Performance Analysis of The Path Selective Decorrelating Detector with a Maximum Likelihood Channel Estimator

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Abstract

The adaptive path selective decorrelating detector requires the knowledge of the channel coefficients for path selection to reduce both system complexity and noise enhancement. Generally, the channel coefficients are assumed to be known at the receiver. However, this is not realistic. Therefore, we propose a maximum-likelihood channel estimation method that makes use of both known spreading sequences and short training sequences for the estimation of channel coefficients. We derive an expression for the Mean Square Error (MSE) of the channel estimate and extend the semi-analytic Bit Error Rate (BER) analysis of the path selective receiver to include estimation errors. Results show that with a fairly short training sequence (8-16 bits) the system performs very close to the known channel case.

1. Introduction

With the ever-increasing requirements for more flexibility, higher capacity and resistance to propagation impairments, Direct Sequence Code Division Multiple Access (DS-CDMA) has become one of the favorite candidates for future mobile radio systems. Although CDMA based systems provide high power efficiency and moderate error rates, Multiple Access Interference (MAI) and intersymbol interference (ISI) due to multipath propagation are two most significant factors limiting the performance of the wireless CDMA systems. Multipath fading is a result of transmission through a multipath channel, whereas the MAI is a result of multiple users sharing the same channel. A receiver structure that consists of the RAKE receiver [1] followed by a Decorrelating Detector (DD) [2, 3] and multipath combiner can be used, first, to separate the resolved parts of the channel, next, to eliminate MAI and, finally, to combine all the paths using channel knowledge [2, 3]. That structure, known as the Multipath-Decorrelating Detector (MD), is computationally demanding and has noise enhancement problem. Recently a path selective scheme has been proposed [4] to reduce the complexity of MD and improve system performance by decreasing noise enhancement, despite some increase in MAI due to unselected paths. However, the path selection needs channel coefficients and, in [4], they are assumed to be known.

In cases where the channel is not known, we need a channel estimator. It is not uncommon to do channel estimation using training sequences known at both receiver and transmitter sides. The downside is the reduction of channel efficiency. So, a method that allows short training sequences with a fairly good performance is worthy of developing. In this paper, we use a maximum likelihood method that allows a fairly short training sequence to be used along with the known spreading sequences to estimate the channel coefficients. Furthermore, we derive an expression for the channel estimation error and incorporate it into the semi-analytic performance analysis of the path selective receiver. We provide analytical results that help to make a reasonable choice of system parameters with insignificant loss in performance compared to the known channel case.

The paper is organized as follows. In the next section, the assumed communication system model is presented. Section 3 describes the channel estimation method with short training sequences. Adaptive path selection of the channel and the proposed detector are presented in section 4. The performance analysis of the proposed system with the channel estimation errors is explained in section 5. Finally, the numerical results illustrating the performance of the path adaptive decorrelating detector and conclusion are given.

2. System Model

In a CDMA system, several users transmit simultaneously over a common channel. The received baseband signal at a single receiver from K asynchronous users can be represented as:

$$r(t) = \sum_{k=1}^{K} \sqrt{w_k} S_k(t) + n(t) \tag{1}$$

where subscript k denotes user index, w_k is the transmitted power, $n(t)$ is complex zero-mean AWGN whose real and imaginary parts are independent and each have power spectral density $N_o/2$. The multipath-fading channel can be expressed as [1]:

$$c_k(t) = \sum_{l=1}^{N_p} c_{k,l} \delta(t - k_l) \tag{2}$$

where N_p is the number of resolvable paths of the channel, $c_{k,l}$ is the complex coefficient of l-th path, k_l is the delay of l-th propagation path and $\delta(t)$ is the Dirac delta function. The channel coefficients, $c_{k,l}$, in (2) are independent zero-mean complex valued Gaussian random variables and the channel vector of user k, $\mathbf{c}_k=[c_{k,1}\ c_{k,2}\ \dots\ c_{k,Np}]^T$ is assumed to be constant within a packet transmission.

The received signal $S_k(t)$ for each user can be represented as:

$$S_k(t) = \sum_{p=1}^{P} b_k(p) \sum_{l=1}^{N_p} c_{k,l} s_k(t-(l-\tau_k-1)T_c-(p-1)T) \tag{3}$$

where P is the length of the packet containing P_t preamble bits, $b_k(p) \in \{\pm 1\}$ denotes the transmitted bit p of the user k, T is the symbol duration, $s_k(t)$ is the real-valued unit-energy spreading sequence of length L with support $[0, T]$, τ_k is the relative transmitter delay where $0 < \tau_1 < \tau_2 < \ldots < \tau_K < T$ and T_c is the chip period of the spreading sequence.

3. Channel Estimation

We assume a training sequence of P_t symbols. By sampling the received signal $r(t)$ at chip rate (T_c), over the training period, the received signal vector $\mathbf{A} = [r(0) \quad r(1) \quad \ldots \quad r(LP_t - 1)]^T$ can also be expressed as:

$$\mathbf{A} = \sqrt{w_1}\mathbf{G}_1\mathbf{c}_1 + \sqrt{w_2}\mathbf{G}_2\mathbf{c}_2 + \cdots + \sqrt{w_K}\mathbf{G}_K\mathbf{c}_K + \mathbf{N} \tag{4}$$

where

$$\mathbf{G}_k = [\mathbf{F}_k(\tau_k) \quad \mathbf{F}_k(\tau_k+1) \quad \cdots \quad \mathbf{F}_k(\tau_k+N_p-1)] \tag{5}$$

and

$$\mathbf{F}_k(n) \overset{\text{def}}{=} [\underbrace{0 \quad \cdots \quad 0}_{n} \quad \mathbf{f}_{k,1}(L) \quad \mathbf{f}_{k,2}(L) \quad \cdots \quad \mathbf{f}_{k,P_t}(L-n)]^T \tag{6}$$

here $(\cdot)^T$ denotes the transpose operation and,

$$\mathbf{f}_{k,p}(x) \overset{\text{def}}{=} b_k(p)[s_k(1) \quad s_k(2) \quad \cdots \quad s_k(x)] \quad x \le L \tag{7}$$

The vector $\mathbf{A}$ given in (4) can be rearranged as follows:

$$\mathbf{A} = \underbrace{[\mathbf{G}_1 \quad \mathbf{G}_2 \quad \cdots \quad \mathbf{G}_K]}_{\mathbf{G}} \underbrace{\begin{bmatrix} \sqrt{w_1}\mathbf{c}_1 \\ \sqrt{w_2}\mathbf{c}_2 \\ \vdots \\ \sqrt{w_K}\mathbf{c}_K \end{bmatrix}}_{\mathbf{C}} + \mathbf{N} \tag{8}$$

Here, the transmitted powers are incorporated into the channel model. The maximum-likelihood estimate of the channel together with the amplitudes, which amounts to the multiplication of **A** by the pseudoinverse of **G**, is given by [5]:

$$\hat{\mathbf{C}} = \mathbf{G}^{\dagger}(\mathbf{GC} + \mathbf{N}) = \mathbf{C} + \mathbf{G}^{\dagger}\mathbf{N} \tag{9}$$

The expected value of $\mathbf{G}^{\dagger}\mathbf{N}$ is $E[\mathbf{G}^{\dagger}\mathbf{N}]=0$. Thus $\hat{\mathbf{C}}$ is an unbiased linear estimate of **C**. $\hat{\mathbf{C}}$ is also the maximum-likelihood estimate of **C** from **A** [6]. The error in the estimation is $\mathbf{G}^{\dagger}\mathbf{N}$. Thus, the estimation error covariance matrix **E** can be given as:

$$\mathbf{E}=E[(\mathbf{G}^{\dagger}\mathbf{N})\,(\mathbf{G}^{\dagger}\mathbf{N})^{H}]=\mathbf{G}^{\dagger}\,E\,[\mathbf{N}\mathbf{N}^{H}](\mathbf{G}^{\dagger})^{H} \tag{10}$$

where $(\cdot)^{H}$ denotes Hermitian transpose. The Mean Square Error (MSE) of user k is the summation of the corresponding N_p diagonal terms of **E**. Since $E[\mathbf{N}\mathbf{N}^{H}]=2\sigma^{2}\mathbf{I}$ [6], where σ^2 is the noise variance, the corresponding MSE of the k-th user can be calculated as:

$$\mathrm{MSE}_k = 2\sigma^2 \sum_{l=(k-1)N_p+1}^{kN_p} g_l^{\dagger}(g_l^{\dagger})^{H} \tag{11}$$

where $g_l^{\dagger}$ is the l-th row of $\mathrm{G}^{\dagger}$.

4. Adaptive Path Selective Receiver Structure

The path selection is done by means of estimated channel coefficients. The number of paths, N_p, which is constant and equal to the number of resolvable paths in the conventional MD, differs for each user and is denoted as $M_p=[M_{p1} \ldots M_{pK}]$. The total number of paths used in the system is $T_p = \sum_{k=1}^{K} M_{pk}$ and varies between K and $K \times N_p$. The receiver model is set up as in Figure 1 with only the selected branches [7]. The signal at the output of the matched filters for the path selective receiver can be written as:

$$\mathbf{y} = \mathbf{R_s C_s b_s} + \mathbf{R_n C_n b_n} + \mathbf{n_c} \tag{12}$$

where subscripts s and n represent the matrices produced by selected and unselected paths respectively. Therefore, $\mathbf{C_s} = diag\left(\mathbf{c}_{s1}^{T}(1)\ldots\mathbf{c}_{sK}^{T}(1)\ldots\mathbf{c}_{s1}^{T}(P) \quad \ldots\mathbf{c}_{sK}^{T}(P)\right)$ and $\mathbf{C_n} = diag\left(\mathbf{c}_{n1}^{T}(1)\ldots\mathbf{c}_{nK}^{T}(1)\ldots\mathbf{c}_{n1}^{T}(P)\ldots\mathbf{c}_{nK}^{T}(P)\right)$ are the channel coefficients where $\mathbf{c}_{sk}$ and $\mathbf{c}_{nk}$ are the selected and unselected channel coefficients where the transmitted powers are incorporated into the channel model, respectively of user k, $\mathbf{b_s}=\left(b_1(1)\mathbf{1}^T_{M_{p1}}\ldots b_K(1)\mathbf{1}^T_{M_{pK}}\ldots b_1(P)\mathbf{1}^T_{M_{p1}}\ldots\ b_K(P)\mathbf{1}^T_{M_{pK}}\right)$ and $\mathbf{b_n}=\left(b_1(1)\mathbf{1}^T_{N_p-M_{p1}}\ldots b_K(1)\mathbf{1}^T_{N_p-M_{pK}}\ldots b_1(P)\mathbf{1}^T_{N_p-M_{p1}}\ldots\ b_K(P)\mathbf{1}^T_{N_p-M_{pK}}\right)$ are the transmitted bits, $\mathbf{R_s}$ and $\mathbf{R_n}$ are the correlation matrices associated with the selected and unselected paths respectively and $\mathbf{n_c}$ is the correlated noise. Here $\mathbf{1}^{\mathrm{T}}{}_n$ is a column of n ones.

The $PT_p \times PT_p$ symmetric block Toeplitz matrix $\mathbf{R_s}$, can be defined as [8]:

$$\mathbf{R}_{\mathrm{S}}=\begin{pmatrix} \mathbf{R}_{\mathrm{s}}(0) & \mathbf{R}_{\mathrm{s}}(-1) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{R}_{\mathrm{s}}(1) & \mathbf{R}_{\mathrm{s}}(0) & \mathbf{R}_{\mathrm{s}}(-1) & & \vdots \\ \mathbf{0} & \mathbf{R}_{\mathrm{s}}(1) & \mathbf{R}_{\mathrm{s}}(0) & \ddots & \mathbf{0} \\ \vdots & & \ddots & \ddots & \mathbf{R}_{\mathrm{s}}(-1) \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{R}_{\mathrm{s}}(1) & \mathbf{R}_{\mathrm{s}}(0) \end{pmatrix} \tag{13}$$

where $\mathbf{0}$ is a $T_p \times T_p$ zero matrix and $T_p \times T_p$ cross correlation matrices, $\mathbf{R_s}(l)$, can be calculated as:

$$\mathbf{R}_{\mathrm{s}}(l)=\int_{-\infty}^{\infty} s_s(t)s_s{}^{\mathrm{T}}(t+lT)dt \tag{14}$$

where $s_s(t)$ is the spreading sequence generated by using only the selected branches and their corresponding delays as:

$$\begin{aligned} s_s(t)=[&s_1(t-\tau_1)\ldots s_1(t\text{-}(M_{p1}\text{-}1)T_c\text{-}\tau_1)\ldots \\ &\ldots s_K(t-\tau_K)\ldots s_K(t\text{-}(M_{pK}\text{-}1)T_c\text{-}\tau_K)]^T \end{aligned} \tag{15}$$

The adaptive path selective linear MD filter output is:

$$\mathbf{z}=\mathbf{R}_{\mathrm{s}}^{-1}\mathbf{y} \tag{16}$$

The output of the path selective MD filter $\mathbf{z}$ associated with the k-th user can be shown as:

$$\mathbf{z}_k=\mathbf{c}_{sk}b_k+\zeta_k \tag{17}$$

where ζ_k is the new noise term at the filter output which is the correlated noise together with the residual MAI coming from the unselected paths. The residual MAI can be modeled as a Gaussian noise and considered as uncorrelated with the noise. As shown in [9], the increase in noise by residual MAI in addition to the noise depends on the magnitude of the eliminated paths and can be considered as the Signal-to-Noise Ratio (SNR) degradation due to the residual MAI. The sum of the variances of residual MAI and noise gives the variance of the total noise. The ratio of this sum to the variance of the noise gives the factor by which the SNR is decreased due to residual MAI. This factor, say D_r, is given by [3, 4, 9]:

$$D_r = 1 + \frac{2}{3L}\sum_{k=1}^{K}\sum_{l\notin A_k}\left|c_{k,l}\right|^2 \frac{E_k}{N_o} \tag{18}$$

where L is the length of the signature waveform, E_k is the energy of user k and A_k is the set of used paths, which are selected according to the threshold value, for each user.

Since the estimated channel coefficients are available at the combiner, $\mathbf{z}_k$ can be written in terms of the estimated channel coefficients, knowing that $\mathbf{c}_{sk} = \hat{\mathbf{c}}_{sk} + \boldsymbol{\xi}_k$ where $\boldsymbol{\xi}_k$ is the estimation error of the channel, as follows:

$$\mathbf{z}_k = \hat{\mathbf{c}}_{sk} b_k + \boldsymbol{\xi}_k b_k + \boldsymbol{\zeta}_k = \hat{\mathbf{c}}_{sk} b_k + \boldsymbol{\gamma}_k \tag{19}$$

where $\boldsymbol{\gamma}_k$ is the noise term at the filter output together with the estimation error of the channel. To find the optimum whitening filter for the noise term $\boldsymbol{\gamma}_k$, the covariance matrix of $\boldsymbol{\gamma}_k$ can be computed as follows:

$$\mathbf{Q}_k = E\{\boldsymbol{\xi}_k \boldsymbol{\xi}_k^H\} + E\{\boldsymbol{\zeta}_k \boldsymbol{\zeta}_k^H\} \tag{20}$$

By assuming that the noise and the channel estimation error are uncorrelated, the covariance matrix of channel estimation error and the noise at the output of the filter can be written as $\mathbf{Q}_k$=$\mathbf{E}_k$+$\mathbf{N}_k$ where $\mathbf{E}_k$=$\mathbf{E}_{k,k}$ (the $M_{pk}\times M_{pk}$ (k, k)th subblock) and $\mathbf{N}_k$=$[N_o D_r \mathbf{R}_s^{-1}]_{k,k}$ (the $M_{pk}\times M_{pk}$ (k, k)th subblock) [7]. For optimum combining, first the noise is whitened using the filter $(\mathbf{T}^H)^{-1}$ obtained through Cholseky decomposition of $\mathbf{Q}_k$=$\mathbf{T}^H\mathbf{T}$ and next the signals are weighted by $\hat{\mathbf{c}}_{sk}^H \mathbf{T}^{-1}$ and input to a maximal ratio combiner. The output of the combiner is $\hat{d}_k(p) = \hat{\mathbf{c}}_{sk}^H \mathbf{T}^{-1} (\mathbf{T}^H)^{-1} [\mathbf{z}]_{(M_{pk}(k-1)+1:M_{pk}k)}$ where $[\mathbf{x}]_{(a:b)}$ denotes elements of $\mathbf{x}$ from a through b. Finally, the estimated bits are obtained by applying the hard decision mechanism to the real part of $\hat{d}_k(p)$.

5. Performance Analysis with Channel Estimation Error

The probability of error for p-th bit of user k, conditioned on the knowledge of the channel coefficients $\hat{\mathbf{c}}_k$, can be obtained as a Q-function by taking the ratio of the signal and noise powers at the output of the combiner as [4]:

$$Pe_{k/\hat{\mathbf{c}}_k}(p, N_o) = Q\left(\sqrt{\frac{\hat{\mathbf{c}}_{sk}^H(p)\left(\mathbf{Q}_{[pk,pk]}\right)^{-1}\hat{\mathbf{c}}_{sk}(p)}{\frac{1}{2}}}\right) \tag{21}$$

The conditional probability of error of user *k* by the knowledge of $\hat{\mathbf{c}}_k$ can be obtained by taking the average of (21) over the packet length as:

$$Pe_{k/\hat{\mathbf{c}}_k}(N_o) = \frac{1}{P}\sum_{p=1}^{P} Pe_{k/\hat{\mathbf{c}}_k}(p, N_o) \tag{22}$$

where the probabilities of all the bits within a packet are assumed to be identical.

6. Numerical Results

The numerical analysis was done for a 10 user system with m-sequence signature waveforms of length L=63. Channels are randomly realized with N_p=6 paths and exponential power delay profile with a dynamic range of 20 dB. We assume that channel coefficients are Gaussian distributed and their variances are determined by the exponential power profile and norm of the channel vector $\mathbf{c}_k$, is one. The performance curves were obtained by averaging the conditional error probability in (22) over 1000 independent channel realizations. The Root Mean Square Error (RMSE) performance of the estimation method is presented in Figure 2 with respect to the number of preamble bits. The curves were analytically obtained for different SNR using (11). The performance improvement by increasing the number of preamble bits used is obvious. Figure 3 illustrates the effect of preamble size on Bit Error Rate (BER) of user 1 at 14 and 20 dB SNR. In this analysis, the BER curve is obtained by (22) for different preamble sizes. It is worth mentioning that the BER performance levels off at a bit length much smaller than that of the MSE performance. Since the ultimate performance is the BER, we conclude that a small number of bits (8-16) are sufficient for successful training. Therefore, the preamble length is chosen as 16 for the rest of the analysis. Figure 4 exhibits the BER behavior of user 1 with respect to SNR at several conditions reported in the figure itself. The tradeoff between noise enhancement and induced MAI can be easily seen from this family of curves. It is noticed from the behavior of the curves that whenever less number of channel paths are used at higher threshold values, the gain coming from the noise enhancement is less compared to the residual MAI producing high BER of user 1. Then around –5 dB, the number of paths used is increased causing the residual MAI to be less and giving the optimum BER of user 1. After –5 dB threshold, the number of paths used is increased further which produce less residual MAI but more noise enhancement, giving again higher BER of user 1. The family of curves in Figure 5 shows the aforementioned behavior from another perspective. To be able to demonstrate how close the channel estimates to the actual channels are, the results with actual channels together with the estimated channels are given.

7. Conclusion

In this paper, the effect of the channel estimation errors on the performance of an adaptive path selective decorrelating detector is examined. It is shown that the necessary channel response knowledge for the path selection of the system can be successfully obtained by using short training sequences together with the known spreading sequences.

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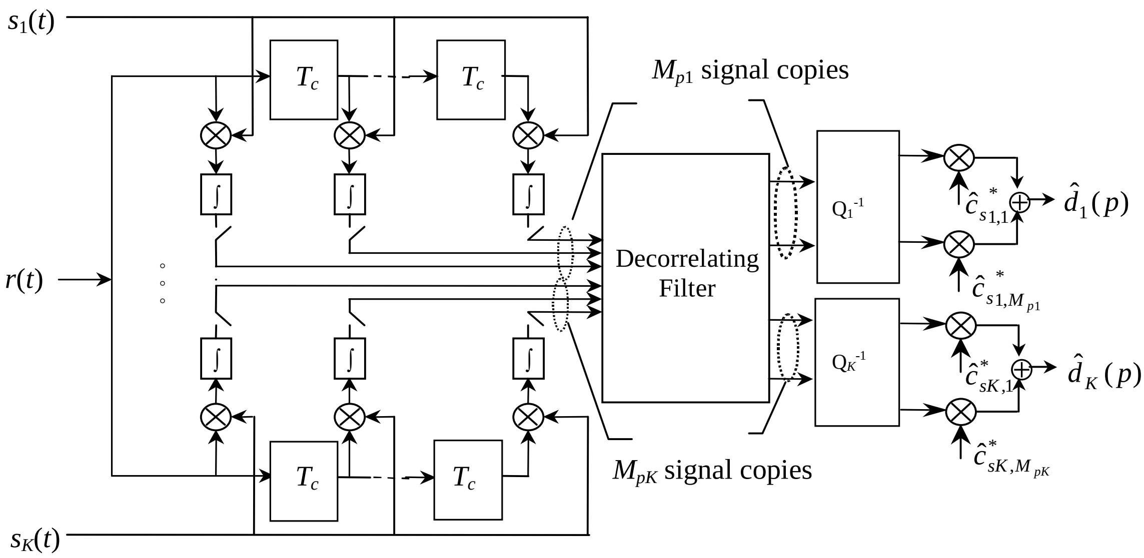

Fig. 1 Multipath-Decorrelating receiver model with selected paths

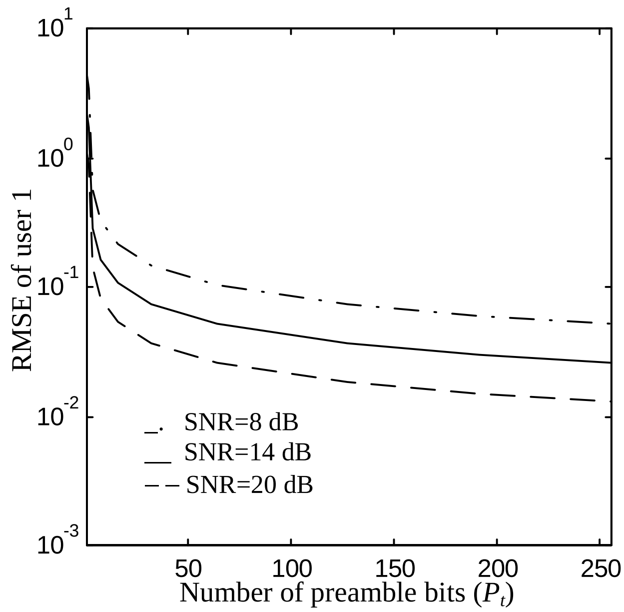

Fig. 2 RMSE of user 1 versus P_t (Ei/E1=0 dB, K=10, L=63, N_p=6)

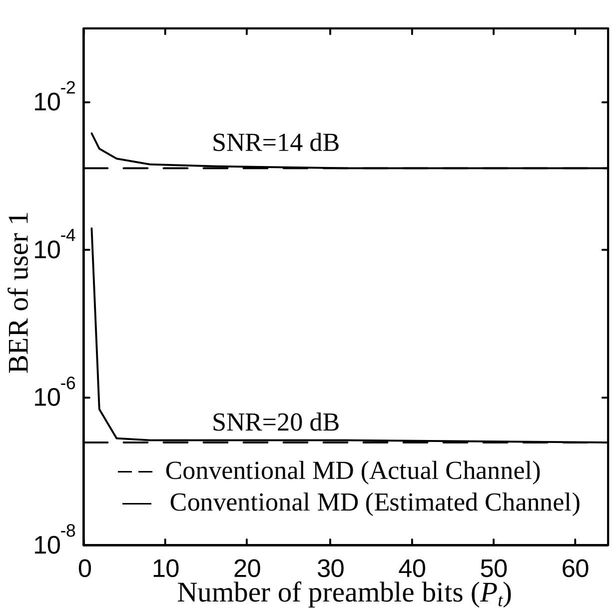

Fig. 3 BER of user 1 versus P_t (Ei/E1=0 dB, K=10, L=63, N_p=6)

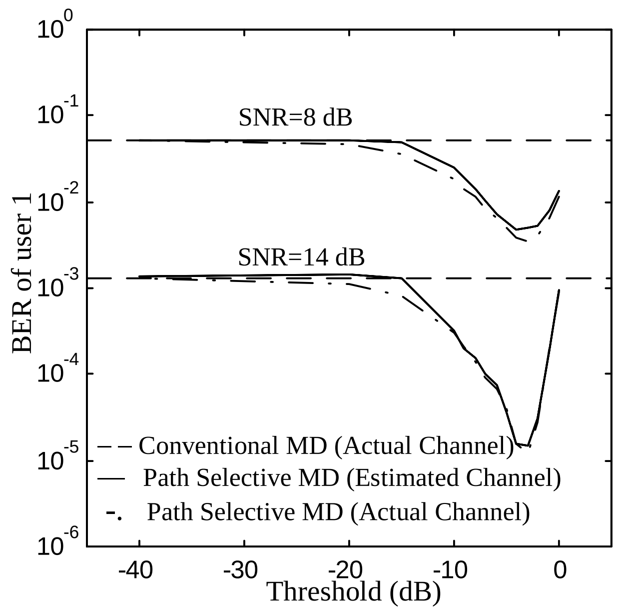

Fig. 4 BER of user 1 versus threshold (Ei/E1=0 dB, K=10, L=63, N_p=6, P_t=16)

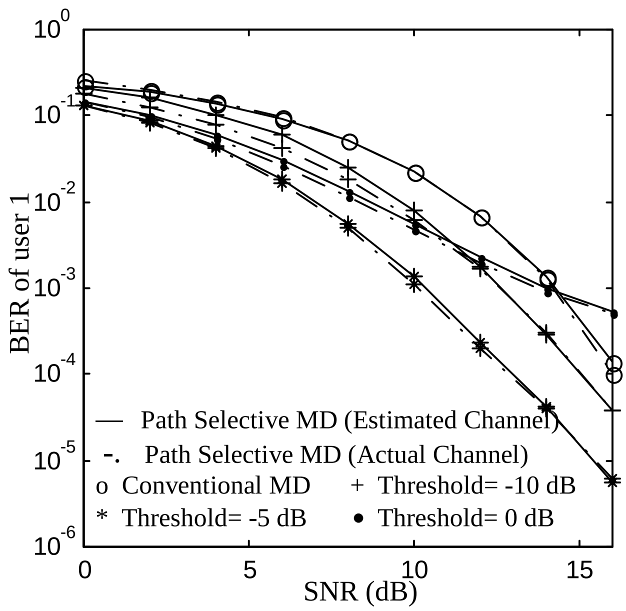

Fig. 5 BER of user 1 versus SNR at different threshold values (Ei/E1=0 dB, K=10, L=63, N_p=6, P_t=16)