

Emergent cosmological constant from colliding electromagnetic waves

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In this study we advocate the view that the cosmological constant is of electromagnetic (em) origin, which can be generated from the collision of em shock waves coupled with gravitational shock waves. The wave profiles that participate in the collision have different amplitudes. It is shown that, circular polarization with equal amplitude waves does not generate cosmological constant. We also prove that the generation of the cosmological constant is related to the linear polarization. The addition of cross polarization generates no cosmological constant. Depending on the value of the wave amplitudes, the generated cosmological constant can be positive or negative. We show additionally that, the collision of nonlinear em waves in a particular class of Born-Infeld theory also yields a cosmological constant.

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I. INTRODUCTION

The subject of colliding plane waves (CPW) in general relativity constitutes one of the important topics that the effects of the nonlinearity of the Einstein's equations manifests itself explicitly. The basic results of CPW's are not limited to find exact solutions, but rather its connections with other predictions of the theory of general relativity such as the spacetime singularities and the black hole interiors. (see [1], for a general review of related works). Although the collision of plane waves assumes idealized situations (the waves that participate in the collision are assumed to be plane symmetric, having an infinite extent in transverse directions), the dynamic nature of CPW spacetime may provide a theoretical background to experimental observations.

For example, according to the Standard Big Bang Cosmological Model in which the universe contains a cosmological constant, the universe went through an exponential growth called inflation and caused the formation of ripples propagating at the speed of light in the fabric of spacetime, called the gravitational waves within a tiny fraction of time after the big bang. It is now well understood that electromagnetic (em) radiation decoupled from free electrons about 380,000 years after the big bang [2]. Once they formed, as a requirement of the Einstein's theory of relativity, em waves are coupled with primordial gravitational waves and naturally their nonlinear interactions started to shape the em distribution.

The findings of BICEP-2, can be given as an example to such phenomena [3]. The observed B-mode in the polarization vector of the cosmic microwave background (CMB) radiation may be explained as a result of interaction with the primordial gravitational waves originated during the inflationary phase of the universe. This prob-

lem can be considered within the context of CPW and the exact analytic solution to the Einstein-Maxwell equations. The nonlinear interaction between plane gravitational waves and shock em waves with cross polarization is of utmost importance. Here, the primordial gravitational waves are assumed to be impulsive and shock types for the sake of an analytic exact solution. It has been shown in [4], that the Faraday rotation in the polarization vector of em waves can be attributed to the encounters with the strong gravitational waves with cross polarization.

On the other hand, the cosmological constant in the Standard Model of Big Bang Cosmology has been associated with the dark energy. Understanding the origin of the cosmological constant, its role in the universal vacuum energy, its repulsive effect in the accelerating expansion of the universe and related matters all constitute a vast literature in modern cosmology. Although experimental observations revealed much information about the evolution of the universe, the origin of the cosmological constant still lacks a satisfactory answer.

Recently, within the framework of CPW, one possible mechanism about the origin of the cosmological constant has been introduced by Barrabes and Hogan [5]. In this study, it has been shown that, the cosmological constant emerges as a result of nonlinear interaction of plane electromagnetic (em) shock waves accompanied by gravitational shock waves. As it was given in [5], this is a *special solution* in the sense that, there is only one component of electric and magnetic fields of the combined em waves that participate in the collision. On the other hand, the fundamental solution in this context is the Bell-Szekeres (BS) [6] solution which describes the collision of plane em shock waves in which there are two components of equal amplitudes of electric and magnetic fields that participate in the collision. Thus, the solution given in [5], is a special case that has no BS limit of equal em amplitudes [6].

Our motivation in this paper is to explore in detail the effects of polarization and wave amplitudes on the emer-

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gence of the cosmological constant as a result of nonlinear interaction of plane em shock waves coupled with gravitational shock waves. That is, our strategy from the outset, is not to introduce a cosmological constant in the initial data of the problem but rather to obtain it emergent as a result of colliding em data. Owing to the importance of the problem, we wish to extend the solution presented in [5] further in various directions. First, we consider the nonlinear interaction of em waves with different amplitudes in the initial data of BS solution. We show that emergent cosmological constant relates only to the linear polarization context of the waves with different amplitudes. We obtain that the cosmological constant ($= \lambda_0$) $= \alpha^2 - \beta^2$, where α and β denote the amplitude constants of electric field components along x and y directions, respectively. Such a theoretical prediction probably may be verified by experimental observations and this naturally necessitates to reevaluate the polarization data of CMB.

Extension of the BS solution to cross-polarized collision with single essential parameter was also found [7, 8]. In this problem the two incoming waves have non-aligned polarization vectors prior to the collision and naturally give rise to an off-diagonal component in the metric. This is analogous to the relation of Khan-Penrose [9] and Nutku-Halil [10] metrics. Secondly, in order to understand the effect of polarization together with different amplitudes, we extend the linear polarization problem to the case of cross-polarization, however, this doesn't yield a pure cosmological constant term. Instead, we obtain a general energy-momentum without immediate interpretation but yet it can be considered as a conversion of em energy into other forms.

In addition, we answer the question whether colliding em waves accompanied with gravitational waves in gravity coupled nonlinear electrodynamics [11] give rise to cosmological constant or not. Our finding for the Born-Infeld (BI) [12] theory is positive, however, this leaves the case of different nonlinear electromagnetic models open.

The work described in this paper and also in [5], i.e. generation of cosmological constant in the interaction region can be explained as a result of the re-distribution of the incoming energies in the waves that participate in the collision. Furthermore, as a by product besides the cosmological constant, two light-like shells are also generated on the null boundaries accompanying the impulsive gravitational waves. To recall a similar scenario and seek support from a different (i.e. quantum) domain of physics we refer to the theoretical side to the historic Breit-Wheeler analysis [13] of matter creation from the process of photon collisions. On the experimental side, this has been taken seriously in recent times through energetic laser photon collisions to materialize the idea at a grand scale [14]. If this quantum picture has any reflections in our macroworld it must correspond with our approach of colliding em plane waves, which is entirely classical.

Let us add that in addition to Ref. [5], Barabes and

Hogan also gave a method to generate a cosmological constant from collision of pure gravitational shock waves [15]. In this work the energy-momentum created with the cosmological constant balances with the emergent null currents on the boundaries of the collision. Hence, the consistency of the Einstein's equations hold.

The organization of the paper is as follows. Section II explores the collision of shock waves in Einstein-Maxwell (EM) theory. Section III considers collision of waves in nonlinear electrodynamic. The paper ends with Conclusion in Section IV. In Appendix A / B, we provide all Ricci / curvature components. Appendix C shows the effect of cross polarization while Appendix D presents energy-momenta and Einstein tensor components of the non-linear electromagnetic model used.

II. COLLIDING SHOCK WAVES IN EINSTEIN-MAXWELL (EM) THEORY

The spacetime describing colliding em shock waves with general polarization is summarized by [6]

$$ds^2 = 2e^{-M} dudv - e^{-U} (\cosh W (e^V dx^2 + e^{-V} dy^2) - 2 \sinh W dx dy). \quad (1)$$

Here (u, v) are the null coordinates while M, U, V and W are metric functions depending on both u and v in the interaction region. In the incoming regions, however, the metric functions depend only on one (either u or v) of the null coordinates. We must add that since the waves are moving at the speed of light, their collision problem can be best described in the null coordinates. The null coordinates are related to (t, z) coordinates by $\sqrt{2}u = t + z$ and $\sqrt{2}v = t - z$. The incoming waves are moving along $\pm z$ and they collide at $t = z = 0$ (or $u = v = 0$). Since em waves are transverse in this picture, we expect to have the x and y components of \mathbf{E} (electric) and \mathbf{B} (magnetic) vectors to be non-zero. The Maxwell and Einstein-Maxwell (EM) equations must be satisfied with the appropriate boundary conditions. For the EM waves these were formulated by O'Brien and Synge [16], but in the present problem, these conditions will be relieved. We wish also to comment that the metric function W carries the information about the second (or cross, or relative) polarization of the incoming waves. Care should be taken that a coordinate rotation of the (x, y) coordinates must not yield parallelly polarized vectors in the two incoming regions. Otherwise the waves are still linearly polarized so that the metric function W can be set to zero by a coordinate rotation. For a similar situation in colliding gravitational waves, one may consult [17]. Non-aligned polarization vectors in the incoming regions is therefore crucial to obtain a genuine solution in the interaction region with $W \neq 0$. As a matter of fact, Bell and Szekeres gave the exact solution only with $W = 0$ [6]. With the exception of the case considered in Appendix C in this paper, we shall restrict ourselves entirely to linear

polarization. The line element

$$ds^2 = 2dudv - \cos^2 \alpha (u_+ - v_+) dx^2 - \cos^2 \beta (u_+ + v_+) dy^2 \quad (2)$$

with the em potential 1-form

$$A(u, v) = \sin \alpha (u_+ - v_+) dx + \sin \beta (u_+ + v_+) dy \quad (3)$$

solves the problem of colliding shock waves in EM theory subject to the following information:

Here α and β are amplitude constants of the em waves; $u_+ = u\theta(u)$ and $v_+ = v\theta(v)$, where $\theta(u) / \theta(v)$ is the Heaviside unit step function. Note that we have the freedom to scale $u \rightarrow au$ and $v \rightarrow bv$ for constants a and b . Since we can absorb ab into the x and y coordinates and for the sake of simplicity, we shall make the choice $a = b = 1$ throughout the paper. For $u > 0, v > 0$, the line element (1) represents the geometry of interaction region (region IV). The incoming region II, for $v < 0, u > 0$ is given by, (see Fig. 1)

$$ds^2 = 2dudv - \cos^2(\alpha u_+) dx^2 - \cos^2(\beta v_+) dy^2. \quad (4)$$

For $u < 0, v > 0$ we obtain the incoming region III from (1), which is similar to II with $u \rightarrow v$. The non-zero em field components are obtained from (2) as follows;

$$F_{ux} = \alpha\theta(u) \cos \alpha (u_+ - v_+), \quad (5)$$

$$F_{vx} = -\alpha\theta(v) \cos \alpha (u_+ - v_+), \quad (6)$$

$$F_{uy} = \beta\theta(u) \cos \beta (u_+ + v_+), \quad (7)$$

$$F_{vy} = \beta\theta(v) \cos \beta (u_+ + v_+). \quad (8)$$

The Newman-Penrose (NP) quantities [18] in the null basis 1-forms

$$\ell = du, \quad n = dv, \quad (9)$$

$$\sqrt{2}m = \cos \alpha (u_+ - v_+) dx + i \cos \beta (u_+ + v_+) dy, \quad (10)$$

$$\sqrt{2}\bar{m} = \cos \alpha (u_+ - v_+) dx - i \cos \beta (u_+ + v_+) dy \quad (11)$$

and their Ricci tensor connections are given in the Appendix A. From (5-8) we can easily read the incoming em waves in region II (with $v < 0$) and region III (with $u < 0$). Equivalently, we find the NP components of the em field by

$$\Phi_2(u) = F_{\mu\nu} \bar{m}^\mu n^\nu = \frac{1}{\sqrt{2}} (\alpha + i\beta) \theta(u) \quad (\text{region II}) \quad (12)$$

$$\Phi_0(v) = F_{\mu\nu} \ell^\mu m^\nu = \frac{1}{\sqrt{2}} (\alpha + i\beta) \theta(v) \quad (\text{region III}). \quad (13)$$

The gravitational shock waves are also given in Appendix B as

$$\Psi_4(u) = \frac{1}{2} (\alpha^2 - \beta^2) \theta(u) \quad (\text{region II}) \quad (14)$$

$$\Psi_0(v) = \frac{1}{2} (\alpha^2 - \beta^2) \theta(v) \quad (\text{region III}). \quad (15)$$

From (5-8), the electric and magnetic components of our fields are

$$E_x = \frac{\alpha}{2} (\theta(u) - \theta(v)) \cos \alpha (u_+ - v_+) \quad (16)$$

$$E_y = \frac{\beta}{2} (\theta(u) + \theta(v)) \cos \beta (u_+ + v_+) \quad (17)$$

$$B_x = \frac{\beta}{2} (\theta(u) - \theta(v)) \cos \beta (u_+ + v_+) \quad (18)$$

$$B_y = -\frac{\alpha}{2} (\theta(u) + \theta(v)) \cos \alpha (u_+ - v_+). \quad (19)$$

Since the incoming em waves move along $\pm z$ -directions, they are transverse. Therefore, their E_x, E_y and B_x, B_y components are all non-zero. This is due to the fact that the chosen ansatz (3) for the vector potential is quite general. We note that our convention to define the polarization of the em waves is based on the electric field vector \mathbf{E} . An alternative choice can be made by using the magnetic field vector \mathbf{B} . From our ansatz for the em potential (3) and the derived field vectors (5-8), it is observed that the waves in region II and III are linearly polarized. The x -components of \mathbf{E} from region II and III yield opposite signs, but this doesn't change the polarization. We remind that em wave is a spin-1 field so that a π -rotation in axis, i.e. from $+x$ to $-x$ is allowed. We finally note that $E_x \neq E_y$ leads to elliptical polarization which reduces to circular polarization for $E_x = E_y$, but these are still linearly polarized along a line in the xy -plane. It is readily seen from (16-19) that after the collision, we have $E_x = B_x = 0$. Prior to the collision, regions II and III both have the em components elliptically polarized in the orthonormal frame $\{u, v, \bar{x}, \bar{y}\}$. To see this, we choose $d\bar{x} = g_{xx} dx$ and $d\bar{y} = g_{yy} dy$ so that

$$\frac{E_{\bar{x}}^2}{\alpha^2} + \frac{E_{\bar{y}}^2}{\beta^2} = 1. \quad (20)$$

Also note that, after the collision both the electric and magnetic fields are polarized in y -direction, i.e. the

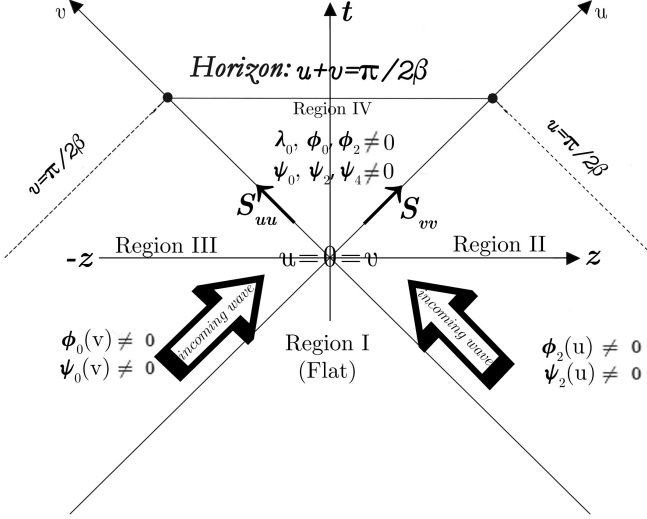


FIG. 1: The spacetime diagram for colliding (em+grav) waves. The region I ($u < 0, v < 0$) is flat, i.e. no-wave region. The incoming region II, ($u > 0, v < 0$), has $\phi_2(u) \neq 0 \neq \psi_4(u)$ while region III ($u < 0, v > 0$) has $\phi_0(v) \neq 0 \neq \psi_0(v)$. The interaction region IV ($u > 0, v > 0$) has the non-zero components $\phi_0(u, v), \phi_2(u, v), \psi_0(u, v), \psi_4(u, v), \psi_2(u, v)$ and λ_0 . The equivalent non-zero Ricci tensor components are given in Appendix A. It can easily be seen that for ($\alpha = \beta$) in the waves sets $\psi_4(u) = 0 = \psi_0(v)$ in the incoming regions and $\lambda_0 = 0 = \psi_2$ in the interaction region. This reduces the problem to colliding pure em waves problem of Bell and Szekeres. We wish to draw attention in particular to the null sources $S_{uu} = \delta(u) (\beta \tan \beta v_+ - \alpha \tan \alpha v_+)$ and $S_{vv} = \delta(v) (\beta \tan \beta u_+ - \alpha \tan \alpha u_+)$ emerging after the collision on the null boundaries. These vanish for the choice $\alpha = \beta$. From S_{uu} and S_{vv} we can read the null singularities as: $u = 0, v_1 = \frac{\pi}{2\alpha}, v_2 = \frac{\pi}{2\beta}$ and $v = 0, u_1 = \frac{\pi}{2\alpha}, u_2 = \frac{\pi}{2\beta}$.

process of collision acts as a polarizer. Throughout the spacetime, EM field equations are given by

$$R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} = -T_{\mu\nu} + S_{\mu\nu} + \lambda_0 g_{\mu\nu} \quad (21)$$

where the energy-momentum tensor of the em field is

$$T_{\mu\nu} = F_{\mu\lambda}F_{\nu}^{\lambda} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} \quad (22)$$

and $S_{\mu\nu}$ stands for the energy-momentum on the null-hypersurfaces [19]. From Appendix A and Eqs. (21-22) we read $S_{uu} = \delta(u) (\beta \tan \beta v_+ - \alpha \tan \alpha v_+)$ and $S_{vv} = \delta(v) (\beta \tan \beta u_+ - \alpha \tan \alpha u_+)$. The latter contains the delta functions of the Ricci tensor as displayed in the Appendix. Let us add that upon suppressing the infinite energy contributions from the planar (x, y) directions the integral of S_{ab} can be shown to contribute finite to the (u, v) plane. This is a result of the integral

$$2 \int_{-\delta}^{\delta} du \int_0^{\epsilon} dv \sqrt{-g} S_{uu} = \frac{\beta - \alpha}{\beta + \alpha} (1 - \cos \epsilon (\alpha + \beta)) + \frac{\beta + \alpha}{\beta - \alpha} (1 - \cos \epsilon (\beta - \alpha)) \quad (23)$$

in which $\delta > 0$ and $\epsilon > 0$ are small parameters. A similar result follows also from the S_{vv} integral. The constant λ_0 is identified as the cosmological constant which turns out to be

$$\lambda_0 = (\alpha^2 - \beta^2) \theta(u) \theta(v). \quad (24)$$

Obviously λ_0 emerges in region IV for $u > 0$ and $v > 0$ and depending on whether $\alpha^2 > \beta^2$ or $\alpha^2 < \beta^2$ it can be positive or negative. It is not difficult to speculate that the waves may start with $\alpha^2 > \beta^2$ but the y -mode can build up by superposition or other mechanisms to suppress the x -mode in successive collisions to make $\alpha^2 < \beta^2$. Thus, the emergent cosmological constant through colliding waves has the potential to change sign in accordance with the dominance of linear x/y modes.

From the Weyl scalars Ψ_4 (Ψ_0) (see Appendix B) it can easily be seen that null singularities occur at $u = 0$ ($v_1 = \frac{\pi}{2\alpha}$ and $v_2 = \frac{\pi}{2\beta}$) and $v = 0$ ($u_1 = \frac{\pi}{2\alpha}$ and $u_2 = \frac{\pi}{2\beta}$) that is, they double in number of the BS solution. When $\alpha = \beta$, the incoming Weyl curvatures disappear and we recover the problem of colliding em shock waves of BS [6].

The effect of second polarization on the formation of the cosmological constant has also been considered in this study. The solution presented in [8] is generalized to different amplitude wave profiles. Our analysis has shown that the addition of second polarization does not yield a cosmological constant. The related metric and Ricci scalar Λ are given in Appendix C.

III. COLLIDING WAVES IN EINSTEIN-NONLINEAR ELECTROMAGNETISM

Let's consider a general form of the nonlinear Maxwell Lagrangian as $\mathcal{L}(\mathcal{F})$ in which $\mathcal{F} = F_{\mu\nu}F^{\mu\nu}$. Hence the Einstein nonlinear Maxwell action reads ($16\pi G = 1$)

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + \mathcal{L}(\mathcal{F}) \right]. \quad (25)$$

The line element is chosen to be (note that in this section we use more appropriately the commonly used +2 signature)

$$ds^2 = -2dudv + \cos^2 \xi dx^2 + dy^2 \quad (26)$$

in which

$$\xi = \epsilon(u_+ - v_+). \quad (27)$$

Our em potential ansatz is $\mathbf{A} = a_0 \sin \xi dx$ so that the em field 2-form is

$$\mathbf{F} = \alpha \cos \xi (\theta(u) du - \theta(v) dv) \wedge dx, \quad (28)$$

where the constant α is $\alpha = a_0 \epsilon$. Its dual ${}^*\mathbf{F}$ is given by

$${}^*\mathbf{F} = -\alpha (\theta(u) du + \theta(v) dv) \wedge dy \quad (29)$$

and

$$\mathcal{F} = 4\alpha^2 \theta(u) \theta(v). \quad (30)$$

The final form of \mathcal{F} implies that \mathcal{F} is non-zero only in the interaction region i.e. $v > 0$ and $u > 0$ (i.e. the incoming em fields are null) and it is a constant. We must add that choosing the field ansatz as in (28) guarantees that the other Maxwell invariant is zero i.e., $\mathcal{G} = -\frac{1}{4}F_{\mu\nu} *F^{\mu\nu} = 0$. Therefore the general form of the nonlinear Lagrangian depends only on \mathcal{F} . For instance, in the case of BI theory, Lagrangian becomes

$$\mathcal{L}(\mathcal{F}) = 2b^2 \left(1 - \sqrt{1 + \frac{\mathcal{F}}{b^2}} \right) \quad (31)$$

which reduces to the linear Maxwell Lagrangian in the limit $b \rightarrow \infty$. We note that $b \neq 0$ is called the BI parameter with dimension of mass. The nonlinear Maxwell's equation in the interaction region ($u > 0$ and $v > 0$) is given by

$$d \left(* \mathbf{F} \frac{d\mathcal{L}(\mathcal{F})}{d\mathcal{F}} \right) = 0. \quad (32)$$

or effectively for (29) and (31) it means

$$d(*\mathbf{F}) = 0, \quad (33)$$

which is obviously satisfied. On the boundaries, however, this gives null currents i.e.

$$d \left(* \mathbf{F} \frac{d\mathcal{L}(\mathcal{F})}{d\mathcal{F}} \right) = * \mathbf{J} \quad (34)$$

where $*\mathbf{J}$ is the current 3-form given by

$$*\mathbf{J} = \frac{2\alpha^3}{b^2} \frac{(\delta(u)\theta(v) - \delta(v)\theta(u))}{\left(1 + \frac{4\alpha^2}{b^2}\theta(u)\theta(v)\right)^{3/2}} du \wedge dv \wedge dy, \quad (35)$$

which occurs similar to S_{ab} on the null boundaries. The energy momentum tensor of this nonlinear field and its explicit components are given in Appendix D. Plugging these into the field equations inside the interaction region

$$G_\mu^\nu - \lambda_0 \delta_\mu^\nu = T_\mu^\nu + S_\mu^\nu \quad (36)$$

yields $\lambda_0 = -(\mathcal{L} - 2\mathcal{F} \frac{d\mathcal{L}}{d\mathcal{F}})$ and to have consistency with the other equations, we must have $\epsilon^2 = -4\alpha^2 \frac{d\mathcal{L}(\mathcal{F})}{d\mathcal{F}}$ which is a constant (note that $\frac{d\mathcal{L}(\mathcal{F})}{d\mathcal{F}} < 0$). The fact that λ_0 emerges as a constant is by virtue of the chosen Lagrangian (31) and the solution (26-28).

Breton considered the following line element [11]

$$ds^2 = -2dudv + \cos^2 \xi dx^2 + \cos^2 \eta dy^2 \quad (37)$$

in which

$$\xi = \epsilon(u_+ + v_+) \quad (38)$$

and

$$\eta = \kappa(u_+ - v_+) \quad (39)$$

together with the em 2-form (here ϵ and κ are amplitude constants analogous to our α and β in the linear Maxwell theory)

$$\mathbf{F} = \alpha_0 \cos \xi (dx \wedge du + dx \wedge dv) - \beta_0 \cos \eta (dy \wedge du - dy \wedge dv). \quad (40)$$

Note that the constants α_0 and β_0 are related to ϵ and κ through the field equations. The nonlinear Lagrangian used here is the more general BI Lagrangian given by

$$\mathcal{L} = 2b^2 \left(1 - \sqrt{1 + \frac{\mathcal{F}}{b^2} - \frac{\mathcal{G}^2}{b^4}} \right). \quad (41)$$

As it was shown in Ref. [11] in the limit $b \rightarrow \infty$ it reduces rightly to the Bell-Szekeres solution. In this general case we also find that there is emergent cosmological constant in the interaction region. Emergence of null currents on the null boundaries after collision, however, from the nonlinear Maxwell equation is also inevitable in [11]. For the case of different nonlinear electromagnetic models other than BI, a similar conclusion remains to be seen.

IV. CONCLUSION

In this study we propose that cosmological constant is of em origin. Collision of linearly polarized em waves accompanied by appropriate gravitational shock waves gives rise to cosmological constant $\lambda_0 = \alpha^2 - \beta^2$ in the interaction region ($u > 0$ and $v > 0$). Here α and β are amplitude constants of the incoming em waves. For $\alpha = \beta$ we have the typical collision of em shock waves derived first by Bell and Szekeres [6] in which the interaction region has only em field with null singularities on the null boundaries. Two null em waves collide and turn into a non-null em field which is isometric to the Bertotti-Robinson spacetime (see [1]). Now, an interesting situation, observed by Barrabes and Hogan [5] arises: When the wave amplitudes along two space directions are unequal (i.e. $\alpha \neq \beta$), a cosmological constant emerges in the interaction region. We use this observation to speculate about the possible origin of the cosmological constant. We prove that this happens when the em waves are linearly polarized. We do this by extending the problem to cross polarized collision where the em energy transforms into a general form of energy-momentum which can't be identified as a cosmological constant. (See Appendix C)

We also show that the collision of em waves in a nonlinear electromagnetism, specifically a reduced version of the BI theory, similar trace of cosmological constant emerges. It should be added that the nonlinear Maxwell equations are satisfied modulo the currents on the null boundaries, after the collision, much like the sources S_{ab} of the linear theory on the null-boundaries.

Appendix A:

From the metric (1) and NP null-tetrad (8-10) we obtain the following Ricci components

$$2\phi_{22} = R_{uu} = (\alpha^2 + \beta^2) \theta(u) + \delta(u) (\beta \tan(\beta v_+) - \alpha \tan(\alpha v_+)) \quad (42)$$

$$2\phi_{00} = R_{vv} = (\alpha^2 + \beta^2) \theta(v) + \delta(v) (\beta \tan(\beta u_+) - \alpha \tan(\alpha u_+)) \quad (43)$$

$$2\phi_{02} = R_{\mu\nu} m^\mu m^\nu = (\alpha^2 + \beta^2) \theta(u) \theta(v) \quad (44)$$

$$R_{uv} = (\beta^2 - \alpha^2) \theta(u) \theta(v) \quad (45)$$

$$R_{xx} = 2\alpha^2 \theta(u) \theta(v) \cos^2 \alpha(u-v) \quad (46)$$

$$R_{yy} = -2\beta^2 \theta(u) \theta(v) \cos^2 \beta(u+v) \quad (47)$$

$$R = -24\Lambda = 4(\beta^2 - \alpha^2) \theta(u) \theta(v) = -4\lambda_0 \quad (48)$$

($\delta(u)$ and $\delta(v)$ are Dirac delta functions and $u_+ = u\theta(u)$ and $v_+ = v\theta(v)$)

Appendix B:

The non-zero Weyl components ψ_2, ψ_4 and ψ_0 are as follows:

$$6\psi_2 = (\alpha^2 - \beta^2) \theta(u) \theta(v) \quad (49)$$

$$2\psi_4 = (\alpha^2 - \beta^2) \theta(u) - \delta(u) (\alpha \tan(\alpha v_+) + \beta \tan(\beta v_+)) \quad (50)$$

$$2\psi_0 = (\alpha^2 - \beta^2) \theta(v) - \delta(v) (\alpha \tan(\alpha u_+) + \beta \tan(\beta u_+)) \quad (51)$$

Appendix C:

Colliding em waves with cross polarization:

Collision of linearly polarized em waves was generalized to include the second polarization in [7, 8]. The situation is analogous to Khan-Penrose [9] and Nutku-Halil [10], or Schwarzschild-Kerr relation. The latter solutions contain one extra parameter so that when the parameter vanishes, we obtain the former solutions. In the wave collision problem, the parameter is the angle of relative polarization of the two incoming waves, say α_0 . The metric of the interaction region induces a cross-term g_{xy} which is proportional to $\sin \alpha_0$ so that $g_{xy} \rightarrow 0$, when the waves are linearly polarized. With reference to [7],

it is not difficult to give the spacetime of the interaction region in oblate-spheroidal type coordinates [10]

$$ds^2 = F \left(\frac{d\tau^2}{\beta^2 \Delta} - \frac{d\sigma^2}{\alpha^2 \delta} \right) - \frac{\delta}{F} dx^2 - \left(\Delta F + \frac{\delta}{F} (\tau \sin \alpha_0)^2 \right) dy^2 + \frac{2\tau\delta}{F} \sin \alpha_0 dx dy. \quad (52)$$

The notation here goes as follows

$$\tau = \sin \beta(u+v), \quad \sigma = \sin \alpha(u-v), \quad (53)$$

$$\Delta = 1 - \tau^2, \quad \delta = 1 - \sigma^2, \quad (54)$$

$$2F = \sqrt{1 + \sin^2 \alpha_0} (1 + \sigma^2) + 1 - \sigma^2. \quad (55)$$

Note that

$$\frac{d\tau^2}{\beta^2 \Delta} - \frac{d\sigma^2}{\alpha^2 \delta} = 4dudv \quad (56)$$

and in the limit $\alpha_0 \rightarrow 0$ we recover the BS metric of linear polarization. The existence of $\alpha \neq \beta$, however, makes the curvature components rather complicated so that a pure cosmological constant doesn't arise in the present case. To verify this we make use of the null-tetrad basis 1-forms

$$\sqrt{2}l = \sqrt{F} \left(\frac{d\tau}{\beta\sqrt{\Delta}} - \frac{d\sigma}{\alpha\sqrt{\delta}} \right) \quad (57)$$

$$\sqrt{2}n = \sqrt{F} \left(\frac{d\tau}{\beta\sqrt{\Delta}} + \frac{d\sigma}{\alpha\sqrt{\delta}} \right) \quad (58)$$

$$\sqrt{2}m = \sqrt{\frac{\delta}{F}} (dx - \tau \sin \alpha_0 dy) + i\sqrt{\Delta F} dy \quad (59)$$

and the complex conjugate of m . It suffices to compute the NP scalar Λ which is

$$\Lambda = \frac{(\alpha^2 - \beta^2)}{48F^3} (4F^2 - \delta \sin^2 \alpha_0). \quad (60)$$

From the trace of equation (20), we see that the expected cosmological 'constant' $\lambda_0 = -\frac{R}{4}$ is not a constant. In the linear polarization limit $\alpha_0 = 0$ ($F = 1$) we obtain that $\lambda_0 = \frac{1}{2}(\alpha^2 - \beta^2)$. (Note that the extra $\frac{1}{2}$ factor comes from $4dudv$ in (3) instead of $2dudv$). Therefore we conclude that emergence of cosmological constant is related to the linear polarization property of the colliding waves. We recall that the cross polarization of waves through the Faraday rotation may be instrumental in the detection of gravitational waves [4].

Appendix D:

The energy-momentum tensor T_μ^ν in region IV with its explicit components and Einstein's terms in all regions are given as follow:

$$T_{\mu}^{\nu} = \mathcal{L}\delta_{\mu}^{\nu} - 4F_{\mu\lambda}F^{\nu\lambda}\frac{d\mathcal{L}}{d\mathcal{F}} \quad (61)$$

$$T_u^u = \mathcal{L} - \mathcal{F}\frac{d\mathcal{L}}{d\mathcal{F}}, \quad (62)$$

$$T_v^v = \mathcal{L} - \mathcal{F}\frac{d\mathcal{L}}{d\mathcal{F}}, \quad (63)$$

$$T_u^v = 4\alpha^2\theta(u)\frac{d\mathcal{L}}{d\mathcal{F}}, \quad T_v^u = 4\alpha^2\theta(v)\frac{d\mathcal{L}}{d\mathcal{F}}, \quad (64)$$

$$T_x^x = \mathcal{L} - 2\mathcal{F}\frac{d\mathcal{L}}{d\mathcal{F}}, \quad T_y^y = \mathcal{L}. \quad (65)$$

$$G_u^u = -\epsilon^2\theta(u)\theta(v), \quad G_v^v = -\epsilon^2\theta(u)\theta(v), \quad (66)$$

$$G_u^u = -\epsilon^2\theta(u) - \epsilon\tan\xi\delta(u), \quad (67)$$

$$G_v^u = -\epsilon^2\theta(v) + \epsilon\tan\xi\delta(v), \quad (68)$$

$$G_x^x = 0, \quad G_y^y = -2\epsilon^2\theta(u)\theta(v). \quad (69)$$

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