A Low-Cost, High-Speed Algorithm for Mobile Positioning

Amir Hossein Akhavan Salmasi

Submitted to the
Institute of Graduate Studies and Research
in partial fulfillment of the requirements for the Degree of

Master of Science
in
Computer Engineering

Eastern Mediterranean University
September 2011
Gazimağusa, North Cyprus

Approval of the Institute of Graduate Studies and Research

_____
Prof. Dr. Elvan Yılmaz
Director

I certify that this thesis satisfies the requirements as a thesis for the degree of Master of Science in Computer Engineering.

_____
Assoc. Prof. Dr. Muhammed Salamah
Chair, Department of Computer Engineering

We certify that we have read this thesis and that in our opinion it is fully adequate in scope and quality as a thesis for the degree of Master of Science in Computer Engineering.

_____
Assoc. Prof. Dr. Muhammed Salamah
Supervisor

Examining Committee

1. Assoc. Prof. Dr. Muhammed Salamah   _____

2. Asst. Prof. Dr. Adnan Acan   _____

3. Asst. Prof. Dr. Gürcü Öz   _____

ABSTRACT

Location determination in cellular networks is one of the most important features of current 3rd generation (3G) wireless communication systems and is important criteria in Federal Communications Commission (FCC) ruling regarding the wireless emergency calling system (E911) in United States. This technology empowers many location based services namely Intelligent Transportation Systems (ITS) and electronic warfare positioning. Using the received signal strength measurement, several methods can be applied to estimate the cellular mobile station position. As an example, Angle of Arrival (AOA) and Time of Arrival (TOA) can be named. To estimate the MS location using these methods two smart antennas need to be implemented, although using more than two antennas lead to higher accuracy and the estimated location will be more precise. The used estimation methods so far, are trigonometric equation based methods, which are software based and sophisticated to be implemented in location determination aims.

In this thesis, we presented a low-cost, high speed hybrid TOA/AOA method that combines TOA and AOA methods and use the both method's advantages. In traditional hybrid AOA/TOA method which is software-oriented, trigonometric equations are used that makes the total computational cost to be increased, hence, is improper to be implemented in hardware. In our proposed method, we used simple add, subtract and shift operations that makes this method proper to be implemented in hardware e.g. FPGA. The comparison between the Traditional software-oriented hybrid AOA/TOA and our proposed method will be carried out in the chapters.

For these methods analysis and simulation, we used Matlab 7 package. Simulation results demonstrate that our proposed algorithm outperforms the AOA, TOA and software-oriented hybrid AOA/TOA methods in terms of total computation cost, derived accuracy and implementation simplicity.

ÖZ

Hücresel şebekeler konum belirleme, 3. nesil (3G) kablosuz haberleşme sistemlerinin en önemli özelliklerinden biridir ve Federal İletişim Komisyonu (FCC), Amerika Birleşik Devletleri kablosuz acil çağrı sistemi (E911) ile ilgili yönetici önemli kriterdir. Bu teknoloji, yani pek çok konum tabanlı servisler, Akıllı Ulaşım Sistemleri (ITS) ve elektronik harp konumlandırma güçlendirir. , Alınan sinyalin gücünü ölçme kullanarak, hücresel mobil istasyon konumunu tahmin etmek için çeşitli yöntemler uygulanabilir. Bir örnek olarak, Varış Varış (AOA) ve Zaman (TOD) Açısı adlandırılabilir. İkiden fazla anten kullanarak yüksek doğruluk ve tahmini konumu daha hassas olacak yol olsa da, iki akıllı antenler uygulanması gerekir, bu yöntemleri kullanarak MS konumu tahmin etmek için. Şimdiye kadar kullanılan tahmin yöntemleri, yazılım tabanlı ve konum belirleme uygulanmak üzere gelişmiş hedefliyor trigonometrik denklem tabanlı yöntemler.

Bu tezde, ÖY ve AOA yöntemleri birleştirir ve ÖY / AOA yöntemi hibrit yöntemi hem avantajları kullanmak, düşük maliyetli, yüksek hızlı sundu. Yazılım odaklı geleneksel hibrid AOA / ÖY yöntemi, trigonometrik denklemler, toplam maliyet hesaplama donanım uygulanmak üzere uygunsuz, bu nedenle, artan olması yaptığı kullanılır. Önerilen yöntem, toplama, çıkarma ve işlemleri yapar bu yöntem uygun donanım örneğin uygulanacak vardiya basit kullandı FPGA. Geleneksel yazılım odaklı hibrid AOA / TOA ve önerilen yöntem arasında karşılaştırma bölümlerde yapılacaktır.

Bu yöntemlerin analiz ve simülasyon için, Matlab 7 paketi kullanıldı. Simülasyon sonuçları önerilen algoritma AOA, TOD ve yazılım odaklı hibrid AOA / TOA yöntemleri, toplam hesaplama maliyet, elde edilen doğruluk ve uygulama basitliği açısından daha iyi performans gösteriyor.

Anahtar Kelimeler: Kablosuz yönlendirme, sinyal isleme, TOD, AOA,

Hibrid konumlandırma, Yerellestirme

To My Family

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

BS   Base Station

MS   Mobile Station

LBS   Location Based Services

ITS   Intelligent Transportation Systems

LOS   Line of Sight

NLOS   Non Line of Sight

AOA   Angle of Arrival

TOA   Time of Arrival

TDOA   Time Difference of Arrival

DOA   Direction of Arrival

RISC   Reduced Instruction Set Computer

FPGA   Field Programmable Gate Array

RTLS   Real-Time Locating Systems

FCC   Federal Communications Commission

USDOD   U.S Department of Defence

PPS   Precise Positioning Service

SPS   Standard Positioning Service

TTFF   Decresing the Time to First Fix

VOR   Very high frequency Omni-directional Range

GPS   Global Positioning System

AGPS   Assisted Global Positioning System

USCG	United States Coast Guard
CCG	Canadian Coast Guard
DR	Dead Reckoning
CID	Cell Identification

Chapter 1

INTRODUCTION

The popularity of cellular mobile phones has increased dramatically and its trend illustrates this increment will be continued in the upcoming years. In the last decade, in August 2000, mobile phone user's population size was approximately 570 millions, while in 2010; BBC News announced there exist Over 5 billion cellular phone users all around the world[1]. This increase represents this technology's inevitable importance in nowadays lives. This technology entered the markets in the late 70's. After about 3 decades, cellular phone user's population conquered the landline ones[2]. This user population needs to be supported with their requisites, e.g.first aid delivery in times ofcrisis. To hand such services, emergency service provider has to be aware of the person approximate location[3]. In some cases the service demandantdoes not have information about his geographical location. Insuchcircumstances, the internet service provider must provide the emergency service provider with the applicant geographical location, but how?!

Mobile user positioning attractedmuch attentionandbecome oneofthe ambiguousissuesin the field ofwireless networks, recently. Various application and services based on mobile location are presented; such services are called Location-Based Services (LBS)[4]. LBSs are services attained by mobile device through mobile network utilizing mobile phone's

geographical location finding ability. Mobile location can be found either by Mobile Station (MS)-based or MS-assisted algorithms running on both MS and Base Station (BS) cooperating with each other to find MS's location. Some location based service and application examples are E-911 service in north america,E-112 in Europe, navigation systems, mobile commerce, weather services and Intelligent Transportation Systems (ITS) . Enhanced 911(E-911) was an improvement over the basic 911, equipping public safety services with knowledge of applicant approximate location. Location based systems concept is not compatible with real-time locating systems standard and related local services (RTLS) as was indicated in ISO/IEC 19762-5 and ISO/IEC 24730-1.

By October 1,2001, U.S. Federal Communications Commission (FCC) determined a rule, under which, public safety answering point (PSAP) attendants of wireless communications networks must be able to know 911 caller's phone number for calling them back and their location so that the call can be transferred to an appropriate PSAP. FCC declared E911, a mandatory requirement for wireless communications services such as cellular telephone, wideband (broadband) personal communications services (PCS), and geographic area specialized mobile radio (SMR). FCC delimited location accuracy to 125 meter in first phase, and then in September 15, 1999, tightened first one implementing phase II to bound the location accuracy to 100 meters for 67% of calls. Table 1.1, illustrates FCC location accuracy level.

Table 1.1. FCC phase II location accuracy

Solutions	67% of Calls	95% of Calls
Handset-Based	50 meters	150 meters
Network-Based	100 meters	300 meters

FCC Asked mobile phone manufacturers and vendors to equip their handsets with location-capable tools no later than March 1, 2001.

Location determination approach is also an important issue for enhanced network operations. In microscopic level point of view, accurately determining MS location movement causes cellular communication network controller to make more accurate decisions when to hand over (Also called handoff) from the BS which MS is getting service to it's neighbor when it's Received Signal Strength(RSS) degrades below a certain threshold.

Currently there are three most commonly used location technologies:

1. Satellite Based
2. Terrestrial radio based
3. Stand-alone

For satellite-based location technology, Global Positioning System (GPS) can be named. This technology will be reviewed in details next chapter.

A typical terrestrial radio-based technology is Long Range Navigation, "C" configuration (LORAN-C). LORAN uses low frequency transmitters in multiple

deployments to determine the location and moving speed. LORAN-C operates in low frequency portion from 90 KHz to 110 KHz. Many countries used this technology in their navigation systems e.g. US, Japan, UK, France and etc. Russia uses another navigation system in the same frequency range named CHAYKA. United States Coast Guard (USCG), Canadian Coast Guard (CCG) and Russia stopped using these systems 2010. Cellular Networks are terrestrial radio based communication systems. It is clear that emitted radio signal between MS and BS can be used to locate MS geographical location using smart antennas and signal processing algorithms used by. Smart antennas are antenna arrays using smart signal processing algorithms to find MS receiving Direction of Arrival (DOA) and use them to calculate and estimate MS's location, e.g. received signal Angle of Arrival (AOA). Different algorithm are used to locate the mobile's position, using the time-stamp signals received at network-side(TOA),Direction of received signals(AOA) or the time difference signals are received at different BSs(TDOA) can be named. This approach will also be investigated in details next chapter.

In stand-alone issue, dead reckoning can be exampled. Dead Reckoning (DR) is the process of location MS position based on previous determined positions. Dead reckoning traditional methods are not held as primary means of navigation and modern inertial navigation systems anymore. This technology's disadvantage is since new MS estimated locations are calculated from previous positions, estimated location error is cumulative of previous positions, hence, estimated error grows by time.

Chapter 2

LOCATION DETERMINATION BACKGROUND

2.1   Location Determination Technologies

In this chapter since here, I explained the radio communication specifications and radio signal types. In this chapter, I specifically focus on the technologies to find the MS location using the articles that have been previously described.

Currently there are four different location determination types

1. Handset based

2. Handset based/Network assisted

3. Network based

4. Network based/Handset assisted

In the handset based technology, location determination algorithm is executed in the handset, while in the 2nd group, handset communicating the network side and the BS, estimates its geographical position. In network based technology, MS location calculation process is executed in the network side of the communication and when it is found, it will be sent to the handset. In handset based/network assisted and network based/ handset assisted, two network sides must cooperate to estimate the position. In network based methods, since the location estimation algorithm is executed in the network, no modification in the handset is needed. But in the case that the handset must determine the location, some software or hardware modification, depending on the used

method, must be done in handset. There are different methods in these four groups estimating geographical location. As an example AOA, TOA, Cell ID (CID), short range beacons, Signal strength and GPS can be named.

2.1.1 Cell-ID (CID)

The simplest MS location determination is finding the cell area where the cellular phone is being used. The antenna location then can be used as criteria to estimate the MS location. Since the antenna location is already specified and somewhere constant, the MS location can be estimated instantaneously. The estimated location error depends on the cell coverage area, which according to the cellular antenna type (e.g. Macro cell, Micro cell, Femto cell,…), it varies from 50 meters indoors to 150 meters in urban and up to 30 km in rural areas. Hence, the estimated location is not precise enough to be used and does not satisfy FCC E-911 mandatory accuracy level. This method also has another weak point, which is the serving cell is not always the closest one to the MS, which decreases this method's accuracy level and makes this method unreliable. Although this method is not accurate enough to be implemented, it has some advantages over the other methods. This method can be used in all existing networks; no soft or hardware modification is needed in both sides. Also since there are no algorithms to be executed in this method, its computational cost is zero and gives the results so fast, typically 3 seconds depending to the network size.

2.1.2 Signal Strength Level

This technique is generally based on the fact that the received signal strength (RSS) degrades moving far from the BS. In ideal conditions, signal level in the circle around and close to the BS has the perfect strength. Hence, the MS located close to the BS receives higher signal power compared with the further MS. These two received signals may affect each other's strength and act like an interference. This effect is called near-far problem.

This method determines the MS location by finding where three different antenna signal level intersect.
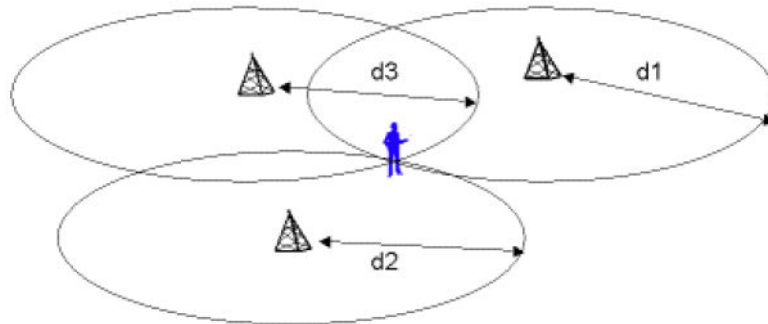
Figure2.1. Determining MS location using signal strength

Figure 2.1. shows signal strength method, location estimation using three different BSs. This method works like TOA method that will be explained by the following methods. Different factors effect received signal strength. The obstacles facing the emitted signals, causes signal diffraction, multipath effect, etc. These phenomenon makes the signal to fluctuate even the MS is not moving. This signal strength variations causes error in estimated location.

2.1.3　Short Range Beacons

Another location determination method is short range beacon transmission. Beacon signals are periodically exchanged across the communication link to catch the location information. BS periodically (1 signal per second) broadcasts the beacon signal to all its coverage area MSs. The receiving MS, if it was the first time it receives the beacon signal, it adds it to its *active beaconkernel table*. This beacon signal contains some parameters:

- cellular network identifier
- timestamp
- gateway address ID of the paging area
- etc

2.1.4   GPS

The GPS project was started in 1973 to rectify the previous method's limitations and became fully functional in 1994. This system was created by the U.S. Department of Defense (USDOD) and consists of 24 satellites spinning around the earth. Any of this 24 satellites orbit the same track and configuration over any point each 24 hours. As of March 2008, 31 actively broadcasting satellites are functioning in GPS system. There exist six orbital planes and four satellites par any plane. It makes five up to eight satellites visible for the MS at any point of the earth all the time. All these satellites are equipped with atomic clock, computer and radio signals to frequently calculate it transmit their positions. On the ground, GPS receivers receive the satellite signals and process them to estimate their own position and moving speed.

A satellite tracking stations located around the world, checks the signal's accuracy. This tracking system checks and computes the precise orbital data and clock corrections for all the satellites to be time synchronized. The master control facility located at Schriever Air Force Base in US, upload these data to all satellites frequently. Figure 2.2 depicts spinning satellites covering the ground in all points. The user on the earth can communicate with 8 different satellites at a time. The satellite orbits are arranged so that at least nine satellites are always visible and in line of sight (LOS) to the ground GPS receivers.
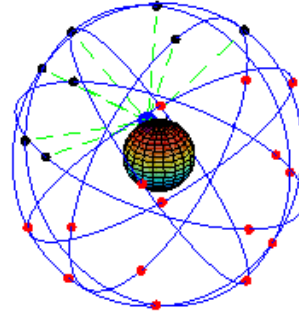
Figure 2.2. Satellite orbital model

This positioning service, offers two services:

- Precise Positioning Service (PPS): This method offers more accurate positioning service, 22 meter horizontally and 27.7 meter vertically. This service is available for authorized users with cryptographic equipment and keys. U.S. military, some authorized U.S. government agencies and civil users permitted by U.S. government are allowed to use PPS service.

- Standard Positioning Service (SPS): This service offers an accuracy of 100 meter horizontally and 156 meter vertically. All civil users around the world are allowed to use this service without any charge or restrictions.

Since March 2000, all SPS users are allowed to use PPS service.

To determine the MS location using this method, three satellites seems to be enough, but since the small clock error multiplied by light speed (speed that satellite signals propagate) leads to a high location estimation error. Hence, using four satellites is more accurate and that clock error would be abandoned.

2.1.5   Assisted GPS (AGPS)

The AGPS exploits the availability of the bidirectional link to uphold the cellular phone functionality that contains a partial or full GPS receiver. This technology basically consists of three parts:

- Location server
- Mobile Station (MS) which is partial or full GPS receiver
- Wireless communication link

Using the AGPS brings some benefits over the non-assisted GPS. Some of these benefits are:

- Increasing the GPS receiver sensitivity by equipping the GPS receiver with the additional positioning information to make sure that the positioning can be done even in bad environments or bad climates.
- Decreasing the Time To First Fix (TTFF) from more than 30 seconds to just a few seconds.
- Increasing the accuracy level up to that of DGPS by using assisted information.

Currently, AGPS localization method divides into two groups:

a) MS- Assisted GPS

In this method, MS assists the network to find the location by providing the data to the location server. The MS receives the satellite radio signals and estimates the corresponding error and real distance between the satellite location and the MS. Then this time stamped satellite psuedoranges will be sent to the location server, which is responsible for location determination calculations.

In this method, location server, shows all the satellite's locations and estimates the MS locations by calculating its distance with several satellites. The transmitted information from the MS to the location server includes:

- Location request

- Rough position of the user

- Time tagged GPS pseudoranges

The transmitted information from location server to MS includes:

- Satellites in reach to that MS

- Doppler frequency (f_D) correction for the satellites in reach

b) MS- Based GPS

In this type of GPS based method, location server provides the assistance information to the MS, so MS itself can calculate its geographical location. Location server also is responsible for monitoring satellites and provides helpful information for the specified users.

The transmitted radio signals from MS to the location server include:

- Location request

- Uneven location of the MS

- Calculated location of the MS

Another radio link which is from the location server to the MS, include:

- List of visible satellites

- Doppler frequency correction for the visible satellites

- Satellites astronomical calendar

- GPS differential corrections

2.1.6 Angle of arrival (AOA)

This location determination technique is based on obtained angle calculation of the received signal at the BS. To find the location using this method, at least two BSs need to be implemented, while using the 3rd antenna array will increase its accuracy. A calibrated antenna array must be used in each signal receiving sides to calculate the MS's angle. As an example, VOR system (Very high frequency Omni directional Range) used for aircraft navigation and military battlefield systems can be named for this method. This technique can be implemented both in handheld and network part, but since it is difficult to implement the necessary directional antenna on a handheld cellular phone, this system is dedicated to be used in network side and in BS only. Hence, no modification in handset device is needed. Figure 2.3.depicts AOA location determination procedure. Both antennas, using the built in algorithms, find the received signal's angle. Using this angle, two antenna signal arrays will overlap in a point. That point is where the MS is located.
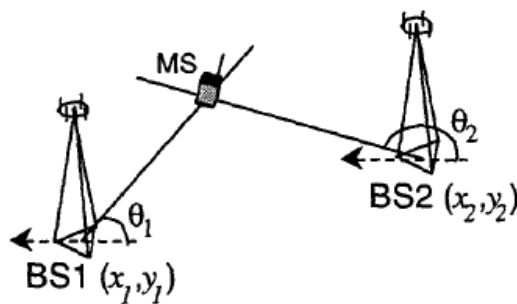

Figure2.3. Location estimation using AOA method

By moving far from the BSs, the estimated error will increase. Using smart antenna will limit this deduced error level. To use this method, antenna must be equipped with AOA location estimation algorithm.

2.1.7   Time of Arrival (TOA)

TOA method is based on measuring the time difference of the traversed signals along the MS and BSs. In this technique, the transmission exact starting time has to be known and all the BSs needs to be time synchronized, because even 1 micro second difference in BS time stamps will lead to 300 meters error in estimated location, which is so high according to FCC mandatory rule. BSs calculate the MS's distance by dividing the radio signal traversed time by 2 and multiply it by the light speed which is the radio signal's propagation speed. Deduced result is how far the MS is located to the BS. Which distance equation gives a range which plots a circle having the specified distance around the BS. Two different BS range circles will overlap in two individual pints that are the estimated MS location. 3rd antenna array will cross through one of two different points which is the final determined MS location.
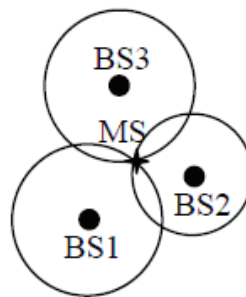
Figure2.4. TOA location determination method

Figure 2.4. illustrates TOA location estimation using three antenna arrays. For this location determination method, GPS can be named as an example.

Table 2.1. shows these six methods and the technologies used, mentioned earlier.

Table 2.1. Location Estimation Methods and the Technologies Used

Technology	Handset based	Handset based/ Netwotk assisted	Network based	Network based/ Handset assisted
Cell-ID (CID)			X	
Signal Level	X		X	
GPS	X			
AOA			X	
TOA			X	
AGPS		X		X

This table shows that both AOA and TOA methods are network base services and the location calculations are executed in the network, so handhelds are not needed to be configured to match and function in these two methods. In GPS method, calculations are done in the handset, while in AGPS method; both handset and network cooperate to find the MS location. Since in AGPS, some computations must be executed in MS, some

modifications must take place in it. This hardware modifications and the connection link to the satellite consumes the energy which is a bottleneck in wireless systems. Hence in some situations depending on the desired accuracy and the facilities, we incline to use network based location estimation methods.

Chapter 3

HYBRID AOA/TOA LOCATION DETERMINATION ALGORITHMS

In this chapter, I will explain the algorithm I used to compare our proposed algorithm with, its derived computational cost in details followed by our proposed algorithm and its characteristics, derived error and computational cost. In this thesis, I proposed a hybrid location estimation method, which combines AOA and TOA that I mentioned in the previous chapter.

Hybrid location estimation algorithm combines AOA/TOA methods together. Using this technique,two antenna arrays need to be implemented, one running AOA algorithm and the other executing TOA algorithm to find MS location. Although this algorithm implemented TOA method, it has less complexity since two antenna arrays are required, while in TOA method, two antenna arrays give two possible MS geographical locations and the 3rd antenna array coverage area circle, passed through the minimum distance geographical estimated location, which is MS determined location.

In first part I will explain the algorithm proposed in Dec 2007 that I called traditional hybrid location estimation method. In second part I will define our proposed algorithm and explain it in details. In next chapter I will illustrate derived simulation results in term of error and accuracy level and total computational cost.

3.1 Traditional Hybrid AOA/TOA Algorithm

MS location can be determined using single reference device consisting of two antennas, spacing in two locations $A1 = [-l,0]^T$ and $A2 = [l,0]^T$, where antenna locations are symmetric to the coordinate origin and MS with coordinates $P = [x, y]^T$, is located by distance d_1 from the antenna A1 and d_2 from 2nd antenna, where two antenna vectors overlap[7].
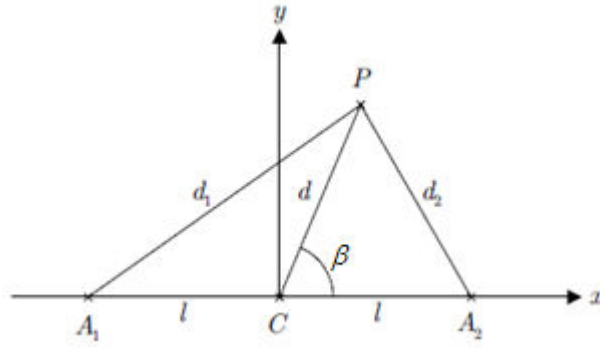
Figure 3.1. Traditional hybrid AOA/TOA method

Figure 3.1. depicts traditional hybrid TOA/AOA location estimation method. MS location distance from the origin point (d) can be calculated using the equation below:

$$d = \sqrt{\left(\frac{d_1^2 - d_2^2}{4l}\right)^2 + \left(d_1^2 - \left(\frac{d_1^2 - d_2^2}{4l} + l\right)^2\right)} \qquad (1)$$

The antenna location is spaced in specific coordinates and its x value (l) is constant. Hence in the equation above, d value can be calculated easily.

Depending on the quadrant of this coordinate system determined by the sign of $d_{12} = d_1 - d_2$, the AOA angle (β) can be calculated by:

$$\beta = 90° \pm \arcsin\left(\frac{\sqrt{4\left(l^2 + d^2\right) - \left(2\left(l^2 + d^2\right) - d_{12}^2\right)^2}}{4ld}\right)$$

(2)

Now d and α values are found. Using triangulation, the mobile station (P) coordinates can be calculated. Hence:

$$P = [x, y]^T = [d\cos(\beta), d\sin(\beta)]^T$$

(3)

3.1.1 The Proposed Low-Cost, High- Speed Hybrid TOA/AOA Algorithm

In our proposed algorithm, simple add, subtract and shift operations are used. This simplicity makes this algorithm attractive to the RISC (Reduced Instruction Set Computer) processors and FPGA chips. In signal processing projects, since RISC processors run faster, easier to design, test and manufacture, it is preferred to use them to process the algorithm. Our proposed algorithm is hardware-oriented location estimation method and is suitable to be implemented in FPGA and RISC processors, whichis a privilege over the other positioning methods. In the traditional software-oriented hybrid location determination method, during the execution cycle, many operations must be executed to locate MS position, hence is not proper to be used in FPGA and RISC processors.

It must be noted that our method will locate the MS for the fixed location. If the MS moves to another location, the algorithm needs to be executed again to estimate the MS

new location. Hence, the MS moving speed does not affect our method's estimated location accuracy.

In the proposed method, two antenna arrays must be implemented, any of them responsible for one operation, while in TOA method at least three antenna arrays are required to estimate the MS location. Using less number of antenna arrays compared with another positioning methods, is our proposed method's another privilege.

Our proposed algorithm has two different hardware-oriented phases:

- Vector circular rotation

- Vector length incrementation

These two phases must be executed in parallel to determine the MS location.

In first phase, according to the TOA algorithm, the vector will rotate around the origin having the rotation angle constant α radius to plot the circle of the possible MS location with the specified distance[5]. The circular rotation matrix M can be written as:

$$M = \begin{bmatrix} Cos\alpha & -Sin\alpha \\ Sin\alpha & Cos\alpha \end{bmatrix} \quad (4)$$

In the above equation, $Sin\,\alpha$ can be defined as:

$$Sin\alpha = 2^{-k} = b \quad (5)$$

And:

$$Cos\alpha = 1 - 2^{-(2k+1)} \quad (6)$$

We have the rotation matrix (M) so the rotated x and y coordinates would be:

$$\begin{bmatrix} x_{i+1} \\ y_{i+1} \end{bmatrix} = \begin{bmatrix} Cos\alpha & -Sin\alpha \\ Sin\alpha & Cos\alpha \end{bmatrix} \cdot \begin{bmatrix} x_i \\ y_i \end{bmatrix} \quad (7)$$

Hence to circular vector rotation equation can be written as:

$$x_{i+1} = x_i - x_i a + y_i b \quad (8)$$

$$y_{i+1} = y_i - y_i a - x_i b \quad (9)$$

The x and y coordinates initial values are:

$$x_{1,0} = 1$$

$$y_{1,0} = 0$$

In the other phase, the AOA antenna is aware of MS location angle [6]. Hence, the received signal angle is a constant value and any time this angle value changes (MS moving), we have to run the algorithm once more to find its new location according to its new angle. Knowing the location angle, the coverage vector must be incremented by 2^{-k} until its vector converge with TOA rotated point. The vector length incrementation can be written as:

$$X_{i+1} = x_i + 2^{-k} Cos\theta \quad (10)$$

$$Y_{i+1} = y_i + 2^{-k} Sin\theta \quad (11)$$

Figure 3.2. shows phases, Vector circular rotation and Vector length incrementation.
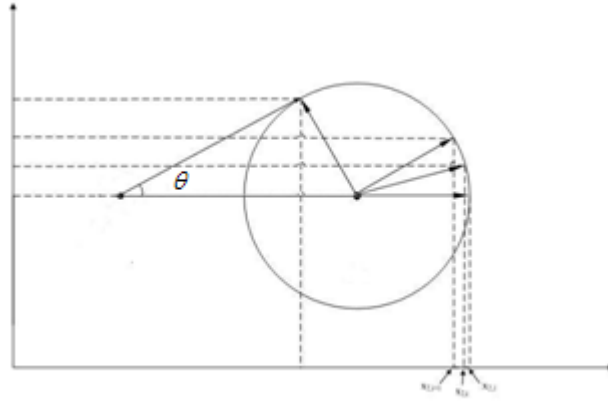
Figure 3.2. Circular rotation and vector incrementation

Our poposed location determination algorithm works as:

While $\Delta x_i = |x_{1,i} - x_{2,i}| > \varepsilon$

Rotate d_1 by step angle α $\begin{cases} x_{i+1} = x_i - x_i a + y_i b \\ y_{i+1} = y_i - y_i a - x_i b \end{cases}$

While $\Delta y_j = |y_{1,j} - y_{2,j}| > \varepsilon$

Increment d_2 by incrementation unit α $\begin{cases} X_{i+1} = x_i + 2^{-k} Cos\theta \\ Y_{i+1} = y_i + 2^{-k} Sin\theta \end{cases}$

End while

End while.
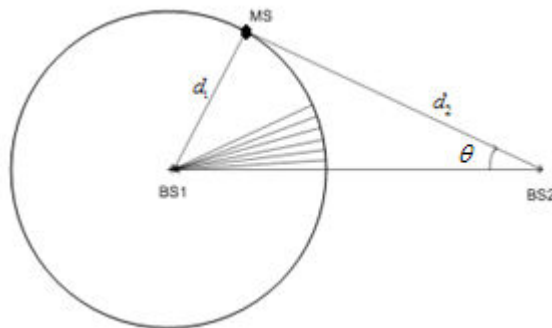
Figure 3.3. Vector d_1 rotation and d_2 incrementation

The algorithm loop will be repeated until the vector rotation and incrementation values converge in a point. This convergence will occur when:

$$\Delta x_i = \left| x_{1,i} - x_{2,i} \right| > \varepsilon \qquad (12)$$

$$\Delta y_j = \left| y_{1,j} - y_{2,j} \right| > \varepsilon \qquad (13)$$

In the rotation and incrementation equations, k is the accuracy level. In vector rotation equations, k is the rotation step size and in in vector incrementation, k is the incrementation value. In my thesis simulations, I assigned k equal to 5 up to 11. It is straight forward that with smaller rotation and incrementation k values ($k = 11$), our accuracy level will be increased out the determined geographical location will be more accurate.In next chapter, these estimated accuracies and its simulations will be demonstrated.

3.2   Hardware Implementation of Our ProposedAlgorithm

In the previous chapters it was declared this algorithm uses simple add, subtract and shift operations, hence can be implemented in hardware easily (e.g. FPGA). This simplicity makes the proposed algorithm proper in hardware implementation, since the number of operations and the complexity decreased in contrast with the traditional hybrid location determination and software-oriented algorithms. In this part I will explain our proposed algorithm in hardware implementation point of view.

In the proposed algorithm, two different procedures are executing, as it was denoted earlier:

- Vector circular rotation
- Vector length incrementation

In any of these processes, the mentioned equations are running to gain the result. In the first procedure, the MS locations will be estimated with the specific radius to the first antenna using these two formulas.

$$x_{i+1} = x_i - x_i a + y_i b \ (14)$$

$$y_{i+1} = y_i - y_i a - x_i b \quad (15)$$

Hardware implementation of these two rotation equations can be demonstrated in the Figure 3.4
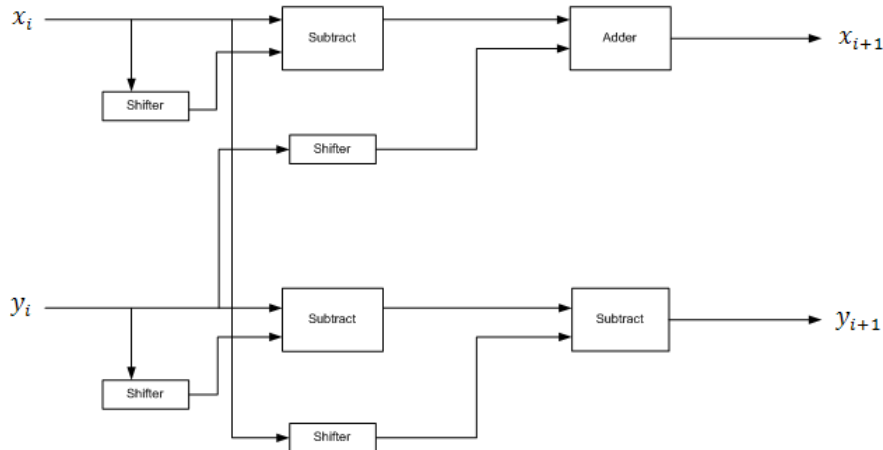
Figure 3.4. The Circular Rotator

Second antenna is responsible for the AOA location determination process. Knowing the MS angle, using these two formulas we can find the MS location by increasing the

vector length one unit until it converges the desired location which is approximated MS

location.

$$X_{i+1} = x_i + 2^{-k} Cos\theta \quad (16)$$

$$Y_{i+1} = y_i + 2^{-k} Sin\theta \qquad (17)$$

These equations hardware implementation scheme has been illustrated inFigure 3.5.
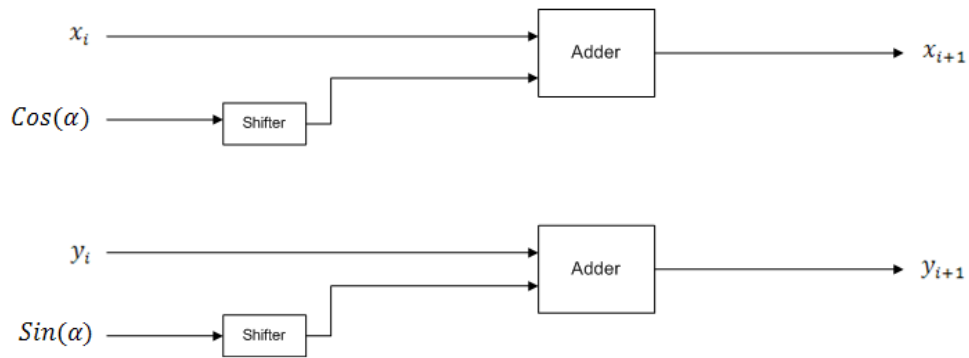
Figure 3.5. The Vector Incrementor

Shifter will increment the vector length by the specific step size where k is the rotation

and incrementation accuracy that we took this value from 5 up to 11.

The two figures above demonstrate our hardware algorithm scheme. It can be seen these

are simple operations and are appropriate to be implemented in RISC processors.

Chapter 4

# SIMULATION AND ANALYSIS RESULTS

To show our proposed algorithm simulation results, we used MATLAT 7 package. We used this package to compare our results with individual AOA and TOA and also traditional hybrid TOA/AOA algorithm. We run our simulation to find the location of the mobile in the coverage area of AOA and TOA BSs.

We compared the proposed algorithm with the traditional one, in terms of computational cost required to find the MS location and the estimated location error and its accuracy level. It was earlier mentioned that our proposed algorithm uses simple add, subtract and shift operations and since trigonometric equations have not been used, it is likely to be implemented in FPGA chips.

In this chapter we first compare the derived weight of operations, then its accuracy and estimated location error will be discussed respectively.

4.1   Computational Cost Simulation Results

The algorithm computational cost can be calculated by multiplying the number of operations by its corresponding weight [8]. Using Table 4.1, our proposed algorithm total computation cost for each step size for 20 bit accuracy can be calculated.

Table 4.1. Weights of the Operations

Operation	Weight
Addition	1
Subtraction	1
Shift	1
Multiplication	40
Division	40
Square Root	100
Sin	404
Arcsin	404
Cos	404
Tan	1448
Arctan	1448

Figure 4.1.shows our proposed algorithm compared with hardware-oriented TOA, hardware oriented AOA and the traditional algorithm.
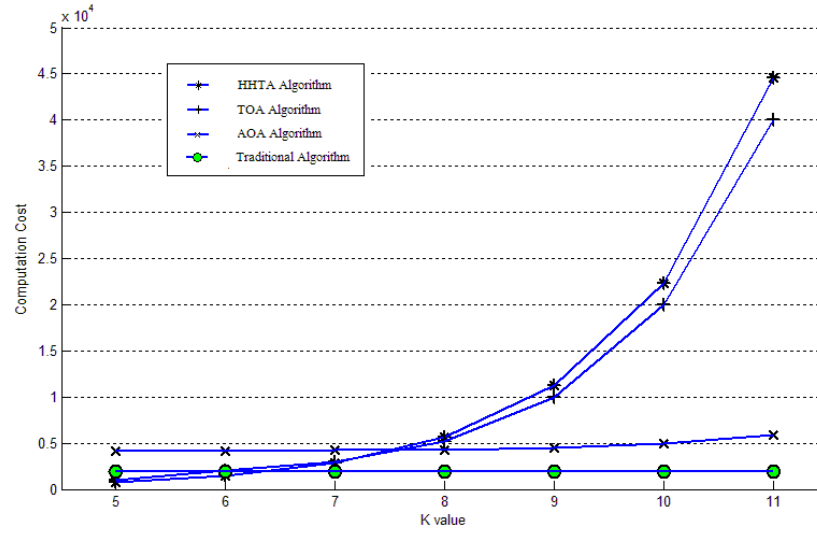
Figure4.1.Computational cost of our proposed, TOA, AOA and traditional algorithms

As it can be observed, our proposed algorithm computational cost outperforms the traditional algorithm for k value equal to 5, and almost close to it for k=6. Hence, it takes shorter time for the algorithm to be processed. During one processor clock, one operation will be processed so less number of operations will use fewer clocks to be processed so the algorithm will be executed faster. Our method's weight of operation increases with increasing the k value (decreasing the step size). This method's computational cost is almost equal to TOA algorithm although the proposed algorithm computational cost increased higher for k values more than 8. Our proposed algorithm got lower computation cost than hardware-oriented AOA for k values less and equal to 7, so more likely to be implemented in FPGA than AOA for such values.

It can be concluded that for k value equal to 5 (step size 2^{-5}), our algorithm earned the least computational cost compared to other three methods.

Running the proposed algorithm and getting the estimated results will take a little amount of time, varies depending on the step size. As the k increases, the time takes for the algorithm to be fully executed will increase. The table 4.2 illustrates the CPU time spent to process and execute our proposed algorithm and estimate the MS location.

Table 4.2 The proposed Algorithm execution CPU Time

K value	CPU Time (s)
5	0.1250
6	0.1875
7	0.3750
8	0.7656
9	1.6720
10	3.2500
11	6.5470

4.2   Estimated Location Error Simulation Results

It was earlier mentioned that FCC made a mandatory rule for all wireless service providers to provide emergency service 911 callers and Public Safety Answering Points (PSAP) with the location determination services, such that the caller asking for emergency service will be located within 100 meters of accuracy for 67 percent of calls, and within 300 meters for 95 percent of calls. Handheld GPS equipped devices also must

provide PSAPs with reports on their locations within 50 meters of accuracy for 67 percent of calls, and within 100 meters for 95 percent of calls.

Our proposed algorithm satisfied the FCC mandatory accuracy level. Figure 4.2.shows our method's location estimation error compared with two other hardware-oriented methods. The hardware-oriented AOA method has high error and hence low accuracy level while two other positioning methods perform better for all step sizes. Our proposed and hardware-oriented TOA method has almost close error, while our method'sresults show it outperforms to TOA method.
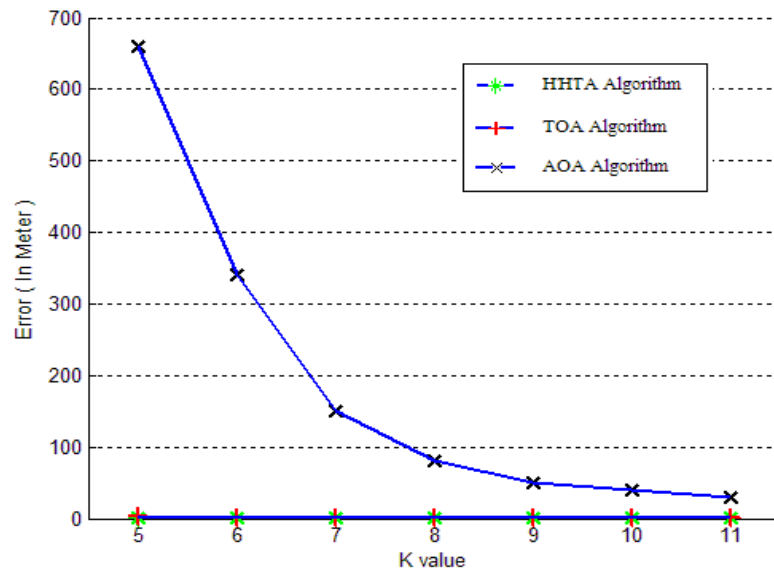

Figure4.2. Error comparison of the Proposed Method with TOA and AOA methods

To make the our method and TOA difference more clear, in Figure 4.3.we illustrate these two methods simulation results.
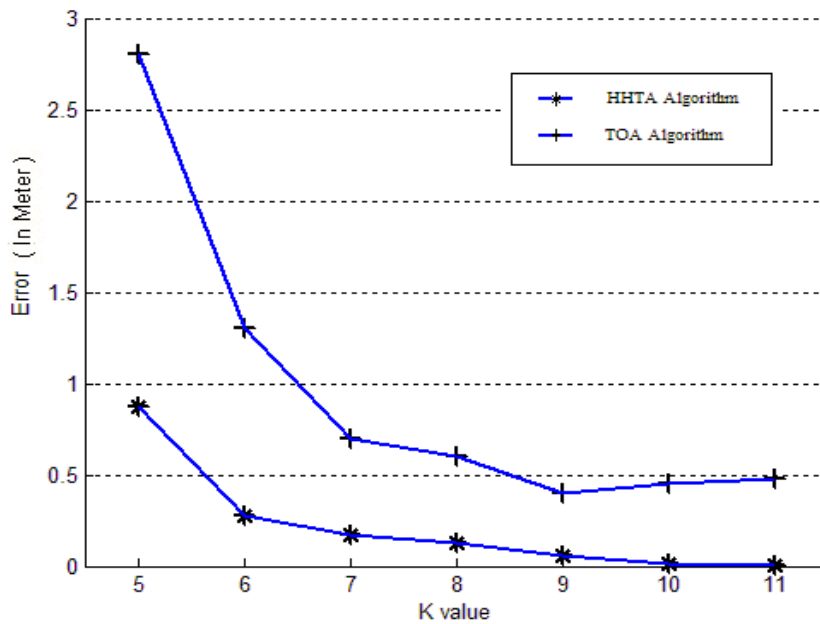
Figure4.3.Our Proposed Method and TOA Error comparison

This figure shows the proposed method in worst case (step size 2^{-5}) has approximately

0.9 meter error which is about one third of the TOA method for the same step size. By

decreasing our method's step size, its estimated error will dramatically decreased to the

point close to zero. For all step sizes, our proposed method got more accurate results

than hardware-oriented TOA method. Since our hybrid method uses simple operations,

not the trigonometric, and locates the MS position with higher accuracy, this method is

suitable to be implemented in FPGA. The estimated coordinates and the accuracy will be

indicated in next parts.

Chapter 5

CONCLUSION

In this thesis, we proposed a hardware oriented geographical location estimation algorithm using the cellular radio signals. This algorithm uses simple add, subtract and shift operations which make this algorithm proper to be used in hardware e.g. FPGA. This algorithm is a hybrid positioning method combining AOA and TOA positioning methods and clearly makes use of improvements these two algorithms brought. The MS located in the two smart antennas line of sight, each running one of these two algorithms can be geographically estimated using our hardware oriented algorithm with higher precision.

Energy consumption is a bottleneck in mobile systems; hence algorithm computational cost must kept as low as possible to save the power. To achieve this goal, we have to minimize the instructions and functions that the MS must execute in location estimation process. Since the RISC (Reduced Instruction Set Computer) processors are used in MS, the number of processing instructions will be reduced by using a simple hardware oriented algorithm. HTTA algorithm had the least computational cost for two k values (k=5, 6) compared with the AOA, TOA and the traditional algorithms.

This thesis offered a new algorithm which is hardware oriented one and since simple add, subtract and shift operations are used, is appropriate to be implemented in FPGA.

This algorithm execution simplicity makes it outperform other algorithms in terms of execution time and the estimated location accuracy.

The method we proposed, is hybrid AOA/ TOA method and outperforms these two hardware oriented algorithms in terms of location estimation accuracy, the dedicated results depicts its higher accuracy compared with AOA and TOA methods and satisfies the E-911 standards. This algorithm has less computational cost than other methods for k value less than 7.Our algorithm also outperforms the traditional algorithm in both software and hardware implementation point of view. The traditional hybrid algorithm requires some trigonometric calculations which are not appropriate to implement in hardware.

Currently, we are inquiring on the use of this algorithm in mobile phone tracking purposes.Another study point could be to modify this algorithm for finding the position of a MS in 3-Dspace. Integrating our proposed algorithm with GPS systems is also remarkable.

REFERENCES

[1] Yilin Zhao: Standardization of mobile phone positioning for 3G systems: Communications Magazine, IEEE 40, 108-116 (2002)

[2] Hung-Chi Chu, Rong-Hong Jan: Cell-based positioning method for wireless networks: Parallel and Distributed Systems ,375-380 (2002)

[3] Guolin Sun; Jie Chen; Wei Guo; Liu, K.J.R.: Signal processing techniques in network-aided positioning: a survey of state-of-the-art positioning designs, IEEE Signal Processing Society 22, 12-23 (2005)

[4] Bellavista, P.; Kupper, A.; Helal, S.: Location-Based Services: Back to the Future: Pervasive Computing, IEEE Computer Society 7, 85-85 (2008)

[5] M. Salamah, and E. Doukhnitch: An efficient algorithm for mobile objects localization: Int. Journal of Communication Systems 21, 301-310 (March 2008)

[6] M. Salamah, E. Doukhnitch, and C. Bayramer: A Dynamic Hardware-Oriented Algorithm for Angle of Arrival Positioning Technique: International Conference on Signal Processing and Communication (ICSPC07), Dubai, UAE, 201-204 (Nov. 2007)

[7] Galler, S., Gerok, W., Schroeder, J., Kyamakya, K., Kaiser, T.: Combined AOA/TOA UWB localization: Communications and Information Technologies, 2007. ISCIT '07 , Sydney, NSW, 1049-1053 (2007)

[8] Jean-Michael Muller: Elementary Function Algorithms and Implementation, Birkhauser, (1997)

[9] Amir Hossein Akhavan Salmasi, Evgueni Doukhnitch, Muhammed Salamah, 'A Hybrid TOA/AOA Hardware-Oriented Algorithm for Mobile Positioning', Accepted in Sixth International Conference on Broadband and Wireless Computing, Communication and Applications,BWCCA-2011 , Spain, October 2011

APPENDICES

APPENDIX A: The Proposed Location Determination Method's Source Code

```
clear

clc

hold on

for k=5:11                      % defining the k value range

x1=1;                           % BS1 coordinates

y1=0;

x2=25;                          % BS2 coordinates

y2=0;

angle=0.7;                      % Received signal angle

alpha=0;

b=2^(-k);

a=2^(-(2*k+1));

mx=10;          % MS real coordinates

my=6.5;

my=mx*tan(0.7);

d1=sqrt((x2-mx)^2+(my-y2)^2);  % MS distance to BS2

x3=5*x1;

y3=5*y1;

x4=d1;
```

```
y4=0;

while ((25-x4-x3)>=a)% First loop repeats until the AOA and TOA, x coordinated overlap

while ((y3-y4)>=a) % Secondloop repeats until the AOA and TOA, y coordinated overlap

x41=x4-(x4*a)-(y4*b);                % The TOA circular rotation equation

    y41=y4-(y4*a)+(x4*b);

x4=x41

    y4=y41

end

x33=x3+b*cos(alpha);              % The AOA incrementation equation

    y33=y3+b*sin(alpha);

x3=x33

    y3=y33

end

plot(x3,y3,'k','LineWidth',2,'MarkerEdgeColor','m','MarkerFaceColor','g','MarkerSize',10

)

hold

pause

end
```

APPENDIX B: Hardware-Oriented AOA Location Determination program

```
for j=1:10

hold on;

counter=0;

delta_alpha=0;

alpha_one=angle(j);              % Received signal angle to BS1

alpha_two=angle(j);              % Received signal angle to BS2

kk=8;

   u=2.^(-(2.*kk+1));               %  Defining the incrementation step size for 5≤k≤11

   v=2.^(-kk);

counter=counter+2;

alpha=0;

xxi=1;

yyi=0;

xx=1;

yy=0;

ifalpha_one>pi/2         % Check if the RSS angle is more than 90

counter=counter+1;

delta=pi-alpha_one;            % If the received signal angle is more than 90, decrease it

flg=1;                   % by 90

else
```

```
delta=alpha_one;

flg=0;

end

whiledelta_alpha>2.^(-11)

counter=counter+1;

xxi=xx-xx.*u-yy.*v;                    % Rotate BS1 Coordinates

yyi=yy-yy.*u+xx.*v;

alpha=alpha+(2.^(-kk));                % Add alpha by one step size unit

counter=counter+9;

xx=xxi;

yy=yyi;

end

ifflg==1                % Check the new coordinated to see whether it passed MS or not

cosalpha_one=-xxi          % If it passed, go back to the previous coordinates

sinalpha_one=yyi

else

cosalpha_one=xxi

sinalpha_one=yyi

end


alpha=0;

xxi=1;

yyi=0;

xx=1;
```

```
yy=0;

ifalpha_two>pi/2              % Check the received signal angle

delta=pi-alpha_two;        % If it was more than 90, decrease it by 90 degree

teta=delta;

flg=1;

counter=conter+1;

else

flg=0;

teta=pi-alpha_two;

delta=alpha_two;

counter=counter+1;

end

whiledelta_alpha>2.^(-11)             % Check and repeat until the BS signal array

counter=counter+1;                    %  overlaps with MS location

xxi=xx-xx.*u-yy.*v;                     % Circular rotation equations

yyi=yy-yy.*u+xx.*v;

alpha=alpha+(2.^(-kk));

counter=counter+9;

xx=xxi;

yy=yyi;

end

ifflg==1

cosalpha_two=-xxi       % If the BS coordinates passes the MS location, go back

sinalpha_rwo=yyi
```

41

```
else

cosalpha_two=xxi

sinalpha_two=yyi

end


x1=0.5*cosalpha_one;

x2=0.5*cosalpha_two;

   y1=0.5*sinalpha_one;

   y2=0.5*sinalpha_two;

   x1i=x1;

   y1i=y1;

   x2i=x2;

   y2i=y2;

   d=0.5;

   k=5;

counter=counter+4;

teta=pi-alpha_two;

   a1=(2.^(-k))*cosalpha_one;

   b1=(2.^(-k))*sinalpha_one;

   a2=(2.^(-k))*cosalpha_two;

   b2=(2.^(-k))*sinalpha_two;

ifalpha_one>teta

while(y1i-y2i)>2.^(-11);       % Check and repeat until the y coordinates overlap

        x1i=x1-a1;
```

```matlab
        y1i=y1-b1;

counter+counter+3;

x1=x1i;

        y1=y1i;

end

end

ifteta>alpha_one

while(y2i-y1i)>2.^(-11)    % Check and repeat until the y coordinates overlap

        x2i=x2-a2;

        y2i=y2-b2;

counter=counter+3;

        y2=y2i;

x2=x2i;

end

end

while((x1i-x2i)-d>2.^(-11))   % Check and repeat until the x coordinates overlap

counter=counter+1;

ifalpha_one>teta

        x1i=x1-a1;

        y1i=y1-b1;

counter=counter+2;

x1=x1i;

        y1=y1i;

while(y2i-y1i)>2.^(-11)         % Check and repeat until the y coordinates overlap
```

```matlab
        x2i=x2-a1;

        y2i=y2-b2;

counter=counter+3;

x2=x2i;

        y2=y2i;

end

end

ifteta>alpha_one

        x2i=x2-a2;

        y2i=y2-b2;

counter=counter+2;

x2=x2i;

        y2=y2i;

while(y1i-y2i)>2.^(-11)                    % Repeats until the y coordinated overlap

        x1i=x1-a1;

        y1i=y1-b1;

counter=counter+3;

x1=x1i;

        y1=y1i;

end

end

end

while((x1i-x2i)-d<2.^(-11))&(x1i*sinalpha_one<0.5)

counter=counter+1;
```

```
ifalpha_one>teta

        x1i=x1+a1;

        y1i=y1+b1;

counter=counter+2;

x1=x1i;

        y1=y1i;

while(y2i-y1i)>2.^(-11)    % Check and repeat until the y coordinates overlap

          x2i=x2+a2;

          y2i=y2+b2;

counter=counter+3;

x2=x2i;

          y2=y2i;

end

end

ifteta>alpha_one     % Check to see when the angle is less than received signal angle

        x2i=x2+a2;

        y2i=y2+b2;

counter=counter+2;

x2=x2i;

        y2=y2i;

while(y1i-y2i)>2.^(-11)     % Check and repeat until the y coordinates overlap

          x1i=x1+a1;

          y1i=y1+b1;

counter=counter+3;
```

```
x1=x1i;

        y1=y1i;

end

end

end

disp(x1);                    % Show the estimated x coordinate

disp(y1);                    % Show the estimated y coordinate

hold off;

end
```

APPENDIX C: Hardware-Oriented TOA Location Determination program

```
clear;

clc;

axis equal;

hold on;

title(' Location Of Mobile Station ');

xlabel(' X coordinates ');

ylabel(' Y coordinates ');

x1=9.8;                        % The BS1 coordinates

y1=9.7;

plot(x1,y1,'x');

x2=26.3;                       % The BS2 coordinates

y2=14.7;

plot(x2,y2,'x');

x3=13.6;                       % The BS3 coordinates

y3=26.6;

plot(x3,y3,'x');

display('start');

b=1/32;                        % Define the circular rotation accuracy

a=(b*b)/2;
```

```matlab
mx=15;

my=20;

plot(mx,my,'k*');

d1=sqrt(((my-y1)*(my-y1))+((mx-x1)*(mx-x1)));     %Find MS, BS1 distance

d2=sqrt(((y2-my)*(y2-my))+((x2-mx)*(x2-mx)););));   %Find the MS, BS2 distance

d3=sqrt(((y3-my)*(y3-my))+((x3-mx)*(x3-mx)));     % Find the MS,BS3 distance

r1=d1;

c1=[x1,y1];

r2=d2;

c2=[x2,y2];

r3=d3;

c3=[x3,y3];

circle1=rsmak('circle',r1,c1);

circle2=rsmak('circle',r2,c2);

circle3=rsmak('circle',r3,c3);

fnplt(circle1);                          % Plot the first antenna signal radius

fnplt(circle2);                          % Plot the 2nd antenna signal radius

fnplt(circle3);                          % Plot the 3rd antenna signal radius

ox1=0;

oy1=0;

ox2=x2-x1;

oy2=y2-y1;

plot(ox1,oy1,'g*');

plot(ox2,oy2,'g*');
```

48

```
c1=[ox1,oy1];

c2=[ox2,oy2];

circle1=rsmak('circle',r1,c1);

circle2=rsmak('circle',r2,c2);

fnplt(circle1,'r-');              % Plot the new circle1 coordinates

fnplt(circle2,'r-');             % Plot the new circle 2 coordinates

vx1=d1;

vy1=0;

plot(vx1,vy1,'m*');

rvx2=ox2-d2;

rvy2=oy2;

plot(rvx2,rvy2,'m*');

mxv2=d2;

myv2=0;

cnt=0;

if(y1>y2)                  % Check if the BS1 and BS2 coordinates do not overlap

display('...y1>y2');

while(vx1>=rvx2)                  % Rotate the coordinates until they overlap

    rmx2=mxv2-(mxv2*a)-(myv2*b);   % Circular rotation equations

    myv2=myv2-(myv2*a)+(mxv2*b);

    mxv2=rmx2;


    rvx2=ox2-mxv2

    rvy2=oy2+myv2
```
49

```matlab
plot(rvx2,rvy2,'g');

while(rvy2>vy1)        % Check and repeat until the BS1 and BS2,y coordinates overlap

        b=1/(64*32);        % Circular rotation accuracy

        a=(b*b)/2;

        vx1r=vx1-(vx1*a)-(vy1*b);      % Rotate the 2nd antenna coordinates

        vy1=vy1-(vy1*a)+(vx1*b);

        vx1=vx1r;

end

cnt=cnt+1;

plot(vx1,vy1,'m');    % Plot the new coordinates after the circular rotation

plot(rvx2,rvy2,'g');

    rmx1=vx1+x1;

    rmy1=vy1+y1;

end

display('2nd intersection point');

  vx1=d1;

  vy1=0;

  rvx2=ox-d2;

  rvy2=oy2;

  mxv2=d2;

  myv2=0;

  b=1/32;

  a=(b*b)/2;

while(vx1>=rvx2)% Rotate the x coordinates to find 2nd overlap points
```

```
        rmx2=mxv2-(mxv2*a)+(myv2*b);% Rotation equations for the 2nd overlap point

        myv2=myv2-(myv2*a)-(mxv2*b);

        mxv2=rmx2;

rvx=ox2-mxv2

        rvy2=oy2+myv2

plot(rvx2,rvy2);

while(rvy2>vy1)          % Repeat until the y coordinates overlap to find 2nd point

        b=1/(64*32);

        a=(b*b)/2;

        vx1r=vx1-(vx1*a)+(vy1*b);   % Circular rotation equations

        vy1=vy1-(vy1*a)-(vx1*b);

        vx1=vx1r;

end

cnt=cnt+1;

plot(vx1,vy1,'c');

plot(rvx2,rvy2);

if(mod(cnt,50)==0)

end

        rmx2=vx1+x1;

        rmy2=vy1+y1;

end

else

display('..y2>y1');

while(vx1>=rvx2)
```

```matlab
    vx1r=vx1-(vx1*a)-(vy1*b);

    vy1=vy1-(vy1*a)+(vx1*b);

    vx1=vx1r;

while(rvy2<vy1)                        % Check the y coordinates and rotate

        b=1/(64*32);

        a=(b*b)/2;

        rmx2=mxv2-(mxv2*a)-(myv2*b);    % Circular rotation equations

        myv2=myv2-(myv2*a)+(mxv2*b);

        mxv2=rmx2;

        rvx2=ox2-mxv2;

        rvy2=oy2+myv2;

plot(rvx2,rvy2,'g');

end

cnt=cnt+1;

plot(vx1,vy1,'m');

plot(rvx2,rvy2,'g');

    rmx1=vx1+x1;

    rmy1=vy1+y1;

end

display('2nd intersection point');

    vx1=d1;

    vy1=0;

    rvx2=ox2-d2,

    rvy2=oy2;
```

52

```matlab
    mxv2=d2;

    myv2=0;

    b=1/32;

    a=(b*b)/2;

while(vx1>=rvx2)              % Find the 2nd overlap point

    rmx2=mxv2-(mxv2*a)+(myv2*b);

    myv2=myv2-(myv2*a)-(mxv2*b);

    mxv2=rmx2;

    rvx2=ox2-mxv2;

    rvy2=oy2+myv2;

plot(rvx2,rvy2);

while(rvy2<vy1)              % Repeat until the y coordinates overlap

        b=1/(64*32);

        a=(b*b)/2;

        vx1r=vx1-(vx1*a)+(vy1*b);     % Circular rotation equations

        vy1=vy1-(vy1*a)-(vx1*b);

        vx1=vx1r;

end

cnt=cnt+1;

plot(vx1,vy1,'c');

plot(rvx2,rvy2);

    rmx2=vx1+x1;

    rmy2=vy1+y1;

end
```

```
end

plot(rmx1,rmy1,'ro');               % Plot the 1st overlap point

plot(rmx2,rmy2,'ro');               % Plot the 2nd overlap point

p1=abs(rmx1-x3)+abs(rmy1-y3);    % Find the 1st overlap point, BS3 distance

p2=abs(rmx2-x3)+abs(rmy2-y3);

if(p1<p2)                           % The point which is closer to BS3 is MS location

rmx=rmx1;

rmy=rmy1;

else

rmx=rmx2;

rmy=rmy2;

end

display(' position of the mobile ');

plot(rmx,rmy,'r*');                 % Plot the MS estimated coordinates

display(' END ');

pause ;

hold off;
```

Appendix D: Outputs of the Proposed Method's Simulation

Error	Computation Cost
0.4692	189
0.2994	301
0.4176	413
0.4895	523
0.8576	664
0.8692	770
1.2133	909
2.5179	1132

Table 1 Simulation results for K=5

Error	Computation Cost
0.0900	364
0.0662	561
0.1321	781
0.1532	995
0.1438	1205
0.2722	1444
0.3551	1677
0.4071	1906

Table 2 Simulation results for K=6

Error	Computation Cost
0.0447	703
0.0661	1120
0.0653	1531
0.0857	1959
0.1439	2408
0.2059	2849
0.2547	3315
0.2989	3771

Table 3 Simulation results for K=7

Error	Computation Cost
0.0412	1402
0.0661	2238
0.0618	3058
0.0858	3916
0.1065	4783
0.1311	5663
0.2049	6591
0.2488	7503

Table 4 Simulation results for K=8

Error	Computation Cost
0.0173	2769
0.0224	4410
0.0258	6071
0.0274	7752
0.0436	9482
0.0601	11234
0.0805	13012
0.1127	14824

Table 5 Simulation results for K=9

Error	Computation Cost
0.0038	5501
0.0069	8779
0.0090	12099
0.0073	15455
0.0113	18878
0.0144	22343
0.0184	25854
0.0196	29392

Table 6 Simulation results for K=10

Error	Computation Cost
0.0026	10967
0.0037	17494
0.0043	24124
0.0035	30861
0.0050	37672
0.0086	44561
0.0118	51542
0.0202	58567

Table 7 Simulation results for K=11

To compare our proposed method with the traditional method computational cost, Table 8 shows the derived simulation results. As it was noted earlier, our proposed method earned better results than the traditional method for k equal to 5 and 6.

k value	Proposed Method Computational Cost	Traditional Method Computational Cost
5	770	1980
6	1444	1980
7	2849	1980
8	5663	1980
9	11234	1980
10	22343	1980
11	44561	1980

Table 8 Comparing the Proposed Method and the Traditional Method Computational Cost

APPENDIX E: Confidence Interval

For sample size equal to 8, with 95% confidence level, we can say the MS is located in this confidence interval. Confidence interval upper and lower boundaries are illustrated in Table 8.

K	Mean Value	Variance (S^2)	Standard Deviation (S)	Lower Boundary	Upper Boundary
5	4.9305	19.1660	4.3779	1.2694	8.5916
6	0.2024	0.0159	0.1262	0.0969	0.3079
7	0.1456	0.0093	0.0964	0.0650	0.2262
8	0.1182	0.0375	0.1936	-0.0436	0.2801
9	0.0487	0.0011	0.0333	0.0209	0.0765
10	0.0113	0.000030	0.0055	0.0067	0.0159
11	0.0074	0.000035	0.0059	0.0025	0.0123

Table 9 Confidence interval for 8 samples

The confidence interval results for different k values are illustrated in the table above.

As an example in the following, we will calculate the confidence interval for k=7.

The confidence interval for the samples can be calculated using this formula:

$$\bar{y} - t_{\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}} \leq \mu \leq \bar{y} + t_{\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}}$$

Where $\bar{y}$ is the sample mean and:

$$\bar{y} = \frac{\sum_{i=1}^{8} y_i}{8} = 0.14565$$

Where $y_1, y_2, \ldots, y_n$ represents a sample of size n.

S is standard deviation and $S = \sqrt{S^2}$ where:

$$S^2 = \frac{\sum_{i=1}^{8}(y_i - \bar{y})^2}{n-1} = 0.0093$$

So:

$$S = \sqrt{0.0093} = 0.0964$$

Since $t_{\frac{0.05}{2},7} = 2.365$, using the confidence interval, the boundaries can be calculated. So

we have:

$$0.14565 - (2.365).\frac{0.0964}{\sqrt{8}} \leq \mu \leq 0.14565 + (2.365).\frac{0.0964}{\sqrt{8}}$$

Therefore:

$$0.06505 \leq \mu \leq 0.22625$$