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Black holes and the classical model of a particle in Einstein non-linear electrodynamics theory

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ABSTRACT

Modified by a logarithmic term, the non-linear electrodynamics (NED) model of the Born–Infeld (BI) action is reconsidered. Unlike the standard BI action, this choice provides interesting integrals of the Einstein-NED equations. It is found that the spherical matching process for a regular black hole entails indispensable surface stresses that vanish only for a specific value of the BI parameter. This solution represents a classical model of an elementary particle whose radius coincides with the horizon. In flat spacetime, a charged particle becomes a conducting shell with a radius proportional to the BI parameter.
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1. Introduction

Non-linear Born–Infeld (BI) theory was introduced to resolve the Coulomb divergences of classical electrodynamics [1]. With the advent of quantum electrodynamics, it was all but forgotten until its reemergence within the context of string theory. However, the original BI theory was later extended to cover more general non-linear electrodynamics (NED) theories [2]. The NED action, with its square root term restricted to real values, provides a natural way to avoid the Coulomb field's singularity. This is reminiscent of the relativistic particle Lagrangian that restricts the speed of a particle to less than the speed of light.

It was expected that the therapeutic effect of the BI action played a non-trivial role when coupled with other fields. Gravity is no exception, and a search for regular black hole solutions of the full theory attracted much interest [3]. Specifically, the existence of regular, purely electrically-charged black holes continued to be a source of discussion [4]. Within the context of the full Einstein–Yang–Mills–Born–Infeld theory it was shown that regular magnetic black holes are a reality, while the pure electrical ones remained on questionable footing [5]. Our present results use a new method that suggests the latter class, although not generic, are quite real as well. Past efforts to study NED introduced a dual structure, through a Legendre transformation, in which the NED solutions were readily available.

In this Letter, without invoking any dual structures, we extend the BI action by a novel non-polynomial term that admits regular black holes. In the absence of gravity, it is clear that our NED model describes a charged elementary particle of finite field energy with a natural cut-off, which turns out to be the radius of the particle. This corresponds to the classical glue-balls of Yang–Mills (YM) theory [5] with the important difference that the non-linear YM field is replaced here by the NED [6]. We concern ourselves entirely with spherically symmetric NED. We glue two spacetimes together in such a manner that continuity of metric and certain first derivatives are satisfied. As could be expected, this imposes severe restrictions on the component metrics and the BI electric field. It is possible, however, with the choice of a Bertotti–Robinson (BR) type ($\widetilde{\mathrm{BR}}$) metric for interior and a Reissner–Nordstrom (RN) type ($\widetilde{\mathrm{RN}}$) metric for the exterior [7,8]. With a particular choice of the BI parameter, it is shown both from the time-like and the null-shell formalisms that the surface stress–energy tensor, i.e., the Lanczos tensor, $S_{\nu}^{\mu}=0$. Intriguingly, this corresponds to a case where the matching surface coincides with the double horizon of a regular black hole.

We organize the Letter as follows. In Section 2, we consider NED in a flat spacetime. Section 3 covers the gluing BR type and RN type spacetimes, resulting in a regular solution. We conclude with a discussion of interpretation in Section 4.

2. NED in flat spacetime

With unit conventions assumed such that ($c=\hbar=k_B=8\pi G=\frac{1}{4\pi\epsilon_\circ}=1$) our action S and line element are

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$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\mathcal{L}(F, {}^\star F), \tag{1}$$

$$ds^2 = -dt^2 + dr^2 + r^2\,d\Omega^2, \tag{2}$$

where

$$d\Omega^2 = d\theta^2 + \sin^2\theta\,d\varphi^2, \tag{3}$$

and

$$\mathcal{L} = -\frac{2}{b^2}\left\{1 - \sqrt{1 + 2b^2F - b^4({}^\star F)^2} + \ln\left[\frac{1}{2}\left(1 + \sqrt{1 + 2b^2F - b^4({}^\star F)^2}\right)\right]\right\} \tag{4}$$

in which b is the BI parameter, $F = F_{\mu\nu}F^{\mu\nu}$, ${}^\star F = F_{\mu\nu}{}^\star F^{\mu\nu}$ and $\star$ stands for duality. Since we shall confine ourselves entirely to the electrostatic problem the $F_{\mu\nu}{}^\star F^{\mu\nu}$ term under the square root vanishes and will be ignored in the subsequent sections. The parameter b is such that

$$\lim_{b\to 0}\mathcal{L} = F_{\mu\nu}F^{\mu\nu} \quad \text{(Maxwell case)}, \tag{5}$$

$$\lim_{b\to\infty}\mathcal{L} = 0 \quad \text{(zero action)}. \tag{6}$$

The electric field 2-form with the radial electric field $E(r)$ is chosen as

$$\mathbf{F} = E(r)\,dt\wedge dr, \tag{7}$$

which leads to $F = F_{\mu\nu}F^{\mu\nu} = -2E(r)^2$. This must satisfy the NED equation

$$d(\mathcal{L}_F{}^\star\mathbf{F}) = 0, \tag{8}$$

where ${}^\star\mathbf{F} = E(r)r^2\sin\theta\,d\theta\wedge d\varphi$. Integrating the latter equation and considering the line element (2) one finds

$$\frac{\sqrt{-2F}}{1 + \sqrt{1 + 2b^2F}} = \frac{C}{r^2}, \tag{9}$$

where $C \in \mathbb{R}^+$ is a constant of integration. It is not difficult to show that this equation gives a non-trivial solution

$$F = \frac{-2C^2r^4}{(C^2b^2 + r^4)^2}, \tag{10}$$

which upon substitution into Eq. (9) implies

$$\frac{2Cr^2}{(C^2b^2 + r^4) + |C^2b^2 - r^4|} = \frac{C}{r^2}, \tag{11}$$

which is valid only for $r > \sqrt{Cb}$ if $C \neq 0$. This solution corresponds to the electric field

$$E(r) = \frac{Cr^2}{(C^2b^2 + r^4)}, \tag{12}$$

which after using the Maxwell limit

$$\lim_{b\to 0}E = \frac{C}{r^2} \tag{13}$$

suggests identifying the constant C as the charge of the particle, i.e., $C = q$. To find the charge distribution one may look at the region $r < r_\circ$ ($r_\circ = \sqrt{qb}$), where the only possible solution of (8) under the spherically symmetric flat spacetime and spherically symmetric electric field corresponds to $C = 0$, or equivalently, a zero electric field. Note that the existence of the absolute value in (11), which arises from the square root term, makes this choice indispensable. That is, $|C^2b^2 - r^4| = C^2b^2 - r^4$ for $r^4 < C^2b^2$ ($= r^4 - C^2b^2$ for $r^4 > C^2b^2$). When this is employed in (11), for the consistency of the solution, we must choose $C = 0$, leading automatically to $E(r) = 0$ for $r^4 < C^2b^2$. Whenever $C \neq 0$, on the other hand, (12) becomes the only acceptable solution for $r^4 > C^2b^2$. These results lead to a surface charge distribution of the particle of $\rho = \frac{q\delta(r - r_\circ)}{4\pi r_\circ^2}$ in which $\delta(r - r_\circ)$ denotes the Dirac delta function. Consequently, one can easily show that the electric potential of the particle is a constant value inside ($r < r_\circ$) and

$$\phi(r) = \frac{\sqrt{2}q}{4r_\circ}\left[\tanh^{-1}\left(\frac{\sqrt{2}rr_\circ}{r^2 + r_\circ^2}\right) + \tan^{-1}\left(\frac{\sqrt{2}rr_\circ}{r^2 - r_\circ^2}\right)\right] \tag{14}$$

for the outside ($r > r_\circ$) region. For $b \to 0$, we recover the Coulomb field for a charge located at $r = 0$, and $r = r_\circ \neq 0$ provides a natural cut-off for the particle. The total energy density is $u = \frac{1}{2}\mathbf{E}\cdot\mathbf{D}$ ($\mathbf{D} = \epsilon\mathbf{E}$, with $\epsilon = \frac{\partial\mathcal{L}}{\partial F} = 1 + (\frac{r_\circ}{r})^4$) with total energy

$$U = 4\pi\int_{r_\circ}^{\infty} u(r)r^2\,dr = 5.45\frac{q^2}{r_\circ}. \tag{15}$$

This amounts to a hard-core particle with charge density ρ. Identifying $U = M$, $r_\circ$ is determined from the energy of the particle. If $\frac{q^2}{2M}$ is identified as the classical electromagnetic radius, r_e, then $r_\circ = 10.90r_e$.

3. Regular electric black holes in Einstein-NED theory

In this section, a composite spacetime will be established consisting of a region ($r \leqslant r_\circ$) of uniform electric field glued at $r = r_\circ$ to an outside region ($r > r_\circ$). The proper junction condition will dictate that $r_\circ$ must coincide with the horizon of the entire spacetime. For this purpose, we choose our action as

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\big[R - \mathcal{L}(F)\big], \tag{16}$$

in which R is the Ricci scalar, and the given Lagrangian (4) is free of magnetic fields. The Einstein-NED equation is

$$G_\mu{}^\nu = T_\mu{}^\nu = -\frac{1}{2}\big[\mathcal{L}(F)\delta_\mu{}^\nu - 4\mathcal{L}_F(F)F_{\mu\lambda}F^{\nu\lambda}\big] \tag{17}$$

in which the electromagnetic field 2-form (7) satisfies the NED equation (8). The static, spherically symmetric spacetimes satisfying the foregoing equations and being glued at $r = r_\circ$ are

$$ds^2 = -\tilde{f}(r)\,dt^2 + \frac{dr^2}{\tilde{f}(r)} + r_\circ^2\,d\Omega^2 \quad (r \leqslant r_\circ), \tag{18}$$

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\,d\Omega^2 \quad (r > r_\circ). \tag{19}$$

The choice of these metrics can be traced back to the form of the stress–energy tensor (17), which satisfies $T_0^0 - T_1^1 = 0$ and consequently $G_0^0 - G_1^1 = 0$, whose explicit form, on integration, gives $|g_{00}g_{11}| = C =$ constant. We need only choose the time scale at infinity to make this constant equal to unity.

Nevertheless, for a spherically symmetric charge in EM theory the external solution is known uniquely to be the RN metric. Therefore, to recover the RN metric in the Maxwell limit ($b \to 0$), we must consider an RN type metric ansatz for $r > r_\circ$. Further, since the outer RN metric was glued consistently with the inner BR metric [7], it is natural to seek a similar ansatz in the present problem as well. On the hypersurface $r = r_\circ$, the continuity of metrics is assumed, whereas some metric derivatives are allowed to be discontinuous to allow for physical sources.

The field equations combined with the junction conditions will determine the metric functions $f(r)$, $\tilde{f}(r)$ and the electric field

$E(r)$. We note from NED equation (8) that the electric field is uniform in the region $r < r_\circ$. Our solution can be summarized as follows

$$E(r) = \begin{cases} \frac{q}{2r_\circ^2}, & r \leqslant r_\circ, \\ \frac{qr^2}{r^4+r_\circ^4}, & r > r_\circ, \end{cases} \tag{20}$$

$$f(r) = 1 - \frac{2M}{r} + \frac{q^2}{3r_\circ^4} r^2 \ln\left(\frac{r^4}{r^4+r_\circ^4}\right) + \frac{q^2\sqrt{2}}{3rr_\circ} \tan^{-1}\left(\frac{\sqrt{2}rr_\circ}{r^2-r_\circ^2}\right) - \frac{q^2\sqrt{2}}{6rr_\circ} \ln\left[\frac{r^2+r_\circ^2-\sqrt{2}rr_\circ}{r^2+r_\circ^2+\sqrt{2}rr_\circ}\right], \quad r > r_\circ, \tag{21}$$

$$\tilde{f}(r) = C_0 r^2 + C_1 r + C_2, \quad r \leqslant r_\circ, \tag{22}$$

where

$$C_0 = \frac{q^2}{r_\circ^4}(1-\ln 2),$$
$$C_1 = \frac{2M}{r_\circ^2} + \frac{q^2}{6r_\circ^3}\left(8\ln 2 - 12 - \sqrt{2}\left(\pi - \ln(3-2\sqrt{2})\right)\right),$$
$$C_2 = 1 - \frac{4M}{r_\circ} - \frac{q^2}{3r_\circ^2}\left(2\ln 2 - 3 - \sqrt{2}\left(\pi - \ln(3-2\sqrt{2})\right)\right), \tag{23}$$

in which M is a mass related constant, $q =$ charge and $r_\circ = \sqrt{bq}$. Furthermore, as a result of satisfying the field equations, $r_\circ$ and q are constrained by the condition

$$r_\circ = q\sqrt{\ln 2}. \tag{24}$$

At this stage it is important to state that the metric function $f(r)$ and $\tilde{f}(r)$ satisfy

$$f(r_\circ) = \tilde{f}(r_\circ) = 0. \tag{25}$$

These conditions eliminate the possibility of quasi-black holes (QBH), which are defined as objects on the verge of being extremal black holes. In order to create such QBH, we would have to consider matching conditions $f(r_\circ) = \tilde{f}(r_\circ) \neq 0$. These will not be our concern here. The conditions (25) dictate that

$$\frac{M}{r_\circ} = \frac{1}{2} + \frac{1}{12\ln 2}\left[\sqrt{2}\left(\pi - \ln(3-2\sqrt{2})\right) - 2\ln 2\right],$$
$$C_1 = \frac{2}{r_\circ}\left(1 - \frac{1}{\ln 2}\right),$$
$$C_2 = \frac{1}{\ln 2} - 1, \tag{26}$$

which casts $\tilde{f}(r)$ into

$$\tilde{f}(r) = (r - r_\circ)^2 \tag{27}$$

for the specific choice

$$r_\circ^2 = \frac{1}{\ln 2} - 1. \tag{28}$$

Let us add that a combination of (24), (28) and $r_\circ = \sqrt{bq}$ determines the value of the BI parameter as $b = 0.55$. The finite scalar invariants, such as the Ricci and Kretschmann of the line element (18), for $r < r_\circ$ are given respectively by

$$R = \frac{2}{r_\circ^2}\left(1 + r_\circ^2\right), \qquad K = \frac{4}{r_\circ^4}\left(1 + r_\circ^4\right). \tag{29}$$

Letting now

$$r_\circ - r = \frac{1}{\bar{r}}, \qquad t = \bar{t}, \tag{30}$$

transforms the metric into

$$ds^2 = \frac{-d\bar{t}^2 + d\bar{r}^2}{\bar{r}^2} + r_\circ^2 \, d\Omega^2 \quad (r \leqslant r_\circ). \tag{31}$$

This is a Bertotti–Robinson (BR) [9,10] type metric with a specific radius that will be referred to here as the $\widetilde{\mathrm{BR}}$ spacetime. Similarly we label the metric (19) for $r > r_\circ$, as the $\widetilde{\mathrm{RN}}$. It is well known that the BR metric is not a black hole solution. However, our present $\widetilde{\mathrm{BR}}$ is a part of a composite system of spacetimes, with an event horizon at $r_\circ$, where it corresponds to an accelerated frame in a conformally flat background with a unit acceleration in the present context [11]. Let us note that our result of $\widetilde{\mathrm{BR}}$ for $(r < r_\circ)$ is not contradicted by a theorem proved long ago by Bronnikov and Shikin [12]. This theorem proved the non-existence of a regular center, which is still satisfied in the case of our $\widetilde{\mathrm{BR}}$ spacetime in the Einstein-NED theory.

In order to determine if our matching of inner $\widetilde{\mathrm{BR}}$ to outer $\widetilde{\mathrm{RN}}$ is smooth, we compute the surface stress tensor S_μ^ν on $r = r_\circ$. This can be expressed in terms of the extrinsic curvature tensor in accordance with

$$8\pi S_\mu^\nu = \left[K_\mu^\nu\right] - \delta_\mu^\nu [K], \tag{32}$$

where $[.] = (.)_+ - (.)_-$, with $K = K_\mu^\mu$ and $\mu, \nu = \{t, \theta, \varphi\}$. Here $(.)_+$ and $(.)_-$ refer to the outer $(r > r_\circ)$ and the inner $(r < r_\circ)$ metrics, respectively. The components of S_μ^ν are given by [7]

$$8\pi S_0^0 = \frac{2}{r}\left[(r_+)' - (r_-)'\right], \tag{33}$$

$$8\pi S_2^2 = 8\pi S_3^3 = \frac{(r\sqrt{f(r)})'_+}{r\sqrt{f(r)}} - \frac{(r-r_\circ)'_-}{(r-r_\circ)}, \tag{34}$$

where a prime $'$ denotes $\frac{d}{d\ell}$, defined by

$$\frac{d}{d\ell} = \begin{cases} (\frac{d}{d\ell})_- = (r - r_\circ)\frac{d}{dr}, \\ (\frac{d}{d\ell})_+ = \sqrt{f(r)}\frac{d}{dr}. \end{cases} \tag{35}$$

We observe that the S_0^0 component, proportional to the proper mass, vanishes, i.e., $S_0^0 = 0$. This can also be checked from the continuity of the general mass formula

$$m(r) \equiv \frac{r}{2}\left(1 - (\nabla r)^2\right), \tag{36}$$

which gives

$$m_- = m(r_\circ - 0) = m_+ = m(r_\circ + 0) = \frac{r_\circ}{2}. \tag{37}$$

The surface pressures on the other hand become

$$8\pi S_2^2 = 8\pi S_3^3 = \frac{1}{r_\circ}\frac{d}{dr}\left(r\sqrt{f(r)}\right) - 1. \tag{38}$$

In order to evaluate this expression we need to expand $f(r)$ in powers of $(r - r_\circ)$. A detailed expansion process gives

$$f(r) = (r-r_\circ)^2 - \frac{2}{3}\frac{(r-r_\circ)^3}{r_\circ} + \frac{1-2\ln 2}{3\ln 2}\frac{(r-r_\circ)^4}{r_\circ^4} - \frac{1-10\ln 2}{15\ln 2}\frac{(r-r_\circ)^5}{r_\circ^5} + \frac{7-60\ln 2}{90\ln 2}\frac{(r-r_\circ)^6}{r_\circ^6} + \cdots. \tag{39}$$

From this expression, as the terms suggest, we can retain the quadratic term as the leading order so that

$$f(r) \tilde{=} (r - r_\circ)^2. \tag{40}$$

Substituting this into (38) for the surface pressures, we obtain under (28) that

$$8\pi S_2^2 = 8\pi S_3^3 = 0. \tag{41}$$

At this point, it is instructive to calculate the charge to mass ratio for such a particle (i.e., a black hole). In SI units we have

$$\begin{aligned}\left(\frac{q}{m}\right)_{\text{SI}} &= 4\pi\sqrt{2G\epsilon_\circ}\left(\frac{q}{m}\right)_{\text{geom.}}\\ &= 8\pi\sqrt{\frac{2G\epsilon_\circ}{\ln 2}} = 1.04\times 10^{-9}\ \frac{\text{C}}{\text{kg}},\end{aligned} \tag{42}$$

which, predictably has a huge gap from the value of an electron ($\sim 1.7\times 10^{11}\ \frac{\text{C}}{\text{kg}}$).

Finally, we invoke the null-shell formalism [13,14], where the metrics are cast into Kruskal form

$$ds^2 \tilde{=} -F(u,v)\,du\,dv + r^2\,d\Omega^2. \tag{43}$$

Here $F(u,v)$ is a bounded function on the horizon, and the null coordinates are defined by

$$t - r_\star = u, \qquad t + r_\star = v \tag{44}$$

for $r_\star = \int \frac{dr}{f(r)}$. By employing the expansion (39) once more and, adopting its first term, we obtain the null coordinates. The smooth matching on $u = 0$ requires that [14]

$$\left(\frac{\partial r}{\partial u}\right)_+ = \left(\frac{\partial r}{\partial u}\right)_- \tag{45}$$

implying in our case that it is satisfied for $r_\circ^2 = \frac{1}{\ln 2} - 1$, which is nothing but the condition (28) that renders smooth matching possible.

4. Conclusion

Employing a modified version of the BI action, consisting of non-polynomial, logarithmic parts, we obtain a class of regular, electrically-charged black holes in Einstein-NED theory, which were previously unknown [4]. Other choices of boundary conditions, which we have not taken into consideration in this Letter, may give rise to what are called quasi-black holes (QBH). The particular choice of the action provides a particle-like structure in flat spacetime whose electric charge resides on its surface, while the particle radius provides a natural cut-off for the electric field. This includes the case of a massless particle whose entire mass derives from the electric field energy. A similar picture applies to the curved space as well. Remarkably, we uncover a regular class of purely electrically-charged black hole solutions where for $r < r_\circ$, we have a uniform electric field with $S_\mu^\nu = 0$ at $r = r_\circ$. This class consists of the extremal black hole in which the horizon, Born–Infeld parameter and charge are related. Smooth gluing of a BR core to an outside RN was also known in the Maxwell electrodynamics [7]. The novel feature here is that the horizon coincides with the specific value $r_\circ = \sqrt{\frac{1}{\ln 2} - 1}$. This gives in SI units, $q_{\text{SI}} = 1.50\times 10^{-18}$ C and $m_{\text{SI}} = 1.44\times 10^{-9}$ kg for such a black hole.

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