Effect of the cosmological constant in the Hawking radiation of 3D charged dilaton black hole

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Abstract

This paper deals with the semiclassical radiation spectrum of static and circularly symmetric 3D charged dilaton black holes with cosmological constant Λ in non-asymptotically flat spacetimes. We first review the 3D charged dilaton black holes which are solution to low-energy string action. The wave equation of a massless scalar field is shown to be exactly solvable in terms of hypergeometric functions. Thus, the radiation spectrum and its corresponding temperature are obtained, precisely. Computations at high frequency regime show that the radiation spectrum yields the Hawking temperature of the black hole with no charge. Unlike the chargeless case, the Hawking temperature of the charged dilaton black holes is derived from the radiation spectrum at the low frequencies. The utmost importance of the Λ in the latter result is highlighted.

I. INTRODUCTION

Since the seminal works of [12–14] and [4], it is well understood that a black hole (BH) can emit particles from its event horizon with a certain temperature, so-called the Hawking temperature (T_H), which is proportional to the surface gravity, κ, of the BH. Therefore, a distant observer could detect a thermal radiation, i.e. Hawking radiation, that spreads throughout the BH surroundings. In fact, this is the way that a BH ends. Today there are various methods to compute the T_H in the literature, (see for instance [2, 3, 8, 11, 19–21], and it is always high on the agenda to find alternative derivations. As for us, none of the methods is catchy as the Hawking's original study [13, 14] in which the Hawking temperature is revealed by the computation of the Bogoliubov coefficients between in and out states for a collapsing BH. Another alternative method which is conclusive as the method of the Bogoliubov coefficients is the computation of the reflection coefficient of an incident wave by the BH. Thereby, the thermal radiation spectrum and hence the T_H are obtained precisely [7]. The latter method especially serves the purpose when the wave equation on the associated geometry is solved exactly. This method is known as the "semi-classical radiation spectrum method", and we call it as *SCRSM* throughout the paper like in [16].

Recent studies [7, 16] have shown us that when the *SCRSM* is applied for the non-asymptotically flat (NAF) charged massive dilaton BHs without cosmological constant Λ in which they have only a single (event) horizon, one can reach to the T_H which is in general computed by the conventional method: $T_H = \frac{\kappa}{2\pi}$. The way followed so far for this purpose in the *SCRSM* was to take merely the high frequency limit of the obtained temperature $T(\omega)$ of the radiation spectrum in which ω represents the frequency of the scalar wave. The main aim of this paper is to explore the effect of the Λ on the radiation spectrum of the charged dilaton BHs. Namely, we intend to fill a gap in this line of thought. To this end, we will consider another NAF static and charged dilaton BH. Unlike the previous studies, the considered BH is a three dimensional (3D) solution to the Einstein-Maxwell-Dilaton theory with cosmological constant Λ (EMDΛ theory), which is found by [6]. As implied above, the inclusion of the Λ enables the BH to possess two horizons instead of one. Here, after a straightforward calculation of the *SCRSM*, it is seen that the high frequency limit of the computed $T(\omega)$ does not lead it directly to the T_H of the charged dilaton BHs as such in the former studies [7, 16]. For matching, it is not only enough to consider the low frequency regime but one needs to impose some special conditions on the wave as well. Another interesting point which comes in this regard is that the Λ remains as a decisive factor on the associated conditions about the wave. All these points together with supporting plots will be examined in the following sections.

The lay out of the paper is as follows: In Sect. II, we start with a brief review of the 3D charged dilaton BH solution in the EMDΛ theory. The next section is devoted to the analytical computation of the $T(\omega)$ via the *SCRSM* for the associated BHs. Some remarks about how the $T(\omega)$ goes to the T_H of the charged dilaton BH are made, and these remarks are illustrated by the various plots. We draw our conclusions in Sect. IV.

II. CHARGED DILATON BLACK HOLES IN 3D

In this section we will review the geometrical and thermodynamical properties of the 3D charged dilaton BHs. The associated BHs were found long ago by [6]. These BHs are exact solutions to the equations of motion of the EMDΛ action which is conformably related to the low-energy string action. This action is defined by

$$I = \int d^3x\sqrt{-g}\left(\Re + 2e^{b\phi}\Lambda - \frac{B}{2}\left(\nabla\phi\right)^2 \right.$$
$$\left. -e^{-4a\phi}F_{\mu\nu}F^{\mu\nu}\right), \tag{1}$$

where $F_{\mu\nu}$ stands for the Maxwell field, the constants a, b, B are arbitrary constants, ϕ is the dilaton field, $\Re$ is the Ricci scalar in 3D and Λ, as already mentioned in the previous section, represents the cosmological constant; $\Lambda > 0$ stands for the anti-de Sitter (AdS) spacetime, and $\Lambda < 0$ represents the de Sitter (dS) spacetime. The corresponding static circularly symmetric solution [6] to the above action is given by

$$ds^2 = -f(r)dt^2 + 4\frac{r^{\frac{4}{N}-2}}{N^2\gamma^{\frac{4}{N}}}f(r)^{-1}dr^2 + r^2d\theta^2, \tag{2}$$

where the metric function is

$$f(r) = -Ar^{\frac{2}{N}-1} + \frac{8\Lambda r^2}{(3N-2)N} + \frac{8Q^2}{(2-N)N}, \tag{3}$$

One can readily see that the spacetime (2) possesses a NAF geometry, therefore it does not behave asymptotically as dS or AdS [6]. This is due to the presence of the nontrivial dilaton field ϕ. The corresponding dilaton field ϕ and the non-zero maxwell tensor F_{tr} are

$$\phi = \frac{2k}{N} \ln\left[\frac{r}{\beta(\gamma)}\right],$$
$$F_{tr} = \frac{2}{N}\gamma^{-\frac{2}{N}} r^{\frac{2}{N}-2} Q e^{4a\phi}, \tag{4}$$

where Q is related with electric charge, γ and whence β are constants, and k is another constant which is governed by

$$k = \pm\sqrt{\frac{N}{B}(1-\frac{N}{2})}, \qquad 4ak = bk = N-2,$$
$$\therefore 4a = b, \tag{5}$$

As it was stated in the paper [6], when $(\Lambda, B) > 0$ and $2 > N > \frac{2}{3}$ the solutions (2) and (3) represent BHs. In this case, the constant A of the metric function (3) is related with the quasilocal mass M [5, 15] of the BH. By employing the Brown-York formalism [5], one can find the following relationship

$$A = -\frac{2M}{N}. \tag{6}$$

From the above expression, one can infer that for $M > 0$, A should take negative values. Similar to the work [17], throughout this paper we restrict attention to the BHs described by $N = \gamma = \beta = a = \frac{b}{4} = \frac{B}{8} = -4k = 1$. These choices modify the metric function, the dilaton field and the Maxwell field to

$$f(r) = -2Mr + 8\Lambda r^2 + 8Q^2,$$

$$\phi = -\frac{1}{2}\ln(r), \tag{7}$$

$$F_{tr} = \frac{2Q}{r^2}.$$

When $M \geq 8Q\sqrt{\Lambda}$, the spacetime (2) represents a BH. For $M > 8Q\sqrt{\Lambda}$, two horizons of the BH are found in the following locations

$$r_+ = M\frac{1+Y}{8\Lambda}, \qquad r_- = M\frac{1-Y}{8\Lambda}, \tag{8}$$

where

$$Y = \sqrt{1 - \frac{64Q^2\Lambda}{M^2}} \qquad (0 < Y < 1), \tag{9}$$

In (8), r_+ and r_- denote outer and inner horizons of the BH, respectively. Besides, there is a timelike singularity at $r = 0$. These charged BHs possess a Hawking temperature (T_{Hch}) that it can be computed from the definition given by [22]

$$\begin{aligned} T_H \equiv T_{Hch} &= \frac{\kappa}{2\pi} = \frac{|g'_{tt}|}{4\pi} \sqrt{-g^{tt}g^{rr}}\Big|_{r=r_+}, \\ &= \frac{M}{4\pi r_+}Y, \end{aligned} \tag{10}$$

The prime symbol in the foregoing equation, and in the following sections, denotes the derivative with respect to the variable "r". For the extreme BHs with $M = 8Q\sqrt{\Lambda}$, which is equivalent to $Y = 0$, one can immediately verify that T_{Hch} vanishes. Furthermore, for the uncharged ($Q = 0$ or $Y = 1$) BHs the Hawking temperature (T_{Huch}) becomes

$$T_H \equiv T_{Huch} = \frac{\Lambda}{\pi}. \tag{11}$$

III. CALCULATION OF $T(\omega)$ VIA THE *SCRSM*

In this section, by using the *SCRSM* [16], we will make more precise calculation of the temperature $T(\omega)$ of the radiation for the 3D charged dilaton BHs which are introduced in the previous section. After obtaining the $T(\omega)$, its T_{Huch} and T_{Hch} limits will be discussed in detail. For this purpose, we will first develop the wave equation for a massless scalar field in the background of the charged dilaton BH. Although the calculations made here share partial similarities with the studies [9, 10], for the sake of completeness we would like to describe the details.

The general equation for a massless scalar field in a curved spacetime is written as

$$\Box\Phi = 0, \tag{12}$$

where the d'Alembertian operator $\Box$ is given by

$$\Box = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu), \tag{13}$$

The scalar wave function Φ of (12) can be separated into a radial equation by letting

$$\Phi = P(r)e^{-i\omega t}e^{\pm m\theta}, \tag{14}$$

where m is a complex integration constant. Meanwhile, before proceeding to the derivation of the radial equation, in order to facilitate the computations, it is better to rewrite the metric function $f(r)$ (7) as

$$f(r) = 8\Lambda(r - r_+)(r - r_-), \tag{15}$$

After inserting the ansatz (14) into the wave equation (12), the radial equation becomes

$$f(r)P(r)'' + f'(r)P(r)' + 4\left(\frac{r^2\omega^2}{f(r)} + m^2\right)P(r) = 0, \tag{16}$$

The above equation can be solved in terms of hypergeometric functions. Here, we give the final result as

$$\begin{aligned} P(r) &= C_1(r - r_+)^{i\tilde{\omega}r_+}(r - r_-)^{-i\tilde{\omega}r_-}F\left[\tilde{a}, \tilde{b}; \tilde{c}; y\right] \\ &+ C_2(r - r_+)^{-i\tilde{\omega}r_+}(r - r_-)^{-i\tilde{\omega}r_-} \\ &\times F\left[\tilde{a} - \tilde{c} + 1, \tilde{b} - \tilde{c} + 1; 2 - \tilde{c}; y\right]. \end{aligned} \tag{17}$$

where $y = \frac{r_+ - r}{r_+ - r_-}$. The parameters of the hypergeometric functions are

$$\begin{aligned} \tilde{a} &= \frac{1}{2} + i(\frac{\omega}{4\Lambda} + \sigma), \\ \tilde{b} &= \frac{1}{2} + i(\frac{\omega}{4\Lambda} - \sigma), \\ \tilde{c} &= 1 + 2i\tilde{\omega}r_+, \end{aligned} \tag{18}$$

where

$$\sigma = \frac{1}{4\Lambda}\sqrt{\omega^2-\rho}, \quad \rho = 4\Lambda\left(\Lambda - 2m^2\right), \quad \tilde{\omega} = \omega\lambda,$$
$$\lambda = \frac{1}{4\Lambda(r_+ - r_-)}. \tag{19}$$

and σ is assumed to have positive real values. Next, setting

$$r - r_+ = \exp(\frac{x}{\lambda r_+}), \tag{20}$$

one gets the behavior of the partial wave near the outer horizon ($r \to r_+$) as

$$\Phi \simeq C_1 e^{i\omega(x-t)} + C_2 e^{-i\omega(x-t)}. \tag{21}$$

We assign the constants C_1 and C_2 as the amplitudes of the near-horizon outgoing and ingoing waves, respectively.

To obtain the asymptotic solution ($r \to \infty$) of (17), one can perform a transformation of the hypergeometric functions of any argument (let us say z) to the hypergeometric functions of its inverse argument ($1/z$) which is given as follows [1]

$$\begin{aligned} F(\bar{a},\bar{b};\bar{c};z) = {} & \frac{\Gamma(\bar{c})\Gamma(\bar{b}-\bar{a})}{\Gamma(\bar{b})\Gamma(\bar{c}-\bar{a})}(-z)^{-\bar{a}} \\ & \times F(\bar{a},\bar{a}+1-\bar{c};\bar{a}+1-\bar{b};1/z) \\ & + \frac{\Gamma(\bar{c})\Gamma(\bar{a}-\bar{b})}{\Gamma(\bar{a})\Gamma(\bar{c}-\bar{b})}(-z)^{-\bar{b}} \\ & \times F(\bar{b},\bar{b}+1-\bar{c};\bar{b}+1-\bar{a};1/z). \end{aligned} \tag{22}$$

Applying this transformation to (17), one can find the solution to the wave equation in the asymptotic region as follows

$$\begin{aligned} \Phi \simeq {} & (r-r_-)^{-i\tilde{\omega}r_-}(r-r_+)^{-\frac{1}{2}} \\ & \times \left\{ D_1 \exp i\left[\frac{x}{\lambda r_+}(\sigma + \tilde{\omega}r_-) - \omega t\right] \right. \\ & \left. + D_2 \exp i\left[\frac{x}{\lambda r_+}(-\sigma + \tilde{\omega}r_-) - \omega t\right] \right\}. \end{aligned} \tag{23}$$

On the other hand, since we consider the case of $r \to \infty$, the overall-factor term behaves as

$$(r - r_-)^{-i\tilde{\omega}r_-} \cong \exp i(-\frac{x\omega r_-}{r_+}), \tag{24}$$

Therefore, the wave near the infinity (23) reduces to

$$\Psi \simeq (r - r_+)^{-\frac{1}{2}} \left[D_1 \exp i \left(\frac{x}{\lambda r_+}\sigma - \omega t \right) \right.$$
$$\left. + D_2 \exp i \left(-\frac{x}{\lambda r_+}\sigma - \omega t \right) \right], \tag{25}$$

where D_1 and D_2 correspond to the amplitudes of the asymptotic outgoing and ingoing waves, respectively. One can derive the relations between D_1, D_2 and C_1, C_2 as the following.

$$D_1 = C_1 \frac{\Gamma(\tilde{c})\Gamma(\tilde{a} - \tilde{b})}{\Gamma(\tilde{a})\Gamma(\tilde{c} - \tilde{b})} + C_2 \frac{\Gamma(2 - \tilde{c})\Gamma(\tilde{a} - \tilde{b})}{\Gamma(\tilde{a} - \tilde{c} + 1)\Gamma(1 - \tilde{b})}, \tag{26}$$

$$D_2 = C_1 \frac{\Gamma(\tilde{c})\Gamma(\tilde{b} - \tilde{a})}{\Gamma(\tilde{b})\Gamma(\tilde{c} - \tilde{a})} + C_2 \frac{\Gamma(2 - \tilde{c})\Gamma(\tilde{b} - \tilde{a})}{\Gamma(\tilde{b} - \tilde{c} + 1)\Gamma(1 - \tilde{a})}. \tag{27}$$

According to the *SCRSM*, Hawking radiation is considered as the inverse process of scattering by the BH in which the outgoing mode at the spatial infinity should be absent [7]. In short, $D_1 = 0$ in (26). This yields the coefficient of reflection R of the wave by the BH as

$$R = \frac{|C_1|^2}{|C_2|^2} = \frac{\left|\Gamma(\tilde{c} - \tilde{b})\right|^2 |\Gamma(\tilde{a})|^2}{\left|\Gamma(1 - \tilde{b})\right|^2 |\Gamma(\tilde{a} - \tilde{c} + 1)|^2}, \tag{28}$$

which corresponds to

$$R = \frac{\cosh\left[\pi\left(\sigma - \frac{\omega}{4\Lambda Y}\right)\right]\cosh\left[\pi\left(\sigma - \frac{\omega}{4\Lambda}\right)\right]}{\cosh\left[\pi\left(\sigma + \frac{\omega}{4\Lambda Y}\right)\right]\cosh\left[\pi(\sigma + \frac{\omega}{4\Lambda})\right]}, \tag{29}$$

Since the resulting radiation spectrum is

$$N = \left(e^{\frac{\omega}{T}} - 1\right)^{-1},$$
$$= \frac{R}{1 - R} \to \ T = T(\omega) = \frac{\omega}{\ln(\frac{1}{R})}, \tag{30}$$

one can easily read the more precise value of the temperature as

$$T(\omega)=\omega/\ln\left\{\frac{\cosh\left[\pi\left(\sigma+\frac{\omega}{4\Lambda Y}\right)\right]}{\cosh\left[\pi\left(\sigma-\frac{\omega}{4\Lambda Y}\right)\right]}\right.$$
$$\left.\times\frac{\cosh\left[\pi(\sigma+\frac{\omega}{4\Lambda})\right]}{\cosh\left[\pi\left(\sigma-\frac{\omega}{4\Lambda}\right)\right]}\right\}. \quad (31)$$

After analyzing the above result, it is seen that there are two prominent cases depending on which frequency regime (high/low) is taken into consideration.

Case I: In the high frequency regime with the condition of $\Lambda-2m^2\geq 0$ (this case is also possible with a shift $m\rightarrow im$ in the ansatz (14) of the wave function), the term $\frac{\omega}{4\Lambda Y}$ always predominates the physical parameter σ i.e.,($\frac{\omega}{4\Lambda Y}>\sigma$), and the temperature (31) reduces to

$$\begin{aligned}T_I &= \lim_{(\frac{\omega}{4\Lambda Y}>\sigma)\rightarrow\infty} T(\omega),\\ &\simeq \frac{\omega}{\ln\left\{\frac{\exp\left[\pi\frac{\omega}{4\Lambda}\left(\frac{1}{Y}+1\right)\right]\exp\left(\pi\frac{\omega}{2\Lambda}\right)}{\exp\left[\pi\frac{\omega}{4\Lambda}\left(\frac{1}{Y}-1\right)\right]}\right\}},\\ &= \frac{\omega}{\ln\left[\exp(\frac{\pi\omega}{\Lambda})\right]},\\ &= \frac{\Lambda}{\pi},\end{aligned} \quad (32)$$

which smears out the $\omega-$dependence, and the spectrum results in a isothermal radiation. In addition to this, one can immediately observe that the above result is nothing but the T_{Huch} (11), and the temperature of the radiation becomes independent from the horizons of the charged dilaton BHs at the high frequencies. In other words, in the high frequency regime the charge completely loses its effectiveness on the temperature of the radiation. Another interesting aspect of this case is that there is no possibility to retrieve the T_{Hch} (10) contary to the former studies [7, 16] in which the high frequency regime was considered as the master regime in obtaining the conventional Hawking temperature ($\frac{\kappa}{2\pi}$) of a BH.

Case II: In the low frequency regime with the conditions of $\Lambda-2m^2<0$ (this case can not be obtained by shifting $m\rightarrow im$ in the ansatz (14) of the wave function) and $\sigma>\frac{\omega}{4\Lambda Y}$, the ratios of the hyperbolic cosines behave as

$$\lim_{\sigma>\frac{\omega}{4\Lambda Y}}\frac{\cosh\left[\pi\left(\sigma+\frac{\omega}{4\Lambda Y}\right)\right]}{\cosh\left[\pi\left(\sigma-\frac{\omega}{4\Lambda Y}\right)\right]}\simeq\frac{\exp\left[\pi\frac{\omega}{4\Lambda}\left(1+\frac{1}{Y}\right)\right]}{\exp\left[\pi\frac{\omega}{4\Lambda}\left(1-\frac{1}{Y}\right)\right]}, \quad (33)$$

and

$$\lim_{\sigma > \frac{\omega}{4\Lambda Y}} \frac{\cosh\left[\pi\left(\sigma + \frac{\omega}{4\Lambda}\right)\right]}{\cosh\left[\pi\left(\sigma - \frac{\omega}{4\Lambda}\right)\right]} \simeq \exp\left(\pi\frac{\omega}{2\Lambda}\right), \tag{34}$$

Thus for this case, the $T(\omega)$ given by (31) becomes

$$\begin{aligned} T_{II} &= \lim_{\sigma > \frac{\omega}{4\Lambda Y}} T(\omega), \\ &\simeq \frac{\omega}{\ln\left\{\frac{\exp\left[\pi\frac{\omega}{4\Lambda}\left(1+\frac{1}{Y}\right)\right]\exp\left(\pi\frac{\omega}{2\Lambda}\right)}{\exp\left[\pi\frac{\omega}{4\Lambda}\left(1-\frac{1}{Y}\right)\right]}\right\}}, \\ &= \frac{2\Lambda}{\pi}\left(\frac{Y}{1+Y}\right), \\ &= \frac{\Lambda}{\pi}\left(1-\frac{r_-}{r_+}\right), \\ &= \frac{M}{4\pi r_+}Y. \end{aligned} \tag{35}$$

which exactly matches with the result of the conventional Hawking temperature of the charged dilaton BHs i.e., T_{Hch} (10). The case II has also the limit of T_{Huch} (11) when $Y = 1$ ($Q = 0$).

Furthermore, we would like to represent the most interesting figures about the spectrum temperature $T(\omega)$ (31). For this purpose, we plot $T(\omega)$ versus wave frequency ω of the 3D charged dilaton BHs with $r_-, r_+ \neq 0$ for both cases. Fig. 1 and Fig. 2 illustrate the thermal behaviors of the charged dilaton BHs in the cases (I) and (II), respectively. As it can be seen from both figures, at the high frequencies the temperature $T(\omega)$ exhibits similar behaviors such that it approaches to T_{Huch} (11) while $\omega \to \infty$. However, in the low frequencies the picture changes dramatically. In this regard, if we first consider the Fig. 1, it is obvious that as $\omega \to 0$, the $T(\omega)$ tends to diverge. Besides, Fig. 1 presents that there is no chance to have T_{Hch} (10) from the $T(\omega)$ if $\Lambda - 2m^2 \geq 0$. The minimum value of the $T(\omega)$ that it can drop, which is also equal to T_{Huch}, is higher than the value of the T_{Hch}. However, when we look at the Fig. 2, for a specific range of the low frequencies in which the conditions for the case (II) hold it is seen that $T(\omega)$ coincides with T_{Hch}. To our knowledge, the latter result that reveals a relationship between the Hawking temperature, the cosmological constant and the low frequencies (or long wavelengths) of the scalar waves is completely new for the literature.

IV. SUMMARY AND CONCLUSIONS

In this paper, we have explored the frequency dependent temperature $T(\omega)$ of the radiation spectrum for the 3D charged dilaton BHs in the EMDΛ theory. For this purpose, we have made use of the previously obtained method, *SCRSM*. Its related studies [7, 16] showed us that whenever the wave equation admits an exact solution, this method is powerful enough to obtain the Hawking temperature of a considered BH. By using this fact, we have first obtained the massless scalar wave equation in that geometry. After finding the exact solution of the radial part in terms of the hypergeometric functions and using their one of the linear transformation expressions, we have obtained the $T(\omega)$. From here on, we have analyzed in detail the behavior of $T(\omega)$ with various parameters. It is seen that two cases come to the forefront during the analysis. The associated cases, which are called as "case I" and "case II", mainly depend on the which frequency regime (high/low) is considered, the sign of the term $\Lambda - 2m^2$ and the domination of the terms σ and $\frac{\omega}{4\Lambda Y}$ against to each other. It is deduced from the calculations and their associated figures that in the case I ($\Lambda - 2m^2 \geq 0$ and $\frac{\omega}{4\Lambda Y} > \sigma$ within the high frequency regime) $T(\omega)$ does not yield the standard Hawking temperature of the 3D charged dilaton BHs (T_{Hch}), it has only the T_{Huch} limit. On the other hand, the case II in which the frequency regime is low and $\Lambda - 2m^2 < 0$ with $\frac{\omega}{4\Lambda Y} < \sigma$ makes the *SCRSM* agree with the T_{Hch}. This result, by contrast with the previous studies [7, 16], represents that the presence of the Λ allows us to obtain the desired Hawking temperature T_{Hch} in the low frequency (or long wavelength) regime. The latter remark might also play a crucial role in the detection of the charged dilaton black holes with the cosmological constant in the future.

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