

Thin-shell wormholes from the regular Hayward black hole

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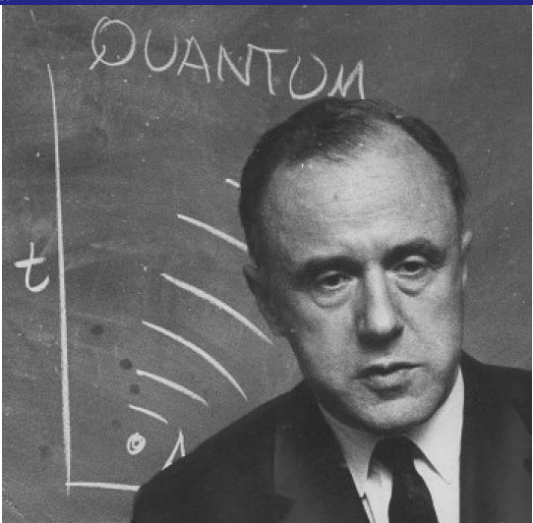


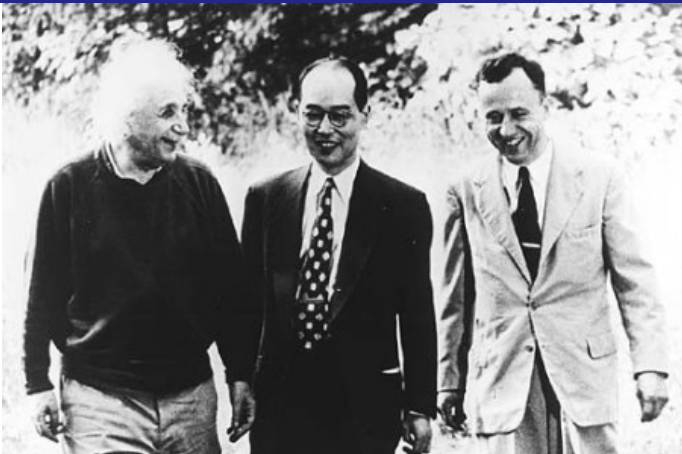


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- In the first part of the paper we reconsider the Hayward regular black hole in 4-dimensional spherical symmetric static spacetime with the source of nonlinear magnetic field.
- In the second part of the paper we establish a spherical thin-shell with inside the Hayward black hole and outside simply Schwarzschild spacetime.
- The Surface stress are determined using the Darmois-Israel formalism at the wormhole throat.
- We analyze the stability of the thin-shell considering linear gas(LG), chaplygin gas(CG), phantom-energy or generalised Chaplyggin gas (GCG), modified generalized chaplygin gas(MGCG) and logarithmic gas(LG) equation of states for the exotic matter at the throat.
- In the last part of the paper we establish a thin-shell wormhole in the Hayward regular black hole spacetime and show the stability regions of potentials.

What is a Wormhole?

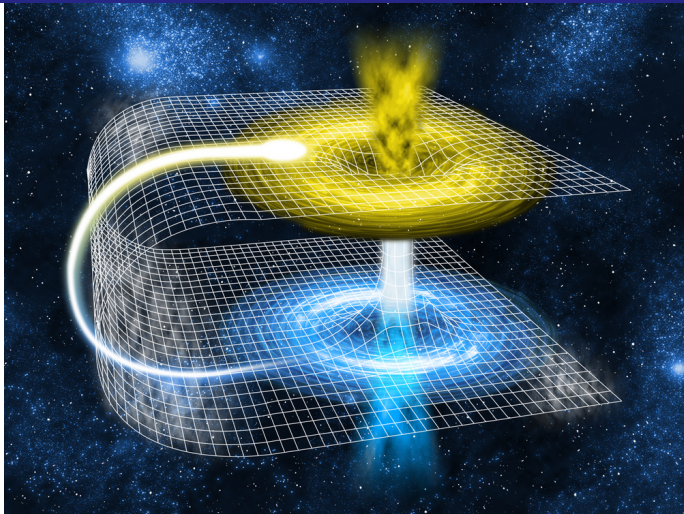


Figure: "Folded" space-time illustrates how a wormhole bridge might

form with at least two mouths that are connected to a single throat.

Theories of wormhole metrics describe the spacetime geometry of a wormhole and serve as theoretical models for time travel. An example of a (traversable) wormhole metric is the MORRIS-THORNE WORMHOLE :

$$ds^2 = -c^2 dt^2 + dl^2 + (k + l)(d\theta^2 + \sin(\theta)^2 d\phi^2)$$

The parameter k defines the size of the throat of the wormhole, and l represents the proper length radius.

One type of non-traversable wormhole metric is the Schwarzschild solution:

$$ds^2 = -c^2(1 - 2M/rc^2)dt^2 + \frac{dr^2}{(1 - 2M/rc^2)} + r^2(d\theta^2 + \sin(\theta)^2 d\phi^2)$$

Singularity of black hole is an acknowledged difficulty in general relativity. At the singular point, the curvature will be divergent, so it means all the physics laws fail at the point. Firstly Bardeen then Hayward proposed a new idea to avoid the singular point.

The spherically symmetric static Hayward nonsingular black hole introduced in is given by the following line element

$$ds^2 = - \left(1 - \frac{2mr^2}{r^3 + 2ml^2} \right) dt^2 + \quad (1)$$

$$\left(1 - \frac{2mr^2}{r^3 + 2ml^2} \right)^{-1} dr^2 + r^2 d\Omega^2 \quad (2)$$

in which m and l are two free parameters and

$$d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2. \quad (3)$$

One particular boundary that captures the idea of a black hole and that can be defined locally is the so-called Trapping Horizon

[Hayward, 1994], defined as the boundary of a space-time region in which initially divergent light rays eventually converge. This boundary can be located by investigating the behaviour of light cones in the region.

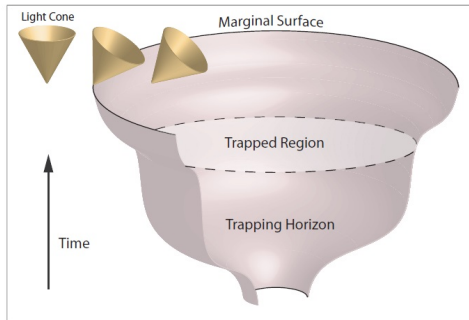


Figure: A trapping horizon is shown here. Inside trapped regions

outwardly directed light signals momentarily fall inwards.

The metric function of this black hole $f(r) = \left(1 - \frac{2mr^2}{r^3 + 2ml^2}\right)$ at large r behaves as Schwarzschild spacetime

$$\lim_{r \rightarrow \infty} f(r) \rightarrow 1 - \frac{2m}{r} + \mathcal{O}\left(\frac{1}{r^4}\right) \quad (4)$$

at small r behaves as de-Sitter black hole

$$\lim_{r \rightarrow 0} f(r) \rightarrow 1 - \frac{r^2}{l^2} + \mathcal{O}(r^5). \quad (5)$$

The curvature invariant scalars are all finite at $r = 0$. The Hayward black hole admits event horizon which is the largest real root of the following equation

$$r^3 - 2mr^2 + 2ml^2 = 0. \quad (6)$$

Setting $r = m\rho$ and $l = m\lambda$ (x) becomes

$$\rho^3 - 2\rho^2 + 2\lambda^2 = 0 \quad (7)$$

which admits no horizon (regular particle solution) for $\lambda^2 > \frac{16}{27}$, one horizon (regular extremal black hole) for $\lambda^2 = \frac{16}{27}$ and two horizons (regular black hole with two horizons) for $\lambda^2 < \frac{16}{27}$.

Therefore the important is the ration of $\frac{l}{m}$ with critical ratio at $(\frac{l}{m})_{crit.} = \frac{4}{3\sqrt{3}}$ but not l and m individually. This suggests to set $m = 1$ in the sequel without lose of generality i.e., $f(r) = 1 - \frac{2r^2}{r^3 + 2l^2}$. For $l^2 < \frac{16}{27}$ the event horizon ("the point of no return") is given by

$$r_h = \frac{1}{3} \left(\sqrt[3]{\Delta} + \frac{4}{\sqrt[3]{\Delta}} + 2 \right) \quad (8)$$

in which $\Delta = 8 - 27l^2 + 3\sqrt{27l^2(3l^2 - 2)}$. For the case of extremal black hole i.e. $l^2 = \frac{16}{27}$ the single horizon accrues at $r_h = \frac{4}{3}$. For the case $l^2 \leq \frac{16}{27}$ the standard Hawking temperature at

the event horizon is given by

$$T_H = \frac{f'(r_h)}{4\pi} = \frac{1}{4\pi} \left(\frac{3}{2} - \frac{2}{r_h} \right) \quad (9)$$

which clearly for $l^2 = \frac{16}{27}$ vanishes and for $l^2 < \frac{16}{27}$ is positive (One must note that $r_h \geq \frac{4}{3}$). Considering the standard definition for the entropy of the black hole $S = \frac{\mathcal{A}}{4}$ in which $\mathcal{A} = 4\pi r_h^2$ one finds the heat capacity of the black hole which is defined as

$$C_I = \left(T_H \frac{\partial S}{\partial T_H} \right)_I \quad (10)$$

which is determined as

$$C_I = 4\pi r_h^3 \left(\frac{3}{2} - \frac{2}{r_h} \right) \quad (11)$$

which is clearly non-negative. Latter shows that thermodynamically the black hole is stable.

Let's consider the following action

$$\mathcal{I} = \frac{1}{16\pi} \int d^4x \sqrt{-g} (R - \mathcal{L}(F)) \quad (12)$$

in which R is the Ricci scalar and

$$\mathcal{L}(F) = - \frac{6}{l^2 \left[1 + \left(\frac{\beta}{F} \right)^{3/4} \right]^2} \quad (13)$$

is the nonlinear magnetic field Lagrangian density with $F = F_{\mu\nu} F^{\mu\nu}$ the Maxwell invariant and l and β two constant positive parameters. The magnetic field two form is given by

$$\mathbf{F} = P \sin^2 \theta d\theta \wedge d\phi \quad (14)$$

in which P is the magnetic monopole charge. Latter field form together with the line element (x) imply

$$F = \frac{2P^2}{r^4}. \quad (15)$$

Einstein field equations are

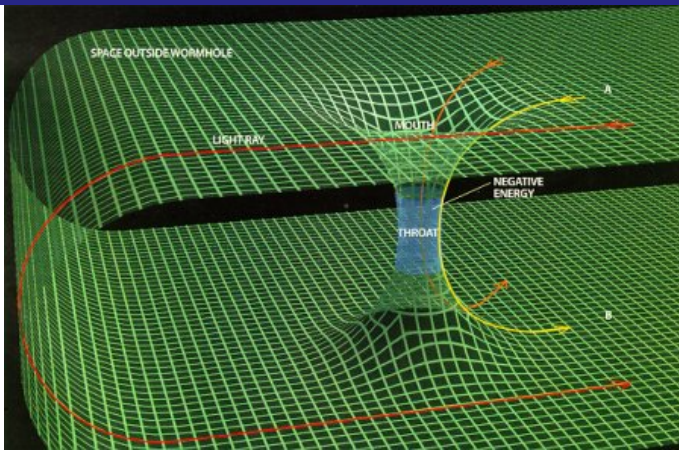
$$G_{\mu}^{\nu} = T_{\mu}^{\nu} \quad (16)$$

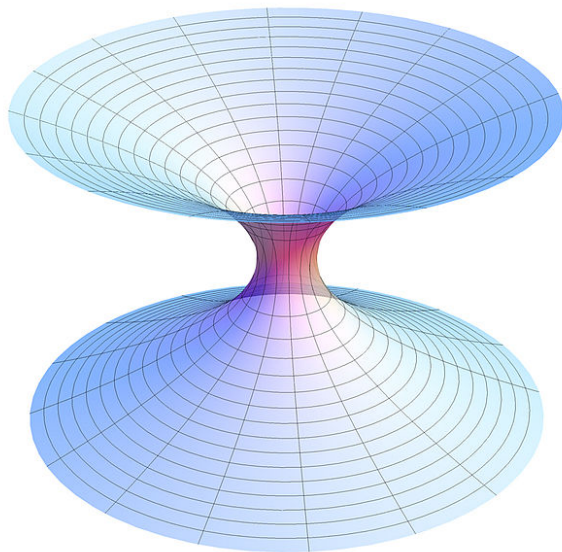
in which

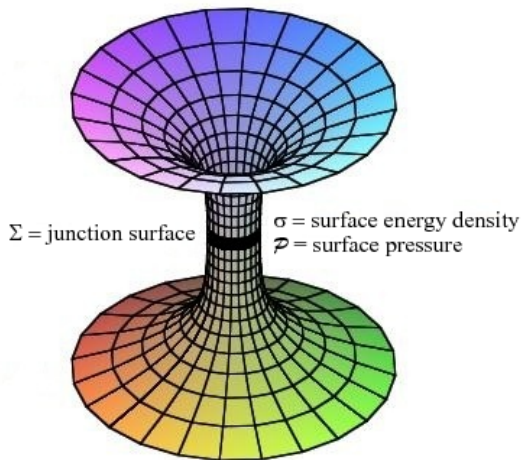
$$T_{\mu}^{\nu} = -\frac{1}{2} \left(\mathcal{L} \delta_{\mu}^{\nu} - 4F_{\mu\lambda} F^{\lambda\nu} \mathcal{L}_F \right) \quad (17)$$

in which $\mathcal{L}_F = \frac{\partial \mathcal{L}}{\partial F}$. One can show that using $\mathcal{L}(F)$ given in (x), the Einstein equation admit the Hayward regular black hole metric providing $\beta = \frac{2P^2}{(2ml^2)^{4/3}}$. The weak field limit of the Lagrangian (x) can be found by expanding the Lagrangian about $F = 0$ which reads

$$\mathcal{L}(F) = -\frac{6F^{3/2}}{l^2 \beta^{3/2}} + \frac{12F^{9/4}}{l^2 \beta^{9/4}} + \mathcal{O}(F^3). \quad (18)$$







In the second part of the paper we use the standard method of making a timelike thin-shell wormhole and make a timelike thin-shell located at $r = a$ ($a > r_h$) by cut $r < a$ from the Hayward regular black hole and past two copy of it at $r = a$. On the shell the spacetime is chosen to be

$$ds^2 = -d\tau^2 + a(\tau)^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (19)$$

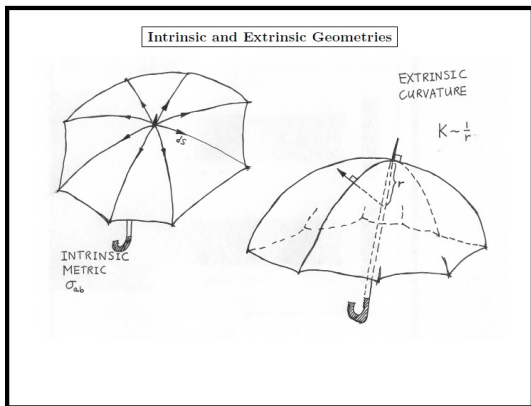
in which τ is the proper time on the shell.

To make a consistent $2 + 1$ -dimensional timelike shell with the two $3 + 1$ -dimensional we have to fulfill the Lanczos conditions which are the Einstein equations on the shell

$$[K_i^j] - [K] \delta_i^j = -8\pi S_i^j \quad (20)$$

in which $[X] = X_2 - X_1$ and K_i^j is the extrinsic curvature tensor in each part of the thin-shell and K denotes its trace. S_i^j is the

energy momentum tensor on the shell such that $S_T^T = -\sigma$ or energy density $S_\theta^\theta = p = S_\phi^\phi$ is the pressure density.



K_i^j is the extrinsic curvature defined by

$$K_{ij}^{(\pm)} = -n_\gamma^{(\pm)} \left(\frac{\partial^2 x^\gamma}{\partial \xi^i \partial \xi^j} + \Gamma_{\alpha\beta}^\gamma \frac{\partial x^\alpha}{\partial \xi^i} \frac{\partial x^\beta}{\partial \xi^j} \right)_{r=a} \quad (21)$$

with the normal unit vector

$$n_\gamma^{(\pm)} = \left(\pm \left| g^{\alpha\beta} \frac{\partial f}{\partial x^\alpha} \frac{\partial f}{\partial x^\beta} \right|^{-1/2} \frac{\partial f}{\partial x^\gamma} \right). \quad (22)$$

are found as follows

$$n_t = \pm \left(\left| g^{tt} \left(\frac{\partial a(\tau)}{\partial t} \right)^2 + g^{rr} \right|^{-1/2} \frac{\partial F}{\partial t} \right)_{r=a}. \quad (23)$$

Upon using

$$\left(\frac{\partial t}{\partial \tau} \right)^2 = \frac{1}{f(a)} \left(1 + \frac{1}{f(a)} \dot{a}^2 \right), \quad (24)$$

it implies

$$n_t = \pm (-\dot{a}). \quad (25)$$

Similarly one finds that

$$n_r = \pm \left(\left| g^{tt} \frac{\partial F}{\partial t} \frac{\partial F}{\partial t} + g^{rr} \frac{\partial F}{\partial r} \frac{\partial F}{\partial r} \right|^{-1/2} \frac{\partial F}{\partial r} \right)_{r=a} \quad (26)$$

$$= \pm \left(\frac{\sqrt{f(a) + \dot{a}^2}}{f(a)} \right), \text{ and } n_{\theta_i} = 0, \text{ for all } \theta_i. \quad (27)$$

Note that $\langle K \rangle = \text{Trace} \langle K_i^j \rangle$ and $S_i^j = \text{diag}(\sigma, p_y, p_\phi)$ is the energy momentum tensor on the thin-shell. The parametric equation of the hypersurface Σ is given by

$$f(x, a(\tau)) = x - a(\tau) = 0. \quad (28)$$

After the unit d -normal, one finds the extrinsic curvature tensor components from the definition

$$K_{ij}^{\pm} = -n_{\gamma}^{\pm} \left(\frac{\partial^2 x^{\gamma}}{\partial \xi^i \partial \xi^j} + \Gamma_{\alpha\beta}^{\gamma} \frac{\partial x^{\alpha}}{\partial \xi^i} \frac{\partial x^{\beta}}{\partial \xi^j} \right)_{r=a}. \quad (29)$$

It follows that

$$\begin{aligned} K_{\tau\tau}^{\pm} &= -n_t^{\pm} \left(\frac{\partial^2 t}{\partial \tau^2} + \Gamma_{\alpha\beta}^t \frac{\partial x^{\alpha}}{\partial \tau} \frac{\partial x^{\beta}}{\partial \tau} \right)_{r=a} - n_r^{\pm} \left(\frac{\partial^2 r}{\partial \tau^2} + \Gamma_{\alpha\beta}^r \frac{\partial x^{\alpha}}{\partial \tau} \frac{\partial x^{\beta}}{\partial \tau} \right)_{r=a} \\ &= -n_t^{\pm} \left(\frac{\partial^2 t}{\partial \tau^2} + 2\Gamma_{tr}^t \frac{\partial t}{\partial \tau} \frac{\partial r}{\partial \tau} \right)_{r=a} - n_r^{\pm} \left(\frac{\partial^2 r}{\partial \tau^2} + \Gamma_{tt}^r \frac{\partial t}{\partial \tau} \frac{\partial t}{\partial \tau} + \Gamma_{rr}^r \frac{\partial r}{\partial \tau} \frac{\partial r}{\partial \tau} \right)_{r=a} \end{aligned} \quad (30)$$

$$= \pm \left(-\frac{f' + 2\ddot{a}}{2\sqrt{f + \dot{a}^2}} \right), \quad (31)$$

Also

$$K_{\theta_i \theta_i}^{\pm} = -n_{\gamma}^{\pm} \left(\frac{\partial^2 x^{\gamma}}{\partial \theta_i^2} + \Gamma_{\alpha\beta}^{\gamma} \frac{\partial x^{\alpha}}{\partial \theta_i} \frac{\partial x^{\beta}}{\partial \theta_i} \right)_{r=a} = \pm \sqrt{f(a) + \dot{a}^2} h h'. \quad (32)$$

and therefore

$$K = \text{Trace} \langle K_i^j \rangle = \langle K_i^i \rangle = \frac{f' + 2\ddot{a}}{\sqrt{f + \dot{a}^2}} + 4\sqrt{f(a) + \dot{a}^2} \frac{h'}{h}. \quad (33)$$

The surface energy-momentum components of the thin-shell are

$$S_i^j = -\frac{1}{8\pi} \left(\langle K_i^j \rangle - K \delta_i^j \right) \quad (34)$$

which yield

$$\sigma = -S_r^r = -\frac{1}{2\pi} \left(\sqrt{f(a) + \dot{a}^2} \frac{h'}{h} \right), \quad (35)$$

$$S_{\theta_i}^{\theta_i} = p_{\theta_i} = \frac{1}{8\pi} \left(\frac{f' + 2\ddot{a}}{\sqrt{f(a) + \dot{a}^2}} + 2\sqrt{f(a) + \dot{a}^2} \frac{h'}{h} \right). \quad (36)$$

One can explicitly find

$$\sigma = -\frac{1}{2\pi a} \sqrt{f(a) + \dot{a}^2} \quad (37)$$

and

$$p = \frac{1}{4\pi} \left(\frac{\sqrt{f(a) + \dot{a}^2}}{a} + \frac{\ddot{a} + f'(a)/2}{\sqrt{f(a) + \dot{a}^2}} \right). \quad (38)$$

Consequently the energy and pressure densities in a static

configuration at $a = a_0$ are given by

$$\sigma_0 = -\frac{1}{2\pi a_0} \sqrt{f(a_0)} \quad (39)$$

and

$$p_0 = \frac{1}{4\pi} \left(\frac{\sqrt{f(a_0)}}{a_0} + \frac{f'(a_0)/2}{\sqrt{f(a_0)}} \right). \quad (40)$$

To investigate the stability of such wormhole we apply a linear perturbation in which after that the following state equation

$$p = \psi(\sigma) \quad (41)$$

with an arbitrary equation $\psi(\sigma)$ is followed by the thin shell. In addition to this relation between p and σ the energy conservation law also imposes

$$S^j_{;j} = 0 \quad (42)$$

which in closed form it amounts to

$$S^j_j + S^{kj}\Gamma_{kj}^{i\mu} + S^{ik}\Gamma_{kj}^j = 0 \quad (43)$$

or equivalantly, after the line element (x),

$$\frac{\partial}{\partial\tau} (\sigma a^2) + p \frac{\partial}{\partial\tau} (a^2) = 0. \quad (44)$$

This equation can be rewritten as

$$\dot{a}^2 + V(a) = 0 \quad (45)$$

in $V(a)$ is given by

$$V(a) = f - 4\pi^2 a^2 \sigma^2 \quad (46)$$

and σ is the energy density after the perturbation. Eq. (x) is a one dimensional equation of motion in which the oscillatory motion for

a in terms of τ about $a = a_0$ is the consequence of having $a = a_0$ the equilibrium point which means $V'(a_0) = 0$ and $V''(a_0) \geq 0$. In the sequel we consider $f_1(a_0) = f_2(a_0)$ and therefore at $a = a_0$, one finds $V_0 = V'_0 = 0$. To investigate $V''(a_0) \geq 0$ we use the given $p = \psi(\sigma)$ to find

$$\sigma' \left(= \frac{d\sigma}{da} \right) = -\frac{2}{a} (\sigma + \psi) \quad (47)$$

and

$$\sigma'' = \frac{2}{a^2} (\sigma + \psi) (3 + 2\psi'). \quad (48)$$

Herein $\psi' = \frac{d\psi}{d\sigma}$. Finally

$$V''(a_0) = f''_0 - 8\pi^2 \left[(\sigma_0 + 2p_0)^2 + 2\sigma_0 (\sigma_0 + p_0) (1 + 2\psi'(\sigma_0)) \right] \quad (49)$$

which we have used $\psi_0 = p_0$.

Recently two of us analyzed the effect of the Gauss-Bonnet parameter in the stability of TSW in higher dimensional EGB gravity . In that paper some specific model of matter has been considered such as LG, CG, GCG, MGCG and LG. In this work we go closely to the same state functions and we analyze the effect of Hayward's parameter in the stability of the TSW constructed above.

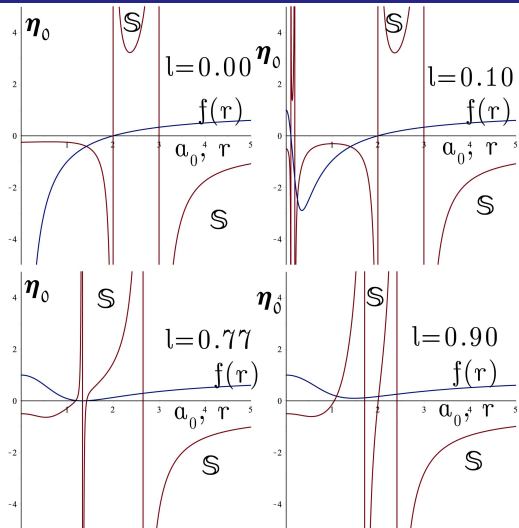


Figure: Stability of TSW supported by LG in terms of a_0 and η_0 for

Linear gas (LG)

$\ell = 0.00, 0.10, 0.77$ and 0.90 . The value of $m = 1$. The effect of Hayward's constant is to increase the stability of the TSW. We note that the stable regions are shown by \mathcal{S} and the metric function is plotted too.

In the case of a linear state function i.e.,

$$\psi = \eta_0 (\sigma - \sigma_0) + p_0$$

in which η_0 is a constant parameter, one finds $\psi'(\sigma_0) = \eta_0$. Fig. 1 displays the region of stability in terms of η_0 and a_0 for different values of Hayward's parameter.

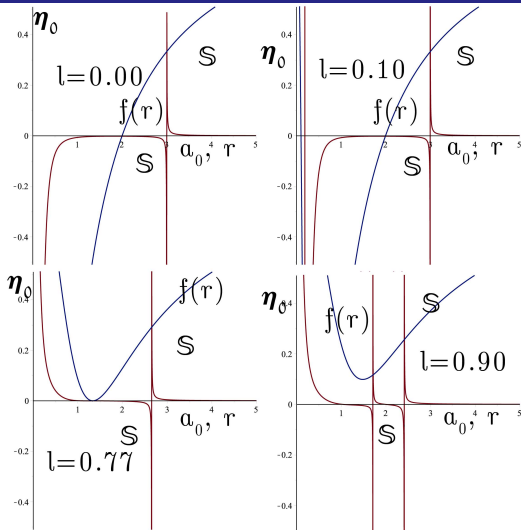


Figure: Stability of TSW supported by CG in terms of a_0 and η_0 for

Chaplygin gas (CG)

$\ell = 0.00, 0.10, 0.77$ and 0.90 . The value of $m = 1$. The effect of Hayward's constant is to increase the stability of the TSW. We also plot the metric function to compare the horizon of the black hole and the location of the throat.

For Chaplygin gas (CG) the state function is given by

$$\psi = \eta_0 \left(\frac{1}{\sigma} - \frac{1}{\sigma_0} \right) + p_0$$

where η_0 is a constant parameter, implies $\psi'(\sigma_0) = -\frac{\eta_0}{\sigma_0^2}$. In Fig. 2 we plot the stability region in terms of η_0 and a_0 for different values of ℓ .

Generalized Chaplygin gas (GCG)

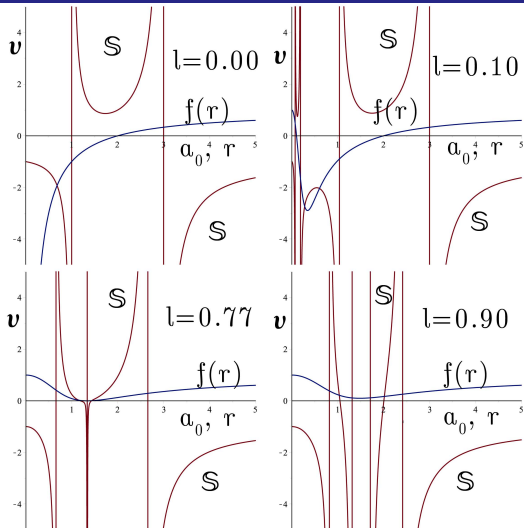


Figure: Stability of TSW supported by GCG in terms of a_0 and ν for

$\ell = 0.00, 0.10, 0.77$ and 0.90 . The value of $m = 1$. The effect of Hayward's constant is to increase the stability of the TSW. We also plot the metric function to compare the horizon of the black hole and the location of the throat.

The state function of the Generalized Chaplygin gas can be cast into

$$\psi(\sigma) = \eta_0 \left(\frac{1}{\sigma^\nu} - \frac{1}{\sigma_0^\nu} \right) + p_0$$

in which ν and η_0 are constant. To see the effect of parameter ν in the stability we set the constant η_0 such that ψ becomes

$$\psi(\sigma) = p_0 \left(\frac{\sigma_0}{\sigma} \right)^\nu.$$

After we found $\psi'(\sigma_0) = -\frac{p_0}{\sigma_0} \nu$, in Fig. 3 we plot the stability regions of the TSW supported by a GCG in terms of ν and a_0 with various values of ℓ .

Modified Generalized Chaplygin gas (MGCG)

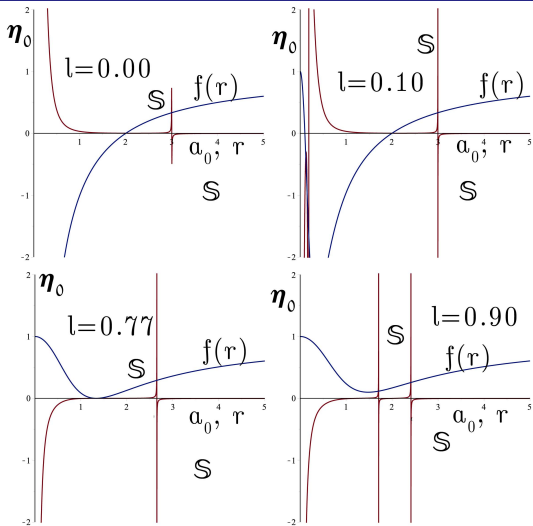


Figure: Stability of TSW supported by MGCG in terms of a_0 and η_0 for

Modified Generalized Chaplygin gas (MGCG)

$\ell = 0.00, 0.10, 0.77$ and 0.90 . The value of $m = 1$ and $\xi_0 = \eta_0 = 1$. The effect of Hayward's constant is to increase the stability of the TSW. We also plot the metric function to compare the horizon of the black hole and the location of the throat.

A more generalized form of CG is called the Modified Generalized Chaplygin gas (MGCG) which is given by

$$\psi(\sigma) = \xi_0(\sigma - \sigma_0) - \eta_0 \left(\frac{1}{\sigma^\nu} - \frac{1}{\sigma_0^\nu} \right) + p_0$$

in which ξ_0 , η_0 and ν are free parameters. One then, finds

$$\psi'(\sigma_0) = \xi_0 + \eta_0 \frac{\eta_0 \nu}{\sigma_0^{\nu+1}}.$$

To go further we set $\xi_0 = 1$ and $\nu = 1$ and in Fig. 4 we show the stability regions in terms of ν and a_0 with various values of ℓ .

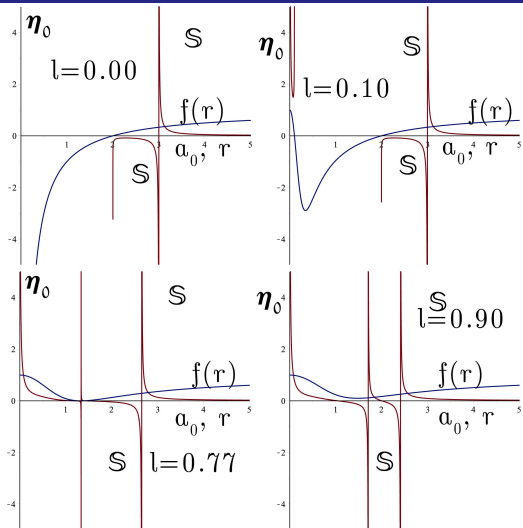


Figure: Stability of TSW supported by LG in terms of a_0 and η_0 for

Logarithmic gas (LG)

$\ell = 0.00, 0.10, 0.77$ and 0.90 . The value of $m = 1$. The effect of Hayward's constant is to increase the stability of the TSW. We also plot the metric function to compare the horizon of the black hole and the location of the throat.

In our last equation of state we consider the Logarithmic gas (LG) which is given by

$$\psi(\sigma) = \eta_0 \ln \left| \frac{\sigma}{\sigma_0} \right| + p_0$$

in which η_0 is a constant. For LG one finds

$$\psi'(\sigma_0) = \frac{\eta_0}{\sigma_0}.$$

In Fig. 5 we plot the stability region for the TSW supported by LG and the effect of Hayward's parameter is shown clearly.

Conclusion We have studied various aspects of thin-shell wormholes constructed from the Hayward regular black hole in 4-dimensional spherical symmetric static spacetime with the source of nonlinear magnetic monopole field. In a nonlinear Magnetic field. We have plotted σ and p for different values of parameters M , Q and $a = a_0$ to show the presence of exotic matter confined within the shell of the wormhole. The nature of the wormhole (attractive and repulsive) has been investigated for fixed values of parameters M and Q . The amount of exotic matter required to support the wormhole is always a crucial issue. We have shown the variation of exotic matter graphically with respect to charge and mass.



A word cloud where the words are arranged to form the shape of the number '10'. The largest words are 'THANK' and 'YOU'. Other prominent words include 'GRACIAS', 'ARIGATO', 'SHUKURIA', 'JUSPAXAR', 'DANKSCHEEN', 'TASHAKKUR ATU', 'YAQHANYELAY', 'SUKSAMA', 'MEHRBANI', 'BOLZIN', 'MERCİ', 'BIYAN', and 'SHUKRIA'. Smaller words include 'SPASIBO', 'SNACHALHYVA', 'NURUN', 'CHALTU', 'WABEEJA', 'MAITEKA', 'HUI', 'YUSPAGARATAM', 'DHIWYBUDAD', 'ASHU', 'ATTO', 'SHIBES', 'SPASIBO', 'DENKAU-JA', 'NENACHALHYVA', 'UNALCHEESH', 'HATUR GI', 'EKO-JU', 'SIKOMO', 'TAVTAPUCH', 'MEDAWAGGE', 'GOZAIMASHITA', 'EFCHARISTO', 'AGUYJE', 'FAKARUE', 'KOMAPSUMNIDA', 'MAAKE', 'LAN', 'MERASTAWHY', 'GAEJTHO', 'PALDIES', 'MINMONCHAR', and 'MAKETAM'.