Fuzzy Game Theory for Decision Analysis

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ABSTRACT

The purpose of this thesis is to consider the synergy of fuzzy logic theory and game theory for the analysis of the decision making process. The different techniques of fuzzy game theory versus their crisp prototypes are described. The properties of the crisp and fuzzy cooperative and non-cooperative games are compared. The fuzzy mixed strategy, fuzzy dominant strategy, and fuzzy Nash equilibrium are investigated.

Keywords: Fuzzy Logic, Game Theory, Fuzzy Cooperative and Non-cooperative Games, Fuzzy Mixed Strategy, Fuzzy Dominant Strategy, Fuzzy Nash Equilibrium

ÖZ

Bu tezin amacı bulanık mantık ve oyun teorilerinin sinerjisini karar verme sürecinde araştırmaktır. Bulanık oyun teorisinin farklı teknikleri ile onların karşılıklı klasik prototipleri incelenir. Klasik ve bulanık işbirlikli ve işbirliksiz oyunların özellikleri kıyaslanır. Bulanık karma gengüdüm, bulanık başat gengüdüm, ve bulanık Nash dengesi araştırılır.

Anahtar Kelimeler: Bulanık Mantık, Oyun Teorisi, Bulanık Işbirlikli ve Işbirliksiz Oyunlar, Bulanık Karma Gengüdüm, Bulanık Başat Gengüdüm, Bulanık Nash Dengesi

I am dedicating this thesis to who supported me through my studies.

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TABLE OF CONTENTS

LIST OF FIGURES

Chapter 1

INTRODUCTION

Game theory is one of the topics of Artificial Intelligence and is used as a mathematical analysis for conditions interest and cooperation between intelligent rational decision makers to take the best decision. These decisions are based on optimization of utilities by the maximization of the profit and behavior strategies of participants. The rationality of decision makers consists in availability of all the possible alternatives and outcomes to implement.

Game theory was founded by economist Oskar Morgenstern and mathematician John von Neumann in 1944 as the result of their collaboration, and was firstly published in the book entitled “The Theory of Games and Economic Behavior”. Game theory is successfully implemented in different areas such as economics, political sciences, computer science and logic, biology etc.

In game theory two or more players take a decision which affects the outcome of each other or another player on the contrary with traditional decision making which the decision is made by one player.

Different types of games are known: cooperative and non-cooperative games, sequential game, constant sum, static and dynamic games.

There are three forms to represent the interaction between players: 1) extensive form describes the situation of the game, the motivation, details and the available information. It also gives conditions for the movements of the players and show different stages for the interaction between players; 2) strategic form gives the possible strategies that can be used by the players along the game, and the payoff of these strategies are chosen by the players; 3) characteristic function form describes the interaction between players to represent coalitions which are used in cooperative games.

In contrast to conventional logic, Prof. Lotfi Zadeh proposed a fuzzy logic in his paper published in 1965. The advantage of the latter logic is being applicable for manipulation of the information and knowledge represented by linguistic terms. The classification of objects in terms of their belonging to a set is partially true or false, i.e. the objects are related to a set with different membership functions.

The set of objective functions in the game may have uncertain values. The uncertainty is due to the unknown decisions of the opponents. The degree of uncertainty is reduced through the assumption that each player knows the desire of the other player(s) and the assumption goals. Another way to deal with uncertainty associated the payoff functions is to use the concept of fuzzy game. The behavior of the players in the game depends on the structure of the game being played. This involves the decisions they face and the information they have when making decision and how their decisions determine the outcome, as well as the preference they have over the outcomes [1-2]. The player may change his/her strategy several times depending on the strategies that the opponents use. The players can't take rational decision in complex environment which is represented by ambiguity, vagueness, imprecision, and uncertainty, and in such cases using fuzzy reasoning is important [3].

The advantage of fuzzy game theory is that the payoff of the game doesn’t need to be precise value, and for that the fuzzy approach is more suitable to represent the real life problems [5].

In this thesis the fuzzy logic approach based game theory is presented. The differences between the conventional and fuzzy types of games are analyzed. Some examples for the calculation of the payoffs of the players and appropriate decisions to be taken are given.

Chapter 2

REVIEW OF EXISTING LITERATURE ON FUZZY GAME THEORY AND ITS APPLICATIONS

In [1] the fuzzy approach, based on multicriteria decision-making method for solving strategic game that defines the optimal strategy, is proposed. This strategy is more advantageous than classic strategy, and shows better performance for the famous “prisoner's dilemma” problem.

In contrary to crisp game, the fuzzy logic based game is very powerful in managing the uncertainties. In [2] the fuzzy logic is used to measure the player’s preference of one payoff to others. A least deviation method is applied to obtain a fuzzy preference relation, and the priority for each player is calculated. In the proposed method the fuzzy payoffs, fuzzy satisfaction functions, and satisfaction degree from each payoff are defined. The calculation of similarity between satisfaction functions enables making crisp game from the fuzzy one.

In [3] the theory of fuzzy moves (TFM) is developed by the merging of theory of moves and the theory of fuzzy sets. The theory of fuzzy moves is used to make the better fuzzy moves. To make more reasonable moves, the fuzzy sets with higher granularity for fuzzy reasoning are used. The computer simulation shows that TFM with fuzzy reasoning shows better and more reasonable performance compare to theory of moves with precise reasoning.

The theory of moves (TOM) is a type of game in which the players should decide the appropriateness of the move to be made. The fuzzy game theory of moves is presented in [4], and the value of moves and their interrelationships are described.

[5] is about fuzzy theory based double action model that is used to represent the auction participant's bidding wills. Using Bellman and Zadeh's concept of confluence of fuzzy decisions shows better results compare to Nash equilibrium game theory. The solved problem shows the usefulness of the proposed method in practical auctions.

[6] considers the extension of P-cores concept in cooperative fuzzy game theory. This concept is extended from P-cores and P-stable sets to generalized P-cores, and generalized P-stable sets. The developed concept provides more rational distribution schemes. The value of generalized P-cores for cooperative fuzzy game is defined.

The extension of concept of the bargaining sets from classical game theory to non-transferable utility fuzzy game is offered in [7]. It is proven that the relation between above theories exists in both super additive non-transferable and non-transferable fuzzy games. The importance and significant contribution of the proposed concept is described.

Basic properties, presented in [8], define two-stage production games. For the solving of the production games the hybrid algorithm, which is the combination of genetic algorithm (GA), neural network (NN) and approximating method, is designed. The distribution games example solved shows the feasibility of the new algorithm.

Usually in conventional game theory each player has a strategy with well-defined outcome. Nowadays the complexity of problems in many areas does not reflect the correctness of the initial assumption to be accepted, because a crisp payoff is defined with a big difficulty. In [9] offered fuzzy numbers are used to incorporate the results of strategies. To define payoffs, the creditability measure is used.

[10] describes the cooperative fuzzy games which deal with the fuzzy coalitions and infinite players. The natural class of fuzzy games with Choquet integral has several rational properties such as convexity, supper additive and monotonicity.

Sometimes using crisp game theory does not lead to effective modeling the incorporation some of the subjective attitudes of the decision makers because of the vague and ambiguous types of information. In [11] the fuzzy approach is presented to solve the Prisoner's Dilemma in which the decision makers should decide whether or not to cooperate. The fuzzy procedure considers subjective attitudes of the decision makers to act under uncertain and risky types of situations.

In case of incompleteness, ambiguity, vagueness and impreciseness of the situations the decision maker can't model a conflict to get a feasible preference, and it affects the overall equilibria to be predicted. In [12] developed method is intended for uncertainty modeling to resolve the conflict in the preferences of the decision maker. In order to illustrate the importance of the developed approach to find the realistic equilibria, the fuzzy preference methodology is applied on prisoner's dilemma problem of game theory.

[13] presents a new model of interval fuzzy cooperative games with Choquet integral form. The relationship between fuzzy convex form of Choquet integral and interval Shapley value is described. It is mentioned that improved Shapley value is very important in fuzzy games with interval fuzzy number.

In [14] some possible two-agent decision making problems are represented which involve perceptions of one agent about the other agent. The importance of defining information links between the agents is explained. The case, when players have fuzzy but close to true criteria, is investigated. It is shown that both players expect actual values from their calculated strategies similar to while making their fuzzy hypotheses.

A new approach proposed in [15] is used to incorporate a hybrid game strategy in Markov-game-based fuzzy control. The universal controller is designed to show an ability of a good performance against disturbance and environment variations. The hybrid control based on experiential information obtains reasonable performance against above variations in Markov-game-based control.

In [16] the fuzzy linguistic preference relation in game theory is described. The priorities of Nash equilibrium are investigated. In order to compare fuzzy variable, two measures are represented by using fuzzy extension principle.

As it is known in game theory, the players' main task consists in maximizing their payoffs. It is difficult to perform this task in the presence of fuzzy and uncertain natures. [17] considers a novel approach to analyze the games with fuzzy payoffs method to find pure strategy Nash equilibrium. The priorities of payoffs are determined by ranking fuzzy numbers.

In [18] two different fuzzy methods for the studying 2×2 game model are proposed. In the first method the multicriteria decision analysis is investigated to obtain optimal strategies of the players. In the second method the application of the theory of fuzzy moves (TFM) for the Chicken game is considered. The importance of using theory of moves consists in the presence of factors to look ahead to improve the decision making process. It is also observed that using above fuzzy methods provide better result for the game of Chicken and demonstrate their effectiveness in the presence of uncertain and vague information.

The new non-cooperative model of a normal form game is introduced in [19]. Bellman and Zadeh's principle of a decision theory is extended to game theory. The conditions for the existence of equilibrium are investigated.

Most Internet transactions are modeled using terms of traditional game theory. Price negotiations, competition for customers, and online auctions can be given as examples. In case of dealing with uncertain values, these games become examples of fuzzy game theory. In [20] proposed fuzzy approach for the game theory is applied to consider some specific peculiarities of e-commerce.

The development of negotiation model in electronic commerce has become an important issue to implement trade-off. In [21] the fuzzy set theory based negotiation model is established which is used to solve the following problems: the normalization process is performed for the goals to define the weight vectors and payoff matrix; the negotiation of multi-goals is realized; and the strategy for the negotiation process is set.

The classical game theory method is not appropriate for using in uncertain environment in which most negotiation processes for the development of E-Commerce systems take place. For this reason in [22] the fuzzy logic based approach in game theory is proposed that overcomes the complexity of negotiation process in E-Commerce system. The implementation of the above method shows better performance to achieve benefits in negotiation parties.

Chapter 3

FUZZY COOPERATIVE AND NON-COOPERATIVE GAMES

3.1 Crisp Cooperative Game Versus Fuzzy Cooperative Game

Cooperative game is a coalitional game that may contain finite number of participators who agree to coordinate their strategies to optimize payoff of the players. The payoff of the game is determined by the combination of the strategies. The target of the game should satisfy the player's objective which is required from the game [5].

The target of each player in the coalition is to maximize his/her own outcomes and the other target is to maximize the outcomes of the other players in the coalition. These coalitions are mostly important in political science and international relations. For example, assuming that the players are several parties in parliament and each of these parties has different degree of power depending on the number of seats they have for the members of the party [32].

Suppose there are two companies A and B. These companies should decide whether to cooperate or not to cooperate according to the payoffs given in Figure 1:

		Company B	
		Cooperation	Non-cooperation
	Cooperation	(5,5)	(8,1)
Company A	Non-cooperation	(1,8)	(3,3)

Figure 1: Crisp Payoff Matrix of a Cooperative Game

There are totally four possible situations to deal with:

1) If company A cooperates, then it is better for company B to cooperate.

2) If company A does not cooperate, then it is better for company B to cooperate.

3) If company B cooperates, then it is better for company A to cooperate.

4) If company B does not cooperate, then it is better for company A to cooperate.

So the best decision is reached when both the companies A and B decide to cooperate.

Most crisp cooperative games can be transferred into fuzzy form. In contrast to crisp cooperative game in which the players take part in a game fully or don’t take part at all, fuzzy cooperative games are represented by the partial coalition between players in which the levels of their participation are taken from the interval [0, 1]. The real valued function used in fuzzy game theory can assign real values to each coalition [23].

We consider the cooperative fuzzy game with two players. If we consider a fuzzy cooperative game in which two players are involved, then in order to decide whether to cooperate one should take into account the values of the participation levels of both the players that are at least 1/2 [23]:

$$v(s_1, s_2) = \begin{cases} 1, if\ s_1 \geq 1/2, s_1 \geq 1/2 \\ 0, otherwise \end{cases} \quad (3.1)$$

where s_1 and s_2 are the participation levels of the players 1 and 2, respectively.

Let's illustrate the above participation levels of a fuzzy cooperative game in example. In Figure 2 each cell represents levels of participation of two players A and B involved in a game:

		Company B	
		Cooperation	Non-cooperation
Company A	Cooperation	(0.8,0.8)	(0.6, 0.3)
	Non-cooperation	(0.3,0.6)	(0.2,0.2)

Figure 2: Fuzzy Matrix of a Cooperative Game

As it can be seen from the table, the optimal solution of the problem (according to the values of the participation levels of the players) is reached when both the players agree to cooperate.

3.2 Crisp Non-cooperative Game Versus Fuzzy Non-cooperative Game

In non-cooperative game the players analyze their strategic choices in decision making process. There is no agreement between players before the game, i.e. neither of the players agrees to cooperate. In non-cooperative game the players are acting in self-interest. Each player chooses the best outcome for him/her no matter what another player undertakes to act.

Non-cooperative game theory is mostly applied in bargaining which produce a specific process that determine who would get an offer for the choices at a specific time [9].

According to the payoffs of companies A and B represented in Figure 3, it is possible to see that if either the company A or B decides to cooperate, their payoffs get less if they choose the option of non-cooperation. And the best outcome for both the companies A and B is reached when the decision is not to cooperate.

Using the formula (3.1), it is possible to apply fuzzy approach to non-cooperative game. As it is seen from the Figure 4, the optimal decision undertaken is reached when both the companies decide not to cooperate (according to the values of participation levels).

Company B

		Cooperation	Non-cooperation
	Cooperation	(3,3)	(2,6)
Company A	Non-cooperation	(6,2)	(8,8)

Figure 3: Crisp Payoff Matrix of a Non-cooperative Game

Company B

		Cooperation	Non-cooperation
	Cooperation	(0.3, 0.3)	(0.2, 0.6)
Company A	Non-cooperation	(0.6, 0.2)	(0.9, 0.9)

Figure 4: Fuzzy Matrix of a Non-cooperative Game

Chapter 4

FUZZY MIXED STRATEGY, FUZZY DOMINANT STRATEGY, AND FUZZY NASH EQUILIBRIUM

4.1 Mixed strategy. Fuzzy Mixed Strategy

A mixed strategy is the strategy that allows the player to assign probability to each of the pure strategy used, and this strategy is used when the opponent can guess the next move to make benefit from it.

Let's consider the following example related to the mixed strategy. Suppose there are two companies A and B, and two strategies: strategy 1 and strategy 2. If both the companies A and B choose the strategy 1, the payoffs are (9,9), respectively.If company A chooses the strategy 1, and the company B chooses the strategy 2, the payoffs are (4,6), respectively. If the company A chooses the strategy 2, and the company B chooses the strategy 1, the payoffs are (6,4), respectively. And finally if both the companies A and B choose the same strategy 2, the payoffs are (2,2), respectively. The above payoffs are represented in the form of matrix in Figure 5:

		Company B	
		Strategy 1	Strategy 2
	Strategy 1	(9,9)	(4,6)
Company A	Strategy 2	(6,4)	(2,2)

Figure 5: Payoff Matrix of a Game with Mixed Strategy

Let both the companies A and B predict choosing strategies for each other in an equilibrium manner, i.e. they choose strategies 1 and 2 with probabilities 50% each. Then the expected utilityof the company A would be

0.5*9+0.5*4=6.5 by choosing the strategy 1

0.5*6+0.5*2=4 by choosing the strategy 2

Since 6.5>4,in a mixed strategy the decision for the company A is to choose the strategy 1.

The expected utilityof the company B would be same:

0.5*9+0.5*4=6.5 by choosing the strategy 1

0.5*6+0.5*2=4 by choosing the strategy 2

Since 6.5>4, in a mixed strategy the decision for the company B is also to choose the strategy 1. So it is concluded that in the above game the best decision for both the companies A and B is to choose the strategy 1.

In most cases while using mixed strategy in a game, it is necessary to find the probability for each strategy both players take. In Figure 6 the payoff matrix of a game involving two companies A and B are described:

		Company B	
		Strategy 1	Strategy 2
Company A	Strategy 1	(9,1)	(2,8)
	Strategy 2	(3,7)	(6,4)

Figure 6: Payoff Matrix in a Mixed Strategy Game

Suppose the company B uses the strategies 1 and 2 with the probabilities q and 1-q, respectively, and the company A uses the strategies 1 and 2 the probabilities p and 1-p, respectively. Then the expected payoff of the company A will be

$9*q+2*(1-q) = 7q+2$ from the strategy 1

$3*q+6*(1-q) = 6-3q$ from the strategy 2

Making above equations equal we can find the probability q:

$$7q+2=6-3q$$

$$q=0.4$$

The expected payoff of the company B will be

$1*p+7*(1-p) = 7-6p$ from the strategy 1

$8*p+4*(1-p) = 4p+4$ from the strategy 2

Making above equations equal we can find the probability p:

$$7-6p=4p+4$$

$$p=0.3$$

So in a mixed strategy for the above game the probabilities should be ((0.3,0.7), (0.4, 0.6)).

In a crisp mixed strategy there is always at least one equilibrium point, but we can't say the same about the fuzzy mixed strategy.

It is to mention that sometimes obtaining expected payoffs becomes impossible, since the process is carried out under uncertainty. In [24] an intuitionistic fuzzy programming to a two-person bi-matrix game for the solution with mixed strategies is considered. For this reason linear membership and non-membership functions are presented. The optimal solution of the problem is reached by maximization of the degree of acceptance and minimization of the degree of the rejection of objectives and constraints. It is underlined that because the players in a game with fuzzy mixed strategy don't exactly know which strategy he/she will use, the probabilities for the possible alternatives will be chosen according to intuitions the players consider in order to calculate the expected payoffs of the players.

Another approach to calculate fuzzy payoffs is proposed in [25]. It is mentioned that obtaining crisp payoffs in the presence of insufficient information is impossible, so the payoffs are described as fuzzy sets. The difficulty in calculation of payoffs, and also being time-consuming process requires the application of fuzzy approach. The ranking of fuzzy numbers and the priorities of payoffs are defined. Using mixed strategy in fuzzy bi-matrix game enables grading the membership values to determine how much each strategy is Nash equilibrium.

The mixed strategy can also be successfully applied to fuzzy non-cooperative game [26]. The fuzzy game with mixed strategy is also characterized by the expected value by fuzzy payoff. The problem to find out which properties are required for the fuzzy game to claim that Nash equilibrium exists is considered.

[27] is about using fuzzy Monte-Carlo methods to obtain optimal fuzzy mixed strategies for the fuzzy two-person zero-sum games. The optimal fuzzy values for the players and fuzzy mixed strategies are defined.

Two-person zero-sum game is also investigated in [28]. The fuzzy payoffs based on fuzzy triangular numbers are calculated for the outcomes. The payoffs and strategies are defined by fuzzy variables. The fuzzy expected minimax equilibrium is used, and the feasible strategy for the game is developed.

While considering properties of the fuzzy mixed strategy, in most cases it is necessary to determinewhether the Nash equilibriumin the game is available. This is discussed in more detailed form in the section 4.3.

4.2. Dominant Strategy. Fuzzy Dominant Strategy

A dominant strategy is an optimal strategy for the player regardless of strategy other player(s) use. The strategy is dominant if it can dominate all other strategies.

When the player prefers to chose strategy A rather than chose strategy B, then strategy A dominates strategy B and it is called dominant strategy. When the outcome of strategy A is always larger than the outcome of strategy B, then A strictly dominates B and if the outcome of strategy A is always as least larger than the outcome of strategy B, then A weakly dominates B.

In Figure 7 the two strategies and according payoffs for the companies A and B are represented. Regardless what strategy is used by the company B, the dominant strategy for the company A is to choose the strategy 1, since payoff in this case is 14. And regardless what strategy is used by the company A, the dominant strategy for the company B is to choose the strategy 1, since payoff is 10. So both the companies have the same dominant strategy, and this is called the equilibrium in dominant strategy.

		Company B	
		Strategy 1	Strategy 2
	Strategy 1	(12,10)	(14,1)
Company A	Strategy 2	(5,9)	(11,4)

Figure 7: Payoff Matrix of a Game with Equilibrium in Dominant Strategy

In fuzzy dominant strategy the players take decision according to degrees of nuance and feasibility represented in ordered form [29]. Thestrategy used in this game is called the Linguistic Fuzzy-Logic (LFL). Each of the players has finite set of nuance and feasibility values. It is to underline that the degrees of nuance and feasibility values in this game are always ordered. The rules by using of which the nuance and feasibility matrices are formed are represented in fuzzy form.

Another form of using fuzzy dominant strategy consists in comparison of utility values of the strategies based on the outcomes of different combinations of strategies.Sometimes the payoffs of the players according to strategies they take can't be deterministically forecasted [30]. Suppose the range of crisp payoffs can vary in the interval of integer numbers [0,20], in which the payoffs [0,5], [6,10], [11,15], and [16,20] are assigned the linguistic terms ‘’very low“, “low”, “high”, and “very high”, respectively. The crisp payoff matrix and the corresponding fuzzy payoff matrix with linguistic terms for two different strategies (Strategy 1 and Strategy 2) of companies A and B are represented in Figure 8 and Figure 9, respectively:

Company B

		Strategy 1	Strategy 2
	Strategy 1	(2,7)	(8,13)
Company A	Strategy 2	(17,3)	(14,19)

Figure 8: Crisp Payoff Matrix with Dominant Strategy

		Company B	
		Strategy 1	Strategy 2
	Strategy 1	(very low, low)	(low, high)
Company A	Strategy 2	(very high, very low)	(high, very high)

Figure 9: Fuzzy Payoff Matrix with Linguistic Terms for Dominant Strategy

Both the companies A and B have the same dominant strategy 2. So in fuzzy dominant strategy in which the payoffs are represented in linguistic form, there is a unique equilibrium in the cell which is an intersection of the strategies 2 of both companies.

It is also possible in fuzzy dominant strategy that each cell representing different intersections of the strategies of the companies A and B gets membership functions. But in this case the dominant strategy is defined if the membership functions are greater than 0.5 [30]. Suppose the membership functions for the payoffs given in the Figure 8 are represented in the following form: μ(Strategy 1 for A) = 0.4, μ(Strategy 2 for A) = 0.2, μ(Strategy 1 for B) = 0.3, μ(Strategy 2 for B) = 0.3. Since neither of the above mentioned strategies have membership function exceeding 0,5, it is to conclude that there is no dominant strategy for each of the players.

4.3. Nash Equilibrium. Fuzzy Nash Equilibrium

Nash equilibrium in game theory is the expectation of the official rule for the game and describes the strategies that will be depending by the players which lead to the outcome of the game. The non-cooperative game consists of more than one player and each player knows the equilibrium of strategies of the opponents and the player gains nothing when he/she changes his/her strategy in one-side. If the player changes his/her strategy but the opponent does not benefit from this change and they keep their strategies unchanged, the set of strategies chosen and the outcome corresponding to these strategies construct the Nash equilibrium. In Nash equilibrium the players don't desire to change their strategy because this will lead to worst result.

Let us consider the strategic interactions between two companies, and find out whether any Nash equilibrium exists. Figure 10 depicts the payoff matrix of two companies A and B assuming that both of the companies may use two different strategies: strategy 1 and strategy 2.

As it is seen from the Figure 10, sum of payoffs of two companies while applying same strategies are always greater than if the companies are applying different strategies. So the final decision to find the Nash equilibrium (if any exists) can be chosen from the cell(s) representing intersections of the same strategies.

Apart from that, it is concluded that there are two Nash equilibria in this game: the first Nash equilibrium is Strategy 1 for A, Strategy 1 for B; and the second Nash equilibrium is Strategy 2 for A, Strategy 2 for B. If one of the companies deviates to another strategy, the payoff reduces. So the final decision is that both the companies must coordinate their games.

		Company B	
		Strategy 1	Strategy 2
	Strategy 1	(7,6)	(3,3)
Company A	Strategy 2	(2,2)	(6,7)

Figure 10: Crisp Payoff Matrix with Nash Equilibria

Since in real life the utility payoffs of the players are uncertain and vague that can change the decision strictly, fuzzy game theoretic approach is inevitable to be applied taking the uncertain utility payoffs into consideration [30].

Presence or absence of fuzzy Nash equilibrium is defined according to the membership functions of the strategy combinations of the players. Let's consider two game matrices to find out whether any fuzzy Nash equilibrium exist(s). In Figure 11 and Figure 12 the membership functions according to different strategy combinations of the players are given.

It is seen from the Figure 11 that because neither of the membership functions representing strategy combinations exceeds the threshold 0.5, so there is no Nash equilibrium in this game.

But two strategy combinations in Figure 12 exceed 0.5: μ(Strategy 1 for A, Strategy 2 for B) = 0.6, and μ(Strategy 2 for A, Strategy 1 for B) = 0.7. So we conclude that there are two Nash equilibria, but since 0.7 > 0.6, the strategy combination μ(Strategy 2 for A, Strategy 1 for B) = 0.7 is more likely to be Nash equilibrium.

		Company B	
		Strategy 1	Strategy 2
	Strategy 1	0.4	0.3
Company A	Strategy 2	0.4	0.2

Figure 11: Fuzzy Game Matrix without Nash Equilibrium

		Company B	
		Strategy 1	Strategy 2
	Strategy 1	0.3	0.6
Company A	Strategy 2	0.7	0.2

Figure 12: Fuzzy Game Matrix with Nash Equilibria

Fuzzy Nash equilibrium strategy is also applied to the game with three players in fuzzy constraint satisfaction problem [31]. Assume that three players are X, Y, Z, and each of the players can use two strategies: a and b. There are following constraints and payoffs:

C_{xy} = {(aa,0.4), (ab,0.3), (ba,0.9), (bb,0.2)}

C_{yz} = {(aa,0.3), (ab,0.6), (ba,0.2), (bb,0.8)}

The Nash equilibrium of the game is found according to the higher value of the preference defined in the following form:

(aaa)= min(aa,aa)=min(0.4,0.3)=0.3

(aab)= min(aa,ab)=min(0.4,0.6)=0.4

(aba)= min(ab,ba)=min(0.3,0.2)=0.2

(abb)= min(ab,bb)=min(0.3,0.8)=0.3

(baa)= min(ba,aa)=min(0.9,0.3)=0.3

(bab)= min(ba,ab)=min(0.9,0.6)=0.6

(bba)= min(bb,ba)=min(0.2,0.2)=0.2

(bbb)= min(bb,bb)=min(0.2,0.8)=0.2

We find the unique Nash equilibrium which is (bab), since its membership function is maximum.

Chapter 5

CONCLUSION

In this thesis the fuzzy game theory for decision making process is analyzed. It is mentioned that using fuzzy game techniques under unclearness, and incompleteness of information is more preferable compare to classical game techniques. Fuzzy cooperative and fuzzy non-cooperative games are discussed. Such techniques as fuzzy mixed strategy, fuzzy dominant strategy, and fuzzy Nash equilibrium are considered.

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