Nonsingular colliding wave solutions in Einstein-Maxwell-dilaton-axion theory

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The local isometry between black holes and colliding plane waves is employed to derive new colliding wave solutions in the Einstein-Maxwell-dilaton-axion theory. The technique is applied to the asymptotically nonflat linear dilaton black holes. We obtain two new metrics which we label (from the language of black holes) as Kerr and Newman-Unti-Tamburino (NUT) types. The NUT type turns out to be type *D* while the Kerr type belongs to the general class. Both types share the common feature that, instead of an all encompassing generic singularity, Cauchy horizons develop in the process of collision.

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I. INTRODUCTION

Chandrasekhar and Xanthopoulos (CX) first observed that a particular metric of colliding plane waves (CPWs) transforms into the trapped region between horizons of the Kerr black hole (BH) [1]. The reason for this local isometry is simple: in that region the Kerr black hole admits two spacelike Killing vectors, the same as required by the space of CPWs. A coordinate transformation maps the one problem into the other provided the boundary conditions are satisfied. By this it is meant that continuous matching of the different wave regions holds, such that no source currents arise at the boundaries. A special case covers naturally the Schwarzschild BH where for $r<2m$ (i.e., inside the horizon) it admits two spacelike Killing vectors and the corresponding CPW spacetime can easily be derived [2]. Extension of the Kerr BH to the Kerr-Newman case and the associated CPW metric in Einstein-Maxwell (EM) theory was also given by CX [3]. The same idea of local isometry has also been used to obtain CPW solutions in Einstein-dilaton-axion (EDA) theory [4]. More recently, we have given an example of the CPW metric in the Einstein-Maxwell-dilaton-axion (EMDA) theory in the limit of zero dilaton field, which also employs an isometry between the throat region of extremal BHs and CPWs [5]. This example suggests that the local isometry in question has a larger scope than envisaged. In a separate work we showed the exact equivalence of the near horizon geometry of extremal BHs and CPWs in the EM theory [6]. All BH solutions alluded to so far share the common feature that they are asymptotically flat. A new type of BH in the linear dilaton gravity has been introduced, on the other hand, which fails to satisfy asymptotic flatness [7–11]. Since the space of CPWs also shares this latter condition, the local isometry between such BHs and CPW spacetimes must be expected in a more natural way. In addition to the linear dilaton, these asymptotically non-flat BHs admit electromagnetic (em) and axion fields, which enable us in this paper to obtain new CPW metrics in the EMDA theory. From the physics standpoint our solutions are important since they are free of physical singularities. Singularities, which used mostly to doom the interaction region of CPWs, are replaced here by extendable Cauchy horizons. By standard solution generation techniques, new solutions in the EMDA theory can be obtained, but singularity-free solutions are not guaranteed [12,13]. Finally, we wish to comment that we can add massless scalar fields to the already existing dilaton, axion, and em fields by using a method which we have developed recently [14,15].

The organization of the paper is as follows. In Sec. II we review the linear dilatonic BH and its extension to stationary form. Section III covers the derivation of our CPW metrics whose details are tabulated in Appendixes B and C. We conclude the paper in Section IV with a conclusion and discussion.

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II. LINEAR DILATON BLACK HOLES

The field equations in the EMDA theory can be generated from the action

$$S=\frac{1}{16\pi}\int d^4x|g|^{1/2}\bigg[-R+2(\nabla\phi)^2+\frac{1}{2}e^{4\phi}(\nabla\kappa)^2 -e^{-2\phi}F_{\mu\nu}F^{\mu\nu}-\kappa F_{\mu\nu}\tilde{F}^{\mu\nu}\bigg] \tag{1}$$

where ϕ is the dilaton, κ is the (pseudoscalar) axion, and $F_{\mu\nu}$ stands for the em field tensor. The dual field tensor is defined by $\tilde{F}^{\mu\nu}=\frac{1}{2}|g|^{-1/2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ in which we choose $\epsilon^{0123}=+1$. In addition to the Einstein equations

$$G_{\mu\nu}=-8\pi T_{\mu\nu} \tag{2}$$

the remaining EMDA field equations are

$$\partial_\mu[|g|^{1/2}(e^{-2\phi}F^{\mu\nu}+\kappa\tilde{F}^{\mu\nu})]=0, \tag{3}$$

$$2\Box\phi=e^{4\phi}(\nabla\kappa)^2+e^{-2\phi}F_{\mu\nu}F^{\mu\nu},$$

$$|g|^{-1/2}\partial_\mu(|g|^{1/2}e^{4\phi}g^{\mu\nu}\kappa_{,\nu})=-F_{\mu\alpha}\tilde{F}^{\mu\alpha},$$

in which $\Box$ stands for the covariant Laplacian. In Appendix A we give the total energy-momentum tensor $T_{\mu\nu}$ in terms of both the fields and the tetrad scalars. The following diagonal metric solves the EMDA equations without the axion [9]:

$$ds^2=\left(1-\frac{r_+}{r}\right)dt^2-\left(1-\frac{r_+}{r}\right)^{-1}dr^2 -r^2\left(1-\frac{r_-}{r}\right)(d\theta^2+\sin^2\theta d\phi^2). \quad (4)$$

The dilaton and Maxwell two-form ($F=dA$) are

$$e^{2\phi}=e^{2\phi_\infty}\left(1-\frac{r_-}{r}\right),$$

$$F=\frac{Qe^{2\phi_\infty}}{r^2}dr\wedge dt, \quad (5)$$

respectively, where ϕ_∞ is the asymptotic value of the dilaton and the mass (M) and electric charge (Q) of the BH are

$$M=\frac{1}{2}r_+,$$

$$Q=e^{-\phi_\infty}\sqrt{\frac{r_+r_-}{2}}. \quad (6)$$

In the string frame the dilaton is a linear function of distance, and in the near horizon limit this solution of EMD theory transforms into

$$ds^2=\frac{r-b}{r_0}dt^2-\frac{r_0}{r-b}dr^2-r_0r(d\theta^2+\sin^2\theta d\phi^2) \quad (7)$$

with

$$e^{2\phi}=\frac{r}{r_0},$$

$$F=\frac{1}{\sqrt{2}r_0}dr\wedge dt.$$

The new constants b and r_0 that arise in the near horizon geometry are related to the mass ($b=2M$) and the electric charge ($Q=r_0/\sqrt{2}$) of the BH. The distinctive feature of this BH, as can be observed easily, is that it fails to satisfy the asymptotic flatness.

Stationary generalization of this BH in the EMDA theory is achieved through the sigma model representation [9–11]. In this method the metric ansatz is taken as

$$g_{\mu\nu}=\begin{pmatrix} f & -fw_i \\ -fw_i & -\frac{1}{f}h_{ij}+fw_iw_j \end{pmatrix}. \quad (8)$$

The em vector potential A_μ is parametrized by the potentials v (electric) and u (magnetic) in accordance with

$$F_{io}=\frac{1}{\sqrt{2}}v_i,$$

$$e^{-2\phi}F^{ij}+\kappa\tilde{F}^{ij}=\frac{f}{\sqrt{2h}}\epsilon^{ijk}u_k, \quad (9)$$

in which a subscript implies a partial derivative. Further, a twist potential χ is introduced through the differential relation (for details we refer to Ref. [9])

$$\chi_i+vu_i-uv_i=-\frac{f^2}{\sqrt{h}}h_{ij}\epsilon^{jkl}w_{i,k}. \quad (10)$$

Thus the six potentials, namely, f, χ, u, v, ϕ, and κ parametrize overall the target space apt for the EMDA theory. The Kerr Newman-Unti-Tamburino (NUT) extension of the static metric (7) is obtained accordingly as

$$ds^2=\frac{\tilde{\Delta}-a^2\sin^2\theta}{\Gamma}(dt-wd\phi)^2 -\Gamma\left(\frac{dr^2}{\tilde{\Delta}}+d\theta^2+\frac{\tilde{\Delta}\sin^2\theta}{\tilde{\Delta}-a^2\sin^2\theta}d\phi^2\right) \quad (11)$$

(we note that we put a tilde over Δ in order to avoid any confusion with the Δ that we shall be using in the next section). The dilaton, axion, and (u,v) potentials are

$$e^{2\phi}=\frac{r^2+(N+a\cos\theta)^2}{\Gamma},$$

$$\kappa=\frac{r_0}{M}\frac{N(r-M)-aM\cos\theta}{r^2+(N+a\cos)^2},$$

$$v=\frac{r^2+(N+a\cos\theta)^2}{\Gamma},$$

$$u=\frac{r_0}{M}\frac{N(r-M)-aM\cos\theta}{\Gamma}, \quad (12)$$

where a and N are the Kerr (rotation) and NUT parameters, respectively, while other abbreviations are as follows:

$$\tilde{\Delta}=r^2-2Mr+a^2-N^2,$$

$$\Gamma=\frac{r_0}{M}(Mr+N^2+aN\cos\theta),$$

$$w=-\frac{r_0}{M}\frac{N\tilde{\Delta}\cos\theta+a(Mr+N^2)\sin^2\theta}{\tilde{\Delta}-a^2\sin^2\theta}. \quad (13)$$

It is observed that the diagonal metric (7) is asymptotic to the off-diagonal one (11). This implies that in the limit $r\to\infty$ the metric (11) goes to (7) in which the axion no longer exists.

III. CPW SOLUTIONS FROM LINEAR DILATON BH

The general metric for CPWs is represented by [1]

$$ds^2=X\left(\frac{d\tau^2}{\Delta}-\frac{d\sigma^2}{\delta}\right)-(Ydx^2+Zdy^2\pm 2Wdxdy) \quad (14)$$

where $\Delta=1-\tau^2$, $\delta=1-\sigma^2$, and the metric functions depend only on the variables (τ,σ). Next, by introducing null coordinates (u,v) through

$$\tau=\sin(au+bv),$$

$$\sigma=\sin(au-bv)$$

$$(a,b=\text{const}), \quad (15)$$

we observe that the line element is cast into the standard form suitable for CPWs. The colliding wave formulation of the problem follows by the substitutions $u\rightarrow u\,\theta(u)$, $v\rightarrow v\,\theta(v)$, where θ is the Heaviside unit step function. The problem of local isometry requires that the Kerr-NUT metric (11) and (12) be transformed into the form of CPWs such that $X>0$, $Y\geqslant 0$, and $Z\geqslant 0$ necessarily. Vanishing of metric functions signals singularities of the coordinate type or generic curvature singularities. We observe that the (r,θ) sector of the BH metric (11) can consistently be mapped into the (τ,σ) form provided

$$\Gamma(r)\left(\frac{dr^2}{N^2-a^2+2Mr-r^2}-d\theta^2\right)=\Gamma(r(\tau))\left(\frac{d\tau^2}{\Delta}-\frac{d\sigma^2}{\delta}\right) \quad (16)$$

is satisfied. Beside identifying $\sigma=\cos\theta$ this tantamounts to

$$\int^r\frac{dr}{\sqrt{N^2-a^2+2Mr-r^2}}=\pm\sin^{-1}\tau \quad (17)$$

or equivalently, by choosing one of the signs,

$$r=M+\sqrt{N^2+M^2-a^2}\,\tau. \quad (18)$$

We note that an analytic expression of τ in terms of r may not be available in all problems where we demand identifications such as Eq. (16). In a large class of problems, however, including BHs in higher dimensions, de Sitter cosmology, and quintessence problems, our prescription works perfectly, implying that a corresponding CPW metric can be found. The linear dilaton BH solution (11) now transforms into CPWs by employing the transformation

$$\sigma=\cos\theta,$$

$$r=M+\sqrt{N^2+M^2-a^2}\,\tau,$$

$$x=t,$$

$$y=\varphi. \quad (19)$$

By imposing appropriate scaling of the coordinates (adapting $M=r_0=1$) and defining $q=a$ (or $q=N$) with a related parameter $p\geqslant 1$ such that

$$p^2-q^2=1, \quad (20)$$

we obtain the following metrics of CPWs in the EMDA theory. For completeness we consider also separately as a third class the case of the EMD metric of CPWs corresponding to $a=0=N$.

(1) The Kerr-type CPW metric ($N=0$, $q=a$)

$$ds^2=(p+\tau)\left[\frac{d\tau^2}{\Delta}-\frac{d\sigma^2}{\delta}-\delta\left(dy-\frac{q}{\tau+p}dx\right)^2\right]-\frac{\Delta}{\tau+p}dx^2. \quad (21)$$

The dilaton, axion, and em potential one-form are

$$e^{2\phi}=\frac{(\tau+p)^2+q^2\sigma^2}{\tau+p},$$

$$\kappa=\frac{-q\sigma}{(\tau+p)^2+q^2\sigma^2},$$

$$A=\frac{1}{\sqrt{2}}\left[\frac{(\tau+p)^2+q^2\sigma^2}{\tau+p}dx+q\,\delta dy\right]. \quad (22)$$

Now substitution of Eq. (15) and insertion of the step functions with the null coordinates we obtain the interaction (collision) region (region IV) of our metric. The incoming region (region II) for $v<0$ becomes

$$ds_{II}^2=(p+\sin au)\left[4ab\,du\,dv-\cos^2 au\left(dy-\frac{q\,dx}{p+\sin au}\right)^2\right]-\frac{\cos^2 au}{p+\sin au}dx^2 \quad (23)$$

(and a similar metric with ds_{III}^2 with $au\leftrightarrow bv$ for region III). For $u<0$, $v<0$, we get the flat metric (region I)

$$ds_I^2=4abp\,du\,dv-\frac{1}{p}dx^2-p\left(dy-\frac{q}{p}dx\right)^2 \quad (24)$$

expressed in a scaled coordinate system. The dilaton, axion, and em fields can also be easily obtained in the incoming regions. By inverting the problem, this information constitutes our initial data which naturally all vanish, as it should in the flat region $u<0$, $v<0$. In Appendix B we give the nonzero curvature and Ricci components of this spacetime. The interesting property is that it is not singular. All Weyl scalars are regular and the singularities at $\tau=1(\sigma=1)$ are spurious coordinate singularities. Another interesting property is that in contrast to the Kerr metric our Kerr-type metric (21) is not type D. This becomes evident after we compute

$$\Psi_0\Psi_4-9\Psi_2^2=\frac{q^2\delta\Delta}{16(\tau+p)^6}\neq 0. \quad (25)$$

In the limit $q\rightarrow 0$ $(p\rightarrow 1)$, which corresponds to the CPW generated from the static dilaton metric (7), it becomes type D.

(2) The NUT-type CPW metric $(a=0, q=N\neq 0)$

$$ds^2=(p+\tau)\left[\frac{d\tau^2}{\Delta}-\frac{d\sigma^2}{\delta}-\delta dy^2\right]-\frac{\Delta}{p+\tau}(dx+q\sigma dy)^2. \tag{26}$$

The dilaton, axion, and Maxwell potential one-form are as follows:

$$e^{2\phi}=\frac{p(1+\tau^2)+2\tau}{\tau+p},$$

$$\kappa=\frac{q\tau}{p(1+\tau^2)+2\tau},$$

$$A=\frac{1}{\sqrt{2p}}\left[\frac{p(1+\tau^2)+2\tau}{\tau+p}dx-\frac{pq\sigma\Delta}{\tau+p}dy\right]. \tag{27}$$

In the incoming region $(v<0)$ (region II) our metric takes the form [with $u=u\theta(u)$]

$$ds_{II}^2=(p+\sin au)[4abdudv-\cos^2 au dy^2] -\frac{\cos^2 au}{p+\sin au}(dx+q\sin au dy)^2 \tag{28}$$

and a similar form (by $au\leftrightarrow bv$) follows for the region III incoming metric. For $u<0$, $v<0$ we obtain the flat metric

$$ds^2=p(4abdudv-dy^2)-\frac{1}{p}dx^2. \tag{29}$$

The initial data for our incoming fields can also easily be found from Eq. (27). We present the details of this metric in Appendix C. The Weyl scalars suggest that, similar to the Kerr-type metric (21), the NUT-type metric (26) is also regular. $\tau=1$ $(\sigma=1)$ are coordinate singularities that can be removed. The significant difference between the NUT and Kerr types is that the NUT type turns out to be type D. The Weyl curvatures (Appendix C) in the interaction region $(u>0, v>0)$ satisfy

$$9\Psi_2^2=\Psi_0\Psi_4. \tag{30}$$

(3) The general static EMD metric was given in Eq. (4). We wish now to obtain the corresponding CPW in this case as well. For this purpose we identify

$$\frac{dr^2}{(r-r_-)(r-r_+)}-d\theta^2=\frac{d\tau^2}{\Delta}-\frac{d\sigma^2}{\delta}, \tag{31}$$

which leads to the relation

$$2r=r_++r_-+(r_+-r_-)\tau. \tag{32}$$

The coordinate transformation toward our CPW metric is accomplished by this condition and $\sigma=\cos\theta$, $x=t$, and $y=\varphi$. The resulting metric is (up to an overall constant rescaling)

$$ds^2=(1+\tau)(\alpha_0+\beta_0\tau)\left[\frac{d\tau^2}{\Delta}-\frac{d\sigma^2}{\delta}-\delta dy^2\right] -\frac{1-\tau}{\alpha_0+\beta_0\tau}dx^2, \tag{33}$$

where $\alpha_0=r_++r_-$ and $\beta_0=r_+-r_-$. This is the CPW metric corresponding to a more general EMD theory without the axion. In the extremal case we choose $\beta_0=0$ and (after rescaling the x coordinate) we obtain

$$ds^2=(1+\tau)\left[\frac{d\tau^2}{\Delta}-\frac{d\sigma^2}{\delta}-\delta dy^2\right]-(1-\tau)dx^2. \tag{34}$$

This is precisely the limiting case $(p=1, q=0)$ of both the Kerr (21) and NUT (26) type $(a=0=N)$ metrics for CPWs. The metric (33) describes collision of waves in EMD theory, which is both regular and type D.

IV. CONCLUSION AND DISCUSSION

The local equivalence between the inner horizon region of BHs and the spacetime of CPWs has been fruitful in the generation of physically significant solutions in the colliding EMDA theory. For sample BHs we have chosen Kerr-NUT-type BHs in a linear dilaton background. As expected, the initial data for dilaton, axion, and em fields cannot be arbitrary but are dictated by the original BH solution. The freedom to eliminate the axion reduces the metric to diagonal and it leads to a regular CPW solution in the EMD theory. The incoming plane waves (i.e., a holographic boundary in the string language) consisting of a mixture of dilaton, axion, and em waves extend smoothly into the interaction region. We realize once more (as in Ref. [5]) that the axion survives within the second polarization context of the colliding waves. We add, finally, that our technique applies also in higher dimensional BHs and colliding branes. One signaling problem in higher dimensions, however, is that the plane waves may propagate in lower dimensional backgrounds. It remains to be seen whether this feature may lead to the creation of extra dimensions via colliding waves.

APPENDIX A

The total energy-momentum tensor is given by

$$4\pi T_{\mu\nu}=\phi_\mu\phi_\nu-\frac{1}{2}g_{\mu\nu}(\nabla\phi)^2 +\frac{1}{4}e^{4\phi}\left(\kappa_\mu\kappa_\nu-\frac{1}{2}g_{\mu\nu}(\nabla\kappa)^2\right) +e^{-2\phi}\left(F_{\mu\alpha}F_\nu^\alpha+\frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right). \tag{A1}$$

In terms of the null tetrad formalism of Newman and Penrose (NP), the energy-momentum is expressed as follows:

$$\begin{aligned}4\pi T_{\mu\nu}=&\phi_{00}n_\mu n_\nu+\phi_{22}l_\mu l_\nu+\phi_{02}\bar{m}_\mu\bar{m}_\nu+\phi_{20}m_\mu m_\nu\\&-\phi_{01}n_\mu\bar{m}_\nu-\phi_{10}n_\mu m_\nu-\phi_{12}l_\mu\bar{m}_\nu-\phi_{21}l_\mu m_\nu\\&+(\phi_{11}+3\Lambda)(l_\mu n_\nu+n_\mu l_\nu)\\&+(\phi_{11}-3\Lambda)(m_\mu\bar{m}_\nu+\bar{m}_\mu m_\nu).\end{aligned}\tag{A2}$$

APPENDIX B

The null tetrad basis one-forms for the Kerr-type metric (21) are

$$\begin{aligned}\sqrt{2}l&=\sqrt{p+\tau}\left(\frac{d\tau}{\sqrt{\Delta}}-\frac{d\sigma}{\sqrt{\delta}}\right),\\ \sqrt{2}n&=\sqrt{p+\tau}\left(\frac{d\tau}{\sqrt{\Delta}}+\frac{d\sigma}{\sqrt{\delta}}\right),\\ \sqrt{2}m&=i\sqrt{\frac{\Delta}{p+\tau}}dx+\sqrt{\delta(p+\tau)}\left(dy-\frac{q\,dx}{p+\tau}\right).\end{aligned}\tag{B1}$$

The nonzero NP Ricci and Weyl scalars are

$$\begin{aligned}\phi_{11}&=-3\Lambda=\frac{ab\,\theta(u)\theta(v)}{16(\tau+p)^3}(\Delta-q^2\delta),MM\\ \phi_{02}&=\phi_{20}=\frac{ab\,\theta(u)\theta(v)}{4(\tau+p)}\left(1+\frac{q^2\sigma^2}{(\tau+p)^2}\right),\\ \phi_{22}&=\frac{b^2\theta(v)}{8(\tau+p)^2}\left(\tau+3p+\frac{q^2\sigma^2}{(\tau+p)}\right),\\ \phi_{00}&=\frac{a^2\theta(u)}{8(\tau+p)^2}\left(\tau+3p+\frac{q^2\sigma^2}{(\tau+p)}\right),\\ \Psi_2+2\Lambda&=\frac{ab\,\theta(u)\theta(v)}{8(\tau+p)^3}(\tau+p+iq\sigma)^2,\\ \Psi_2&=\frac{ab\,\theta(u)\theta(v)}{12}K,\\ \Psi_4&=bG_1(u)\delta(v)+\frac{b^2\theta(v)K}{4},\\ \Psi_0&=aF_1(v)\delta(u)+\frac{a^2\theta(u)K}{4},\end{aligned}\tag{B2}$$

with

$$K=\frac{1}{(\tau+p)^3}[2(1+\tau p)+q^2\delta+(\tau+p)(\tau+3iq\sigma)]$$

and $G_1=\lambda_v$ and $F_1=-\sigma_u$, where the spin coefficients λ and σ are

$$\begin{aligned}\lambda&=\frac{1}{2\sqrt{2}(\tau+p)^{3/2}}\left[\frac{1+p\tau}{\sqrt{\Delta}}+\frac{\sigma}{\sqrt{\delta}}(p+\tau)-iq\sqrt{\delta}\right],\\ \sigma&=\frac{1}{2\sqrt{2}(\tau+p)^{3/2}}\left[-\frac{1+p\tau}{\sqrt{\Delta}}+\frac{\sigma}{\sqrt{\delta}}(p+\tau)-iq\sqrt{\delta}\right].\end{aligned}\tag{B3}$$

APPENDIX C

The null-tetrad basis one-forms for the NUT-type metric (26) are

$$\begin{aligned}\sqrt{2}l&=\sqrt{p+\tau}\left(\frac{d\tau}{\sqrt{\Delta}}-\frac{d\sigma}{\sqrt{\delta}}\right),\\ \sqrt{2}n&=\sqrt{p+\tau}\left(\frac{d\tau}{\sqrt{\Delta}}+\frac{d\sigma}{\sqrt{\delta}}\right),\\ \sqrt{2}m&=\sqrt{\delta(p+\tau)}dy+i\sqrt{\frac{\Delta}{p+\tau}}(dx+q\sigma dy).\end{aligned}\tag{C1}$$

The nonzero NP scalars are

$$\begin{aligned}\phi_{11}&=-3\Lambda=\frac{ab\,\theta(u)\theta(v)}{16(\tau+p)^3}p^2\Delta,\\ \phi_{02}&=\phi_{20}=\frac{ab\,\theta(u)\theta(v)}{4(\tau+p)}\left(1+\frac{q^2\sigma^2}{(\tau+p)^2}\right),\\ \phi_{22}&=\frac{b^2\theta(v)}{8(\tau+p)^2}\left(\tau+3p+\frac{q^2\sigma^2}{(\tau+p)}\right),\\ \phi_{00}&=\frac{a^2\theta(u)}{8(\tau+p)^2}\left(\tau+3p+\frac{q^2\sigma^2}{(\tau+p)}\right),\end{aligned}$$

$$\Psi_2=\frac{ab\,\theta(u)\,\theta(v)}{12(\tau+p)^3}[(1+p\tau+iq)^2+(p\tau+1)(1+iq)],$$

$$\Psi_4=bG_2(u)\,\delta(v)+\frac{b^2\theta(v)}{4(p+\tau)^3}[-p^2\Delta+3(1+p\tau)(1+iq)],$$

$$\Psi_0=aF_2(v)\,\delta(u)+\frac{a^2\theta(u)}{4(p+\tau)^3}[-p^2\Delta+3(1+p\tau)(1+iq)], \tag{C2}$$

where the impulsive components are

$$G_2=\lambda_v \quad\text{and}\quad F_2=-\sigma_u,$$

in which the spin coefficients are

$$\lambda=\frac{1}{2\sqrt{2}(\tau+p)^{3/2}}\left[\frac{1+p\tau}{\sqrt{\Delta}}+\frac{\sigma}{\sqrt{\delta}}(p+\tau)+iq\sqrt{\Delta}\right],$$

$$\sigma=-\frac{1}{2\sqrt{2}(\tau+p)^{3/2}}\left[\frac{1+p\tau}{\sqrt{\Delta}}-\frac{\sigma}{\sqrt{\delta}}(p+\tau)+iq\sqrt{\Delta}\right]. \tag{C3}$$

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