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FORECASTING SOUTH AFRICAN MACROECONOMIC VARIABLES WITH A MARKOV-SWITCHING SMALL OPEN-ECONOMY DYNAMIC STOCHASTIC GENERAL EQUILIBRIUM MODEL

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Abstract

The aim of this paper is to investigate structural changes in the South African economy using an estimated small open-economy dynamic stochastic general equilibrium (DSGE) model. The structure of the model follows recent work in this area and incorporates the expectations of agents and a number of shocks that are assumed to affect the economy at various points in time. In addition, the dynamic linkages between the respective variables in the model may be explained in terms of the microfoundations that characterise the behaviour of firms, households and the central bank. After estimating the model, we allow for the parameters in a number of different structural equations to change periodically over time. Different versions of the model are assessed using various statistical criteria to identify the model that is able to explain the changing dynamics in the South African economy. The results suggest that the central bank has responded in a consistent manner over the sample period; however, there are periods of time where it does not focus too greatly on output pressure. This impacts on some of the impulse response functions where we note that a monetary policy shock has a slightly larger effect on inflation, while the risk-premium shock has a larger effect on output, inflation and interest rates.

JEL Classifications: E32, E52, F41.

Keywords: Monetary policy, inflation targeting, Markov-switching, dynamic stochastic general equilibrium model, Bayesian estimation, small open-economy.

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1 Introduction

South African macroeconomic data incorporates a number of structural breaks due in part to political transitions, changes in policy frameworks and economic crises. This would suggest that an appropriate modelling framework for macroeconomic phenomena within this country should allow for some form of regime-switching, which could also be used to consider changes to the reaction function of a central bank. The incorporation of a stochastic Markov process within a macroeconometric model for South Africa would present a particularly attractive proposition as it would allow for the data to identify the changes in the respective regimes.

Early contributions to the literature that consider the use of Markov-switching in a reduced-form vector autoregressive (VAR) modelling framework for multiple variables include Sims and Zha (2004), Sims and Zha (2006), and Sims *et al.* (2008). These papers consider both whether-or-not and how monetary policy has changed in the United States.[1] This work also suggests that regime switching models should be used for describing monetary policy over relatively long periods of time, particularly in cases where the framework has changed (from one that considers monetary aggregates to one that is primarily concerned with prices). In addition, they note that policy changes are not monotonic and should be treated as probabilistic outcomes that recognise the degree of uncertainty about their nature and timing.

The use of regime-switching models that allow for structural changes in South African data is considered in quite a large number of recent studies (Naraidoo and Gupta (2010); Naraidoo and Raputsoane (2010, 2011, 2015); Kasai and Naraidoo (2011, 2012, 2013); Naraidoo and Paya (2012)). These papers model various kinds of asymmetric behaviour in the preferences of the central banker (South African Reserve Bank, SARB), leading to nonlinear reduced-form Taylor-type rules. Nonlinearities are not only considered in the output-gap and inflation, but also in a financial conditions index to capture changes in the financial state of the South African economy. In general, all these studies suggest that the fit of the regime-switching models is superior to that of linear models in both in-sample and out-of-sample evaluations. In addition, these papers also suggest that the SARB does respond to financial conditions, especially during episodes of crises.

While these findings are of significant interest, the use of reduced-form models for monetary policy investigations have been criticized by Lucas (1976) for not incorporating forward-looking behaviour, while Galí (2008) and Christiano *et al.* (2010) note that reduced-form models have been largely unable to describe some of the essential features of monetary policy. This motivated for the use of theoretical models, which were pioneered by the seminal contribution of Kydland and Prescott (1982), and there continued use has also been supported by Smets and Wouters (2007), who suggest that modern dynamic stochastic general equilibrium (DSGE) models are able to provide impressive forecasting results.

The use of Markov-switching behaviour in a DSGE model is described in Liu *et al.* (2009), Farmer *et al.* (2009), Farmer *et al.* (2011), Liu and Mumtaz (2011), Liu *et al.* (2011) and Alstadheim *et al.* (2013). These models allow for the analysis of samples with multiple regime changes, where they are largely focused on the way in which the central bank reacts to various factors that influence the policy rule. In addition, Alstadheim *et al.* (2013) consider how changes in the volatility of the respective shocks may influence the behaviour of the central bank. Most of these studies suggest that the assumption of a time-invariant central bank reaction function (as well as constant volatility) may bias the results.

To the best of our knowledge, this is the first application that considers the use of a

[1] These studies extend the work of Clarida *et al.* (2000) and Lubik and Schorfheide (2004), by considering the application of Markov-switching behaviour to this phenomena. Computational details that describe a robust method for the calculation of the posterior density for the complex likelihood function are contained in Sims and Zha (2004) and Sims *et al.* (2008).

MS-DSGE model for South Africa. The rest of the paper is organized as follows. Section 2 describes the methodology, while section 3 provides details of the data. The in-sample results are discussed in section 4 and out-of-sample results are discussed in section 5. The conclusion is contained in section 6.

2 Methodology

2.1 Theoretical model

The structure of the model follows that of Alpanda *et al.* (2011), which incorporates several small open-economy features of the South African economy.[2] After all variables are log-linearised around their steady-state, the equations that characterise the equilibrium conditions of the *non* Markov-switching version of model may be expressed as follows.

The domestic household's Euler condition yields a partially forward-looking IS curve in consumption:

$$c_t = \frac{1}{1+\zeta} E_t\left[c_{t+1}\right] + \frac{\zeta}{1+\zeta} c_{t-1} - \frac{1-\zeta}{\sigma(1+\zeta)}\left(i_t - E_t\left[\pi^c_{t+1}\right] - \Theta_t\right) \tag{1}$$

where σ is the inverse intertemporal-elasticity of substitution and habits in consumption are represented by ζ. The exogenous demand shock, is represented by Θ, whose natural logarithm follows an AR(1) process, with persistence parameter ρ_c, and error, $\epsilon_{c,t} \sim \text{i.i.d.} N[0, \sigma_c^2]$. The rate of consumer price inflation is expressed as π_t^c.

The relation between consumption and domestic output can be derived from the goods market clearing condition as:

$$y_t = (1-\alpha)c_t + \left[(1-\alpha)\eta\alpha + \eta\alpha\right] s_t + \alpha y_t^\star + \eta\alpha\psi_{f,t} \tag{2}$$

where α is the share of imports in consumption, η is the elasticity of substitution between domestic and foreign goods, y_t and $y_t^\star$ are domestic and foreign output, respectively, whilst $s_t = p_{f,t} - p_{h,t}$ is the terms of trade, and $\psi_{f,t}$ is the deviation of imported goods prices from the law-of-one-price.

Time differencing the terms-of-trade yields $s_t = s_{t-1} + p_{f,t} - p_{h,t}$, where $p_{h,t}$ and $p_{f,t}$ are inflation rates associated with the domestic and foreign goods prices, respectively. The domestic producer's problem yields a partially forward-looking New Keynesian Phillips curve for domestic price inflation:

$$\pi_{h,t} = \frac{\delta}{1+\delta\beta}\pi_{h,t-1} + \frac{\beta}{1+\delta\beta}E_t[\pi_{h,t+1}] + \frac{(1-\theta_h)(1-\theta_h\beta)}{\theta_h(1+\delta\beta)} mc_t \tag{3}$$

where β is the time-discount parameter, δ determines the degree with which prices are indexed to past domestic price inflation, and θ_h is the probability that the firms cannot adjust their prices in any given period. The above Phillips curve ties current domestic inflation rate to past and expected future inflation as well as the marginal costs of the firm. Marginal cost is given as, $mc_t = \varpi_t - a_t + \gamma s_t + \eta_t^p$, where ϖ_t is the real wage rate, a_t is the level of productivity in the production function that follows an exogenous AR(1) process, and η_t^p is a domestic cost-push shock that also follows an AR(1) process.

Similarly, foreign goods price inflation follows a forward-looking Phillips curve:

$$\pi_{f,t} = \beta E[\pi_{f,t+1}] + \frac{(1-\theta_f)(1-\theta_f\beta)}{\theta_f}\psi_{f,t} \tag{4}$$

[2] See, Alpanda *et al.* (2010*a*) and Alpanda *et al.* (2010*b*) for further details of the derivation of the model.

where θ_f is the probability that the importers cannot adjust their prices in any given period. Overall consumer price inflation in the domestic country is given by $\pi_t = (1-\alpha)\pi_{h,t} + \alpha\pi_{f,t}$.

Staggered wage setting by households yields the following wage inflation Phillips curve:

$$\pi_{w,t} - \varphi_w \pi_{t-1} = \beta E_t[\pi_{w,t+1}] - \varphi_w \beta \pi_t + \frac{(1-\theta_w)(1-\theta_w\beta)}{\theta_w(1+\xi_w\gamma)}\mu_t^w \tag{5}$$

where $\pi_{w,t}$ is the nominal wage inflation, φ_w is a parameter determining the degree of inflation indexation of nominal wage inflation, γ is the inverse of the elasticity of labour supply, and ϵ_w is the elasticity of substitution between differentiated labour services of households in the labour aggregator function. The wedge between the real wage and the marginal rate of substitution between consumption and labour in the household's utility function is μ_w, which may be expressed as,

$$\mu_t^w = \frac{\sigma}{1-\zeta}(c_t - \zeta c_{t-1}) + \gamma(y_t - a_t) - \varpi_t + \eta_t^w \tag{6}$$

where η_t^w is a wage cost-push shock that follows an AR(1) process. The relationship between nominal wage inflation and real wages can be expressed as $\pi_{w,t} = \varpi_t - \varpi_{t-1} + \pi_t$.

The uncovered interest parity (UIP) condition is then given by,

$$E[q_{t+1}] - q_t = (r - E[\pi_{t+1}]) - (r_t^\star - E_t[\pi_{t+1}^\star])) + \phi_t \tag{7}$$

where $q_t = e_t + p_t^\star - p_t$ is the real exchange rate. This is related to the terms-of-trade and the gap from the law-of-one-price, which is expressed as, $q_t = (1-\alpha)s_t + y_{f,t}$. Time differencing the real exchange rate yields the relationship between real and nominal depreciation rates, where $q_t - q_{t-1} = \Delta e_t + \pi_t^\star - \pi_t$. The variable $\phi_t = \mu_t^\phi + \chi \cdot nfa_t$ captures the time-varying country risk-premia. It is determined by the sum of an exogenous component, μ_t^ϕ, which follows an AR(1) process, and the net foreign asset position of the country, nfa_t, where χ is an elasticity parameter. The net asset position of the country evolves over time according to

$$nfa_t - \frac{1}{\beta} nfa_{t-1} = y_t - c_t - \alpha(s_t - \phi_{f,t}). \tag{8}$$

The central bank then makes use of the nominal interest rate as its policy instrument in an open-economy Taylor rule that allows for the inclusion of the exchange rate in its reaction function. In addition, we assume that the central bank targets the expected future value of inflation, and as such we make use of an expectational operator for this critical variable. Hence,

$$i_t = \rho\, i_{t-1} + (1-\rho)\left[\varrho_\pi E_t\left(\pi_{t+1}^c\right) + \varrho_y \tilde{y}_t + \varrho_d d_t\right] + \varepsilon_{i,t} \tag{9}$$

The rest of the world is modelled as a closed-economy version of the domestic economy, which can be represented by the representative IS curve (where the use of the * denotes foreign versions of the domestic counterparts):

$$y_t^\star = \frac{1}{1+\zeta}E_t[y_{t+1}^\star] + \frac{\zeta}{1+\zeta}y_{t-1}^\star - \frac{1-\zeta}{\sigma^\star(1+\zeta)}\left(r_t^\star - E_t[\pi_{t+1}^\star] + \mu_t^{d\star}\right) \tag{10}$$

a New Keynesian Phillips curve,

$$\pi_t^\star = \frac{\delta^\star}{1+\delta^\star\beta}\pi_{h,t-1} + \frac{\beta}{1+\delta^\star\beta}E_t[\pi_{h,t+1}^\star] + \frac{(1-\theta^\star)(1-\theta^\star\beta)}{\theta^\star(1+\delta^\star\beta)}mc_t^\star \tag{11}$$

where the foreign marginal cost is given by,

$$mc_t^\star = \left(\frac{\sigma^\star}{1-\zeta} + \gamma^\star\right)y_t^\star - \left(\frac{\sigma^\star\zeta}{1-\zeta}\right)y_{t-1}^\star - (1+\gamma^\star)a_t^\star + \mu_t^{w,\star} \tag{12}$$

and a foreign Taylor rule that is specified as,

$$i_t^\star = \rho^\star i_{t-1}^\star + (1-\rho^\star)\left[\varrho_\pi^\star \pi_t^\star + \varrho_y^\star \tilde{y}_t^\star\right] + \epsilon_t^{i\star} \tag{13}$$

2.2 Markov-switching

In the version of the model that incorporates Markov-switching in the domestic monetary policy reaction function, the Taylor rule in (9) may be expressed as,

$$i_t = \rho_\kappa \, i_{t-1} + (1-\rho_\kappa)\left[\varrho_{\kappa,\pi} E_t\left(\pi^c_{t+1}\right) + \varrho_{\kappa,y}\tilde{y}_t + \varrho_{\kappa,d} d_t\right] + \varepsilon_{i,t} \tag{14}$$

where κ is used to denote a two-state discrete Markov process taking values $\kappa \in \{1,2\}$ with transition probabilities $p_{ij},\ i,j = 1,2$, that influence the current state of the two regime model, which are influenced by the response of the central bank to the various factors that are contained in the monetary rule. In this case we denote the low response regime as $\kappa = 1$, while the high response regime is denoted by $\kappa = 2$.

In addition to the above specification, we also consider the effects of a change in the volatility of the shocks. This results in the inclusion of an additional ten parameters, where the notation ς^i_ϑ would refer to the volatility in the corresponding monetary policy shock, $\varepsilon_{i,t} \sim \text{i.i.d.}N[0, \varsigma^i_\vartheta]$, where ϑ is a two-state discrete Markov process with state indices in $\{1,2\}$.[3] As in the previous case, we denote the low volatility regime as $\vartheta = 1$, while the high volatility regime is denoted $\vartheta = 2$.

In addition to these two models, that incorporate Markov-switching and constant volatility, and Markov-switching in volatility only; we also consider the results for a model that allows for Markov-switching in both the policy reaction function and volatility, where each of these phenomena is controlled by separate (independent) chains. The set of models that we consider is then further augmented with a model that makes use of Markov-switching in both the policy reaction function and volatility, but where both chains are controlled by the chain in volatility, ϑ.[4]

2.3 Solution and estimation

As the solution in each state, is a function of the solution in the other states (and vice-versa), traditional solution methods for constant-parameter linear rational expectations models may not be used. Therefore, we make use of the methods developed in Svensson (2005), Farmer *et al.* (2011), Maih (2012) and Foerster *et al.* (2014) that seek to identify the minimum state variable solutions after applying the concept of mean square stability. This characterisation allows us to specify the general form of the Markov-switching rational expectations model as,

$$E_t\left\{A^+_{s_{t+1}} x_{t+1}\left(\bullet, s_t\right) + A^0_{s_t} x_t\left(s_t, s_{t-1}\right) + A^-_{s_t} x_{t-1}\left(s_{t-1}, s_{t-2}\right) + B_{s_t}\varepsilon_t\right\} = 0 \tag{15}$$

where x_t is a $n \times 1$ vector of endogenous (observed and unobserved) variables, and $\varepsilon_t \sim N\left(0, \varsigma_\vartheta\right)$ is the vector of structural exogenous shocks. The stochastic regime index s_t switches between a finite number of possibilities with cardinality h, such that $s_t = 1, 2, \ldots, h$. These probabilities may change over time, where s_t denotes the state of the system today and s_{t-1} denotes the state in the previous period.

The parameters in the model are estimated with Bayesian techniques, where all the unobserved variables, states of the Markov chains, and parameter values are treated as random variables. In this case the filter that is used to compute values for the unobserved processes would need to incorporate information up to the present time period, which include information relating to the states of the Markov chains (which is not incorporated in the traditional Kalman or particle filter). Therefore, we implement a version of the Hamilton

[3] Similar notation is used for the volatility in the other stochastic shocks.

[4] The results from these additional models are available on request from the authors.

(1989) filter that limits the number of states that are carried forward after each iteration, as in Farmer *et al.* (2008).

After computing the likelihood function with the aid of the procedures that are mentioned above, we are able to derive the posterior kernel, which we maximize to get the mode of the posterior distribution. Thereafter, we are able to initialize the Markov Chain Monte Carlo (MCMC) procedure that is used to construct the full posterior distribution and marginal data density. Details of the prior parameter values that are used in the calculation of the posterior estimates are similar to those that were used in Alpanda *et al.* (2011) and are provided along with all the posterior estimates in Table 2.

3 Data

The dataset extends over the period 1989q1 to 2014q4. The start date of the sample is motivated by the findings of Du Plessis & Kotzé (2010; 2012), who suggest that there is a significant structural change in most macroeconomic variables that would impact on the measure of the business cycle during the mid-1980s.[5]

Essentially, we estimate the model with ten observed variables for measures of: domestic output growth, $\tilde{y}$, GDP-deflator inflation, π, consumer inflation, π^c, nominal interest rate, i, nominal wage inflation, π^w, nominal productivity, z, nominal currency depreciation, d, foreign output growth, y^*, foreign GDP-deflater inflation, π^*, and foreign nominal interest rate, i^*.

All of the data for the South African economy was obtained from the South African Reserve Bank, with the exception of consumer prices, which was obtained from Statistics South Africa.[6] The data for the United States economy was obtained from the Federal Reserve System. Measures of output, inflation, productivity and currency depreciation are transformed to growth rates, while interest rates are expressed as annualised rates.

4 Results

4.1 In-sample statistics

Table 1 displays the in-sample statistics for the base-line model, which does not include Markov-switching, along with the model that allows for switching in the policy parameters and volatility of the shocks. These statistics would appear to suggest that there is little difference in the in-sample fit of the respective models.

	No-switching	Markov-switching
log-posterior:	3329	3422
log-likelihood:	3395	3424
log-prior:	-66.06	-2.084
log-MDD (Laplace)	3195	3107

Table 1: In-sample estimation statistics

[5]Hence, if the sample period started prior to this structural break the Markov-switching model would possibly only pick up on this behaviour and leave the remaining sample as one that is characterised as a single regime.

[6]To create a single measure of consumer price inflation we combine the respective measures that existed prior to 2008 with that which was established under the current methodology, using the monthly weighting procedure that is discussed in Du Plessis *et al.* (2015).

4.2 Parameter estimates

Table 2 provides details of the prior and posterior parameter estimates for the two models. In this case, we show the results for the model that does not include switching behaviour under regime one (although these results would obviously apply to both regimes).

Parameter	Distribution	Prior Mean	Prior Std.	No-switching	Markov-switching
$\rho(\kappa = 1)$	beta	0.75	0.1	0.82	0.87
$\rho(\kappa = 2)$	beta	0.75	0.1		0.90
$\varrho_\pi(\kappa = 1)$	gamma	1.5	0.25	1.62	1.00
$\varrho_\pi(\kappa = 2)$	gamma	1.5	0.25		1.16
$\varrho_y(\kappa = 1)$	gamma	0.25	0.12	0.57	0.00
$\varrho_y(\kappa = 2)$	gamma	0.25	0.12		1.20
$\varrho_d(\kappa = 1)$	gamma	0.12	0.05	0.06	0.00
$\varrho_d(\kappa = 2)$	gamma	0.12	0.05		0.00
κ^{1-2}	beta	0.9	0.1		0.12
κ^{2-1}	beta	0.9	0.1		0.08

Table 2: Prior and posterior parameter estimates - Monetary Policy Rule

When considering these results we note that the smoothing coefficient, ρ, in the two models differ slightly. In the model that does not include any switching we have a coefficient of 0.82, which is similar to the value that was obtained in Alpanda *et al.* (2011). In the Makov-switching model the posterior estimate for the interest rate smoothing coefficients are $\rho(\kappa = 1) = 0.87$ and $\rho(\kappa = 2) = 0.90$, which allows for greater smoothing in the interest rate. The values of these smoothing coefficients need to be taken into account when interpreting the response of the central bank to inflation, output and exchange rate movements.

When calculating $(1 - \rho)\varrho_\pi$, we note that with no switching, the value for the central bank response to inflation is 0.29, while under regime-one and two in the switching model, the values are 0.13 and 0.11 respectively. These results for the Markov-switching model would suggest that the central bank does not respond as aggressively to changes in inflation, when allowing for regime-switching behaviour.

The central bank response to output would suggest that when in regime-one, the central bank does not respond to output, where $(1 - \rho)\varrho_y(\kappa = 1) = 0$. In addition, when in regime-two the central bank would appear to respond to changes in output in a manner that is slightly similar to that of the case where we do not allow for regime-switching, where $(1-\rho)\varrho_y(\kappa = 1) = 0.12$ and $(1-\rho)\varrho_y = 0.10$. The response of the central bank to changes in the exchange rate suggest that in all cases, the response to the exchange rate is rather small, where in both regimes of the Markov-switching model, the coefficient approaches zero.

To summarise these results, we firstly note that the coefficients for the model that does not include switching are similar to those of Alpanda *et al.* (2010*a*,*b*, 2011), Steinbach *et al.* (2009) and Ortiz and Sturzenegger (2007). From the results of the Markov-switching model, we note that the central bank favours a greater degree of smoothing when in regime-one. In addition, when in this regime, its response to inflation is smaller and it does not respond to changes in output. In contrast with the results for regime-one, when in regime-two of the model we note that the central bank responds more aggressively to changes in output.

Parameter	Distribution	Prior Mean	Prior Std.	No-switching	Markov-switching
$\varsigma^{z}(\vartheta=1)$	weibull	0.23	0.3	0.008	0.008
$\varsigma^{z}(\vartheta=2)$	weibull	0.23	0.3		0.006
$\varsigma^{c}(\vartheta=1)$	weibull	0.23	0.3	0.001	0.002
$\varsigma^{c}(\vartheta=2)$	weibull	0.23	0.3		0.007
$\varsigma^{h}(\vartheta=1)$	weibull	0.23	0.3	0.015	0.012
$\varsigma^{h}(\vartheta=2)$	weibull	0.23	0.3		0.012
$\varsigma^{f}(\vartheta=1)$	weibull	0.23	0.3	0.073	0.041
$\varsigma^{f}(\vartheta=2)$	weibull	0.23	0.3		0.072
$\varsigma^{w}(\vartheta=1)$	weibull	0.23	0.3	0.017	0.016
$\varsigma^{w}(\vartheta=2)$	weibull	0.23	0.3		0.023
$\varsigma^{d}(\vartheta=1)$	weibull	0.23	0.3	0.004	0.017
$\varsigma^{d}(\vartheta=2)$	weibull	0.23	0.3		0.063
$\varsigma^{i}(\vartheta=1)$	weibull	0.23	0.3	0.002	0.001
$\varsigma^{i}(\vartheta=2)$	weibull	0.23	0.3		0.005
$\varsigma^{y*}(\vartheta=1)$	weibull	0.23	0.3	0.007	0.005
$\varsigma^{y*}(\vartheta=2)$	weibull	0.23	0.3		0.014
$\varsigma^{\pi*}(\vartheta=1)$	weibull	0.23	0.3	0.002	0.002
$\varsigma^{\pi*}(\vartheta=2)$	weibull	0.23	0.3		0.003
$\varsigma^{i*}(\vartheta=1)$	weibull	0.23	0.3	0.001	0.001
$\varsigma^{i*}(\vartheta=2)$	weibull	0.23	0.3		0.003

Table 3: Prior and posterior parameter estimates - volatility of shocks

Table 3 contains the parameter estimates for the volatility in the shocks, where we note that the case of the largest difference between the two models relates to the ς^{d} parameter, which describes the volatility in the risk premium. Allowing for Markov-switching behaviour ensures that this coefficient increases by between four to seventeen times.

4.3 Transition probabilities

The smoothed transition probabilities for the central bank reaction function in the model that incorporates Markov-switching features in both the reaction function and the volatility of the shocks are displayed in Figures 1 and 2. These probabilities have been plotted against the respective variables that are included in the central bank reaction function, where a probability value of one (on the right-hand axis) corresponds to regime two (i.e. where $\kappa = 2$). The first thing to note about the probabilities in policy reaction function in Figure 1, is that there is no level shift in these probabilities. This would imply that the monetary policy reaction function would appear to be fairly consistent over the sample. This is also supported by the fact that at each point in time, the reaction function is determined by a combination of the two regimes, as the probabilities do not take on a value of zero or one at any particular point in time.

When we turn our attention to the transition probabilities in volatility, which are presented in Figure 2, we note that the state of the model would be in $\vartheta = 2$ during of time that corresponds with the emerging market crisis, the Russian crisis, and the global financial crisis. During these periods of time the volatility in the risk premium is almost seventeen times that of the model that does not allow for Markov-switching. Note also that large depreciations in the exchange rate (positive values in d) are associated with movements into state, $\vartheta = 2$. Hence, these results would suggest that the effect of the risk-premium could be larger than would be the case when we only allow for a single state of the economy.

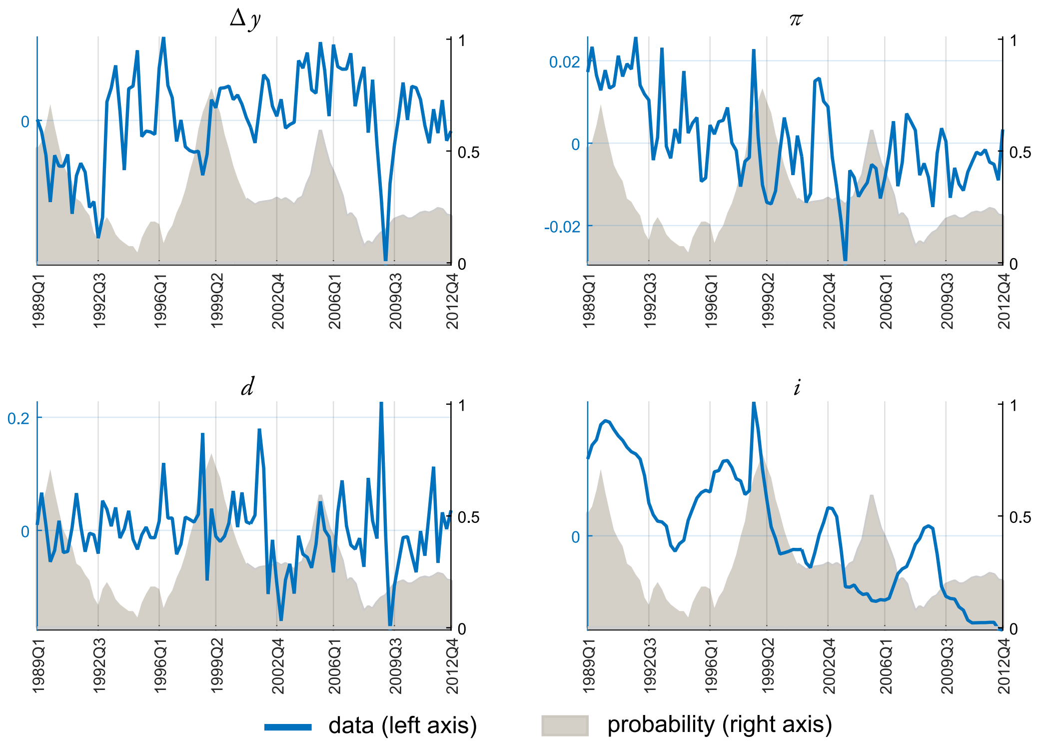

Figure 1: Smoothed transition probabilities - policy parameters

4.4 Generalised impulse response functions

While most of the impulse response functions in the two models are relatively similar, the response of the variables to a monetary policy and risk-premium shock, display some interesting differences. Figure 3 contains the results for the generalised impulse response functions for the two models that experience a monetary policy shock. In both cases, output and inflation decline following a rise in interest rates, where inflation declines by more than output. In addition, the currency also strengthens on impact, as denoted by the decline in the depreciation rate of the currency. When comparing the impulse response functions of the two models, we note that the response of output and inflation is greater when using the Markov-switching model and the sacrifice ratio is significantly lower.

When turning our attention to the generalised impulse response functions for a risk-premium shock, which is shown in Figure 4, we note that the currency depreciation increases, which contributes towards increased inflationary pressure. The central bank would respond to the rising consumer prices by increasing the nominal interest rate. The change in the external value of the currency would result in a decrease in the net-exports-to-output ratio and as a result, domestic output would increase by a relatively small amount. Note that the response of all the variables is much larger in the case of the Markov-switching model, which would suggest that the risk-premium shock is more prominent when the model allows for more than one possible state.

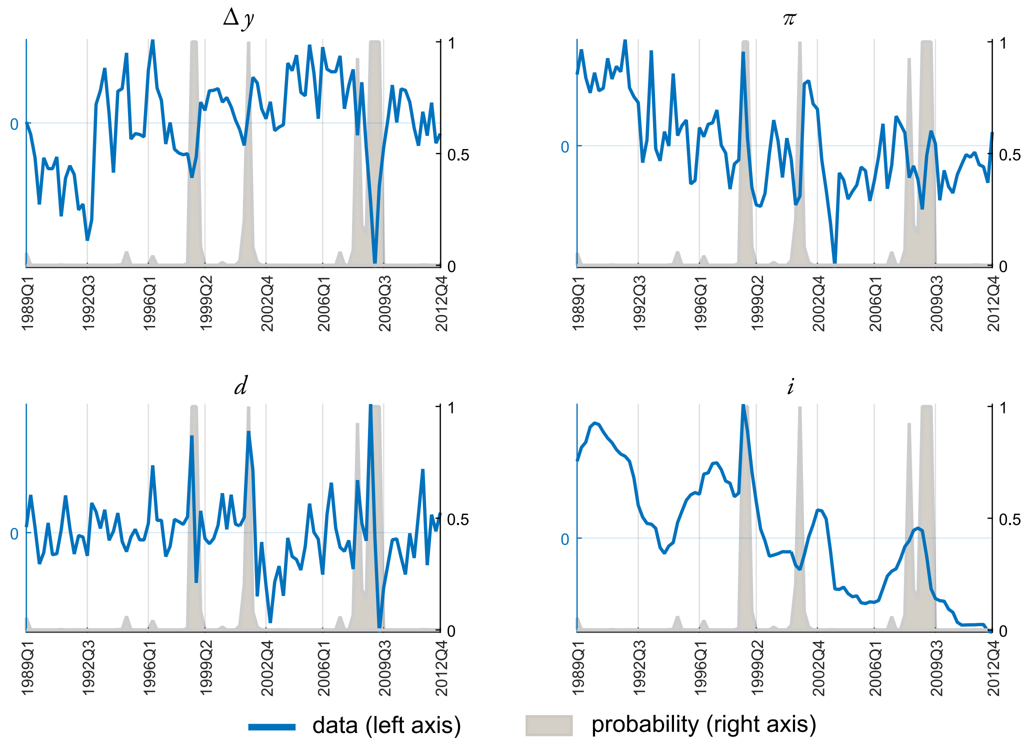

Figure 2: Smoothed transition probabilities - volatilities

5 Forecasting

The results of the out-of-sample forecasting exercise are contained in Table 4. To generate the first of these forecasts, we estimate the model using an in-sample period that ends in 2001q4. We then generate a one- to eight-step ahead forecast, before we update the in-sample data to 2002q1 for the subsequent re-estimation and forecast generation. The evaluation of the forecasts is conducted after calculating the root-mean squared-error (RMSE) for the one- to eight-step ahead forecasts over the entire out-of-sample period which extends over ten years. In addition, we also employ the statistic of Diebold and Mariano (1995), which may be used to describe the significance of the differences in the respective RMSE. In each of these tables, bold entries indicate the minimum RMSEs, and where the Diebold-Mariano statistic exceeds the ± 1.96 confidence interval, we attach a [$\star$] to those values.

After taking the average over time for the one- to eight-step ahead RMSEs, the forecasts of output suggest that the model that does not include any switching behaviour may provide slightly better out-of-sample results over the short-term. These results are contained in Table 4, which shows that as the horizon increases, the differences in the RMSEs become very small, where the Markov-switching model provides a slightly better RMSE at the six-step ahead horizon. In addition, the Diebold-Mariano statistics for each step-ahead forecast suggest that none of the forecasting errors are significantly different from one another (i.e. the results are within the confidence intervals).

The results for the short-term inflation forecasts are similar to those of output. However, in this case the RMSEs for the Markov-switching model are also inferior over longer horizons. These results are also contained in Table 4, where we note that the Diebold-Mariano statistics suggests that the forecasting performance of the model without Markov-switching is significantly better than its counterpart at the seven-step ahead horizon.

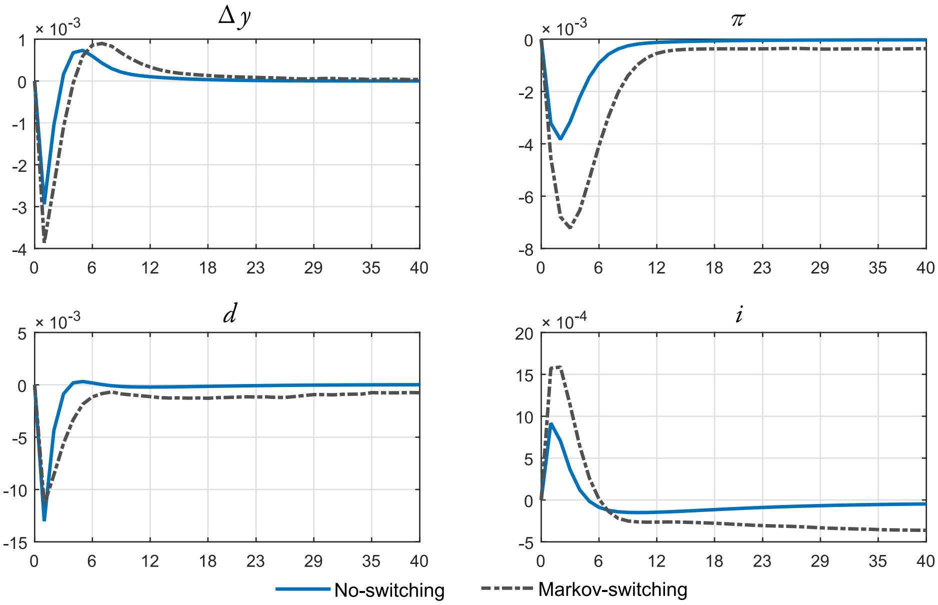

Figure 3: Generalised impulse response function - monetary policy shock

Then lastly, the out-of-sample results for interest rates are particularly poor for the Markov-switching model, where the RMSEs at each step are relatively high and the Diebold-Mariano statistics suggest that the difference between these results at the short and medium-term horizons are in most cases significant.

6 Conclusion

This paper considers the use of a Markov-switching DSGE model for the South African economy. The results suggest that there is little evidence of a level shift in the transition probabilities in the central bank reaction function. This would imply that the central bank has been fairly consistent with the application of policy over this sample period. The instances where the model switches into a second regime possibly reflect those cases where the central bank does not react strongly to changes in economic output, thereby focusing on inflationary pressure.

The model can also be used to identify changes in the volatility of shocks, where we note that it identifies most of the periods where there is a change in the risk-premium. When turning our attention to the behaviour of the impulse response functions, we note that the response of inflation to a monetary policy shock is greater in the Markov-switching model, and that both inflation and interest rates respond more aggressively to a change in the risk-premium.

The out-of-sample forecasting results suggest that in most instances, the model with a single-state provides more accurate results. This is more evident in the case of short-term forecasts of output and over most horizons for inflation and interest rates. When comparing the differences between these forecasting errors, we note that in most cases the difference is largely insignificant, except for forecasts for South African interest rates (where the realised values of the variable have remained relatively stable over the out-of-sample period).

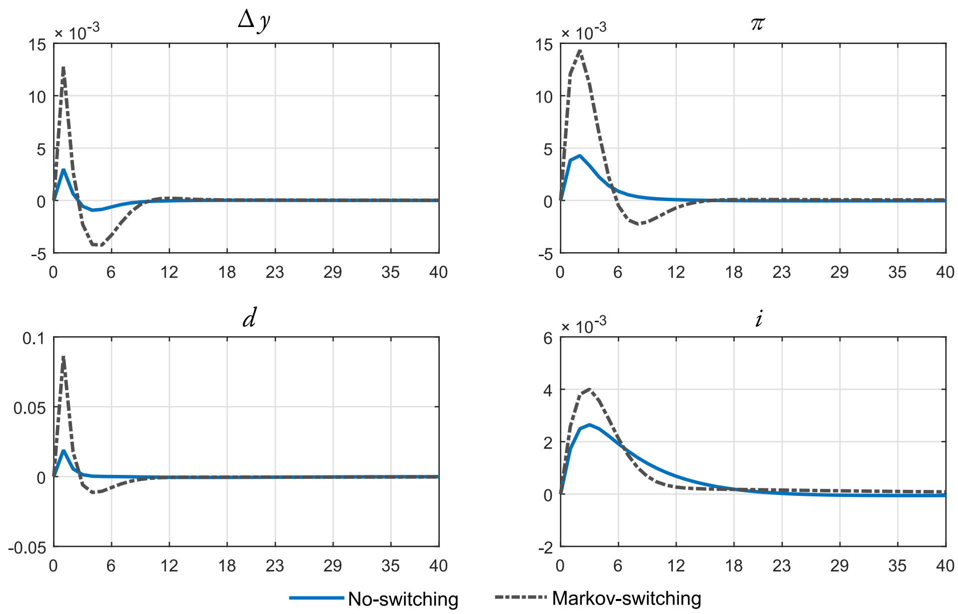

Figure 4: Generalised impulse response function - risk-premium shock

	Forecast Horizons							
	1 step	2 step	3 step	4 step	5 step	6 step	7 step	8 step
Output								
Markov-switching	0.055	0.045	0.062	0.055	0.047	**0.045**	0.043	0.044
No-Switching	**0.031**	**0.037**	**0.044**	**0.047**	0.047	0.046	0.043	**0.043**
DM-statistic	1.620	1.959	1.903	1.105	0.089	-0.279	-0.011	1.391
Inflation								
Markov-switching	0.131	0.171	0.146	0.089	0.057	0.048	0.047	0.044
No-Switching	**0.058**	**0.064**	**0.062**	**0.053**	**0.049**	**0.045**	**0.043**	**0.042**
DM-statistic	1.035	1.00	0.997	1.057	1.072	0.982	3.083⋆	0.828
Interest Rates								
Markov-switching	0.028	0.035	0.041	0.045	0.048	0.050	0.053	0.056
No-Switching	**0.015**	**0.024**	**0.030**	**0.034**	**0.038**	**0.042**	**0.045**	**0.048**
DM-statistic	3.188⋆	3.468⋆	3.55⋆	3.278⋆	2.497⋆	1.954	1.713	1.655

Table 4: Root-Mean Squared-Errors and Diebold-Mariano statistics (2002q1-2012q4)

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