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2 + 1-dimensional electrically charged black holes in Einstein-power-Maxwell theory

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A large family of new black hole solutions in 2 + 1-dimensional Einstein-power-Maxwell gravity with prescribed physical properties is derived. We show with particular examples that according to the power parameter k of the Maxwell field, the obtained solutions may be asymptotically flat for $1/2 < k < 1$ or nonflat for $k > 1$ in the vanishing cosmological constant limit. We study the thermodynamic properties of the solution with two different models, and it is shown that thermodynamic quantities satisfy the first law. The behavior of the heat capacity indicates that by employing the 1 + 1-dimensional dilaton analogy the local thermodynamic stability is satisfied.

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I. INTRODUCTION

It is well known that the main motivation to study nonlinear electrodynamics (NED) was to overcome some of the difficulties that occur in the standard linear Maxwell theory. Divergence in self-energy due to point charges was one such difficulty that kept physics community busy for decades. The Born-Infeld NED model was developed with the hope of removing these divergences; see e.g. [1–6], and by a similar trend NED was employed to eliminate black hole singularities in general theory of relativity. A striking example of regular black hole solutions in (3 + 1) dimensions was given in [7] that considers Einstein field equations coupled with NED which satisfies the weak energy condition and recovers the Maxwell theory in the weak field limit. The use of NED for eliminating singularities was also proved successful in (2 + 1) dimensions [8].

During the last decade, (2 + 1)-dimensional spacetimes admitting black hole solutions have attracted much attention. As a matter of fact, the $d = 3$ case singles out among other ($d \geq 4$) spacetimes with special mass and charge dependence. The first example in this regard is the Banados-Teitelboim-Zanelli (BTZ) black hole [9]. Later on, Einstein-Maxwell [10] and Einstein-Maxwell-dilaton [11] extensions were also found. The black hole solutions found in this context include all typical characteristics that can be found in (3 + 1) or higher dimensional black holes such as horizon(s), black hole thermodynamics and Hawking radiation. The black hole solution derived in [12] is another example for (2 + 1) dimensions within the context of a restricted class of NED in which the Maxwell scalar has a power in the form of $(F_{\mu\nu}F^{\mu\nu})^{3/4}$. This particular power results from imposing traceless condition on the energy-momentum tensor.

The main objective of the present study is twofold. First, we construct a large class of black hole solutions sourced by the power Maxwell field in which the Maxwell scalar has the form $(F_{\mu\nu}F^{\mu\nu})^k$. Here, the power k is a real rational number which will be restricted to some intervals as a requirement of the energy conditions. In general, for d-dimensional spacetimes, the specific choice of $k = \frac{d}{4}$ yields a traceless Maxwell's energy-momentum tensor [13], which is known to satisfy the conformal invariance condition. In recent years, the use of power Maxwell fields has attracted considerable interest. It has been used for obtaining solutions in d-spacetime dimensions [14], Ricci flat rotating black branes with a conformally Maxwell source [15], Lovelock black holes [16], Gauss-Bonnet gravity [17], and the effect of power Maxwell field on the magnetic solutions in Gausss-Bonnet gravity [18]. Therefore in [12], the power $3/4$ of the Maxwell scalar in (2 + 1) dimensions is the unique case that results from this traceless condition. Our first motivation in this study is to find the most general solution in (2 + 1)-dimensional Einstein-power-Maxwell (EPM) spacetime without imposing the traceless condition. Stated otherwise, choosing a traceful energy-momentum tensor amounts to treating k as a new parameter and we wish to investigate this freedom as much as we can. However, our analysis on the obtained solutions has revealed that the power parameter k can not be arbitrary. For a physically acceptable solution it must be a rational number. Hence, our general solution overlaps with the solution presented in [12], if one takes the power parameter $k = \frac{3}{4}$. Depending on the value of k, however, the resulting metric displays different characteristics near $r = 0$ which makes the present study more interesting. With the freedom of k we explore a rich possibility in the structure of singularities. For values $1/2 < k < 1$, for instance, the resulting spacetime becomes asymptotically flat in the vanishing cosmological constant ($\Lambda = 0$), and for $\Lambda > 0$, it is the asymptotically de Sitter spacetime. For $k > 1$ the resulting spacetime is nonasymptotically flat. Furthermore, the resulting metric depends not only on the parameter k but also on the mass M, the charge Q, and the cosmological constant Λ. When $\Lambda > 0$, the solution describes a charged de Sitter black hole spacetime

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with inner and outer horizons for $1/2 < k < 1$, and a cosmological horizon for $k > 1$. For specific values of these parameters the resulting spacetime singularity at $r = 0$ is naked whose strength becomes k dependent. When the cosmological constant $\Lambda < 0$, the resulting spacetime corresponds to charged anti de Sitter with a cosmological horizon in the range of the power $1/2 < k < 1$. For $k > 1$, the resulting charged de Sitter spacetime becomes naked singular at $r = 0$.

Another important issue in black hole physics is the concept and analysis of thermodynamic properties. This issue has gained a significant momentum not just in the linear Maxwell theory but also in the NED. As an example, in [19], higher dimensional gravity coupled to NED sourced by power Maxwell field has been analyzed thermodynamically for $d > 3$. The local and global thermodynamic stability is investigated by calculating the Euclidean action with the appropriate boundary term in the grand canonical ensemble. Our second objective in this study is to investigate the local thermodynamic stability of the resulting black holes. This is achieved by employing the method presented in [20], in which the local Hawking temperature is found from the Unruh effect. We calculate the heat capacities and show that our solution conditionally displays local thermodynamic stability. For specific values of the parameters, the calculated specific heat capacity at constant charge and electric potential both change sign at particular points. This behavior indicates that there may be a possible phase change in the black hole state. Alternatively, for a thorough thermodynamical analysis we appeal to the dilatonic analogy established in $1 + 1$ dimensions [21–23].

The organization of the paper is as follows. Section II, introduces the theory of EPM with solution and spacetime structure. The thermodynamic properties of the solution are considered in Sec. III. We complete the paper with a conclusion in Sec. IV.

II. EINSTEIN- POWER-MAXWELL SOLUTIONS AND SPACETIME STRUCTURE

The three-dimensional action for EPM theory with cosmological constant Λ is given by ($c = k_B = \hbar = 8G = 1$)

$$I = \int dx^3 \sqrt{-g}\Big(\frac{1}{2\pi}\Big(R - \frac{2}{3}\Lambda\Big) - L(\mathcal{F})\Big), \tag{1}$$

in which $L(\mathcal{F}) = |\mathcal{F}|^k$ and $\mathcal{F}$ is the Maxwell invariant

$$\mathcal{F} = F_{\mu\nu}F^{\mu\nu},$$

while the parameter k is arbitrary for the time being. Variation with respect to the gauge potential $\mathbf{A}$ yields the Maxwell equations

$$\mathbf{d}({}^\star\mathbf{F}L_{\mathcal{F}}) = 0 \rightarrow \mathbf{d}({}^\star\mathbf{F}|\mathcal{F}|^{k-1}) = 0, \tag{2}$$

where $\star$ denotes duality. Variation of the action with respect to the spacetime metric $g_{\mu\nu}$ yields the field equations

$$G^\nu_\mu + \frac{1}{3}\Lambda\delta^\nu_\mu = \pi T^\nu_\mu, \tag{3}$$

where

$$T^\mu{}_\nu = \frac{1}{2}(4(F_{\nu\lambda}F^{\mu\lambda})L_{\mathcal{F}} - \delta^\mu{}_\nu L), \tag{4}$$

is the energy-momentum tensor of the power Maxwell field and explicitly reads

$$T^\mu{}_\nu = \frac{|\mathcal{F}|^k}{2}\Big(\frac{4k(F_{\nu\lambda}F^{\mu\lambda})}{\mathcal{F}} - \delta^\mu{}_\nu\Big). \tag{5}$$

Our metric ansatz for $(2 + 1)$ dimensions, is chosen as

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2d\theta^2. \tag{6}$$

Static, electrically charged potential ansatz is given by

$$\mathbf{A} = A(r)dt,$$

which leads to

$$\mathbf{F} = \mathbf{dA} = E(r)dr \wedge dt, \tag{7}$$

with its dual

$${}^\star\mathbf{F} = E(r)rd\theta \tag{8}$$

and

$$\mathcal{F} = F_{\mu\nu}F^{\mu\nu} = -2E(r)^2. \tag{9}$$

Accordingly, the Maxwell equation reads now

$$\mathbf{d}(E(r)rd\theta[2E(r)^2]^{k-1}) = 0, \tag{10}$$

which leads to the solution as

$$rE(r)^{2k-1} = \text{constant}, \tag{11}$$

or equivalently

$$E(r) = \frac{\text{constant}}{r^{(1/2k-1)}}. \tag{12}$$

By using the latter result in (7) and choosing the integration constant proportional to the electric charge Q, one obtains the potential

$$A(r) = \begin{cases} Q\ln r & k = 1 \\ \frac{Q(2k-1)}{2(k-1)} r^{(2(k-1)/(2k-1))} & k \neq 1, \frac{1}{2} \end{cases}. \tag{13}$$

The resulting energy-momentum tensor follows from (5) as

$$T^\mu{}_\nu = \frac{1}{2}|\mathcal{F}|^k \text{diag}(\xi, \xi, -1), \tag{14}$$

where $\xi = (2k - 1)$ and the explicit form of $\mathcal{F}$ is given by

$$\mathcal{F} = -\frac{Q^2}{r^{(2/2k-1)}}, \tag{15}$$

in which we recall that Q is a constant related to the charge of the black hole. One can show that the weak energy condition (WEC) and the strong energy condition (SEC) restrict us to the set $k \in (\frac{1}{2}, \infty)$ (see the Appendix). The tt component of Einstein equations (3) reads

$$\frac{1}{2r}f'(r) + \frac{1}{3}\Lambda = 4\pi\xi|\mathcal{F}|^k, \tag{16}$$

whose integration gives,

$$f(r) = D + \frac{r^2}{l^2} - \frac{\pi(2k-1)^2}{2(k-1)}Q^{2k}r^{(2(k-1)/2k-1)}, \tag{17}$$

in which D is an integration constant, $\Lambda = -1/l^2$ and $k \neq 1$. The choice $k = 1$, which we exclude here gives the known charged BTZ black hole solution in Einstein-Maxwell theory [10]. We note that for $k = 1$, $\mathcal{F} = -\frac{Q^2}{r^2}$ diverges at $r = 0$ which is weaker than the cases for $k < 1$. This behavior, however, turns opposite for the choice $k > 1$. In order to illustrate this important effect of the power parameter k, we calculate the Kretschmann invariant for $k = 1$, $k < 1$ and $k > 1$. Since the resulting expressions for any $k > 1$ or $k < 1$ is too complicated, we prefer to calculate the Kretschmann invariant for specific values of k;

$$\begin{aligned}
\mathcal{K} &= \frac{12}{l^4} - \frac{4Q^2}{r^2}\left(\frac{2}{l^2} - \frac{Q^2}{r^2}\right) \quad \text{for } k = 1,\\
\mathcal{K} &= \frac{12}{l^4} - \frac{8Q^2}{r^6} \quad \text{for } k = 3/4,\\
\mathcal{K} &= \frac{12}{l^4} - \frac{2\pi Q^4}{r^{4/3}}\left(\frac{10}{l^2} - \frac{19\pi Q^4}{2r^{4/3}}\right) \quad \text{for } k = 2.
\end{aligned} \tag{18}$$

It is clear from these results that the rate of divergence of the Kretschmann invariant for $k = \frac{3}{4}$ is faster than the cases $k \geq 1$. The case when the power of r, $\frac{2(k-1)}{2k-1} < 0$, bounds the value of k to $\frac{1}{2} < k < 1$ which is also consistent with the energy conditions. Note that this case corresponds to asymptotically flat spacetime if one takes $\Lambda = 0$.

The integration constant D can be associated with the mass of the black hole, i.e., D can be expressed in terms of mass at infinity by employing the Brown-York [12,24,25] formalism. Following the quasilocal mass formalism it is known that, a spherically symmetric three-dimensional metric solution as

$$ds^2 = -F(r)^2dt^2 + \frac{1}{G(r)^2}dr^2 + r^2d\theta^2 \tag{19}$$

admits a quasilocal mass M_{QL} defined by

$$M_{\mathrm{QL}} = \lim_{r_b\to\infty} 2F(r_b)[G_r(r_b) - G(r_b)]. \tag{20}$$

Here $G_r(r_b)$ is an arbitrary non-negative reference function, which yields the zero of the energy for the background spacetime, and r_b is the radius of the spacelike hypersurface. According to our line element we get

$$F(r)^2 = G(r)^2 = D + \frac{r^2}{l^2} - \frac{\pi(2k-1)^2}{2(k-1)}Q^{2k}r^{(2(k-1)/2k-1)}, \tag{21}$$

$$G_r(r)^2 = \frac{r^2}{l^2} - \frac{\pi(2k-1)^2}{2(k-1)}Q^{2k}r^{(2(k-1)/2k-1)} \tag{22}$$

which yield

$$\begin{aligned}
M_{\mathrm{QL}} = \lim_{r_b\to\infty} 2\Bigg(\frac{r_b^2}{l^2}\left(1 + \frac{l^2}{2r_b^2}D - \frac{l^2 a r_b^{(2(k-1)/2k-1)}}{2r_b^2}\right)\\
- \left(D + \frac{r_b^2}{l^2} - a r_b^{(2(k-1)/2k-1)}\right)\Bigg).
\end{aligned} \tag{23}$$

Here we expanded the square roots and $a = \frac{\pi(2k-1)^2}{2(k-1)}Q^{2k}$. We note that since $\frac{2(k-1)}{2k-1} < 1$ for all values of $1/2 < k < \infty$, this limit results in-D, independent of the value of k. We would like to add that the same result may be found by applying the method introduced in [26,27] with a proper choice of the background metric.

Therefore the metric function, irrespective of the power of r, for $M > 0$ is given by

$$f(r) = -M + \frac{r^2}{l^2} - \frac{\pi(2k-1)^2}{2(k-1)}Q^{2k}r^{(2(k-1)/2k-1)}. \tag{24}$$

Finally, in this section we give the Ricci scalar

$$R = -\frac{6}{l^2} + \frac{\pi Q^{2k}(4k-3)}{r^{(2k/2k-1)}}, \tag{25}$$

which indicates the occurrence of true curvature singularity for any $k > \frac{1}{2}$. Although the particular choice $k = \frac{3}{4}$ shows R to be regular at $r = 0$ this is not supported by the Kretschmann scalar expression in (18). Nevertheless, the energy conditions (i.e., at least WEC and SEC)—given in the Appendix—always result in negative exponents in radial coordinate r therefore $r = 0$ is a true curvature singularity.

III. THERMODYNAMICS

A. Analysis with finite boundary model

In this section, we study the thermodynamical properties of the solution (24). A similar analysis for d-dimensional charged black holes with a NED sourced by power Maxwell fields was considered in [11], by employing Euclidean action with a suitable boundary term in the grand canonical ensemble. The analysis was carried out for spacetime dimensions $d > 3$.

In this study, we follow an alternative method as demonstrated in [19] to find the local Hawking temperature by using the Unruh effect in curved spacetime which is equivalent to finding the periodicity in the time coordinate in the Euclidean version of the metric covering the outer region of the black hole. In the Unruh effect, an observer outside the black hole experiences a thermal state with local temperature defined by

$$T_H(r) = \frac{2f'(r_h)}{\pi\sqrt{-\chi_\alpha\chi^\alpha}} = \frac{32}{\pi\sqrt{f(r)}}\left\{\frac{r_h}{l^2} - \frac{\pi(2k-1)Q^{2k}}{2r_h^{(1/2k-1)}}\right\}, \tag{26}$$

where χ^α is the Killing vector field generating the outer horizon and the location of the horizons are given by the roots of $f(r_h) = 0$, which implies

$$M = \frac{r_h^2}{l^2} - \frac{\pi(2k-1)^2}{2(k-1)} Q^{2k} r_h^{(2(k-1)/2k-1)}. \tag{27}$$

It should be noted that the power parameter k in the analysis of thermodynamic properties is assumed to satisfy $1/2 < k < 1$. It is remarkable to note that in the limits, $T_H(r)\mid_{r\to r_h}\to\infty$ and $T_H(r)\mid_{r\to\infty}\to 0$. This is expected because the solution given in (24) is nonasymptotically flat, hence, we have a vanishing temperature [i.e., from (26)] at infinity. Following the same procedure as demonstrated in [19], we define the reenergized temperature as $T_\infty = T_H(r)\sqrt{-\chi_\alpha\chi^\alpha} = \frac{f'(r_h)}{4\pi}$, which gives

$$T_\infty = \frac{4}{\pi}\left\{\frac{r_h}{l^2} - \frac{\pi(2k-1)Q^{2k}}{2r_h^{(1/2k-1)}}\right\}. \tag{28}$$

The internal energy of the system on a constant t hypersurface can be defined from the Brown-York [12,24,25] quasilocal energy formalism as

$$E(r_b) = -2\left(\sqrt{f(r_b)} - \frac{r_b}{l}\right), \tag{29}$$

where $r = r_b$ is a finite boundary of the spacetime. One may vary the internal energy $E(r_b)$ with respect to r_h and Q which leads to the first law of thermodynamics in the following form:

$$dE(r_b) = T_H(r_b)dS + \Phi(r_b)de, \tag{30}$$

in which the entropy S, after our unit convention ($c = k_B = \hbar = 8G = 1$) is given by

$$S = 4\pi r_h, \tag{31}$$

and e is the electric charge

$$e = \frac{\sqrt{2}}{4}\pi k(2k-1)^{(2k-1/2k)} Q^{2k-1}. \tag{32}$$

Here,

$$T_H(r_b) = \frac{1}{2\pi\sqrt{f(r_b)}}\left\{\frac{r_h}{l^2} - \frac{\pi}{2}(2k-1)Q^{2k} r_h^{-(1/2k-1)}\right\} \tag{33}$$

is the Hawking temperature at the boundary of the black hole spacetime and

$$\Phi(r_b) = \frac{2(2k-1)^{(1/2k)}\sqrt{2}Q}{(1-k)\sqrt{f(r_b)}}\left(r_h^{(2(k-1)/2k-1)} - r_b^{(2(k-1)/2k-1)}\right) \tag{34}$$

is the electric potential difference between the horizon and boundary r_b. On the other hand, the electric potential difference between the boundary and infinity is given by

$$\Psi(r_b) = \frac{2(2k-1)^{(1/2k)}\sqrt{2}Q}{(1-k)\sqrt{f(r_b)}}(r_b^{(2(k-1)/2k-1)}). \tag{35}$$

We consider the black hole inside a box bounded by $r = r_b$ and calculate the heat capacity of the black hole at constant Q, Φ and Ψ. The local thermodynamical stability conditions are determined by the sign of the heat capacities calculated at constant quantities in the limit of large values of r_b. which is defined by

$$C_X \equiv T\left(\frac{\partial S}{\partial T}\right)_X \geq 0, \tag{36}$$

in which $T = T_H(r_b)$ and X is the quantity to be held constant. Note that, we consider $S = S(r_h)$ and $T = T(r_h, Q^{2k})$; therefore, Eq. (36) can be written as

$$\begin{aligned} C_X \equiv T\left(\frac{\partial S}{\partial T}\right)_X &= T\left(\frac{\partial T}{\partial S}\right)_X^{-1} \\ &= T\left\{\left(\frac{\partial T}{\partial r_h}\right)\left(\frac{\partial r_h}{\partial S}\right) + \left(\frac{\partial T}{\partial Q^{2k}}\right)\left(\frac{\partial Q^{2k}}{\partial S}\right)_X\right\}^{-1}, \end{aligned} \tag{37}$$

and with $\frac{\partial S}{\partial r_h} = 4\pi$, latter equation reduces to

$$C_X = \frac{4\pi T}{\left\{\left(\frac{\partial T}{\partial r_h}\right) + \left(\frac{\partial T}{\partial Q^{2k}}\right)\left(\frac{\partial Q^{2k}}{\partial r_h}\right)_X\right\}}. \tag{38}$$

The heat capacities for constant Q, Φ, and Ψ are calculated by using Eq. (38). Because of the complexity of the resulting expressions we prefer to give only the expressions for large values of r_b; hence, the limiting heat capacities as $r_b \to \infty$ are

$$\begin{aligned} C_Q &= T\left(\frac{\partial S}{\partial T}\right)_Q \simeq \frac{4\pi r_h\left[r_h^{(2k/2k-1)} - \frac{\pi l^2 Q^{2k}(2k-1)}{2}\right]}{\left(r_h^{(2k/2k-1)} + \frac{\pi l^2 Q^{2k}}{2}\right)}, \\ C_\Phi &= T\left(\frac{\partial S}{\partial T}\right)_\Phi \simeq \frac{4\pi r_h\left[r_h^{(2k/2k-1)} - \frac{\pi l^2 Q^{2k}(2k-1)}{2}\right]}{(r_h^{(2k/2k-1)} + \pi l^2 Q^{2k}(2k-1)^2)}, \\ C_\Psi &= T\left(\frac{\partial S}{\partial T}\right)_\Psi \simeq \frac{4\pi r_h\left[r_h^{(2k/2k-1)} - \frac{\pi l^2 Q^{2k}(2k-1)}{2}\right]}{\left(r_h^{(2k/2k-1)} + \frac{\pi l^2 Q^{2k}}{2}\right)}. \end{aligned} \tag{39}$$

One observes that since, from (28), $f'(r_h) > 0$, i.e.,

$$r_h^{(2k/2k-1)} - \frac{\pi l^2 Q^{2k}(2k-1)}{2} > 0, \tag{40}$$

thermodynamically our solution indicates a locally stable black hole.

B. Analysis with 1 + 1-dimensional dilaton gravity model

In this section, we employ the method presented in [21–23] to study the thermodynamics of the EPM black hole found above. In this method the solution given in Eq. (24) will be obtained from the dilaton and its potential of two-dimensional dilaton gravity through dimensional reduction. Now we consider

$$ds^2 = g_{ab}dx^a dx^b = \tilde{g}_{\mu\nu}d\tilde{x}^\mu d\tilde{x}^\nu + \phi^2(\tilde{x})d\theta^2, \tag{41}$$

where ϕ denotes the radius of the circle S^1 in $M_3 = M_2 \times S^1$. The Greek indices represent the two-dimensional spacetime. After the Kaluza-Klein dimensional reduction, the action (1) reads as

$$S_{2D} = 2\pi \int d\tilde{x}^2 \sqrt{-\tilde{g}}\phi\left(\frac{\tilde{R} - 2\tilde{\Lambda}}{2\pi} - L(\mathcal{F})\right), \qquad L(\mathcal{F}) = |\mathcal{F}|^k, \tag{42}$$

in which $\tilde{R}$ is the Ricci scalar of M_2 and $\tilde{\Lambda} = \Lambda/3$. Varying the above action leads to the following field equations:

$$d(\phi L_{\mathcal{F}}{}^\star\mathbf{F}) = 0, \tag{43}$$

$$\nabla^2\phi + 2\phi\tilde{\Lambda} = 2\pi\phi(L(\mathcal{F}) - 2\mathcal{F}L_{\mathcal{F}}), \tag{44}$$

$$\tilde{R} - 2\tilde{\Lambda} = -2\pi L(\mathcal{F}). \tag{45}$$

Herein, the electric field 2− form is given by $\mathbf{F} = E(\phi)dt \wedge d\phi$ and its dual becomes 0-form ${}^\star\mathbf{F} = E(\phi)$. Note that, ϕ is effectively one of our coordinates. The electric field invariant $F_{ab}F^{ab}$ is

$$\mathcal{F} = -\tfrac{1}{2}E(\phi)^2, \tag{46}$$

which implies from (43)

$$E\phi(E^2)^{k-1} = \text{constant}. \tag{47}$$

The latter equation yields the following electric field:

$$E(\phi) = \frac{q}{\phi^{(1/2k-1)}}, \tag{48}$$

where q is an integration constant. Then, the Lagrangian $\mathcal{L}(F)$ can be written as

$$L(\mathcal{F}) = \frac{1}{2^k}\frac{q^{2k}}{\phi^{(2k/2k-1)}}, \tag{49}$$

and

$$L_{\mathcal{F}} = \frac{-k}{2^{(k-1)}}\frac{q^{2(k-1)}}{\phi^{(2(k-1)/2k-1)}}. \tag{50}$$

The rest of the field equations are given by

$$\nabla^2\phi = V(\phi) = -2\phi\tilde{\Lambda} + 2\pi\phi\left(\frac{1}{2^k}\frac{q^{2k}}{\phi^{(2k/2k-1)}}\right)(1-2k) \tag{51}$$

and

$$\tilde{R} = -V'(\phi) = 2\tilde{\Lambda} - 2\pi\left(\frac{1}{2^k}\frac{q^{2k}}{\phi^{(2k/2k-1)}}\right). \tag{52}$$

It is remarkable to observe that, these equations correspond to the two-dimensional field equations of dilaton gravity with an action

$$S_{2D} = \int_{M_2} d\tilde{x}dt\sqrt{-\tilde{g}}(\phi\tilde{R} + V(\phi)) \tag{53}$$

and the line element

$$ds^2 = -f(\tilde{x})dt^2 + \frac{d\tilde{x}^2}{f(\tilde{x})}. \tag{54}$$

After manipulating Eqs. (51) and (52), one finds

$$\nabla^2\phi = f\phi'' + f'\phi' = V(\phi) \tag{55}$$

and

$$\tilde{R} = -f'' = -V'(\phi), \tag{56}$$

in which a prime means derivative with respect to the argument. Our dilaton ansatz

$$\phi = \tilde{x} \tag{57}$$

admits

$$f' = V(\phi), \tag{58}$$

such that

$$f(\phi) = J(\phi) - \mathcal{C} \tag{59}$$

in which

$$\begin{aligned} J(\phi) &= \int V(\phi)d\phi \\ &= \frac{\phi^2}{\ell^2} - \frac{2\pi(1-2k)^2 q^{2k}}{2^{k+1}(k-1)} \\ &\quad \times \left(\frac{1}{\phi^{(2(1-k)/2k-1)}} - \frac{1}{\phi_0^{(2(1-k)/2k-1)}}\right). \end{aligned} \tag{60}$$

Herein ϕ_0 is a reference potential, $\tilde{\Lambda} = -\frac{1}{\ell^2}$ and $\mathcal{C}$ represents the Arnowitt-Deser-Misner mass of the EPM black hole [21–23]. Also the line element (54) becomes

$$ds^2 = -f(\phi)dt^2 + \frac{d\phi^2}{f(\phi)}. \tag{61}$$

As was introduced in [21–23], the extremal value of ϕ_+ is obtained from $V(\phi_+ = \phi_e) = 0$, which yields

$$\phi_e^{(2k/2k-1)} = \frac{2\pi q^{2k}\ell^2}{2^{k+1}}(2k-1). \tag{62}$$

This implies that the extremal mass [from (60)] is given by

$$M_e = J(\phi_e), \tag{63}$$

in which for $M \geq M_e$ the metric function admits at least one horizon ϕ_+ that indicates the outer horizon. The Hawking temperature at the outer horizon, the heat capacity, and free energy are given, respectively, by

$$T_H = \frac{V(\phi_+)}{4\pi} = \frac{\phi_+}{4\pi}\left(\frac{2}{\ell^2} + 2\pi\left(\frac{1}{2^k}\frac{q^{2k}}{\phi_+^{(2k/2k-1)}}\right)(1-2k)\right), \tag{64}$$

$$C_q(\phi_+) = 4\pi\left(\frac{V(\phi_+)}{V'(\phi_+)}\right) = \frac{4\pi\phi_+(\frac{2}{\ell^2} + 2\pi(\frac{1}{2^k}\frac{q^{2k}}{\phi_+^{(2k/2k-1)}})(1-2k))}{\frac{2}{\ell^2} + 2\pi(\frac{1}{2^k}\frac{q^{2k}}{\phi^{(2k/2k-1)}})} \tag{65}$$

and

$$F(\phi_+) = \frac{\phi_+^2}{\ell^2} - \frac{2\pi(1-2k)^2 q^{2k}}{2^{k+1}(k-1)} \times\left(\frac{1}{\phi_+^{(2(1-k)/2k-1)}} - \frac{1}{\phi_0^{(2(1-k)/2k-1)}}\right) - J(\phi_e) - \phi_+^2\left(\frac{2}{\ell^2} + 2\pi\left(\frac{1}{2^k}\frac{q^{2k}}{\phi_+^{(2k/2k-1)}}\right)(1-2k)\right). \tag{66}$$

In summary, in this section the thermodynamic analysis of the EPM black hole is investigated by two entirely different methods. Our analysis reveals that by rescaling the constant q we recover the results obtained in so (40) that two different approaches for thermodynamic stability are in agreement.

IV. CONCLUSION

In this study, the most general solution in (2 + 1)-dimensional spacetime in EPM theory, without imposing the traceless condition on the energy-momentum tensor is derived. The obtained solutions describe black holes sourced by the power Maxwell fields. From a physics standpoint and in analogy with the self-interacting scalar fields, k can be interpreted as the measure of self-interaction that electromagnetic field undergoes. As such, it alters much of physics and, in particular, the black hole/singularity formations. We have shown with particular examples that the power parameter k has a significant effect on the physical interpretation of the obtained solutions. For specific values of parameter k, it is possible to obtain asymptotically flat (with $\Lambda = 0$) or nonasymptotically flat solutions. As it has been shown in the Appendix, for the choice $\frac{2}{3} < k < 1$, all energy conditions (WEC, SEC and DEC [dominant energy condition]) are satisfied, as well as the causality condition. The character of the singularity at $r = 0$ is timelike, since a new coordinate defined by $r_* = \int \frac{dr}{f}$ is finite as $r \to 0$. In solutions admitting black holes, this timelike singularity is covered by horizon(s). But, in some cases it remains naked and violates the cosmic censorship hypothesis. It becomes worthful therefore to investigate the structure of this singularity in quantum mechanical point of view. For $\frac{2(k-1)}{2k-1} > 0$ the resulting spacetime geometry is very similar to the BTZ black hole whose quantum singularity structure is investigated in [28] by quantum test particles obeying the Klein-Gordon and Dirac equations. The results reported in [28] are; for massive scalar fields the spacetime is quantum singular but for massless scalar bosons and for fermions, the spacetime is quantum regular. On the other hand, naked singularity that occurs for $\frac{2(k-1)}{2k-1} < 0$ is structurally similar to the solution given in [12]. The quantum nature of this singularity is recently investigated in [29] with the test particles obeying the Klein-Gordon and Dirac equations. It was shown that the spacetime is quantum singular for massless scalar particles obeying Klein-Gordon equation but quantum regular for fermions obeying Dirac equation. Therefore, these results are also applicable to the solutions presented in this study. Thermodynamic quantities such as Hawking temperature, entropy and specific heat capacity are also calculated by two different methods in which we obtain the same result for stability. Magnetically charged nonblack hole EPM solutions are considered in a recent study [30] in which singularities, both classically and quantum mechanically are investigated thoroughly. The fact that by employing NED in 2 + 1 dimensions one can construct regular black holes through cutting and pasting method has also been shown in a separate study [31]. Finally, we note that by choosing $k \neq 1$ in the flat spacetime electrodynamics, we avoid the logarithmic potential, once and for all.

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APPENDIX: ENERGY CONDITIONS

When a matter field couples to any system, energy conditions must be satisfied for physically acceptable solutions. This is achieved by following the steps as given in [32,33]. $T_\nu^\mu = \frac{1}{2}|\mathcal{F}|^k \mathrm{diag}(\xi, \xi, -1)$.

1. Weak energy condition

The energy-momentum tensor given in Eq. (14) implies

$$\rho = -T_t^t = \frac{1}{2}|\mathcal{F}|^k\xi, \qquad p_r = T_r^r = \frac{1}{2}|\mathcal{F}|^k\xi, \qquad p_\theta = -\frac{1}{2}|\mathcal{F}|^k,$$

in which ρ is the energy density and p_i are the principal pressures.

The WEC states that

$$\rho \geq 0 \quad \text{and} \quad \rho + p_i \geq 0 \qquad (i = 1, 2), \tag{A1}$$

which imposes $k > \frac{1}{2}$.

2. Strong energy condition

This condition states that

$$\rho + \sum_{i=1}^{2} p_i \geq 0 \quad \text{and} \quad \rho + p_i \geq 0, \tag{A2}$$

which yields $k \geq 0$. The SEC together with the WEC constraint the parameter k to $k > \frac{1}{2}$.

3. Dominant energy condition

DEC states that $p_{\text{eff}} \geq 0$, in which

$$p_{\text{eff}} = \frac{1}{2} \sum_{i=1}^{2} T_i^i, \tag{A3}$$

and this gives the constraint $k \leq 1$. One can show that DEC, together with SEC and WEC, imposes $\frac{1}{2} < k \leq 1$.

4. Causality condition

Beside the energy conditions, one can impose the causality condition

$$0 \leq \frac{p_{\text{eff}}}{\rho} < 1, \tag{A4}$$

which implies $\frac{2}{3} < k \leq 1$. Therefore if the causality condition is imposed, naturally all other conditions are satisfied.

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