AIP Journal of Mathematical Physics

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Citation: Journal of Mathematical Physics **47**, 012502 (2006); doi: 10.1063/1.2157051
View online: http://dx.doi.org/10.1063/1.2157051
View Table of Contents: http://scitation.aip.org/content/aip/journal/jmp/47/1?ver=pdfcov
Published by the AIP Publishing

JOURNAL OF MATHEMATICAL PHYSICS **47**, 012502 (2006)

Colliding wave solutions from five-dimensional black holes and black *p*-branes

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(Received 22 July 2005; accepted 9 November 2005; published online 13 January 2006)

We consider both the five-dimensional Myers-Perry and Reissner-Nordstrom black holes (BHs) and black p-branes in $(4+p)$-dimensions. By employing the isometry with the colliding plane waves (CPWs) we generate Cauchy-Horizon (CH) forming CPW solutions. From the five-dimensional vacuum solution through the Kaluza-Klein reduction the corresponding Einstein-Maxwell-dilaton solution is obtained. This CH forming cross polarized solution with the dilaton turns out to be a rather complicated nontype D metric. Since we restrict ourselves to the five-dimensional BHs we obtain exact solutions for colliding 2- and 3-form fields in $(p+4)$-dimensions for $p \geqslant 1$. By dualizing these forms we can obtain also colliding $(p+1)$- and $(p+2)$-forms which are important processes in the low energy limit of the string theory. All solutions obtained are CH forming, implying that an analytic extension beyond is possible.
[DOI: 10.1063/1.2157051]

I. INTRODUCTION

Black holes (BHs) are known to have region isometric to the space of colliding plane waves (CPW).[1,2] This may be either in between the two horizons (i.e., inner and outer) or given the case with single horizon the inner region of the event horizon. Such an isometry renders it possible to generate CPW solutions from known solutions of BHs. In a recent paper[3] we gave a prescription for generating CPW solutions from an Einstein-Maxwell-Dilaton-Axion (EMDA) theory. In this theory the dilaton was linear and the BH was not asymptotically flat. In this solution the axion arises as the cross polarizing agent for the CPWs. This means that the limit of linear polarization removes the axion leaving behind only the Einstein-Maxwell-Dilaton (EMD) theory. Another solution with similar features but valid only in the zero dilaton limit was obtained previously.[4] Interesting physical property shared by both of these solutions is that the space-time subsequent to the collision of waves emerges free of physical singularities. Horizon forming CPW[1,2,5–7] solutions in the EMDA theory are naturally of utmost important to the string theory. Since the idea of higher dimensions has already gained enough momentum it is important to investigate the collision of waves in higher dimensions.[8,9] It is known already that the four-dimensional EMDA theory is equivalent to the six-dimensional Ricci flat, vacuum solution.[10,11]

In this paper we restrict ourselves to the five- (and four-) dimensional space-times and their extension through the brane world. We consider first the five-dimensional collision of gravitational (impulse and shock) waves obtained from the isometry with the Myer-Perry black hole (MPBH).[12,13] This particular BH contains two rotation parameters in addition to the mass. For simplicity we make the special choice in which the two angular momenta are equal. Then we identify the (r,θ) sector of the BH at hand with the null coordinates sector (u,v) of the colliding

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waves and accompany this with the necessary coordinate transformation. Inclusion of the Heaviside step functions along with the null coordinates must guarantee that no additional current sheets are created at the boundaries. In the standard Einstein and Einstein-Maxwell (EM) theories these are summarized in the O'Brian-Synge[14] boundary conditions, respectively. Similar arguments rightly follow in the higher dimensional space-times as well.

The static MPBH is transferred to the linearly polarized CPWs, which turns out to be a nonsingular type-D solution. The rotating MPBH transforms through the Kaluza-Klein (KK) reduction procedure to the CPW space-time with cross polarization in the four-dimensional EMD theory. This space-time is also regular but it does not belong to the type-D class. As a matter of fact the dilaton involved cross polarization (instead of axion, as it arises in the above-mentioned solutions) makes the space-time structure rather involved.

As a second example we consider the Reissner-Nordstrom (RN) BH[15] in five dimensions from which we obtain CPW solution in the five-dimensional EM theory. Similar to the CPW solution obtained from the MPBH this one also is singularity free. Our examples of five-dimensional BHs exclude the extremal limits because in such a limit which removes the isometric region of BHs with CPWs the equivalence fails to work. Under such circumstances an alternative transformation, analogous to the RN-Bertotti-Robinson equivalence, must be pursed which is out of our scope in this paper.

As a third example we consider the black-branes in the $(d+p)$-dimensional brane world. We find regular CPW solutions to colliding 3-form fields in higher dimensions. Another solution that we obtain from the same black-brane metric is colliding (EM) (2-form) fields in higher dimensions.

Our study may lay the foundation for promoting the string theory in approximation in low energies from single plane wave background to the more realistic CPW background. The regular initial data of CPWs provides a natural choice among the nonunique Penrose limits of space-times.[16–18] It is known that the incoming region of a CPW space-time admits automatically a Penrose limit of the interaction region. The advantage is that we have the double Penrose limits which are both well-defined initial data. Any Penrose limit does not qualify as an initial data toward construction of the interaction region.

Finally we wish to express the view that our technique can be extrapolated to higher dimensions provided some associated difficulties are overcome. The most important problem is the analytic integration of the radial coordinate (r) of BHs in terms of the prolate-type coordinate (τ) of CPWs. And as the second major difficulty we cite of the necessary proper representation of the higher dimensional spherical line element suitable for the ideals of the geometry of CPWs.

The organization of the paper is as follows. In Sec. II we obtain CPW solutions from the five-dimensional BHs, whose details are tabulated in Appendixes A, B, and C. Section III investigates the physical properties of the metrics obtained in Sec. II. Section IV contains solutions for colliding 3-form fields in higher dimensions and their KK reductions. Section V focuses attention on a class of by-product solutions of colliding EM shock waves in higher dimensions. We dualize our 2-form fields of Sec. V in Sec. VI to obtain colliding $(p+2)$-form fields in $(p+4)$-dimensions. Our conclusion and discussion is in Sec. VII.

II. CPW SOLUTIONS FROM FIVE-DIMENSIONAL BHs

In this section we concentrate on the two well-known types of BHs in five-dimensions. First we consider the MPBH and then RNBH. Our analysis applies, however, to any five-dimensional BHs that possesses two nonoverlapping horizons albeit the technical difficulties. We comment on this point at the end of the section in concentration with the Schwarzschild-de(-anti) Sitter BH.

(A) The MPBH in five dimensions with two equal angular momenta is given

$$ds_5^2 = \tilde{g}_{AB}\,dx^A\,dx^B \quad (A,B,\ldots=0,\ldots 4),$$

$$ds_5^2 = dt^2 - \frac{\mu}{\rho^2}\left[dt + \frac{\bar{a}}{2}(d\varphi - \cos\theta\, d\eta)\right]^2 - \frac{\rho^4\, d\rho^2}{\rho^4 - \mu\rho^2 + \mu\bar{a}^2} - \rho^2\, d\Omega_3^2, \tag{1}$$

where μ and $\bar{a}$ are, respectively, proportional to the mass and angular momentum of the BH. We note that five-dimensional suffices are denoted by capital italic letters while four dimensional ones by greek letters. A tilde over specifies also the five-dimensional geometrical object. For the three-dimensional metric of S^3 we choose the representation

$$d\Omega_{(3)}^2 = \tfrac{1}{4}(d\theta^2 + d\eta^2 + d\varphi^2 - 2\cos\theta\, d\eta\, d\varphi), \tag{2}$$

where $0 < \theta < \pi$, and the angles η and φ are defined modulo 2π. The static MPBH corresponds to $a=0$, while the extreme case is defined by $\mu = 4\bar{a}^2$. The CPW form in five dimensions is obtained by imposing the identification of the (ρ,θ) sector in the above metric with the (τ,σ) sector of CPW as follows:[3]

$$\left(\frac{4\rho^2\, d\rho^2}{\mu\rho^2 - \rho^4 - \mu\bar{a}^2} - d\theta^2\right) = \left(\frac{d\tau^2}{1-\tau^2} - \frac{d\sigma^2}{1-\sigma^2}\right). \tag{3}$$

In the sequel, for simplicity we choose $\mu=1$ leading us to the solution

$$2\rho^2 = 1 + \sqrt{1 - 4\bar{a}^2}\tau, \qquad \cos\theta = \sigma \tag{4}$$

implying further that we impose $|\bar{a}| < \frac{1}{2}$. Supplementing this transformation with the identifications

$$t \rightarrow x, \qquad \varphi \rightarrow y, \qquad \eta \rightarrow z, \qquad a_o^2 = 2\bar{a}^2 \tag{5}$$

followed by an appropriate rescaling of coordinates we obtain the five-dimensional vacuum metric apt for CPWs:

$$ds_5^2 = F\left(\frac{d\tau^2}{\Delta} - \frac{d\sigma^2}{\delta}\right) - \frac{1}{F}[Z_o(dy - 2\sigma\, dz)dy + Z\, dz^2 + 4a_o(dy - \sigma\, dz)dx - (1 - k\tau)dx^2]. \tag{6}$$

Our abbreviations in this metric stand as follows:

$$\Delta = 1 - \tau^2,$$

$$\delta = 1 - \sigma^2,$$

$$F = 1 + k\tau,$$

$$Z = F^2 + 2a_o^2\sigma^2,$$

$$Z_o = F^2 + 2a_o^2,$$

$$k = \sqrt{1 - 2a_o^2},$$

$$0 < k \leqslant 1, \tag{7}$$

where the coordinates (τ,σ) are related to the null coordinates (u,v) through

$$\tau = \sin(au + bv),$$

$$\sigma = \sin(au - bv), \tag{8}$$

$$(a,b\text{:constants}).$$

Now, the crucial point toward the interpretation of this metric as a CPW metric is by making the substitutions

$$u \to u\theta(u),$$
$$v \to v\theta(v) \tag{9}$$

in the metric functions, where θ stands for the Heaviside unit step function. This process must not create currents (sources) on the null boundaries $u=0=v$. Alternatively this implies that the five-dimensional Ricci terms all vanish globally

$$\widetilde{R}_{AB} = 0$$
$$(x^A{:}u,v,x,y,z). \tag{10}$$

The Riemann tensor components $\widetilde{R}_{ABCD}$, however, involve Dirac delta functions, indicative of impulsive gravitational waves in addition to the shock waves required commonly by the step functions. In Appendix A we tabulate all components exhaustively from which we can easily identify the nonvanishing ones in the incoming regions. By setting $v<0(u<0)$ we restrict ourselves to the incoming region II (III), comprising of five-dimensional gravitational plane waves alone. Obviously, for both $u<0$ and $v<0$ we obtain the region I which is a five-dimensional flat space-time given by

$$\mathrm{d}s_5^2 = 4ab\,\mathrm{d}u\,\mathrm{d}v - (1 + a_o^2)\mathrm{d}y^2 - \mathrm{d}z^2 - 4a_o\,\mathrm{d}x\,\mathrm{d}y - \mathrm{d}x^2. \tag{11}$$

The Kretschmann scalar in the interaction region (region IV, $u>0, v>0$) turns out to be

$$K = \widetilde{R}_{ABCD}\widetilde{R}^{ABCD} = \frac{6}{(1 + k\tau)^6}[4k^2(k+\tau)^2 - (1 + k\tau)^2] \tag{12}$$

which is free of singularities.

We wish now to apply the KK reduction procedure to our five-dimensional metric (6) in order to obtain CPWs of the EMD theory in four dimensions. We follow the KK reduction procedure through the identification

$$\widetilde{g}_{AB} = \phi^{-1/3}\begin{pmatrix} g_{\mu\nu} + 4\phi A_\mu A_\nu & 2\phi A_\mu \\ 2\phi A_\nu & -\phi \end{pmatrix}. \tag{13}$$

This makes it possible to read both the four-dimensional metric $g_{\mu\nu}$, as well as the dilaton ϕ, and the EM potential A_μ. The results are as follows:

$$\mathrm{d}s_4^2 = (FZ)^{\frac{1}{2}}\left[\frac{\mathrm{d}\tau^2}{\Delta} - \frac{\mathrm{d}\sigma^2}{\delta} - \frac{1}{Z}\left(\frac{L}{F}\mathrm{d}x^2 + \delta Z_o\,\mathrm{d}y^2 - 4a_o\delta\,\mathrm{d}x\,\mathrm{d}y\right)\right],$$

$$\phi = \left(\frac{Z}{F}\right)^{\frac{3}{2}},$$

$$A_\mu = \frac{\sigma}{Z}\left(0,0,a_o,\frac{1}{2}Z_o\right), \tag{14}$$

where the notations are as in (7), and in addition we have labelled $L=2F-Z$. The action of the resulting four-dimensional EMD theory is

$$S = \frac{1}{16\pi}\int |g|^{\frac{1}{2}}\,\mathrm{d}x^4\left[-R - \phi F_{\mu\nu}F^{\mu\nu} + \frac{(\nabla\phi)^2}{6\phi^2}\right] \tag{15}$$

so that the dilaton and Maxwell equations take the respective forms

$$\Box(\ln\phi) = -3\phi F_{\mu\nu}F^{\mu\nu}, \tag{16}$$

$$\nabla_\alpha(\phi F^\alpha_\mu) = 0, \tag{17}$$

in which $\Box$ stands for the covariant Laplacian. To complete the set of Einstein equations we need also the Ricci tensor which is given by

$$R_{\mu\nu} = \frac{1}{6}\frac{\phi_{,\mu}\phi_{,\nu}}{\phi^2} - 2\phi\left(F_{\mu\alpha}F^\alpha_\nu - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right). \tag{18}$$

We note that this representation of dilaton is different from the standard one expressed as an exponential function in the action. This more familiar latter form is related to the present one by the substitution

$$\phi = e^{-2a\sigma} \tag{19}$$

which casts the action into

$$S = \frac{1}{16\pi}\int |g|^{\frac{1}{2}}\,\mathrm{d}x^4[-R + 2(\nabla\sigma)^2 - e^{-2a\sigma}F_{\mu\nu}F^{\mu\nu}], \tag{20}$$

where the dilatonic parameter is $\sqrt{3}$. The physical properties of the EMD space-time obtained hitherto will be studied in the next section.

(B) The five-dimensional RNBH is given by

$$\mathrm{d}s_5^2 = \left(1 - \frac{m}{r^2} + \frac{q^2}{r^4}\right)\mathrm{d}t^2 - \left(1 - \frac{m}{r^2} + \frac{q^2}{r^4}\right)^{-1}\mathrm{d}r^2 - r^2\,\mathrm{d}\Omega^2_{(3)}, \tag{21}$$

where m and q are, respectively, related to the mass and charge of the BH. The EM vector potential one-form is given by

$$A = A_\mu\,\mathrm{d}x^\mu = \frac{\sqrt{3}q}{2r^2}\mathrm{d}t. \tag{22}$$

Here also, similar to the MP case we choose the S^3 line element as in (2). The transition to the CPW metric is accomplished here by the identification

$$\frac{4\,\mathrm{d}r^2}{m - r^2 - \dfrac{q^2}{r^2}} - \mathrm{d}\theta^2 = \frac{\mathrm{d}\tau^2}{1-\tau^2} - \frac{\mathrm{d}\sigma^2}{1-\sigma^2}. \tag{23}$$

A possible integral for $r(\tau)$ is readily available as

$$r^2 = \frac{m}{2}(1 + l\tau), \tag{24}$$

where

$$l = \sqrt{1 - \frac{4q^2}{m^2}} > 0.$$

By choosing $m=1$ in addition to the identifications

$$\sigma = \cos\,\theta = \sin(au - bv),$$

$$t \to x, \varphi \to y, \eta \to z,$$

$$\tau = \sin(au + bv),$$

$$(a,b = \text{constants}) \tag{25}$$

we obtain the metric (after rescaling of coordinates)

$$\mathrm{d}s_5^2 = (1 + l\tau)(4ab\,\mathrm{d}u\,\mathrm{d}v - \mathrm{d}y^2 - \mathrm{d}z^2 - 2\sin(au - bv)\mathrm{d}y\,\mathrm{d}z) - \frac{\Delta}{(1 + l\tau)^2}\mathrm{d}x^2. \tag{26}$$

The EM vector potential one-form takes the form under the above transformation

$$A = \frac{\sqrt{3}q}{2\sqrt{2}l(1 + l\tau)}\mathrm{d}x. \tag{27}$$

The interpretation of this metric as a CPW is completed by inserting the step functions into the null coordinates u and v. This metric represents collision of EM plane waves in five dimensions. For $l=1$ (or $q=0$) it reduces to the CPW metric of the five-dimensional pure gravity and coincides with the $a_o=0$ case of the metric (6). Thus, (26) is the EM extension while (6) was the rotational extension of the same CPW metric obtained from the five-dimensional Schwarzschild metric. The Riemann components of the metric (26) are given in Appendix B from which we compute the Kretschmann scalar to find (for $u>0$ and $v>0$).

$$K = \frac{127l^4 + 180l^3\tau - 2l^2 - 72\Delta l^2 + 19 - 36l\tau}{4(1 + l\tau)^4} \tag{28}$$

which is also regular to the future of the collision point $u=0=v$.

Finally we wish to comment on other BHs and corresponding CPW solutions in five dimensions. Although our method applies to any such BH that admits inner and outer horizons such that the region in between possesses two spacelike Killing vectors technically some cases are not tractable. As an example we cite the Schwarzschild-de(-anti) Sitter BH given by the line element

$$\mathrm{d}s_5^2 = h(r)\mathrm{d}t^2 - h(r)^{-1}\,\mathrm{d}r^2 - r^2\,\mathrm{d}\Omega_{(3)}^2, \tag{29}$$

where $h(r)=k-(m/r^2)\pm(r^2/l^2)$, in which $k=\pm1$, m is related to mass and l to the cosmological constant. To obtain the associated CPW solution we demand now that

$$\int \frac{\mathrm{d}\rho}{\sqrt{\rho}\sqrt{m - k\rho \pm \frac{\rho^2}{l^2}}} = \pm \sin^{-1} \tau, \tag{30}$$

where we have used $\rho = r^2$. The inversion of such an elliptical integral seems to be beyond analytical calculation which must therefore be handled within the scope of numerical analysis.

III. PROPERTIES OF THE COLLIDING EMD SPACE-TIME

The linearly polarized CPW metric (14) is rather transparent so we restrict ourselves to the case $a_o = 0$, first. Upon rescaling of x and y we have the metric

$$\mathrm{d}s^2 = (1+\tau)^{\frac{3}{2}}\left(2\,\mathrm{d}u\,\mathrm{d}v - \frac{1-\tau}{(1+\tau)^2}\mathrm{d}x^2 - \delta\,\mathrm{d}y^2\right) \tag{31}$$

in which τ and σ are implied with the step functions. By the choice of Newman-Penrose (NP) null-tetrad basis one-forms,

$$l = (1+\tau)^{\frac{3}{4}}\,\mathrm{d}u,$$

$$n = (1+\tau)^{\frac{3}{4}}\,\mathrm{d}v,$$

$$\sqrt{2}m = (1-\tau)^{\frac{1}{2}}(1+\tau)^{-1/4}\,\mathrm{d}x + i\sqrt{\delta}(1+\tau)^{\frac{3}{4}}\,\mathrm{d}y, \tag{32}$$

we obtain all Ricci and Weyl components as tabulated in the Appendix C. It is observed by studying the Weyl scalars Ψ_o, Ψ_2, and Ψ_4 that the space-time is regular everywhere for $(u > 0, v > 0)$. On the boundaries, however, both Ψ_o and Ψ_4 suffer from singularities at $(u=0, bv = \pi/2)$ and $(v=0, au=\pi/2)$, respectively. These are the typical null singularities inherited from the problem of colliding EM shock waves, therefore such singularities in the present problem is not unexpected at all. From the metric (31) we observe that $\tau=1$ and $\sigma=1$ are spurious, removable coordinate singularities since they do not show up in the Weyl scalars. In particular, $\tau=1$ is the location of the horizon in the interaction region beyond which the metric can be extended analytically. The other coordinate singularity $\sigma=1$ is out of question since it does not belong to the interaction region. The incoming EMD waves prior to the collision can also be easily identified from Appendix C. In the region II we have

$$\begin{aligned}\Psi_2 &= -\frac{3}{8}a^2\theta(u)\frac{(5+\sin(au))}{(1+\sin(au))^{\frac{5}{2}}},\\ \Phi_{22} &= \frac{1}{16}a^2\theta(u)\frac{(7+\sin(au))}{(1+\sin(au))^{\frac{5}{2}}},\end{aligned} \tag{33}$$

while in the region III we must replace $au \leftrightarrow bv$ to obtain Ψ_o and Φ_{oo}. The incoming waves that comprise Ricci components $\Phi_{22}(\Phi_{oo})$ is obviously constructed from both the EM and the dilaton parts. Inside the collision region we observe also that the condition

$$9\Psi_2^2 = \Psi_0\Psi_4 \tag{34}$$

holds among the Weyl scalars, showing its type-D character. Direct choice of a Kinnersley type tetrad eliminates both Ψ_o and Ψ_4 components of the Weyl scalars.[19] Such a tetrad is given by the basis one-forms

$$l = \frac{\sqrt{1+\tau}}{1-\tau}\mathrm{d}\tau - \mathrm{d}x,$$

$$2n = \mathrm{d}\tau + \frac{1-\tau}{\sqrt{1+\tau}}\mathrm{d}x,$$

$$\sqrt{2}m = (1+\tau)^{\frac{3}{4}}\left(\frac{\mathrm{d}\sigma}{\sqrt{\sigma}} - i\sqrt{\delta}\,\mathrm{d}y\right). \tag{35}$$

This choice gives now the only nonzero component

$$\psi_2 = -\frac{1}{8}\frac{(5+\tau)}{(1+\tau)^{\frac{5}{2}}} \tag{36}$$

verifying its manifestly type-D character.

Now returning to the general CPW metric (14) with $a_o \neq 0$, we can discuss again the interaction region alone. For this reason we omit all step functions and consider an NP tetrad basis one-form

$$\sqrt{2}l = (FZ)^{\frac{1}{4}}\left(\frac{\mathrm{d}\tau}{\sqrt{\Delta}} + \frac{\mathrm{d}\sigma}{\sqrt{\delta}}\right),$$

$$\sqrt{2}n = (FZ)^{\frac{1}{4}}\left(\frac{\mathrm{d}\tau}{\sqrt{\Delta}} - \frac{\mathrm{d}\sigma}{\sqrt{\delta}}\right),$$

$$\sqrt{2}m = \frac{\sqrt{L}}{(FZ)^{\frac{1}{4}}}\left[\mathrm{d}x + \left(\frac{a_o\delta F}{L} + i\sqrt{\frac{\delta F Z_o}{L}}\right)\mathrm{d}y\right]. \tag{37}$$

The results are rather tedious so we shall refrain from tabulating the Ricci and Weyl components. Instead, relying on a numerical computation we have verified that the condition (34) fails to hold in the present case. Thus, we have seen numerically at least that our space-time is not type-*D*. By the same numerical analysis we conclude that our space-time is not singular. Another approach to study this space-time is to search for a possible Kinnersley tetrad that serves to generalize (35). To attain this goal we define the null vector l^μ out of the geodesics equation. Unfortunately, in contrast to the BH case the choice $\theta=\theta_o=$constant which used to simplify the problem significantly, remains ineffective. By this analysis we obtain a set of Kinnersley-type tetrad as follows:

$$l^\mu = \frac{\sqrt{F}}{\Delta}\{k\Delta S, 0, F, -a_o\},$$

$$2n^\mu = \frac{1}{k^2 F\sqrt{Z}S^2}\{k\Delta S, 0, -F, a_o\},$$

$$\sqrt{2}m^\mu = \frac{F^{\frac{1}{4}}}{\sqrt{\delta}Z^{\frac{3}{4}}S}\left\{0, -\delta S\sqrt{\frac{Z}{F}}, -ia_o\delta, i\right\}, \tag{38}$$

where $S=Z^{-1/2}(F-a_o^2\delta)^{\frac{1}{2}}$.

In the limit $a_o = 0$, this tetrad reduces to (35), as it should. This fact that our space-time is not type-D reflects in the computation of the spin coefficients since none turn out to vanish.

In conclusion, we state that colliding cross-polarized EMD space-time obtained from the five-dimensional MPBH through the KK reduction procedure turns out to be nonsingular in spite of all its complication. This reflects the highly transcendental coupling between the dilaton and the other fields. Linear polarization limit removes all complication and we obtain a much simpler space-time structure.

IV. CPW SOLUTIONS FROM BLACK *p*-BRANES

A class of black p-brane solutions in d-dimensions of the action

$$S = \int d^{(d+p)}x\sqrt{-g}\left(R - \frac{2}{(d-p)!}F_{(d-2)}^2\right) \tag{39}$$

with

$$F_{(d-2)} = \frac{1}{(d-2)!}F_{\mu_1\cdots\mu_{(d-2)}}\,\mathrm{d}x^{\mu_1}\wedge\cdots\wedge\mathrm{d}x^{\mu_{d-2}}$$

is given by the metric.[20]

$$\mathrm{d}s_{(d+p)}^2 = A_d B_d^{(1-p)/(1+p)}\mathrm{d}t^2 - (A_d B_d)^{-1}\,\mathrm{d}r^2 - B_d^{2/(p+1)}\mathrm{d}y\,\mathrm{d}y - r^2\,\mathrm{d}\Omega_{(d-2)}^2 \tag{40}$$

in which

$$A_d = 1 - \left(\frac{r_+}{r}\right)^{d-3},$$

$$B_d = 1 - \left(\frac{r_-}{r}\right)^{d-3}.$$

We consider here only the nonextremal case $r_+ > r_-$, where the region $r_- < r < r_+$ enables us to construct nonsingular CPW solutions. As examples we shall present solutions for $p=1$, $p=6$, and $p\to\infty$, however, our procedure applies for any $p \geqslant 1$. In particular, the six-dimensional magnetically charged metric (40) becomes

$$\mathrm{d}s_6^2 = A_5\,\mathrm{d}t^2 - (A_5B_5)^{-1}\,\mathrm{d}r^2 - B_5\,\mathrm{d}y^2 - r^2\,\mathrm{d}\Omega_{(3)}^2,$$

$$F_{(3)} = Q\epsilon_3, \tag{41}$$

where ϵ_3 is the volume form on the 3-sphere and $Q^2 = 2(r_+r_-)^2$. By the KK reduction procedure the six-dimensional action is reduced to the five-dimensional one

$$S = \int d^5x\sqrt{-g}\left[R - 2(\nabla\phi)^2 - \frac{1}{3}e^{-2c\phi}F_{(3)}^2\right], \tag{42}$$

where $c = \sqrt{\frac{2}{3}}$ and the metric, dilaton and 3-form fields are

$$\mathrm{d}s_5^2 = B_5^{\frac{1}{3}}[A_5\,\mathrm{d}t^2 - (A_5B_5)^{-1}\,\mathrm{d}r^2 - B_5\,\mathrm{d}y^2 - r^2\,\mathrm{d}\Omega_{(3)}^2],$$

$$e^{c\phi} = B_5^{-\frac{1}{3}},$$

$$F_{(3)} = \sqrt{2}(r_+ r_-)\epsilon_3. \tag{43}$$

The corresponding CPW solutions are obtained by the transformation

$$\begin{aligned} r^2 &= \tfrac{1}{2}(a_o + b_o\tau), \\ \cos\theta &= \sigma, \\ t &= 2x, \\ \varphi &= z, \\ \eta &= w, \end{aligned} \tag{44}$$

in which we have adopted the representation for $\mathrm{d}\Omega_{(3)}^2$ and introduced the abbreviations

$$\begin{aligned} a_o &= r_+^2 + r_-^2, \\ b_o &= r_+^2 - r_-^2. \end{aligned} \tag{45}$$

The resulting CPW metric, dilaton and the 3-form fields are

$$\begin{aligned} \mathrm{d}s_5^2 &= \left(\frac{1+\tau}{k+\tau}\right)^{1/3}\left\{(k+\tau)\left[2\,\mathrm{d}u\,\mathrm{d}v - \mathrm{d}z^2 - \mathrm{d}w^2 + 2\sigma\,\mathrm{d}z\,\mathrm{d}w - \frac{1-\tau}{k+\tau}\mathrm{d}x^2\right]\right\}, \\ e^{c\phi} &= \left(\frac{1+\tau}{k+\tau}\right)^{-1/3}, \\ F_{(3)uzw} &= Qa\,\theta(u)\sqrt{\delta}, \\ F_{(3)vzw} &= -Qb\,\theta(v)\sqrt{\delta}, \end{aligned} \tag{46}$$

where $Q=(1/\sqrt{2})(k^2-1)$ and $k=a_o/b_o>1$. Our notation for τ, σ, and δ are as in the preceding sections and in transforming (43) into (46) we used the freedom of rescaling of x and $\mathrm{d}s^2$. This metric has the scalar curvature (for $u>0$, $v>0$)

$$R = \frac{4}{3}\frac{ab(k-1)}{(1+\tau)^{4/3}(k+\tau)^{5/3}} \tag{47}$$

which is regular in the interaction region. The colliding 3-form metric corresponding to (41) becomes

$$\mathrm{d}s_6^2 = (k+\tau)(2\,\mathrm{d}u\,\mathrm{d}v - \mathrm{d}z^2 - \mathrm{d}w^2 + 2\sigma\,\mathrm{d}z\,\mathrm{d}w) - \frac{1}{k+\tau}[(1-\tau)\mathrm{d}x^2 + (1+\tau)\mathrm{d}y^2], \tag{48}$$

whereas the 3-form field preserves its form. This metric represents the collision of 2-form fields in flat background. The 3-form field is obviously obtained from the 2-form potential by

$$F_{(3)} = \mathrm{d}A_{(2)}, \tag{49}$$

where

$$A_{(2)} = \tfrac{1}{2}A_{zw}\,\mathrm{d}z \wedge \mathrm{d}w,$$

$$A_{zw} = Q\sin(au\theta(u) - bv\theta(v)).$$

By a similar analysis we obtain the collision of these 3-form fields in 11-dimensional space. The result is

$$\mathrm{d}s_{11}^2 = (k+\tau)(2\,\mathrm{d}u\,\mathrm{d}v - \mathrm{d}z^2 - \mathrm{d}w^2 + 2\sigma\,\mathrm{d}z\,\mathrm{d}w) - \left(\frac{1+\tau}{k+\tau}\right)^{\frac{2}{7}}\sum_{i=1}^{6}(\mathrm{d}y^i)^2 - \left(\frac{1-\tau}{k+\tau}\right)\left(\frac{1+\tau}{k+\tau}\right)^{-5/7}\mathrm{d}x^2 \tag{50}$$

which has the regular scalar curvature,

$$R = -\frac{5}{7}\frac{ab(k^2-1)}{(k+\tau)^3}. \tag{51}$$

The KK reduction of this 11-dimensional metric to the fifth dimension is expressed by

$$\mathrm{d}s^2 = \left(\frac{1+\tau}{k+\tau}\right)^{\frac{4}{7}}\left\{\begin{array}{c}(k+\tau)[2\,\mathrm{d}u\,\mathrm{d}v - \mathrm{d}z^2 - \mathrm{d}w^2 + 2\sigma\,\mathrm{d}z\,\mathrm{d}w]\\ -\left(\frac{1-\tau}{k+\tau}\right)\left(\frac{1+\tau}{k+\tau}\right)^{\frac{-5}{7}}\mathrm{d}x^2\end{array}\right\},$$

$$e^{c\phi} = \left(\frac{1+\tau}{k+\tau}\right)^{-4/7}, \tag{52}$$

and $F_{(3)}$ components are as in (49).

V. COLLIDING EM WAVE SOLUTION IN ANY HIGHER DIMENSION

The action for the $(4+p)$-branes is given by

$$S = \int d^{(4+p)}x\sqrt{-g}(R - F_{(2)}^2), \tag{53}$$

in which $F_{(2)}$ stands for the *EM* 2-form.

Solution is given by[20]

$$\mathrm{d}s_{4+p}^2 = AB^{(1-p)/(1+p)}\,\mathrm{d}t^2 - B^{2/(p+1)}\mathrm{d}y\,\mathrm{d}y - (AB)^{-1}\,\mathrm{d}r^2 - r^2\,\mathrm{d}\Omega_{(2)}^2, \tag{54}$$

where

$$A = 1 - \frac{r_+}{r},$$

$$B = 1 - \frac{r_-}{r},$$

and

$$F = Q\epsilon_2.$$

Now, the transformation (with $r_+ = a_o + b_o$ and $r_- = a_o - b_o$)

$$r = a_o + b_o\tau,$$

$$\cos\theta = \sigma,$$

$$\varphi = z,$$

$$t = x, \tag{55}$$

yields the CPW metric,

$$\mathrm{d}s_{4+p}^2 = (k+\tau)^2(2\,\mathrm{d}u\,\mathrm{d}v - \delta\,\mathrm{d}z^2) - \left(\frac{1+\tau}{k+\tau}\right)^{2/(p+1)}\sum_{i=1}^{p}(\mathrm{d}y^i)^2 - \left(\frac{1-\tau}{k+\tau}\right)\left(\frac{1+\tau}{k+\tau}\right)^{(1-p)/(1+p)}\mathrm{d}x^2 \tag{56}$$

in which we have introduced $k = a_o/b_o > 1$, and rescaled the coordinates. The EM potential 1-form is given by

$$A = -Q\sin(au\theta(u) - bv\theta(v))\mathrm{d}z.$$

So that the nonzero field 2-form components are

$$\begin{aligned} F_{uz} &= -Qa\theta(u)\sqrt{\delta}, \\ F_{vz} &= Qb\theta(v)\sqrt{\delta}. \end{aligned} \tag{57}$$

It is observed now, that it is a simple matter to obtain the CPW metrics for an arbitrary $p \geqslant 1$. In particular, for $p=1$ and $p=7$ we have

$$\mathrm{d}s_5^2 = (k+\tau)^2(2\,\mathrm{d}u\,\mathrm{d}v - \delta\,\mathrm{d}z^2) - \frac{1}{k+\tau}[(1+\tau)\mathrm{d}y^2 + (1-\tau)\mathrm{d}x^2] \tag{58}$$

and

$$\mathrm{d}s_{11}^2 = (k+\tau)^2(2\,\mathrm{d}u\,\mathrm{d}v - \delta\,\mathrm{d}z^2) - \left(\frac{1+\tau}{k+\tau}\right)^{\frac{1}{4}}\sum_{i=1}^{7}(\mathrm{d}y^i)^2 - \left(\frac{1-\tau}{k+\tau}\right)\left(\frac{1+\tau}{k+\tau}\right)^{-3/4}\mathrm{d}x^2, \tag{59}$$

respectively. By letting $p \to \infty$, we can easily obtain also the colliding EM wave solutions in the ∞-brane world. The KK reduction to the fourth dimension for an arbitrary p-brane is

$$\mathrm{d}s_4^2 = \left(\frac{1+\tau}{k+\tau}\right)^{p/(p+1)}\left\{(k+\tau)^2(2\,\mathrm{d}u\,\mathrm{d}v - \delta\,\mathrm{d}z^2) - \left(\frac{1-\tau}{k+\tau}\right)\left(\frac{1+\tau}{k+\tau}\right)^{(1-p)/(1+p)}\mathrm{d}x^2\right\} \tag{60}$$

with

$$e^{c\phi} = \left(\frac{1+\tau}{k+\tau}\right)^{-p/[2(p+1)]},$$

$$c = \sqrt{\frac{p}{p+2}},$$

$$F_{uz} = -Qa\theta(u)\sqrt{\delta},$$

$$F_{vz} = Qb\theta(v)\sqrt{\delta},$$

$$Q^2 = \frac{1}{2}(k^2-1)\left(\frac{p+2}{p+1}\right).$$

This metric describes the collision of dilaton coupled EM waves in four dimensions.

VI. COLLIDING (p+2)-FORMS IN (p+4)-DIMENSION

In Sec. V we have constructed CPW solutions for the 2-form fields in $(p+4)$-dimension. By applying the duality principle we can obtain $(p+2)$-form fields and consider their collision at equal ease. We define the duality

$$\tilde{F}^{\mu_1\cdots\mu_k} = \frac{|g|^{-1/2}}{(n-k)!}\epsilon^{\mu_1\cdots\mu_k\mu_{k+1}\cdots\mu_n}F_{\mu_{k+1}\cdots\mu_n}, \tag{61}$$

where $(n>k)$ and $F_{\mu_{k+1}\cdots\mu_n}$ is assumed known.

The permutation symbol $\epsilon^{\mu_1\cdots\mu_n}$ satisfies[21]

$$\epsilon^{\mu_1\cdots\mu_n}\epsilon_{\mu_1\cdots\mu_n} = (-1)^l n! \ , \tag{62}$$

where l=number of minus signs in $g_{\mu\nu}$. Since we have readily available 2-form at hand we define its dual

$$\tilde{F}^{\mu_1\cdots\mu_{p+2}} = \frac{1}{2!}|g_{p+4}|^{-1/2}\epsilon^{\mu_1\cdots\mu_{p+4}}F_{\mu_{p+3}\mu_{p+4}} \tag{63}$$

in $(p+4)$-dimension. The action of (gravity $+\tilde{F}_{p+2}$) is taken as

$$S = \int d^{4+p}x\sqrt{g}\left(R - \frac{2}{(p+2)!}\tilde{F}^2_{p+2}\right) \tag{64}$$

with the field equations

$$R_{\mu\nu} = \frac{2}{(p+2)!}\left(\tilde{F}_{\mu\mu_1\cdots\mu_{p+1}}\tilde{F}_{\nu}^{\mu_p\cdots\mu_{p+1}} - \frac{(p+1)}{(p+2)^2}g_{\mu\nu}\tilde{F}^2\right)$$

$$\partial_\mu(|g_{p+4}|^{1/2}\tilde{F}^{\mu\mu_1\cdots\mu_{p+1}}) = 0, \tag{65}$$

where $\tilde{F}^2=\tilde{F}_{\mu_1\cdots\mu_{p+2}}\tilde{F}^{\mu_p\cdots\mu_{p+2}}$.

We proceed with two particular examples, $p=1$ and $p=6$.

(i) $p=1$ case. The 3-form field $\tilde{F}_3$ is from the metric (58) and $F_{uz}=-Qa\theta(u)\sqrt{\delta}$, $F_{vz}=-Qb\theta(v)\sqrt{\delta}$. It is given by

$$\tilde{F}_3 = \frac{Q\sqrt{\Delta}}{(k+\tau)^2}(a\theta(u)\mathrm{d}u + b\theta(v)\mathrm{d}v)\wedge \mathrm{d}x\wedge \mathrm{d}y \tag{66}$$

which can be associated through $\tilde{F}_3=\mathrm{d}\tilde{A}_2$ to the 2-form potential

$$\tilde{A}_2 = -\frac{Q}{(k+\tau)}\mathrm{d}x\wedge \mathrm{d}y. \tag{67}$$

The incoming region (II) metric and 3-form fields can also be expressed in the Brinkmann form since they are given here in the Rosen form. For this we define new coordinates (U,V,X,Y,Z) as follows:

$$U = \int (k + \sin au)^2 \, \mathrm{d}u,$$

$$X = A(u)x, \quad Y = B(u)y, \quad Z = C(u)z,$$

$$V = v + \frac{x^2}{2}AA_u + \frac{y^2}{2}BB_u + \frac{z^2}{2}CC_u,$$

where

$$A^2(u) = \frac{1 - \sin au\theta}{k + \sin au\theta},$$

$$B^2(u) = \frac{1 + \sin au\theta}{k + \sin au\theta},$$

$$C = (k + \sin au\theta)\cos au\theta. \tag{68}$$

The relation between U and u can be chosen as

$$U = \frac{3}{2}u + \frac{2}{a}(1 - \cos au) - \frac{1}{4a}\sin 2au, \tag{69}$$

so that $u=0$ and $U=0$ coincide. Further, the graph of $U(u)$ reveals that in the interval $0 < au < \pi/2$, $u > 0$ implies that $U > 0$. However, as it is observed we cannot invert u in terms of U, and this enforces us to keep the Brinkman form in an implicit form. We have ultimately

$$\mathrm{d}s^2 = 2\,\mathrm{d}U\,\mathrm{d}V - \mathrm{d}X^2 - \mathrm{d}Y^2 - \mathrm{d}Z^2 - 2H(u(U),X,Y,Z)\mathrm{d}U^2,$$

where

$$H(u(U),X,Y,Z) = \frac{a}{2}\delta(U)\left[Y^2 - X^2 + \frac{1}{k}(2Z^2 - X^2 - Y^2)\right] + \frac{a^2\theta(U)}{4(k + \sin au)^2}[(k+1)(3 - k + 2\sin au)X^2 - (k-1)(3 + k - 2\sin au)Y^2 - (k + \sin au)(k + 4\sin au)Z^2]. \tag{70}$$

We recall that a general class of metrics given by

$$\mathrm{d}s^2 = 2\,\mathrm{d}U\,\mathrm{d}V - \left(\sum_{i,j} A_{ij}(U)X^iX^j\right)\mathrm{d}u^2 - \sum_i (\mathrm{d}X^i)^2,$$

where A_{ij}=constant, is known as Cahen-Wallach space.[22]

It is clear that we have $\nabla^2 H \neq 0$ in our case, indicating the presence of energy momentum for the 3-form field as it should. The 3-form field, in the Brinkman form for region II is

$$\widetilde{F}_3 = \frac{aQ\theta(U)}{(k + \sin au)^3}\mathrm{d}U \wedge \mathrm{d}X \wedge \mathrm{d}Y, \tag{71}$$

where the inversion of the expression (69) is implied.

(ii) p=6 case. The 8-form field $\widetilde{F}_8$ is found from the metric (56),

$$\widetilde{F}_8 = \widetilde{F}_{uxy^1\ldots y^6}\,\mathrm{d}u \wedge \mathrm{d}x \wedge \mathrm{d}y^1 \wedge \cdots \wedge \mathrm{d}y^6 + \widetilde{F}_{vxy^1\ldots y^6}\,\mathrm{d}v \wedge \mathrm{d}x \wedge \mathrm{d}y^1 \wedge \cdots \wedge \mathrm{d}y^6, \tag{72}$$

where

$$\tilde{F}^{uxy^1\cdots y^6} = \frac{Qb\theta(v)\sqrt{\delta}}{|g_{10}|^{1/2}},$$

$$\tilde{F}^{vxy^1\cdots y^6} = \frac{Qa\theta(u)\sqrt{\delta}}{|g_{10}|^{1/2}}$$

and we have chosen $\epsilon^{uvzxy^1\cdots y^6}=+1$. The corresponding 7-form potential is

$$\tilde{A}_7 = \frac{-Q}{k+\tau}\mathrm{d}x \wedge \mathrm{d}y^1 \wedge \cdots \wedge \mathrm{d}y^6 \tag{73}$$

which derives $\tilde{F}_8$ through $\tilde{F}_8=\mathrm{d}\tilde{A}_7$.

It is seen that the collision problem of these 8-form fields is automatically solved with well-defined incoming states. The solutions, as we stated earlier are regular but our procedure does not allow at the moment to obtain the collision problem of arbitrary n-form fields. Our procedure limits itself only with the 2(3)-form fields and their duals. Different authors addressed themselves to the more general problem but they obtained only perturbative and singular solutions.[23,24]

VII. CONCLUSIONS

In this paper we have concentrated first on two five-dimensional BHs, namely Myers-Perry (MP) and Reissner-Nordstrom (RN). These are both extensions of the five-dimensional Schwarzschild BH, MP with rotation while RN with electric charge. The inherent isometry between the BHs and colliding plane waves (CPWs) yields regular, horizon forming solutions to the latter. By regular, throughout the paper we imply a Cauchy-Horizon (CH) forming space-time with finite curvature invariants. We have not attempted to extend our space-time beyond CH. Once this is done by Chandrasekhar and Xanthopoulos,[1] we may face various singularities ranging from timelike to spacelike ones or no singularities at all. Another issue that we have not addressed ourselves in the paper is the stability of the CH formed in the collision. There are strong arguments that under certain perturbations the CHs of the CPWs transform into curvature singularities.[25] Definitely this matter is far from being conclusive and requires further investigation. We note also that beside the BHs the more general Weyl solutions can be employed in the generating of CPWs.[26]

Our particular cross-polarized dilatonic non-type-*D* metric with CH provides an example to be taken into account other than the singular ones used in string theory.[27,28] Our procedure is extendible to higher dimensional BHs provided technical matters are overcome. One such problem is to find representation for the n-dimensional spherical line element which admits $(n-1)$-dimensional Abelian subspace. Equation (2) performs just this for the three-dimensional sphere. Although we obtain colliding 2(3)-form fields by our procedure through, employing five-dimensional BHs, we can dualize our forms and obtain colliding $(p+1)$- and $(p+2)$-form fields in $(p+4)$-dimensions. Extension of our work to arbitrary form fields will be the next stage of our study. Presumably all these metrics will find application in higher dimensional space-times and low energy limit of the string theory.

APPENDIX A

The nonzero components of the metric (6) are given below

$$\tilde{R}_{uvuv} = -2a^2b^2k^2\theta(u)\theta(v),$$

$$\tilde{R}_{uvyz} = -2abka_o^2\theta(u)\theta(v),$$

$$\widetilde{R}_{uvzx} = 2abka_o\theta(u)\theta(v),$$

$$\widetilde{R}_{uyvy} = abk^2a_o^2\theta(u)\theta(v),$$

$$\widetilde{R}_{uyvx} = abk^2a_o^2\theta(u)\theta(v) = \widetilde{R}_{uxvy},$$

$$\widetilde{R}_{uzvy} = abka_o^2\theta(u)\theta(v) = -\widetilde{R}_{uyvz},$$

$$\widetilde{R}_{uzvz} = -aba_o^2\theta(u)\theta(v),$$

$$\widetilde{R}_{uzvx} = abka_o\theta(u)\theta(v) = -\widetilde{R}_{uxvz},$$

$$\widetilde{R}_{uxvx} = abk^2\theta(u)\theta(v),$$

$$\widetilde{R}_{yzyz} = -\left(\tfrac{1}{2} + 2a_o^4\right)\theta(u)\theta(v),$$

$$\widetilde{R}_{yzzx} = 2a_o^3\theta(u)\theta(v),$$

$$\widetilde{R}_{yxyx} = \tfrac{1}{2}k^2\theta(u)\theta(v),$$

$$\widetilde{R}_{zxzx} = \left(k^2 - \tfrac{1}{2}\right)\theta(u)\theta(v),$$

$$\widetilde{R}_{uyuy} = -a^2\theta(u)Y_1 + a\delta(u)\cos(bv\theta(v))Y_2,$$

$$\widetilde{R}_{vyvy} = -b^2\theta(v)Y_1 + b\delta(v)\cos(au\theta(u))Y_2,$$

$$\widetilde{R}_{uxux} = -2a^2\theta(u)Y_3 - \frac{2ak}{D^2}\delta(u)\cos(bv\theta(v)),$$

$$\widetilde{R}_{vxvx} = -2b^2\theta(v)Y_3 - \frac{2bk}{D^2}\delta(v)\cos(au\theta(u)),$$

$$\widetilde{R}_{uyux} = -a^2a_o\theta(u)Y_4 - \frac{2aa_ok}{D^2}\delta(u)\cos(bv\theta(v)),$$

$$\widetilde{R}_{vyvx} = -b^2a_o\theta(v)Y_4 - \frac{2ba_ok}{D^2}\delta(v)\cos(au\theta(u)),$$

$$\widetilde{R}_{uzuz} = a^2\theta(u)Y_5 + a\delta(u)\cos(bv\theta(v))Y_6,$$

$$\widetilde{R}_{vzvz} = b^2\theta(v)Y_5 + b\delta(v)\cos(au\theta(u))Y_6,$$

$$\tilde{R}_{uzux} = -a^2 a_o \theta(u) Y_7 - 2aa_o \frac{(1+2k\tau)}{D^2} \delta(u)\cos(bv\theta(v)),$$

$$\tilde{R}_{vzvx} = b^2 a_o \theta(v) Y_7 + 2ba_o \frac{(1+2k\tau)}{D^2} \delta(v)\cos(au\theta(u)),$$

$$\tilde{R}_{uyuz} = -a^2 \theta(u) Y_8 - a\delta(u)\cos(bv\theta(v)) Y_9,$$

$$\tilde{R}_{vyvz} = b^2 \theta(v) Y_8 + b\delta(v)\cos(au\theta(u)) Y_9,$$

where we have used the following abbreviations:

$$D = 1 + k\tau,$$

$$Y_1 = \frac{1}{D^3}[4(1+k\tau - a_o^2) - k^2\Delta(3(1+a_o^2)+k\tau)],$$

$$Y_2 = \frac{k^2}{D^3}[\tau(k^2+2) + k(1+3\tau^2+k\tau^3)],$$

$$Y_3 = \frac{a_o^2 - k^2\Delta}{D^3},$$

$$Y_4 = \frac{1}{D^3}[2(1+k\tau) - 3k^2\Delta],$$

$$Y_5 = \frac{1}{D^3}\{-4(1+k\tau - a_o^2) + \Delta[a_o^2(11+12k^2\tau^2) + k\tau(7-6k^2) - 3(1-2k^2)]\},$$

$$Y_6 = \frac{1}{D^3}[k + \tau(3-10a_o^2) + k\tau^2(3-16a_o^2) + k^2\tau^3(1-8a_o^2)],$$

$$Y_7 = \frac{1}{D^3}[2(k+\tau) - k\Delta(5+6k\tau)],$$

$$Y_8 = \frac{1}{D^3}[4(k+\tau(1-a_o^2)) + k\Delta(k^2\Delta - 5 - a_o^2 - 3k\tau(1+2a_o^2))],$$

$$Y_9 = \frac{1}{D^3}[(2-k^2)(1+3k\tau) + k^2\tau^2(5+k\tau-2k^2)].$$

APPENDIX B

The nonzero Riemann components of the metric (26) are with the step functions inserted as follows:

$$\widetilde{R}_{uvuv} = \frac{-2la^2b^2}{C}(l+\tau)\theta(u)\theta(v),$$

$$\widetilde{R}_{uxux} = -\frac{a^2\theta(u)}{C^4}(3l^2+2l\tau-1) - \frac{2a}{C^3}\delta(u)\cos(bv\theta(v))(l+\tau),$$

$$\widetilde{R}_{uxvx} = \frac{ab}{C^4}\theta(u)\theta(v)(2l^2+l\tau-1),$$

$$\widetilde{R}_{uyuy} = -\frac{a^2\theta(u)}{4C}(3l^2+4l\tau+1) + a\delta(u)l\cos(bv\theta(v)),$$

$$\widetilde{R}_{uyuz} = -\frac{a^2\theta(u)\sigma}{4C}(3l^2+4l\tau+1) + a\delta(u)\cos(bv\theta(v)),$$

$$\widetilde{R}_{uyvy} = \frac{ab}{4C}(1-l^2)\theta(u)\theta(v),$$

$$\widetilde{R}_{uyvz} = \frac{ab\sigma}{4C}(1-l^2)\theta(u)\theta(v),$$

$$\widetilde{R}_{uzuz} = -\frac{a^2\theta(u)}{4C}(3l^2+4l\tau+1) + a\delta(u)l\cos(bv\theta(v)),$$

$$\widetilde{R}_{uzvz} = \frac{ab}{4C}(1-l^2)\theta(u)\theta(v),$$

$$\widetilde{R}_{vxvx} = -\frac{b^2\theta(v)}{C^4}(3l^2+2l\tau-1) - \frac{2b}{C^3}\delta(v)\cos(au\theta(u))(1+\tau),$$

$$\widetilde{R}_{vyvy} = -\frac{b^2\theta(v)}{4C}(3l^2+4l\tau+1) + b\delta(v)l\cos(au\theta(u)),$$

$$\widetilde{R}_{vyvz} = -\frac{b^2\theta(v)\sigma}{4C}(3l^2+4l\tau+1) - b\delta(v)\cos(au\theta(u)),$$

$$\widetilde{R}_{vzvz} = -\frac{b^2\theta(v)}{4C}(3l^2+4l\tau+1) + b\delta(v)l\cos(au\theta(u)),$$

$$\widetilde{R}_{xyxy} = \frac{l\Delta}{2C^4}\theta(u)\theta(v)(l+\tau),$$

$$\widetilde{R}_{xyxz} = \frac{l\Delta\sigma}{2C^4}\theta(u)\theta(v)(l+\tau),$$

$$\widetilde{R}_{xzxz} = \frac{l\Delta}{2C^4}\theta(u)\theta(v)(l+\tau),$$

$$\widetilde{R}_{yzyz} = \frac{-\delta\theta(u)\theta(v)}{4C}(1+2l\tau+l^2),$$

where we have used $C=1+l\tau$.

APPENDIX C

The nonzero NP quantities for the metric (31) are

$$\Psi_2 = \frac{1}{8}ab\theta(u)\theta(v)\frac{(5+\tau)}{(1+\tau)^{\frac{5}{2}}},$$

$$\Psi_4 = -\frac{3}{8}a^2\theta(u)\frac{(5+\tau)}{(1+\tau)^{\frac{5}{2}}} + \frac{a}{4}\frac{\delta(u)}{\cos bv(\theta(v))}\frac{(3+\tau)}{(1+\tau)^{3/2}},$$

$$\Psi_0 = -\frac{3}{8}b^2\theta(v)\frac{(5+\tau)}{(1+\tau)^{\frac{5}{2}}} + \frac{b}{4}\frac{\delta(v)}{\cos au(\theta(u))}\frac{(3+\tau)}{(1+\tau)^{3/2}},$$

$$\Phi_{22} = \frac{1}{16}a^2\theta(u)\frac{(7+\tau)}{(1+\tau)^{\frac{5}{2}}},$$

$$\Phi_{00} = \frac{1}{16}b^2\theta(v)\frac{(7+\tau)}{(1+\tau)^{\frac{5}{2}}},$$

$$\Phi_{02} = -\frac{1}{4}\frac{ab\theta(u)\theta(v)}{(1+\tau)^{\frac{3}{2}}},$$

$$\Phi_{11} = \frac{3}{32}ab\theta(u)\theta(v)\frac{(1-\tau)}{(1+\tau)^{\frac{5}{2}}} = -3\Lambda.$$

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