Higher dimensional thin-shell wormholes in Einstein-Yang-Mills-Gauss-Bonnet gravity

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We present thin-shell wormhole solutions in Einstein-Yang-Mills-Gauss-Bonnet (EYMGB) theory in higher dimensions $d \geq 5$. Exact black hole solutions are employed for this purpose where the radius of thin-shell lies outside the event horizon. For some reasons the cases $d = 5$ and $d > 5$ are treated separately. The surface energy-momentum of the thin-shell creates surface pressures to resist against collapse and rendering stable wormholes possible. We test the stability of the wormholes against spherical perturbations through a linear energy-pressure relation and plot stability regions. Apart from this restricted stability we investigate the possibility of normal (i.e. non-exotic) matter which satisfies the energy conditions. For negative values of the Gauss-Bonnet (GB) parameter we obtain such physical wormholes.

Dedicated to the memory of Rev. Ibrahim EKEN (1927-2010) of Turkey.

I. INTRODUCTION

One of the challenging problems in general relativity is to construct viable, traversable wormholes [1, 2] from curvature of spacetime and physically meaningful energy-momenta. Most of the sources to support wormholes to date, unfortunately consists of exotic matter which violates the energy conditions [3]. More recently, however, there are examples of thin-shell wormholes that resist against collapse when sourced entirely by physical (normal) matter satisfying the energy conditions [4]. From this token, it has been observed that pure Einstein's gravity consisting of Einstein-Hillbert (EH) action with familiar sources alone doesn't suffice to satisfy the criteria required for normal matter. This leads automatically to taking into account the higher curvature corrections known as the Lovelock hierarchy [5]. Most prominent term among such higher order corrections is the Gauss-Bonnet (GB) term to modify the EH Lagrangian. There is already a growing literature on Einstein-Gauss-Bonnet (EGB) gravity and wormhole constructions in such a theory.

In this paper we intend to fill a gap in this line of thought which concerns Einstein-Yang-Mills (EYM) theory amended with the GB term. More specifically, we wish to construct thin-shell wormholes that are supported by normal (i.e. non-exotic) matter. To this end, we first construct higher dimensional ($d \geq 5$) exact black hole solutions in EYMGB theory. This we do by employing the higher dimensional Wu-Yang ansatz which has been described elsewhere [6, 7]. The distinctive point with this particular ansatz is that the YM invariant emerges with the same power, irrespective of the spacetime dimensionality. In this regard EYM solution becomes simpler in comparison with the Einstein-Maxwell (EM) solutions. This motivates us to seek for thin-shell wormholes by cutting / pasting method in EYM theory.

Another point of utmost importance is the GB parameter (α), whose sign plays a crucial role in the positivity of energy of the system. Although in string theory this parameter is chosen positive for some valid reasons, when it comes to the subject of wormholes our choice favors the negative values ($\alpha < 0$), for the GB parameter. One more item that we consider in detail in this study is to investigate the stability of such wormholes against linear perturbations when the pressure and energy density are linearly related.

The exact solution to EYMGB gravity that we shall employ in this paper were established before [6, 7]. Our line element is chosen in the form [7]

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-2}^2, \tag{1}$$

in which $f(r)$ is the only metric function and

$$d\Omega_{d-2}^2 = d\theta_1^2 + \sum_{i=2}^{d-2} \prod_{j=1}^{i-1} \sin^2\theta_j \; d\theta_i^2, \tag{2}$$

where

$$0 \le \theta_{d-2} \le 2\pi, 0 \le \theta_i \le \pi, \quad 1 \le i \le d-3.$$

According to the higher dimensional Wu-Yang ansatz the YM potential is chosen as

$$\mathbf{A}^{(a)} = \frac{Q}{r^2} C^{(a)}_{(i)(j)} \; x^i dx^j, \quad Q = \text{YM magnetic charge}, \; r^2 = \sum_{i=1}^{d-1} x_i^2, \tag{3}$$

$$2 \le j+1 \le i \le d-1, \quad \text{and} \; 1 \le a \le (d-2)(d-1)/2,$$
$$x_1 = r\cos\theta_{d-3}\sin\theta_{d-4}...\sin\theta_1, \; x_2 = r\sin\theta_{d-3}\sin\theta_{d-4}...\sin\theta_1,$$
$$x_3 = r\cos\theta_{d-4}\sin\theta_{d-5}...\sin\theta_1, \; x_4 = r\sin\theta_{d-4}\sin\theta_{d-5}...\sin\theta_1,$$
$$...$$
$$x_{d-2} = r\cos\theta_1,$$

where $C^{(a)}_{(b)(c)}$ is the non-zero structure constants [8]. By this choice the YM invariant $\mathcal{F}$ reduces to a simple form

$$\mathcal{F} = \mathbf{Tr}(F^{(a)}_{\lambda\sigma} F^{(a)\lambda\sigma}) = \frac{(d-3)}{r^4} Q^2, \tag{4}$$

which yields the energy-momentum tensor

$$T^\nu_{\;\mu} = -\frac{1}{2}\mathcal{F}\text{diag}\,[1, 1, \kappa, \kappa, .., \kappa], \quad \text{and} \; \kappa = \frac{d-6}{d-2}. \tag{5}$$

Accordingly, the field equations are (without a cosmological term)

$$G^E_{\mu\nu} + \alpha G^{GB}_{\mu\nu} = T_{\mu\nu}, \tag{6}$$

where

$$G^{GB}_{\mu\nu} = 2\left(-R_{\mu\sigma\kappa\tau}R^{\kappa\tau\sigma}_{\quad\nu} - 2R_{\mu\rho\nu\sigma}R^{\rho\sigma} - 2R_{\mu\sigma}R^\sigma_{\;\nu} + RR_{\mu\nu}\right) - \frac{1}{2}\mathcal{L}_{GB}g_{\mu\nu}\,, \tag{7}$$

α is the GB parameter and GB Lagrangian $\mathcal{L}_{GB}$ is given by

$$\mathcal{L}_{GB} = R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta} - 4R_{\mu\nu}R^{\mu\nu} + R^2.$$

The exact solutions which we shall use throughout this paper are [6, 7]

$$f_\pm(r) = \begin{cases} 1 + \frac{r^2}{4\alpha}\left(1 \pm \sqrt{1 + \frac{32\alpha M_{ADM}}{3r^4} + \frac{16\alpha Q^2 \ln r}{r^4}}\right), & d = 5 \\ 1 + \frac{r^2}{2\tilde\alpha}\left(1 \pm \sqrt{1 + \frac{16\tilde\alpha M_{ADM}}{r^{d-1}(d-2)} + \frac{4(d-3)\tilde\alpha Q^2}{(d-5)r^4}}\right), & d \ge 6 \end{cases}, \tag{8}$$

in which $\tilde\alpha = (d-3)(d-4)\alpha$, with the GB parameter α. Here M_{ADM} stands for the usual ADM mass of the black hole and Q is the YM charge. When compared with Ref.s [6] (for $d = 5$) and [7] (for $d > 5$) the meaning of M_{ADM} implies that $M_{ADM} = \frac{3}{2}(m + 2\alpha)$ and $M_{ADM} = \frac{1}{4}m(d-2)$, respectively. Let us also add that in Ref. [7] we set $Q = 1$ through scaling. The crucial point in our solution is that the YM term under the square root has a fixed power $\frac{1}{r^4}$ for all d$\ge$ 6. As it can be checked, the negative branch gives the correct limit of higher dimensional black hole solution in EYM theory of gravity if $\alpha \to 0$, and therefore in the sequel we only consider this specific case. We also notice that for negative α there exists a curvature singularity at $r = r_\circ$ where $r_\circ$ is the smallest radius of which, for $r > r_\circ$, inside the square root is positive. For $\alpha > 0$, although for $d \ge 6$ there is no curvature singularity, for $d = 5$ it depends on the value of the free parameters (i.e., $\tilde\alpha$, Q, M_{ADM}) to result in a curvature singularity.

Here, in order to explore the physical properties of the above solutions we investigate some essential thermodynamic quantities. Since $d = 5$ case has been studied elsewhere [9] we shall concentrate on $d \ge 6$.

Radius of the event horizon (i.e., r_h) of the negative branch black hole $f_-(r)$, with positive α is the maximum root of $f_-(r_h) = 0$. It is not difficult to show that in terms of event horizon radius one can write

$$M_{ADM} = \frac{(d-2)}{4}\left[(\tilde\alpha + r_h^2) - \frac{(d-3)}{(d-5)}Q^2\right]r_h^{d-5}. \tag{9}$$

Also we find the Hawking temperature T_H in terms of r_h, i.e.,

$$T_H = \frac{1}{4\pi} f'(r_h) = \frac{(d-3)\left(r_h^2 - Q^2\right) + \tilde{\alpha}(d-5)}{4\pi r_h \left(2\tilde{\alpha} + r_h^2\right)}. \tag{10}$$

To complete our thermodynamical quantities we use the standard definition of the specific heat capacity with the constant charge

$$C_Q = T_H \left(\frac{\partial S}{\partial T_H}\right)_Q, \tag{11}$$

in which S is the standard entropy defined as

$$S = \frac{A}{4} = \frac{(d-1)\pi^{\frac{d-1}{2}}}{4\Gamma\left(\frac{d+1}{2}\right)} r_h^{d-2}, \tag{12}$$

to show the possible thermodynamical phase transition. After some manipulation we find

$$C_Q = \frac{(d-2)(d-1)\left(2\tilde{\alpha}+r_h^2\right)\pi^{\frac{d-1}{2}}r_h^{d-2}\left[(d-5)\tilde{\alpha}+(d-3)\left(r_h^2-Q^2\right)\right]}{4\Gamma\left(\frac{d+1}{2}\right)\left\{2\tilde{\alpha}[Q^2(d-3)-\tilde{\alpha}(d-5)]+[3Q^2(d-3)-\tilde{\alpha}(d-9)]r_h^2-(d-3)r_h^4\right\}}. \tag{13}$$

The phase transition is taking place at the real and positive root(s) of the denominator, i.e.,

$$2\tilde{\alpha}\left[Q^2(d-3) - \tilde{\alpha}(d-5)\right] + \left[3Q^2(d-3) - \tilde{\alpha}(d-9)\right]r_h^2 - (d-3)r_h^4 = 0. \tag{14}$$

One can show that under the condition

$$\frac{Q^2}{\tilde{\alpha}} < \frac{7d-39}{9(d-3)} \tag{15}$$

there is no phase transition, while if

$$\frac{7d-39}{9(d-3)} < \frac{Q^2}{\tilde{\alpha}} < \frac{d-5}{d-3} \tag{16}$$

we will observe two phase transitions. Finally upon choosing

$$\frac{d-5}{d-3} \le \frac{Q^2}{\tilde{\alpha}} \tag{17}$$

there exists only one phase transition. Also, if $\frac{Q^2}{\tilde{\alpha}} = \frac{7d-39}{9(d-3)}$ one phase transition occurs at $r_h = \sqrt{\frac{6(d-3)}{7d-39}Q^2}$. These results show that the dimensionality of spacetime plays a crucial role in the thermodynamical behavior of the EYMGB system.

For negative α in the negative branch we write $\tilde{\alpha} = -|\tilde{\alpha}|$ and therefore the horizon radius r_h is given by solving

$$1 - \frac{2|\tilde{\alpha}|}{r_h^2} = \sqrt{1 - \frac{16|\tilde{\alpha}|M_{ADM}}{r_h^{d-1}(d-2)} - \frac{4(d-3)|\tilde{\alpha}|Q^2}{(d-5)r_h^4}}. \tag{18}$$

The method of establishing the thin-shell wormhole, based on the black hole solutions given in (8), follows the standard procedure which has been employed in many recent works [4].

II. DYNAMIC THIN-SHELL WORMHOLES IN $d-$DIMENSIONS

The method of establishing a thin-shell wormhole in the foregoing geometry goes as follows. We cut two copies of the EYMGB spacetime

$$M^{\pm} = \{r^{\pm} \ge a, \ a > r_h\} \tag{19}$$

and paste them at the boundary hypersurface $\Sigma^{\pm} = \{r^{\pm} = a, \ a > r_h\}$. These surfaces are identified on $r = a$ with a surface energy-momentum of a thin-shell whose radius coincides also with the throat radius such that geodesic

completeness holds for $M = M^+ \cup M^-$. Following the Darmois-Israel formalism [10] in terms of the original coordinates $x^\gamma = (t, r, \theta_1, \theta_2, ...)$ (i.e. in M) the induced metric $\xi^i = (\tau, \theta_1, \theta_2, ...)$, on Σ is given by (Latin indices run over the induced coordinates i.e., $\{1, 2, 3, .., d-1\}$ and Greek indices run over the original manifold's coordinates i.e., $\{1, 2, 3, .., d\}$)

$$g_{ij} = \frac{\partial x^\alpha}{\partial \xi^i} \frac{\partial x^\beta}{\partial \xi^j} g_{\alpha\beta}. \tag{20}$$

Here τ is the proper time and

$$g_{ij} = \text{diag}\left(-1, a^2, a^2 \sin^2\theta, a^2 \sin^2\theta \sin^2\phi, ...\right), \tag{21}$$

while the extrinsic curvature is defined by

$$K_{ij}^\pm = -n_\gamma^\pm \left(\frac{\partial^2 x^\gamma}{\partial \xi^i \partial \xi^j} + \Gamma_{\alpha\beta}^\gamma \frac{\partial x^\alpha}{\partial \xi^i} \frac{\partial x^\beta}{\partial \xi^j}\right)_{r=a}. \tag{22}$$

It is assumed that Σ is non-null, whose unit d−normal in $M^\pm$ is given by

$$n_\gamma = \left(\pm \left| g^{\alpha\beta} \frac{\partial F}{\partial x^\alpha} \frac{\partial F}{\partial x^\beta} \right|^{-1/2} \frac{\partial F}{\partial x^\gamma}\right)_{r=a}, \tag{23}$$

in which F is the equation of the hypersurface Σ, i.e.

$$\Sigma : F(r) = r - a(\tau) = 0. \tag{24}$$

The generalized Darmois-Israel conditions on Σ determines the surface energy-momentum tensor S_{ab} which is expressed by [11]

$$S_i^j = -\frac{1}{8\pi}\left(\langle K_i^j \rangle - K\delta_i^j\right) - \frac{\alpha}{16\pi}\left\langle 3J_i^j - J\delta_i^j + 2P_{imn}{}^j K^{mn}\right\rangle. \tag{25}$$

Here a bracket implies a jump across Σ. The divergence-free part of the Riemann tensor P_{abcd} and the tensor J_{ab} (with trace $J = J_a^a$) are given by

$$P_{imnj} = R_{imnj} + (R_{mn}g_{ij} - R_{mj}g_{in}) - (R_{in}g_{mj} - R_{ij}g_{mn}) + \frac{1}{2}R(g_{in}g_{mj} - g_{ij}g_{mn}), \tag{26}$$

$$J_{ij} = \frac{1}{3}\left[2KK_{im}K_j^m + K_{mn}K^{mn}K_{ij} - 2K_{im}K^{mn}K_{nj} - K^2 K_{ij}\right]. \tag{27}$$

By employing these expressions through (25) we find the energy density and surface pressures for a generic metric function $f(r)$, with $r = a(\tau)$. The results are given by

$$\sigma = -S_\tau^\tau = -\frac{\Delta(d-2)}{8\pi}\left[\frac{2}{a} - \frac{4\tilde{\alpha}}{3a^3}\left(\Delta^2 - 3\left(1 + \dot{a}^2\right)\right)\right], \tag{28}$$

$$S_{\theta_i}^{\theta_i} = p = \frac{1}{8\pi}\left\{\frac{2(d-3)\Delta}{a} + \frac{2\ell}{\Delta} - \frac{4\tilde{\alpha}}{3a^2}\left[3\ell\Delta - \frac{3\ell}{\Delta}\left(1 + \dot{a}^2\right) + \frac{\Delta^3}{a}(d-5) - \frac{6\Delta}{a}\left(a\ddot{a} + \frac{d-5}{2}\left(1 + \dot{a}^2\right)\right)\right]\right\}, \tag{29}$$

where $\ell = \ddot{a} + f'(a)/2$ and $\Delta = \sqrt{f(a) + \dot{a}^2}$ in which

$$f(a) = f_-(r)|_{r=a}. \tag{30}$$

We note that in our notation a 'dot' denotes derivative with respect to the proper time τ and a 'prime' with respect to the argument of the function. It can be checked by direct substitution from (28) and (29) that the conservation equation

$$\frac{d}{d\tau}\left(\sigma a^{(d-2)}\right) + p\frac{d}{d\tau}\left(a^{(d-2)}\right) = 0. \tag{31}$$

holds true.

Once we know precisely the energy density and surface pressures, we can study the energy conditions and the amount of exotic / normal matter that is to support the above thin-shell wormhole. Let us start with the weak energy condition (WEC) which implies for any timelike vector V_μ we must have $T_{\mu\nu}V^\mu V^\nu \geq 0$. Also by continuity, WEC implies the null energy condition (NEC), which states that for any null vector U_μ, $T_{\mu\nu}U^\mu U^\nu \geq 0$ [2]. It is not difficult to show that in an orthonormal basis these conditions read as

$$\begin{aligned} WEC: \quad &\rho \geq 0, \ \rho + p_i \geq 0, \\ NEC \quad &\rho + p_i \geq 0, \end{aligned} \tag{32}$$

in which $i \in \{2, 3, ..., d-1\}$. Here in the spherical thin-shell wormholes, the radial pressure p_r is zero and $\rho = \delta(r-a)\sigma$ which imply WEC and NEC coincide as $\sigma \geq 0$. Note that $\delta(r-a)$ stands for the Dirac delta-function. By looking at σ given in (28) one may conclude that these conditions reduce to

$$\frac{3}{2}a^2 \leq \tilde{\alpha}\left(f(a) - 2\dot{a}^2 - 3\right). \tag{33}$$

For the static configuration with $\dot{a} = 0, \ddot{a} = 0$ and $a = a_0$ it is not difficult to see that for $\tilde{\alpha} \geq 0$ the latter condition is not satisfied. In other words, both WEC and NEC are violated. This is simply from the fact that the metric function is asymptotically flat and $f(a) < 1$ for $a \geq r_h$. Unlike $\tilde{\alpha} \geq 0$, for the case of $\tilde{\alpha} < 0$ this condition in arbitrary dimensions is satisfied. Direct consequence of these results can be seen in the total matter in supporting the thin-shell wormhole. The standard integral definition of the total matter is given by

$$\Omega = \int (\rho + p_r) \sqrt{-g} d^{d-1}x \tag{34}$$

which gives

$$\Omega = \frac{2\pi^{\frac{d-1}{2}} a_0^{d-2}}{\Gamma\left(\frac{d-1}{2}\right)} \sigma_0 \tag{35}$$

in which

$$\sigma_0 = -\frac{\sqrt{f(a_0)}(d-2)}{8\pi}\left[\frac{2}{a_0} - \frac{4\tilde{\alpha}}{3a_0^3}\left(f(a_0) - 3\right)\right]. \tag{36}$$

It is obvious from Ω that similar to σ_0, in static configuration the total matter which supports the thin-shell wormhole is exotic if $\tilde{\alpha} \geq 0$ and normal if $\tilde{\alpha} < 0$. This result is independent of dimensions and other parameters.

III. STABILITY OF THE THIN-SHELL WORMHOLES FOR $d \geq 5$

To study the stability of the thin-shell wormhole, constructed above, we consider a radial perturbation of the radius of the throat a. After the linear perturbation we may consider a linear relation between the energy density and radial pressure, namely [12]

$$p = p_0 + \beta^2(\sigma - \sigma_0). \tag{37}$$

Here the constant σ_0 is given by (36) and p_0 reads as

$$p_0 = \frac{\sqrt{f(a_0)}}{8\pi}\left\{\frac{2(d-3)}{a_0} + \frac{f'(a_0)}{f(a_0)} - \frac{4\tilde{\alpha}}{a_0^2}\left[\frac{f'(a_0)}{2} - \frac{f'(a_0)}{2f(a_0)} + \frac{f(a_0)(d-5)}{3a_0} - \frac{d-5}{a_0}\right]\right\}. \tag{38}$$

The constant parameter β^2 for the wormhole supported by normal matter is related to the speed of sound. By considering (37) in (31), one finds

$$\sigma(a) = \left(\frac{\sigma_0 - p_0}{\beta^2 + 1}\right)\left(\frac{a_0}{a}\right)^{(d-2)\left(\beta^2+1\right)} + \frac{\beta^2\sigma_0 - p_0}{\beta^2 + 1} \tag{39}$$

in which a_0 is the radius of the throat in static equilibrium wormhole and $\sigma_0(p_0)$ is the static energy density (pressure) on the thin-shell. By equating the latter expression and the one found by using Einstein equation on the shell (28), we find the equation of motion of the wormhole which reads

$$\dot{a}^2 + V(a) = 0, \tag{40}$$

where

$$V(a) = f(a) - \left(\left[\sqrt{\mathbb{A}^2 + \mathbb{B}^3} - \mathbb{A} \right]^{1/3} - \frac{\mathbb{B}}{\left[\sqrt{\mathbb{A}^2 + \mathbb{B}^3} - \mathbb{A} \right]^{1/3}} \right)^2 \tag{41}$$

and

$$\mathbb{A} = \frac{3\pi a^3}{2(d-2)\tilde{\alpha}} \left[\left(\frac{\sigma_0 + p_0}{\beta^2 + 1} \right) \left(\frac{a_0}{a} \right)^{(d-2)(\beta^2+1)} + \frac{\beta^2 \sigma_0 - p_0}{\beta^2 + 1} \right], \tag{42}$$

$$\mathbb{B} = \frac{a^2}{4\tilde{\alpha}} + \frac{1 - f(a)}{2}. \tag{43}$$

Here $V(a)$ is called the potential of the wormhole's motion and it helps us to figure out the regions of stability for the wormhole under our linear perturbation. According to the standard method of stability of thin-shell wormholes, we expand $V(a)$ as a series of $(a - a_0)$. One can show that both $V(a_0)$ and $V'(a_0)$ vanish and the first non-zero term in this expansion is $\frac{1}{2}V''(a_0)(a - a_0)^2$. Now, in a small neighborhood of the equilibrium point a_0 we have

$$\dot{a}^2 + \frac{1}{2}V''(a_0)(a - a_0)^2 = 0, \tag{44}$$

which implies that with $V''(a_0) > 0$, $a(\tau)$ will oscillate about a_0 and make the wormhole stable. At this point it will be in order also to clarify the status of parameter β since ultimately the three-dimensional (i.e. $V''(a_0) > 0$, β, a_0) stability plots will make use of it. First of all although in principle $\beta < 0$ is possible we shall restrict ourselves only to the case $\beta > 0$. Unfortunately β can only be expressed implicitly as a function of a_0, through (37) and expressions for p, σ, p_0 and σ_0. It turns out that the usual expression for stability, namely $V''(a_0) > 0$, can be plotted as a projection onto the plane formed by β and a_0. This must not give the impression, however, that the relation $\beta = \beta(a_0)$ is known explicitly.

A. $d = 5$

Let us first eliminate α from the equations, by using the solution given in (8). To do so we introduce new variables and parameters as

$$\tilde{a} = \frac{a}{\sqrt{|\alpha|}}, \tilde{\tau} = \frac{\tau}{\sqrt{|\alpha|}}, \tilde{Q}^2 = \frac{Q^2}{|\alpha|},$$

$$\tilde{m} = \frac{2M_{ADM}}{3|\alpha|} + \frac{Q^2}{2|\alpha|} \ln|\alpha|. \tag{45}$$

Upon these changes of variables, the other quantities change according to

$$f(a) = f(\tilde{a}), \sigma(a) = \frac{\sigma(\tilde{a})}{\sqrt{|\alpha|}}, p(a) = \frac{p(\tilde{a})}{\sqrt{|\alpha|}},$$

$$\mathbb{A}(a) = \mathbb{A}(\tilde{a}), \mathbb{B}(a) = \mathbb{B}(\tilde{a}), V(a) = V(\tilde{a}). \tag{46}$$

Finally the wormhole equation reads

$$\left(\frac{d\tilde{a}}{d\tilde{\tau}} \right)^2 + \tilde{V}(\tilde{a}) = 0. \tag{47}$$

Now, we consider two distinct cases, for $\alpha > 0$ and $\alpha < 0$, separately.

1. with $\alpha > 0$

In this section we consider $\alpha > 0$, such that the negative branch of the EYM black hole solution reads

$$f(\tilde{a}) = 1 + \frac{\tilde{a}^2}{4} \left(1 - \sqrt{1 + \frac{16\tilde{m}}{\tilde{a}^4} + \frac{16\tilde{Q}^2 \ln \tilde{a}}{\tilde{a}^4}} \right), \tag{48}$$

in which the condition

$$1 + \frac{16\tilde{m}}{\tilde{a}^4} + \frac{16\tilde{Q}^2 \ln \tilde{a}}{\tilde{a}^4}\bigg|_{\tilde{a}=\tilde{a}_0} \geq 0, \tag{49}$$

and

$$\mathbb{A}^2 + \mathbb{B}^3\big|_{\tilde{a}=\tilde{a}_0} \geq 0 \tag{50}$$

must hold. The latter equation automatically is valid and the final relation between the parameters reduces to (49). Based on this solution we find $\tilde{V}''(\tilde{a}_0)$ in terms of the other parameters. Fig. 1 shows the stability regions and also $f(\tilde{a})$ and $\sigma(\tilde{a}_0)$.

2. with $\alpha < 0$

Next, we concentrate on the case $\alpha < 0$. With this choice negative branch of the EYM black hole solution reads

$$f(\tilde{a}) = 1 - \frac{\tilde{a}^2}{4} \left(1 - \sqrt{1 - \frac{16\tilde{m}}{\tilde{a}^4} - \frac{16\tilde{Q}^2 \ln \tilde{a}}{\tilde{a}^4}} \right). \tag{51}$$

Based on this solution we study $\tilde{V}''(\tilde{a}_0)$ in terms of the other parameters. Fig.s 4 and 5 show the stability regions and also $f(\tilde{a})$ and $\sigma(\tilde{a}_0)$.

In this case also we have some constraint on the parameters in order to get $f(\tilde{a}_0) \geq 0$, $\sigma(\tilde{a}_0) \geq 0$, and $\mathbb{A}^2 + \mathbb{B}^3\big|_{\tilde{a}=\tilde{a}_0} \geq 0$. It is not difficult to see that all these conditions reduce to

$$0 \leq \frac{1}{4} \sqrt{1 - \frac{16\tilde{m}}{\tilde{a}_0^4} - \frac{16\tilde{Q}^2 \ln \tilde{a}_0}{\tilde{a}_0^4}} \leq \frac{4}{\tilde{a}_0^2} - 1, \tag{52}$$

and

$$1 - \frac{16\tilde{m}}{\tilde{a}_0^4} - \frac{16\tilde{Q}^2 \ln \tilde{a}_0}{\tilde{a}_0^4} \geq 0. \tag{53}$$

After some manipulation, the parameters must satisfy the following constraint

$$\tilde{a}_0^4 \geq 16 \left(\tilde{m} + \tilde{Q}^2 \ln \tilde{a}_0^2 \right) \tag{54}$$

where $0 \leq \tilde{a}_0^2 \leq 4$. The stability region for this case is given in Fig. 2.

B. $d \geq 6$

Here also we eliminate $\tilde{\alpha}$ from the equations. By introducing

$$\tilde{a} = \frac{a}{\sqrt{|\tilde{\alpha}|}}, \tilde{\tau} = \frac{\tau}{\sqrt{|\tilde{\alpha}|}}, \tilde{Q}^2 = \frac{Q^2}{|\tilde{\alpha}|},$$

$$\tilde{m} = \frac{M_{ADM}}{|\tilde{\alpha}|^{\frac{d-3}{2}}}. \tag{55}$$

the other quantities become

$$f(a) = f(\tilde{a}), \sigma(a) = \frac{\sigma(\tilde{a})}{\sqrt{|\alpha|}}, p(a) = \frac{p(\tilde{a})}{\sqrt{|\alpha|}},$$

$$\mathbb{A}(a) = \mathbb{A}(\tilde{a}), \mathbb{B}(a) = \mathbb{B}(\tilde{a}), V(a) = V(\tilde{a}), \tag{56}$$

and the wormhole equation is given by

$$\left(\frac{d\tilde{a}}{d\tilde{\tau}} \right)^2 + \tilde{V}(\tilde{a}) = 0. \tag{57}$$

1. with $\alpha > 0$

In this section we consider $\alpha > 0$, such that the negative branch of the EYMGB black hole solution reads

$$f_-(\tilde{a}) = 1 + \frac{\tilde{a}^2}{2}\left(1 - \sqrt{1 + \frac{16\tilde{m}}{\tilde{a}^{d-1}(d-2)} + \frac{4(d-3)\tilde{Q}^2}{(d-5)\tilde{a}^4}}\right) \tag{58}$$

Here we comment that constraints always restrict our free parameters. In the case of $\alpha > 0$ the first constraint is given by

$$\mathbb{A}^2 + \mathbb{B}^3\big|_{\tilde{a}=\tilde{a}_0} \geq 0, \tag{59}$$

which upon substitution and manipulation automatically is satisfied for all value of parameters. Based on this solution we find $\tilde{V}''(\tilde{a}_0)$ in terms of the other parameters. Fig.s 3-5 shows the stability regions and also $f(\tilde{a})$ and $\sigma(\tilde{a}_0)$ for dimensions $d = 6, 7$ and 8.

2. with $\alpha < 0$

Next, we concentrate on the case $\alpha < 0$. With this choice negative branch of the EYMGB black hole solution reads

$$f_-(\tilde{a}) = 1 - \frac{\tilde{a}^2}{2}\left(1 - \sqrt{1 - \frac{16\tilde{m}}{\tilde{a}^{d-1}(d-2)} - \frac{4(d-3)\tilde{Q}^2}{(d-5)\tilde{a}^4}}\right). \tag{60}$$

Based on this solution we study $\tilde{V}''(\tilde{a}_0)$ in terms of the other parameters. Fig. 6 show the stability regions and also $f(\tilde{a})$ and $\sigma(\tilde{a}_0)$. In order to set $f(\tilde{a}_0) \geq 0$, $\sigma(\tilde{a}_0) \geq 0$, and $\mathbb{A}^2 + \mathbb{B}^3\big|_{\tilde{a}=\tilde{a}_0} \geq 0$ it is enough to satisfy

$$0 < \sqrt{1 - \frac{16\tilde{m}}{\tilde{a}_0^{d-1}(d-2)} - \frac{4(d-3)\tilde{Q}^2}{(d-5)\tilde{a}_0^4}} < \frac{2}{\tilde{a}_0^2} - 1, \tag{61}$$

and

$$1 - \frac{16\tilde{m}}{\tilde{a}^{d-1}(d-2)} - \frac{4(d-3)\tilde{Q}^2}{(d-5)\tilde{a}^4} > 0, \tag{62}$$

where $0 < \tilde{a}_0^2 < 2$.

IV. CONCLUSION

We have investigated the possibility of thin-shell wormholes in EYMGB theory in higher ($d \geq 5$) dimensions with particular emphasis on stability against spherical, linear perturbations and normal (i.e. non-exotic) matter. For this purpose we made use of the previously obtained solutions that are valid in all dimensions. The case $d = 5$ is considered separately from the cases $d > 5$ because the solution involves a logarithmic term apart from the power-law dependence. For $d = 5$ we observe (Fig. 2) the formation of a narrow band of positive energy region that attains a stable wormhole only for $\alpha < 0$. On the contrary, for $\alpha > 0$ although a large region of stability (i.e. $V''(a_0) > 0$) forms, the energy turns out to be exotic. For $d > 5$ also, we have more or less a similar picture. That is, whenever the GB parameter $\alpha > 0$, negative energy shows itself versus the stability requirements. We have analyzed the cases $d = 6, 7$ and 8 as examples. Our technique is powerful enough to apply in any higher dimensions, however, for technical reasons we had to be satisfied with these selected dimensions. We must admit also that for non-spherical perturbations a similar analysis remains to be seen. In our study we were able also to observe a stability region which employs $0 < \beta < 1$, which can be interpreted as a case corresponding to less than the speed of light. In conclusion, we state that formation of stable, positive energy thin-shell wormholes in EYMGB are possible only with a GB parameter $\alpha < 0$. Without the GB term whatever source is available the situation is always worse. The indispensable character of the GB parameter

toward useful wormhole constructions invites naturally the Lovelock hierarchy [5] for which GB term constitutes the first member.

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(This reference contains unfortunate errors which invalidate its overall conclusions. Eq.s (15) and (19) for instance, should read

$$\sigma \;=\; -S_\tau^\tau = -\frac{1}{8\pi}\left[\frac{6\sqrt{B(b)}}{b} - 2\alpha\sqrt{B(b)}\left(\frac{4B(b)}{b^3} - \frac{12}{b^3}\right)\right],$$

$$W \;=\; \pi^2 b^3 \sigma = -\frac{3\pi b^2\sqrt{B(b)}}{4} + \pi\alpha\sqrt{B(b)}\left(B(b) - 3\right)$$

respectively.)
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Figure captions:

Figure 1: Region of stability (i.e. $V''(a_0) > 0$) for the thin-shell in $d = 5$ and for $\alpha > 0$. The $f(r)$ and σ_0 plots are also given. It can easily be seen that the energy density σ_0 is negative which implies exotic matter.

Figure 2: For $d = 5$ and $\alpha < 0$ case with the chosen parameters $f(r)$ has no zero but σ_0 has a small band of positivity with the presence of normal matter. We note also that $\beta < 1$ in a small band.

Figure 3: For $d = 7$ and $\alpha > 0$ the stability region is plotted which is seen to have exotic matter alone.

Figure 4: For $d = 6$ and $\alpha > 0$ also a region of stability is available but with $\sigma_0 < 0$. Note that $d = 6$ is special, since from Eq. (5) in the text we have $\kappa = 0$ and the energy-momentum takes a simple form.

Figure 5: For $d = 8$ with $\alpha > 0$ exotic matter is seen to be indispensable.

Figure 6: For $d = 6$ with $\alpha < 0$ there are two disjoint regions of stability for the thin-shell and in contrast to the $\alpha > 0$ case in Fig. 4, we have $\sigma_0 > 0$. We notice in this case also that $\beta < 1$ is possible.

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