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Stability of thin-shell wormholes supported by normal matter in Einstein-Maxwell-Gauss-Bonnet gravity

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Recently in [Phys. Rev. D **76**, 087502 (2007) and Phys. Rev. D **77**, 089903 (2008)] a thin-shell wormhole has been introduced in five-dimensional Einstein-Maxwell-Gauss-Bonnet gravity which was supported by normal matter. We wish to consider this solution and investigate its stability. Our analysis shows that for the Gauss-Bonnet parameter $\alpha < 0$, stability regions form for a narrow band of finely tuned mass and charge. For the case $\alpha > 0$, we iterate once more that no stable, normal matter thin-shell wormhole exists.

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I. INTRODUCTION

Whenever the agenda is about wormholes, exotic matter (i.e. matter violating the energy conditions) continues to occupy a major issue in general relativity [1]. It is a fact that Einstein's equations admit wormhole solutions that require such matter for its maintenance. In quantum theory, temporary violation of energy conditions is permissible but in classical physics this can hardly be justified. One way to minimize such exotic matter, even if we can not ignore it completely, is to concentrate it on a thin shell. This seemed feasible, because general relativity admits such thin-shell solutions and by employing these shells at the throat region may provide the necessary repulsion to support the wormhole against collapse. The ultimate aim, of course, is to get rid of exotic matter completely, no matter how small. In the four-dimensional general relativity with a cosmological term, however, such a dream never turned into reality. For this reason the next search should naturally cover extensions of general relativity to higher dimensions and with additional structures. One such possibility that received a great deal of attention in recent years, for a number of reasons, is the Gauss-Bonnet (GB) extension of general relativity [2]. In the braneworld scenario our universe is modeled as a brane in a 5D bulk universe in which the higher order curvature terms, and therefore the GB gravity comes in naturally. Einstein-Gauss-Bonnet gravity, with additional sources such as Maxwell, Yang-Mills, dilaton, etc., has already been investigated extensively in the literature [3]. Not to mention that all these theories also admit black hole, wormhole [4], and other physically interesting solutions. As it is the usual trend in theoretical physics, each new parameter invokes new hopes and from that token, the GB parameter α does the same. Although the case $\alpha > 0$, has been exalted much more than the case $\alpha < 0$ in Einstein-Gauss-Bonnet gravity so far [5] (and references cited therein), it turns out here in the stable, normal matter thin-shell wormholes that the latter comes first time to the fore.

Construction and maintenance of thin-shell wormholes has been the subject of a large literature, so that we shall provide only a brief review here. Instead, one class [6] that made use of nonexotic matter for its maintenance attracted our interest and we intend to analyze its stability in this paper. This is the 5D thin-shell solution of Einstein-Maxwell-Gauss-Bonnet (EMGB) gravity, whose radius is identified with the minimum radius of the wormhole. For this purpose we employ radial, linear perturbations to cast the motion into a potential-well problem in the background. In doing this, a reasonable assumption employed routinely, which is adopted here also, is to relate pressure and energy density by a linear expression [7]. For special choices of parameters we obtain islands of stability for such wormholes. To this end, we make use of numerical computation and plotting since the problem involves highly intricate functions for an analytical treatment.

The paper is organized as follows. In Sec. II the five-dimensional (5D) EMGB thin-shell wormhole formalism has been reviewed briefly. We perturb the wormhole through radial linear perturbation and cast the problem into a potential-well problem in Sec. III. In Sec. IV we impose constraint conditions on parameters to determine possible stable regions through numerical analysis. The paper ends with a Conclusion which appears in Sec. V.

II. A BRIEF REVIEW OF 5D EMGB THIN-SHELLS

The action of EMGB gravity in 5D (without cosmological constant, i.e. $\Lambda = 0$) is

$$S = \kappa \int \sqrt{|g|} d^5x \left(R + \alpha \mathcal{L}_{\mathrm{GB}} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) \quad (1)$$

in which κ is related to the 5D Newton constant and α is the GB parameter. Beside the Maxwell Lagrangian the GB Lagrangian $\mathcal{L}_{\mathrm{GB}}$ consists of the quadratic scalar invariants in the combination

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$$\mathcal{L}_{\rm GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \tag{2}$$

in which $R =$ scalar curvature, $R_{\mu\nu} =$ Ricci tensor, and $R_{\mu\nu\rho\sigma} =$ Riemann tensor. Variational principle of S with respect to $g_{\mu\nu}$ yields

$$G_{\mu\nu} + 2\alpha H_{\mu\nu} = \kappa^2 T_{\mu\nu} \tag{3}$$

where the Lovelock ($H_{\mu\nu}$) and Maxwell ($T_{\mu\nu}$) tensors, respectively, are

$$H_{\mu\nu} = 2(-R_{\mu}{}^{\sigma\kappa\tau}R_{\nu\sigma\kappa\tau} - 2R_{\mu\rho\nu\sigma}R^{\rho\sigma} - 2R_{\mu\sigma}R^{\sigma}{}_{\nu} + RR_{\mu\nu}) - \tfrac{1}{2}g_{\mu\nu}\mathcal{L}_{\rm GB}, \tag{4}$$

$$T_{\mu\nu} = F_{\mu\alpha}F^{\alpha}_{\nu} - \tfrac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}. \tag{5}$$

The Einstein tensor $G_{\mu\nu}$ is to be found from our metric ansatz

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta(d\phi^2 + \sin^2\phi d\psi^2)), \tag{6}$$

in which $f(r)$ will be determined from (3). A thin-shell wormhole is constructed in EMGB theory as follows. Two copies of the spacetime are chosen from which the regions

$$M_{1,2} = \{r_{1,2} \le a, a > r_h\} \tag{7}$$

are removed. We note that a will be identified in the sequel as the radius of the thin shell and r_h stands for the event horizon radius. (Note that our notation a corresponds to b in Ref. [6]. Other notations all agree with those in Ref. [6]). The boundary, timelike surface $\Sigma_{1,2}$ of each $M_{1,2}$, accordingly will be

$$\Sigma_{1,2} = \{r_{1,2} = a, a > r_h\}. \tag{8}$$

Next, these surfaces are identified on $r = a$ with a surface energy-momentum of a thin shell such that geodesic completeness holds. Following the Darmois-Israel formalism [8] in terms of the original coordinates $x^\gamma = (t, r, \theta, \phi, \psi)$, we define $\xi^a = (\tau, \theta, \phi, \psi)$, with τ the proper time. The GB extension of the thin-shell Einstein-Maxwell theory requires further modifications. This entails the generalized Darmois-Israel boundary conditions [9], where the surface energy-momentum tensor is expressed by $S_a^b = \mathrm{diag}(\sigma, p_\theta, p_\phi, p_\psi)$. We are interested in the thin-shell geometry whose radius is assumed a function of τ, so that the hypersurface becomes

$$\Sigma\colon f(r,\tau) = r - a(\tau) = 0. \tag{9}$$

The generalized Darmois-Israel conditions on Σ take the form

$$2\langle K_{ab} - Kh_{ab}\rangle + 4\alpha\langle 3J_{ab} - Jh_{ab} + 2P_{acdb}K^{cd}\rangle = -\kappa^2 S_{ab}, \tag{10}$$

where a bracket implies a jump across Σ, and $h_{ab} = g_{ab} - n_a n_b$ is the induced metric on Σ with normal vector n_a and the coordinate set $\{X^a\}$. K_{ab} is the extrinsic curvature (with trace K), defined by

$$K^{\pm}_{ab} = -n^{\pm}_c\left(\frac{\partial^2 X^c}{\partial\xi^a\partial\xi^b} + \Gamma^c_{mn}\frac{\partial X^m}{\partial\xi^a}\frac{\partial X^n}{\partial\xi^b}\right)_{r=a}. \tag{11}$$

The remaining expressions are as follows. The divergence-free part of the Riemann tensor P_{abcd} and the tensor J_{ab} (with trace J) are given by

$$P_{abcd} = R_{abcd} + (R_{bc}h_{da} - R_{bd}h_{ca}) - (R_{ac}h_{db} - R_{ad}h_{cb}) + \tfrac{1}{2}R(h_{ac}h_{db} - h_{ad}h_{cb}), \tag{12}$$

$$J_{ab} = \tfrac{1}{3}[2KK_{ac}K^c_b + K_{cd}K^{cd}K_{ab} - 2K_{ac}K^{cd}K_{ab} - K^2K_{ab}]. \tag{13}$$

The EMGB solution that will be employed as a thin-shell solution with a normal matter [6] is given by (with $\Lambda = 0$)

$$f(r) = 1 + \frac{r^2}{4\alpha}\left(1 - \sqrt{1 + \frac{8\alpha}{r^4}\left(\frac{2M}{\pi} - \frac{Q^2}{3r^2}\right)}\right) \tag{14}$$

with constants, $M =$ mass and $Q =$ charge. For a black hole solution the inner (r_-) and event horizons ($r_+ = r_h$) are

$$r_{\pm} = \sqrt{\frac{M}{\pi} - \alpha \pm \left[\left(\frac{M}{\pi} - \alpha\right)^2 - \frac{Q^2}{3}\right]^{1/2}}. \tag{15}$$

By employing the solution (14) we determine the surface energy-momentum on the thin-shell, which will play the major role in the perturbation. We shall address this problem in the next section.

III. RADIAL, LINEAR PERTURBATION OF THE THIN-SHELL WORMHOLE WITH NORMAL MATTER

In order to study the radial perturbations of the wormhole we take the throat radius as a function of the proper time, i.e., $a = a(\tau)$. Based on the generalized Birkhoff theorem, for $r > a(\tau)$ the geometry will be given still by (6). For the metric function $f(r)$ given in (14) one finds the energy density and pressures as [6]

$$\sigma = -S^\tau_\tau = -\frac{\Delta}{4\pi}\left[\frac{3}{a} - \frac{4\alpha}{a^3}(\Delta^2 - 3(1 + \dot a^2))\right], \tag{16}$$

$$S^\theta_\theta = S^\phi_\phi = S^\psi_\psi = p = \frac{1}{4\pi}\left[\frac{2\Delta}{a} + \frac{\ell}{\Delta} - \frac{4\alpha}{a^2}\left(\ell\Delta - \frac{\ell}{\Delta}(1 + \dot a^2) - 2\ddot a\Delta\right)\right], \tag{17}$$

where $\ell = \ddot a + f'(a)/2$ and $\Delta = \sqrt{f(a) + \dot a^2}$ in which

$$f(a) = 1 + \frac{a^2}{4\alpha}\left(1 - \sqrt{1 + \frac{8\alpha}{a^4}\left(\frac{2M}{\pi} - \frac{Q^2}{3a^2}\right)}\right). \tag{18}$$

We note that in our notation a "dot" denotes derivative with respect to the proper time τ and a "prime" implies differentiation with respect to the argument of the function. By a simple substitution one can show that, the conservation equation

$$\frac{d}{d\tau}(\sigma a^3) + p\frac{d}{d\tau}(a^3) = 0. \tag{19}$$

is satisfied. The static configuration of radius a_0 has the following density and pressures:

$$\sigma_0 = -\frac{\sqrt{f(a_0)}}{4\pi}\left[\frac{3}{a_0} - \frac{4\alpha}{a_0^3}(f(a_0) - 3)\right], \tag{20}$$

$$p_0 = \frac{\sqrt{f(a_0)}}{4\pi}\left[\frac{2}{a_0} + \frac{f'(a_0)}{2f(a_0)} - \frac{2\alpha}{a_0^2}\frac{f'(a_0)}{f(a_0)}(f(a_0) - 1)\right]. \tag{21}$$

In what follows we shall study small radial perturbations around the radius of equilibrium a_0. To this end we adapt a linear relation between p and σ as [7]

$$p = p_0 + \beta^2(\sigma - \sigma_0). \tag{22}$$

Here since we are interested in the wormholes which are supported by normal matter, β^2 is the speed of sound. By virtue of Eqs. (19) and (22) we find the energy density in the form

$$\sigma(a) = \left(\frac{\sigma_{0+}p_0}{\beta^2 + 1}\right)\left(\frac{a_0}{a}\right)^{3(\beta^2+1)} + \frac{\beta^2\sigma_{0-}p_0}{\beta^2 + 1}. \tag{23}$$

This, together with (16) lead us to the equation of motion for the radius of the throat, which reads

$$-\frac{\sqrt{f(a) + \dot{a}^2}}{4\pi}\left[\frac{3}{a} - \frac{4\alpha}{a^3}(f(a) - 3 - 2\dot{a}^2)\right] = \left(\frac{\sigma_{0+}p_0}{\beta^2 + 1}\right)\left(\frac{a_0}{a}\right)^{3(\beta^2+1)} + \frac{\beta^2\sigma_{0-}p_0}{\beta^2 + 1}. \tag{24}$$

After some manipulation this can be cast into

$$\dot{a}^2 + V(a) = 0, \tag{25}$$

where

$$V(a) = f(a) - \left([\sqrt{A^2 + B^3} - A]^{1/3} - \frac{B}{[\sqrt{A^2 + B^3} - A]^{1/3}}\right)^2 \tag{26}$$

in which the functions A and B are

$$A = \frac{\pi a^3}{4\alpha}\left[\left(\frac{\sigma_{0+}p_0}{\beta^2 + 1}\right)\left(\frac{a_0}{a}\right)^{3(\beta^2+1)} + \frac{\beta^2\sigma_{0-}p_0}{\beta^2 + 1}\right], \tag{27}$$

$$B = \frac{a^2}{8\alpha} + \frac{1 - f(a)}{2}. \tag{28}$$

We notice that $V(a)$, and more tediously $V'(a)$, both vanish at $a = a_0$. The stability requirement for equilibrium reduces therefore to the determination of $V''(a_0) > 0$, and it is needless to add that, $V(a)$ is complicated enough for an immediate analytical result. For this reason we shall proceed through numerical calculation to see whether stability regions or islands develop or not. Since the hopes for obtaining thin-shell wormholes with normal matter when $\alpha > 0$, have already been dashed [5], we shall investigate here only the case for $\alpha < 0$.

In order to analyze the behavior of $V(a)$ (and its second derivative) we introduce new parametrization as follows:

$$\tilde{a}^2 = -\frac{a^2}{\alpha}, \qquad m = -\frac{16M}{\pi\alpha}, \qquad q^2 = \frac{8Q^2}{3\alpha^2}, \qquad \tilde{\sigma}_0 = \sqrt{-\alpha}\sigma_0, \qquad p_0 = \sqrt{-\alpha}p_0. \tag{29}$$

Accordingly, our new variables $f(\tilde{a})$, $\tilde{\sigma}_0$, $\tilde{p}_0$, A, and B take the following forms:

$$f(\tilde{a}) = 1 - \frac{\tilde{a}^2}{4} + \frac{\tilde{a}^2}{4}\sqrt{1 - \frac{m}{\tilde{a}^4} + \frac{q^2}{\tilde{a}^6}} \tag{30}$$

and

$$\tilde{\sigma}_0 = -\frac{\sqrt{f(\tilde{a}_0)}}{4\pi}\left[\frac{3}{\tilde{a}_0} + \frac{4}{\tilde{a}_0^3}(f(\tilde{a}_0) - 3)\right], \tag{31}$$

$$\tilde{p}_0 = \frac{\sqrt{f(\tilde{a}_0)}}{4\pi}\left[\frac{2}{\tilde{a}_0} + \frac{f'(\tilde{a}_0)}{2f(\tilde{a}_0)} + \frac{2}{\tilde{a}_0^2}\frac{f'(\tilde{a}_0)}{f(\tilde{a}_0)}(f(\tilde{a}_0) - 1)\right], \tag{32}$$

$$A = -\frac{\pi\tilde{a}^3}{4}\left[\left(\frac{\tilde{\sigma}_{0+}\tilde{p}_0}{\beta^2 + 1}\right)\left(\frac{\tilde{a}_0}{\tilde{a}}\right)^{3(\beta^2+1)} + \frac{\beta^2\tilde{\sigma}_{0-}\tilde{p}_0}{\beta^2 + 1}\right], \tag{33}$$

$$B = -\frac{\tilde{a}^2}{8} + \frac{1 - f(\tilde{a})}{2}. \tag{34}$$

Following this parametrization our Eq. (25) takes the form

$$\left(\frac{d\tilde{a}}{d\tau}\right)^2 + \tilde{V}(\tilde{a}) = 0, \tag{35}$$

where

$$\tilde{V}(\tilde{a}) = -\frac{V(\tilde{a})}{\alpha}. \tag{36}$$

In the next section we explore all possible constraints on our parameters that must satisfy to materialize a stable, normal matter wormhole through the requirement $V''(\tilde{a}) > 0$.

IV. CONSTRAINTS VERSUS FINELY TUNED PARAMETERS AND SECOND DERIVATIVE PLOTS OF THE POTENTIAL

(i) Starting from the metric function we must have

$$1-\frac{m}{\tilde{a}_0^4}+\frac{q^2}{\tilde{a}_0^6}\geq 0. \tag{37}$$

(ii) In the potential, the reality condition requires also that

$$A^2+B^3\geq 0. \tag{38}$$

At the location of the throat this amounts to

$$\left(-\frac{\pi\tilde{a}_0^3}{4}\tilde{\sigma}_0\right)^2+\left(-\frac{\tilde{a}_0^2}{8}+\frac{1-f(\tilde{a}_0)}{2}\right)^3\geq 0 \tag{39}$$

or after some manipulation it yields

$$f(\tilde{a}_0)-2+\frac{\tilde{a}_0^2}{2}\leq 0. \tag{40}$$

This is equivalent to

$$0\leq 1-\frac{m}{\tilde{a}_0^4}+\frac{q^2}{\tilde{a}_0^6}\leq\left(\frac{4}{\tilde{a}_0^2}-1\right)^2. \tag{41}$$

(iii) Our last constraint condition concerns, the positivity of the energy density, which means that

$$\tilde{\sigma}_0>0. \tag{42}$$

This implies, from (31) that

$$\left[\frac{3}{\tilde{a}_0}+\frac{4}{\tilde{a}_0^3}(f(\tilde{a}_0)-3)\right]<0 \tag{43}$$

or equivalently

$$0\leq 1-\frac{m}{\tilde{a}_0^4}+\frac{q^2}{\tilde{a}_0^6}<4\left(\frac{4}{\tilde{a}_0^2}-1\right)^2. \tag{44}$$

It is remarkable to observe now that the foregoing constraints (i–iii) on our parameters can all be expressed as a single constraint condition, namely,

$$0\leq 1-\frac{m}{\tilde{a}_0^4}+\frac{q^2}{\tilde{a}_0^6}\leq\left(\frac{4}{\tilde{a}_0^2}-1\right)^2. \tag{45}$$

We plot $\tilde{V}''(\tilde{a})$ from (26) for various fixed values of mass and charge, as a projection into the plane with coordinates β and $\tilde{a}_0$. In other words, we search and identify the regions for which $\tilde{V}''(\tilde{a})>0$, in three-dimensional figures considered as a projection in the $(\beta, \tilde{a}_0)$ plane. The metric function $f(r)$ and energy density $\tilde{\sigma}_0>0$, behavior also are given in Figs. 1–4. It is evident from Figs. 1–4 that for increasing charge the stability regions shrink to smaller domains and tends ultimately to disappear completely. For smaller $\tilde{a}_0$ bounds we obtain fluctuations in $\tilde{V}''(\tilde{a})$, which is smooth otherwise.

In each plot it is observed that the maximum of $\tilde{V}''(\tilde{a})$ occurs at the right-below corner (say, at $a_{\max}$) which decreases to the left (with $\tilde{a}_0$) and in the upward direction (with β). Beyond certain limit (say $a_{\min}$), the region of instability takes the start. The proper time domain of

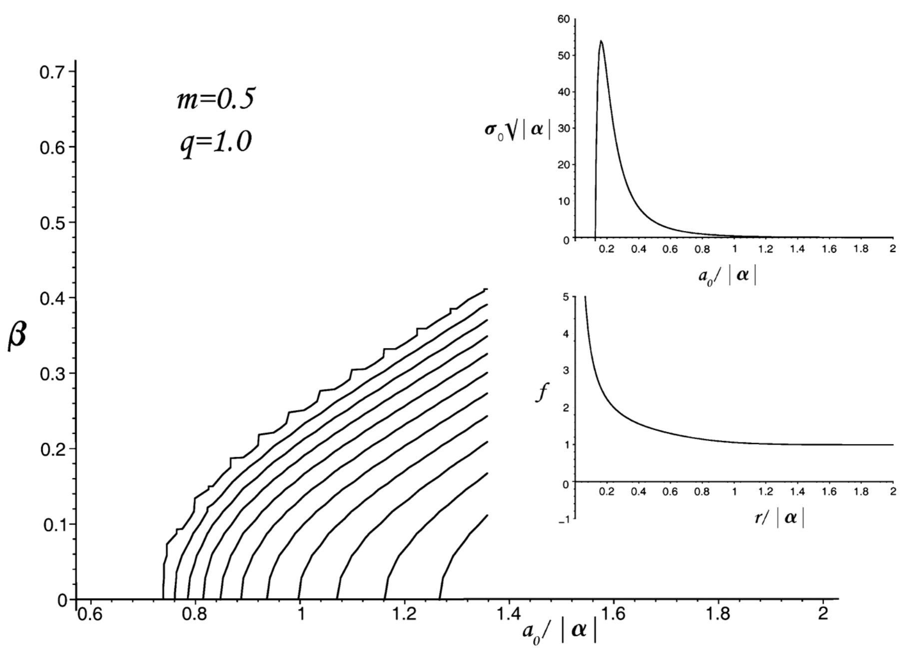

FIG. 1. $\tilde{V}''(\tilde{a})>0$ region ($m=0.5$, $q=1.0$) for various ranges of β and $\tilde{a}_0$. The lower and upper limits of the parameters are evident in the figure. The metric function $f(\tilde{r})$ and $\tilde{\sigma}_0>0$, are also indicated in the smaller figures.

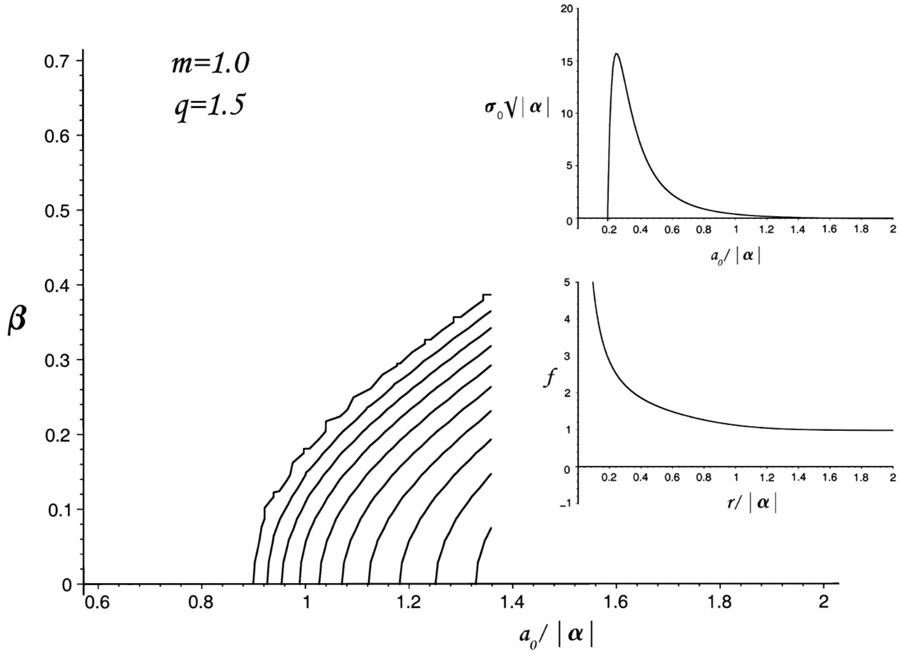

FIG. 2. $\tilde{V}''(\tilde{a}) > 0$ plot for $m = 1.0$, $q = 1.5$. The stability region is seen clearly to shrink with the increasing charge. This effect reflects also to the $\tilde{\sigma}_0 > 0$, behavior.

stability can be computed from (35) as

$$\Delta\tau = \int_{a_{\min}}^{a_{\max}} \frac{d\tilde{a}}{\sqrt{-V(\tilde{a})}}. \tag{46}$$

From a distant observer's point of view the timespan Δt can be found by using the radial geodesics Lagrangian which admits the energy integral

$$f\left(\frac{dt}{d\tau}\right) = E_\circ = \text{const.} \tag{47}$$

This gives the lifetime of each stability region determined by

$$\Delta t = \frac{1}{E_\circ}\int_{a_{\min}}^{a_{\max}} \frac{d\tilde{a}}{f(\tilde{a})\sqrt{-V(\tilde{a})}}. \tag{48}$$

Once $a_{\min}$ ($a_{\max}$) are found numerically, assuming that no

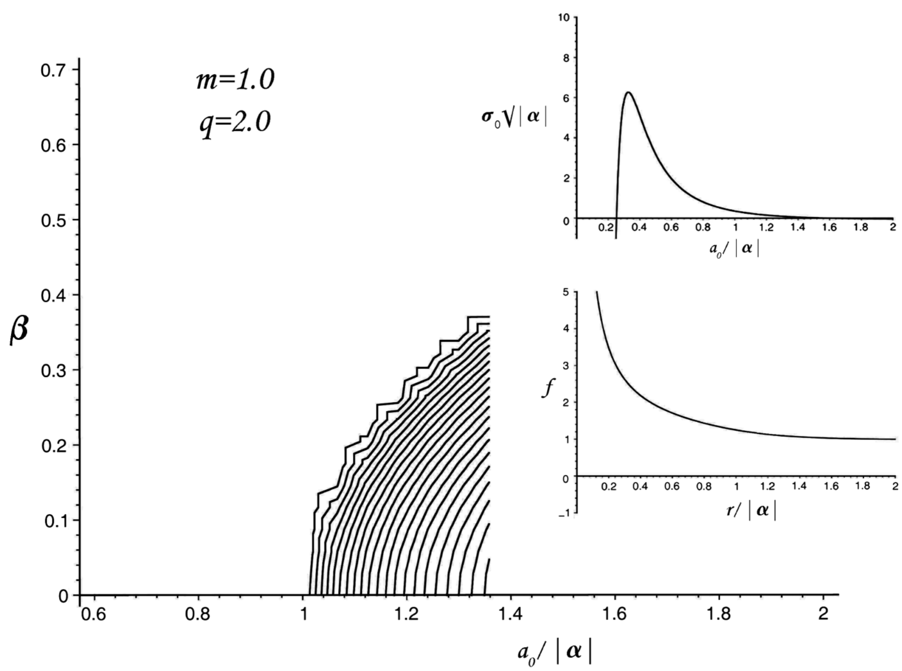

FIG. 3. The stability region for $m = 1.0$, $q = 2.0$, is seen to shift outward and get smaller.

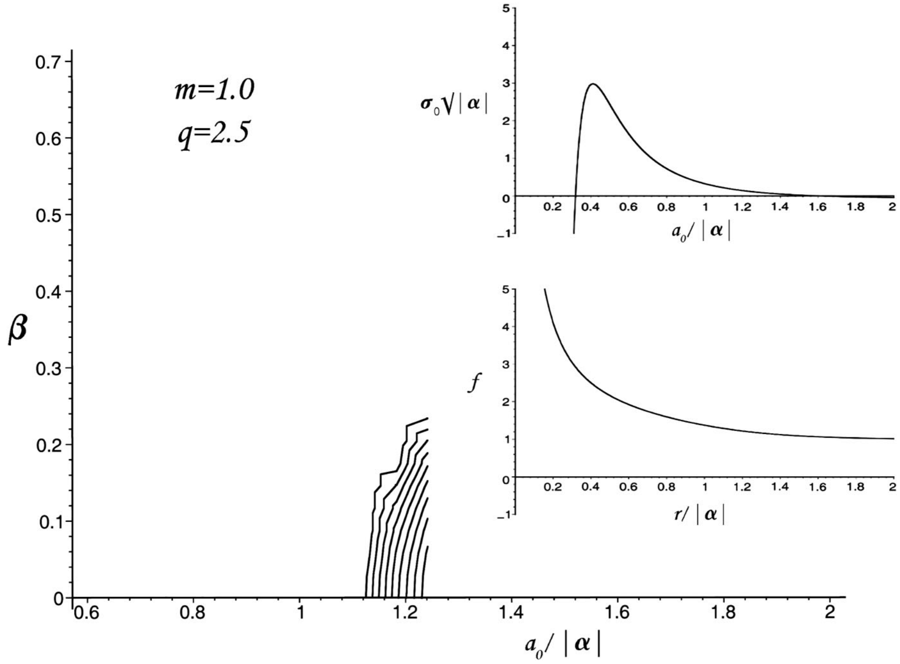

FIG. 4. For fixed mass $m = 1.0$ but increased charge $q = 2.5$ it is clearly seen that the stability region and the associated energy density both get further reduced.

zeros of $f(\tilde{a})$ and $V(\tilde{a})$ occurs for $a_{\min} < a < a_{\max}$, the lifespan of each stability island can be determined. We must admit that the mathematical complexity discouraged us to search for possible metastable region that may be triggered by employing a semiclassical treatment.

V. CONCLUSION

Our numerical analysis shows that for $\alpha < 0$, and specific ranges of mass and charge the 5D EMGB thin-shell wormholes with normal matter can be made stable against linear, radial perturbations. The fact that for $\alpha > 0$ there are no such wormholes is well known. The magnitude of α is irrelevant to the stability analysis. This reflects the universality of wormholes in parallel with black holes, i.e., the fact that they arise at each scale. Stable regions develop for each set of finely tuned parameters which determine the lifespan of each such region. Beyond those regions instability takes the start. Our study concerns entirely the exact EMGB gravity solution given in Ref. [6]. It is our belief that beside EMGB theory in different theories also such stable, normal-matter wormholes are abound, which will be our next venture in this line of research.

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