result (i.e., thermal efficiency) obtained by using the developed equations compared with the result obtained by using the EES software was around 0.1 % which indicated that the developed equations are accurate.

Keywords: Curve fitting, air properties, ship propulsion, EES software.

ÖZ

Havanın mükemmel gaz varsayımıyla özelliklerini hesaplamak için denklemler geliştirilmiştir. Denklemler sıcaklığın fonksiyonu olarak sunulmuştur. Özellik tablolarını kullanabilecek çeşitli yazılımlar olmasına rağmen örneğin EES, VisSim vb., diğerleri, FORTRAN gibi yazılımlar özellik hasaplamaları için denklemlerin kullanımını gerektirir. Denklemler "CurveFitting Expert" yazılımını kullanarak geliştirildi. Sıcaklığın fonksiyonu olarak sunulan havanın özellik bağıntıları; özgül entalpi, özgül iç enerji, özgül entropi, indirgenmiş basınç ve sanki-indirgenmiş özgül hacimdir. Birçok denklem geliştirilmiş ancak korelasyonu yüksek olanlar (en yüksek R^2'li denklemler) sunuldu. Ayrıca, hesaplanan özelliklerin tablo değerleri incelendi ve yüzdelik sapması 1'den fazla olan denklemler ihmal edildi. Sıcaklık hesaplamaları için indirgenmiş basınç ve özgül entalpinin fonksiyonu olarak iki farklı denklem sunuldu. Bu çalışmada geliştirilen özellik denklemleri, küçük bir askeri geminin araısıtıcılı ve rejenaratörlü 320 BG gücünde gerçek gaz türbin motorunu simüle etmek için kullanıldı. EES, MATLAB yazılımları ve geliştirilmiş denklemler kullanılarak havanın özellikleri ve gaz turbininin çevrim verimliliğini hesaplamada kullanıldı. EES yazılımı kullanılarak elde edilen sonuçlar (ısıl verim) denklemlerin kullanılmasıyle elde edilen sonuçlar karşılaştırıldığında farkın % 0.1 civarında bulunması, geliştirilen denklemlerin oldukça doğru olduğunu göstermektedir.

Anahta Kelimeler: Eğri uydurma, hava özellikleri, gemi tahrik gaz türbini, EES yazılım.

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R^2	coefficient of determinations
h	enthalpy
T	temperature
pr	reduced pressure
u	internal energy
Vr	reduced volume
S^0	standard entropy
T_amb & T_{amb}	ambient temperature
P_atm & P_{atm}	ambient pressure
T_t_in & $T_{t,in}$	turbine inlet temperature
rp_c & rp_c	compressor pressure ratio
rp_pt & rp_{pt}	power turbine pressure ratio
eta_c & η_c	compressor efficiency
eta_gt & η_{gt}	gasifier turbine efficiency
eta_pt & η_{pt}	power turbine efficiency
DT_rec	recuperator approach temperature difference
S	entropy
s_s	entropy balance on reversible equipment
h_s & h_i	enthalpy leaving reversible equipment
Q_dot_rec & $\dot{Q}_{rec}$	recuperator heat transfer per unit mass
eff_rec & η_{rec}	effectiveness of recuperator
w_dot_out & $\dot{w}_{out}$	output work of the both turbines
w_dot_in & $\dot{w}_{in}$	input work of the compressor

q_dot_in & $\dot{q}_{\text{in}}$	input thermal energy
w_dot_net & $\dot{w}_{\text{net}}$	net output energy of the system
eta_th & η_{th}	efficiency

Chapter 1

INTRODUCTION

Problem solving in Thermodynamics, fluid mechanics and heat transfer require values of fluid properties, such as air, water, refrigerants, carbon dioxide, etc. The properties of substances are presented as tables. This is due to the complex thermodynamics property relations of substances and usually equations developed for finding properties are not simple.

The property tables for various substances are readily available and easy to use. On the other hand property equations are useful in computer applications where the use of tables are not possible (i.e., not all software are capable to use tables such as FORTRAN) or use of equations are desirable. Thermodynamics systems simulations require the properties of the substances used in the systems. In engineering system design, it is required to design and produce more efficient parts compared to the available products. Computer simulation is an effective and efficient way to improve the design efficiency. For example, it is not economically feasible to build a huge steam power plant condenser and test its efficiency by experimentation, but computer simulations can be employed for designing an efficient condenser. Using computer simulations to improve the design in small systems is also more effective compared to real experiments. Reynolds in his book presented equations for finding the properties of various substances [1]. Recently, Zhao et al. [2] in their study developed equations by curve fitting for calculating the properties of refrigerants

(CO_2 and R410A) in supercritical region and indicated that the calculation time is 100 times faster than those using more accurate methods whereas the total mean relative deviation is less than 1%. Researchers have divided the calculation methods of thermodynamic properties of refrigerants into two; the first one is accurate method in which equations of states are used and the second one is fast method where curve fitting method is used Lui et al. [3] indicated that the time taken for the simulation of a heat exchanger was more than 10 h if accurate method was employed and the simulation time for optimization will be very long. In the fast method the computer simulation time can be shortened effectively [4] have developed a dimensionless implicit curve fitting method (i.e., fast property calculation method) for two phase properties of R407C.

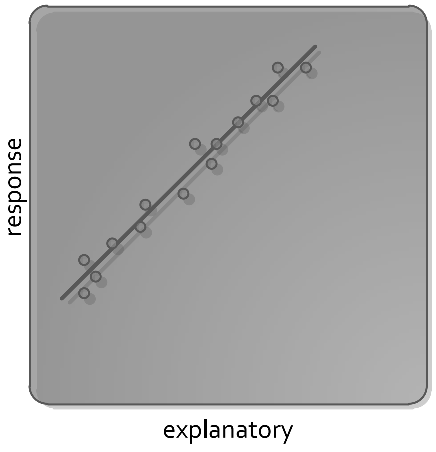
Curve fitting is important in engineering applications. Extensive studies on curve fitting were done and different approaches were used. Even though the appearance of some curves may look similar to each other, a curve may be an exponential, polynomial, or a complicated logarithmic function. The main aim of this study is to obtain curves for air properties. In this study different curve fitting methods were used to obtain equations of the air properties. The equations developed can be used for calculating the properties of air for computer simulation that involves the use of air properties such as gas power cycles. The results obtained from the equations developed were compared with the EES software solutions. Equations for finding the properties such as carbon dioxide, carbon monoxide, nitrogen and other substances can be obtained in a similar manner.

Chapter 2

CURVE FITTING OR REGRESSION

2.1 Curve Fitting

Curve fitting or regression is a statistical technique for investigation of the relations between data and variables that express the variable value as a function of the other value. The curve fitting operation has two principal branches, such as linear regression which approximate the best straight line through the variables and non-linear regression that approximate the relationship of a best curve. Where in the two dimensional diagrams two kinds of variables are defined the explanatory (or independent variable) and the response (or dependent variable), see Fig. 2.1.



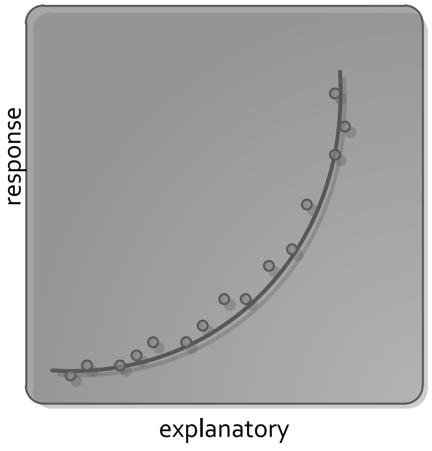


Figure 2.1a: (Left diagram) illustrate a straight line regression

Figure 2.1b: (Right diagram) shows non-linear regression by a cure.

2.2 Why Curve Fitting is Necessary

The aim of curve fitting is to describe the experimental data in theoretical ways by modeling them in equations or in functions; moreover to find related variable with these functions and equations.

The fitted experimental data is to acquire a specific function to determine interpolation, the first and second derivatives between the data. Unfortunately sometimes there are dramatically differences between the experimental data and the data that obtained by fitted curves [5].

2.3 Different types of curve fitting

As mentioned earlier linear regression is one type of curve fitting in which a straight line was used for regression. Polynomial regression could be a very accurate approximation for the regression function by increasing the power of polynomial. The non-linear method which mostly calculate more accurate fitted equations with the lowest deviation from the experimental data or variables, is divided in various kind; the power law fitting, the exponential fitting, logarithmic fitting, sigmoidal model, in which some of them have similar diagrams that may cause misdiagnosis between them.

2.4 Correlation Coefficient (The goodness of fitting)

The parameter which presents the quality of curve fitting is the correlation coefficient (usually marked as R) that shows how closely one variable related to the other variable. The range of R is from (-1) to (+1) which it is perfectly correlated in beginning and the end of the range in negative and positive direction and when it is equals to zero the variables are not correlated[6].

It could be illustrated by coefficient of determinations that is pronounced as R-squared; it is a number between 0 and 1 that represents how the regression line is accurate and fitted on the experimental points [7]. The R^2 is expressed by the following equation.

$$R^2 = 1 - \frac{\sum_{1}^{n} (Y_i - \hat{Y}_i)^2}{\sum_{1}^{n} (Y_i - \overline{Y})^2} \tag{2. 1}$$

Where, $\bar{Y}$ is the mean of Y which is the related variable obtained from experimental data and "$\hat{Y}$" is the predicted values which were obtained from the fitted equation [7].

One of the objectives of this study is to develop accurate functions by employing the R^2 for air property calculations.

2.5 Fitting Functions

Functions such as polynomial, power law, power regression and etc., are employed in curve fitting. The functions used in this study are briefly explained in the following sub-sections.

2.5.1 Polynomial Functions

Polynomial function fits data into the curve to the form of:

$$Y=A+B*X+C*X^2+\ldots+K*X^{10}+\ldots \tag{2.2}$$

Where A, B, C… K and etc. are the constants that should be calculated. X and Y are the independent and dependent variables respectively [5]. The more complex and higher polynomial order equations the accuracy of the fitted data would be higher.

2.5.2 Power Law Functions

The Power family includes rising the independent parameter or some parameters to the power of independent variable, and there is no min/max or fluctuation. They are mostly concave or convex curves.

2.5.2.1 Power Regression

Power Curve fitting fit the data through the following functions:

$$Y=A*X^{B} \tag{2.3}$$

where A and B are the constants that they should be calculated; X and Y are the explanatory and response variables respectively. The variables should not be zero or negative.

2.5.2.2 Shifted Power Regression

The shifted power regression is also similar to power regression; however before the effect of the power (constant C) on the independent value, the specific amount would be subtract from it (constant B). The function is defined as:

$$Y=A*(X-B)^{C} \tag{2.4}$$

Where “X” is independent and “Y” is dependent variables.

2.5.2.3 Hoerl Regression

The Hoerl Model is complicated, both raising the independent variable (i.e., X) to the power of a constant and a constant to the power of independent variable, exist in this regression such as in the following function:

$$Y=A*B^{X}X^{C} \tag{2.5}$$

Where, A, B and C are constants and Y and X are response and explanatory variables.

2.5.3 Sigmoidal Growth Models

The sigmoidal model is an “S” shape function that has various types; Morgan-Mercer-Flodin (MMF) Regression is from sigmoidal regression family that frequently approximated more accurate predictions in this study which is explained by the below function:

$$Y = \frac{A*B+C*X^D}{B+X^D} \qquad (2.6)$$

Where A, B, C, and D are constants and X is the explanatory variable and Y is response value [8].

2.5.4 Decline Models

The Decline curve fitting is the old and commonly used in industrial and production which relate the production rate to the time; the hyperbolic decline method function is as follows:

$$q_t = \frac{q_0}{\left(1+bD_t t\right)^{\frac{1}{b}}} \qquad (2.7)$$

Where q_t is response variable and *t* is explanatory variable. q_0, b and D_t are constants [9].

In the following chapter equations developed for air gas properties by using the functions explained above are presented.

Chapter 3

METHODOLOGY IN THERMODYNAMIC AIR PROPERTIES CURVE FITTING

3.1 Ideal and Actual Air Treatment

Air is a mixture of different gasses and contained some liquid and solid components; it is mainly 78% of nitrogen and 21% of oxygen and the remaining 1% is including Argon, Helium, Neon, H_2O and etc. Various gases are differing in their behavior even in very small proportion; which are caused by intermolecular forces and the atomic weight. The gases behave much similar to each other, so the idea of an ideal gas was developed. In the concept of ideal gas there are no forces between molecules and molecules volume are not considered. However the estimated properties are closely like actual gases under the most conditions [10].

3.2 Thermodynamics Properties

Each thermodynamic property is identified by several different manners. There are three specific ways to distinguish the properties; such as the measured properties which are calculated for example, volume, temperature, pressure and etc. The fundamental properties which are directly related to the fundamental thermodynamics laws such as internal energy and entropy these properties can be identified in laboratories. Moreover, the properties that should be derived by some specific relations for example enthalpy and etc., these are also could not be measured in the laboratories [11].

Some of the thermodynamics properties are directly related to the size of the system, like volume but the value of some of those properties are unrelated, such as pressure and temperature which they are called extensive and intensive properties respectively.

3.3 Different Methods to Obtain Thermodynamic Properties

The thermodynamics properties of air can be determined from five different methods. Primary method is the usage of thermodynamics equations of state. Second method is by using the thermodynamics tables. Finding the property value by using thermodynamic charts is the third method. The forth method is to obtain the property value by direct experimental measurements. The final method to determine property is by employing formulae developed from statistical thermodynamics.

In this study thermodynamic properties of ideal gas air are obtained by curve fitting of air table values. By curve fitting the air property values obtained from the thermodynamic tables equations would be obtained which can be used in the analyses of thermodynamic problems instead of using the tables.

These equations could be employed in various software for designing and analyses of thermodynamic issues. However the accuracy and the simplicity of these equations are important.

3.4 Curve Fitting of the Gas Properties of Air

The regression process is performed by using the “Curvefitting Expert” software, the Curvefitting Expert software has a wide database of the variety of the functions, and easy to use. The equation that has the nearest amount of R-squared to “one” has to be chosen. The plot of the best curve along the original data also gives visual feedback about the curve fit.

As the air can be treated as an ideal gas, thermodynamic properties of air are functions of temperature only. (Air property table is presented in the appendix B). In this study, the air properties as a function of temperature are presented; and in order, to increase the accuracy, the temperature was divided into several intervals as needed.

Calculating the deviation experienced by each calculated parameter could illustrate the amount of error; in this study less than one percent deviation was used as a benchmark for developing the equations i.e., equations having more than 1 percent deviation were rejected. The deviations were calculated as follows.

$$\text{Deviation} = \left|\frac{(Value\ of\ experimental\ Parameter)-(Value\ of\ obtained\ parameter)}{(Value\ of\ experimental\ Parameter)}\right| \times 100 \qquad (3.1)$$

Equations are calculated with the lowest deviation and with an R-squared value greater than 0.999 were obtained and presented.

One of the aims is to obtain simple and accurate equations for the air properties. However, accurate equations may not be very simple. The equations obtained are presented in the following tables having the R—squared values of one or near one, are slightly complicated; however, these equations can be employed easily in several software such as FORTRAN, MATHLAB, VisSim, Excel and etc., to calculate air properties for design and analyses of thermodynamic systems.

The thermodynamic variables are divided in two or three parts in order to obtain firstly, more accurate equations and secondly to obtain more simple equations. The equations such as the power regression, has the simplest form, by dividing the data into several intervals simple and relatively accurate equations can be obtained. The equations obtained by curve fitting are presented in the tables 3.1 to 3.7.

3.5 The Methodology for Calculating the Equations

The Fig.3.1 illustrates the flow chart of the methodology for the calculation of the equations. These equations can be used instead of the ideal gas air property thermodynamic tables. The parameters “X” and “Y” in this flowchart represent the explanatory and the response variables of the air properties from the tables, moreover the parameter “$\hat{Y}$” is the mount of the value of response variable obtained by using the developed equations. All the obtained equations are presented in the next chapter.

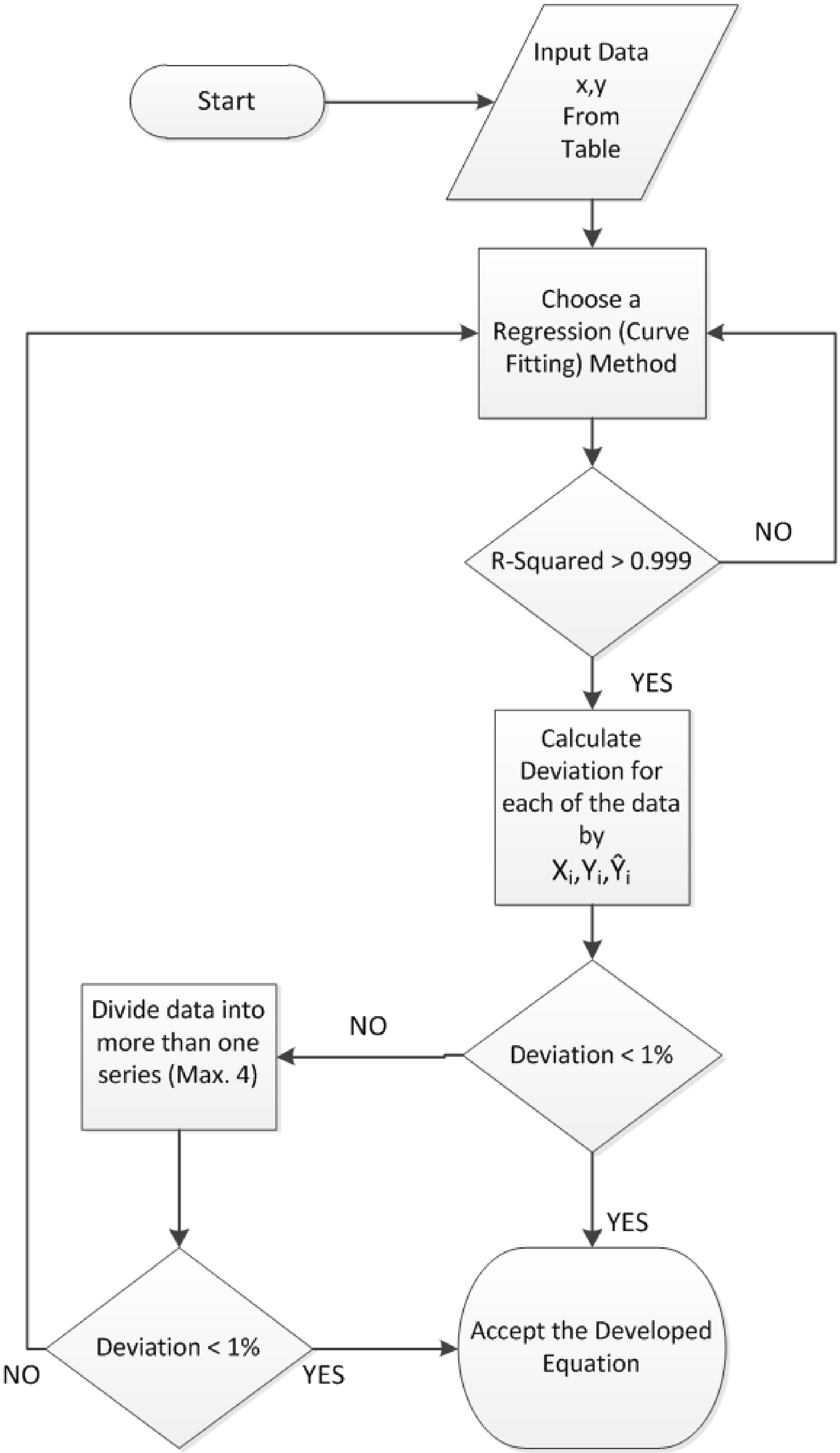


Figure 3.1. The flow chart of the methodology for calculating the equations

Chapter 4

EQUATIONS DEVELOPED FOR CALCULATING IDEAL GAS-AIR PROPERTIES

The developed equations are presented in the following sections as tables. The Table 4.1 shows the relation between enthalpy and the temperature of the air where temperature is the explanatory and the enthalpy is the response variable. Table 4.2 illustrates the relation among the reduced pressure and the temperature. The reduced pressure can be calculated by inserting the temperature into the developed equations between the ranges from 250 to 1500 Kelvin. Table 4.3 represents the relationship of the internal energy and the temperature that temperature is independent variable and the internal energy is dependent variable in those equations. Table 4.4 indicates the equations which relate the reduced volume to the temperature by using the property table and the curve fitting method where temperature and reduced volume are explanatory and response variables respectively. Tables 4.5 demonstrate equations which standard entropy is dependent to the temperature.

Tables 4.6 and 4.7 unlike the previous tables; temperature is not the explanatory variable because in some part of thermodynamic problem solving the enthalpy should be calculated by using the reduced pressure. The table 4.6 illustrates the relation between reduced pressure and the enthalpy and the table 4.7 shows the enthalpy as explanatory and temperature as response variable which mostly used in the thermodynamic cycles problems.

Table 4.1: Property equations for air, T – h* table

Equations	Regression Method	Average Deviation (Percent)	R^2
$h = 13.2818824 + 919.093 \times 10^{-3}\,T + 123.4542653 \times 10^{-6}\,T^2 - 9.6778505 \times 10^{-9}\,T^3$ $200 \le T \le 1500$	*Polynomial degree of 3*	*0.0941932*	*0.999998*
$h = 9.4649778 + 937.5687231 \times 10^{-3}\,T + 98.7740772 \times 10^{-6}\,T^2$ $200 \le T \le 500$	*Polynomial degree of 2*	*0.190688*	*0.999997*
$h = 1.0469353 + 953.8210000 \times 10^{-3}\,T + 91.3465559 \times 10^{-6}\,T^2$ $500 \le T \le 1500$		*0.07109133*	*0.999999*
$h = 1.09450727 \times 1.000105777152^{T} \times T^{0.9780635945}$ $200 \le T \le 1500$	*Hoerl*	*0.1365502*	*0.999994*
$h = 0.96736098220213\,T^{1.0059942128}$ $200 \le T \le 525$	*Power*	*0.0737011*	*0.999991*
$h = 0.6368407038627\,T^{1.0726866461}$ $525 \le T \le 1500$		*0.4402566*	*0.99977*

* Table Parameters Unit: T (degree of Kelvin), h (kJ/kg)

Table 4.2: Property equations for air table, T – Pr* table

Equations	**Regression Method**	**Average Deviation (Percent)**	**R^2**
$Pr = 1.1328623 - 11.3723411 \times 10^{-3} T + 38.3805511 \times 10^{-6} T^2 - 19.0563752 \times 10^{-9} T^3 + 79.5689260 \times 10^{-12} T^4 + 25.4252534 \times 10^{-15} T^5$ $250 \le T \le 1500$	*Polynomial* *Degree of 5*	*0.003163633*	*1.000000*
$Pr = 114.476 \times 10^{-6} \left(1 + 36.7919818 \times 10^{-3} T\right)^{3.7791609}$ $250 \le \mathrm{T} \le 650$	*Hyperbolic* *Decline*	*0.488669183*	*0.99981*
$Pr = 23.07 \times 10^{-3} \left(1 + 4.9068494 \times 10^{-3} T\right)^{4.7890197}$ $650 \le T \le 1500$		*0.076939259*	*0.99999*
$Pr = 9.228466 \times 10^{-9} \times 1.000653233^{T}\, T^{3.267167}$ $250 \le T \le 750$	*Hoerl*	*0.15728075*	*0.999999*
$Pr = 3.9206 \times 10^{-9} \times 1.000561503^{T}\, T^{3.407010356}$ $750 \le T \le 1500$		*0.1109775*	*0.999998*
$Pr = 2.9853848 \times 10^{-9}\, T^{3.49892662062}$ $250 \le T \le 400$	*Power*	*0.0933774*	*0.99999*
$Pr = 1.82799619965 \times 10^{-9}\, T^{3.58090341235}$ $400 \le T \le 600$		*0.2463938*	*0.99998*
$Pr = 4.427663223 \times 10^{-10}\, T^{3.8015654126}$ $600 \le T \le 950$		*0.5001761*	*0.99979*
$Pr = 5.9390432956 \times 10^{-11}\, T^{4.0940332738}$ $950 \le T \le 1500$		*0.4669045*	*0.99985*

* Table Parameters Unit: T (degree of Kelvin), pr (-)

Table 4.3: Property equations for air table, T – u* table

Equations	**Regression Method**	**Average Deviation (Percent)**	**R^2**
$u = 10.0208553 + 649.0183445 \times 10^{-3}\,T + 99.5613344 \times 10^{-6}\,T^2$ $250 \le T \le 1500$	*Polynomial degree of 2*	*0.117*	*0.999995*
$u = 0.823963804 \times 1.0001496893^{T}\,T^{0.9662646128}$ $250 \le T \le 1500$	*Hoerl*	*0.189312*	*0.999995*
$u = 30.8361045\left(1 + 13.3289392 \times 10^{-3}\,T\right)^{1.2038600}$ $250 \le T \le 1500$	*Hyperbolic Decline*	*0.22815*	*0.999982*
$u = 0.1704418202047\,(T + 75.0246909149)^{1.2038599986}$ $250 \le T \le 1500$	*Shifted Power*	*0.22815*	*0.999982*

* Table Parameters Unit: T (degree of Kelvin), u (kJ/kg)

Table 4.4: Property equations for air table, T – Vr* table

Equations	Regression Method	Average Deviation (Percent)	R^2
$Vr = 16.3779242 \times 10^{3} - 185.3304161\,T + 986.87 \times 10^{-3}\,T^{2} - 3.1483772 \times 10^{-3}\,T^{3} + 6.5602831 \times 10^{-6}\,T^{4} - 9.2561765 \times 10^{-9}\,T^{5} + 8.9203031 \times 10^{-12}\,T^{6} - 5.7869862 \times 10^{-15}\,T^{7} + 2.4169985 \times 10^{-18}\,T^{8} - 586.8730057 \times 10^{-24}\,T^{9} + 62.9358124 \times 10^{-27}\,T^{10}$ $250 \leq T \leq 1500$	*Polynomial degree of 10*	*0.948635163*	*0.999997*
$Vr = \dfrac{9.756956485 \times 10^{8} - 5.51218185\,T^{2.49852359}}{10924.48536721 + T^{2.49852359}}$ $250 \leq T \leq 600$	*MMF*	*0.105557*	*0.999994*
$Vr = \dfrac{2.5382604 \times 10^{9} - 2.7246498\,T^{2.6520597}}{29.6835548 \times 10^{3} + T^{2.6520597}}$ $600 \leq T \leq 1500$			*0.999843*
$Vr = \dfrac{4.218090861 \times 10^{8} \times 0.9994502192952^{T}}{T^{2.325175719758}}$ $250 \leq T \leq 480$	*Hoerl*	*0.132753*	*0.999994*
$Vr = \dfrac{2.67368675261013 \times 10^{8} \times 0.9992729922^{T}}{T^{2.236336682036}}$ $480 \leq T \leq 1500$		*0.255969*	*0.99997*

* Table Parameters Unit: T (degree of Kelvin), Vr (-)

Table 4.5: Property equations for air table, T –S^0 * table

Equations	Regression Method	Average Deviation (Percent)	R^2
$S^0 = 1.5182413 \times 10^{-1} + 7.7136561 \times 10^{-3} T - 1.1145342 \times 10^{-5} T^2 + 1.0203442 \times 10^{-8} T^3 - 4.9002046 \times 10^{-12} T^4 + 9.4465845 \times 10^{-16} T^5$ $250 \leq T \leq 1500$	*Polynomial degree of 5*	*0.05006*	0.999994
$S^0 = 0.31024645483\,(T - 135.89734834675)^{0.333798344934}$ $250 \leq T \leq 1500$	*Shifted power*	*0.1055307*	*0.999971*
$S^0 = \dfrac{-45.95743308 + 16.981540473\, T^{0.274237709055}}{15.88727341324 + T^{0.274237709055}}$ $250 \leq T \leq 1500$	*MMF*	*0.106002*	*0.999973*
$S^0 = 0.07113442802057 \times 0.9998165492855^{T}\, T^{0.56732441234046}$ $250 \leq T \leq 1500$	*Hoerl*	*0.437117*	*0.99955*

* Table Parameter Unit: T (degree of Kelvin), S^0 (kJ/kg. k)

Table 4.6: Property equations for air table, Pr – h* table

Equations	**Regression Method**	**Average Deviation (Percent)**	**R^2**
$h = \dfrac{279.1522686 + 13319.743517\, pr^{0.299578}}{48.769311 + pr^{0.299578}}$ $o \le pr \le 600$	*MMF*	*0.17794*	*1.00000*
$h = 275.4627599 \times 0.999948564^{pr}\, pr^{0.2828553754}$ $o \le pr \le 600$	*Hoerl*	*0.2115*	*0.999989*
$h = 273.52503\,(pr - 0.00236346)^{0.286189}$ $0 \le pr \le 20$	*Shifted Power*	*0.039738*	*0.999998*
$h = 283.5347093756\,(pr - 0.097539691634)^{0.27470816112}$ $20 \le pr \le 600$		*0.276147*	*0.999988*
$h = 275.185562\, pr^{0.2826645}$ $0 \le pr \le 190$	*Power*	*0.325978*	*0.99988*
$h = 281.025957\, pr^{0.275304247782}$ $190 \le pr \le 600$		*0.4654198*	*0.999855*

* Table Parameter Unit: Pr (-), h (kJ/kg)

Table 4.7: Property equations for air table, h-T * table

Equations	**Regression Method**	**Average Deviation (Percent)**	**R^2**
$T = -5.7559645569 + 1.03468305147\,h - 0.0000376039987284\,h^2 - 5.044969502941 \times 10^{-8}\,h^3 + 1.88651774609433 \times 10^{-11}\,h^4$ $200 \leq h \leq 1650$	*Polynomial Degree of 4*	*0.039572*	*0.999999*
$T = 0.963362526937 \times 0.99991593073^{h} \times h^{1.011631542987}$ $200 \leq h \leq 1650$	*Hoerl*	*0.163978*	*0.99999*
$T = 2.26157693959988\,(h - 43.65192520636)^{0.8813620503243}$ $250 \leq h \leq 1650$	*Shifted power*	*0.248486*	*0.999984*

* Table Parameters Unit: T (degree of Kelvin), h (kJ/kg)

4.1 The Accuracy of the Obtained equations

The tables demonstrate equations obtained which have the average deviation less than one percent from the original values furthermore, the R-squared in the entire equations are equal to one or almost one; however, they should be investigated in a real problem which deal with air tables such as a gas turbine power plant or a gas power cycle and compare the results obtained by using the developed equations with the results which are calculated by using the gas air tables.

EES is a engineering software which the thermodynamic tables are defined in it, and it has the ability to solve the thermodynamic problem by the tables. In the following chapter a case study is perused and the results are compared.

Chapter 5

CASE STUDY

5.1 Gas Power Cycle

The system that transforms the thermal energy to shaft work to generate power is known as power cycle. If the working fluid is gaseous throughout the cycle, than the cycle is a gas power cycle. In this study a small gas turbine efficiency calculation, where the working fluid is air, is presented as a case study.

The schematic of the thermodynamic of gas turbine engine power plant is shown in Fig.5.1. This is for a small military ship which needs 320 hp power output. At the beginning, the fresh air goes into the compression stage, where the temperature and the pressure are increased. As the exhaust gas temperature from the turbine in cycle is extremely higher than the air temperature which leaves the compressor the exhaust turbine hot air flow could transfer its heat to the leaving high pressure air from the compressor, in a counter flow heat exchanger. This heat exchanger always is known as a recuperator or a regenerator [12]. The consequence of using the regeneration of the power cycle is to increase the thermal efficiency of the cycle. The portion of the heat in the exhaust air is used to preheat the air entering the combustor.

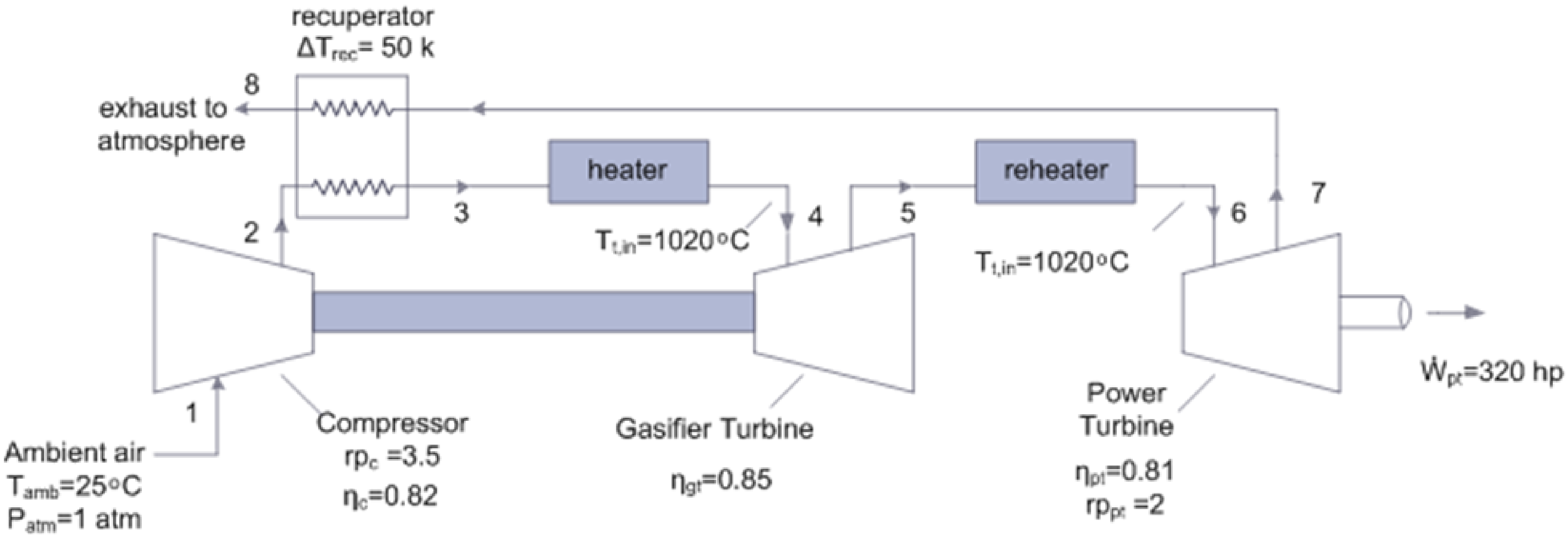


Figure 5.1: Actual gas power turbine engine of the small military ship with reheater and recuperator

The high-pressure air proceeds into the combustion chamber (heater); the fuel is burned at constant pressure. The resulting high-temperature gases then enter the gasifier turbine equipment, the power produced by the gasifier turbine is equal to the power consumed by the compressor (the gasifier turbine provide the energy that the compressor consumed to compressing the air). The air flow entered to another combustion chamber to reheat and rise the working fluid temperature till the previous entrance temperature; then in the power turbine they expand to the atmospheric pressure and net-work is produced.

5.2 Gas Power Turbine details

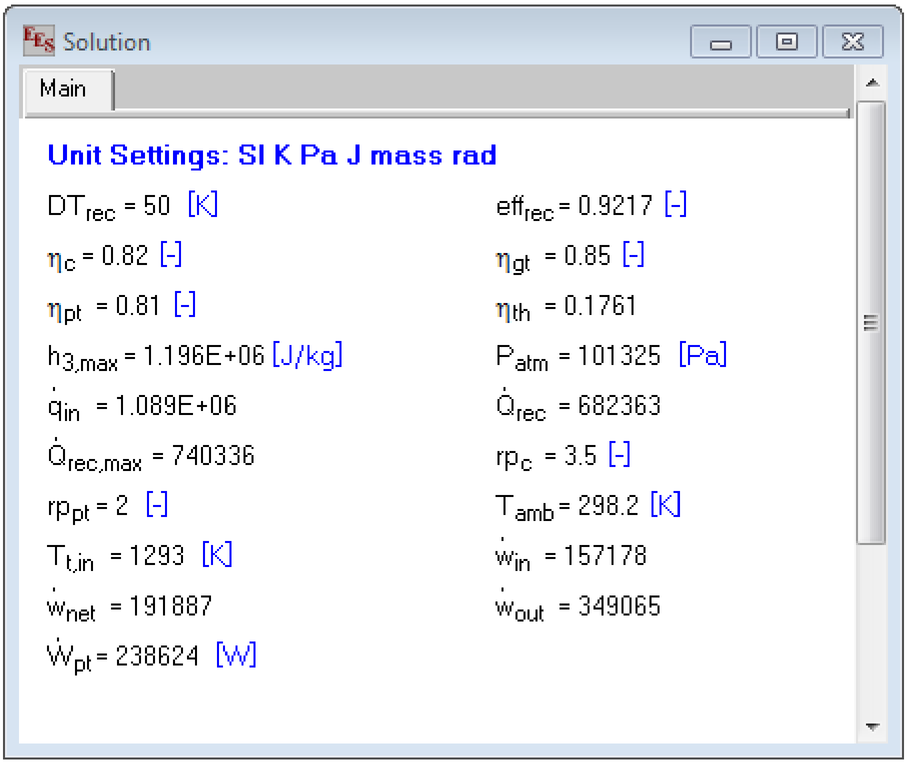
The gas turbine engine for ship propulsion is working in the ambient air, the temperature is 25^{o}C in and the pressure is one atmosphere at the compressor inlet (state 1). The compressor and power turbine pressures are 3.5 and 2 respectively, and the efficiency of the compressor is 82%. The air evacuates the compressor at the state 2 and it enters the regenerator (recuperator), the temperature difference at the hot end of recuperator is 50 ^{o}C. The air enters at the first heater (i.e., combustion chamber) at the state 3 and the air temperature is 1020 ^{o}C at the exit of the combustion chamber. Then the air enters to the gasifier (state 4), the work produced by the gasifier turbine runs the compressor; the gasifier turbine isentropic efficiency is 85%. The expanded working fluid leaving gasifier turbine enters to the reheater and heated to 1020 ^{o}C. The reheated air enters to the power turbine (state 6), the air expands through the power turbine and executes propulsion power for the ship; the efficiency of the power turbine is 81%. The hot air leaves the turbine to the regenerator for preheating the compressed air at the state 3 and it released to the atmosphere at the state 8 [13].

5.3 Analysis of the Gas Power Turbine

The gas power turbine is examined by using the EES software. The EES was used to obtain the following parameters: thermal efficiency, properties at various state points. Air table properties available within the EES were used in the cycle analysis. Then, the EES software again was used with the equations developed for air property calculations to calculate the same parameters. The programs developed by using EES for the analyses of the cycle are presented in the following sub-sections. The analyses of the gas power cycle by using the equations developed are presented in the Fig.5.3. and Fig.5.5.

EES software is an engineering software and mostly used in thermodynamics and mechanics. The benefit of the equations is they could be used in any software with high accuracy. This problem is also solved in the MATLAB software which it is not engineering software (Appendix A. part c) and the results are presented in the table 5.1 as it shows, for an illustration the deviation for thermal efficiency is zero.

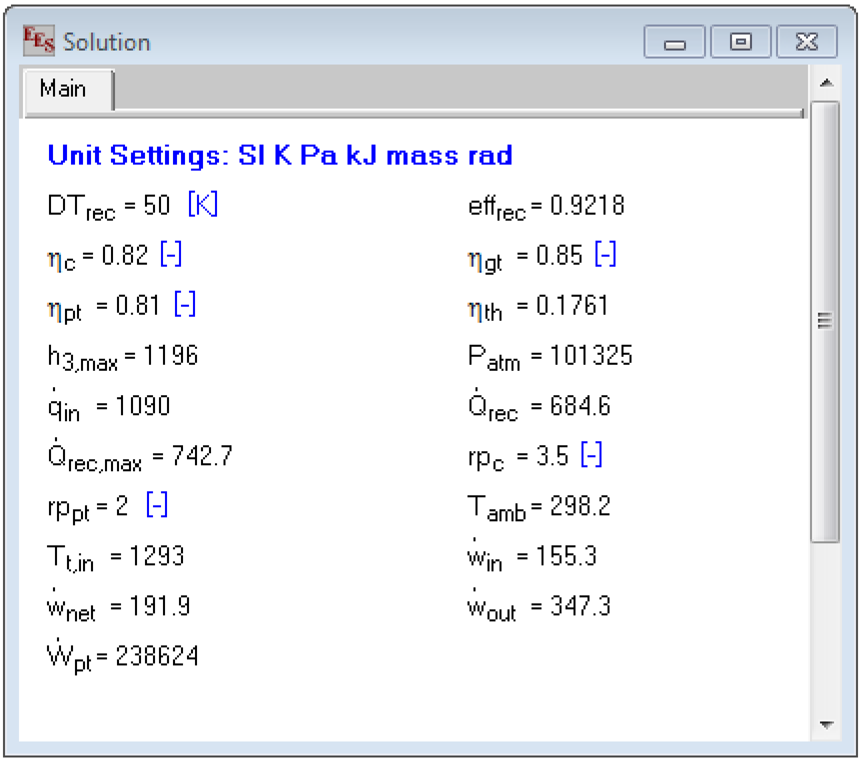
All the deviations are less than 1%; therefore, the developed equations can be used in the analyses of thermodynamic problems involving air properties.



Unit Settings: SI K Pa J mass rad

$DT_{rec} = 50$ [K]
$eff_{rec} = 0.9217$ [-]
$\eta_c = 0.82$ [-]
$\eta_{gt} = 0.85$ [-]
$\eta_{pt} = 0.81$ [-]
$\eta_{th} = 0.1761$
$h_{3,max} = 1.196E+06$ [J/kg]
$P_{atm} = 101325$ [Pa]
$\dot{q}_{in} = 1.089E+06$
$\dot{Q}_{rec} = 682363$
$\dot{Q}_{rec,max} = 740336$
$rp_c = 3.5$ [-]
$rp_{pt} = 2$ [-]
$T_{amb} = 298.2$ [K]
$T_{t,in} = 1293$ [K]
$\dot{w}_{in} = 157178$
$\dot{w}_{net} = 191887$
$\dot{w}_{out} = 349065$
$\dot{W}_{pt} = 238624$ [W]

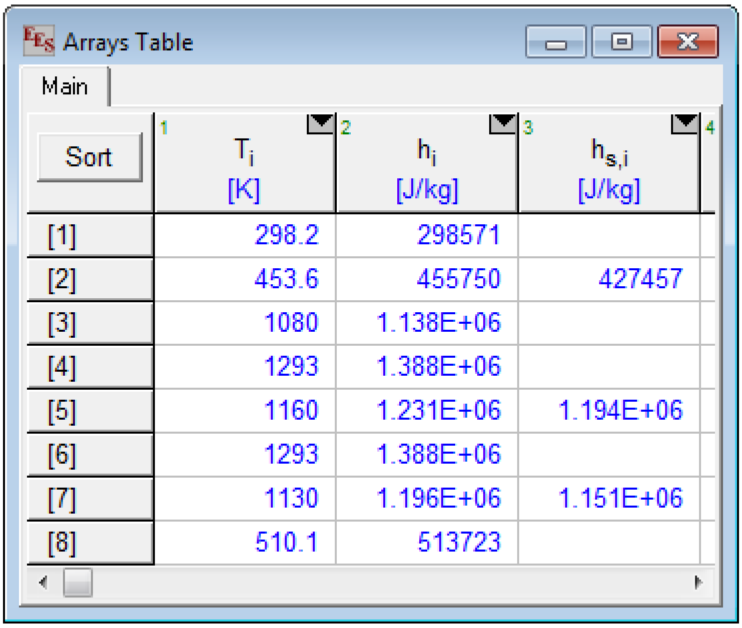
Figure 5.2: EES result window for cycle analysis by using the thermodynamic air tables.



Unit Settings: SI K Pa kJ mass rad

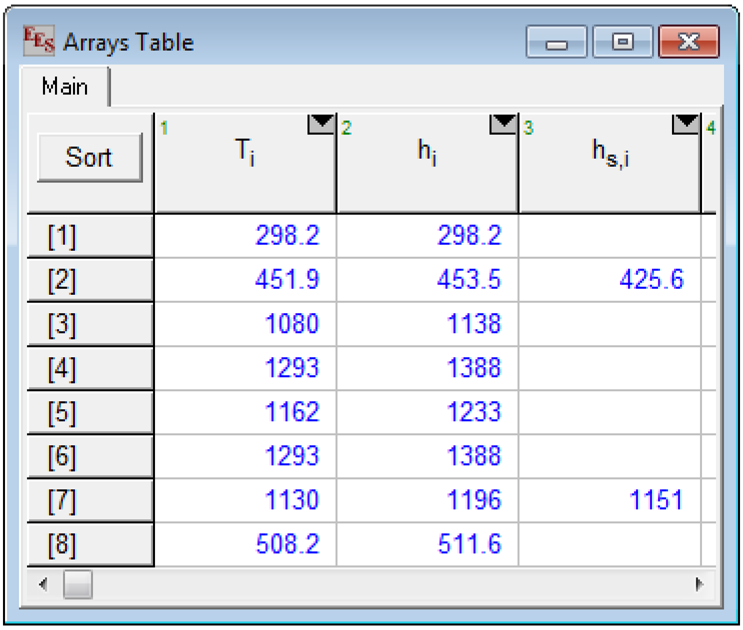
$DT_{rec} = 50$ [K]
$eff_{rec} = 0.9218$
$\eta_c = 0.82$ [-]
$\eta_{gt} = 0.85$ [-]
$\eta_{pt} = 0.81$ [-]
$\eta_{th} = 0.1761$
$h_{3,max} = 1196$
$P_{atm} = 101325$
$\dot{q}_{in} = 1090$
$\dot{Q}_{rec} = 684.6$
$\dot{Q}_{rec,max} = 742.7$
$rp_c = 3.5$ [-]
$rp_{pt} = 2$ [-]
$T_{amb} = 298.2$
$T_{t,in} = 1293$
$\dot{w}_{in} = 155.3$
$\dot{w}_{net} = 191.9$
$\dot{w}_{out} = 347.3$
$\dot{W}_{pt} = 238624$

Figure 5.3: EES result window for the cycle analysis by using obtained equations



	T_i [K]	h_i [J/kg]	$h_{s,i}$ [J/kg]
[1]	298.2	298571	
[2]	453.6	455750	427457
[3]	1080	1.138E+06	
[4]	1293	1.388E+06	
[5]	1160	1.231E+06	1.194E+06
[6]	1293	1.388E+06	
[7]	1130	1.196E+06	1.151E+06
[8]	510.1	513723	

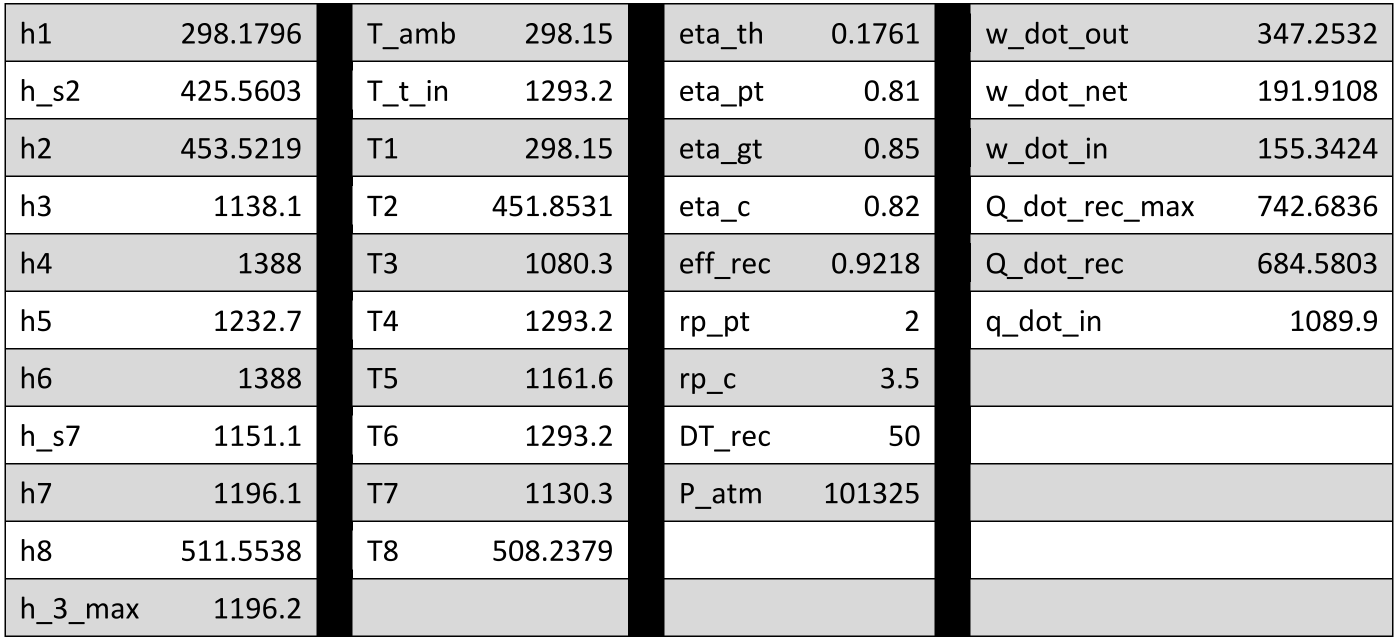
Figure 5.4: Enthalpy and temperature obtained by using table and EES equations



	T_i	h_i	$h_{s,i}$
[1]	298.2	298.2	
[2]	451.9	453.5	425.6
[3]	1080	1138	
[4]	1293	1388	
[5]	1162	1233	
[6]	1293	1388	
[7]	1130	1196	1151
[8]	508.2	511.6	

Figure 5.5: Enthalpy and temperature obtained by using developed equations

Table 5.1: collected results by using obtained equations in MATLAB software



h1	298.1796	T_amb	298.15	eta_th	0.1761	w_dot_out	347.2532
h_s2	425.5603	T_t_in	1293.2	eta_pt	0.81	w_dot_net	191.9108
h2	453.5219	T1	298.15	eta_gt	0.85	w_dot_in	155.3424
h3	1138.1	T2	451.8531	eta_c	0.82	Q_dot_rec_max	742.6836
h4	1388	T3	1080.3	eff_rec	0.9218	Q_dot_rec	684.5803
h5	1232.7	T4	1293.2	rp_pt	2	q_dot_in	1089.9
h6	1388	T5	1161.6	rp_c	3.5		
h_s7	1151.1	T6	1293.2	DT_rec	50		
h7	1196.1	T7	1130.3	P_atm	101325		
h8	511.5538	T8	508.2379				
h_3_max	1196.2						

Chapter 6

DISCUSSION AND CONCLUSION

Curve fitting is a method used to calculate values of different variables which they don’t have mathematical relation with each other; however the numerical relation between them is presented by experimental methods which generally calculated in laboratories (Property tables). By the contribution of curve fitting and the numerical relations between variables some equations could be determined that if the curve is fitted very carefully the equations could be used to find response variable by having the explanatory value. Some of these relations are complicated but only one relation would be needed to assume the dependent variable value in the defined range that it is suggested not to be used in the manual calculation and problem solving it would be better to use them in some software such as FORTRANT, MATLAB and etc., whereas some of the equations are less complex and they could be used in manual calculation but usually those equations have less accuracy to prevent accuracy drop in equation the defined ranges are splitted into two or three separate ranges to keep the deviation of each parameter less than one in whole range of variables and also the using of the simple equation.

Thermal efficiency and the effectiveness are approximately equal as seen in Figs. 5.2 and 5.3 plus table 5.1, also the utilization of the equations in the EES software are simple. The equations can be also employed in other software. The deviations in thermal efficiency and effectiveness of the recuperator are less than 0.01% which indicates that the equations developed are accurate.

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APPENDICES

Appendix A: Programming By EES and MatLab

a. Solving the Problem by using the Tables and EES equation

```
"Input conditions"
T_amb=converttemp(C,K,25 [C])                     "ambient temperature"
P_atm=1 [atm]*convert(atm,Pa)                     "ambient pressure"
T_t_in=converttemp(C,K,1020 [C])                  "turbine inlet temperature"
rp_c=3.5[-]                                       "compressor pressure ratio"
rp_pt=2[-]                                        "power turbine pressure ratio"
W_dot_pt=320 [hp]*convert(hp,W)                   "power delivered to ship propulsion
system"

"Performance parameters"
eta_c=0.82[-]                                     "compressor efficiency"
eta_gt=0.85[-]                                    "gasifier turbine efficiency"
eta_pt=0.81[-]                                    "power turbine efficiency"
DT_rec=50 [K]                                     "recuperator approach temperature
difference"

"State 1"
T[1]=T_amb                                        "temperature"
P[1]=P_atm                                        "pressure"
s[1]=entropy(Air,T=T[1],P=P[1])                   "entropy"
h[1]=enthalpy(Air,T=T[1])                         "enthalpy"
```

```
"State 2"

P[2]=rp_c*P[1]                               "pressure"

s_s[2]=s[1]                                  "entropy balance on reversible

compressor"

h_s[2]=enthalpy(Air,s=s_s[2],P=P[2])         "enthalpy leaving reversible

compressor"

h[2]=h[1]+((h_s[2]-h[1])/eta_c)              "enthalpy leaving actual compressor"

s[2]=entropy(Air,h=h[2],P=P[2])              "entropy leaving actual compressor"

T[2]=temperature(Air,h=h[2])                 "temperature leaving actual

compressor"


"State 4"

T[4]=T_t_in                                  "temperature"

P[4]=P[2]                                    "pressure"

h[4]=enthalpy(Air,T=T[4])                    "enthalpy"

s[4]=entropy(Air,T=T[4],P=P[4])              "entropy"


"State 5"

s_s[5]=s[4]                                  "entropy balance on reversible gasifier

turbine"

h_s[5]=enthalpy(Air,s=s_s[5],P=P[5])         "enthalpy leaving reversible gasifier

turbine"
```

```
h[5]=h[4]-(h[2]-h[1])                          "enthalpy leaving actual gasifier
turbine"
s[5]=entropy(Air,h=h[5],P=P[5])                "entropy leaving actual gasifier
turbine"
T[5]=temperature(Air,h=h[5])                   "temperature leaving actual gasifier
turbine"
P[5]=P[6]


"State 6"
T[6]=T_t_in                                    "temperature"
P[6]=rp_pt*P[7]                                "pressure"
h[6]=enthalpy(Air,T=T[6])                      "enthalpy"
s[6]=entropy(Air,T=T[6],P=P[6])                "entropy"



"State 7"
P[7]=P_atm                                     "power turbine exit pressure"
s_s[7]=s[6]                                    "entropy balance on reversible power
turbine"
h_s[7]=enthalpy(Air,s=s_s[7],P=P[7])           "enthalpy leaving reversible power
turbine"
h[7]=h[6]-(eta_pt*(h[6]-h_s[7]))               "enthalpy leaving actual power
turbine"
s[7]=entropy(Air,h=h[7],P=P[7])                "entropy leaving actual power turbine"
```

```
T[7]=temperature(Air,h=h[7])                    "temperature leaving actual power
turbine"


"State 3"
T[3]=T[7]-DT_rec                                "temperature"
P[3]=P[2]                                       "pressure"
h[3]=enthalpy(Air,T=T[3])                       "enthalpy"
s[3]=entropy(Air,T=T[3],P=P[3])                 "entropy"


"State 8"
h[2]+h[7]=h[3]+h[8]
P[8]=P[7[                                       "pressure"
T[8]=temperature(Air,h=h[8])                    "temperature"
s[8]=entropy(Air,h=h[8],P=P[8])                 "specific entropy"



Q_dot_rec=h[3]-h[2]                             "recuperator heat transfer per unit
mass"
h_3_max=enthalpy(Air,T=T[7])                    "maximum enthalpy leaving cold side"
Q_dot_rec_max=h_3_max-h[2]                      "maximum recuperator heat transfer
per unit mass"
eff_rec=Q_dot_rec/Q_dot_rec_max                 "effectiveness of recuperator"
```

```
"efficiency"

w_dot_out=(h[4]-h[5])+(h[6]-h[7])                 "the output work of the both turbines"

w_dot_in=(h[2]-h[1])                              "the input work of the compressor"

q_dot_in=(h[6]-h[5])+(h[4]-h[2]                   "input thermal energy"

w_dot_net=w_dot_out-w_dot_in                      "the net output energy of the system"

eta_th=w_dot_net/q_dot_in                         "efficiency"
```

b. Solving The Problem By Using The Obtained Equations In EES

```
"Input Conditions"
T_amb=converttemp(C,K,25[C])                         "ambient temperature"
P_atm=1 [atm]*convert(atm,Pa)                        "atmospheric Pressure"
T_t_in=converttemp(C,K,1020 [C])                     "turbine inlet"
rp_c=3.5[-]                                          "compressor pressure ratio"
rp_pt=2[-]                                           "power turbine pressure ratio"
W_dot_pt=320 [hp]*convert(hp,W)                      "power delivered to ship propulsion
system"

"performance parameters"
eta_c=0.82[-]                                        "compressor efficiency"
eta_gt=0.85[-]                                       "gasifier turbine efficiency"
eta_pt=0.81[-]                                       "power turbine efficiency"
DT_rec=50 [K]                                        "recuperator approach temperature
defference"

"state 1"
T[1]=T_amb                                           "temperature"
P[1]=P_atm                                           "pressure"
h[1]=2.42510557613+(0.992324*T[1])-
((T[1]^2)*0.000035351212993866)+((T[1]^3)*(1.264724545582e-7))-((T[1]^4)*4.004439991e-
11)                                                  "specific enthalpy"
```

```
Pr[1]=1.13286229442-1.137234107131*10^(-2)*T[1]+3.8380551092451*10^(-5)*T[1]^2-
1.9056375223946*10^(-8)*T[1]^3+7.956892602*10^(-11)*T[1]^4+2.5425253418641*10^(-
14)*T[1]^5                                          "reduced pressure at the state 1"


"state 2"
P[2]=rp_c*P[1]                                      "pressure"
Pr[2]=rp_c*Pr[1]                                    "reduced pressure at state 2"
h_s[2]=((5.723933*48.769311+(13319.743517*Pr[2]^0.299578))/(48.769311+Pr[2]^0.299578))
                                                    "specific enthalpy leaving reversible
compressor"
h[2]=h[1]+((h_s[2]-h[1])/eta_c)                     "specific enthalpy leaving actual
compressor"


T[2]=-5.7559645569+1.03468305147*h[2]-0.0000376039987284*h[2]^2-5.044969502941E-
08*h[2]^3+1.88651774609433E-11*h[2]^4
                                                    “eaving actual compressor"
"state 4"
T[4]=T_t_in                                         "tempreture"
P[4]=P[2]                                           "pressure"
h[4]=2.42510557613+(0.992324*T[4])-
((T[4]^2)*0.000035351212993866)+((T[4]^3)*(1.264724545582e-7))-((T[4]^4)*4.004439991e-
11)                                                 "specific enthalpy"
```

```
Pr[4]=1.13286229442-1.137234107131*10^(-2)*T[4]+3.8380551092451*10^(-5)*T[4]^2-
1.9056375223946*10^(-8)*T[4]^3+7.956892602*10^(-11)*T[4]^4+2.5425253418641*10^(-
14)*T[4]^5                                        "reduced pressure at the state 4"


"state 5"
h[5]=h[4]-(h[2]-h[1])
T[5]=-5.7559645569+1.03468305147*h[5]-0.0000376039987284*h[5]^2-5.044969502941E-
08*h[5]^3+1.88651774609433E-11*h[5]^4             "temperature leaving actual gasifier
turbine"
Pr[5]=1.13286229442-1.137234107131*10^(-2)*T[5]+3.8380551092451*10^(-5)*T[5]^2-
1.9056375223946*10^(-8)*T[5]^3+7.956892602*10^(-11)*T[5]^4+2.5425253418641*10^(-
14)*T[5]^5                                        "reduced pressure at the state 5"


"state 6"
T[6]=T_t_in                                       "tempreture"


Pr[6]=1.13286229442-1.137234107131*10^(-2)*T[6]+3.8380551092451*10^(-5)*T[6]^2-
1.9056375223946*10^(-8)*T[6]^3+7.956892602*10^(-11)*T[6]^4+2.5425253418641*10^(-
14)*T[6]^5                                        "reduced pressure at the state 6"


h[6]=2.42510557613+(0.992324*T[6])-
((T[6]^2)*0.000035351212993866)+((T[6]^3)*(1.264724545582e-7))-((T[6]^4)*4.004439991e-
11)                                               " specific enthalpy"
```

```
"state 7"
Pr[7]=(1/rp_pt)*Pr[6]                                  "reduced pressure at the state 6"
h_s[7]=((5.723933*48.769311+(13319.743517*Pr[7]^0.299578))/(48.769311+Pr[7]^0.299578))
                                                       "specific enthalpy leaving reversible

power turbine"

h[7]=h[6]-(eta_pt*(h[6]-h_s[7]))                       "specific enthalpy leaving actual
power turbine"

T[7]=-5.7559645569+1.03468305147*h[7]-0.0000376039987284*h[7]^2-5.044969502941E-
08*h[7]^3+1.88651774609433E-11*h[7]^4                  "temperature leaving actual power
turbine"

"state 3"
T[3]=T[7]-DT_rec                                       "temperature"
h[3]=13.281882397+(T[3]*0.919093)+(0.00012345426528*T[3]^2)-(9.677850513449*10^(-
9)*T[3]^3)                                             "specific enthalpy"
Pr[3]=1.13286229442-1.137234107131*10^(-2)*T[3]+3.8380551092451*10^(-5)*T[3]^2-
1.9056375223946*10^(-8)*T[3]^3+7.956892602*10^(-11)*T[3]^4+2.5425253418641*10^(-
14)*T[3]^5                                             "reduced pressure at the state 5"

"state 8"
h[2]+h[7]=h[3]+h[8]                                     "energy balance on recuperator"
```

```
T[8]=-5.7559645569+1.03468305147*h[8]-0.0000376039987284*h[8]^2-5.044969502941E-
08*h[8]^3+1.88651774609433E-11*h[8]^4                    "tempreture"
Q_dot_rec=h[3]-h[2]                                      "recuperator heat transfer per unit
mass"

h_3_max=2.42510557613+(0.992324*T[7])-
((T[7]^2)*0.000035351212993866)+((T[7]^3)*(1.264724545582e-7))-((T[7]^4)*4.004439991e-
11)                                                      “maximum enthalpy leaving cold side"
Q_dot_rec_max=h_3_max-h[2]                               "maximum recuperator heat transfer
per unit mass"

eff_rec=Q_dot_rec/Q_dot_rec_max                          "effectiveness of recuperator"

"efficiency"
w_dot_out=(h[4]-h[5])+(h[6]-h[7])                        "the output work of the both turbines"
w_dot_in=(h[2]-h[1])                                     "the input work of the compressor"
q_dot_in=(h[6]-h[5])+(h[4]-h[2])                         "input thermal energy"
w_dot_net=w_dot_out-w_dot_in                             "the net output energy of the system"
eta_th=w_dot_net/q_dot_in                                "efficiency"
```

c. Solving the problem by using the obtained equations in MATLab

```
T_amb=298.15        ;
P_atm=101325        ;
T_t_in=1293.15;
rp_c=3.5      ;
rp_pt=2 ;

eta_c=0.82    ;
eta_gt=0.85 ;
eta_pt=0.81 ;
DT_rec=50   ;

T1=T_amb ;
P1=P_atm ;
h1=2.42510557613+(0.992324*T1)-((T1^2)*0.000035351212993866)+((T1^3)*(1.264724545582e-7))-((T1^4)*4.004439991e-11)     ;
Pr1=1.13286229442-1.137234107131*10^(-2)*T1+3.8380551092451*10^(-5)*T1^2-1.9056375223946*10^(-8)*T1^3+7.956892602*10^(-11)*T1^4+2.5425253418641*10^(-14)*T1^5;

P2=rp_c*P1;
Pr2=rp_c*Pr1;
h_s2=((5.723933*48.769311+(13319.743517*Pr2^0.299578))/(48.769311+Pr2^0.299578));
h2=h1+((h_s2-h1)/eta_c)        ;
T2=-5.7559645569+1.03468305147*h2-0.0000376039987284*h2^2-5.044969502941E-08*h2^3+1.88651774609433E-11*h2^4;

T4=T_t_in    ;
```

```
P4=P2       ;
h4=2.42510557613+(0.992324*T4)-
((T4^2)*0.000035351212993866)+((T4^3)*(1.264724545582e-7))-((T4^4)*4.004439991e-11)
    ;
Pr4=1.13286229442-1.137234107131*10^(-2)*T4+3.8380551092451*10^(-5)*T4^2-
1.9056375223946*10^(-8)*T4^3+7.956892602*10^(-11)*T4^4+2.5425253418641*10^(-
14)*T4^5;

h5=h4-(h2-h1);
T5=-5.7559645569+1.03468305147*h5-0.0000376039987284*h5^2-5.044969502941E-
08*h5^3+1.88651774609433E-11*h5^4;
Pr5=1.13286229442-1.137234107131*10^(-2)*T5+3.8380551092451*10^(-5)*T5^2-
1.9056375223946*10^(-8)*T5^3+7.956892602*10^(-11)*T5^4+2.5425253418641*10^(-
14)*T5^5;

T6=T_t_in   ;
Pr6=1.13286229442-1.137234107131*10^(-2)*T6+3.8380551092451*10^(-5)*T6^2-
1.9056375223946*10^(-8)*T6^3+7.956892602*10^(-11)*T6^4+2.5425253418641*10^(-
14)*T6^5;
h6=2.42510557613+(0.992324*T6)-
((T6^2)*0.000035351212993866)+((T6^3)*(1.264724545582e-7))-((T6^4)*4.004439991e-11);

Pr7=(1/rp_pt)*Pr6;
h_s7=((5.723933*48.769311+(13319.743517*Pr7^0.299578))/(48.769311+Pr7^0.299578));
h7=h6-(eta_pt*(h6-h_s7));
T7=-5.7559645569+1.03468305147*h7-0.0000376039987284*h7^2-5.044969502941E-
08*h7^3+1.88651774609433E-11*h7^4;


T3=T7-DT_rec;
```

```
h3=13.281882397+(T3*0.919093)+(0.00012345426528*T3^2)-(9.677850513449*10^(-9)*T3^3)     ;
Pr3=1.13286229442-1.137234107131*10^(-2)*T3+3.8380551092451*10^(-5)*T3^2-1.9056375223946*10^(-8)*T3^3+7.956892602*10^(-11)*T3^4+2.5425253418641*10^(-14)*T3^5;

h8=h2+h7-h3;
T8=-5.7559645569+1.03468305147*h8-0.0000376039987284*h8^2-5.044969502941E-08*h8^3+1.88651774609433E-11*h8^4;

Q_dot_rec=h3-h2      ;
h_3_max=2.42510557613+(0.992324*T7)-((T7^2)*0.000035351212993866)+((T7^3)*(1.264724545582e-7))-((T7^4)*4.004439991e-11)      ;
Q_dot_rec_max=h_3_max-h2  ;
eff_rec=Q_dot_rec/Q_dot_rec_max      ;

w_dot_out=(h4-h5)+(h6-h7)    ;
w_dot_in=(h2-h1)      ;
q_dot_in=(h6-h5)+(h4-h2)      ;
w_dot_net=w_dot_out-w_dot_in          ;
eta_th=w_dot_net/q_dot_in     ;
```

Appendix B: Thermodynamic Air Table

Appendix B Table 1: Air thermodynamic property table, T-Pr [12]

T	Pr	T	Pr	T	Pr	T	Pr	T	Pr
200	0.3363	460	6.245	720	32.02	980	105.2	1240	272.3
213	0.41979	473	6.8998	733	34.254	993	110.92	1253	284.325
226	0.51622	486	7.6016	746	36.61	1006	116.82	1266	296.68
239	0.62672	499	8.3523	759	39.078	1019	122.93	1279	309.42
252	0.75442	512	9.1616	772	41.718	1032	129.34	1292	322.7
265	0.89975	525	10.027	785	44.45	1045	135.95	1305	336.3
278	1.06292	538	10.954	798	47.31	1058	142.84	1318	350.34
291	1.24624	551	11.94	811	50.412	1071	150.12	1331	365.04
304	1.45208	564	12.996	824	53.592	1084	157.58	1344	380.06
317	1.68152	577	14.116	837	56.8485	1097	165.32	1357	395.53
330	1.9352	590	15.31	850	60.345	1110	173.4	1370	411.65
343	2.218	603	16.586	863	63.9735	1123	181.71	1383	428.145
356	2.5272	616	17.936	876	67.802	1136	190.42	1396	445.24
369	2.8654	629	19.3859	889	71.8195	1149	199.45	1409	462.875
382	3.237	642	20.884	902	75.966	1162	208.7	1422	480.89
395	3.6435	655	22.495	915	80.36	1175	218.45	1435	499.675
408	4.0836	668	24.194	928	84.942	1188	228.52	1448	518.98
421	4.5613	681	25.994	941	89.666	1201	238.84	1461	538.685
434	5.0818	694	27.894	954	94.684	1214	249.69	1474	559.29
447	5.6421	707	29.906	967	99.87	1227	260.86	1487	580.385
								1500	601.9

Appendix B Table 2: Air thermodynamic property table, T-h [12]

T	h	T	h	T	H	T	h	T	h
200	199.97	460	462.02	720	734.82	980	1023.25	1240	1324.93
213	212.97	473	475.315	733	748.866	993	1038.0635	1253	1340.283
226	226	486	488.64	746	762.95	1006	1052.895	1266	1355.657
239	239.02	499	501.992	759	777.091	1019	1067.7475	1279	1371.0555
252	252.058	512	515.382	772	791.29	1032	1082.666	1292	1386.478
265	265.1	525	528.805	785	805.51	1045	1097.6025	1305	1401.9175
278	278.126	538	542.276	798	819.758	1058	1112.559	1318	1417.381
291	291.162	551	556.683	811	834.0665	1071	1127.5265	1331	1432.872
304	304.214	564	569.338	824	848.4	1084	1142.526	1344	1448.378
317	317.278	577	582.905	837	862.765	1097	1157.593	1357	1463.9065
330	330.34	590	596.52	850	877.175	1110	1172.675	1370	1479.465
343	343.441	603	610.173	863	891.6135	1123	1187.7735	1383	1495.037
356	356.544	616	623.854	876	906.102	1136	1202.912	1396	1510.624
369	369.661	629	637.574	889	920.6265	1149	1218.0775	1409	1526.229
382	382.792	642	651.344	902	935.175	1162	1233.262	1422	1541.847
395	395.93	655	665.155	915	949.7675	1175	1248.485	1435	1557.4925
408	409.092	668	679.006	928	964.396	1188	1263.72	1448	1573.158
421	422.277	681	692.152	941	979.0515	1201	1278.966	1461	1588.838
434	435.502	694	706.82	954	993.761	1214	1294.254	1474	1604.542
447	448.743	707	720.809	967	1008.495	1227	1309.577	1487	1620.253
								1500	1635.97

Appendix B Table 3: Air thermodynamic property table, h-Pr [12]

h	Pr	**h**	**Pr**	**h**	**Pr**	**h**	**Pr**	**h**	**Pr**
199.97	0.3363	462.02	6.245	734.82	32.02	1023.25	105.2	1324.93	272.3
212.97	0.41979	475.315	6.8998	748.866	34.254	1038.0635	110.92	1340.283	284.325
226	0.51622	488.64	7.6016	762.95	36.61	1052.895	116.82	1355.657	296.68
239.02	0.62672	501.992	8.3523	777.091	39.078	1067.7475	122.93	1371.0555	309.42
252.058	0.75442	515.382	9.1616	791.29	41.718	1082.666	129.34	1386.478	322.7
265.1	0.89975	528.805	10.027	805.51	44.45	1097.6025	135.95	1401.9175	336.3
278.126	1.06292	542.276	10.954	819.758	47.31	1112.559	142.84	1417.381	350.34
291.162	1.24624	556.683	11.94	834.0665	50.412	1127.5265	150.115	1432.872	365.04
304.214	1.45208	569.338	12.996	848.4	53.592	1142.526	157.58	1448.378	380.06
317.278	1.68152	582.905	14.116	862.765	56.8485	1157.593	165.315	1463.9065	395.53
330.34	1.9352	596.52	15.31	877.175	60.345	1172.675	173.4	1479.465	411.65
343.441	2.218	610.173	16.586	891.6135	63.9735	1187.7735	181.71	1495.037	428.145
356.544	2.5272	623.854	17.936	906.102	67.802	1202.912	190.42	1510.624	445.24
369.661	2.8654	637.574	19.386	920.6265	71.8195	1218.0775	199.445	1526.229	462.875
382.792	3.237	651.344	20.884	935.175	75.966	1233.262	208.7	1541.847	480.89
395.93	3.6435	665.155	22.495	949.7675	80.36	1248.485	218.45	1557.4925	499.675
409.092	4.0836	679.006	24.194	964.396	84.942	1263.72	228.52	1573.158	518.98
422.277	4.5613	692.152	25.994	979.0515	89.666	1278.966	238.835	1588.838	538.685
435.502	5.0818	706.82	27.894	993.761	94.684	1294.254	249.69	1604.542	559.29
448.743	5.6421	720.809	29.906	1008.495	99.87	1309.577	260.86	1620.253	580.385
								1635.97	601.9

Appendix B Table 4: Air thermodynamic property table, T-u [12]

T	u	T	u	T	u	T	u	T	u
250	178.28	390	278.93	590	427.15	820	608.59	1220	951.09
260	185.45	400	286.16	600	434.78	840	624.95	1240	968.95
270	192.6	410	293.43	610	442.42	860	641.4	1260	986.9
280	199.75	420	300.69	620	450.09	880	657.95	1280	1004.76
285	203.33	430	307.99	630	457.78	900	674.58	1300	1022.82
290	206.91	440	315.3	640	465.5	920	691.28	1320	1040.88
295	210.49	450	322.62	650	473.25	940	708.08	1340	1058.94
298	212.64	460	329.97	660	481.01	960	725.02	1360	1077.1
300	214.07	470	337.32	670	488.81	980	741.98	1380	1095.26
305	217.67	480	344.7	680	496.62	1000	758.94	1400	1113.52
310	221.25	490	352.08	690	504.45	1020	776.1	1420	1131.77
315	224.85	500	359.49	700	512.33	1040	793.36	1440	1150.13
320	228.42	510	366.92	710	520.23	1060	810.62	1460	1168.49
325	232.02	520	374.36	720	528.14	1080	827.88	1480	1186.95
330	235.61	530	381.84	730	536.07	1100	845.33	1500	1205.41
340	242.82	540	389.34	740	544.02	1120	862.79		
350	250.02	550	396.86	750	551.99	1140	880.35		
360	257.24	560	404.42	760	560.01	1160	897.91		
370	264.46	570	411.97	780	576.12	1180	915.57		
380	271.69	580	419.55	800	592.3	1200	933.33		

Appendix B Table 5: Air thermodynamic property table, T-Vr [12]

T	Vr	T	Vr	T	Vr	T	Vr	T	Vr
250	979	390	321.5	590	110.6	820	44.84	1220	13.747
260	887.8	400	301.6	600	105.8	840	41.85	1240	13.069
270	808	410	283.3	610	101.2	860	39.12	1260	12.435
280	738	420	266.6	620	96.92	880	36.61	1280	11.835
285	706.1	430	251.1	630	92.84	900	34.31	1300	11.275
290	676.1	440	236.8	640	88.99	920	32.18	1320	10.747
295	647.9	450	223.6	650	85.34	940	30.22	1340	10.247
298	631.9	460	211.4	660	81.89	960	28.4	1360	9.78
300	621.2	470	200.1	670	78.61	980	26.73	1380	9.337
305	596	480	189.5	680	75.5	1000	25.17	1400	8.919
310	572.3	490	179.7	690	72.56	1020	23.72	1420	8.526
315	549.8	500	170.6	700	69.76	1040	23.29	1440	8.153
320	528.6	510	162.1	710	67.07	1060	21.14	1460	7.801
325	508.4	520	154.1	720	64.53	1080	19.98	1480	7.468
330	489.4	530	146.7	730	62.13	1100	18.896	1500	7.152
340	454.1	540	139.7	740	59.82	1120	17.886		
350	422.2	550	133.1	750	57.63	1140	16.946		
360	393.4	560	127	760	55.54	1160	16.064		
370	367.2	570	121.2	780	51.64	1180	15.241		
380	343.4	580	115.7	800	48.08	1200	14.47		

Appendix B Table 6: Air thermodynamic property table, T-S^0[12]

T	S0	T	S0	T	S0	T	S0	T	S0
250	1.51917	390	1.96633	590	2.3914	820	2.74504	1220	3.19834
260	1.55848	400	1.99194	600	2.40902	840	2.7717	1240	3.21751
270	1.59634	410	2.01699	610	2.42644	860	2.79783	1260	3.23638
280	1.63279	420	2.04142	620	2.44356	880	2.82344	1280	3.2551
285	1.65055	430	2.06533	630	2.46048	900	2.84856	1300	3.27345
290	1.66802	440	2.0887	640	2.47716	920	2.87324	1320	3.2916
295	1.68515	450	2.11161	650	2.49364	940	2.89748	1340	3.30959
298	1.69528	460	2.13407	660	2.50985	960	2.92128	1360	3.32724
300	1.70203	470	2.15604	670	2.52589	980	2.94468	1380	3.34474
305	1.71865	480	2.1776	680	2.54175	1000	2.9677	1400	3.362
310	1.73498	490	2.19876	690	2.55731	1020	2.99034	1420	3.37901
315	1.75106	500	2.21952	700	2.57277	1040	3.0126	1440	3.39586
320	1.7669	510	2.23993	710	2.5881	1060	3.03449	1460	3.41247
325	1.78249	520	2.25997	720	2.60319	1080	3.05608	1480	3.42892
330	1.79783	530	2.27967	730	2.61803	1100	3.07732	1500	3.44516
340	1.8279	540	2.29906	740	2.6328	1120	3.09825		
350	1.85708	550	2.31809	750	2.64737	1140	3.11883		
360	1.88543	560	2.33685	760	2.66176	1160	3.13916		
370	1.91313	570	2.35531	780	2.69013	1180	3.15916		
380	1.94001	580	2.37348	800	2.71787	1200	3.17888		