Robot Move Scheduling In an FMC

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Submitted to the
Institute of Graduate Studies and Research
In partial fulfillment of the requirements for the Degree of

Master of Science
in
Industrial Engineering

Eastern Mediterranean University
Jun 2012
Gazimağusa, North Cyprus

Approval of the Institute of Graduate Studies and Research

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ABSTRACT

In this study we deal with a cyclic schedule (the robot performs a set of activities and when the system returns to the initial state, the cycle is completed). A Flexible Manufacturing Cell (FMC) with few numbers of machines is considered which processes parts in which the loading and unloading of the machines is made by a robot, this is a Flow Shop technology and robot serves the machines in a cyclic manner. The type of FMC is of Robot centered type. FMC may produce the same type of parts or different types of parts. The robot is scheduled and its move can be cyclic, so that we can define which type of movement of robot (as a result, loading / unloading sequence of machines) is more appropriate for our purpose (minimization the cycle time and maximization of outcomes).

This study contains a complete mathematical theory of determination of cycle time, which is missing from the literature**.** In Chapter 3, the optimality condition of each strategy of robot cyclic move is discussed and finally in last chapter, a lower bound for the feasible scheduling strategy is obtained.

Keywords: Flexible manufacturing system, robot centered type FMS, job scheduling, robot cyclic move scheduling.

ÖZ

Bu çalışma döngüsel bir program ile ilgilidir(robot bir dizi etkinlikten gerçekleştirir ve sistem başlangıç durumuna döndüğünde, döngü tamamlanır).Robotlar tarafından yapılan yükleme, boşaltma ve işlemler Esnek Üretim Hücre (FMC) makinelerinin birkaçıyla dikkate alınır.Bu BİR akış dükkanı teknolojisidir ve robot servis makineleri dairesel şeklindedir.FMC'nın tipi Robot merkezli tiptedir. MYK parça veya parçaları farklı türde aynı tip üretebilir.

Robot planlanıyor ve onun haraketleri dairesel olabilir bu yüzden robotun haraketini tanımlayabiliriz ki (sonuç olarak, makinelerin yükleme boşaltma düzenleri) daha uygundur bizim amacımız için (minimum döngü süresi ve maximum sonuçlar) Bu çalışma literatürde eksik döngü zamanı, belirlenmesi tam bir matematiksel teorisi içerir.Uygun planlama stratejisi için Bölüm 3, robot döngüsel hareket her stratejisinin eniyilik durumu tartışılmış ve nihayet son bölümde ise, bir alt sınırı elde edilir. Anahtar Kelimeler:

Anahtar Kelimeler: Esnek üretim sistemi, robot merkezli tipi FMS, iş planlaması, robot döngüsel hareket zamanlama.

To ***Jonathan.H.Ch***

The one who always remembers me, whom her warm embrace is the safest place for the tears. Whom who is the meaning of the true love.

To 3 friends; ***Fruzsina, Shahed*** *and* ***Mazyar*** *for always being real*

To Karina, for her sweet bitterness

And to ***Mezhgona, the one who will be remembered forever***

ACKNOWLEDGMENT

Being at this point could not be possible without helps and supports of many people.

My first and biggest gratitude is to my supervisor, Prof. Bela Vizvari. For his passion for the science, inspiration and his great efforts to explain and clarify the most complicated issues and concepts. He helped me to develop this study with his fresh innovative ideas and with all the mathematical and non-mathematical part of this thesis. His scientific sagacity and passion of research encouraged me not to give up and finish this path. Beside the academic issues, he, with his vast knowledge, has been a valuable advisor for my personal life within these 3 years. I am truly indebted to him.

I would like to thank Asst. Prof. Dr. Gökhan İzbırak for his continuous support and guidance during these years of mine in the department and for his patience and tolerance in the hard times of the Industrial Engineering department and for his passion to the big family of Industrial Engineering department members. I am indebted to him for providing a loving learning and developing environment.

Also I am thankful to Gizem Kaya who helped me by providing the ÖZ part for without her help it was too difficult for me.

I sincerely acknowledge Mazyar Ghadirineghad for his advises and useful comments who supported me as a real friend.

I have to gratefully thank the industrial engineering department members at Eastern Mediterranean University, all my colleagues, my professors, staff and friends for their assistance in many different ways.

I owe my family specially my parents who encouraged me all throughout my studies. I am grateful to them for they brought me up, supported me and loved me.

In the end my appreciation goes to every single person who helped me during this thesis in any way, and my apology for I am not able to name them one by one.

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INTRODUCTION

1.1. Flexible Manufacturing System (FMS)

A group of interconnected machines through an automated transportation system which are controlled by a central control system (normally a central computer) form a Flexible Manufacturing System (FMS).

It is also true to say a FMS is a group of interconnected work stations by means of an automated transportation system. This transportation system is responsible for the material handling and storage task. The control of the machines and the transportation system is by an integrated computerized controlling system [7].

The term of flexibility in one hand is due to the reason that he system is able to process a variety of different part types simultaneously and on the other hand it is capable to work with different volume of production and therefore it respond to changing demand patterns.

1.1.1. The basic components of FMS

The main components of a Flexible Manufacturing System are the working stations, automated material handling and storage system and the central computer control system [1].

1.1.2. Industrial FMS Communication

In an Industrial Flexible Manufacturing System (FMS) the robots, Computer-controlled Machines, Numerical controlled machines (CNC), are the main components of the system as well as computers, sensors, and inspection machines. The use of robots in the manufacturing industries provides various rang of production type from high utilization to high volume production. Each Robotic cell or node is located along a material handling system such as a conveyer. The production of different part type is possible through a different combination of manufacturing nodes. The movement of parts is done through a robot or an automated material handling system. Finally the finished parts will be sent to an inspection node, and will be unloaded from the System.

A robotic Flow shop is made of m machines M_j, J=1, 2,…, n and an input and output station and at least one robot. The robot is responsible for performing all material handling tasks, loading and unloading the machines is also the robots task. All parts available at the beginning of the sequence and must be processed through all machines.

1.2. Flexibilities in FMS

Based on [6] Flexibility in manufacturing is the ability of dealing with different combination of type and volume of product for allowing the variation in parts assembly and process sequence to provide the chance of making change in volume and the design of the products whenever it is required.

Based on [1] Figure 1 illustrates the relation between the product variety and the product volume in different production systems, as it is shown the maximum flexibility in producing the different part types belongs to stand alone NC machines while the maximum volume belongs to the Transfer Line system.

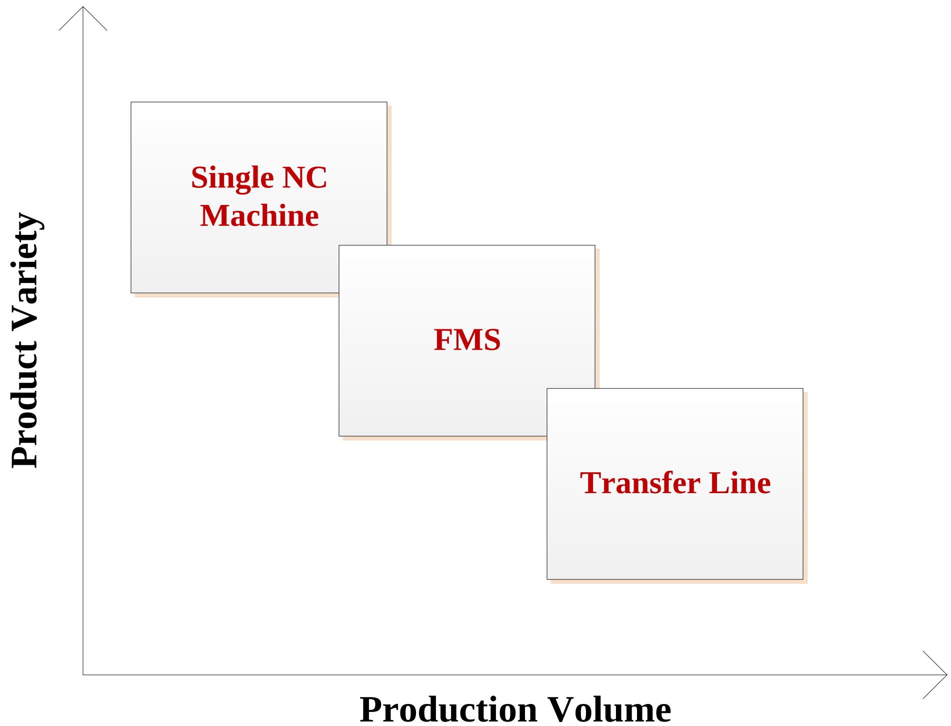

Figure 1. The characteristic of a FMS

1.2.2. The Types of Flexibilities in FMS

Based on [1] the types of flexibilities can be discussed in 3 categories in an FMS; Machine Flexibilities, System Flexibilities and Aggregated Flexibility. In following each of them is briefly discussed;

a. Basic Flexibility;

The machine flexibility: The machine is capable of performing various operations provided that the required tools are available in its tool magazine.

The material handling flexibility: Transferring different part types to the machines is possible.

The operation flexibility: the sequence of the operations is not fixed. The system is able to use various alternative operation sequences for processing a part type.

b. System Flexibilities;

Volume flexibility: The system is flexible enough to operate at different volume of production.

Expansion flexibility: The system can be increasingly expanded.

Routing flexibility: it is possible to use different alternative paths to process a part type effectively. In means for a given process plan using a certain path is not a must.

Process flexibility: Without increasing any setup, the system is able to produce different volume of part type.

Product flexibility: With some minor setup, the system is capable to produce different part types.

c. Aggregate flexibilities

Program flexibility: In the case that there is no external interrupt, the system is able to run for a long period of time.

Market flexibility: The system is flexible enough to adopt the changes in market conditions.

In the next table the meaning of Flexibility in different approaches in an FMS is discussed.

Table 1. Flexibility Concept in Different Approaches

Approach	Flexibility meaning
Manufacturing	The system is capable to produce different part provided that the required tools are already installed in its tool magazine. The company is able to use the existing facilities to converts its process (es) from an old line of products to produce a new product. The production schedule is flexible to change, to modify a part as well as to handle multiple parts.
Operational	The products can be highly customized
Customer	The system is able to provide a fast delivery to the customers.
Strategic	Various types of products can be offered to the customers.
Capacity	Making increase or decrease in production level is fast and easy, also shifting the capacity of one product or servise to the other one is possible.

1.3. The History of FMS

By the middle years of 1960's, the market competition turned more intense, from 1960 to 1970 for a period of nearly 10 years the cost was the main concern for the companies and producers, somehow later the quality got the higher priority. By increasing the complexity of the markets, the delivery speed turned to be an important factor for the customers as well [1].

To respond to all these changes a new strategy known as customizability was formulated. The companies were in need of having a flexible environment in operations to be able to satisfy the different segments of the market.

The very first idea of a flexible machining system that could operate without human operators 24 hours a day under computer control was proposed in England (1960s) known as "System 24", somehow at first, the emphasis was more on automation rather than reorganizing the work stations.

The early FMSs were too large, complicated and expensive, and they were controlled by very complex software. Only a few numbers of industries were able to invest in those types of FMSs.

1.4. Numerical Control (NC)

The term “Numerical control” or simply “CNC” refers to the automation of the machines which are controlled trough a handle wheels or levers or mechanically automated via cam. 1940s and 1950s are the time of early NC machines, at that time the operation plan used to be fed to the machine on punched tapes. Not much later, as the computer systems were developing rapidly, these early punch tape operating servomechanisms turned to analog and digital computers and what we know as modern **Computer Numerical Control (CNC)** got born. In these modern systems, by using the Computer Aided Design (CAD) and Computer Aided Manufacturing (CAM) program, designing the end-to-end component is highly automated. Today CNCs are one of the most important components of the modern FMSs.

1.4.1. Direct Numerical Control (DNC)

DNC is also refereeing to **Distributed Numerical Control**. It is a common term for a network of CNC machines. In this system the program which already is stored in a computer is sent directly to the operating machine, if there are a number of machines, then the program is distributed to them where it is required. To have DNCs working properly, having suitable software is required, normally the manufacturer of the DNCs provide the suitable software as well. In the case that CAM program must be run on some of the CNC machines, the networking of the DNC is required.

1.5. Flexible Manufacturing Cell

The common trend of today in FMS is to use a small version of the FMS which is limited to a cell. It is known as Flexible Manufacturing Cell (FMC). At least two Flexible Manufacturing Cell, form a Flexible manufacturing system [8].

FMS is a technology and a philosophy which provides a systematic view of manufacturing. This concept is one of the ways that enables the manufacturer to achieve agility and agility is the key element of success in highly competitive market of today where satisfying the customers is more difficult than any other time in the history.

1.6. Robot Centered Type of FMS

Based on [1] 5 different types of FMS are defined so far, sequential, random FMS, dedicated FMS, engineered FMS and modular FMS. Modular FMS itself is divided to 5 subsystems; progressive or Line Type, Loop Type, Ladder Type, Open field type and Robot centered type. The last one is relatively the newest type of flexible system. In this type of system usually one or more robots are used as the material handling system, as long as the robots are equipped with suitable grippers they are capable of handling of rotational parts.

In our study, the Flexible Manufacturing cell is a robot centered FMS type. A robot in the center and 3 CNC machines are served with the robot consequently. Our aim is to schedule the robot to serve the machines in order to maximize the output and minimize the sequence long run. Figure 2 illustrates the general state of a Robot centered FMS type.

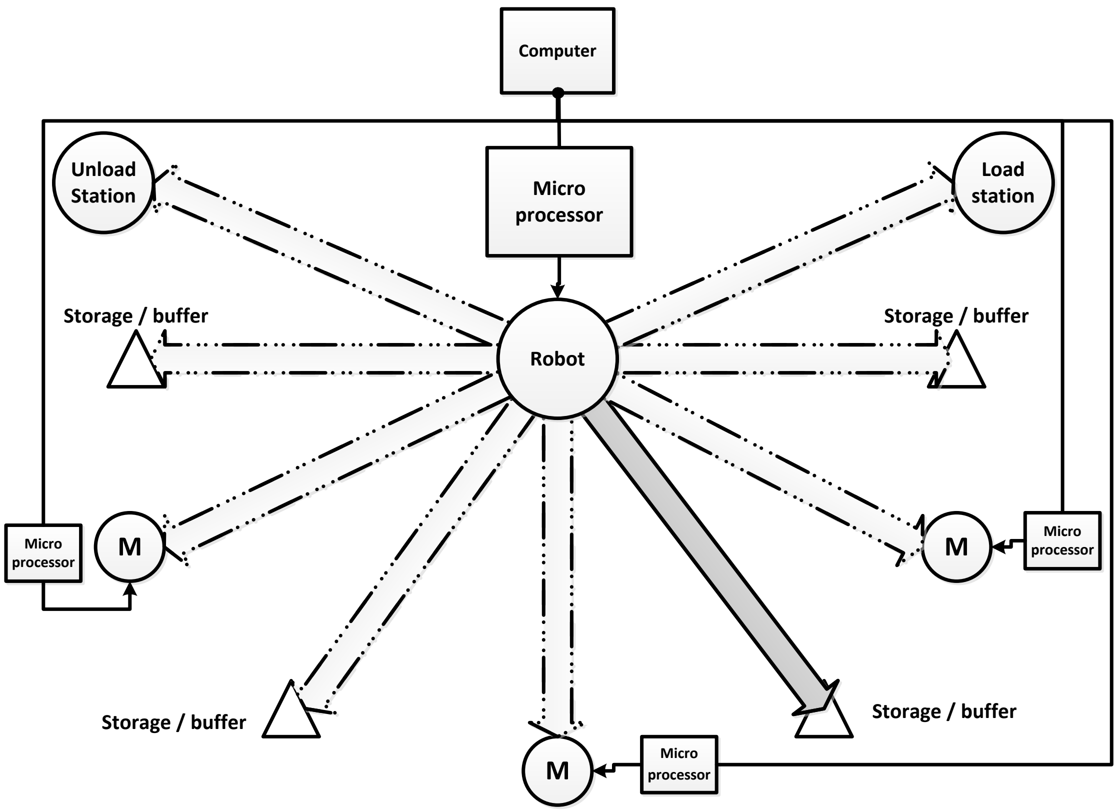

Figure 2. Conceptual robot centered type FMS

Chapter 2

CYCLES of an FMC

2.1. Introduction

In this study, we deal with a small flexible manufacturing system with only one production cell so that our FMS will turn to FMC. The load and unload process, and transportation of the parts between machines is made by a robot. The machines used in the FMC are of DNC type of machines which are flexible enough to perform several types of operations provided that the required tools are installed on them. Machines are placed along a line, the moves of the robot can be considered as movements along a line.

However the moves of the robot can be explained easier such that the load and unload stations and the machines are located around a circuit.

Figure 3 illustrates the robot move status through a circuit which is equivalent to the linear move type.

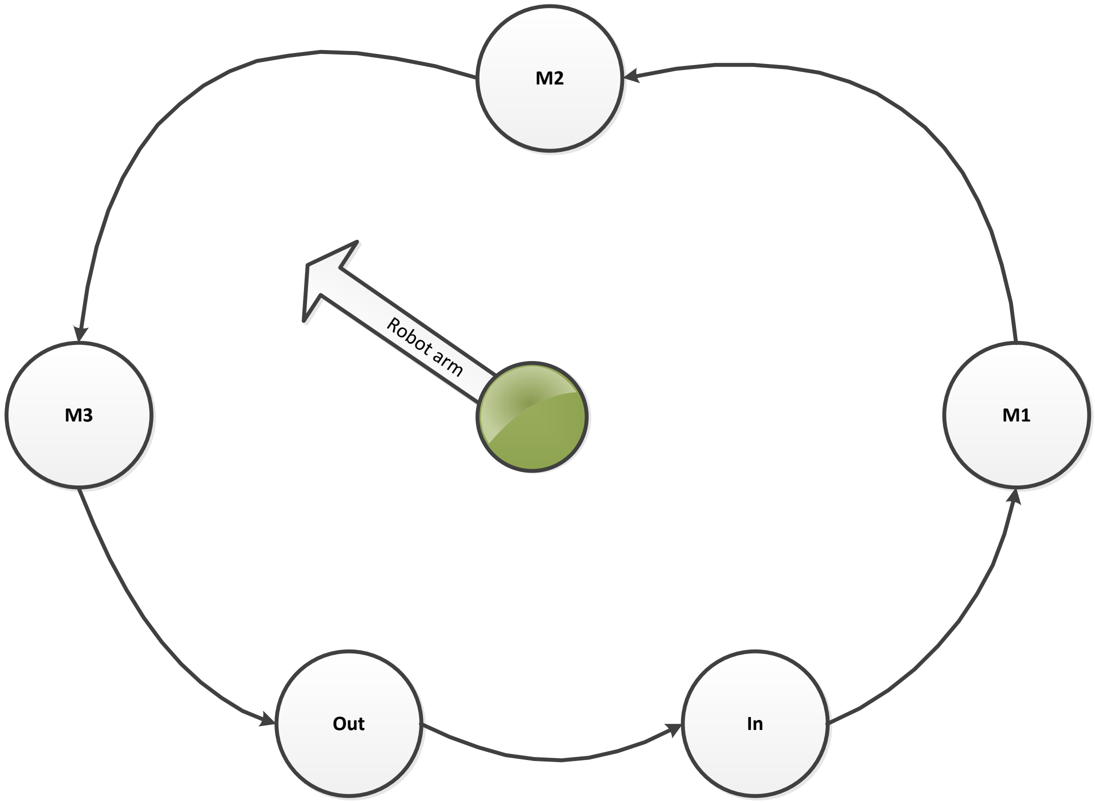

Figure 3. The route of the products in an FMC of 3 machines

In the basic version of the problem, the FMC produces only one type of products. The parts cannot be stored between the machines, i.e. when a part arrives to a machine, the machine must be empty [4]. The FMC has a flow shop – like technology, i.e. a part visits all machines in the fixed order; it goes first to M_1, then to M_2 and finally to M_3 [9].

It is provided that the system is completely automatized and therefore the robot moves in a cyclic way. The cycles are completely determined by the order of the stations from where the part is transported to the next machine, thus the possible number of cycles is $3! = 6$ (statistically it is proved that since we have 3 machines and no preemption is allowed, for choosing the first machine we have 3 choice, for choosing the second one, two choices and finally for the last one only one choice, it gives us 3! possible strategy). The 6 different possible cycles are denoted by: $C_1, C_2, \dots, C_6$.

2.2 Previous Researches

S.P. Sethi *et al.* in the paper [3] dealt with the problem of sequencing the parts and robot moves in a robotic cell where the robot was used to feed the machines in the cell. The aim was to maximize the long-run average throughput of the system subject to the constraint that parts are produced in a proportion of their demand. The cycle time formulas were developed and analyzed for that purpose for cells producing a single part type using two or three machines [2]. However the complete proof of these formulas is still missing in the literature.

[3] And [2] contain the proof of only one case and it is not obvious how the same method can be applied to the other cases. In our study a complete mathematical proof of the theorem is provided.

2.3. Formula description of Cycles

The states of system are defined by a 4-tuple E (e_1, e_2, e_3, e_4) Where:

$$e_1 \begin{cases} 1 & M_1 \textit{ is occupied} \\ 0 & \textit{otherwise} \end{cases}$$

$$e_2 \begin{cases} 1 & M_2 \textit{ is occupied} \\ 0 & \textit{otherwise} \end{cases}$$

$$e_3 \begin{cases} 1 & M_3 \textit{ is occupied} \\ 0 & \textit{otherwise} \end{cases}$$

$$
e_4 \begin{cases} \text{In} & \textit{the robot has just arrived to input station.} \\ \text{Out} & \textit{the robot has just dropped the part in unload station.} \\ & \\ M_i^+ & \textit{indicates the robot's position is right after Machine i is loaded.} \\ M_j^- & \textit{indicates the robot's position is right after Machine i is unloaded.} \end{cases}
$$

$\boldsymbol{\delta}$: *time for robot move between two consecutive machines*

$\boldsymbol{\epsilon}$: *the loading/unloading time.*

The following notations are also used:

$$\boldsymbol{\alpha} = 4\delta + 4\epsilon$$

$$\boldsymbol{\beta} = 8\delta + 4\epsilon$$

$\boldsymbol{w_j}$ = *the robot's waiting time for the completion of the part on* M_i

The cycles consist of the following states. The description of each cycle starts with the movement where M_3 was just loaded is as follows:

$$
\begin{cases} C_1\text{: In, } M_1, M_2, M_3. \\ C_2\text{: In, } M_2, M_1, M_3. \\ C_3\text{: In, } M_1, M_3, M_2. \\ C_4\text{: In, } M_3, M_1, M_2. \\ C_5\text{: In, } M_2, M_3, M_1. \\ C_6\text{: In, } M_3, M_2, M_1. \end{cases}
$$

Table 2. Robot movement strategy for each cycle C_j

$\mathbf{C_1}$	$(0,0,1,M_3^+)(0,0,0,\text{Out})(0,0,0,\text{In})(1,0,0,M_1^-)(1,0,0,M_1^+)(0,1,0,M_2^-)(0,1,0,M_2^+)(0,0,1,M_3^-)(0,0,1,M_3^+)$
$\mathbf{C_2}$	$(0,1,1,M_3^+)(0,1,0,\text{Out})(0,1,0,\text{In})(1,1,0,M_1^-)(1,1,0,M_2^+)(1,0,1,M_3^-)(1,0,1,M_1^+)(0,1,1,M_2^-)(0,1,1,M_3^+)$
$\mathbf{C_3}$	$(0,1,1,M_3^+)(0,1,0,\text{Out})(0,1,0,M_2^+)(0,0,1,M_3^-)(0,0,1,\text{In})(1,0,1,M_1^-)(1,0,1,M_1^+)(0,1,1,M_2^-)(0,1,1,M_3^+)$
$\mathbf{C_4}$	$(1,0,1,M_3^+)(1,0,0,\text{Out})(1,0,0,M_1^+)(0,1,0,M_2^-)(0,1,0,M_2^+)(0,0,1,M_3^-)(0,0,1,\text{In})(1,0,1,M_1^-)(1,0,1,M_3^+)$
$\mathbf{C_5}$	$(1,0,1,M_3^+)(1,0,0,\text{Out})(1,0,0,M_1^+)(0,1,0,M_2^-)(0,1,0,\text{In})(1,1,0,M_1^-)(1,1,0,M_2^+)(1,0,1,M_3^-)(1,0,1,M_3^+)$
$\mathbf{C_6}$	$(1,1,1,M_3^+)(1,1,0,\text{Out})(1,1,0,M_2^+)(1,0,1,M_3^-)(1,0,1,M_1^+)(0,1,1,M_2^-)(0,1,1,\text{In})(1,1,1,M_1^-)(1,1,1,M_3^+)$

In the following the figures 4 to 9, illustrate the robot move strategy for all cycles C_j.

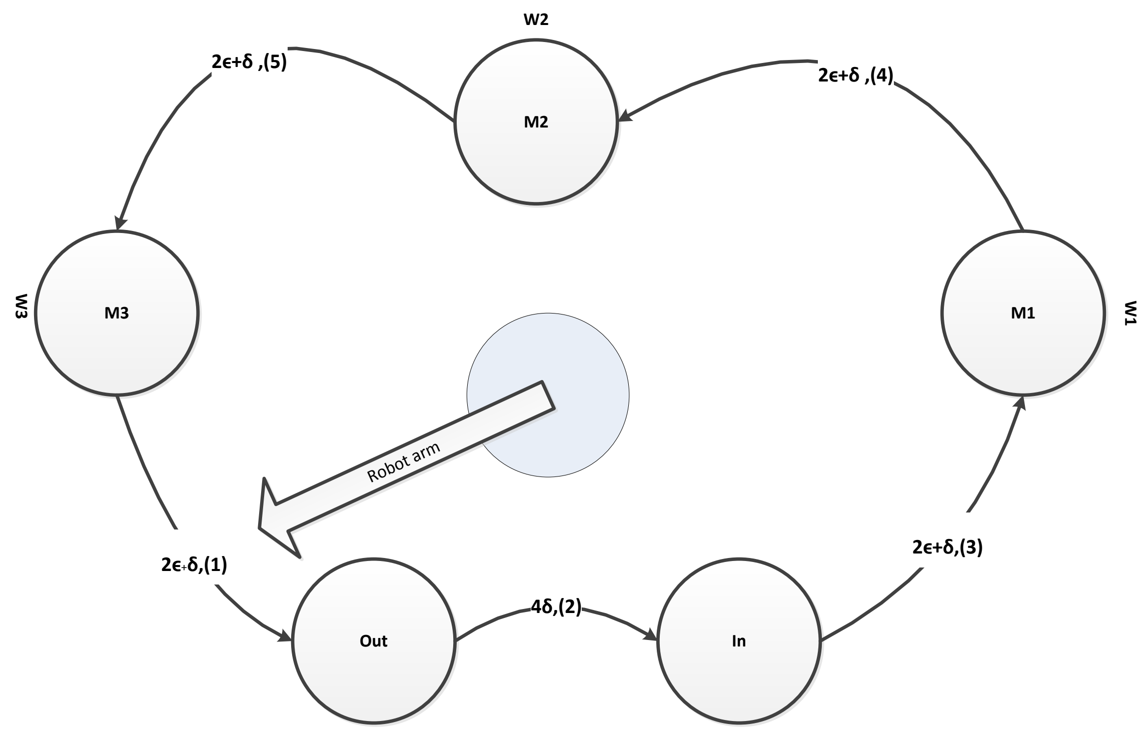

Figure 4. Cyclic sequence C_1 with $T_1 = 2\alpha + a + b + c$

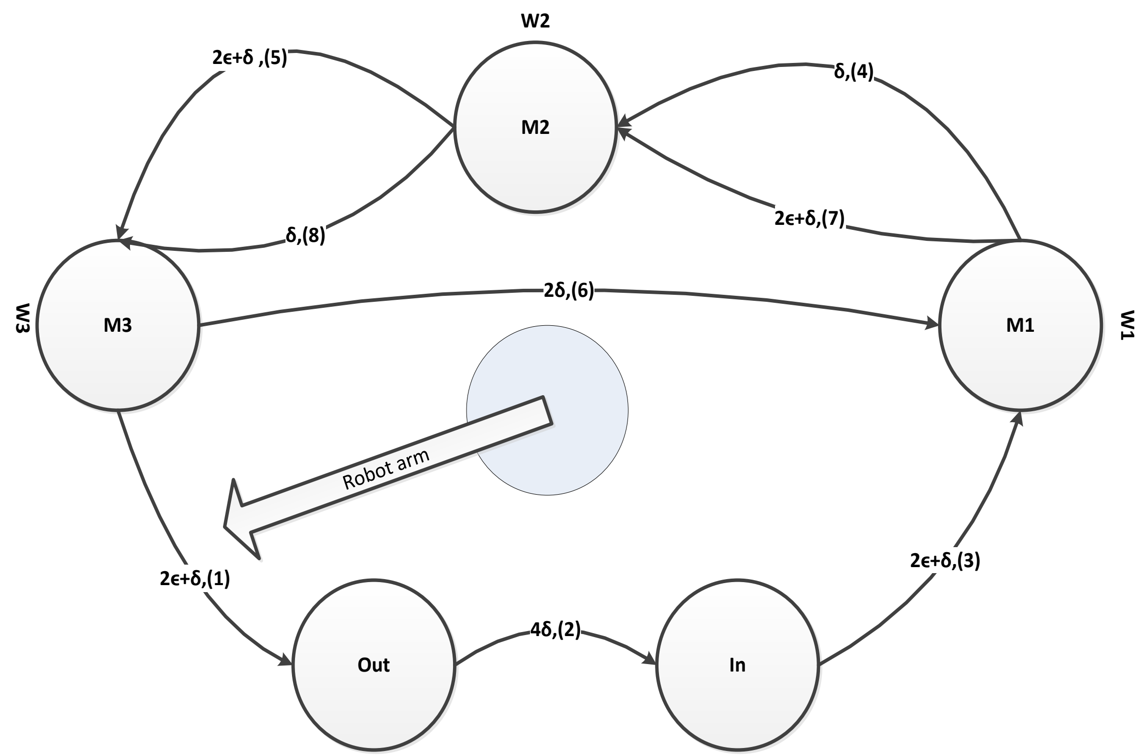

Figure 5. Cyclic sequence C_2 with $T_2 = \alpha + \max\{\beta, b, \beta/2 + a, \beta/2 + c, (a + b + c)/2\}$

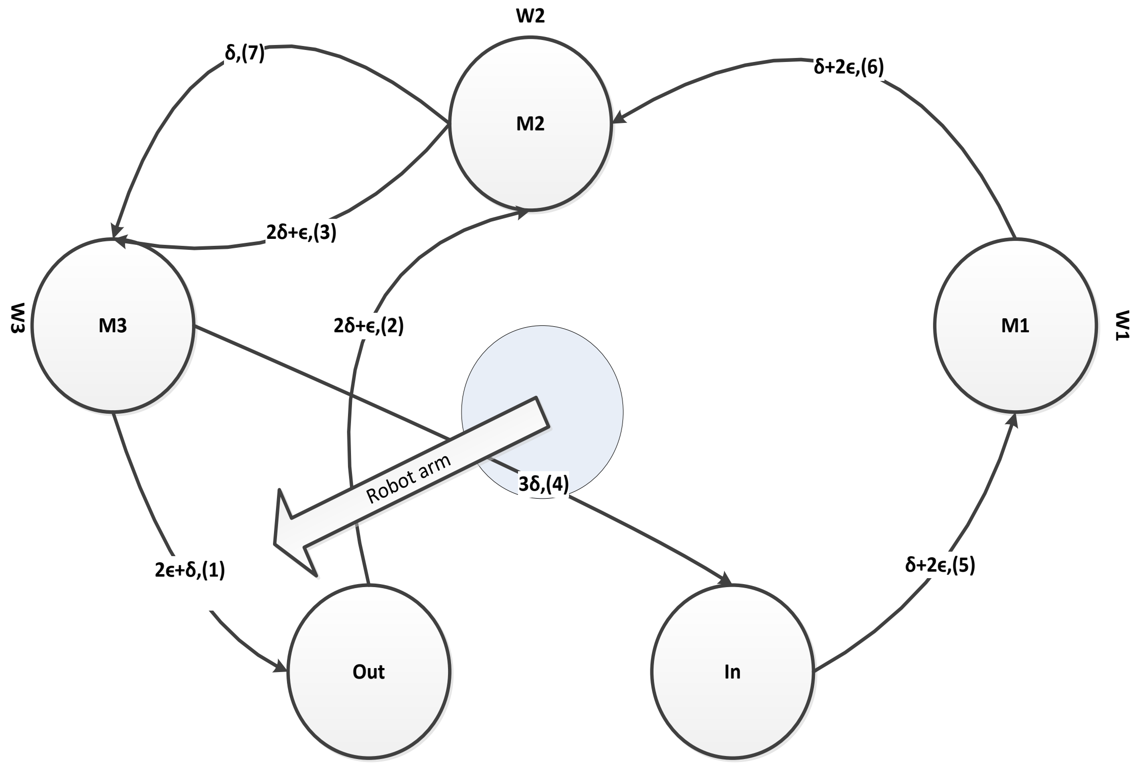

Figure 6. Cyclic sequence C_3 with $T_3 = \alpha + \max\{c, \alpha + a + 2\delta, \alpha/2 + a + b\}$

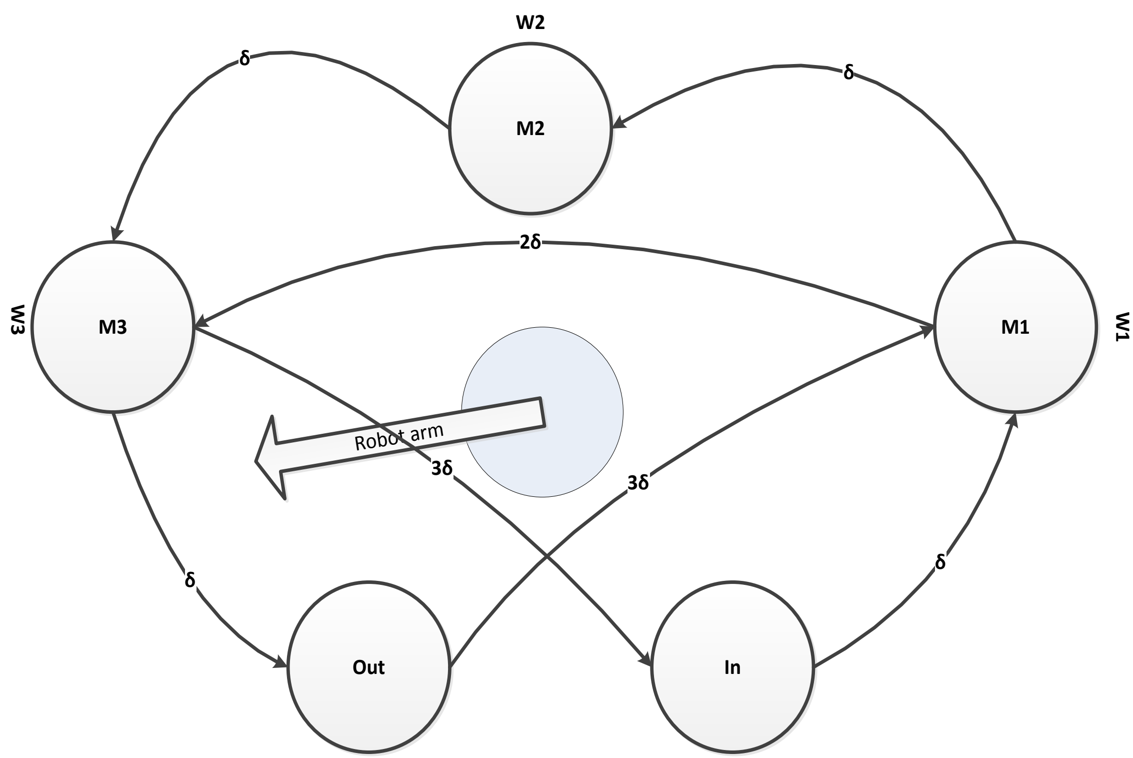

Figure 7. Cyclic sequence C_4 with $T_4 = \alpha + \max\{\beta + b, \alpha/2 + a + b, \alpha/2 + b + c\}$

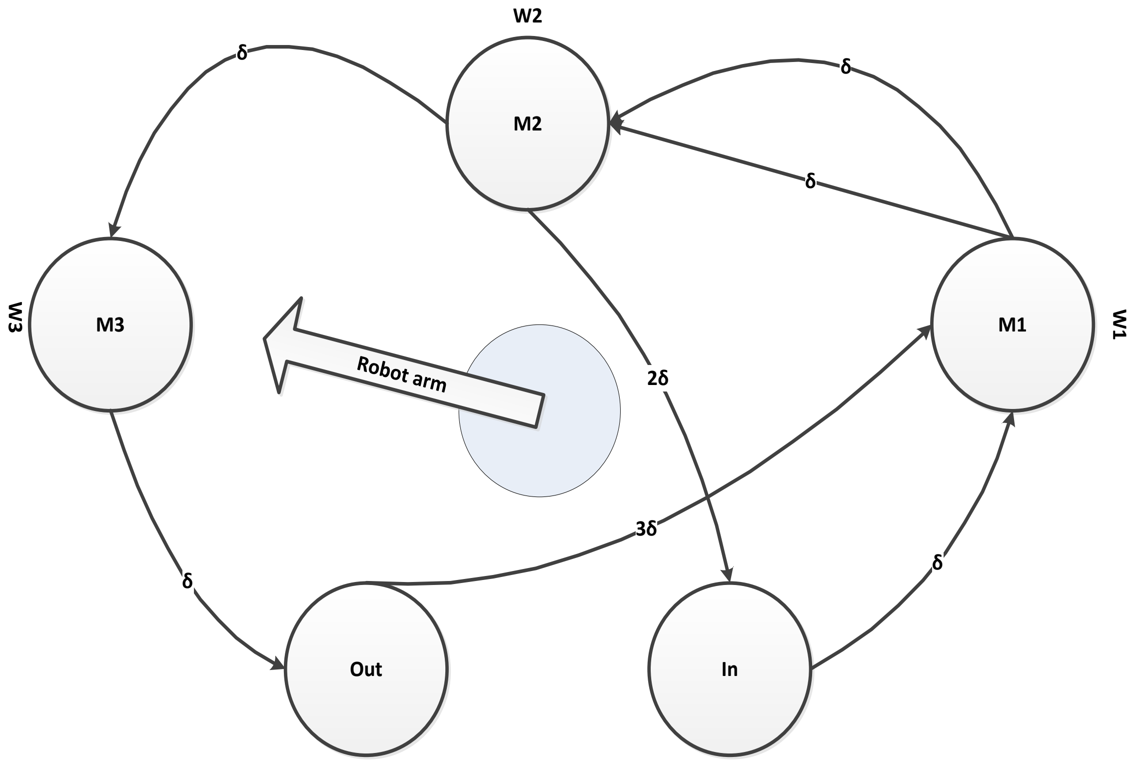

Figure 8. Cyclic sequence C_5 with $T_5 = \alpha + \max\{a, \alpha + c + 2\delta, \alpha/2 + b + c\}$

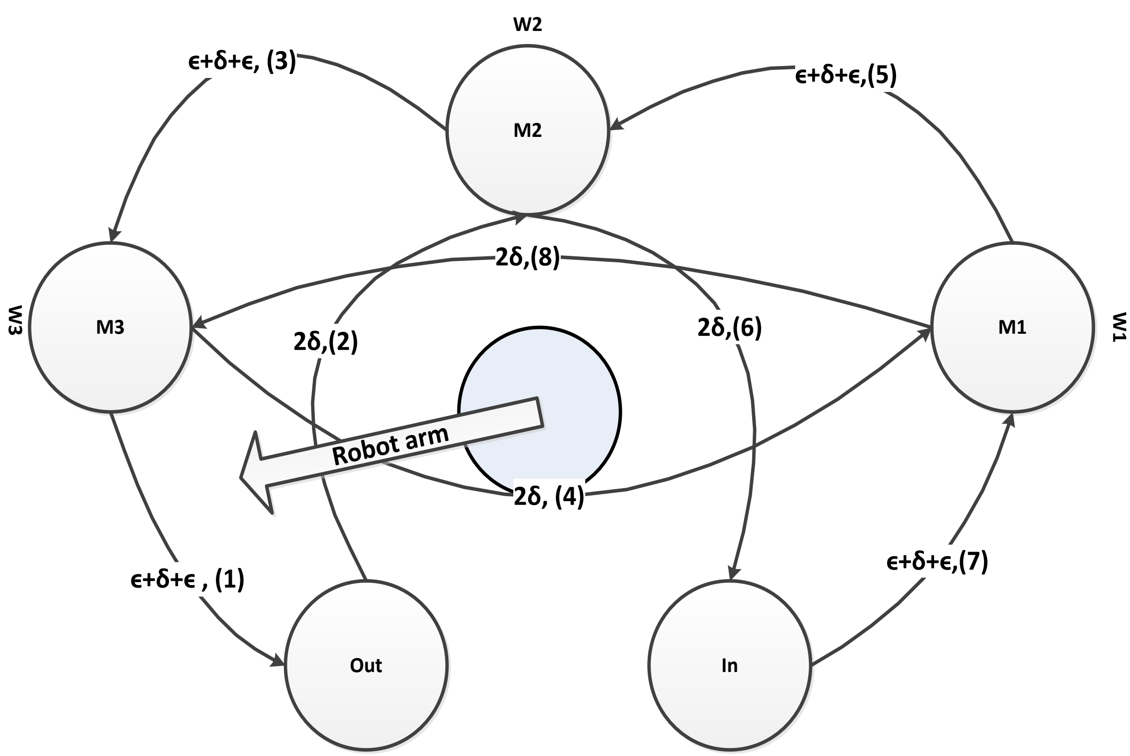

Figure 9. Cyclic sequence C_6 with $T_6 = \alpha + \max\{\beta, a, b, c\}$

The waiting time in all cases is the positive time difference between completion time of the job on machines and the returning time of the robot to the same machine [5]. If the robot returns earlier than the completion time of the part, then the time deference is positive and positive waiting time exist. Otherwise the waiting time is zero which means the difference is either negative or zero.

2.4. The Main Research

What is done by us is resolving the model again and determining the waiting times and cycle times with a different method and different proof for all of the cycles as follows. The process is to find the optimal route (policy) for the robot to serve the machines. Optimal means that the productivity of the system is maximal, i.e. the length of the cycle is minimal, so that we must find the optimal cycle among these available cycles such that the cycle time is minimal.

The final result for the cycle times obtained from Sethi *et al.* [3] studies is as follows:

THEOREM: let T_i be the cycle time of cycle C_j, then:

$$\boldsymbol{T_1} = 2\alpha + a + b + c$$

$$\boldsymbol{T_2} = \alpha + \max\{\beta, b, \beta/2 + a, \beta/2 + c, (a + b + c)/2\}$$

$$\boldsymbol{T_3} = \alpha + \max\{c, \alpha + a + 2\delta, \alpha/2 + a + b\}$$

$$\boldsymbol{T_4} = \alpha + \max\{\beta + b, \alpha/2 + a + b, \alpha/2 + b + c\}$$

$$\boldsymbol{T_5} = \alpha + \max\{a, \alpha + c + 2\delta, \alpha/2 + b + c\}$$

$$\boldsymbol{T_6} = \alpha + \max\{\beta, a, b, c\}$$

2.5. Proof for each cycle time

First of all recall that the robot starting position is right after loading the M_3. T_1 represents the cycle time for the first strategy of cyclic movement of the robot. In this strategy the robot does the following actions:

The first step of the robot is waiting on M_3 until the process is finished. The waiting times are the processing times on all machines as the robot loads the machine and waits until the part is completed in all of the cases. In the second step the robot unloads the part from M_3 and takes it to unload station, the 3rd step is returning to the load station, in the 4th step the robot takes a part from load station and moves to M_1, and loads it. 5th step is waiting on M_1 for completion of the process, after the process is done in the 6th step the robot unloads M_1 and takes the part to M_2 and loads it, again the 7th step is waiting on M_2 for completion of the process, and finally at step 8, the robot unloads M_2 and takes the part to M_3 and loads the M_3, step 9 is actually repeating the first step so the cycle is completed.

To simplify these explanations, the following format will be used to describe each cycle (Note that the time of each step is at the end of the description of the step):

Robot moves sequences of C_1 with duration of T_1;

Step 1: The robot is at M_3 and waits until the process is finished: $\boldsymbol{w_3 = c}$

Step 2: The robot unloads the part from M_3 and takes it to the unload station and unloads it: $\boldsymbol{2\epsilon + \delta}$

Step 3: The robot moves to loading station: $\boldsymbol{4\delta}$

Step 4: The robot takes a part and moves to M_1, and loads it: $\boldsymbol{2\epsilon + \delta}$

Step 5: The robot waits at M_1 until the process is finished: $\boldsymbol{w_1 = a}$

Step 6: The robot unloads M_1 and takes the part to M_2 and loads M_2**:** $\boldsymbol{2\epsilon + \delta}$

Step 7: The robot waits at M_2 until the process is finished: $\boldsymbol{w_2 = b}$

Step 8: The robot unloads the part and takes it to M_3 and loads M_3: $\boldsymbol{2\epsilon + \delta}$

Step 9: The robot waits until the process of job at M_3 is done. (It is step 1 again)

The total cycle time of C_1 is$\boldsymbol{2\alpha + a + b + c}$. The sum of the step's duration time is equal to the value given in the statement.

Robot moves strategy of C_2 with duration of $\boldsymbol{T_2}$;

Step 1: The robot waits on M_3 (waiting to unload it) while M_1 and M_2 are already loaded: $\boldsymbol{w_3}$

Step 2: After the process of part on M_3 is finished, the robot unloads the machine and takes the part to unload station and unloads it: $\boldsymbol{2\epsilon + \delta}$

Step 3: The robot goes to loading station: $\boldsymbol{4\delta}$

Step 4: It picks up another part, goes to M_1 and loads it: $\boldsymbol{2\epsilon + \delta}$

Step 5: The robot moves to M_2, waits if necessary until the process is done: $\boldsymbol{\delta + w_2}$

Step 6: The Robot unloads the part from M_2 and takes it to M_3 (which is empty now) and loads M_3: $\boldsymbol{2\epsilon + \delta}$

Step 7: It returns to M_1 and waits if necessary until the process is finished: $\boldsymbol{2\delta + w_1}$

Step 8: The robot unloads the part and takes it to M_2 and loads it: $\boldsymbol{2\epsilon + \delta}$

Step 9: Then goes to M_3: $\boldsymbol{\delta}$

The duration for T_2 is as follows:

$$T_2 = w_3 + 2\epsilon + \delta + 4\delta + 2\epsilon + \delta + \delta + w2 + 2\epsilon + \delta + 2\delta + w_1 + 2\epsilon + \delta + \delta = (12\,\delta + 8\,\epsilon + w_3 + w_1 + w_2).$$

Note that:

$a = w_1 + w_2 + 4\delta + 2\epsilon$ implies that: $w_1 = max\,\{0, a - (w_2 + 4\delta + 2\,\epsilon)\}$ [*follow steps 4 to 7*].

$b = w_2 + w_3 + 8\delta + 4\epsilon$ implies that: $w_2 = max\,\{0, b - (w_3 + 8\delta + 4\epsilon)\}$ [*follow steps 8 to 6*].

$c = w_3 + w_1 + 4\delta + 2\epsilon$ implies that: $w_3 = max\,\{0, c - (w_1 + 4\delta + 2\epsilon)\}$ [*follow steps 6 to 1*].

Recall that:

$\beta = 8\delta + 4\epsilon$

$\beta/2 = 4\delta + 2\epsilon$

$\alpha = 4\delta + 4\epsilon$

$\alpha/2 = 2\delta + 2\epsilon$

Notation: in the case of all cycles, 8 possible scenarios can be distinguished according to the sign of the waiting times as follows:

- $w_1 = w_2 = w_3 = 0$
- $w_1 = w_2 = 0$, $w_3 > 0$
- $w_1 = w_3 = 0$, $w_2 > 0$
- $w_1 > 0$, $w_1 = w_3 = 0$
- $w_3 = 0$, w_1, $w_2 > 0$
- $w_2 = 0$, w_1, $w_3 > 0$

- $w_1, w_2, w_3 > 0$
- $w_1 = 0$, $w_2, w_3 > 0$

Each of the 8 scenarios will be investigated separately here and in the case of the following cycles.

Case 1: $\boldsymbol{w_1 = w_2 = w_3 = 0}$

Then:

$a - (w_2 + 4\delta + 2\epsilon) \leq 0$ implies that $a \leq 4\delta + 2\epsilon$.

$b - (w_3 + 8\delta + 4\epsilon) \leq 0$ implies that $b \leq 8\delta + 4\epsilon$.

$c - (w_1 + 4\delta + 2\epsilon) \leq 0$ implies that $c \leq 4\delta + 2\epsilon$.

Note: since w_1= max $\{0, a - (w2 + 4\delta + 2\epsilon)\}$ and we decided $w_1 = 0$, it automatically implies that $a - (w_2 + 4\delta + 2\epsilon) \leq 0$. The same logic is used in all cases.

Hence:

$a, b, c \leq \beta$ and $a + b + c \leq 2\beta$ implies that $(a + b + c)/2 \leq \beta$, by using the same logic it could be said that: $a + \beta/2 \leq \beta, c + \beta/2 \leq \beta$. Thus it is obvious that β is the maximum between $(b, \beta/2 + a, \beta/2 + c, (a + b + c)/2)$, the suggested statement is $T2 = \alpha + max\{\beta, b, \beta/2 + a, \beta/2 + c, (a + b + c)/2\}$, and what we obtained through our calculations is: $T2 = \alpha + \beta$.

The sum of the step's duration time, i.e. $T2 = 12\delta + 8\epsilon + w_1 + w_2 + w_3$, is equal to the value given in the statement, hence the statement is true.

Case 2: w_1= w_2= 0, w_3> 0

Thus:

$T_2 = 8\epsilon + 12\delta + w_3$.

$a - (w_2 + 4\delta + 2\epsilon) \le 0$ implies that: $a \le (4\delta + 2\epsilon) = \beta/2$.

$c - (w_1 + 4\delta + 2\epsilon) \ge 0$ implies that: $c \ge (4\delta + 2\epsilon) = \beta/2 \; and \; w_3 = c - \beta/2$.

$b - (w_3 + 8\delta + 4\epsilon) \le 0$ implies that: $b \le (w_3 + 8\delta + 4\epsilon) = \beta + w_3 = c + \beta/2$.

As a result $T_2 = 8\epsilon + 12\delta + c - \beta/2 = \alpha + c + \beta/2$, furthermore:

$a \le \beta/2$ Implies that $a + \beta/2 \le \beta$ and $c \ge \beta/2$ implies that $c + \beta/2 \ge \beta$.

Hence:

$\beta/2 + c + \beta/2 \ge a + b$. Thus, $(c + \beta)/2 \ge (a + b)/2 \; and \; c + \beta/2 \ge (a + b + c)/2$.

Thus; c+ $\beta/2$ is the maximum among the terms of the maximum in the expression of T_2. Thus, the sum of the step's duration time is equal to the value given in the statement.

Case 3: w_1= w_3= 0, w_2 > 0

Thus:

$T_2 = 8\epsilon + 12\delta + w_2$.

$b - (w_3 + 8\delta + 4\epsilon) \ge 0$ implies that: $w2 = b - \beta \; and \; b \ge (8\delta + 4\epsilon)$ and hence $b \ge \beta$.

$a - (w_2 + 4\delta + 2\,\epsilon) \leq 0$ Implies that: $a \leq (b - \beta + 4\delta + 2\epsilon)$ Thus $a + \beta \leq b + \beta/2$ thus $a + \beta/2 \leq b$.

$c - (w_1 + 4\delta + 2\epsilon) \leq 0$ Implies that: $c \leq (4\delta + 2\epsilon)$ thus $c \leq \beta/2$ accordingly $c + \beta/2 \leq \beta \leq b$.

Hence:

$a + c \leq b - \beta/2 + \beta/2 = b$.

Thus $a + c + b \leq 2b$ this implies that: $(a + b + c)/2 \leq b$.

Then we came to conclusion that b is the maximum among all other parameters.

What we obtained through our calculations is: $T_2 = 8\epsilon + 12\delta + b - \beta = 4\epsilon + 4\delta + b = \alpha + b$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Case 4: $w_1 > \mathbf{0}$**,** $\boldsymbol{w_2} = \boldsymbol{w_3}$ **= 0**

Then:

$T_2 = 8\epsilon + 12\delta + w_1$.

$a - (w_2 + 4\delta + 2\,\epsilon) \geq 0$ implies that: $w1 = a - \beta/2\ and\ a \geq (4\delta + 2\epsilon)$.

Thus $a \geq \beta/2$ implies that $a + \beta/2 \geq \beta$.

$b - (w_3 + 8\delta + 4\epsilon) \leq 0$ implies that: $b \leq (8\delta + 4\epsilon)$ thus $b \leq \beta$.

$c - (w_1 + 4\delta + 2\epsilon) \leq 0$ implies that:

$c \leq (a - \beta/2 + 4\delta + 2\epsilon)\ thus\ c \leq a$.

Thus:

$a + \beta/2 > b, a \geq c$ Implies $a/2 \geq c/2, \beta \geq b$ thus $\beta/2 \geq b/2$ and $a/2 + \beta/2 \geq c/2 + b/2$ hence $a + \beta/2 \geq (a + b + c)/2$.

The conclusion is that *a*+ *β/2* is the maximum among all the other involved parameters. What we obtained through our calculations is: $T_2 = 8\epsilon + 12\delta + w_1 = 8\epsilon + 12\delta + a - \beta/2 = \alpha + \beta + a - \beta/2 = \alpha + a + \beta/2$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Case 5: w_3=0, w_1, w_2> 0

Then:

$T_2 = 8\epsilon + 12\delta + w_1 + w_2$.

$b - (w_3 + 8\delta + 4\epsilon) \geq 0$ Implies that: $b \geq (0 + 8\delta + 4\epsilon)$ thus $b \geq \beta$ and $w_2 = b - \beta$.

$a - (w_2 + 4\delta + 2\epsilon) \geq 0$ Implies that: $a \geq (b - \beta + 4\delta + 2\epsilon)$ thus $a + \beta/2 \geq b$ and $w2 = a - b + \beta/2$.

$c - (w_1 + 4\delta + 2\epsilon) \leq 0$ Implies that: $c \leq (a + \beta/2 - b + 4\delta + 2\epsilon)$ thus $c + b \leq a + \beta \leq a + b$ implying that: $(a + b + c)/2 \leq a + \beta/2$ and $c \leq a$.

Hence:

$a + \beta/2$ is obviously the maximum among all the other parameters. Our calculation leaded to the following result:

$T_2 = 8\epsilon + 12\delta + a - b + \beta/2 + b - \beta + 0 = 8\epsilon + 12\delta + a - \beta/2 = \alpha + \beta + a - \beta/2 = \alpha + a + \beta/2$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Case 6: w_2=0, w_1, w_3 > 0

Then:

$T_2 = 8\epsilon + 12\delta + w1 + w_3$.

$a - (w_2 + 4\delta + 2\,\epsilon) \geq 0$ Implies that: $w1 = a - \beta/2 \; and \; a \geq \beta/2$.

$c - (w_1 + 4\delta + 2\epsilon) \geq 0$ Implies that $c - a + \beta/2 + \beta/2 \geq 0 \; and \; w_3 = c - a$ at accordingly $c \geq a$.

$b - (w_3 + 8\delta + 4\epsilon) \leq 0$ Implies that $b - c + a - \beta \leq 0, thus \; a + b \leq c + \beta$ implies that $(a + b + c)/2 \leq c + \beta/2$.

On the other hand:

$a \geq \beta/2$ And $a + b \leq c + \beta$ implies that $\beta/2 + b \leq c + \beta$ hence $b \leq c + \beta/2$.

Thus:

$c + \beta/2 \geq (a + b + c)/2, \beta, b, a + \beta/2,$ i.e. $(c + \beta/2)$ is the maximum among all the other parameters. Our calculation leaded to the following result:

$T_2 = \alpha + \beta + a - \beta/2 + c - a = \alpha + \beta + c - \beta/2 \; thus \; T2 = \alpha + c + \beta/2$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Case 7: w_1= 0, w_2, w_3 > 0

Then:

$T_2 = 8\epsilon + 12\delta + w_2 + w_3$.

$c - (w_1 + 4\delta + 2\epsilon) \geq 0$ implies that $c - \beta/2 \geq 0$ *thus* $c \geq \beta/2$.

$b - (w_3 + 8\delta + 4\epsilon) \geq 0$ implies that $b - c + \beta/2 - \beta \leq 0$ thus

$b - c - \beta/2 \geq 0$ implying $b \geq c + \beta/2$ *and* $b \geq \beta$.

$a - (w_2 + 4\delta + 2\epsilon) \leq 0$ implies that $a - (b - c + \beta/2 - \beta + 4\delta +$

$2\epsilon) \leq 0$, thus $a - b + c \leq 0$ implies that $b \geq a + c$ thus $b \geq (a + b +$

$c)/2$.

Hence:

$b \geq a + c \geq a + \beta/2;$

Thus b is the maximum among all the parameters. Our calculation leaded to the following result: $T_2 = \alpha + \beta + c - \beta/2 + b - c - \beta/2 = \alpha + b$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Case 8: w_1, w_2, w_3 > 0

Then:

$T_2 = 8\epsilon + 12\delta + w_1 + w_2 + w_3$.

$a - (w2 + 4\delta + 2\epsilon) \geq 0$ implies that $a \geq \beta/2 + w_2$ and $w_1 =$

$a - \beta/2 - w_2$.

$b - (w_3 + 8\delta + 4\epsilon) \geq 0$ implies that $b \geq \beta + w3$ and $w2 = b - \beta - w_3$.

$c - (w_1 + 4\delta + 2\epsilon) \geq 0$ implies that $c \geq \beta/2 + w_1$and $w_3 =$

$c - \beta/2 - w_{3.}$

$a + b + c \geq 2\beta + w_1 + w_2 + w_3$ and $(a + b + c)/2 \geq \beta + (w_1 + w_2 +$

$w_3)/2$ implying that$(a + b + c)/2 \geq \beta$.

By replacing the equivalents of the w_js in to the equations the following equations are obtained:

$w_1 = a - \beta/2 - b + w_3 + \beta$.

$w_3 = c - \beta/2 - a + \beta/2 + w_2 = c - a + w_2$.

$w_2 = b - \beta - c + a - w_2$thus $w_2 = (b - \beta - c + a)/2$.

Hence:

$w_1 = a - \beta/2 + c - (b - \beta - c + a)/2 = (a - b + c)/2 \geq 0$ Thus $a/2 +$

$c/2 \geq b/2\ thus\ a + c \geq b$.

$w_2 = b/2 - \beta/2 - c/2 + a/2 \geq 0$ Thus $a + b \geq c + \beta$.

$w_3 = c - a + b/2 - \beta/2 - c/2 + a/2 \geq 0$ Thus $c/2 + b/2 \geq a/2 +$

$\beta/2$ implies that:

$c + b \geq a + \beta$.

It implies that:

$a/2 + b/2 + c/2 \geq b$.

$(a + b + c)/2 \geq c + \beta/2$.

$(a + b + c) \geq a + \beta/2$.

$a + b - c \geq \beta, c + b - a \geq \beta$.

Thus by adding the last two inequalities we have: $2b \geq 2\beta$ thus $b \geq \beta$.

$(a + b + c)/2$ is the maximum among all the other parameters. Our calculation leaded to the following result: $T_2 = \alpha + \beta + (a + b + c)/2 - \beta = \alpha + (a + b + c)/2$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

In the tables 3 to 7 the summary of each strategy for cycle time calculation can be seen.

Table 3. The summary table of 8 possible case of the waiting times for C_2 strategy

Possible Scenarios	**The Maximum Among** $\{\boldsymbol{\beta}, \boldsymbol{b}, \boldsymbol{a} + \boldsymbol{\beta}/2, \boldsymbol{c} + \boldsymbol{\beta}/2, (\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})/2\}$	**Cycle Time**
Case1: $w_1 = w_2 = w_3 = 0$	β	$T_2 = \alpha + \beta$
Case2: $w_1 = w_2 = 0, w_3 > 0$	$c + \beta/2$	$T_2 = \alpha + c + \beta/2$
Case3: $w_1 = w_3 = 0, w_2 > 0$	b	$T_2 = \alpha + b$
Case 4: $w_1 > 0,\ w_2 = w_3 = 0$	$a + \beta/2$	$T_2 = \alpha + a + \beta/2$
Case 5: $w_3 = 0, w_1, w_2 > 0$	$a + \beta/2$	$T_2 = \alpha + a + \beta/2$
Case 6: $w_2 = 0, w_1, w_3 > 0$	$c + \beta/2$	$T_2 = \alpha + c + \beta/2$
Case 7: $w_1 = 0, w_2, w_3 > 0$	b	$T_2 = \alpha + b$
Case 8: $w_1, w_2, w_3 > 0$	$(a + b + c)/2$	$T_2 = \alpha + (a + b + c)/2$
$T_2 = \alpha + \max\{\beta, b, a + \beta/2, c + \beta/2, (a + b + c)/2\}$		

Robot moves strategy of C_3 with duration of T_3;

Step 1: The robot waits on M_3 until the process is finished: $\boldsymbol{w_3}$

Step 2: Then it unloads the part and takes it to output station: **$2\epsilon+\delta$**

Step 3: Then it returns to M_2, and waits on it if necessary until the process is finished: **$2\delta + w_2$**

Step 4: Then the robot unloads the part and takes it to M_3, and load M_3: **$2\epsilon+\delta$**

Step 5: Then it goes back to input station: **3δ**

Step 6: Picks up a part and moves to M_1, loads it: **$2\epsilon+\delta$**

Step 7: It waits at it until the process is done w_1

Step 8: It takes the part to M_2, and loads it: **$2\epsilon+\delta$**

Step 9: Finally the robot returns to M_3 and waits until the process is finished: **δ**

The total cycle time of C_3 is: $T_3 = 10\delta + 8\epsilon + w1 + w2 + w3$.

$c = w_3 + (3\delta + 2\epsilon + \delta + w_1 + 2\epsilon + \delta + \delta)$ implies that $w_3 = max\,\{0, c - (6\delta + 4\epsilon + w_1)\}$ [follows up step 5 to 9].

$b = w_2 + (\delta + w_3 + 2\epsilon + \delta + 2\delta)$ implies that w_2= max $\{0, b - (4\delta + 2\epsilon + w_3)\}$ [follow up steps 9 to 3].

$a = w_1$ [look at steps 6 and 7].

Case 1: $\boldsymbol{w_1}$= a, $\boldsymbol{w_2}$= $\boldsymbol{w_3}$= 0

Then:

$T_3 = 10\delta + 8\epsilon + a = \alpha + \alpha + a + 2\delta.$

$c - (6\delta + 4\epsilon + w_1) \le 0$ hence $c \le 6\delta + 4\epsilon + a$ implies that $c \le \alpha + 2\delta + a.$

$b - (4\delta + 2\epsilon + w_3) \leq 0$ hence $b \leq 4\delta + 2\epsilon$ implies that $b \leq \alpha/2 + 2\delta$.Thus $b \leq \alpha/2 + 2\delta$. imply that $\alpha + 2\delta + a \geq b + a + \alpha/2$ which means that the maximum among $c, \alpha + a + 2\delta$ and $\alpha/2 + a + b$ is the last one. Our calculation returns the following:

$T_3 = 10\delta + 8\epsilon + a = \alpha + \alpha + a + 2\delta.$

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Case 2: w_1= a, w_2= 0, w_3> 0

Then:

$c - (6\delta + 4\epsilon + w_1) \geq 0$, hence $c \geq 6\delta + 4\epsilon + a$ implies that $c \geq \alpha + 2\delta + a$ and $w_3 = c - \alpha - 2\delta - a$.

$b - (4\delta + 2\epsilon + w_3) \leq 0$ hence $b \leq 4\delta + 2\epsilon + c - (6\delta + 4\epsilon + a)$ implies that $a + b - c + 2\delta + 2\epsilon \leq 0$ thus $c \geq a + b + \alpha/2$

Thus *c* is the maximum between $c, \alpha + a + 2\delta$ and $\alpha/2 + a + b$.

Our calculation returns the following result:

$T_3 = 10\delta + 8\epsilon + a + c - 6\delta - 4\epsilon - a = 4\delta + 4\epsilon + c = c + \alpha.$

The sum of the step's duration time is equal to the value given in the statement, hence the statement is proved.

Case 3: w_1=a, w_2>0, w_3=0

Then:

$b - (4\delta + 2\epsilon + w_3) \geq 0$ *hence* $b \geq 4\delta + 2\epsilon$ implies that b ≥ α/2+2δ and w_2= b – α/2 - 2δ.

$c - (6\delta + 4\epsilon + w_1) \leq 0$ hence $c \leq 6\delta + 4\epsilon + a$ implies that $c \leq \alpha + 2\delta + a$.

$b \geq \alpha/2 + 2\delta$ implies that $\alpha/2 + b + a \geq \alpha + 2\delta + a$. Hence $\alpha/2 + b + a$ *is* the maximum among $c, \alpha + a + 2\delta$ and $\alpha/2 + a + b$. Our calculation returns the following result: $T_3 = 10\delta + 8\epsilon + a + b - 4\delta - 2\epsilon + 0 = 6\delta + 6\epsilon + a + b = \alpha + \alpha/2 + a + b$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Case 4: w_1= a, w_2, w_3>0

Then:

$c - (6\delta + 4\epsilon + w1) \geq 0$ implies that $c \geq 6\delta + 4\epsilon + a$ thus $c \geq \alpha/2 + 2\delta + a$ and $w_3 = c - \alpha/2 - 2\delta - a$.

$b - (4\delta + 2\epsilon + w_3) \geq 0$ thus $b \geq 4\delta + 2\epsilon + c - 6\delta - 4\epsilon - a$ implies that $b + \alpha/2 + a \geq c$ and $w_2 = b - c + a + \alpha/2$.

These statements imply that:

$b + \alpha/2 + a \geq c, c \geq \alpha/2 + 2\delta + a$ thus $b + \alpha/2 + a$ is the maximum among $c, \alpha + a + 2\delta$ and $\alpha/2 + a + b$.

Our calculation returns us the following result:

T$_3$ = $10\delta + 8\epsilon + a + w_2 + w_3$ = T$_3$ = $10\delta + 8\epsilon + a + (b - 4\delta - 2\epsilon - w_3) + w_3 = 10\delta + 8\epsilon + a + b - 4\delta - 2\epsilon = 6\delta + 6\epsilon + a + b = \alpha + \alpha/2 + a + b$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Table 4. The summary table of 4 possible waiting times for C_3 strategy

Possible Scenarios	**The Maximum Among** $\boldsymbol{c, \alpha + a + 2\delta, \alpha/2 + a + b}$	**Cycle Time**
Case1: w_1= a, $\boldsymbol{w_2}$= $\boldsymbol{w_3}$= 0	$\alpha + a + 2\delta$	T$_3$ = $\alpha + \alpha + a + 2\delta$
Case2: w_1= a, $\boldsymbol{w_2}$= 0, $\boldsymbol{w_3}$> 0	α	T$_3$ = $c + \alpha$
Case3: w_1=a, $\boldsymbol{w_2}$>0, $\boldsymbol{w_3}$= 0	$\alpha/2 + a + b$	T$_3$ = $\alpha + \alpha/2 + a + b$
Case4: w_1=a, $\boldsymbol{w_2}$, $\boldsymbol{w_3}$> 0	$\alpha/2 + a + b$	T$_3$ = $\alpha + \alpha/2 + a + b$
T$_3$ = α +max$\{c, \alpha + a + 2\delta, \alpha/2 + a + b\}$		

Robot moves strategy of C_4 with duration of T_4;

Step 1: The robot is at M_3 (M_1 is loaded), it waits on M_3 until the process is finished, $\boldsymbol{w_3}$.

Step 2: It takes the part to the output station, **2ϵ+δ.**

Step 3: Then goes back to M_1 and waits if necessary, till the process is finished, **3δ**+w_1.

Step 4: It unloads M_1 and takes the part to M_2, **2ϵ+δ.**

Step 5: The robot waits at M_2 until it finishes the process, $\boldsymbol{w_2}$.

Step 6: The robot then unloads M_2 and takes the part to M_3 and loads M_3, **2ϵ+δ.**

Step 7: It returns to input station, **3δ.**

Step 8: The robot takes a part and moves to M_1 and loads it, **2ϵ+δ.**

Step 9: The robot goes back on M_3, **2δ.**

The total cycle time of C_4 is $T_4 = 12\delta + 8\epsilon + w_1 + w_2 + w_3$.

a = $2\delta + w_3 + 2\epsilon + \delta + 3\delta + w_1$ implies that w_1=max $\{0, a - (6\delta + 2\epsilon + w_3)\}$ [follow step 8 to 3].

$b = w_2$ [look at step 4 and 5].

$c = 3\delta + 2\epsilon + \delta + 2\delta + w_3$ implies that w_3= max $\{0, c - (6\delta + 2\epsilon)\}$ [follow step 7 to 1].

Case 1: $\boldsymbol{w_1}$= $\boldsymbol{w_3}$= 0, $\boldsymbol{w_2}$= b

Then:

$a \leq 6\delta + 2\epsilon + w_3 = 6\delta + 2\epsilon.$

$c \leq 6\delta + 2\epsilon.$

Hence;

$\alpha/2 + a + b \leq \alpha/2 + 6\delta + 2\epsilon + b = \beta + b$, similarly $\alpha/2 + c + b \leq \beta + b$. then $\beta + b$ is the maximum among the terms of the maximum in the format of T_4 and hence the statement is true.

Case 2: w_1= 0, w_2= b, w_3> 0

Then:

$w_3 = c - 6\delta - 2\epsilon = c - \beta + \alpha/2 > 0$. Implies that $c \geq 6\delta + 2\epsilon$ thus

$c + \alpha/2 + b \geq \beta + b$.

$a \leq 6\delta + 2\epsilon + w_3 = c$ thus $a \leq 6\delta + 2\epsilon + c - 6\delta - 2\epsilon$ thus:

$a \leq c$ implies that $a + \alpha/2 + b \leq c + \alpha/2 \geq \beta$

From these statements it is implied that $c + \alpha/2 + b$ is the maximum among $\beta+b$, $\alpha/2 + a + b, \alpha/2 + b + c$. Our calculation returns the following result:

$T_4 = \alpha + \alpha/2 + b + c$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Case 3: w_1> 0, w_3= 0, w_2= b

Then:

$c \leq 6\delta + 2\epsilon$ thus $c \leq \beta - \alpha/2$ thus $c + \alpha/2 \leq \beta$ implies $that\ c + \alpha/2 + b \leq \beta + b$.

$a \geq 6\delta + 2\epsilon + w_3 = 6\delta + 2\epsilon$. Hence $a \geq c\ and\ a + b + \alpha/2 \geq c + b + \alpha/2$.

On the other hand:

$a \geq 6\delta + 2\epsilon$ thus $a \geq \beta - \alpha/2$ implies that a+α/2 ≥ β thus a +b +α/2 ≥ β +b, and since $a \geq c$ implying $a + \alpha/2 + b \geq c + b + \alpha/2$. Hence is the maximum among $\beta + b, \alpha/2 + a + b, \alpha/2 + b + c$. Our calculation returns the following result: $T_4 = \alpha + \alpha/2 + b + c$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Case 4: w_2= b, w_1, w_3> 0

Then:

$w_3 = c - 6\delta - 2\epsilon > 0, i.e. c \geq 6\delta + 2\epsilon$. Thus

$c \geq \beta - \alpha/2$ *implies that* $c + \alpha/2 \geq \beta$ *and* $b + c + \alpha/2 \geq \beta + b$.

$a \geq 6\delta + 2\epsilon + w_3$thus $a \geq c$.

Thus the following inequalities hold:

$a \geq c, a + b \geq c + b, a + b + \alpha/2 \geq c + b + \alpha/2$. Hence: $a + b + \alpha/2 \geq \beta + b, c + b + \alpha/2\, c + b + \alpha/2 \geq \beta + b$, *thus* $a + b + \alpha/2$ is the maximum among $\beta + b, \alpha/2 + a + b, \alpha/2 + b + c$. Our calculation returns the following result: $T_4 = \alpha + \alpha/2 + a + b$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Table 5. The abstract table of 4 possible waiting times for C_4 strategy

Possible Scenarios	**The Maximum Among** $\boldsymbol{\beta + b, \alpha/2 + a + b\ , \alpha/2 + b + c}$	**Cycle Time**
Case 1: $w_1 = w_3 = 0$, $w_2 = b$	$\beta + b$	$T_4 = \alpha + \beta + b$
Case 2: $w_1 = 0$, $w_2 = b$, $w_3 > 0$	$\alpha/2+b + c$	$T_4 = \alpha + \alpha/2+b + c$
Case 3: $w_1 > 0$, $w_3 = 0$, $w_2 = b$	$b + \beta$	$T_4 = \alpha+b + \beta$
Case 4: $w_2 = b$, w_1, $w_3 > 0$	$\alpha/2 + a + b$	$T_4 = \alpha + \alpha/2 + a + b$
$T_4 = \alpha+\max\{\beta+b, \alpha/2+a+b, \alpha/2+b+c\}$		

Robot moves strategy of C_5 with duration of T_5;

Step 1: The robot is waiting at M_3 until the process is finished, $\boldsymbol{w_3}$.

Step 2: Then robot unloads M_3 and takes the part to output station, (M_1 is already loaded), **2ϵ+δ.**

Step 3: Then the robot goes back on M_1 and waits if necessary until the process is finished, **3δ+**w_1.

Step 4: the robot unloads M_1 and takes the part to M_2 and loads M_2, **2ϵ+δ.**

Step 5: It returns to input station, **2δ.**

Step 6: the robot, picks up a part, moves to M_1 and loads it, **2ϵ+δ.**

Step 7: the robot goes back on M_2, waits if necessary, **δ+**$\boldsymbol{w_2}$.

Step 8: the robot, takes the part to M_3 and loads it, **2ϵ+δ.**

Step 9: the robot waits at M_3 until the process is finished.

The duration C_5 is: $T_5 = 10\delta + 8\epsilon + w_1 + w_2 + w_3$.

$a = \delta + w_2 + 2\epsilon + \delta + w_3 + 2\epsilon + \delta + 3\delta + w_1$ thus w_1= max $\{0, a - (6\delta + 4\epsilon + w_2 + w_3)\}$ [follow step 7 to 3].

$b = (2\delta + 2\epsilon + \delta + \delta + w_2)$ thus w_2= max $\{0, b - (4\delta + 2\epsilon)\}$ [Follow step 5 to 7].

$c = w_3$ [look at step 8 and 9].

Case 1: w_1**=**$\boldsymbol{w_2}$**=0,** $\boldsymbol{w_3}$**= c**

Then:

$a - (6\delta + 4\epsilon + w_2 + w_3) = a - (6\delta + 4\epsilon + c) \leq 0$. Thus $a \leq 6\delta + 4\epsilon + c = \alpha + 2\delta + c$.

$b - (4\delta + 2\epsilon) \leq 0$ thus $b \leq 4\delta + 2\epsilon$ and $b \leq \alpha/2 + 2\delta$. Thus $b + c \leq \alpha/2 + 2\delta + c$ implies that $b + c + \alpha/2 \leq \alpha + 2\delta + c$. Thus $\alpha + 2\delta + c$ is the maximum among $a, \alpha + c + 2\delta$ and $\alpha/2 + b + c$. Our calculation returns the following result: $T_5 = \alpha + \alpha + c + 2\delta$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Case 2: w_1= 0, w_2> 0, w_3= c

Then:

$w_1 = a - (6\delta + 4\epsilon + w_2 + w_3) = a - (6\delta + 4\epsilon + b - 4\delta - 2\epsilon + c) = a - b - c - \alpha/2 \leq 0$ thus $a \leq 6\delta + 4\epsilon + c$ implies that $a \leq \alpha + 2\delta + c$.

$w_2 = b - (4\delta + 2\epsilon) \geq 0$ thus $b \geq 4\delta + 2\epsilon$ and $b \geq \alpha/2 + 2\delta$ and $b + c + \alpha/2 \geq \alpha + c + 2\delta$.

It is shown that $\alpha + c + 2\delta \geq a, \alpha + 2\delta + c$ and $b + c + \alpha/2 \geq \alpha + c + 2\delta$ implies that $b + c + \alpha/2 \geq a$. Our calculation returns the following result: $T_5 = \alpha + \alpha/2 + b + c$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Case 3: w_1> 0, w_2= 0, w_3= c

Then:

$b - (4\delta + 2\epsilon) \leq 0$ thus $b \leq \alpha/2 + 2\delta$ imply that $b + c + \alpha/2 \leq \alpha + c + 2\delta$.

$a - (6\delta + 4\epsilon + w_2 + w_3) \geq 0$ thus $a \geq 6\delta + 4\epsilon + b - (4\delta + 2\epsilon) +$

$c\ thus: a \geq \alpha/2 + b + c.$

On the other hand:

$b + c + \alpha/2 \leq \alpha + c + 2\delta$ implies that $(\alpha + c + 2\delta), a \geq \alpha/2 + b +$

c and $a \geq \alpha + 2\delta + c$.

a is the maximum among $a, \alpha + c + 2\delta$ and $\alpha/2 + b + c$. Our calculation returns the following result: $T_5 = \alpha + a$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Case 4: w_3=c, w_1, w_2> 0

Then:

$T_5 = \alpha + a$

$b - (4\delta + 2\epsilon) \geq 0$ thus $b \geq 4\delta + 2\epsilon$ implies that $b + c \geq \alpha/2 + 2\delta + c$

thus: $b + c + \alpha/2 \geq \alpha + 2\delta + c$.

$a - (6\delta + 4\epsilon + w_2 + w_3) \geq 0$ thus a ≥ $6\delta + 4\epsilon + b - (4\delta + 2\epsilon) + c$ thus

$a \geq \alpha/2 + b + c$.

Hence $a \geq \alpha/2 + b + c$ and $b + c + \alpha/2 \geq \alpha + 2\delta + c$ it is shown that a is the maximum among $a, \alpha + c + 2\delta$ and $\alpha/2 + b + c$, our calculation shows $T_5 = \alpha + a$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Table 6. The abstract table of 8 possible waiting times for C_5 strategy

Possible Scenarios	**The Maximum Among** $\boldsymbol{a, \alpha + c + 2\delta, \alpha/2 + b + c}$	**Cycle Time**
Case 1: w_1=w_2=0, w_3= c	$\alpha + c + 2\delta$	Exactly case 1
Case 2: w_2=0, w_1, w_3= c	a	$T_5 = \alpha + a$
Case 3: w_1= 0, w_2, w_3= c	$\alpha/2 + b + c$	$T_5 = \alpha + \alpha/2 + b + c$
Case 4: w_1, w_2> 0, w_3= c	a	$T_5 = \alpha + a$
$T_5 = \alpha + \max\{ a, \alpha + c + 2\delta, \alpha/2 + b + c \}$		

Robot moves strategy of C_6 with duration of T_6;

Step 1: Initially all 3 machines are loaded and the robot is on M_3 waiting for the completion of the process, $\boldsymbol{w_3}$.

Step 2: When the process is finished, the robot unloads M_3 and takes the part to output station, **ϵ+δ+ϵ.**

Step 3: Then the robot goes back to M_2, waits until the process is finished (if necessary), **2δ +$\boldsymbol{w_2}$**.

Step 4: When the part is completed, the robot unloads M_2 and moves the part to M_3 and loads it, **2ϵ+δ.**

Step 5: Then the robot goes back on M_1, waits if necessary until the process is finished, **2δ**+w_1.

Step 6: Then the robot unloads the part, moves it to M_2 and loads it, **2ϵ+δ.**

Step 7: The robot returns to the input station, **2δ.**

Step 8: The robot picks up a part, moves to M_1 and loads it, **ϵ+δ+ϵ.**

Step 9: The robot returns on M_3 and waits until the process is finished, **2δ.**

The duration of C_6 is: $T_6 = 12\delta + 8\epsilon + w1 + w2 + w3$.

Recall that $8\delta + 4\epsilon = \beta$;

$a = (2\delta + w3 + 2\epsilon + \delta + 2\delta + w2 + 2\epsilon + \delta + 2\delta + w1)$. Hence w_1= max $\{0, a - (8\delta + 4\epsilon + w2 + w3)\}$ [follow step 9 to 5].

$b = (2\delta + 2\epsilon + \delta + 2\delta + w3 + 2\epsilon + \delta + 2\delta + w2)$. Thus w_2= max $\{0, b - (8\delta + 4\epsilon + w3)\}$ [follow step 7 to 3].

$c = (2\delta + w1 + 2\epsilon + \delta + 2\delta + 2\epsilon + \delta + 2\delta + w3)$. Hence w_3= max $\{0, c - (8\delta + 4\epsilon + w1)\}$ [follow step 5 to 1].

Case 1: w_1=w_2=w_3= 0

Then:

$a - (8\delta + 4\epsilon + w_2 + w_3) \le 0$ thus:

$a - (8\delta + 4\epsilon) \le 0$ *implies that* $a - \beta \le 0$.

$b - (8\delta + 4\epsilon + w_3) \le 0$ thus $b - (8\delta + 4\epsilon) \le 0$ implies that $b - \beta \le 0$.

$c - (8\delta + 4\epsilon + w_1) \le 0$ thus $c - (8\delta + 4\epsilon) \le 0$ implies that $c - \beta \le 0$.

Imply that $\beta \ge a, b, c$ and thus, the maximal among β, a, b and c. Our calculation shows $T_6 = \alpha + \beta$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Case 2: w_1= w_2= 0, w_3> 0

Then:

$c - (8\delta + 4\epsilon + w_1) \ge 0$ thus $w_3 = c - (8\delta + 4\epsilon) \ge 0$ implies that $c - \beta \ge 0$.

$a - (8\delta + 4\epsilon + w_2 + w_3) \le 0$, thus $a - (8\delta + 4\epsilon + w_3) = a - \beta - c + \beta \le 0$ implies that $a - c \le 0$.

$b - (8\delta + 4\epsilon + w_3) \le 0$, thus $b - (8\delta + 4\epsilon + c - \beta) \le 0$ implies that $b - \beta - c + \beta \le 0$.

It follows from the inequalities that $c \ge \beta, b, a$ i.e. c is the maximal element. Our calculation shows $T_6 = \alpha + c$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Case 3: w_1= w_3=0, w_2> 0

Then:

$b - (8\delta + 4\epsilon + w_3) \geq 0$ thus $w_2 = b - (8\delta + 4\epsilon) \geq 0$ implies that $b - \beta \geq 0$.

$a - (8\delta + 4\epsilon + w_2 + w_3) \leq 0$ thus $a - (8\delta + 4\epsilon + b - \beta) \leq 0$ implies that $a - b \leq 0$.

$c - (8\delta + 4\epsilon + w_1) \leq 0$ thus $c - (8\delta + 4\epsilon) \leq 0$ implies that $c - \beta \leq 0$.

Hence $b \geq a, b, c$ and thus the maximum, our calculation shows $T_6 = \alpha + b$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Case 4: w_1 > 0, w_2= w_3= 0

Then:

$a - (8\delta + 4\epsilon + w_2 + w_3) \geq 0$ thus $w_1 = a - (8\delta + 4\epsilon) \geq 0$ implies that $a - \beta \geq 0$.

$b - (8\delta + 4\epsilon + w_3) \leq 0$ thus $b - (8\delta + 4\epsilon) \leq 0$ implies that $b - \beta \leq 0$.

$c - (8\delta + 4\epsilon + w_1) \leq 0$ thus $c - (8\delta + 4\epsilon + a - \beta) \leq 0$ implies that $c - \beta - a + \beta \leq 0$.

Hence $a \geq b, c, \beta$ and i.e. a is the maximal element. Our calculation shows $T_6 = \alpha + a$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Case 5: w_1, w_2> 0, w_3=0

Then:

$b - (8\delta + 4\epsilon + w_3) \geq 0$ $w_2 = b - (8\delta + 4\epsilon) \geq 0$ implies that $b - \beta > 0$.

$a - (8\delta + 4\epsilon + w_2 + w_3) \geq 0$ w_1=thus $a - (8\delta + 4\epsilon + b - \beta) \geq 0$ implies that $a - b > 0$.

$c - (8\delta + 4\epsilon + w_1) \leq 0$ thus $w_3 = c - (\beta + a - b) \leq 0$ implies that $c - \beta - a + b \leq 0$ and thus $c + b \leq a + \beta$, as $b > \beta$ therefore $a > c$. Hence $a \geq b, \beta, c$ *i.e.* a is the maximal element. Our calculation shows $T_6 = \alpha + a$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is proved.

Case 6: w_2=0, w_1, w_3>0

Then:

$w_1 = a - (8\delta + 4\epsilon + w_2 + w_3) \geq 0$, $w_2 = b - (8\delta + 4\epsilon + w_3) \leq 0$,

$c - (8\delta + 4\epsilon + w_1) \geq 0$, by subtracting the value of w_3, the inequality is;

$w_1 = a - (\beta + c - \beta - w_1)$ is obtained. Thus $2w_1 = a - c$ implies that $w_1 = (a - c)/2 > 0$ thus $a \geq c$.

It follows $w_2 = 0$ that $b - (\beta + c - \beta - a/2 + c/2) \leq 0$. Thus $b - c + a/2 - c/2 \leq 0$ implies that $b + a/2 \leq c + c/2$.

As $a/2 \geq c/2$ then $b < c$ implying that $a > b$. $w_3 = c - (\beta + (a - c)/2)$ thus $c - \beta - a/2 + c/2 > 0$ implies that $c + c/2 \geq a/2 + \beta$.

Similarly it follows that $a > \beta$. Hence $a > b, c, \beta$ i.e. a is the maximal element. Our calculation shows:

$T_6 = \alpha + a.$

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Case 7: w_1= 0,w_2, w_3> 0

Then:

$c - (8\delta + 4\epsilon + w_1) \geq 0$ thus w_3= $c - (8\delta + 4\epsilon) = c - \beta \geq 0.$

$b - (8\delta + 4\epsilon + w_3) \geq 0$ thus $w_2 = b - (8\delta + 4\epsilon + c - \beta) = b - \beta - c + \beta \geq 0.$

$a - (8\delta + 4\epsilon + w_2 + w_3) \leq 0$ thus $w_1 = a - (8\delta + 4\epsilon + b - c + c - \beta) = a - b \leq 0$

Imply that $b \geq a, c, \beta$ and thus the maximum, our calculation shows T_6 = $\alpha + a.$

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Case 8: w_1, w_2, w_3> 0

Then:

$a - (8\delta + 4\epsilon + w_2 + w_3) > 0$ Thus $w_1 = a - (\beta + w_2 + w_3) > 0.$

$b - (8\delta + 4\epsilon + w_3) > 0$ thus $w_2 = b - (\beta + w_3) > 0.$ $c - (8\delta + 4\epsilon + w_1) > 0$ thus $w_3 = c - (\beta + w_1) > 0$. Implying that a, b and $c > \beta$.

On the other hand:

By subtracting $w_2$and $w_3$into the formula of $w_1$the equation

$w_1 = a - \beta - w_2 - w_3 = a - \beta - b - \beta + w_3 - w_3 = a - b > 0$ is obtained.

Hence $w_2 = b - \beta - w_3 = b - \beta - c + \beta + a - b = a - c > 0$ implying that $a > c$.

Thus $w_3 = c - \beta - w_1 = c - (\beta + a - b)$. Finally, $w_3 > 0$ if and only if $c + b > a + \beta$. As we know $a > b$, then c must be larger than β.Then a is the maximal among a, b, c, β.

Our calculation returns the following result: $T_6 = \alpha + a$.

The sum of the step's duration time is equal to the value given in the statement, hence the statement is true.

Table 7. The abstract table of 8 possible waiting times for C_6 strategy

Possible Scenarios	**The maximum Among** $\boldsymbol{a, b, c, \beta}$	**Cycle Time**
Case 1: $w_1 = w_2 = w_3 = 0$	β	$T_6 = \alpha + \beta$
Case 2: $w_1 = w_2 = 0$, $w_3 > 0$	c	$T_6 = \alpha + c$
Case 3: $w_1 = w_3 = 0$, $w_2 > 0$	b	$T_6 = \alpha + b$
Case 4: $w_1 > 0$, $w_2 = w_3 = 0$	a	$T_6 = \alpha + a$
Case 5: $w_1, w_2 > 0$, $w_3 = 0$	a	$T_6 = \alpha + a$
Case 6: $w_2 = 0$, $w_1, w_3 > 0$	c	$T_6 = \alpha + c$
Case 7: $w_1 = 0$, $w_2, w_3 > 0$	b	$T_6 = \alpha + b$
Case 8: $w_1, w_2, w_3 > 0$	a	$T_6 = \alpha + a$
$T_6 = \alpha + \max\{\beta, a, b, c\}$		

As the statement is true in all possible cases, it is proved. Q.E.D.

Chapter 3

OPTIMALITY OF T_j

3.1. Optimality Comparison between T_js

In this chapter we are going to determine which of each obtained T_j is feasible and therefore can be taken to consideration in the analysis. It means first it is required to compare these T_j together and see if they return a better result than the other ones. In the following a set of pair wise comparison is shown and in the end the feasible T_js are introduced.

3.1.1. T_6 vs. T_js

T_6 vs. T_1: $\alpha +$ max $\{\beta, a, b, c\}$ vs. $2\alpha + a + b + c$.

In this case if any of a, b, or c of the left hand side is the maximum then $T_6 \le T_1$.

If β is the maximum of the left hand side then we have: $8\delta + 4\epsilon\ vs.\ 4\delta + 4\epsilon + a + b + c$. Hence, if $4\delta \le a + b + c$ then: $T_6 \le T_1$, Otherwise $T_6 > T_1$.

T_6 vs. T_2: $\alpha +$ maximum $\{\beta, a, b, c\}$ vs. $\alpha +$ maximum $\{\beta, b, \beta/2 + a, \beta/2 + c, (a + b + c)/2\}$.

If β is the maximum of the left hand side, then: $\beta \le$ maximum $\{\beta, b, \beta/2 + a, \beta/2 + c, (a + b + c)/2\}$, thus $T_6 \le T_2$.

If a is the maximum of the left hand side, then: $a \le$ maximum $\{\beta, b, \beta/2 + a, \beta/2 + c, (a + b + c)/2\}$. (1)

If b is the maximum of the left hand side, then: $b \leq$ maximum $\{\beta, b, \beta/2 + a, \beta/2 + c, (a\ +\ b\ +\ c)/2\}$, thus $T_6 \leq T_2$. (2)

If c is the maximum of the left hand side, then: $c\ \leq$ maximum $\{\beta, b, \beta/2 + a, \beta/2 + c, (a\ +\ b\ +\ c)/2\}$, thus $T_6 \leq T_2$. (3)

Thus T_6 always dominates T_2.

T_6 vs. T_3: α+ maximum $\{\beta, a, b, c\}$ vs. α+maximum$\{c, \alpha + a + 2\delta, \alpha/2 +\ a\ +\ b\}$.

If any of a, b, or c is the maximum of left hand side, then: $T_6 \leq T_3$. If β is the maximum of the left hand side then we have the followings:

β vs. c thus $T_6 \geq T_3$ (we assumed β is greater than c). (1)

β vs. $\alpha + a + 2\delta$ which implies $8\delta + 4\epsilon$ vs. $6\delta + 4\epsilon + a$. Hence if $2\delta\ \leq\ a$ then $T_6 \leq T_3$, otherwise $T_6 > T_3$. (2)

β vs. $\alpha/2 +\ a\ +\ b$ which implies $8\delta + 4\epsilon$ vs. $2\delta\ + 2\epsilon\ + a\ + b$ thus we have: $6\delta + 2\epsilon$ vs. $a\ +\ b$, as a result if $6\delta\ + 2\epsilon\ \leq\ a\ +\ b$ then $T_6 \leq T_3$ otherwise $T_6 > T_3$. (3)

T_6 vs. T_4: α + maximum $\{\beta, a, b, c\}$ vs. α + maximum $\{\beta + b, \alpha/2 + a + b, \alpha/2 + b + c\}$.

This case is similar to case 2 which means that if any of β, a, b or c are the maximum of the left hand side, the maximum of the right side is always larger or at least equal to them, hence T_6 always dominates T_4.

T_6 vs. T_5: α+ maximum $\{\beta, a, b, c\}$ vs. α+maximum $\{a, \alpha + c + 2\delta, \alpha/2 + b + c\}$.

Again if any of a, b or c is the maximum of the left hand side, then maximum $\{a, \alpha + c + 2\delta, \alpha/2 + b + c\}$ is larger thus $T_6 \leq T_5$ in this cases. But if β is the maximum among $\{\beta, a, b, c\}$ then we have $8\delta + 4\epsilon$ vs. $6\delta + 4\epsilon + c$ and $8\delta + 4\epsilon$ vs. $2\delta + 2\epsilon + b + c$. In this case there are two states:

- If $8\delta + 4\epsilon$ vs. $6\delta + 4\epsilon + c$ and $2\delta \leq c$ thus $T6 \leq T5$ otherwise $T6 > T5$.
- If $8\delta + 4\epsilon$ vs. $2\delta + 2\epsilon + b + c$ and $6\delta + 2\epsilon \leq a + b$ then $T_6 \leq T_5$ otherwise $T_6 > T_5$.

Note: $T_6 > T_5$ if and only if $\beta > \alpha + c + 2\delta$ AND $\beta > \alpha/2 + b + c$. Then $2\delta > c$ AND $6\delta + 2\epsilon > a + b$. if $6\delta + 2\epsilon \leq a + b, 2\delta \leq c, 2\delta \leq a,\ c \leq \alpha + c + 2\delta,\ c \leq \alpha/2 + b + c,\ 4\delta \leq a + b + c$; then imply that if generally $2\delta \leq c, c \leq \alpha + a + 2\delta$ then T_6 is the optimal cycle time.

3.1.2. T_5 vs. T_js

T_5 vs. T_3: a + maximum $\{a, \alpha + c + 2\delta, \alpha/2 + b + c\}$ vs. α +maximum$\{c, \alpha + a + 2\delta, \alpha/2 + a + b\}$.

If a is the maximum on the left hand side, then $a < \alpha + a + 2\delta$ implies that $T_5 \leq T_3$.

If $\alpha + c + 2\delta$ vs. α+maximum$\{c, \alpha + a + 2\delta, \alpha/2 + a + b\}$ then there are the following possibilities; If the two maximums are $\alpha + c + 2\delta$ and c respectively, in this case T_5 cannot be better than T_3. If $\alpha + c + 2\delta$ and $\alpha + a + 2\delta$ are the two maximums, in this case if $c \leq a$ then $T_5 \leq T_3$. Finally if the two maximums are $\alpha + c + 2\delta$ vs. $\alpha/2 + a + b$ then if $\alpha/2 + c + 2\delta \leq a + b$ then it conclude that $T_5 \leq T_3$. If the maximum of left hand side is $\alpha/2 + b + c$ vs. α+maximum$\{c, \alpha + a + 2\delta, \alpha/2 + a + b\}$ the following statements may hold; If $\alpha/2 + b + c$ and $\alpha/2 + a + b$ are the two maximums respectively, if $c \leq a$ then $T_5 \leq T_3$. If the two maximums are $\alpha/2 + b + c$ vs. $\alpha + a + 2\delta$, in this case $if\ b + c \leq \alpha/2 + a + b$ then $T_5 \leq T_3$ and finally if the two maximums are $\alpha/2 + b + c$ vs. c, it obviously conclude that $\alpha/2 + b + c \geq c$, thus in this situation, T_5 cannot be optimal.

T_5 vs. T_1: α + maximum $\{a, \alpha + c + 2\delta, \alpha/2 + b + c\}$ vs. $2\alpha + a + b + c$. Depend on the maximum of the left hand side any of the following statements may hold:

a vs. $a + b + c + \alpha$ implies that $a \leq a + b + c + \alpha$ thus $T_5 \leq T_1$.

$\alpha + c + 2\delta$ vs. $\alpha + a + b + c$ then if $\alpha + 2\delta \leq \alpha + a + b$ then $T_5 \leq T_1$.

$\alpha/2 + b + c$ vs. $\alpha + a + b + c$ then $T_5 \leq T_1$.

3.1.3. T_3 vs. T_js

T_3 vs. T_1: α+maximum$\{c, \alpha + a + 2\delta, \alpha/2 + a + b\}$ vs. $2\alpha + a + b + c$.

If c vs. $\alpha + a + b + c$ then $c \leq \alpha + a + b + c$ and $T_3 \leq T_1$.

If $\alpha + a + 2\delta \leq \alpha + a + b + c$ then $T_3 \leq T_1$ if $2\delta \leq b + c$.

If $\alpha/2 + a + b$ vs. $\alpha + a + b + c$ it conclude that $\alpha/2 + a + b \leq \alpha + a + b + c$ thus $T_3 \leq T_1$.

Finally by comparing all the possible cycles together, it is concluded that any of the T_js might be optimal depend on the parameter of the problem except T_2 and T_4. As it is clearly shown in the comparisons, T_2 and T_4 are always dominated by T_6, as a result we always use the T_6 strategy instead of T_4 or T_2 so far so that using T_2 or T_4 is not feasible in any cases regardless of what the parameters of the problem are.

In the Appendix section, a decision tree is put which provides a better understanding of the condition in which one or more strategy can return a better solution. In this decision tree the comparison between the feasible cycle times in which one state dominates the other one is shown. Also by following each part of the tree, the best cycle time is obtained in a very special state of the problem parameters. For instant in the very first line a comparison between T_1, T_3, T_5 and T_6 is done, it can be seen that T_1 is better than T_6 if $4\delta > a + b + c$. For determination the minimum cycle time, the comparison is between T_1, T_3, $T_{5,}$ again the same sequence is applied and each time, one none optimal cycle time is out of the calculation until the optimal cycle time is obtained.

Chapter 4

FINDING THE INTERVAL of PROPOSED CYCLES

4.1. Obtaining a Lower Band for the Feasible Cycles

In the last chapter of this study, we are going to determine a lower band for the feasible cycle times, we assume that the summation of the process times, a, b and c is constant and equal to T. we have to decide on the amount of each, such that the time duration of the cycle is minimal. To achieve this objective it is required to minimize the maximum of the parameters which are involved in the formula of the cycle time. The cycle time T_2 and T_4 are always dominated by T_6, on the other hand, T_1 is the summation of the parameters, thus it is independent from a, b and c ways of distribution. Thus, we only have to determine this lower band for the cycles T_3, T_5 and T_6. In the following section we are going to discuss them in detail:

4.1.1. The Lower Band for T_3

$T_3 = \alpha + \max\{c, \alpha + a + 2\delta, \alpha/2 + a + b\}$, which means we have to discuss each parameter separately and decide on its lower band.

Let us consider:

$c + \alpha/2 + a + b = \alpha/2 + T$ = constant, the maximum of c and $\alpha/2 + a + b$ is minimal if $c = \alpha/2 + a + b = \alpha/2 + T - c$ implies that $c = \alpha/4 + T/2$. On the other hand having $0 \leq c \leq T$ implies that $0 \leq \alpha/2 + T/2 \leq T$. Thus $\alpha/4 \leq T/2$ implies that $\alpha \leq 2T$.

Assume that $\alpha \leq 2T$, then $\alpha/4 + T/2 = \alpha/2 + a + b$ implies that

$-\alpha/4 + T/2 = a + b$ and we already know $(-\alpha/4 + T/2) \geq 0$.

The minimum of $\alpha + a + 2\delta$ happens when $a = 0$. We have:

$a = 0, b = T/2 - \alpha/4, c = T/2 + \alpha/4$.

The third term is α+2δ. We need to discover whether or not $\alpha + 2\delta > T/2 + \alpha/4$. If $5\delta + 3\epsilon > T/2$ then the term $\alpha + a + 2\delta = 6\delta + 4\epsilon$ is the Maximum.

If $\alpha \leq 2T$ and $5\delta + 3\epsilon \leq T/2$ then $T/2 + \alpha/4 = c = \alpha/2 + a + b$ is the maximum.

And finally if $\alpha > 2T$, then the best feasible value of c is T and $a = b = 0$. In this case $T_3 = \alpha + \max\{T, \alpha + 2\delta, \alpha/2\} = \alpha + \alpha + 2\delta = 2\alpha + 2\delta = 10\delta + 8\epsilon$.

4.1.2. The Lower Bound for T5

$T_5 = \alpha + \max\{a, \alpha + c + 2\delta, \alpha/2 + b + c\}$, which means we have to discuss each parameter separately and decide on its lower band.

Let us consider:

$a + \alpha/2 + c + b = \alpha/2 + T$ = constant, the maximum of a and $\alpha/2 + c + b$ is minimal if $a = \alpha/2 + c + b = \alpha/2 + T - a$ implies that $a = \alpha/4 + T/2$. On the other hand having $0 \leq a \leq T$ implies that $0 \leq \alpha/2 + T/2 \leq T$. Thus $\alpha/4 \leq T/2$ implies that $\alpha \leq 2T$.

Assume that $\alpha \leq 2T$, then $\alpha/4 + T/2 = \alpha/2 + c + b$ implies that

$-\alpha/4 + T/2 = c + b$ and we already know $(-\alpha/4 + T/2) \geq 0$.

The minimum of $\alpha + c + 2\delta$ happens when c = 0. We have:

$c = 0, b = T/2 - \alpha/4, a = T/2 + \alpha/4.$

The third term is $\alpha+2\delta$. We need to discover whether or not $\alpha + 2\delta > T/2 + \alpha/4$. If $5\delta + 3\epsilon > T/2$ then the term $\alpha + c + 2\delta = 6\delta + 4\epsilon$ is the Maximum.

If $\alpha \leq 2T$ and $5\delta + 3\epsilon \leq T/2$ then $T/2 + \alpha/4 = a = \alpha/2 + c + b$ is the maximum.

And finally if $\alpha > 2T$, then the best feasible value of c is T and c $= b = 0$. In this case $T_3 = \alpha + \max\{T, \alpha + 2\delta, \alpha/2\} = \alpha + \alpha + 2\delta = 2\alpha + 2\delta = 10\delta + 8\epsilon$.

4.1.3. The Lower Bound for T6

For T_6 we have: $T_6 = \alpha + max\{\beta, a, b, c\}$. In this case either $\beta > T/3$ or $a = b = c = T/3 \geq \beta$, if $\beta > T/3$, Then the cycle time duration is independent from $T = a + b + c$ and their distribution so far so that $T_6 = \alpha + \beta$ and it is independent from the value of a, b or c.

Chapter 5

CONCLUSION, REMARKS AND FURTHER STUDIES

In this study we dealt with a Robot cyclic schedule in a Flexible Manufacturing Cell (FMC) with few numbers of machines. The loading and unloading of the machines is made by a robot, this is a Flow Shop technology and robot serves the machines in a cyclic manner. The robot is scheduled and its cyclic, so that we defined which type of movement of robot (as a result, loading / unloading sequence of machines) is more appropriate for our purpose (minimization the cycle time and maximization of outcomes).

In this study we provided a complete mathematical theory of determination of the different type of cyclic moves of the robot, which is missing from the literature. In Chapter 3, the optimality condition of each strategy of robot cyclic move is discussed and finally in last chapter, a lower bound for the feasible scheduling strategy is obtained. To provide a better understanding of the system for a decision maker in appendix A, a decision tree is included. This decision tree helps to determine the optimal cyclic robot move strategy based on the parameters of the model. In this study we did not consider any buffer between the machines, however in a real life system it is possible to consider some buffer between the machines, a study that investigates this state of the system can be a topic of a future research. Also considering more number of machines seems to be a good topic for a new research.

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APPENDIX

Appendix A: The Decision Tree for T_js

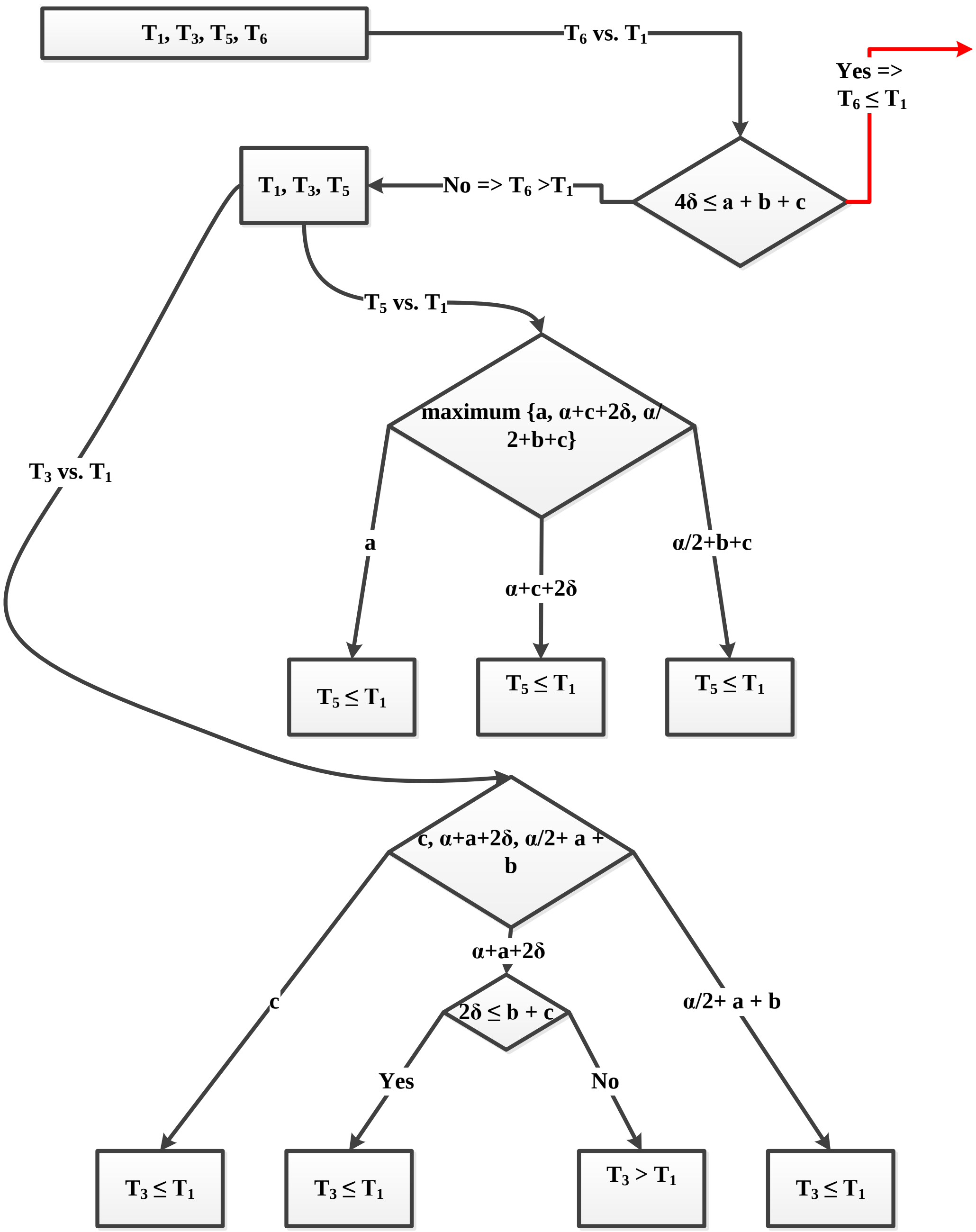

Figure 10. Decision Tree for Tjs - step1

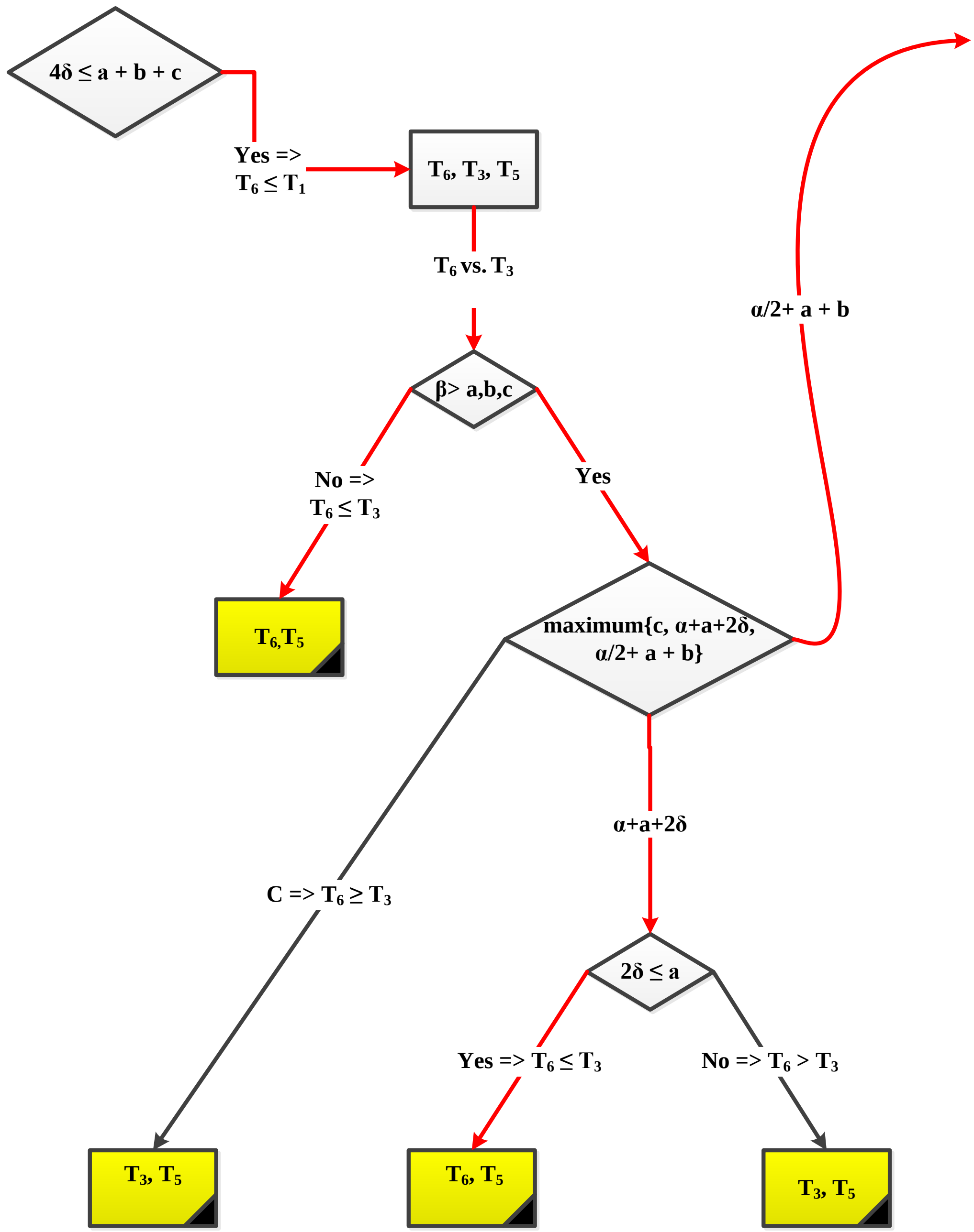

Figure 11. Decision Tree for Tjs - step2

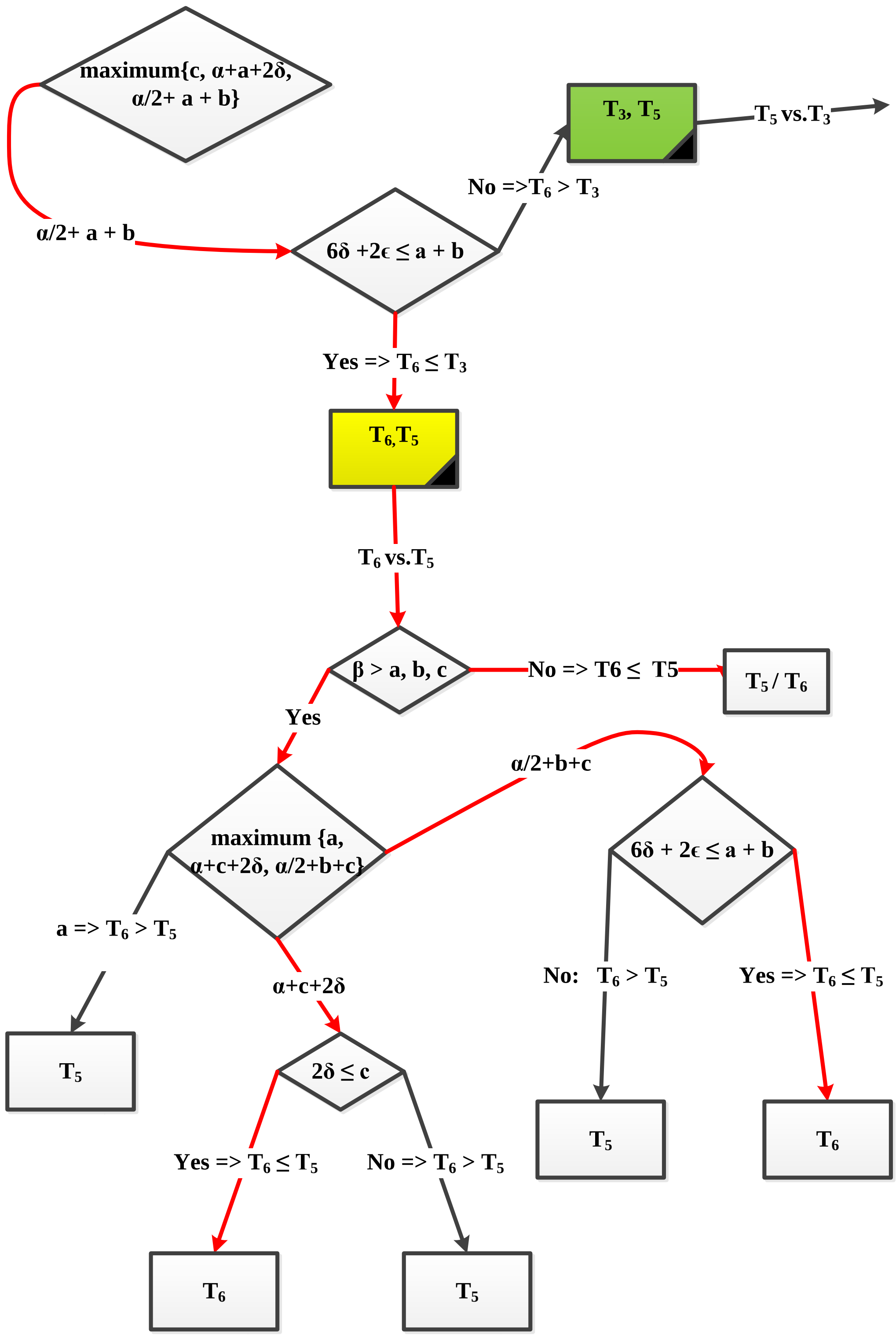

Figure 12. Decision Tree for Tjs – step3

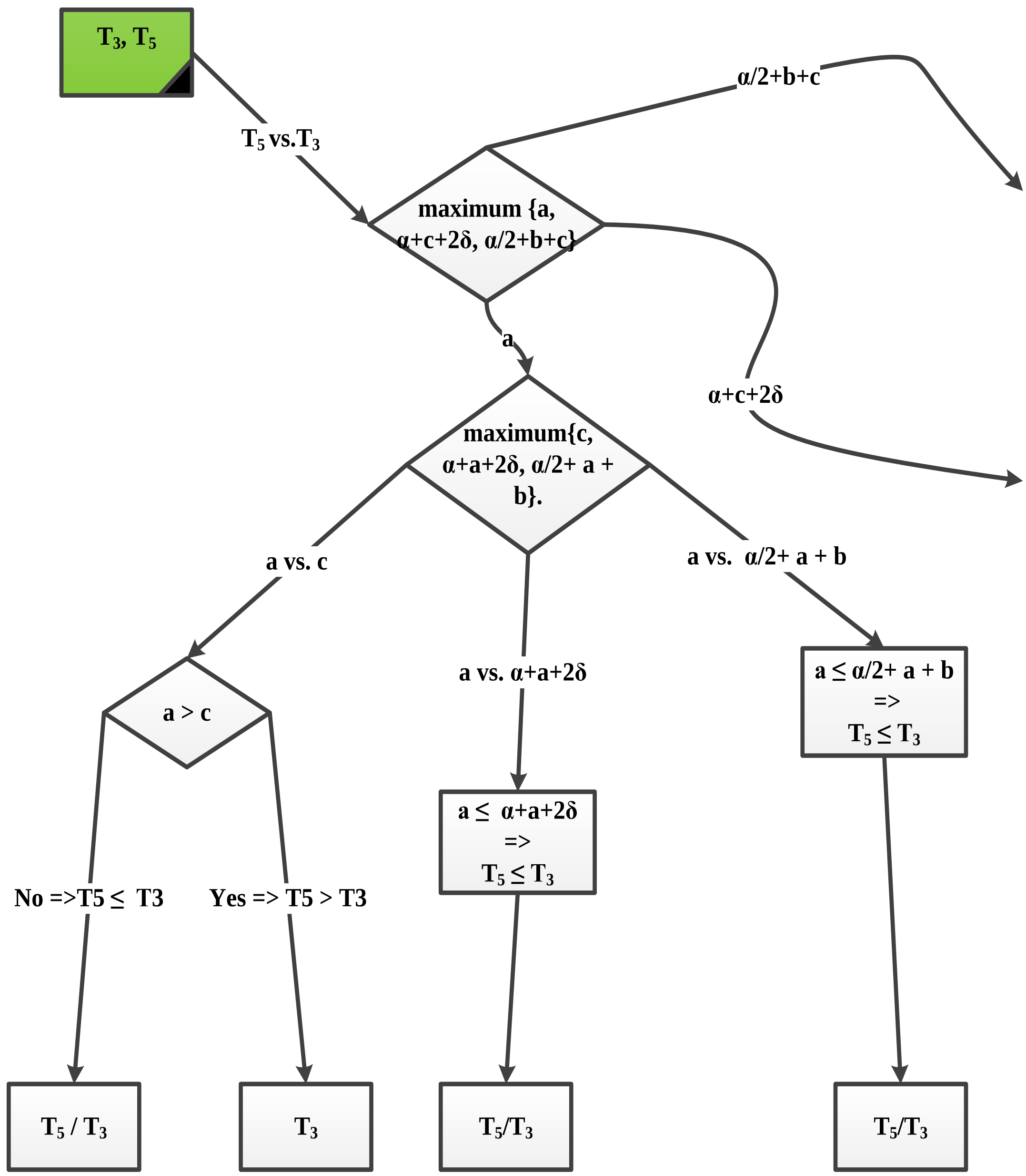

Figure 13. Decision Tree for Tjs – step4

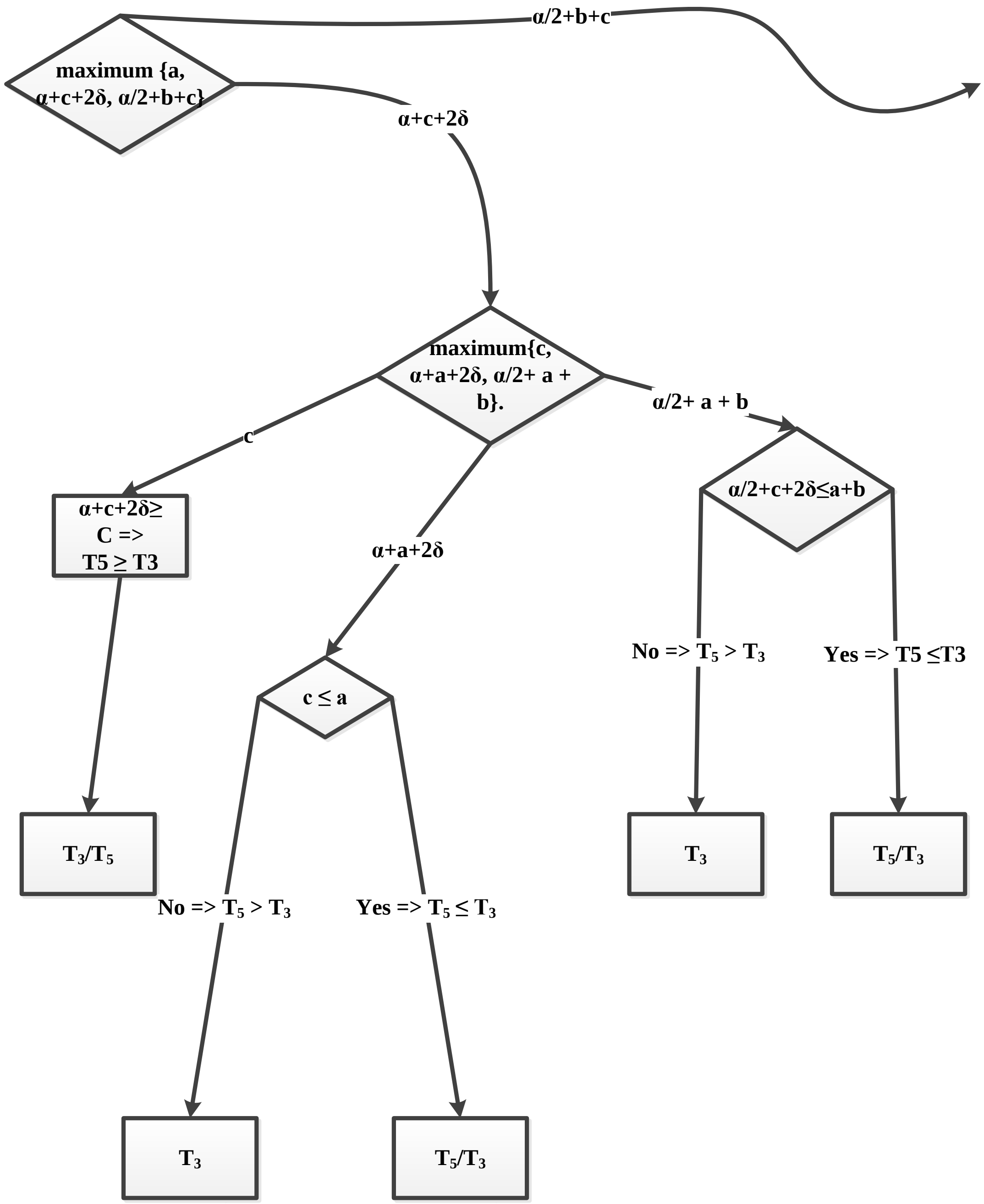

Figure 14. Decision Tree for Tjs – step5

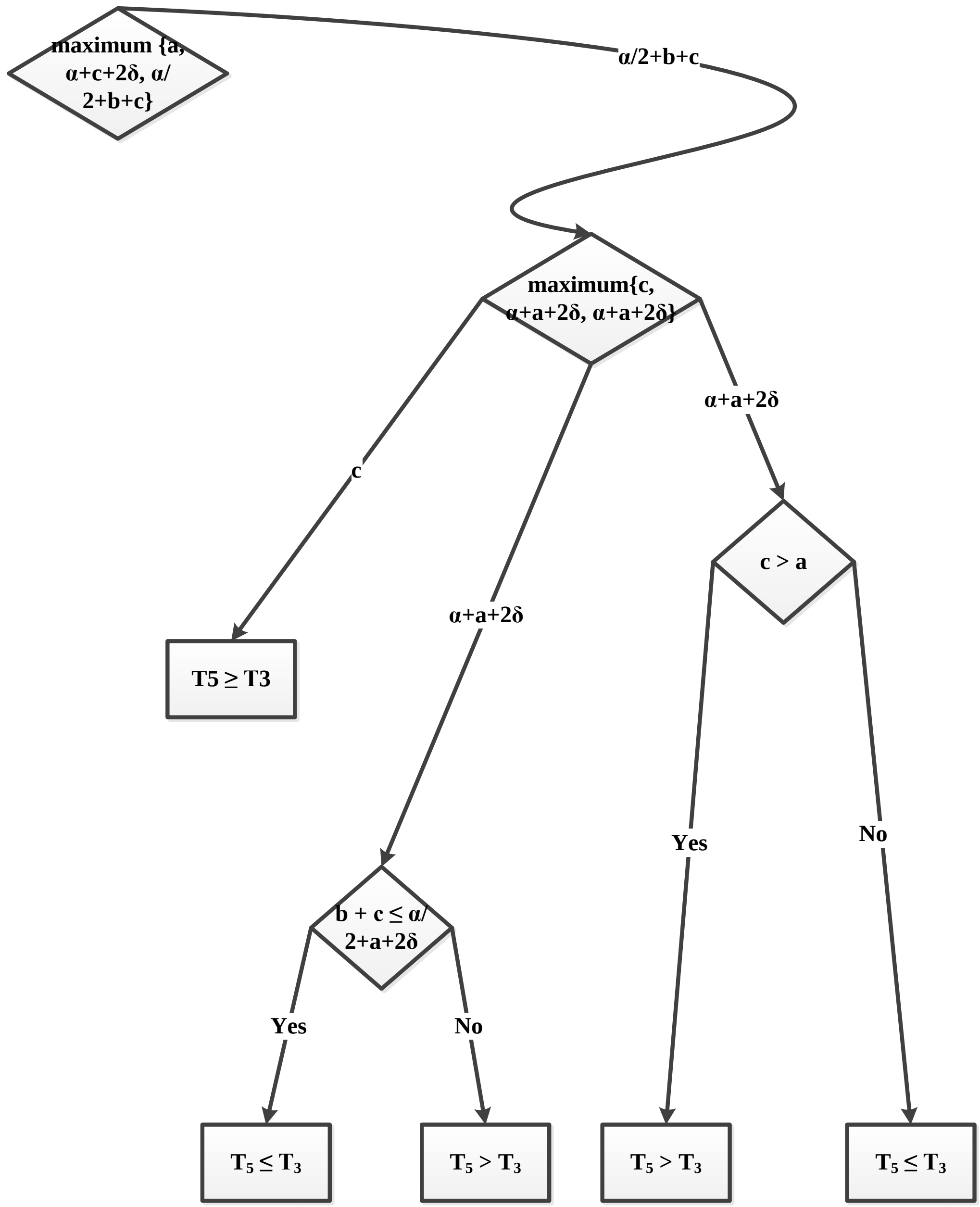

Figure 15. Decision Tree for Tjs – step6