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Hawking Radiation of Linear Dilaton Black Holes in Various Theories

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Abstract Using the Damour-Ruffini-Sannan, the Parikh-Wilczek and the thin film brick-wall models, we investigate the Hawking radiation of uncharged massive particles from 4-dimensional linear dilaton black holes, which are the solutions to Einstein-Maxwell-Dilaton, Einstein-Yang-Mills-Dilaton and Einstein-Yang-Mills-Born-Infeld-Dilaton theories. Our results show that the tunneling rate is related to the change of Bekenstein-Hawking entropy. Contrary to the many studies in the literature, here the emission spectrum is precisely thermal. This implies that the derived emission spectrum is not consistent with the unitarity of the quantum theory, which would possibly lead to the information loss.

Key words Entropy, Linear dilaton black holes, Tunneling effect, Thin film brick-wall model

1 Introduction

Obeying the laws of black hole mechanics [1], Hawking [2,3] proved that a stationary black hole can emit particles from its event horizon with a temperature proportional to the surface gravity. According to this idea, the vacuum fluctuations near the horizon would produce a virtual particle pair, similar to electron-positron pair creation in a constant electric field. When a virtual particle pair is created just inside or outside the horizon, the sign of its energy changes as it crosses the horizon. So after one member of the

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pair has tunneled to the opposite side, the pair can materialize with zero total energy. This discovery also announced the relation between the triple subjects – the quantum mechanics, thermodynamics and the gravitation. After this pioneering study of Hawking, many methods have been proposed to calculate the Hawking radiation for the last three decades.

One of the commonly used methods is known as Damour-Ruffini-Sannan (DRS) [4,5] method. This method is applicable to any Hawking temperature problem in which the asymptotic behaviors of the wave equation near the event horizon are known.

In 2000, Parikh and Wilczek [6] proposed a method based on null geodesics in order to clarify more the Hawking radiation via tunneling across the event horizon. Namely, they treated the Hawking radiation as a tunneling process, and used the WKB approximation to determine the correction spectrum for the black hole's Hawking radiation. In their study, it is supposed that the barrier depends on the tunneling particle itself. The crucial point of this method is not to violate the energy conservation during the process of particle emission and to pass to an appropriate coordinate system at horizon. In general, the tunneling process is not precisely a thermal effect and it explains the modification of the black hole radiation spectrum in which it leads to the unitarity in the quantum theory [7,8,9].

Another possible method to study the statistical origin of the black hole entropy is the brick-wall model initially proposed by t'Hooft [10]. The brick-wall model identifies the black hole entropy by the entropy of a thermal gas of quantum field excitations outside the event horizon. Since then, this method has been satisfactorily applied to many black hole geometries (see for instance [11], and the references therein). Although t' Hooft made significant contribution to clarify the understanding and calculating the entopy of the black holes, there were some drawbacks in his model. Those drawbacks are overcome by the improved form of the original brick-wall model, which is called as thin film brick-wall model [12]. The thin film brick-wall model gives us acceptable and net physical meaning of the entropy calculation. In summary, since the entropy calculated by the thin film brick-wall model is just from a small region (thin film) near the horizon, this improved version of the brick-wall model represents explicitly the correlation between the horizon and the entropy. In this study, we obtain the ultraviolet cut-off distance as 90β, where β is the Boltzmann factor.

Hawking described the black hole radiation as tunneling triggered by the vacuum fluctuations near the horizon. His discovery, which treats the black hole radiation as being pure thermal gave also rise to a new paradox in the black hole physics – the information loss paradox. Although, Parikh and Wilczek's tunneling process [6] is a way to overcome the information loss paradox in the Hawking radiation, the information might not be conserved in some black hole geometries. For instance, if only the tunneling process of the outer horizon of the Reissner-Nordström black hole is considered [13,14], it can be shown that the information loss is possible. The similar violation in the conservation of information happens in the 4-dimensional linear dilaton black holes (LDBHs) in various theories, and we will explain its reason by using the differences in entropies of the black holes before and after the emission.

The paper is organized as follows: In section 2, a brief overview of the 4-dimensional LDBHs in Einstein-Maxwell-Dilaton (EMD), Einstein-Yang-Mills-Dilaton (EYMD) and Einstein-Yang-Mills-Born-Infeld-Dilaton (EYMBID) theories, which they have been recently employed in [15] for calculating the Hawking radiation via the method of semi-classical radiation spectrum is given. Next, we apply the DRS method to find the temperature of the LDBHs and the tunneling rate of the chargeless particles crossing the event horizon. Section 3 is devoted to the calculation of the entropy of the horizon by using all those methods mentioned above. As it is expected, they all conclude with the same result. Finally, we draw our conclusions and discussions.

Throughout the paper, the units $G = c = \hbar = k_B = 1$ are used.

2 LDBHs, Calculation of Their Temperature and Tunneling Rate

The line-element of N-dimensional ($N \geq 4$) LDBHs, which are static spherically symmetric solutions in various theories (EMD, EYMD and EYMBID) have been recently summarized by [15]. However, throughout this paper we restrict ourselves to the 4-dimensional LDBHs and follow the notations of [15].

Consider a general class of static, spherically symmetric spacetime for the LDBHs as

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + A^2 r d\Omega^2, \tag{1}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. Here, the metric function f is given by [15]

$$f = \Sigma r (1 - \frac{r_+}{r}), \tag{2}$$

where r_+ is the radius of the event horizon. The coefficients Σ and A in the metric (1) take different values according to the concerned theory.

Since the present form of the metric represent asymptotically non-flat solutions, one should consider the quasi-local mass definition M of the metric (1). In [15], the relationship between the horizon r_+ and the mass M is explicitly given as

$$r_+ = \frac{4M}{\Sigma A^2}, \tag{3}$$

In the EMD theory [15,16,17], the coefficients Σ and A are found as

$$\Sigma \rightarrow \Sigma_{EMD} = \frac{1}{\gamma^2} \quad \text{and} \quad A \rightarrow A_{EMD} = \gamma, \tag{4}$$

where γ is a constant related to the electric charge of the black hole. Meanwhile, one can match the metric (1) to the LDBH's metric of Clément *et.al.* [17] by setting $\gamma \equiv r_0$. Next, if one considers the EYMD and EYMBID theories [18,19], the coefficients in the metric (1) become

$$\Sigma \rightarrow \Sigma_{EYMD} = \frac{1}{2Q^2} \quad \text{and} \quad A \rightarrow A_{EYMD} = \sqrt{2}Q, \tag{5}$$

and

$$\Sigma \rightarrow \Sigma_{EYMBID} = \frac{1}{Q_C^2}\left[1-\sqrt{1-\frac{Q_C^2}{Q^2}}\right] \quad \text{and} \quad A \rightarrow A_{EYMBID} = \sqrt{2}Q\left(1-\frac{Q_C^2}{Q^2}\right)^{\frac{1}{4}}, \tag{6}$$

where Q and Q_C are YM charge and the critical value of YM charge, respectively. The existence of the metric (1) in EYMBID theory depends strictly on the condition [19]

$$Q^2 > Q_C^2 = \frac{1}{4\tilde{\beta}^2}, \tag{7}$$

where $\tilde{\beta}$ is the Born-Infeld parameter. Meanwhile, it is not necessary to say that values of Σ in equations (4), (5) and (6) are always positive.

By using the definition of the surface gravity [20], we get

$$\kappa = \lim_{r \rightarrow r_+} \frac{f'(r)}{2} = \frac{\Sigma}{2}. \tag{8}$$

Since the surface gravity (8) is positive, one can deduce that it is directed towards the singularity. As a consequence, it is attractive and the matter can only fall into the black hole. This horizon is a future horizon to an observer, who is located outside of it.

In curved spacetime, a massive test scalar field Φ with mass μ obeys the covariant Klein-Gordon (KG) equation, which is given by

$$\frac{1}{\sqrt{-\det g}}\partial_\mu\left(\sqrt{-\det g}g^{\mu\nu}\partial_\nu\Phi\right) - \mu^2\Phi = 0, \tag{9}$$

The massive scalar wave equation Φ in metric (1) can be separated as $\Phi = Y(\theta,\varphi)\psi(t,r)$ in which the radical KG equation (9) satisfies the following equation:

$$\frac{\partial^2\psi}{\partial t^2} + f(\frac{f}{r}+\Sigma)\frac{\partial\psi}{\partial r} + f^2\frac{\partial^2\psi}{\partial r^2} - f(\mu^2 - \frac{l(l+1)}{r})\psi = 0, \tag{10}$$

where l is the angular quantum number. In order to change equation (10) into a standard wave equation at the horizon, we introduce the tortoise coordinate transformation, which is obtained from

$$dr_* = \frac{dr}{f}, \tag{11}$$

After making the straightforward calculation, we find an appropriate r_* as

$$r_* = \frac{1}{2\kappa}\ln(r - r_+), \tag{12}$$

Thus, one can transform the radical equation (10) into the following form

$$\frac{\partial^2\psi}{\partial t^2} - \frac{f}{r}\frac{\partial\psi}{\partial r_*} - \Sigma\frac{\partial\psi}{\partial r_*} + \Sigma\frac{\partial\psi}{\partial r_*} - \frac{\partial^2\psi}{\partial r_*^2} + f[\mu^2 - \frac{l(l+1)}{r}]\psi = 0, \tag{13}$$

While $r \to r_+$ in which $f \to 0$, the transformed radical equation (13) can be reduced to the following standard form of the wave equation as

$$\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi}{\partial r_*^2} = 0, \tag{14}$$

This form of the wave equation reveals that there are propagating waves near the horizon. The solutions of equation (14), which give us the ingoing and outgoing waves at the black hole horizon surface r_+ are

$$\psi_{out} = \exp(-i\omega t + i\omega r_*), \tag{15}$$

$$\psi_{in} = \exp(-i\omega t - i\omega r_*), \tag{16}$$

When we introduce the ingoing Eddington-Finkelstein coordinate, $v = t + r_*$, the line-element (1) of the LDBHs becomes

$$ds^2 = -f dv^2 + 2dvdr + A^2 r d\Omega^2, \tag{17}$$

The present form of the metric does not attribute a singularity to the horizon, so that the ingoing wave equation behaves regularly at the horizon. This yields the solutions of ingoing and outgoing waves at the horizon r_+ as follows

$$\psi_{out} = e^{-i\omega v} e^{2i\omega r_*}, \tag{18}$$

$$\psi_{in} = e^{-i\omega v}, \tag{19}$$

Now, we consider only the outgoing waves. Namely,

$$\psi_{out}(r > r_+) = e^{-i\omega v}(r - r_+)^{\frac{i\omega}{\kappa}}, \tag{20}$$

which has a singularity at the horizon r_+. Therefore, equation (20) can only describe the outgoing particles outside the horizon and strictly cannot describe the particles, which are inside the horizon. In other words, the description of the particles' behavior inside horizon has to be made as well. To this end, the outgoing wave ψ_{out} should be analytically extended from outside to the interior of the black hole by the lower half complex r-plane

$$(r - r_+) \rightarrow |r - r_+|\, e^{-i\pi} = (r_+ - r)e^{-i\pi}, \tag{21}$$

We can derive the solution of outgoing wave inside the horizon as follows

$$\psi_{out}(r < r_+) = \psi_{out}^{'}\,(r < r_+)\, e^{\frac{\omega\pi}{\kappa}}, \tag{22}$$

where

$$\psi_{out}^{'}\,(r < r_+) = e^{-i\omega v}(r_+ - r)^{\frac{i\omega}{\kappa}}, \tag{23}$$

According to the Damour-Ruffini-Sannan (DRS) [4,5] method, it is possible to calculate the emission rate. The total outgoing wave function can be written in a uniform form

$$\psi = N_\omega[\Theta(r - r_+)\psi_{out}\,(r > r_+) + e^{\frac{\omega\pi}{\kappa}}\Theta(r_+ - r)\psi_{out}^{'}\,(r < r_+)], \tag{24}$$

where Θ is the Heaviside step function and N_ω represents the normalization factor. From the normalization condition

$$(\psi, \psi) = \pm 1, \tag{25}$$

we can obtain the resulting radiation spectrum of scalar particles

$$N_\omega^2 = \frac{\Gamma}{1 - \Gamma} = \frac{1}{e^{\frac{\omega}{T}} - 1}, \tag{26}$$

and read the temperature of the horizon as

$$T = \frac{\kappa}{2\pi}, \tag{27}$$

In equation (26) Γ symbolizes the emission or tunneling rate, which is found by the following ratio

$$\Gamma = \left|\frac{\psi_{out}\,(r > r_+)}{\psi_{out}\,(r < r_+)}\right|^2 = e^{\frac{-2\pi\omega}{\kappa}}. \tag{28}$$

One can remark for this section that the resulting temperature (27) obtained from the DRS method is in agreement with the statistical Hawking temperature [20] computed as usual by dividing the surface gravity by 2π.

3 Entropy of the Horizon

In this section, we shall use three different methods in order to show that they all lead to the same entropy result. We first employ the DRS method, which is worked in detail and obtained remarkable results in the previous section. The second method will be the Parikh-Wilczek method [6] describing the Hawking radiation as a tunneling process. Last method that is also going to be used in the calculation of the entropy is the thin film brick-wall model [12].

In the DRS method, the emission rate of outgoing particles is found as in equation (28). Accordingly, the probability of emission can be modified into [13,21] (and references therein)

$$\Gamma = e^{-2\pi \int_0^{\omega} \frac{d\omega'}{\kappa}} = e^{-\int_0^{\omega} \frac{d\omega'}{T}} = e^{\Delta S_{BH}}, \tag{29}$$

where ΔS_{BH} is the difference of Bekenstein-Hawking entropies of the LDBHs before and after the emission of the particle.

On the other hand, the novel study on the tunneling effect is designated by Parikh-Wilczek method [6], which proposes an approach for calculating the tunneling rate at which particles tunnel across the event horizon. They treated Hawking radiation as a tunneling process, and used the WKB method [9]. In classical limit, we can also find the tunneling rate by applying WKB approximation. This relates the tunneling amplitude to the imaginary part of the particle action at stationary phase and the Boltzmann factor for emission at the Hawking temperature.

In the WKB approximation, the imaginary part of the amplitude for outgoing positive energy particle which crosses the horizon outwards from initial radius of the horizon r_{in} to the final radius of the horizon r_{out} could be expressed by

$$\operatorname{Im} I = \operatorname{Im} \int_{r_{in}}^{r_{out}} p_r dr = \operatorname{Im} \int_{r_{in}}^{r_{out}} \int_0^{p_r} dp'_r dr, \tag{30}$$

By using the standard quantum mechanics, the tunneling rate Γ is given in the WKB approximation as [22,23],

$$\Gamma \sim \exp(-2 \operatorname{Im} I), \tag{31}$$

Here we can consider the particle with energy ω as a shell of energy and fix the total mass M (quasi-local mass) and allow the hole mass to fluctuate. Then the Hamilton's equation of motion can be used to write $dp_r = \frac{dH}{\dot{r}}$, and it can be noted that the horizon moves inwards from M to $M-\omega$ while a particle emits. Introducing $H = M - \omega$ and inserting the value of the $\dot{r} \equiv \frac{dr}{dv} = \frac{f}{2}$ obtained from the null geodesic equation into (30), we obtain

$$\operatorname{Im} \int_{r_{in}}^{r_{out}} \int_0^{p_r} dp'_r dr = \operatorname{Im} \int_{r_{in}}^{r_{out}} \int_M^{M-\omega} \frac{dr}{\dot{r}} dH = \operatorname{Im} \int_0^{\omega} \int_{r_{in}}^{r_{out}} \frac{2dr}{\Sigma(r-r_+)} \left(-d\omega'\right), \tag{32}$$

The r-integral can be done by deforming the contour. The deformation of the integral is based on an assumption that the contour semicircles the residue in a clockwise fashion. In this way, one can obtain

$$\operatorname{Im} I = 2\pi \int_0^{\omega} \frac{d\omega'}{\Sigma}, \tag{33}$$

So, the tunneling rate (31) is

$$\Gamma \sim \exp(-2 \operatorname{Im} I) = \exp\left(-4\pi \int_0^{\omega} \frac{d\omega'}{\Sigma}\right) = \exp\left(\Delta S_{BH}\right), \tag{34}$$

Our result (34) is consistent with the results of the other works [6, 24, 25, 26, 27].

Now, we come to the stage to apply the thin film brick-wall model [12], which was based on the brick wall model proposed firstly by t'Hooft [10]. According to this model, the considered field outside the horizon is assumed to be non-zero only in a thin film, which exists in a small region bordered by $r_+ + \varepsilon$ and $r_+ + \varepsilon + \delta$. Here, ε is the ultraviolet cut-off distance and δ is the thickness of the thin film. In summary, both ε and δ are positive infinitesimal parameters. This model treats the entropy as being associated with the field in the considered small region in which the local thermal equilibrium and the statistical laws are valid [28]. That is why one can work out the entropy of the horizon by using this model.

If one redefines the massive test scalar field Φ as being $\Phi = e^{-i\omega t}\psi(r)Y_{lm}(\theta, \phi)$ in the KG equation (9) and considers its radial part only, the wave vector is found with the help of WKB approximation as

$$k^2 = \frac{1}{\Sigma r(1 - \frac{r_+}{r})}\left[\frac{\omega^2}{\Sigma r(1 - \frac{r_+}{r})} - (\mu^2 + \frac{l(l+1)}{A^2 r})\right], \tag{35}$$

Using the quantum statistical mechanics, we calculate the free energy from

$$F = \frac{-1}{\pi} \int_0^{\infty} d\omega \int_r dr \int_l (2l+1) \frac{k}{e^{\beta\omega} - 1} dl, \tag{36}$$

While integrating equation (36) with respect to l, one should consider the upper limit of integration such that k^2 remains positive, and the lower limit becomes zero. Briefly, we get

$$F \cong \frac{-2A^2}{3\pi\Sigma^2} \int_0^{\infty} \frac{d\omega}{e^{\beta\omega} - 1} \int_r \frac{r}{(r - r_+)^2} [\omega^2 - \mu^2 \Sigma (r - r_+)]^{\frac{3}{2}} dr, \tag{37}$$

where β denotes the inverse of the temperature. In equation (37), the integration with respect to r is quite difficult. On the other hand, the thin film brick-wall model imposes us to take only the free energy of a thin layer near horizon of a black hole, and the integration with respect to r must be limited in the region $r_+ + \varepsilon \leq r \leq r_+ + \varepsilon + \delta$. The natural result of this choice sets the coefficient of μ^2 to zero, and whence the integration of equation (37) with respect to ω becomes very simple such that it can be easily found as $\pi^4/15\beta^4$. Finally, the equation (37) reduces to

$$F \cong \frac{-2\pi^3 A^2}{45\beta^4 \Sigma^2} \int_{r_+ +\varepsilon}^{r_+ +\varepsilon+\delta} \frac{r}{(r-r_+)^2} dr, \tag{38}$$

$$\cong \frac{-2\pi^3 A^2 r_+}{45\beta^4 \Sigma^2} \int_{r_+ +\varepsilon}^{r_+ +\varepsilon+\delta} \frac{dr}{(r-r_+)^2}, \tag{39}$$

$$F \cong \frac{-2\pi^3 A^2 r_+}{45\beta^4 \Sigma^2} \frac{\delta}{\varepsilon(\delta+\varepsilon)}, \tag{40}$$

and we can get the entropy

$$S_{BH} = \beta^2 \frac{\partial F}{\partial \beta} = [\frac{8\pi^3 A^2 r_+}{45\beta^3 \Sigma^2}] \frac{\delta}{\varepsilon(\delta+\varepsilon)}, \tag{41}$$

Since the beta is the inverse of the temperature

$$\beta = \frac{1}{T} = \frac{4\pi}{\Sigma}, \tag{42}$$

and if we select an appropriate cut-off distance ε and thickness of thin film δ to satisfy

$$\frac{\delta}{\varepsilon(\delta+\varepsilon)} = 90\beta, \tag{43}$$

the total entropy of the horizon becomes

$$S_{BH} = \frac{1}{4} A_h, \tag{44}$$

where A_h is the area of the the black hole horizon, i.e. $A_h = 4\pi A^2 r_+$. The derivative of the entropy (44) with respect to M is

$$\frac{\partial S_{BH}}{\partial M} = \pi A^2 \frac{\partial r_+}{\partial M} = \frac{4\pi}{\Sigma}, \tag{45}$$

Getting the integral of M, equation (45) becomes to

$$\Delta S_{BH} = \int_M^{M-\omega} \frac{\partial S_{BH}}{\partial M'} dM' = 4\pi \int_M^{M-\omega} \frac{dM'}{\Sigma}, \tag{46}$$

After substituting $M' = M - \omega'$ into the above equation, we obtain

$$\Delta S_{BH} = -\int_0^{\omega} \frac{\partial S_{BH}}{\partial M'} d\omega' = -4\pi \int_0^{\omega} \frac{d\omega'}{\Sigma}, \tag{47}$$

One can easily see that equation (47) is nothing but the results obtained both from Parikh-Wilczek method (34) and the DRS method (29).

On the other hand, for the LDBHs the change of the entropy before and after the radiation is

$$\varDelta S_{BH} = S(M-\omega) - S(M) = -\frac{2\pi\omega}{\kappa}. \tag{48}$$

Since equation (48) contains only ω, we deduce that the spectrum is precisely thermal. In other words, the thermal spectrum does not suggest the underlying unitary theory, and whence we can understand that the conservation of information is violated.

4 Discussion and Conclusion

In this paper, we have effectively utilized three different methods (the DRS model, the Parikh-Wilczek model and the thin film brick wall model) to investigate the Hawking radiation for massive 4-dimensional LDBHs in the EMD, EYMD and EYMBID theories. By considering the DRS method, the tunneling probability for an outgoing positive energy particle or simply the tunneling rate is neatly found. Later on, it is shown that the tunneling rate found from the DRS method can be expressed in terms of the difference of Bekenstein-Hawking entropies $\varDelta S_{BH}$ of the black holes. Beside this, the other two methods i.e. the Parikh-Wilczek method and the thin film brick-wall model attribute also the same $\varDelta S_{BH}$ result. In the thin film brick-wall model, the cut-off factor is found to be 90β, which is exactly same as in the calculation of the entropy for the Schwarzschild black hole [29].

On the other hand, the obtained $\varDelta S_{BH}$ result shows us that the emission spectrum is nothing but a pure thermal spectrum. This result is not consistent with the unitarity principle of quantum mechanics. It also implies the violation of the conservation of information in the LDBHs.

Finally, further application of the Hawking radiation of the charged massive particles via different methods to the case of LDBHs in higher dimensions [15] may reveal more information compared to the present case. This will be our next problem in the near future.

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