Spectroscopy of Black Holes

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ABSTRACT

In this thesis, the Maggiore's method (MM), which evaluates the transition frequency that appears in the adiabatic invariant from the highly damped quasinormal mode (QNM) frequencies, is used to investigate the entropy/area spectra of the Garfinkle-Horowitz-Strominger black hole (GHSBH) and $z = 0$ Lifshitz black hole (ZZLBH). The complex QNM frequencies of the GHSBH and ZZLBH are computed using the confluent hypergeometric (CH) differential equation that arises when the scalar perturbations around the event horizon are considered. Although the entropy/area is characterized by the parameters of black holes (BHs), their quantization is shown to be independent of those parameters. However, both spectra are equally spaced.

We also represent the mass calculations of the associated BHs. In this regard, we compute the mass of the GHSBH by using Komar's mass integral formulation. For the mass of the ZZLBH, we use both Wald's entropy formula and Brown-York (BY)'s quasilocal mass formalism.

Keywords: Garfinkle-Horowitz-Strominger Black Hole, $z = 0$ Lifshitz Black Hole, Maggiore's Method, Black Hole Spectroscopy, Quasilocal Mass.

ÖZ

Bu tezde, Garfinkle-Horowitz-Strominger (GHSBH) kara deliğinin ve $z = 0$ Lifshitz kara deliğinin (ZZLBH) entropi/alan spektrumlarını incelemek için, adyabatik invaryant içerisinde görülen geçiş frekansını, yüksek sönümlenen kuazinormal mod (QNM) frekanslarından elde eden Maggiore'un metodu (MM) kullanılmıştır. Olay ufkunun etrafında skalar pertürbasyonlar düşünüldüğü zaman ortaya çıkan konflüent hipergeometrik (CH) diferansiyel denklemi kullanılarak, GHSBH ve ZZLBH'a ait kompleks kuazinormal mod frekansları hesaplanmıştır. Entropi/alan, kara delik (BH) parametreleri ile karakterize edilmesine rağmen, kuantizasyonun bu parametrelerden bağımsız olduğu gösterilmiştir. Bununla birlikte, her iki spektrumun da eşit aralıklı olduğu gösterilmiştir.

İlgili BH'ların kütle hesapları da ayrıca gösterilmiştir. Bu bağlamda, GHSBH kütlesi Komar'ın kütle integrali kullanılarak hesaplanmıştır. ZZLBH kütlesi için ise hem Wald entropi formülü hem de Brown-York (BY)'un kuazilokal kütle formalizmi kullanılmıştır.

Anahtar Kelimeler: Garfinkle-Horowitz-Strominger Kara Deliği, $z = 0$ Lifshitz Kara Deliği, Maggiore Metodu, Kara Delik Spektroskopisi, Kuazilokal Kütle.

Dedicated to My Family

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INTRODUCTION

Currently, one of the greatest projects in theoretical physics is to unify general relativity (GR) with quantum mechanics (QM). Such a new unified theory is known as the quantum gravity theory (QGT) [1]. Recent developments in physics show that our universe has a more complex structure than that predicted by the standard model [2]. The QGT is considered to be an important tool that can tackle this problem. However, current QGT still requires further extensive development to reach completion. The development of the QGT began in the seventies when Hawking [3,4] and Bekenstein [5-9] considered the black hole (BH) as a quantum mechanical system rather than a classical one. In particular, Bekenstein showed that the area of the BH should have a discrete and equally spaced spectrum:

$$\mathcal{A}_n = \varepsilon n \hbar = 8\pi\xi n\hbar \,, \qquad (n = 1,2 \ldots); \tag{1.1}$$

where ε is the undetermined dimensionless constant and ξ is the order of unity. The above expression also shows that the minimum increase in the horizon area is $\Delta\mathcal{A}_{min} = \varepsilon\hbar$. Bekenstein [7,8] also conjectured that for the Schwarzschild BH (and also for the Kerr-Newman BH) the value of ε is 8π (or $\xi = 1$). Following the seminal work of Bekenstein, various methods have been suggested to compute the area spectrum of the BHs. Some methods used for obtaining the spectrum can admit the value of ε different than that obtained by Bekenstein; this has led to the discussion of this subject in the literature (for a review of this topic, see [10] and references therein). Among those methods, the Maggiore's results [11] show a perfect agreement with Bekenstein's result by modifying the Kunstatter's [12] formula as

$$I_{adb} = \int \frac{dM}{\Delta\omega}\,, \tag{1.2}$$

where I_{adb} denotes the adiabatic invariant quantity and $\Delta\omega = \omega_{n+1} - \omega_n$ represents the transition frequency between the subsequent levels of an uncharged and static BH with the total energy (or mass) M. However, the researchers [13,14,15] working on this issue later realized that the above definition is not suitable for the charged rotating (hairy) BHs that the generalized form of the definition should be given by

$$I_{adb} = \int \frac{TdS}{\Delta\omega}\,, \tag{1.3}$$

where T and S denote the temperature and the entropy of the BH, respectively. Thus, using the first law of BH thermodynamics, Eq. (1.3) can be modified for the considered BH. In contrast, according to the Bohr-Sommerfeld quantization rule, I_{adb} behaves as a quantized quantity $I_{adb} = n\hbar$ while the quantum number n tends to infinity. To determine $\Delta\omega$, Maggiore considered the BH as a damped harmonic oscillator that has a proper physical frequency in the form of $\omega = ({\omega_R}^2 + {\omega_I}^2)^{1/2}$, where ω_R and ω_I are the real and imaginary parts of the frequency, respectively. For the highly excited modes ($n \to \infty$) $\omega_I \gg \omega_R$, and therefore $\Delta\omega \simeq \Delta\omega_I$. Hod [16,17] was the first to argue that the quasinormal modes (QNMs) can be used in the identification of the quantum transitions for the I_{adb}. Subsequently, there have been other published papers that use the Maggiore's method (MM) to achieve similar results (see for instance [18-27]).

In this study, we first focus on the investigation of the Garfinkle-Horowitz-Strominger black hole (GHSBH) [28] spectroscopy. This problem was previously studied by Wei et al. [15]. They used the QNMs of Chen and Jing [29] who studied the monodromy method [30] and obtained an equal spacing of GHSBH spectra at the high frequency modes. The main difference between the present study and that of Wei et al. is the method that we employ for computing the QNMs. There are several methods to calculate the QNMs, such as the WKB method, the phase integral method, continued fractions and direct integrations of the wave equation in the frequency domain [31]. One of our goals in this study is to consider an approximation method (its details are given in the next chapters) for obtaining the QNMs of the GHSBH. Although this method shows some similarities with the monodromy method, especially, the resulting ordinary differential equations of the two methods are different from each other. Thus, we seek to support the study of Wei et al. [15] because we believe that the studies that obtain the same conclusion using different methods are more reliable.

In the low-energy limit of the string theory, there is a family of solutions which covers the GHSBH spacetime. This geometry reveals when the electromagnetic and gravitational fields are enlarged to include a dilaton field. Dilaton generally couples to the gauge field and the metric in non-trivial form. Because of that coupling procedure, the resulting stringy BH is different from the Reissner-Nordström (RN) BH.

To employ the MM, the QNMs (a set of complex frequencies arising from the perturbed BH) of the GHSBH should be computed. To achieve this, we first consider the Klein-Gordon equation (KGE) for a massless scalar field in the background of the GHSBH. After separating the angular and the radial equations, we obtain a Schrödinger-like wave equation, which is the so-called Zerilli equation [32]. Plots of the potential indicate that there exist some cases in which the effective potential may diverge after the BH horizon; thus, the QNMs may not reach the observer in the far region. Therefore, to reliably detect the QNMs, instead of a distant observer, we consider a detector located in the vicinity of the BH. Namely, we assume that there is a probe that receives the frequencies of the QNMs and sends them to the distant observer via a transmission line. In the framework of this scenario, we focus our analysis on the near horizon region. We then derive the QNMs of the GHSBH by using a particular approximation method based on the fact that for the QNMs to exist, the outgoing waves must be terminated at the event horizon. This method is based on the studies [33,34] in which the QNMs are computed using the poles of the scattering amplitude in the Born approximation when the argument of the Gamma function takes a negative integer value. The method is further enhanced by the studies [21,24,25,35,36] in which the near-horizon form of the Zerilli equation [32] is reduced to a confluent hypergeometric (CH) differential equation [37]. After choosing the expedient solution, we consider one of the features of the CH functions that corresponds to the case when its variable is very small. Next, we use the well-known pole structure of the Gamma functions [37] to define the QNMs of the GHSBH. Once the QNMs are obtained, we use their ω_I term in the MM and obtain the GHSBH spectra.

In the sense of Riemannian geometry, conformal gravity (CG) is an exclusive name for the gravity theories that are invariant under Weyl transformations. CG admits static and asymptotically Lifshitz BH solutions. In this thesis, we also consider a particular four dimensional Lifshitz BH with the dynamical exponent $z = 0$ that is the so-called $z = 0$ Lifshitz black hole (ZZLBH). Our interest is to study spectroscopy of this BH using the MM and calculate its mass via Brown-York (BY) formalism [38,39] and Wald's entropy formula [40-42].

The thesis includes the following: In chapter 2, we first introduce the GHSBH metric including its mass calculation with Komar's formula and its thermodynamical features. We then demonstrate the method by which the massless KGE is separated for that geometry. Next, we express the radial equation in the form of the Zerilli equation with an effective potential, and then we compute the QNMs of the GHSBH. In particular, for the near–horizon region, we show how the Zerilli equation reduces to a CH differential equation, which yields the QNMs of the GHSBH spacetime. Finally, we apply the MM to obtain the quantum spectra of the entropy/area of the GHSBH.

Chapter 3 is devoted to the computation of the QNMs and the quantum spectra of the ZZLBH. We briefly introduce the ZZLBH metric. We calculate its mass by following the BY formalism. The obtained mass expression is also verified over the Wald's entropy formula [40-42] which yields the mass via the first law thermodynamics: $dM = TdS$. We separate the KGE on the ZZLBH geometry and obtain the radial wave equation. After having the near–horizon form of the Zerilli equation, we calculate the QNMs of the ZZLBH with the aid of CH differential equation. Subsequently, we employ the MM and derive the entropy/area spectra of the ZZLBH.

We present our conclusions in chapter 4. Throughout the thesis, the units $G = c = k_B = 1$ are used.

Chapter 2

QUANTIZATION OF THE GHSBH

2.1 GHSBH Spacetime

In this section, we represent the geometrical and thermodynamical properties of the GHSBH.

The four-dimensional Einstein-Maxwell-dilaton action (in the low-energy limit of the string field theory) describing the dilaton field ϕ coupled to a U(1) gauge field is given by

$$S = \int d^4x \sqrt{-g}\left[-R + 2(\nabla\phi)^2 + e^{-2\phi}F^2\right], \tag{2.1}$$

where $F^2 = F_{\mu\nu}F^{\mu\nu}$ in which $F_{\mu\nu}$ is the Maxwell field associated with a $U(1)$ subgroup of $E_8 x E_8$ or Spin (32)/Z_2 [28]. Meanwhile, the action is expressed within the Einstein frame. After applying the variational principle to the above action, we obtain the following field equations:

$$\nabla_\mu\left(e^{-2\phi}F^{\mu\nu}\right) = 0, \tag{2.2}$$

$$\nabla^2\phi + \frac{1}{2}e^{-2\phi}F^2 = 0, \tag{2.3}$$

$$R_{\mu\nu} = 2\nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}(\nabla\phi)^2 + 2e^{-2\phi}F_{\mu\rho}F_\nu^\rho - \frac{1}{2}g_{\mu\nu}e^{-2\phi}F^2. \tag{2.4}$$

Their solutions are expressed by the following static and spherically symmetric metric:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + A(r)d\Omega^2, \tag{2.5}$$

where $d\Omega^2$ is the standard metric on the 2-sphere. The metric functions are given by

$$f(r) = 1 - \frac{2M}{r}, \tag{2.6}$$

$$A(r) = r(r - 2a), \tag{2.7}$$

where the physical parameter a is defined by

$$a = \frac{Q^2 e^{-2\phi_0}}{2M}, \tag{2.8}$$

in which Q, M and ϕ_0 describe the magnetic charge, mass and the asymptotic constant value of the dilaton field, respectively. Besides, $r_+ = 2M$ represents the event horizon of the GHSBH. In this spacetime, the dilaton is governed by

$$e^{-2\phi} = e^{-2\phi_0}\left(1 - \frac{2a}{r}\right), \tag{2.9}$$

and the Maxwell field reads

$$F = Q\, sin\, \theta d\theta \wedge d\varphi. \tag{2.10}$$

For the electric charge case, one can simply apply the following duality transformations:

$$\tilde{F}_{\mu\nu} = \frac{1}{2} e^{-2\phi} \epsilon_{\mu\nu}^{\lambda\rho} F_{\lambda\rho} \quad \text{and} \quad \phi \longrightarrow -\phi. \tag{2.11}$$

Since the R^2 part of the GHSBH metric (2.5) is identical to the Schwarzschild BH, the surface gravity [41] naturally coincides with the Schwarzschild's one

$$\kappa = lim_{r \to r_+} \sqrt{-\frac{1}{2} \nabla^\mu \chi^\nu \nabla_\mu \chi_\nu} = \frac{f'(r_+)}{2} = \frac{1}{4M}, \tag{2.12}$$

where the timelike Killing vector is $\chi^\nu = [1, 0, 0, 0]$. Therefore, the Hawking temperature T_H of the GHSBH reads

$$T_H = \frac{\hbar\kappa}{2\pi} = \frac{\hbar}{8\pi M}. \tag{2.13}$$

Therefore, the Hawking temperature of the GHSBH is independent of the amount of the charge. But, the similarity between the GHSBH and the Schwarzschild BH is apparent since the radial coordinate does not belong to the areal radius. So, the entropy of the GHSBH is different than the Schwarzschild BH's entropy:

$$S_{BH} = \frac{\mathcal{A}}{4\hbar} = \frac{\pi r_+(r_+ - 2a)}{\hbar}, \tag{2.14}$$

In fact, at extremal charge $Q = \sqrt{2}Me^{\phi_0}$ i.e. $a = M$, the BH has a vanishing area and hence its entropy is zero. The extremal GHSBH is not a BH in the ordinary sense since its area has become degenerate and singular: it is indeed a naked singularity. Unlike the singularity of RN, which is timelike, this singularity is null and whence outward-directed radial null geodesics cannot hit it. For a detailed study of the null geodesics of the GHSBH, one may refer to [43]. On the other hand, one can easily prove that the first law of thermodynamics ($T_H dS_{BH} = dM - V_H dQ$ in which the electric potential on the horizon is given by $V_H = \frac{a}{Q}$) for the GHSBH is satisfied.

Energy notion always plays an important role in all physical theories. In GR, when we consider a static, asymptotically flat (AF) spacetime with the normalized timelike Killing vector ξ^a, the total mass of the vacuum spacetime in the exterior region (near infinity) can be defined by the following integral [41,44]

$$m = \frac{-1}{8\pi}\int_S \; M_{ab} \,, \tag{2.15}$$

$$M_{ab} = E_{abcd}\mathrm{s}^{cd}, \tag{2.16}$$

$$\mathrm{s}^{cd} = \nabla^c \xi^d = g^{cc}\nabla_c \xi^d = g^{cc}\left(\xi^d_{,c} + \Gamma^d_{ce}\xi^e\right). \tag{2.17}$$

Here s^{ab}, ∇^a and E_{abcd} are the surface tensor, the contravariant derivative and the covariant Levi-Civita tensor, respectively. m is the mass provided by the limit when the two dimensional sphere approaches to infinity. The covariant Levi-Civita tensor is given by [45]

$$E_{abcd} = \sqrt{-g}\varepsilon_{abcd} , \tag{2.18}$$

where ε_{abcd} is the Levi-Civita tensor for Minkowski spacetime and $g = \det[g_{ab}]$ where g_{ab} is the metric tensor.

The non-zero Christoffel symbols that we need for calculating s^{ab} are

$$\Gamma^{r}_{tt} = \frac{f(r)\, f'(r)}{2}, \tag{2.19}$$

$$\Gamma^{t}_{rt} = \frac{f'(r)}{2f(r)} . \tag{2.20}$$

Substituting the timelike Killing vector $\xi^{a} = [1, 0, 0, 0]$ into Eq. (2.17), one can easily find the non-zero components of the surface tensor as follows

$$\mathrm{s}^{rt} = -\mathrm{s}^{tr} = \frac{f'(r)}{2} = \frac{r_{+}}{2r^{2}} = \frac{M}{r^{2}} . \tag{2.21}$$

Thus, from Eq. (2.16), the integrant of Eq. (2.15) becomes

$$M_{\theta\varphi} = -M_{\varphi\theta} = \frac{r_{+}(r-2a)}{r} sin\, \theta. \tag{2.22}$$

After evaluating the integral (2.15), the GHSBH mass is found as

$$m = \frac{1}{8\pi}\int_{0}^{2\pi} d\varphi \int_{0}^{\pi} 2M\, sin\, \theta\, d\theta = \frac{r_{+}}{2} = M. \tag{2.23}$$

So we prove that the M seen in the metric function (2.6) is nothing but the mass of the GHSBH.

2.2 Separation of the KGE on the GHSBH Geometry

In this section, we derive the Zerilli equation and its corresponding effective potential for a massless scalar field propagating in the GHSBH background.

To obtain the GHSBH spectra via the MM, we shall first consider the massless scalar field Ψ satisfying the KGE:

$$\frac{1}{\sqrt{-g}}\partial_\nu\left(\sqrt{-g}g^{\mu\nu}\partial_\mu\Psi\right)=0. \tag{2.24}$$

The chosen ansatz for the scalar field Ψ has the following form

$$\Psi=A(r)^{-1/2}F(r)e^{i\omega t}Y_l^m(\theta,\varphi), \quad Re(\omega)>0; \tag{2.25}$$

in which ω and $Y_l^m(\theta,\varphi)$ represent the frequency of the propagating scalar wave and the spheroidal harmonics with the eigenvalue $-l(l+1)$ respectively. Here, m and l denote the magnetic quantum number and orbital angular quantum number, respectively. After some algebra, the radial equation can be reduced to the following form

$$\left[-\frac{d^2}{dr^{*2}}+V(r)\right]F(r)=\omega^2F(r), \tag{2.26}$$

which is nothing but the Zerilli equation [32]. Employing the tortoise coordinate r^* defined as

$$r^*=\int\frac{dr}{f(r)}, \tag{2.27}$$

we get

$$r^*=r+r_+\,ln\left(\frac{r}{r_+}-1\right), \tag{2.28}$$

or inversely, one can also obtain

$$r=r_+[1+W(u)], \tag{2.29}$$

where $u = e^{\left(\frac{r^*}{r_+}-1\right)}$ and $W(u)$ represents the $LambertW$ function or the $omega$ function [46]. It can be checked that

$$lim_{r\to r_+} r^* = -\infty \qquad \text{and} \qquad lim_{r\to\infty} r^* = \infty. \tag{2.30}$$

The effective or the so-called Zerilli potential $V(r)$ is given by

$$V(r) = \frac{f(r)}{r(r-2a)}\left[l(l+1) - \frac{a^2}{r(r-2a)}f(r) + f'(r)(r-a)\right]. \tag{2.31}$$

Figure 1 shows the effect of the a–parameter on the effective potential. In that figure, we plot $V(r^*)$ versus r^* with $M = 1$, $l = 2$ at various values of a. It is apparent from Figure 1 that when a–parameter assumes values higher than one, the effective potential diverges at a specific point that is within the event horizon and the spatial infinity. This means that the scalar waves do not always reach the observer located at the spatial infinity. Therefore, to obtain the QNMs for each time, it is better to focus our analysis on the near-horizon region of the GHSBH.

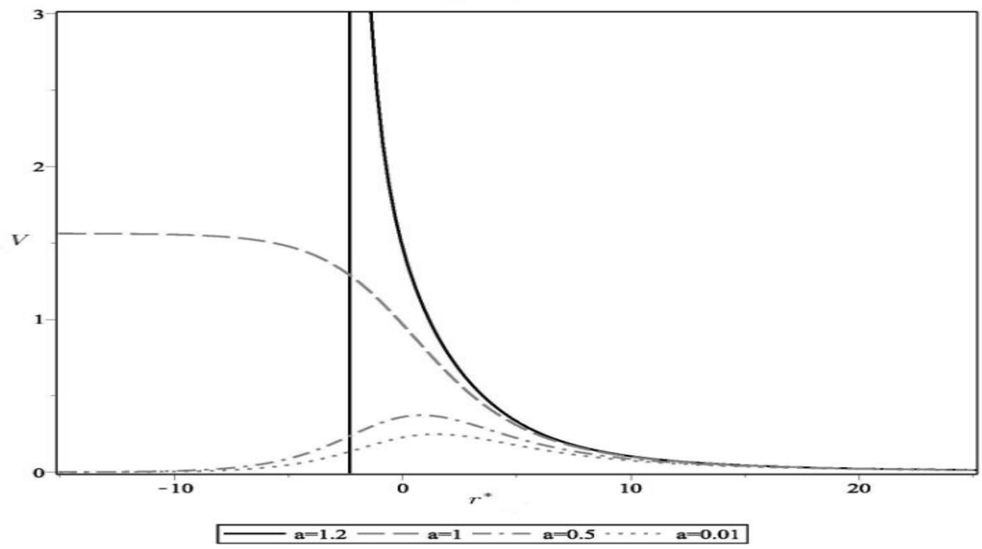

Figure 1: The plot of the effective potential $V(r^*)$ versus r^*. The physical parameters are chosen as $M = 1$ and $l = 2$. Different line styles belong to different a-values.

2.3 Spectroscopy of the GHSBH

The QNMs are conventionally defined as the solutions of the perturbational wave equation, such as Eq. (2.24), which have the following boundary conditions: the waves travel purely inward (ingoing) at the event horizon and purely outward (outgoing) at the spatial infinity. The latter condition is generally suitable for the effective potentials, which have a bumpy shape that dies off at both ends as in the Zerilli potential of the Schwarzschild BH [32]. However, as illustrated in Figure 1, depending on the value of the a–parameter, the potential (2.31) can diverge and thus stop the waves that originate from the BH and propagate toward the spatial infinity. For this reason, in this section we follow up the particular method [21,33-36] that considers only the small perturbations in the vicinity of the event horizon, and then analyze how the outgoing waves terminate there. In particular, we shall derive an analytical formula for the discrete spectrum of the QNM frequencies.

The metric function $f(r)$ can be expanded to series around the event horizon as follows

$$f(r) = f(r_+) + f'(r_+)(r - r_+) + O[(r - r_+)^2] \simeq 2\kappa y, \tag{2.32}$$

where $y = r - r_+$. After substituting this new variable into Eq. (2.31) and performing the Taylor expansion around $= 0$, the near–horizon form of the potential becomes

$$V(y) \simeq 2\kappa y[l(l+1)(C + Dy) + 2\kappa(G + Hy) - 2\kappa y N], \tag{2.33}$$

where the parameters are given by

$$C = \frac{1}{z}, \quad D = -\frac{2x}{z^2}, \quad G = \frac{x}{z}, \quad H = -\frac{x^2+a^2}{z^2}, \quad N = \frac{a^2}{z^2}, \tag{2.34}$$

with

$$x = r_+ - a, \qquad z = r_+(r_+ - 2a). \tag{2.35}$$

Furthermore, the near–horizon limit of the tortoise coordinate (2.28) becomes

$$r^* \simeq \frac{1}{2\kappa} ln\, y, \tag{2.36}$$

which enables us to obtain the near-horizon form of the Zerilli equation (2.26) as follows:

$$-4\kappa^2 y^2 \frac{d^2F(y)}{dy^2} - 4\kappa y^2 \frac{dF(y)}{dy} + V(y)F(y) = \omega^2 F(y). \tag{2.37}$$

The above differential equation has two separate solutions:

$$F(y) = y^{\frac{i\omega}{2\kappa}} e^{-i\lambda y}\left[C_1 M\left(\tilde{a}, \tilde{b}, 2i\lambda y\right) + C_2 U\left(\tilde{a}, \tilde{b}, 2i\lambda y\right)\right], \tag{2.38}$$

where $M\left(\tilde{a}, \tilde{b}, 2i\lambda y\right)$ and $U\left(\tilde{a}, \tilde{b}, 2i\lambda y\right)$ are called CH functions [37] of the first and second kinds, respectively. The parameters of the functions are given by

$$\tilde{a} = \frac{\tilde{b}}{2} - i\frac{\gamma}{4\kappa z\lambda}, \tag{2.39}$$

$$\tilde{b} = 1 + \frac{i\omega}{\kappa}, \tag{2.40}$$

where

$$\gamma = 2\kappa x + l(l+1), \tag{2.41}$$

$$\lambda = \frac{1}{z\sqrt{\kappa}}\sqrt{xl(l+1) + \kappa\left(z + \frac{3}{4\kappa^2}\right)}. \tag{2.42}$$

The following limiting forms of the CH functions are needed for our analysis [47].

$$lim_{z\to 0}\, M\left(\tilde{a}, \tilde{b}, z\right) = 1 + O(z), \tag{2.43}$$

$$lim_{z\to 0}\, U\left(\tilde{a}, \tilde{b}, z\right) = \frac{\Gamma\left(\tilde{b}-1\right)}{\Gamma(\tilde{a})} z^{1-\tilde{b}} + \frac{\Gamma\left(1-\tilde{b}\right)}{\Gamma\left(1+\tilde{a}-\tilde{b}\right)} + O\left(z^{2-\Re\tilde{b}}\right);$$

$$1 \le \Re\tilde{b} < 2\,, \qquad \tilde{b} \ne 1. \tag{2.44}$$

By using them, we obtain the near–horizon ($y \ll 1$) behavior of the solution (2.38) as

$$F(y) \sim \left[C_1 + C_2 \frac{\Gamma(1-\tilde{b})}{\Gamma(1+\tilde{a}-\tilde{b})}\right] y^{\frac{i\omega}{2\kappa}} + C_2 \frac{\Gamma(\tilde{b}-1)}{\Gamma(\tilde{a})} y^{-\frac{i\omega}{2\kappa}}. \tag{2.45}$$

and using Eq. (2.36), we represent it as the superposition of the ingoing and outgoing waves:

$$F(r^*) \sim \left[C_1 + C_2 \frac{\Gamma(1-\tilde{b})}{\Gamma(1+\tilde{a}-\tilde{b})}\right] e^{i\omega r^*} + C_2 \frac{\Gamma(\tilde{b}-1)}{\Gamma(\tilde{a})} e^{-i\omega r^*}. \tag{2.46}$$

Since the QNMs impose that the outgoing waves must spontaneously terminate at the horizon, the second term must be vanished. This is possible with the poles of the Gamma function of the denominator seen in the second term. In short, if we set $\tilde{a} = -n, n = 0, 1, 2, \ldots$ the outgoing waves vanish and hence we read the frequencies of the QNMs of the GHSBH. The result is given by

$$\omega_n = \frac{\gamma}{2z\lambda} + i\kappa(2n+1). \tag{2.47}$$

Now, n is called the overtone quantum number or the so-called resonance parameter [48]. From Eq. (2.47), one can immediately examine why the real part of ω_n depends on the angular momentum quantum number l, unlike the results given in [16,17,29]. Actually, in the literature, it remained unclear whether the l-independence for the real parts of the QNM frequencies is universal or not. Because, there are also some studies that the effect of the angular quantum number on the real part of ω_n is highlighted (see for instance [49-53]).

For the highly excited states ($n \to \infty$ and therefore $\omega_I \gg \omega_R$), we have

$$\Delta\omega \approx \Delta\omega_I = 2\kappa = \frac{4\pi T_H}{\hbar}. \tag{2.48}$$

Substituting this into Eq. (1.3), we obtain

$$I_{adb} = \frac{S_{BH}}{4\pi}\hbar. \qquad (2.49)$$

Acting upon the Bohr-Sommerfeld quantization rule ($I_{adb} = \hbar n$), we find the entropy spectrum as

$$S_n = 4\pi n. \qquad (2.50)$$

Furthermore, since $S = \frac{\mathcal{A}}{4\hbar}$ we can also read the area spectrum:

$$\mathcal{A}_n = 16\pi\hbar n. \qquad (2.51)$$

Thus, the minimum area spacing becomes

$$\Delta\mathcal{A}_{min} = 16\pi\hbar, \qquad (2.52)$$

which represents that the entropy/area spectra of the GHSBH are evenly spaced.

Chapter 3

QUANTIZATION OF THE ZZLBH

3.1 The ZZLBH Spacetime

In this section we give a brief introduction about the four dimensional Lifshitz spacetimes and particularly consider one of their metrics, which is the ZZLBH.

The Lifshitz spacetimes have received considerable attraction for being invariant under anisotropic scale and characterizing gravitational dual of strange metals [54]. Lifshitz spacetimes are described by the following line element

$$ds^2 = -\frac{r^{2z}}{l^{2z}}dt^2 + \frac{l^2}{r^2}dr^2 + \frac{r^2}{l^2}d\vec{x}^2, \tag{3.1}$$

where l is the length scale in the geometry, z denotes the dynamical exponent and $\vec{x}$ shows the spatial vector. The action corresponding to the Einstein-Weyl gravity [55] is given by

$$S = \frac{1}{2\tilde{\kappa}^2}\int \sqrt{-g}d^4x\left(R - 2\Lambda + \frac{1}{2}\alpha|Weyl|^2\right), \tag{3.2}$$

where

$$\tilde{\kappa}^2 = 8\pi, \tag{3.3}$$

$$|Weyl|^2 = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 2R^{\mu\nu}R_{\mu\nu} + \frac{1}{3}R^2, \tag{3.4}$$

and $\alpha = \frac{z^2+2z+3}{4z(z-4)}$. For CG that is the limiting case of the Einstein-Weyl gravity, α goes to infinity. For this condition, it can be seen that the static, asymptotically Lifshitz BH solutions are available for both $z = 0$ and $z = 4$ [55,60]. Besides, it is also found that $z = 3$ and $z = 4$ Lifshitz BHs exist in the Horava-Lifshitz gravity [55,60].

Here, we especially take our attention to $z = 0$ case that produces the four dimensional ZZLBH. Its metric [55] is given by

$$ds^2 = -f(r)dt^2 + \frac{4}{r^2 f(r)}dr^2 + r^2 d\Omega_{2,k}^2, \quad (3.5)$$

where the metric function $f(r)$ is described by

$$f(r) = 1 + \frac{c}{r^2} + \frac{c^2 - k^2}{3r^4}. \quad (3.6)$$

In Eq. (3.5), $d\Omega_{2,k}^2 = \frac{dx^2}{1-kx^2} + (1 - kx^2)dy^2$ corresponds to different geometrical structures depending on the k–value. When $k = 1$, $d\Omega_{2,1}^2$ represents 2–sphere, which is **our choice in this chapter.** Furthermore, one can set $k = -1$ in $d\Omega_{2,k}^2$ and get the unit hyperbolic plane, and $k = 0$ stands for the 2–torus. Spacetime (3.5) has always a curvature singularity at $r = 0$. Moreover, this singularity becomes naked when $k = 0$. There is an event horizon for $k = \pm 1$ solution:

$$r_h = \frac{1}{6}\left[\sqrt{3(4 - c^2)} - 3c\right]. \quad (3.7)$$

Since r_h^2 is positive, it is required to be $-2 \leq c < 1$. As mentioned before, our focus is on "$k = 1$ solution with $c = -1$" which yields that $r_h = 1$. So, now the metric function (3.6) can be rewritten as

$$f(r) = 1 - \frac{{r_h}^2}{r^2}. \quad (3.8)$$

The Ricci scalar ($R = R_j^j$) and Kretschmann scalar ($K = R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$) of the ZZLBH metric are given by

$$R = \frac{r_h^{\,2}}{2r^2} + \frac{2}{r^2} - \frac{3}{2}, \tag{3.9}$$

$$K = \frac{7r_h^{\,4}+8r_h^{\,2}+16}{4r^4} - \frac{r_h^{\,2}+4}{2r^2} + \frac{3}{4}. \tag{3.10}$$

As it is clear from above, while $r \to 0$ both scalars diverge that proves the existence of the singularity at $r = 0$. In addition to this, while $r \to \infty$, $R = -\frac{3}{2}$ and $K = \frac{3}{4}$ which show that ZZLBH is a non-asymptotically flat (NAF) spacetime. By using the following formula of the surface gravity [41]

$$\kappa = \left(-\frac{1}{4} lim_{r\to r_h} g^{tt} g^{rr} g_{tt,r} g_{tt,r}\right)^{1/2}, \tag{3.11}$$

we get

$$\kappa = f'(r_h)\frac{r_h}{4} = \frac{1}{2}. \tag{3.12}$$

Therefore, the Hawking temperature T_H of the ZZLBH reads

$$T_H = \frac{\kappa}{2\pi} = \frac{1}{4\pi}. \tag{3.13}$$

So, the entropy of the ZZLBH becomes

$$S_{BH} = \frac{A_h}{4} = \pi r_h^2. \tag{3.14}$$

3.1.1 Mass Calculation of the ZZLBH via the BY Formalism

The concept of quasilocal mass (M_{QL}) was proposed by physicists about forty years ago to measure the energy of a given compact region by a closed spacelike 2-surface.

GR unifies space and time, and this union, which is the so-called spacetime has a curvature that represents the gravitational force. Energy–mass matchup is fundamental in the GR. The energy/mass of a given compact region by a closed spacelike 2–surface (hypersurface $\sum$; $d\Omega_{2,k}^2$) is measured by the M_{QL} definition. In general, M_{QL} computation is valid for NAF BH geometries.

In this section, we will consider BY formalism for calculating the M_{QL} of the ZZLBH. In this formalism, a spherically symmetric N–dimensional metric solution [38,39] is given by

$$ds^2 = -F(R)^2dt^2 + \frac{dR^2}{G(R)^2} + R^2 d\Omega_{N-2}^2, \tag{3.15}$$

which formulates the M_{QL} with the following definition [56,57]

$$M_{QL} = \frac{N-2}{2} R^{N-3} F(R)\left[G_{ref}(R) - G(R)\right] \tag{3.16}$$

where R is the radius of hypersurface $\sum$ and $G_{ref}(R)$ is an optional non-negative reference function providing zero energy for the spacetime. If we adopt the BY metric (3.15) to our 4–dimensional ZZLBH metric (3.5), we get the following matching.

$$F(R) = \sqrt{f(r)} = \sqrt{1-z}, \quad G(R) = \frac{F(R)}{g(r)} = \frac{R}{2}\sqrt{1-z}, \tag{3.17}$$

where $z = \frac{r_h^2}{r^2}$ and $g(r) = \frac{2}{R}$. The reference G-function is found as

$$G_{ref}(R) = G(R)\Big|_{z=0} = \frac{R}{2}. \tag{3.18}$$

After substituting Eq. (3.17) into Eq. (3.16) and making a straightforward calculation, one can obtain the mass of the ZZLBH as

$$M_{QL} \equiv M = \frac{r_h^2}{4}. \tag{3.19}$$

Therefore, the event horizon reads

$$r_h = 2\sqrt{M}. \tag{3.20}$$

3.1.2 Mass Calculation of the ZZLBH via the Wald's Entropy Formula

In this section, we will introduce the Wald's entropy calculation [40,41]. We shall derive the mass of the ZZLBH by using the first law of the thermodynamics: $dM = TdS$. Then, we will compare this result with the BY's mass (3.19).

It is convenient to start with the timelike Killing vector ξ^{μ} which describes the symmetry of time translation in the spacetime. The Wald's entropy is then defined by

$$S = \frac{2\pi}{\kappa} \int_{\Sigma} d^2x \sqrt{h} \beta, \tag{3.21}$$

where

$$\beta = \epsilon_{\mu\nu} J^{\mu\nu} = \frac{1}{16\pi}, \tag{3.22}$$

$$\epsilon_{\mu\nu} = \frac{1}{2}\left(n_{\mu}u_{\nu} - n_{\nu}u_{\mu}\right), \tag{3.23}$$

where n_{μ} is the outward unit normal vector of hypersurface Σ, which satisfies $n_{\mu}n^{\mu} = 1$. And h is the induced metric on the hypersurface Σ of a horizon, which is now a 2-sphere with $h = r^2 \sin\theta$. On the other hand, u_{μ} is the four-vector velocity defined as the proper velocity of a fiducial observer moving along the orbit of ξ^{μ}. And $J^{\mu\nu}$ is called the Noether potential [58,59]:

$$J^{\mu\nu} = -2\Theta^{\mu\nu\rho\sigma}\left(\nabla_{\rho}\xi_{\sigma}\right) + 4\left(\nabla_{\rho}\Theta^{\mu\nu\rho\sigma}\right)\xi_{\sigma}, \tag{3.24}$$

where

$$\Theta^{\mu\nu\rho\sigma} = \frac{1}{32\pi}\left(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}\right). \tag{3.25}$$

The timelike Killing vector of the metric (3.5) is given by $\xi^{\mu} = \gamma \frac{\partial}{\partial t}$. The normalization constant γ should be normalized at the AF Minkowski spacetime (at spatial infinity) over the condition: $g_{\mu\nu}\xi^{\mu}\xi^{\nu} = -1$. Thus, we obtain

$$\gamma = 1. \tag{3.26}$$

The four-vector velocity u^μ is defined as

$$u^\mu = \frac{1}{\alpha}\xi^\mu, \tag{3.27}$$

with $\alpha = \sqrt{-\xi^\mu \xi_\mu}$. Thus

$$u^t = \frac{r}{\sqrt{r^2 - r_h^2}}. \tag{3.28}$$

Meanwhile, using Eq. (3.26) the surface gravity κ can be calculated by the following expression [42]

$$\kappa = lim_{r \to r_h} \sqrt{\frac{\xi^\mu \nabla_\mu \xi_\nu \xi^\rho \nabla_\rho \xi^\nu}{-\xi^2}} = \frac{\gamma}{2}. \tag{3.29}$$

Since $\gamma = 1$, the surface gravity yields $\kappa = \frac{1}{2}$ as it is found in Eq. (3.12). For the metric (3.5) we obtain the outward unit normal vector as

$$n_r = \frac{1}{n^r} = \frac{2}{\sqrt{r^2 - r_h^2}}. \tag{3.30}$$

Thus, the non-zero components of $\epsilon_{\mu\nu}$ tensor (3.23) are

$$\epsilon_{tr} = -\epsilon_{rt} = -\frac{1}{r}. \tag{3.31}$$

It is worth to note that the Wald's entropy is independent of the normalization of ξ^μ, since the Noether potential and the surface gravity are proportional to the normalization constant of ξ^μ. Hence, this constant does not appear in the entropy formula.

We obtain 24 non-zero components of $\Theta^{\mu\nu\rho\sigma}$:

$\Theta^{\mu\nu\rho\sigma}$	$Value$
$\Theta^{trtr} = \Theta^{rtrt} = -\Theta^{trrt} = -\Theta^{rttr}$	$\frac{1}{32\pi}\left(\frac{-r^2}{4}\right)$
$\Theta^{\theta r\theta r} = \Theta^{r\theta r\theta} = -\Theta^{\theta rr\theta} = -\Theta^{r\theta\theta r}$	$\frac{1}{32\pi}\left[\frac{(r^2-r_h^2)}{4r^2}\right]$
$\Theta^{\varphi r\varphi r} = \Theta^{r\varphi r\varphi} = -\Theta^{\varphi rr\varphi} = -\Theta^{r\varphi\varphi r}$	$\frac{1}{32\pi}\left[\frac{(r^2-r_h^2)}{4r^2\, sin\,\theta^2}\right]$
$\Theta^{\theta\varphi\theta\varphi} = \Theta^{\varphi\theta\varphi\theta} = -\Theta^{\theta\varphi\varphi\theta} = -\Theta^{\varphi\theta\theta\varphi}$	$\frac{1}{32\pi}\left(\frac{1}{r^4\, sin\,\theta^2}\right)$
$\Theta^{t\varphi t\varphi} = \Theta^{\varphi t\varphi t} = -\Theta^{t\varphi\varphi t} = -\Theta^{\varphi tt\varphi}$	$\frac{-1}{32\pi}\left[\frac{1}{(r^2-r_h^2)\, sin\,\theta^2}\right]$
$\Theta^{\theta t\theta t} = \Theta^{t\theta t\theta} = -\Theta^{\theta tt\theta} = -\Theta^{t\theta\theta t}$	$\frac{-1}{32\pi}\left[\frac{1}{(r^2-r_h^2)}\right]$

(3.32)

One can check that the covariant derivative of $\Theta^{\mu\nu\rho\sigma}$

$$\nabla_\rho \Theta^{\mu\nu\rho\sigma} = 0, \tag{3.33}$$

which means the second term of the Noether potential (3.24) is vanished. So, we have

$$J^{\mu\nu} = -2\Theta^{\mu\nu\rho\sigma}\left(\nabla_\rho \xi_\sigma\right) = -2\Theta^{\mu\nu\rho\sigma} C_{\rho\sigma}, \tag{3.34}$$

where $C_{\rho\sigma} = \nabla_\rho \xi_\sigma$ that results in

$$C_{tr} = -C_{rt} = -\frac{r_h^2}{r^3}. \tag{3.35}$$

After substituting Eqs. (3.32) and (3.33) into Eq. (3.34), we compute the non-zero components of the Noether potential as

$$J^{tr} = -J^{rt} = -\frac{r_h^2}{32\pi}. \tag{3.36}$$

Finally, the entropy (3.21) of the ZZLBH is calculated as

$$S = \pi r_h^2 = \frac{\mathcal{A}_h}{4\hbar}, \tag{3.37}$$

which is in consistence with the Bekenstein-Hawking entropy. Hence, the mass of the ZZLBH can now be derived from this entropy by using the following integral

$$M = \int T dS, \tag{3.38}$$

where $T = 1/(4\pi)$ (see Eq. (3.13)). After evaluating the above integral, the mass is found to be

$$M = \frac{r_h^2}{4}, \tag{3.39}$$

which is nothing but the mass (3.19) obtained in the BY formalism. Consequently, we may rewrite the metric function (3.8) as follows

$$f(r) = 1 - \frac{4M}{r^2}\,. \tag{3.40}$$

3.2 Separation of the KGE on the ZZLBH Geometry

In this section, we solve the eigenvalue problem presented by the KGE with the boundary conditions at the horizon and spatial infinity in order to compute the frequency of QNMs, and the entropy/area spectra of the ZZLBH by using the MM [11].

In order to derive the spectra of ZZLBH, it is convenient to start with the massive KGE, which is given by

$$\frac{1}{\sqrt{-g}}\partial_\nu\left(\sqrt{-g}g^{\mu\nu}\partial_\mu\Psi\right) = m^2\Psi. \tag{3.41}$$

The ansatz of the scalar field Ψ can be chosen as

$$\Psi = \frac{1}{r}F(r)e^{i\omega t}Y_l^m(\theta,\varphi), \quad Re(\omega) > 0, \tag{3.42}$$

where $F(r)$ is the radial function. After substituting Eq. (3.42) into Eq. (3.41), the radial part of the equation reduces to the Zerilli equation [32] given by

$$\left[-\frac{d^2}{dr^{*2}}+V(r)\right]F(r)=\omega^2F(r), \tag{3.43}$$

where the effective potential $V(r)$ is obtained as

$$V(r)=f(r)\left\{\frac{l(l+1)}{r^2}+\frac{1}{4}\left[f(r)+r\frac{df(r)}{dr}\right]+m^2\right\}, \tag{3.44}$$

and the tortoise coordinate r^* can be found by the following integral definition

$$r^*=2\int\frac{dr}{rf(r)}, \tag{3.45}$$

which results in

$$r^*=ln\left(\frac{r^2}{r_h^2}-1\right), \tag{3.46}$$

where the above form of r^* is valid for $r>r_h$. One may check that the limits of r^* are as follows

$$lim_{r\to r_h}r^*=-\infty \quad \text{and} \quad lim_{r\to\infty}r^*=\infty. \tag{3.47}$$

Similarly, the limits of the Zerilli potential (3.44) are

$$lim_{r\to r_h}V(r)=0 \quad \text{and} \quad lim_{r\to\infty}V(r)=m^2+\frac{1}{4}. \tag{3.48}$$

It is obvious that the potential never terminates at the spatial infinity. Even, it tends to diverge at the spatial infinity when the scalar particle is very massive ($m\to\infty$).

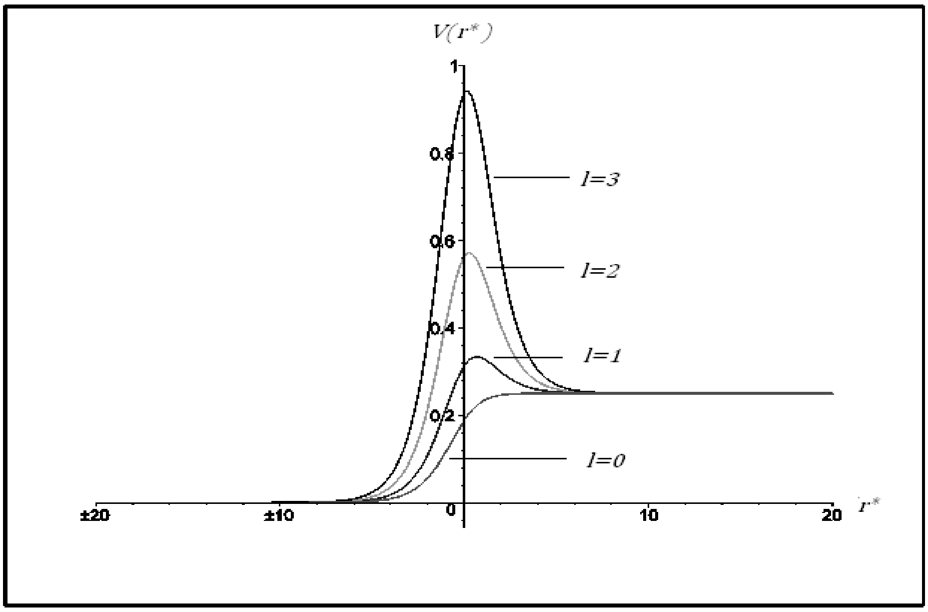

Figure 2: The plot of the effective potential $V(r^*)$ versus r^*. The physical parameters are chosen as $M = 1$ and $m = 0.001$. Different lines belong to different l-value.

3.3 Spectroscopy of the ZZLBH

In this section, we will follow the particular approximation method [36,37,41,43,46], described in detail in the section 2.3, and apply it to the ZZLBH geometry to obtain its entropy/area spectra.

It can be seen that expansion of the function $\frac{f(r)r}{2}$ around the event horizon is

$$\frac{f(r)r}{2} = \frac{r_h}{2} f(r_h) + \frac{d}{dr}\left[\frac{f(r)r}{2}\right]\Big|_{r=r_h} (r - r_h) + O[(r - r_h)^2], \tag{3.49}$$

which can be expressed in terms of the surface gravity

$$\frac{f(r)r}{2} \approx 2\kappa y, \tag{3.50}$$

where $y = r - r_h$. After substituting Eq. (3.50) into Eq. (3.44) and performing Taylor expansion around $y = 0$, we derive the near–horizon form of the Zerilli potential as

$$V(y) \simeq 4\kappa G y[G^2(1 - 2Gy)l(l + 1) + \kappa(1 + 2Gy) + m^2], \tag{3.51}$$

with parameter $G = \frac{1}{r_h}$. Furthermore, the near–horizon limit of the tortoise coordinate (3.46) becomes

$$r^* \simeq \frac{1}{2\kappa} ln\, y, \tag{3.52}$$

which authorizes us to obtain the near–horizon form of the Zerilli equation (3.41) as

$$-4\kappa^2 y^2 \frac{d^2F(y)}{dy^2} - 4\kappa y^2 \frac{dF(y)}{dy} + V(y)F(y) = \omega^2 F(y). \tag{3.53}$$

Eq. (3.53) has two separate solutions:

$$F(y) = y^{\frac{i\omega}{2\kappa}} e^{-z/2}\left[C_1 M\left(\bar{a},\bar{b},z\right) + C_2 U\left(\bar{a},\bar{b},z\right)\right], \tag{3.54}$$

where $M\left(\bar{a},\bar{b},z\right)$ and $U\left(\bar{a},\bar{b},z\right)$ are the CH functions [37] of the first and second kind, respectively. The parameters of the functions are given by

$$\bar{a} = \frac{\bar{b}}{2} + \frac{\gamma}{\sqrt{\beta}}, \tag{3.55}$$

$$\bar{b} = 1 + \frac{i\omega}{\kappa}, \tag{3.56}$$

$$z = 2iG\sqrt{\beta}y, \tag{3.57}$$

where

$$\gamma = \frac{1}{2}\left\{1 + \frac{1}{\kappa}\left[l(l+1)G^2 + m^2\right]\right\}, \tag{3.58}$$

$$\beta = 2\left[l(l+1)\frac{G^2}{\kappa} - 1\right]. \tag{3.59}$$

By using the limiting forms of the CH functions given in Eqs. (2.43) and (2.44), we obtain the near horizon $y \ll 1$ behavior of the solution Eq. (3.54) as follows

$$F(y) \sim \left[C_1 + C_2 \frac{\Gamma(1-\bar{b})}{\Gamma(1+\bar{a}-\bar{b})}\right] y^{\frac{i\omega}{2\kappa}} + C_2 \frac{\Gamma(\bar{b}-1)}{\Gamma(\bar{a})} y^{-\frac{i\omega}{2\kappa}}. \tag{3.60}$$

Using Eq. (3.52), we represent it as the superposition of the ingoing and outgoing waves:

$$F(r^*)\sim\left[C_1+C_2\frac{\Gamma(1-\bar{b})}{\Gamma(1+\bar{a}-\bar{b})}\right]e^{i\omega r^*}+C_2\frac{\Gamma(\bar{\bar{b}}-1)}{\Gamma(\bar{a})}e^{-i\omega r^*}. \tag{3.61}$$

Since the QNMs impose that the outgoing waves spontaneously terminate at the horizon, the second term of Eq. (3.61) vanishes. This scenario is enabled by the poles of the Gamma function in the denominator of the second term. Specifically, if $\bar{a}=-n$, $n=0,1,2,...$, the outgoing waves vanish and we can read the frequencies of the QNMs of the ZZLBH. The result is

$$\omega_n=i\kappa(2n+1)+\frac{2\kappa\gamma}{\sqrt{\beta}}. \tag{3.62}$$

As aforementioned before, n is the resonance parameter [48]. Therefore, the transition frequency between two highly damped neighboring states is

$$\Delta\omega\approx\Delta\omega_I=2\kappa=\frac{4\pi T_H}{\hbar}, \tag{3.63}$$

where $T_H=\frac{\hbar\kappa}{2\pi}$ is the Hawking temperature of the ZZLBH given in Eq. (3.13). Consequently, substituting Eq. (3.63) into the adiabatic invariant quantity (1.3), we obtain the entropy spectrum of the ZZLBH as follows

$$S_n=4\pi n. \tag{3.64}$$

Since $S=\frac{\mathcal{A}}{4\hbar}$, we can also read the area spectrum:

$$\mathcal{A}_n=16\pi\hbar n, \tag{3.65}$$

which yields the minimum spacing of the area as

$$\Delta\mathcal{A}_{min}=16\pi\hbar. \tag{3.66}$$

We deduce that the spacings of the entropy/area spectra are evenly spaced as found by Bekenstein [9]. Our findings encourage the Kothawala et al.'s hypothesis [18] signifying that the BHs have equally spaced area spectrum in Einstein's gravity theory.

Chapter 4

CONCLUSION

In this thesis, the quantum spectra of the GHSBH and ZZLBH are investigated via the MM that is based on the adiabatic invariant formulation (1.3) of the BHs. We have therefore attempted to find the QNM of the GHSBH and the ZZLBH. The massless (massive) KGE for the GHSBH (for the ZZLBH) geometry has been separated into the angular and the radial parts. In particular, the Zerilli equations (2.26) and (3.43) with their effective potentials (2.31) and (3.44) of the associated geometries have been obtained.

From Figure 1 illustrated in chapter 2, it is clear that depending on the value of the a–parameter, the effective potential may diverge beyond the event horizon. In such a case, the scalar waves of the QNMs cannot reach the observer located in the far region. We have therefore employed the particular method for finding the QNMs at the near horizon region. According to this method, the Zerilli equation is well approximated by a CH differential equation. After some straightforward calculations including the limiting forms of the CH functions, we then obtained the QNMs of the GHSBH. To this end, we used the poles of the Gamma functions. We then applied the MM to the highly damped QNMs to derive the entropy/area spectra of the GHSBH. The obtained spectra are equally spaced and are independent of the physical parameters of the GHSBH as concluded in the study of Wei et al. [15].

Moreover, our results support the Kothawala et al.'s conjecture [18] stating that the BHs in Einstein's gravity theory have equi-spaced area spectrum.

In chapter 3, the quantum spectra of the ZZLBH are investigated through the MM. As described in chapter 2, we have considered the near–horizon form of the Zerilli equation of the ZZLBH, which is Eq. (3.53). Then, we have approximated Eq. (3.53) to another CH differential equation and by this way we have managed to read the frequency of the QNMs of the ZZLBH (3.62). The obtained entropy/area spectra, which are represented in Eqs. (3.64) and (3.65) are found to be equi-spaced.

As a final remark, our calculations have revealed that the value of the dimensionless constant ε is 16π. This result may be questioned because it is different from the expected value of 8π. In fact, this difference helps us understand why Bekenstein [8] defined a somewhat ambiguous definition of ξ as of the order of unity. Considering the Heisenberg uncertainty principle, Bekenstein gave a flexible definition for ξ. Thus, a ξ–value of two is acceptable in the computation of the BH spectroscopy. However, as stated by Hod [16,17] in the subject of the BH quantization, the spacings between two neighboring levels may be different depending on which method is applied.

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