A Soft Decision Feedback Equalizer

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ABSTRACT

In this paper, a method to reduce the error propagation in Decision Feedback Equalizers (DFEs) is addressed. An M-level staircase nonlinearity is proposed for the feedback chain of the DFE.

Analytical results are presented to show advantages obtained with increasing M. Finally, the saturation nonlinearity (the limiting case as M tends to infinity) is suggested for the DFE. Simulations are provided to support the proposed DFE. Its performance is found to be better than that of the recently proposed Erasure DFE.

I. INTRODUCTION

Communications systems at high bit rates suffer from Inter-Symbol-Interference (ISI). Both Linear Equalization (LE) and Decision Feedback Equalization (DFE) may be used to suppress the ISI [1]. However, DFE has received more interest than LE for its better error rate performance particularly on channels having spectral nulls (e.g. frequency selective multipath channels).

The DFE consists of two transversal filters, a feedforward filter (FFF) and a feedback filter (FBF). In DFE, the aim is to cancel ISI due to previously detected symbols by subtracting it at the input of the decision device (or slicer). The major problem in this scheme is the so called error propagation; a decision error propagating through the FBF enhances ISI instead of cancelling it. Thus, a single error may cause a burst of errors in subsequent decisions. As reported in [1], the performance loss due to this phenomenon is approximately 2dB for some channels.

Recently, a modification has been made in DFE structure to reduce the effect of error propagation [2]. Instead of using final decisions supplied by the slicer as the symbols fed back, symbols from a different nonlinearity were used. This modified scheme is depicted in Figure 1. With the dead-zone limiter nonlinearity proposed in [2], the modified DFE (called Erasure

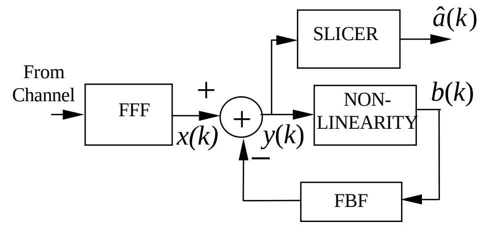

Figure 1: The modified DFE.

DFE) operates as follows

$$b(k) = \begin{cases} \hat{a}(k) & |y(k)| > A \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $\hat{a}(k)$ is the traditionally decided symbol. In this case, an input sample with absolute value below a certain threshold is assumed unreliable and no decision is fed back; so error propagation is reduced by avoiding feedback of the less reliable symbols. Analytical and experimental results have shown that the approach is promising [2].

In this paper, the dead-zone limiter nonlinearity is viewed as a 3-level uniform mid-thread quantizer and the analytical results in [2] are extended to M-level quantizer characteristics. Analytical results have shown a significant improvement in performance as the number of levels, M, is increased. So we suggest a saturation nonlinearity (limiting case as M approaches infinity) for the feedback chain of the DFE.

The paper is organized as follows. The system model and the nonlinearities are presented in section II. Section III presents analytical results for one-memory channels. Extension via simulations to higher memory channels is made in section IV. Conclusions, together with possible future work, are made in the last section.

II. SYSTEM MODEL

The combination of the actual channel and the feedforward filter shown in Figure 1 is assumed to satisfy

$$x(k) = a(k) + \sum_{j=1}^{\infty} h_j a(k-j) + n(k) \tag{2}$$

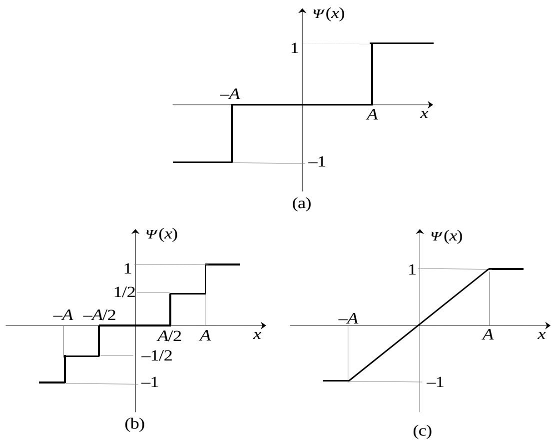

Figure 2: (a) Dead-zone nonlinearity, (b) 5-level staircase nonlinearity, (c) saturation nonlinearity.

where $a(k)$ is the current symbol to be detected, h_j is the j-th channel response sample, $n(k)$ is the additive noise and $x(k)$ is the noisy output of the FFF. Furthermore, $a(k)\epsilon\{+1,-1\}$ with equal probabilities and $n(k)$ is zero mean Gaussian with variance σ^2. Equation (2) assumes the use of an infinite length zero-forcing FFF to remove the precursor ISI [3].

Some of the several possible nonlinearities that can be used are illustrated in Figure 2. Figure 2(a) corresponds to the dead-zone limiter. Figure 2(b) is its extension to 5-levels. The nonlinearity in the limiting case, as the number of levels approach infinity, which we call the saturation nonlinearity, is depicted in Figure 2(c).

Compared to the E-DFE given by (1), the DFE with saturation nonlinearity operates as follows.

$$b(k) = \begin{cases} \hat{a}(k) & |y(k)| > A \\ \frac{1}{A}y(k) & \text{otherwise} \end{cases} \tag{3}$$

For obvious reasons, the latter is called the Soft-DFE (S-DFE).

III. ANALYTICAL RESULTS

We consider a 1-bit memory channel, that is, $h_1 \neq 0$ and $h_j = 0$ for $j > 1$. In contrast to [2], we provide analytical results for arbitrary number of levels. Let us represent the M-level uniform mid-thread quantizer by $\Psi(.)$. The output of the nonlinearity, $b(k)$, is given by

$$b(k) = \Psi[x(k) - h_1 b(k-1)] \tag{4}$$

where

$$x(k) = a(k) + h_1 a(k-1) + n(k) \tag{5}$$

Define the error term as

$$e(k) = a(k) - b(k) \tag{6}$$

Subtract both sides of (4) from $a(k)$ and use (5) and (6) to get

$$e(k) = a(k) - \Psi[a(k) + h_1 e(k-1) + n(k)] \tag{7}$$

Let $s(k) = h_1 e(k)$ be the error state at time k and rewrite (7) as

$$s(k) = h_1[a(k) - \Psi(a(k) + s(k-1) + n(k))] \tag{8}$$

Note that (8) describes a finite state, discrete time Markov chain. The number of states is determined, as will be shown, by the number of levels, M, in $\Psi(.)$.

Let the entire real line, over which $\Psi(.)$ is defined, be partitioned into M regions as $R_1 = (-\infty, r_1]$, $R_2 = (r_1, r_2], ..., R_M = (r_{M-1}, \infty)$. All finite length regions, except the one centered at the origin, $R_{\frac{M+1}{2}}$, are of equal length, which is denoted by Δ_r. Each region has the corresponding level denoted by v_i. For symmetry we set $r_i = -r_{M-i}$ and $v_i = -v_{M+1-i}$, $i = 1, 2, ..., \frac{M-1}{2}$ and $v_{\frac{M+1}{2}} = 0$. We limit the ranges for r_i and v_i setting $r_1 = -r_{M-1} = -A$ and $v_1 = -v_M = -1$. So, for the uniform characteristics we have

$$r_i = -r_{M-i} = (-\frac{M+1}{2} + i)\Delta_r \quad \text{i=1,2,...,}\tfrac{M-1}{2} \tag{9}$$

and

$$v_i = \frac{2i - M - 1}{M - 1} \quad \text{i=1,2, ...,M} \tag{10}$$

where $\Delta_r = \frac{2A}{M-1}$.

The possible values of $e(k)$ in (7) are given by $\{\pm 1 - v_i\}_{i=1}^{M}$. Particularly, for $M = 3$, we have error values $\{2, 1, 0, -1, -2\}$ and the states $\{2h_1, h_1, 0, -h_1, -2h_1\}$, since $v_1 = -1, v_2 = 0$ and $v_3 = 1$. Thus, the number of states is 5. In general, this number is given by $N = 2M - 1$. Let S be the state space of the Markov chain. Note that S is a discrete set with elements $\{s_1, s_2, ..., s_N\}$. The ordering of states is assumed to be as $s_1 = 2h_1, ..., s_M = -2h_1$. The state diagram for $M = 5$ is illustrated in Figure 3. Note that each state can be reached from all states.

In the following we define $p_{ij}(k)$ as the transition probability from the j-th state, s_j, to the i-th state, s_i, at time k. The set of all states can be divided into two subsets. One subset is the set of states that can be reached by $a(k) = +1$ and the other subset is the set of states that can be reached by $a(k) = -1$. We denote the former by S_+ and the latter by S_-. It should be noted that both have the zero state as the common element. Specifically, $S_+ = \{2h_1, h_1, 0\} = \{s_1, s_2, s_3\}$ and $S_- = \{0, -h_1, -2h_1\} = \{s_3, s_4, s_5\}$ for $M = 3$. In general, $s_i, i = 1, 2, ..., M-1$, are elements of $S_+ - \{0\}$ and $s_i, i = M+1, ..., N$ are elements of $S_- - \{0\}$. The zero state $s_M \epsilon S_+ \cap S_-$.

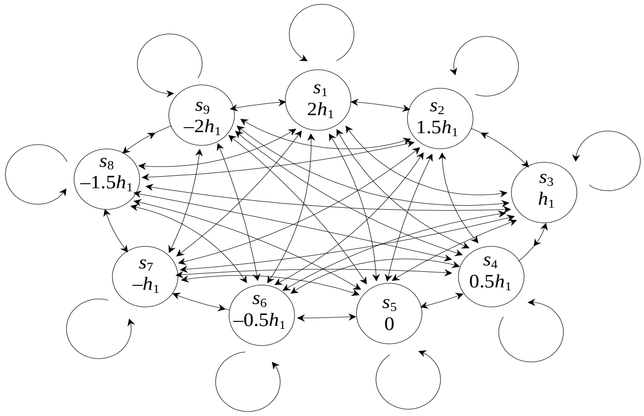

Figure 3: State diagram for the 5-level staircase non-linearity.

It can easily be shown that

$$p_{ij}(k) = \begin{cases} pPr\{1+s_j+n(k)\ \epsilon R_i\} & \text{i=1,2,...,}M\text{-1} \\ pPr\{1+s_j+n(k)\ \epsilon R_M\}+ & \\ \quad qPr\{-1+s_j+n(k)\ \epsilon R_1\} & \\ & i=M \\ qPr\{-1+s_j+n(k)\ \epsilon R_{i-M}\} & \\ & i=M+1,...,N \end{cases}$$

for $j=1,2,...,N$, or equivalently,

$$p_{ij}(k) = \begin{cases} pPr\{r_{i-1} < 1+s_j+n(k) \leq r_i\} \\ \qquad i=1,2,...,M-1 \\ pPr\{r_{M-1} < 1+s_j+n(k) \leq r_M\}+ \\ \quad qPr\{r_0 < -1+s_j+n(k) \leq r_1\} \\ \qquad i=M \\ qPr\{r_{i-M} < -1+s_j+n(k) \leq r_{i-M+1}\} \\ \qquad i=M+1,...,N \end{cases} \tag{11}$$

Note that $r_0 = -\infty$ and $r_M = \infty$. In terms of the Cumulative Distribution Function (CDF) of the noise term $n(k)$, say $F(n)$, we may express (11) as

$$p_{ij}(k) = \begin{cases} p[F(r_i-1-s_j)-F(r_{i-1}-1-s_j)] \\ \qquad i=1,2,...,M-1 \\ p[1-F(r_{M-1}-1-s_j)]+ \\ \quad qF(r_1+1-s_j)] \\ \qquad i=M \\ qF[r_{i-M+1}+1-s_j)- \\ \quad F(r_{i-M}+1-s_j)] \\ \qquad i=M+1,...,N \end{cases} \tag{12}$$

using $F(\infty)=1$ and $F(-\infty)=0$. Note that k is dropped since the CDF does not depend on time. For example, the above equation for case $M=5$ gives the following list of transition probabilities;

$p_{1j} = pF(-A-1-s_j)$
$p_{2j} = p[F(-A/2-1-s_j)-F(-A-1-s_j)]$
$p_{3j} = p[F(A/2-1-s_j)-F(-A/2-1-s_j)]$
$p_{4j} = p[F(A-1-s_j)-F(A/2-1-s_j)]$
$p_{5j} = p[1-F(A-1-s_j)]+qF(-A+1-s_j)]$
$p_{6j} = q[F(-A/2+1-s_j)-F(-A+1-s_j)]$
$p_{7j} = q[F(A/2+1-s_j)-F(-A/2+1-s_j)]$
$p_{8j} = q[F(A+1-s_j)-F(A/2+1-s_j)]$
$p_{9j} = q[1-F(A+1-s_j)]$
where $j=1,2,...,9$.

Define P, with elements p_{ij}, as the one-step transition matrix of the Markov chain. The dimensionality of the matrix depends on the cardinality of the state space S. Thus, it is an $N \times N$ matrix.

The chain is homogenous since the probabilities do not depend on k; is irreducible since every state is accessible from every other state in one step; is aperiodic since a self loop with nonzero probability exist in every state and is trivially recurrent [4]. Then, there exists a limiting probability of states that satisfy

$$\Pi = P\Pi \tag{13}$$

where the elements of vector $\Pi = [p_1, p_2, ..., p_N]^t$ and $p_i = \lim_{k\to\infty} Pr\{s(k)=s_i\}$. The probability of error at equilibrium is given by

$$P_e = \Pi^t W \tag{14}$$

where $W = [w_1, w_2, ..., w_N]$ is the vector with elements that correspond to the error probabilities at each state.

The decision variable for symbol $a(k)$ at state s_j is

$$y(k) = a(k) + s_j + n(k) \tag{15}$$

and the slicer operates as follows:

$$\hat{a}(k) = sign(y(k)) \tag{16}$$

Here, the probability of error is given by

$$\begin{aligned} w_j &= pPr\{y(k)<0|a(k)=+1\} + \\ &\quad qPr\{y(k)>0|a(k)=-1\} \\ &= pF(-1-s_j)+q[1-F(+1-s_j)] \end{aligned} \tag{17}$$

for $j=1,2,...,N$

IV. NUMERICAL RESULTS AND SIMULATIONS

A. Short-Memory Case

This section presents numerical results for one memory channel with respect to the threshold value A, the channel coefficient h_1, the Signal-to-Noise ratio SNR, and the number of levels M. The SNR is defined as $10\log\frac{1}{2\sigma^2}$.

In Figure 4, we present the results for $P_e(A)$ normalized by $P_e(0)|_{A=0}$, which is represented as $P_e(A)/P_e(0)$, with respect to A for several values of M. The other parameters are fixed as $h_1 = -0.7$, $SNR = 6$dB and $p = q = 0.5$. The case $A = 0$ corresponds to the traditional DFE. So, values of $P_e(A)/P_e(0)$ lower than unity indicate improvement relative to DFE. Two observations are made as we increase M;

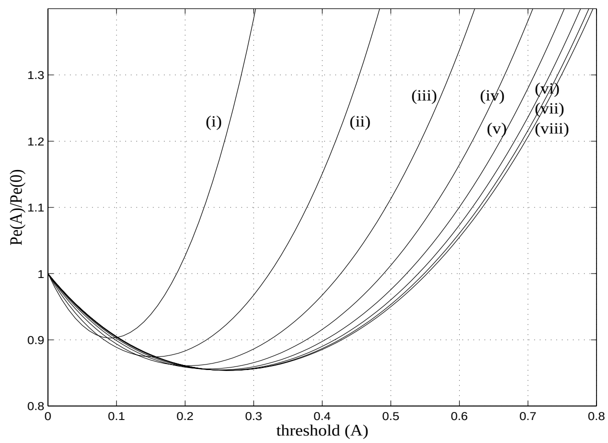

Figure 4: The $P_e(A)/P_e(0)$ with respect to A for E-DFE ($h_1 = -0.7, SNR = 6$ dB) for various number of levels; i) 3, ii) 5, iii) 9, iv) 33, v) 65, vi) 129, vii) 257 and viii) 513.

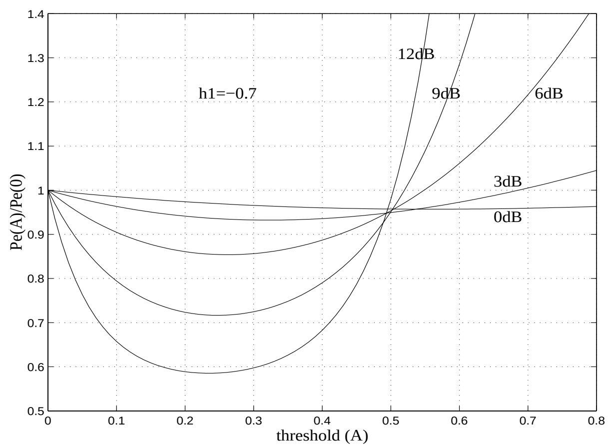

Figure 5: The $P_e(A)/P_e(0)$ with respect to A ($h_1 = -0.7, M = 513$) for several SNR values.

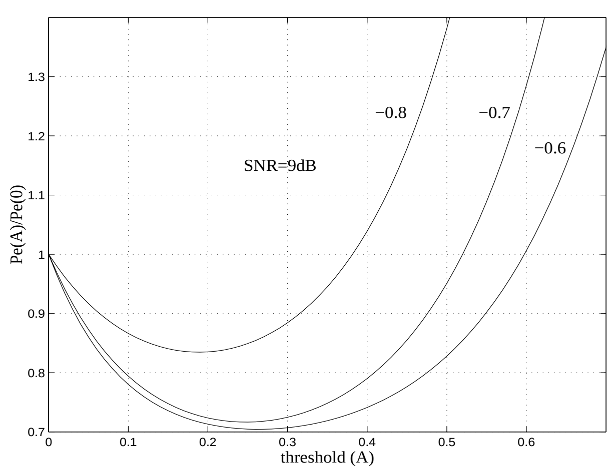

Figure 6: The $P_e(A)/P_e(0)$ with respect to A ($SNR = 9dB, M = 513$) for values of h_1 =-0.6, -0.7 and -0.8.

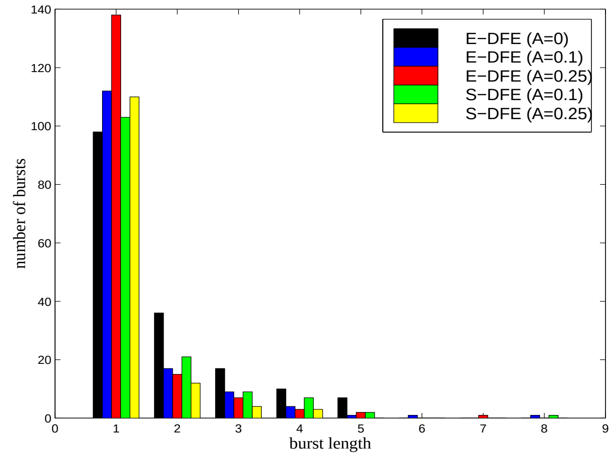

Figure 7: The burst percentage versus burst length histogram for DFE, E-DFE and S-DFE for different values of A with $SNR = 9$dB and channel coefficients $\mathbf{h} = [-0.6\ -0.3\ -0.2\ -0.2\ -0.1]$.

1. The range of A, for which the method improves the performance, increases
2. The optimum performance gets better

It can be easily seen that the optimum value of A depends on the number of levels and it increases as M increases. As Figure 4 indicates, safer choice of A is possible for larger values of M. As M approaches infinity the nonlinearity $\Psi(.)$ approaches to the saturation nonlinearity shown in Figure 2(c). Thus, we suggest to use this nonlinearity in the modified DFE structure because of its better performance for a wide range of A and ease of practical implementation. Hereafter, the DFE with this nonlinearity will be referred to as the Soft DFE (S-DFE).

The optimum value of A also depends on noise variance and channel coefficient. This can be easily observed from Figure 5 and Figure 6.

In Figure 5, we fixed $M = 513$ and presented results for $SNR = 0$, 3, 6, 9 and 12 dB. The results exhibited in Figure 6 are for $h_1 = -0.8, -0.7$ and -0.6 with $M = 513$ and $SNR = 9$dB. The strong dependence of the optimum value of A on the corresponding system parameters is evident from the figures.

B. Long-Memory Case

The exact analysis of S-DFE, even for one-memory channel is difficult, if not impossible. So, in this section we evaluate the performance of the proposed DFE and compare with the DFE and E-DFE through Monte Carlo simulations for a channel memory higher than 1. The simulated channel is $h_1 = -0.6, h_2 = -0.3, h_3 = -0.2, h_4 = -0.2, h_5 = -0.1$ and $h_j = 0$ for $j > 5$. The SNR was set to 9dB. Figure 7 presents the histogram of the length of burst errors for DFE ($A = 0$), E-DFE ($A = 0.1$ and 0.25) and S-DFE ($A = 0.1$ and 0.25). The best result is obtained by S-DFE with $A = 0.25$. Although the number of burst errors of length 1 bit is still higher than that of the

DFE but smaller than that of the E-DFE, the burst errors of length greater than 1 are significantly reduced in comparison to both DFE and E-DFE.

V. CONCLUSION

In this paper, steps towards the proposal of a soft DFE has been introduced. It has been shown that it is possible to reduce the effect of error propagation in DFE, which is a long standing problem. It has been demonstrated that the threshold A strongly dependent on the channel coefficients and the noise level. In the companion paper [5], a method has been suggested to determine the optmum value of A using a quite different point of view based on fuzzy logic. In addition, extension of the S-DFE to higher signal constellations is currently under considerations.

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