

Improved PCA based Face Recognition using Feature based Classifier Ensemble

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ABSTRACT

Automatic face recognition has been a challenging problem in the field of image processing and has received a great attention over the last few decades because of its applications in different cases. Most of the face recognition systems employ single type of data, such as faces, to classify the unknown subject among many trained subjects. Multimodal systems are also available to improve the recognition performance by combining different types of data such as image and speech for the recognition of the subjects. In this thesis, an alternative approach is used where the given face data is used to automatically generate multiple sub feature data sets such as eyes, nose and mouth. Feature extraction is automatically performed by using rough features regions extracted from Viola-Jones face detector followed by Harris corner detector and Hough Transform for refinement. Automatically generated feature sets are used to train separate classifiers which would recognize a person from its respective feature. Given separate feature classifiers, standard data fusion techniques are used in the form of classifier assembling to improve the performance of the face recognition system.

10-Fold cross validation methodology is used to train and test the performance of the respective classifiers, where nine fold is used for training and one fold is used for testing. Principal component analysis (PCA) is employed as a data dimensionality reduction method in each classifier. Five different classifiers for right and left eyes, nose, mouth and face data sets are developed using PCA. The classifiers, of five different features are merged by different data fusion techniques such as Minimum Distance, Majority Voting, Maximum Probability, Sum and Product Rule. Overall,

the proposed algorithm using the Minimum Distance improves the accuracy of state-of-the-art performance from 97.00% to 99.25% using ORL face database.

Keywords: Face Recognition, Face Detection, Cross Validation, Viola-Jones Detection, Feature Extraction, Minimum Distance, Data Fusion, Classifier Ensemble.

ÖZ

Otomatik yüz tanıma, görüntü işleme alanında farklı uygulama olasılıklarından dolayı zorlu bir problem olarak yıllardır büyük bir ilgi çekmektedir. Yüz tanıma sistemlerinin bir çoğu yüz imgeleri gibi tek bir veri kaynağını kullanarak eğitilen şahıslar arasından bir kişiyi tanımaktadır. Multimodal sistemler de bu şahısların tanınması için görüntü ve konuşma gibi farklı veri türlerini birleştirerek tanıma performansını artırmak için kullanılabilir. Bu tezde, eldeki yüz imgelerinin oluşturduğu veri setlerinden otomatik olarak gözler, burun ve ağız gibi birden çok öznitelik veri setleri çıkarılarak alternatif bir yaklaşım için altyapı oluşturulmaktadır. Viola-Jones yüz dedektörü kullanılarak çıkarılan kaba öznitelik bölgelerinden otomatik olarak Harris köşe dedektörü ve Hough dönüşümü ile ince ayar yapılarak otomatik öznitelik çıkarımı gerçekleştirilmektedir. Otomatik olarak oluşturulan öznitelik setleri ayrı ayrı sınıflandırıcıları eğitmek için kullanılmakta ve bir şahıs bu sınıflandırıcılar tarafından tanınabilmektedir. Elde edilen farklı sınıflandırıcılar standard veri füzyon teknikleri kullanılarak sınıflandırıcı topluluğu oluşturulmakta ve yüz tanıma sisteminin performansı arttırılmaktadır.

10-kat çapraz doğrulama yöntemi ile dokuz katı eğitim için, bir katı ise test için olmak üzere veriler, iki farklı gruba ayrılmakta ve bu yaklaşımla sınıflandırıcıların performansı ölçülmektedir. Ana bileşenler analizi (ABA) ile her bir sınıflandırıcının çok boyutlu uzaydaki veri boyutu düşük bir seviyeye indirgenmektedir. Gözler, burun, ağız ve yüz imge settleri ABA yöntemi kullanılarak beş farklı sınıflandırıcı oluşturulmaktadır. Beş farklı sınıflandırıcı, veri birleştirme yöntemleri kullanılarak birleştirilmekte ve bu amaçla en az uzaklık, çoklu oylama, en yüksek olasılık, toplam

ve çarpım kuralları uygulanmaktadır. Sunuç olarak, önerilen yöntemlerden en az uzaklık kullanarak veri füzyonu gerçekleştiren yöntem literatürde ORL yüz veritabanı kullanan alternatif yöntemin performansını %97.00'den %99.25'e çıkarmaktadır.

Anahtar Kelimeler: Yüz Tanıma, Yüz Sezimi, Capraz Validasyon, Viola-Jones Yüz Sezimi, Öznitelik Çıkarımı, Enaz Uzaklık, Veri Füzyonu, Sınıflandırıcı Topluluğu.

DEDICATION

This dissertation is dedicated to my lovely parents for their love, devoting their time to support me. Further, I would like to dedicate this work to my beloved husband for his encouragement and endless support.

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LIST OF SYMBOLS/ABBREVIATIONS

$\ \cdot \ _2$	2-Norm
$\ x_i - y_i\ $	Distance between two vectors
Γ_i	Vector of mapped image
$\lambda_i, \mu_i, \Lambda$	Eigenvalue
σ, SD	Standard deviation
σ^2, S^2, Var	Variance
v_i	Eigenvector
Φ_i	Normalized vector
Ψ	Mean face
$\omega(x, y)$	Weighing function
Cov, C	Covariance
$\det(A)$	Determine of matrix A
I_x	Intensity of Weighing function
Trace	Sum of the elements on the main diagonal of matrix
U_i	Eigenfaces
\bar{X}	Mean of variables
AAM	Active Appearance Models
CIR	Correct Identification Rate
DWT	Discrete Wavelet Transform
FAR	False Acceptance Rate
FIR	False Identification Rate
FRR	False Rejection Rate

ICA	Independent Component Analysis
LCA	Linear Discriminant Analysis
LPP	Locality Preserving Projection
ORL	Olivetti Research Laboratory
PCA	Principle Component Analysis

Chapter 1

INTRODUCTION

1.1 Face Recognition

The face is a sign key in recognizing a certain person. The recognition ability is good in human beings to recognize unknown faces, dealing with large number of unknowns may lead to failure which is inevitable since mankind has limited capabilities. To solve this problem, computers are used because of their high speed and rich memory and computational resources. This process is referred to as face recognition. In rudimentary methods, very simple geometric models were used by face recognition systems. Recently, face recognition methods had been more sophisticated as they employ more complicated mathematical representations. In last decades, innovations in face recognition lead to reveal the broad capacity to do researches in this topic.

1.1.1 History of Face Recognition

Automatic face recognition is almost a new notion and many different industry areas such as video surveillance, human-machine interaction, photo cameras, virtual reality or law enforcement are interested in what it could offer. Engineers started to show interest in face recognition in the 1960's as the first semi-automated system which was designed and implemented by Bledsoe [1]. Mentioned system was in need of an administrator to select some facial coordinates (features) for computers to calculate required processing ranges to some special points and then match them to the reference data. However, there was some problem such as the variation of face

illumination, head rotation, facial expression, and aging were present in such a system that even 50 years later, face recognition systems still suffers from them.

In 1970's, Jay Goldstein, Leon D. Harmon and Ann B. Lesk used the same approach plus introducing a vector with 21 subjective features such as hair color, lip thickness, eyebrow weight, nose length, ear size and between-eye distance as the basis of face recognition using pattern classification method [1]. Later Fischler and Elschanger employed similar automatic feature measurements by using local template matching and a global measure of it to find and obtain facial features. Back then, some other works were conducted to introduce a face as a collection of geometric factors and then define some challenges based on those factors. Kenade in 1973 developed a fully automated system which ran in a computer system designed for this purpose [2]. Sixteen facial factors were automatically extracted in this algorithm and only a small difference was observed in comparison with a human or manual extraction. He proved that better result can be obtained by exclusion of irrelevant features where he got a correct identification rate of 45-75%. In the 1980's face recognition was actively followed and most of these researches continued and completed the pervious works. Some works tried to improve the methods used for measuring subjective features. For example, Mark Nixon introduced a geometric measurement for eye spacing [3]. In this decade some new methods were also invented based on artificial neural network in face recognition algorithms.

In 1986 L. Sirovich and M. kirby propounded the use of eigenfaces in image processing for the first time which become the dominant approach in later years [4]. This method was based on the Principal Component Analysis (PCA). The purpose of this method was to represent an image in a lower dimension without losing much

information, and then reconstructing it [5]. This improvement became the foundation for the most of future suggestions in this field. In the 1990's, eigenfaces method was used as the basis for the state of the art first industrial applications.

In 1992 Mathew Turk and Alex Pentland improved an algorithm for the recognition which used eigenfaces and was able to locate, track and classify a subject's head and the residual error to detect faces in the image [6]. It is worthy of note that the dominant deviation factor in this algorithm was the environmental factors. Later, this method became a base for real-time automatic face recognition, associated with noticeable increase in the number of publications. The public's attention was captured at January 2001 to trial implementation, where face recognition was used to capture surveillance images and compare them with a database of digital mug shot of media. To date, many methods have been presented in this engineering field, leading to different algorithms. Some of the most famous methods are PCA, ICA, LDA, and so on.

1.2 Thesis Contributions

In this thesis, the proposed recognition method is started by face detection; Viola-Jones Detector and Circular Hough Transform are used to point the top of nose, four corners of mouth and iris of eyes. By cutting the extracted features in fixed size, database is produced. The method named 10 Fold Cross Validation builds training and testing set and by Principle Component Analysis (PCA) method the recognition process is performed separately for each feature set and five individual classifiers are made. Combination of the result of classifiers for by Minimum Distance technique can improve the output of the system and present better conclusion.

1.3 Thesis Overview

This thesis includes three main parts to recognize face and at the end some methods are used to combine the result of each parts to improve the method to recognize the person correctly. Chapter 1 as an introduction includes brief review of face recognition evolution and its problems and the methods to solve some of these problems. Chapter 2 deals with definition of face recognition and the methods to perform this application. Chapter 3 considers some classifiers and techniques of face detection, surveying standard methods which are employed in this thesis. Chapter 4 starts to explain the proposed methods and algorithms that are applied in this thesis and introduce the new method to achieve the better result compared with pervious researches. Chapter 5 discusses the methodologies such as available and used database in this thesis, K-fold Cross Validation and performance accuracy of proposed system. Chapter 6 studies the Data fusion Techniques such as the classifiers which are used to combine the results of different steps to present the desirable result. Chapter 7 discusses about the experimental method and compares the proposed method by pervious works. In Chapter 8, the conclusion and future work is explained.

Chapter 2

FACE RECOGNITION

2.1 Introduction

Face recognition and identification is one of the topics that attract interest in so many different fields such as computer vision in the last three decades. In this chapter, the role of face recognition and its achievements are discussed. The implementation of some of the important mercantile face recognition systems are studied as well.

2.2 Face Recognition Applications

The three essential face recognition applications are verification, identification, and watch list. These three applications cause different results because of their dependence to the nature.

2.2.1 Verification

The verification application is used in the supplements which are needed user interplay in the form of personality assertion, such as access application. In verification test, people are classified into two groups:

- The group tries to access applications by their own identity as Client.
- The group tries to access applications by wrong identity or a known identity but not pertaining to them as Imposter.

It reports the False Acceptance Rate (FAR) which shows the percentage of wrong matches of input test templates with available database and the False Rejection Rate

(FRR) which shows the percentage of system failing to find the input test templates among available database.

2.2.2 Identification

The identification is used in supplements which are not needed user interplay, such as surveillance applications.

In identification test all faces in the examination are assumed familiar faces. It reports the Correct Identification Rate (CIR) which shows the numbers of identifications of test pattern among trained database which are done correctly or the False Identification Rate (FIR) which shows the number of wrong identifications in recognition process.

2.2.3 Watch List

The watch list is used in extension of the identification application that involves unknown persons.

In watch list test as in identification test CIR or FIR is reported. The sensitivity of the watch list is shown with FAR and FRR has relating with it. It means that how often the system recognizes an unknown person as one in the watch list.

2.3 Process of Face Recognition

Face recognition is a visual sample of recognition system. It tries to recognize the faces that are affected by illumination, pose, expression etc.

Different types of input sources can be used by face recognition, for example:

- A Single Two-Dimensional Images
- Stereo Two-Dimensional Images

- Three-Dimensional Laser Scans

By considering time dimension, it can be possible to raise the dimensionality of these sources. For example, the video sequence is an image with time dimension property therefore, the performance of recognition a person in video is much better than in image.

In this thesis the first type of sources will be used, but it is possible to make some changes and developments in this method to use video as the source of f recognition system. As shown in Figure 2.1, four steps usually compose face recognition system:

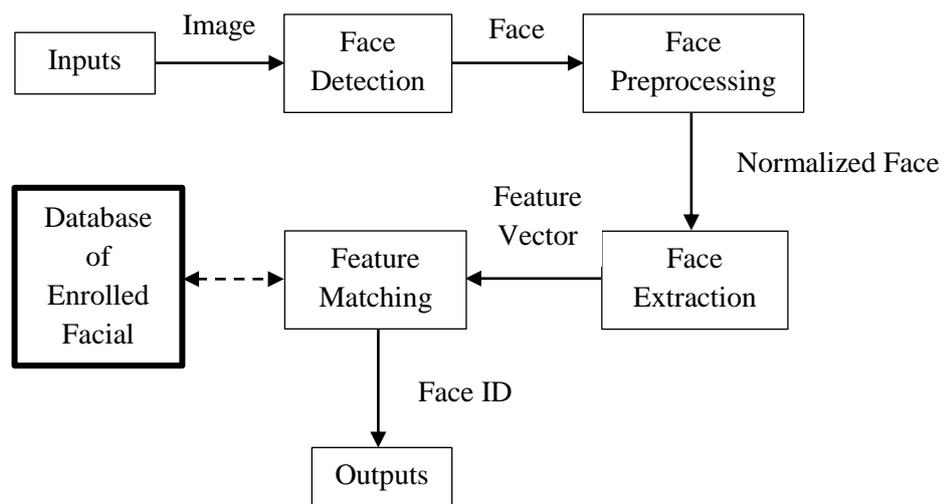


Figure 2.1: General four steps in face recognition

The brief introduction of these steps can be as follow;

- **Step One:** Face Detection localizes the faces in an image. If the source is a video, tracking the face among frames will be more helpful to decrease the time of calculation and memorize the person in frames. For example, Shape Templates, Neural Networks and Active Appearance Models (AAM) are methods that are employed in face detection step.

- **Step Two:** Preprocessing normalizes the large detected face to gain a quality feature extraction. Alignment (translation, rotation, scaling) and light normalization/correlation are methods that are used in face preprocessing.
- **Step Three:** Feature Extraction draws out a set of special personal signs and features of the face. PCA and Locality Preserving Projection (LPP) are the methods applied in feature extraction.
- **Step Four:** Feature Matching is the recognition part. In this step, the eigenvectors captured in pervious step (Feature Extraction) are compared with faces in database then the system tries to find these offered faces among the trained faces in system.

2.4 Face Recognition Methods

Automatic face recognition system can be divided in three methods as;

2.4.1 Holistic Matching Methods

In this method, system uses whole face zone as the input data. Some famous uses of the holistic method are Principal Component Analysis, Linear Discriminant Analysis and Independent Component Analysis which employ eigenfaces in reorganization.

2.4.1.1 Flowchart of Holistic Matching

The holistic method is the combination of some processes that is shown in Figure 2.2:

- Firstly, a set of images as the training set is used to make the eigenfaces and then to contrast with testing set.
- Secondly, the interpersonal features on face construct the eigenfaces. After normalization, the input images and specifies the location of features as mouth, nose and eyes, the output are resized and prepared to calculate the

eigenfaces by doing some mathematical process that is named as Principle Component Analysis (PCA).

- Thirdly, the eigenfaces illustrates weight vectors.
- Finally, the weight vectors of testing set are found and compared with the weight vectors of training set that if it is less than a given threshold or not. If the answer is positive, the identification of test image is done successfully and the closest weight is recognized as a result [7].

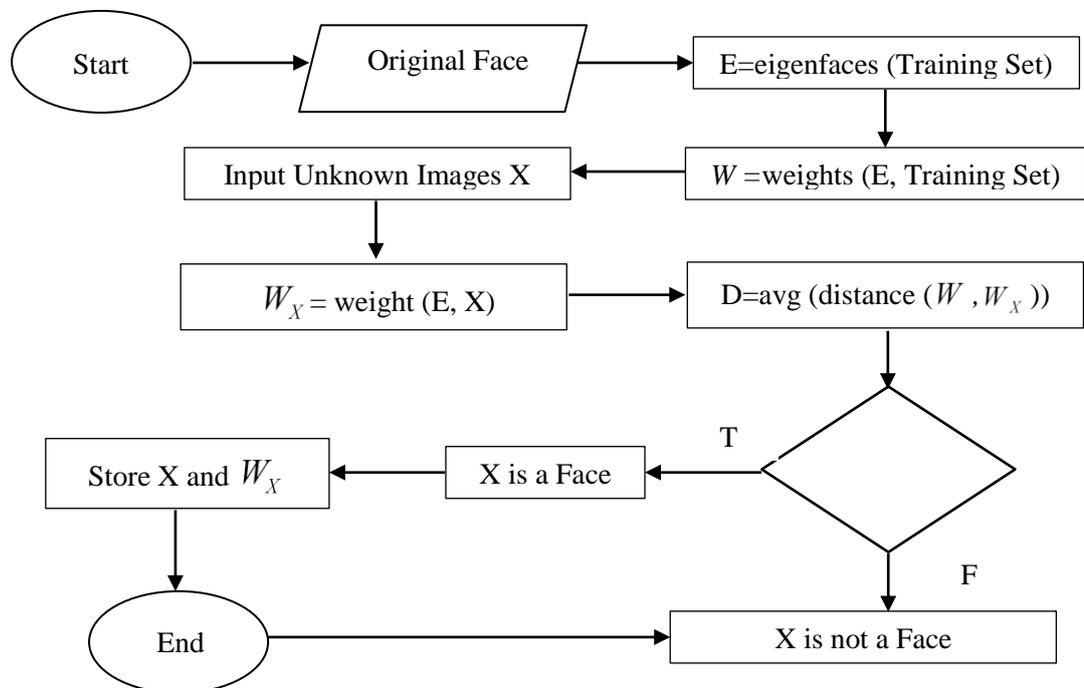


Figure 2.2: Flowchart of Holistic Matching System (Based on Eigenfaces) [8]

2.4.2 Feature-based (Structural) Methods

In this method, firstly, the feature of face like mouth, nose and eyes are detected then their characteristics are extracted to be processed by classifier. The problem of this method is when the features are restored the large variations must be considered for example head can have many positions while the frontal pose is compared with

profile pose [9]. There are different types of extracting method that are arranged into three basic groups:

- The methods based on edges, lines, and curves
- The methods based on feature-template
- The methods based on structural matching by observing the geometrical limitation of the features

2.4.3 Hybrid Methods

Hybrid Method is the combination of Holistic Matching and Feature Extraction methods. The advantage of this method is its ability in using Three-Dimensional images also [9]. By using Three-Dimensional image it is possible to know about the shape of the chin or forehead or the curves of the eye sockets. Depth and an axis of measurement are used by this system therefore; enough information can be taken to compose a whole face. The combinations of processes that make Three-Dimensional systems are:

- **Detection;** in this step, the person's face in an image or sequence of images in real time videos is found.
- **Position;** in this step, all the geometrical properties of detected face like location, size and angle of the head are defined.
- **Measurement;** in this step, the measurements of each curve of face is determined so a format is constructed which can centralize on specific areas such as outside and inside of the eye and the angle of the nose.
- **Representation;** in this step, the processed face is converted into numerical format and became as a code.
- **Matching;** in this step, the collected data is contrasted with the available faces in database.

2.5 Thesis Perspective

In this thesis, all four general areas in face recognition (Face Detection, Preprocessing, Feature Extracting and Feature Matching) are used. For Face Detection and Preprocessing, Viola-Jones method is used. By using Harris Corner Detection and Hough Transform for Circle Detection the features (mouth, nose, right eye and left eye) are extracted and finally the Feature Matching is performed by PCA method. As a used face recognition technique in this thesis, PCA method is going to be introduced.

2.6 Principal Component Analysis

Principal Component Analysis (PCA) is a linear dimension-reduction and statistical method that has been employed in several implementations such as face recognition, signal processing, and data mining etc. The method tries to approximate the projection directions with minimum reconstruction errors to original data by using eigenvectors and eigenvalues [10]. As the facial images are very high in dimensions, it takes a long time to compute and find the classification, therefore PCA method had been presented by Turk and Pentland in 1991 [6]. They applied PCA to reduce dimension of image data for decreasing classification and subsequent recognition time. Before starting to survey PCA method in detail some mathematical concepts which are essential to be known and used in this method is going to review. The following part covers standard deviation, covariance, eigenvectors and eigenvalues.

2.6.1 Background Mathematics

In this part some basic mathematical knowledge that will be important in analysis of the PCA method will be reviewed.

2.6.1.1 Statistic

The purpose of statistics is to understand the dependency between each member of the big set of data.

- **Standard Deviation**

While studying statistic, choosing the sample of a population is very important case, because almost the properties of the entire population can be investigated by studying on the properties of the sample of population [11]. To understand the standard deviation, first of all, mean of sample is introduced. Mean of sample is the average of the data in sample set, but it cannot involve enough information about the set, therefore standard deviation is used to explain about the density of data. Mean and standard deviation, which are shown by \bar{X} and SD or S , are calculated such as equation (2.1) and (2.2):

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n} \quad (2.1)$$

$$SD = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}} \quad (2.2)$$

- **Variance**

Another factor that can measure the density of data in dataset is variance and it is shown by S^2 and calculated such as equation (2.3):

$$S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1} \quad (2.3)$$

- **Covariance**

Standard deviation and variance are one-dimensional factors, but sometimes it is needed to work with more than one-dimensional data sets, in this case,

statistic tries to know if there is any dependence between dimensions. Covariance always deals with two dimensions. If the covariance is calculated for one dimension and itself, the result is looks like the variance as shown in equation (2.4):

$$Var(X) = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{n-1} \quad (2.4)$$

Covariance is shown by *COV* and calculated by equation (2.5):

$$COV(X, Y) = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{n-1} \quad (2.5)$$

2.6.2 Matrix Algebra

Matrix is a rectangular set of numbers, symbols or expressions which are sequenced in rows and columns. In image processing field, each image can be illustrated by matrix. Each array in this matrix represents some special properties of the pixel in selected image. One of the main characteristic properties of a matrix is the possibility of computing and extracting some useful information such as eigenvectors and eigenvalues [12]. Eigenvectors and eigenvalues which are going to discussed more, are exerted in so many different cases as in PCA method and construct the basis of this method.

- **Eigenvectors**

Eigenvectors are particular occasion of multiplying two matrices and presenting the correspondent sizes. To catch the eigenvectors, the square matrix is required. If this matrix is multiplied on the left of a vector, another vector will be resulted. All the other vectors lay on this vector constitute eigenvectors in general. Some conditions and properties which are occurred while studying the eigenvectors can be such as:

- a) The eigenvectors can be calculated just for square matrices

- b) Not all square matrix has eigenvectors
- c) The numbers of computable eigenvectors for $m \times m$ matrix are m .
- d) All the eigenvectors of a matrix are perpendicular to each other, means that eigenvectors are orthogonal

- **Eigenvalues**

In the case of having a nonzero vector like w and a square matrix such as A , if W and AV are parallel, there would be a real number such as λ which is establishing like equation (2.6):

$$AW = \lambda W \quad (2.6)$$

According to equation (2.6) W is an eigenvector and λ is an eigenvalue.

2.6.3 Mathematical Process of PCA

PCA method mathematically follows some steps to perform the recognition task to

Figure out the owner of unknown faces by using trained database;

- Obtain a Dataset
- Extract the Mean of Data
- Compute the Covariance Matrix
- Compute the Eigenvectors and Eigenvalues of Covariance Matrix
- Selecting Components and Creating a Feature Vector
- Establishing a New Dataset
- Calling Back the Old Dataset
- Comparing the New and Old Datasets with Each Other
- Choosing the Closest Data
- Introducing the Resulted Data as the Recolonized Data

2.6.4 The Basic Principle in PCA Algorithm

As mentioned before, PCA algorithm is exerted to reduce dimensions of data while preserves the variation of used database [8], [10], [13] and [14]. The first step is converting each images of database like $I(x,y)$ into vectors like Γ_i , to speed up the calculation and recognition time within this method, what the method performs is extracting the vectors with highest account for the distribution of faces in m images.

As these vectors are the eigenvectors of covariance matrix of original face images and their appearances are face-like, they are named as eigenfaces in PCA method. To explain the process of the method, consider 2-dimensional image like $X(x,y)$ which is $m \times n$, the dimension of representative vector for this image will be mn . If the Learning set is defined as $\Gamma_1, \Gamma_2, \dots, \Gamma_N$ the average face of these set can be calculated by equation (2.7);

$$\Psi = \frac{\sum_{n=0}^M \Gamma_n}{M} \quad (2.7)$$

All converted vectors are normalized to become useable inputs in recognition porocess as egression (2.8)

$$\Phi_i = \Gamma_i - \Psi \quad (2.8)$$

The calculated covariance matrix by equation (2.9) as the expected value of $\Phi\Phi^T$

$$C = \frac{\sum_{n=0}^M \Phi_n \Phi_n^T}{M} = AA^T \quad (2.9)$$

Where the matrix $A = \Phi_1 \Phi_2 \dots \Phi_M$. The covariance matrix C , however is $N^2 \times N^2$ real symmetric matrix, and determining the N^2 eigenvectors and eigenvalues is an intrac-table task for typical image sizes. We need a computationally feasible method to findthese eigenvectors.

$$AA^T v_i = \mu_i v_i \quad (2.9)$$

Where v_i and λ are the eigenvector and eigenvalues corresponding to covariance matrix C . Multiplying both sides by A ,

$$AA^T Av_i = \mu_i v_i \quad (2.10)$$

AA^T generates the new $M \times M$ matrix like L , where $L_{mn} = \Phi_m^T \Phi_n$ and find the M Eigenvectors. These vectors determine linear combinations of the M training set face images to form the eigenfaces U_i

$$U_i = \sum_{k=1}^M v_{ik} \Phi_k, i = 1, 2, \dots, M \quad (2.11)$$

After extracting the eigenvectors and eigenvalues all available database such as training and testing sets are projected into same eigenspace and then in recognition process the nearest trained image to tested one is identified as the owner of face image [15].

Chapter 3

FEATURE EXTRACTION TECHNIQUES

3.1 Introduction

Detecting the face and its features are the basic subject in a face recognition system. Localization and extraction are such tasks that a face detection system performs. But there is a cardinal problem with this system; detecting the object that has a lot of movements, positions and poses is very difficult job and it will become more complicated while considering the changes during the time. Other problems that the system may face, are facial expression, removable features, partial occlusion and three-dimensional position of the face.

Researchers present a lot of methods to detect the face and generally classify them into 4 groups such as:

- Feature invariant approaches
- Knowledge-based methods
- Appearance-based methods
- Template matching methods

The difference between face detection and localization is, face detection finds all faces in an image if there is any, and however, face localization localizes only one face in an image [16], [17]. By using this knowledge, the methods that are applied by face detection are feature invariant and knowledge-based and the methods are used

by face localization and also face detection are appearance-based and template matching.

Table 3.1: Categorization of methods for face detection within a single image [15]

Methods	Relevant Works
Feature Invariant Facial Features Texture Skin Color Multiple Features	Grouping of edges Space Gray-Level Dependence matrix of face pattern Mixture of Gaussian Integration of Skin Color, Size and Shape
Knowledge-Based	Multi Resolution Rule-Based Method
Appearance-Based Eigenfaces and Fisherfaces Neural Network Deformable Models	Eigenvector Decomposition and Clustering Ensemble of Neural Network and Arbitration Schemes Active Appearance models
Template matching Predefined Face Templates Deformable Templates	Shape Templates Active Shape models

3.2 Face Detection Methods Utilized in This Thesis

In this part Viola-Jones, Harris Corner, Hough Transform and Circular Hough Transform algorithms are going to be explained.

3.2.1 Viola-Jones Detector

Paul Viola and Michael Jones presented an algorithm which can be one of the considerable step in object (specially face) detection field. This method is based on machine-learning and tries to choose a collection of the best cases of features then

merges them into an impressive classifier [18]. The properties that made this algorithm more operational, are;

- The high percentage of True-Positive rate and low percentage of False-Positive make the system more robust comparing with others.
- The high speed in processing for example 2 frames per second makes the algorithm usable in real-time frame workings.

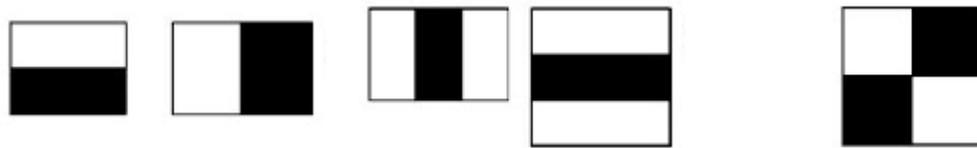
Viola-Jones method mostly is utilized to detect faces from non-faces and prepares the primary usable data for recognition algorithm. Some novelties in mentioned method prepare the field of getting better results over other systems, such as;

- Using Haar-Like Feature to extract the certain features of people
- Using the Integral Images for quickly feature calculation
- Using Ada boost (Adaptive Boosting) learning algorithm to fastest electing of efficient classifiers
- Using the special method to compound different classifiers into a cascade to remove the background areas and gather attention on more important region of picture.

These properties are studied in deyal in the following sub-sections.

3.2.1.1 Haar-Like Features

The human faces have some similar properties. For example, eyes region is darker than upper-cheek or nose region is brighter than eyes region are the similar properties on all faces [19], [20]. In this case, Haar-like features are employed to find the special signs and different characteristics on different faces.



(a) Edge Features (b) Line Features (c) Four-Rectangular Features

Figure 3.1: Some samples of Haar-Like Features

As shown in Figure 3.1, Haar-Like features are rectangular areas which contain black and white regions; each feature presents the special value that is yielded from subtracting the summation of pixels of main image under white region from summation of pixels under black one. Here there is a problem; the large number of features for single images causes the plenty of calculations during extracting features. To face with this problem the Integral Images are applied.

3.2.2 Integral Images

The mass of calculation while working with Haar-Like features, Integral Images are employed to handle the problem. In this case the image is divided in small size areas which involve just four pixels, so the program deals with more logical number of calculation therefore the speed of process becomes faster [19],[20].

3.2.3 Ada Boost (Adaptive Boosting)

While extracting features on Integral Images, some calculated features are incoherent. For example Figure 3.2-a demonstrates the feature which focuses on darkness of eyes region compare with nose and cheeks region or Figure 3.2-b demonstrates the feature which focuses on brightness of nose region compare with eyes region, the problem is electing the best features to solve this problem.



Figure 3.2: Features with different properties like; eyes region is darker than upper-cheek or nose region is brighter than eyes region

The Ada boosting uses all features over learned images and the best threshold is selected to categorize the images to face and non-face groups. While processing, the system may make mistake and some errors will occur, the minimum error rates present the best classifications [21], [22]. In this step, at first, the same weight may be submitted for each image, the Ada boosting causes the increasing the weight of misclassified images, then the error rates are calculated again and the same process is continued till desired accuracy errors are gained. At the end of process, a strong classifier is introduced which is the combination of weak classifiers.

3.2.4 Cascade of Classifiers

As said before, an image involves the face and non-face regions, so it is better to employed the simple method to investigate the face region on image and made the mass of calculation less than before. The Cascade of Classifiers can be the concept which helps the system in this case. Different classifiers are divided into smallest groups and if the program employs the groups one by one the time duration for processing is decreased, for example if a group fails in extracting features for the special part of image then the area is discarded out of calculations, so the remained features are not considered for that region and the process is repeated again for other parts. At the end, the area which is passed all processes can be the face area [23], [24].

3.2.2 Harris Corner Detection

Harris Corner Detector is one of the most common methods, which is used in order to define the location of corner points in a sample image. The large amount of changes in intensity in different directions of the image is the basic rule in Harris Corner Detection [25]. These changes can be examined by studying the oscillations of intensity in special area. Intensity changes around the corner point as the selected

window was shifted in a direction. According to this fact, the Harris Corner Detector utilizes the second moment matrix. This matrix also is called autocorrelation matrix and its values are related with derivative of image intensity [25], [26]. The equation (3.1) shows autocorrelation matrix:

$$A(X) = \sum \omega(p, q) \begin{bmatrix} I_x^2(X) & I_x I_y(X) \\ I_x I_y(X) & I_y^2(X) \end{bmatrix} \quad (3.1)$$

Where the derivatives of intensity and the amounts of weighing function which is shown as $\omega(x, y)$ for a pixel in Cartesian coordinate in selected point like X can be represented by I_x , I_y , p and q

$$\omega(x, y) = g(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad (3.2)$$

In weighting function x and y are the Cartesian coordinate and represent the location of the targeted pixel and σ is standard deviation and utilizes the amount of variation from the mean of variables. The weighting function is employed to average the weighting of local region and presents that the weighting of center and center for a window is higher than other place. The shape of autocorrelation matrix is changed as the location window is shifted, if the eigenvalues of autocorrelation matrix are extracted as λ_1 and λ_2 , these amounts are the representatives of changes of autocorrelation matrix in windows. Harris and Stephens in 1988 found out when the autocorrelation matrix centered on corners, it would present two eigenvalues which are large and positive [25], so Harris submitted the measurement that is based on trace and determinant, which is shown as equation (3.3)

$$R = \det(A) - \alpha \text{trac}^2(A) = \lambda_1 \lambda_2 + (\lambda_1 + \lambda_2)^2 \quad (3.3)$$

In this equation, α is constant value, as mentioned before, the eigenvalues on corner points are large and positive, therefore Harris measure would became large and

known as local maxima Harris measure which are higher than a determined threshold.

$$\mathcal{C} = \{x_c | R(x_c) > R(x_i), \forall x_i \in W(x_c), R(x_c) > t_{threshold}\} \quad (3.4)$$

In equation (3.4), if the Harris measure which is extracted at point like x would be shown by $R(x)$, $W(x)$ is representative of 8-neighbor set around x_c and $t_{threshold}$ defines threshold, then \mathcal{C} can show the set of all corner points.

3.2.3 Hough Transform

The Hough transform is a commonly used method for recognition shapes that is presented by Paul Hough in 1962 [27]. At first this method was used to detect lines and involves information about features in selected place anywhere in the image. The edges of the object are detected directly by Hough Transform which uses the global features. The advantage of Hough Transform is to find lines with using the edge points in short time and also formulates these parameters to use in other cases [28]. But there is a problem with Hough Transform; this method needs a lot of calculations, therefore in the case of dealing with large image the amount of data is became too large and its procedure is grown slowly.

In Cartesian coordinate, the Hough Line Transform is the simplest form of this kind of transformation. To characterize lines, which adjoin edges in two-dimensional image, the Hough Transform is used. Normally the slope-intercept form is used as a representation of a line,

$$y = m \cdot x + c \quad (3.5)$$

As shown in equation (3.5), m is slope and c is the y-intercept. With having m and c all lines that pass any point (x, y) can be achieved.

The Hough Transform can describe any shape by equation (3.6):

$$F(x, y, \vec{\rho}) = 0 \quad (3.6)$$

Where x and y are row and column depending on the space of image. If Hough Transform is used to describe line overviews equation is appeared like equation (3.7)

$$\rho = x \cdot \cos \theta + y \cdot \sin \theta, \quad \rho = \frac{\rho}{\theta} \quad (3.7)$$

3.2.4 Circular Hough Transform

Later, the Hough Transform is improved and the Circular Hough Transform (CHT) method is constructed to detect circular in an image. The Circle Hough Transform extracts circular shapes by inputs that are formed by Canny edge detector [29]. Circle Hough Transform performs the same procedure to extract circle that Hough Transform dose to find line. The main difference between them is the Circular Hough Transform can be done in more than two-dimensional space in different directions. Suppose three-dimensional space such as (x_0, y_0, r) , where (x_0, y_0) are the central coordinate of the circle and r is radius of this circle, the equation (3.8) and Figure (3.3) show all points in that region

$$(x - x_0)^2 + (y - y_0)^2 = r^2 \quad (3.8)$$

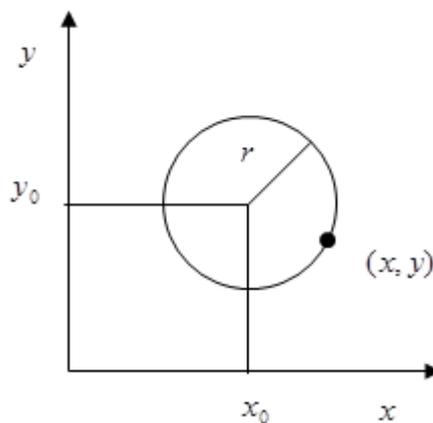


Figure 3.3: The detected region by Circular Hough Transform [29]

The process of Circular Hough Transform is:

- Detecting the edges; this performance helps to decrease the searching region for finding desired objects
- Increasing the signal to noise ratio and localization on edges
- Decreasing the false positive on edges

The last two processes are done by Canny Detector.

The voting and the agglomeration of the voting to cells are the basic methods of Hough Transform. Analyze the dual parameter space based on the resolution of the original image raises the cells. The collections of these cells make the discrete space which is named an accumulator, a voting or a Hough space.

The edges of a coin are detected more successfully by Hough Transform when it is placed on the plain surface. Vary surfaces as a background can affect Hough Transform to detect the edges and cause the mistakes in detecting processing. To solve this problem, Canny Detector with the special threshold is applied to catch the best result.



(a) Original Image



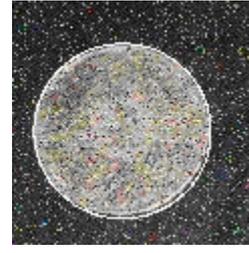
(b) Detected Image

Figure 3.4: Detecting a Coin with any Noise [30]

Figure 3.4 shows the successful Hough Transform detection on an image without noise, also Figure 3.5 shows the same detection on a noisy image.



(a) Corrupted Image with Noise



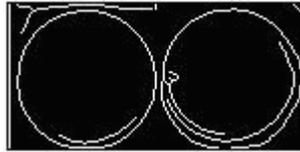
(b) Detected Image

Figure 3.5: Detecting the Coin with Salt and Pepper Noise [30]

In second example, Canny Detector finds out edges at first then by using Hough Transform the accumulator space is determined.



(a) Original image with two coins



(b) Detected edges by Canny



(c) Detected Image

Figure 3.6: Detecting Two Coins with Extra edge in second Coin cause of brightness [30]

Chapter 4

PROPOSED FACE RECOGNITION SYSTEM

4.1 Introduction

As mentioned in Chapter 1, the face recognition system is a combination of several processes such as face detection, feature extraction and face recognition.

Here, the method known as 10-Fold Cross Validation, divides the images of database into two groups namely are training and testing. In second step, Viola-Jones detector is employed to detect the faces in images if there is any then this method also extract the facial features. The extracted features are useless till they are not cropped; to solve this problem, the corner detector and circular detector have applied to find the definite points and crop the environment around points. In this step the main recognition has performed by using Principle Component Analysis (PCA) method to identify face and four facial feathers individually. In this work we propose the novel method called Minimum Distance to improve previous works.

This thesis will explain the general structure of face recognition and the important factors of a face in process of recognition and present the method to catch the best result in this field.

4.2 Stages of Proposed System

In this chapter, the face recognition based on feature detection has been presented. The proposed system also incorporates a systematic way of detecting face and facial

features. Conceptually several stages have been used to extract the data set for system to perform recognition.

The proposed approach is applied to face images from ORL database. The system diagram of the proposed approach is shown in Figure 4.1 to 4.5.

4.2.1 Stage one: Introduce Database, Training and Testing Data

In the first stage, to start proposed recognition technique which is employed in this thesis, the collection of images is required as database. There are several standard collections which are exploited in inquiries as database like FERET, CMU, FIA, PIE, ORL and etc. ORL is the database that is employed in this thesis which contains 400 gray scale images that each 10 images belong to a person, in other word, 40 people have 10 images with different poses. After introducing the database, it is time to arrange the components of database into two groups. These groups are called training and testing groups. The training group is the set of images that trained to system as known people and testing group is the set of images that their owners are going to be found among known people. To test the model in the training group the method named cross validation is applied. 10-Fold Cross Validation algorithm is one of the commonly used branches of cross validation method which is utilized to construct the training and testing sets. As shown in Figure 4.1, aforesaid algorithm randomly subdivides the original set (ORL database) into 10 equal size subset. In every 10 subset, a single image is preserved as validation data for testing the model, and the remaining 9 images are used as training data. In this case, the output of stage will be 360 training and 40 testing images.

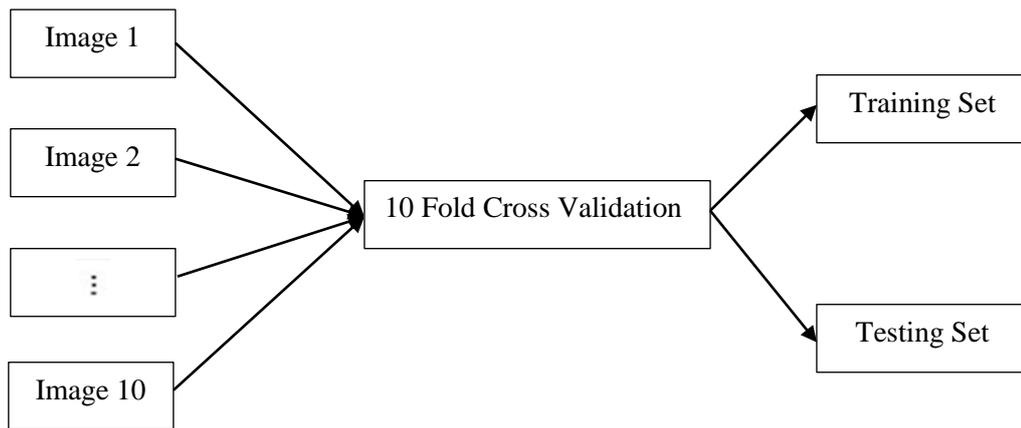


Figure 4.1: Subdivide the images into testing and training groups

4.2.2: Stage Two: Detecting Face and Features

In the second stage, Viola-Jones Detector method is employed to detect the face in image. As aforesaid in Chapter 3, Viola-Jones detection algorithm applies rectangular Haar-Like feature to find the face and facial parts regions in an image. Haar-like feature is defined as the difference of the summation of pixels in a rectangular area with another rectangular area in any position and scale, this scalar amount represents specifications of selected part of the image and checks if there is a face in that part or not. In an image, most of the regions are non-face region and the large number of Haar-like features; it is not affordable to do this amount of calculations all over the image. The notion of Cascade of Classifiers helps the system to solve the problem and focus on regions where there can be the face region. These classifiers regiment the features and employ these groups one by one, if the result of one group is positive the next group is applied to check the region, if not the region is discarded and the processes are not continued for that region. After detecting the faces, facial feature such as right-eye, left-eye, mouth and nose are also indicated for each detected face by Haar-like features. To make the input images utilizable for our system the gray scale input images are converted to RGB. Figure 4.2 represents the

processes of detection of face and features by Viola-Jones detector. The output in this stage contains 5 sets. Detected faces, extracted right-eye, left-eye, mouth and nose for each image are placed into these sets individually. Now there are 5 datasets and the purpose of the program is to identify a person by these sets, by the other word, the program tries for example to know if there is an unknown mouth available, who is the owner of that mouth.

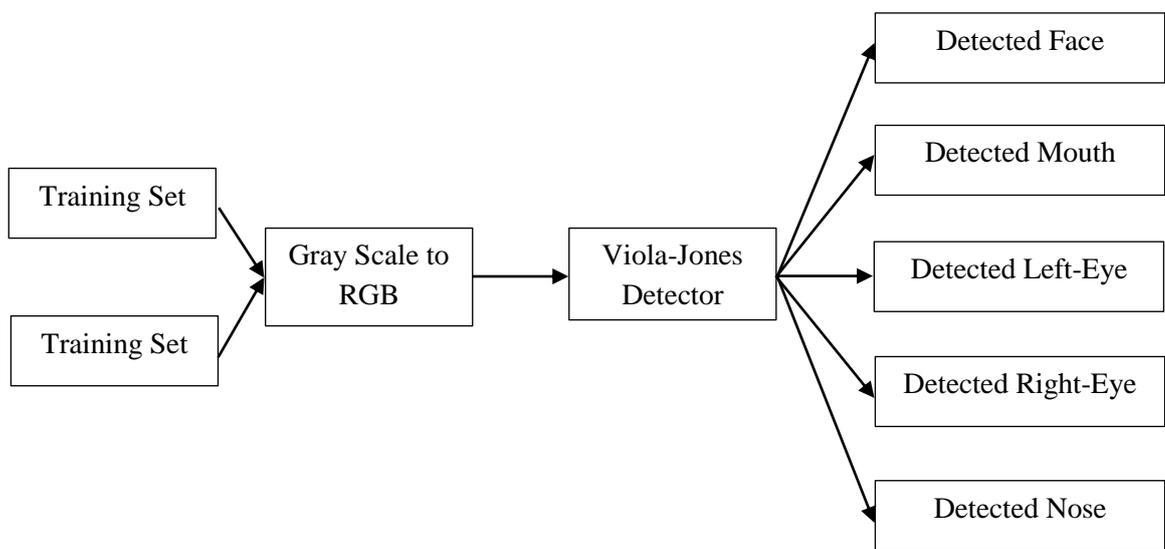


Figure 4.2: Detecting face and facial features by Viola –Jones Detector

4.2.3: Stage Three: Cropping the Extracted Features

In the third stage, the output of the previews stage is 5 sets that involve detected face and facial features, but this data cannot be useable for recognition system as input data because each set comports images in different sizes, to solve this problem some detectors will be used to determine the special points as central point of rectangular around the detected features that will be cropped. To specify those special points Harris Corner Detector and Hough Circular Detector are applied. The Harris Corner Detector indicates central points for mouth and nose features in gray scale. In detected mouth features, at first the detector points four corner of lips, then the

averages of these four points are calculated and introduced as the central points for mouth features. For detected nose features the same processes are done with a difference, instead of finding four points, the detector finds three points as top and side edges, then the same processes that is performed for mouth is continued as well. For right and left eye features, the Hough Circular Detector found the circle of iris then its center is calculated. This detector does not need any special image scale. Now there are four points as four features, these points are the center of rectangular that will be cut out by cropping program. This program employs the extracted points as origin of Cartesian coordinate then, cuts the required environment around the point according to specified x and y. The output of cropping program is 5 sets of images; each contains 400 images in same size and scale also usable for recognition stage. As previously mentioned, these 400 images are randomly divided into two groups; 360 training and 40 testing. In Figure 4.3, the diagram of this stage is shown.

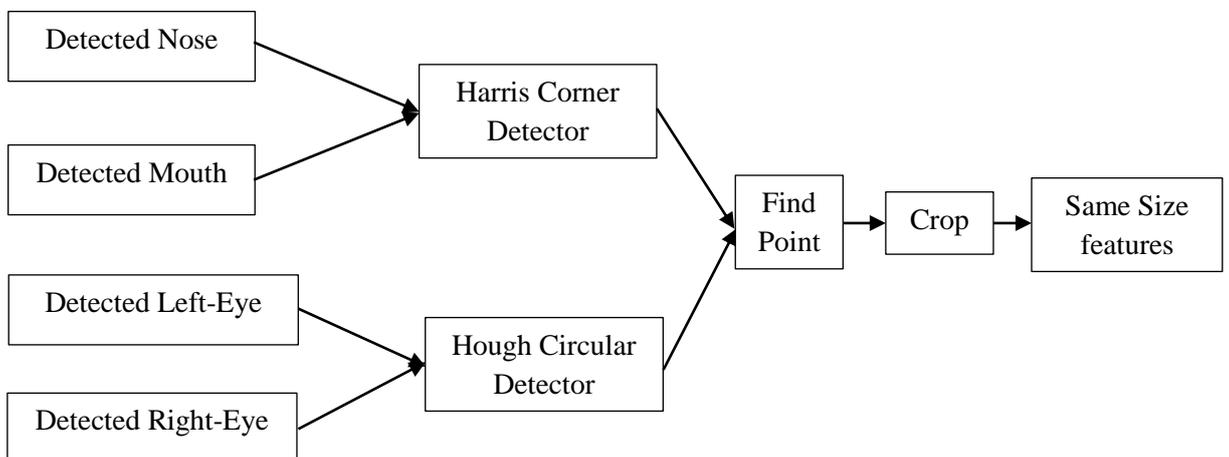


Figure 4.3: Refining extracted features in same size

4.2.4: Stage Four: Recognition by PCA

In stage four, Principle Component Analysis (PCA) algorithm is exerted to recognize the 40 testing samples in each set among 360 training samples. Each sample is

represented as a matrix in image process knowledge. In this stage, PCA method is performed for each 5 sets individually. Before starting the recognition stage all images in both training and testing groups are transmuted to gray scale images, then the matrices of gray scale images are converted to vectors and two new matrices are built. The matrices are named training and testing matrices and contain the training and testing images in the form of the columns of those matrices. Now all requirements are available to perform PCA method in several steps, as shown in Figure 4.4:

- The mean vector for training matrix is extracted.
- The distance of each column of training matrix with mean vector is calculated and a new matrix named mean centered data is introduced.
- Covariance matrix of mean centered data is calculated.
- The eigenvectors of covariance matrix are computed and sorted to select the most dominant eigenvectors then; the matrix of sorted eigenvectors is normalized.
- The eigenfaces matrix is extracted by multiplying the normalized matrix with mean centered data and dimensionally reduction of yield matrix.
- Difference matrices between mean vector and columns of training and testing matrices are found, then each matrix is multiplied by eigenfaces matrix and the Projected-Images and Projected-Test-Images matrices are evaluated.
- The columns of Projected-Test-Images matrix are selected one by one and the norm of the differences of each column with all columns of Projected-Images matrix is calculated, a 1 by 360 matrix is created and called Euclidean Distance. The place of minimum component of this matrix is

found as the person's number that is recognized in that case, the number belongs to an integer set from 1 to 40. The process repeats for all 40 columns of Projected-Test-Image matrix, these 40 results are collected in a set without any special order as the answer of recognition process.

- This step is an optional step and just illustrates the percentage of success in recognition for each part but in this thesis, it is done to see the improvement of recognition results by combining the results, which are caught for each detected part. To determine the recognition percentage for each set, the new set is defined as reference set, which contains the integers in increasing order from one to 40, the achieved set is compared with reference set and the difference of same place component is computed. Number of zeros yield in this computation is the number of success in recognition process for 40 unknown samples. By dividing this number to 40 and multiplying by 100, the efficiency of method is represented in the field of percentage for each detected part.

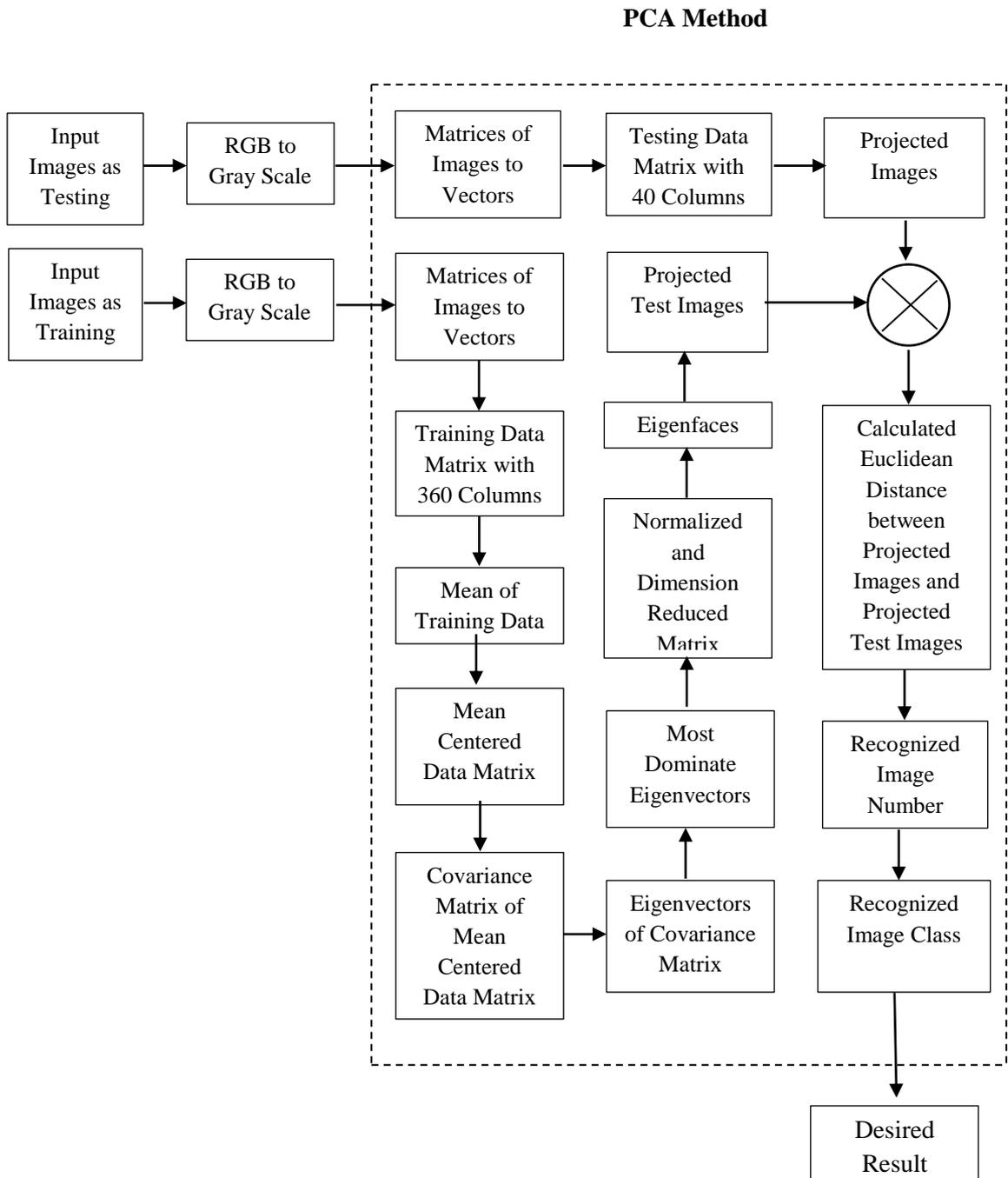


Figure 4.4: Recognized testing images by PCA for each detected and cropped set

4.2.5: Stage Five: Combination of 5 Sets by Minimum Distance Method

In the stage five, the main goal of the program is to extract the result by using the results of recognition for face and its parts individually. As said before, the output of fourth stage is 5 sets as recognized 5 detected subjects. The method which is used in fifth stage is similar to the fourth stage.

- The set of integer number in increasing orders introduced as the reference set.
- The differences of same placed members of recognized sets with reference set are calculated.
- The member with least distance to reference set is chosen as a final recognition result in that place.

The final result is a set that is obtained by combining the 5 individual recognized sets. In Figure 4.5 the steps of the last stage are illustrated.

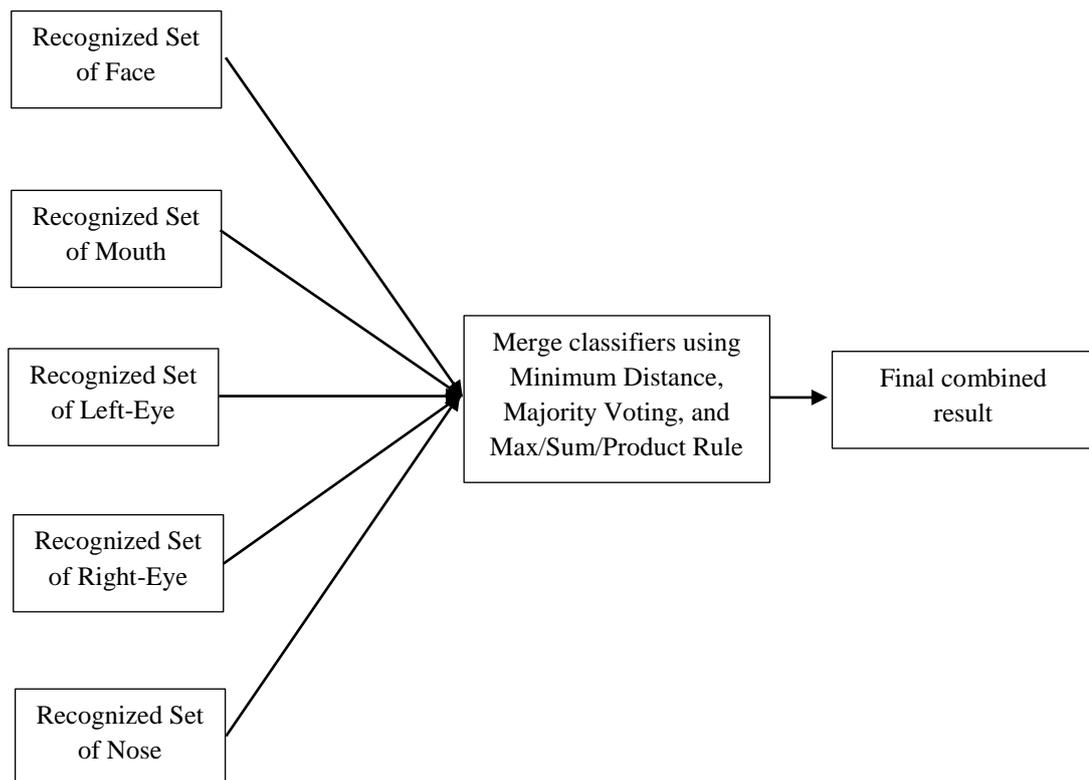


Figure 4.5: Choosing the best result by data fusion methods

4.3 Performance Accuracy

To evaluate and compare the performance of different validation algorithms, the performance accuracy is a good representative. This parameter illustrates the

statistic results of recognized faces and features clearly which helps to have a fair comparison and judgment between one or more recognition algorithms. The performance accuracy of a recognition algorithm can be computed using the following formula:

$$Accuracy \% = \frac{NT}{NT + NF} 100 \quad (4.1)$$

Where:

- NT: Number of True recognized individuals in each iteration
- NF: Number of False recognized individuals in each iteration

Chapter 5

METHODOLOGY

5.1 Introduction

In the pervious chapters the main techniques in the structure of face recognition system are perused, but to set up the system some data and extra methods are required. First of all, the system needs the collection of images as a database which contains appropriate number of the samples. The extent of representative database directly effects on the accuracy of conclusions in this kind of research. In this chapter several common used database are introduced. After presenting the database the for system, some the of images in the database are selected to train the system and some of them to test the system. The technique which is employed to select the training and testing samples in this thesis is 10-Fold Cross Validation.

5.2 Database

In last few decades, face recognition becomes one of the most popular subjects in computer vision field therefore, researchers try to create the various databases which are going to be exerted in face recognition area such as FERET, NIST MID, UMIST, Yale and AT & T (formerly ORL) are some. This section intorducts ORL and its properetes as proposed database.

5.2.1 AT& T (ORL)

AT & T (formerly ORL database) is composed of face images of 40 different individuals, 10 images were taken for each person therefore total number of images in this database is 400. To study the effect of time duration of facial appearance these

images were shot at distinct time against a dark resembling background and slight lighting variation, between 1992 to 1994 by Samaria and Harter [31]. The components of ORL database are 8-bit gray scale images with 112x92 pixel size which are in various facial moods and details such as open or closed eyes, with or without smiling, with or without glasses , etc. The subjects, in this work, were chosen in different ages, genders and colors, these subjects were in up-light, frontal position with about $\pm 15^\circ$ rotational tolerance and $\pm 20^\circ$ pose tolerance. In Figure 5.1, 10 images of a person are shown as example set of this database [32].



Figure 5.1: An example subject from ORL database[32]

5.3 Designating the Training and Testing

After introducing the used database in this thesis now it is time to speak about the method which can be the start of proposed method of this work. Our method is started by determining the training and testing samples for recognition system, then the explained functions are exerted on these samples and the achieved results are perused. The Cross Validation algorithm is the method that determines the learning and testing samples of database.

5.3.1 Introduction of Cross Validation Algorithm

The typical task of data mining is training the available data to a system like regression system or the classifier. In this case, although the adequate prediction capability on the training data during the evaluating the system can be indicated but the future unseen data may not be predicted. In 1930's Larson, S.C made a study on the shrinkage of a regression equation usage for a group to predicate the scale scores for another group [33]. Mosteller and Turkey presented an idea which was homogeneous to current version of K-Fold Cross Validation for the first time [34], in 1960's. And finally in 1970's, Cross Validation Algorithm started to be employed not only just for estimating model performance, but also, for selecting particular parameters by Stone and Geisser [35], [36]. Cross Validation with statistical form is an algorithm which tries to evaluate or compare learning model in several iterations by apportioning data in two groups;

- The group which is employed to train the system
- The group which is employed to validate the system

This algorithm is used in several forms such as K-Fold Cross Validation and special case of K-Fold Cross Validation, 10-Fold Cross Validation is the most common model in data mining or machine learning.

5.3.2 Different Versions of Cross Validation Algorithm

Cross Validation algorithm has two possible purposes such as;

- Estimating the performance of the training algorithm by available data
- Comparing the performance of two or more algorithms to deduce the best one among them

To achieve these purposes some methods are presented.

5.3.2.1 Resubstitution Validation

In this method, all available data is employed to train the system and then the same set is used to test the system. The method has good performance while dealing with available data but has poor performance upon future unseen test data because of overlapping problem.

5.3.2.2 Hold-Out Validation

To solve the overlapping problem which is faced in Resubstitution Validation method, instead of using all available data, the procedure prefers to use two independent sets as training and testing. The system holds out testing set and does not pay attention to it while learning the training sets, therefore, the results pertain to the chosen instances for training and testing sets, because the results can be skewed easily by these sets. Sometime the valuable data is held out as testing samples so prediction performance may skew the achievements, in this case the iteration of Hold-Out Validation for multiple times and averaging the results can be helpful. The problem with this solution is, in each repetition, the choosing process for sets is randomly, thus it can be possible for some data to be selected by some sets and does not find any chance to be chosen as testing set. K-Fold Cross Validation algorithm is presented to deal with this problem and employ the available data to the maximum.

5.3.2.3 K-Fold Cross-Validation

The basis of K-Fold Cross Validation method is, administering all available data into segments or folds with same or almost same size which is equal to K. The method employs 1-K folds to train the system and the remained one fold for prediction performance of trained system, this process is going to be repeated for K times, to avoid the problem which is faced by Hold-Out Validation, in each repetition the different folds are utilized and finally the desired conclusion is extracted by

averaging the results of iterations. In this algorithm, it is usual to classify data before dividing them into folds to make sure that each fold is a good delegate of all data.

5.3.2.4 Leave-One-Out Cross-Validation

As K-Fold Cross Validation algorithm deals with lots of data particularly in bioinformatics, in each repetition, the system utilizes almost all data for training and just one observation for testing. This model is famed as Leave-One-Out Cross-Validation (LOOCV). Although the estimating the accuracy of this model is not affectable but, because of its high variance, the LOOCV method cannot be valid [36].

Chapter 6

DATA FUSION TECHNIQUES

6.1 Introduction

In pervious chapter the processes of purposed face recognition method and the used techniques in each process are explained. The main intent of studying in this field is improvement of existing methods, to attain a desired result, a method is provided as Minimum Distance method which employs the individual recognized results for face and its parts and extracts the new conclusion according to them. In this chapter, some other methods that can have the same performance in system will be discussed. All these methods are based on the Euclidean Distance matrix which will be introduced and explained.

6.2 Euclidean Distance Matrix

As said in Chapter 4, the collection of 5 processes builds our recognition system. The brief studying on these processes is as follow;

1. The training and testing data are specified.
2. The detection procedure is performed for all training and testing samples.
3. Extracted features are cut in same size and represented as dataset for recognition flow.
4. The recognition proceeding is accomplished for detected faces and features exclusively.
5. The method is presented to study the recognized result and selects the best of them.

If the pervious procedure is investigated more accurate, in step four, after constructing the Projected-Images and Projected-Test-Images matrices with using eigenvectors, a new 1 by n (n: number of trained samples) matrix is created by exerting the norm of the differences of each column in Projected-Test-Images matrix with all columns of Projected-Images matrix, this matrix is called Euclidean Distance matrix. In other word, Euclidean Distance matrix in recognition step indicates the amount of resemblance between each testing sample and training data, as this value decreases, the similarity increases. The methods which are going to be introduced use Euclidean Distance matrix in different forms in next step.

6.2.1 The Mathematical Introduction of Euclidean Distance Matrix

The Euclidean Distance Matrix is became one of the important topics in Linear Algebra in recent years, because of its elegance and usability in so many different cases to solve problems [38]. To create Euclidean Distance Matrix, suppose 2-dimensional sets like X and Y, the elements which are extracted by equation (5.1) constructs a new matrix A;

$$a_{ij} = \|x_i - y_j\|^2, x_i \& y_j \in R^2 \quad (5.1)$$

The symmetric matrix $A = (a_{ij})$ with nonnegative elements and zero diagonal is called Euclidean Distance Matrix.

6.2.2 Different forms of Euclidean Distance Matrix

As said before, all combination techniques in step five exploit the Euclidean Matrix, but they are divided into two groups according to how they employ this matrix;

- Group one which use Euclidean Distance matrix directly without any changes in elements of matrix

- Group two, which use the probabilistic form of Euclidean Distance matrix. To create this form of matrix, there is need to find maximum elements of matrix, then divided all of them to maximum amount. The results of subtracting these divisions from one create the probabilistic form of matrix.

6.3 Proposed Methods

Minimum Distance method is the best combination method among several methods such as Majority Voting, Maximum Probability, Sum Rule and Product rule which are going to be introduced and illustrated. According to their usage of Euclidean Distance matrix, Minimum Distance and Majority Voting are placed in group one and Maximum Probability, Sum Rule and Product Rule belong to the other group.

6.3.1 Minimum Distance

Minimum Distance is the basic method used in this thesis and clarified in Chapter 4. The minimum presented by Wolfowitz in 1955 [39], the mathematical proof of the method has been explained in his paper. The algorithm depends on species of continues mapping theorem and is employed to test a hypothesis and estimating parameters to submits supplementary appearance of a statistical model. In our structure, after achieving the recognition process for each feature the combination methods select the most acceptable value among several answers and produce the unique answer. What the Minimum Distance method has done is, firstly, introduces a set of integer number from 1 to 40 in increasing order as Reference set, then compares appurtenances of each recognized results set which involves integer numbers from 1 to 40 without any special order and repeatable property with appurtenances of Reference set and designates the nearest of them. In Table 6.1, the application of this method is demonstrated.

6.3.2 Majority Voting

Majority Voting algorithm is applied on any sequence of votes to assign whether any component is voted more than others, and if so, proposes the component as a result of algorithm. This method is invented by Bob Boyer and J. Stothor Moor in 1980 and tries to solve time liner problems in any given sequence [40]. The application of this algorithm in this thesis is to study on sequences which have 5 or 4 members. These members are same placed components of recognized sets which introduce distinguished person for that placed test sample. As said before, this method employs the results of recognition which is generated by direct usage of Euclidean Distance matrix. Majority-Voting method peruses the components of recognized sets and chooses the one which is voted more. The structure of method is very simple, but sometimes some problems occur like;

- As each set nominates different person for one place therefore, method have no chose.
- As each set nominates two different people for one place therefore, method has more than one chose.

The solution of these problems divided usage of this method into two categories;

- Choose the answer randomly for slightly place.
- Choose the nearest answer to reference set which is propounded in Minimum Distance method.

Table 6.2 represents the application of Majority-Voting method for combining the recognized result sets in the form of two likes of solutions.

6.4 Maximum Probability

Maximum probability combination technique compounds the result of recognition process which is performed by probabilistic form of Euclidean Distance matrix. As explained before, in probabilistic form, the elements in Euclidean Distance matrix for each testing sample are converted to the subtraction of elements over the maximum of them, then a new 40 by 360 matrix is created. To achieve the individual recognition results the place of the maximum element for each row is extracted and situated in recognized set. In combination step, the method compares the same rows of probabilistic matrices and determines the most probable elements, then submits the location of them as result of this method. Table 6.3 mentions combining recognized result sets by Maximum Probability technique.

6.5 Sum Rule

In statistic, suppose two events A and B, these are mutually exclusive or disjoint if they cannot occur at the same time;

$$P(A \text{ and } B) = 0 \quad (5.1)$$

In this condition, the probability that A or B will occur is the sum of the probability of each event;

$$P(A \text{ or } B) = P(A) + P(B) \quad (5.2)$$

This method is famous as Addition or Sum Rule. To investigate the usage of sum rule in this thesis, first of all, the probabilistic form of Euclidean distance matrix for each detected part is calculated and recognition process is performed for each of these matrices individually according to find the most probable element for each row, then in combination step, because of these matrices are exclusive events, as Sum Rule says, the probability that each element occur is sum of all same placed elements in matrices. This rule creates a new matrix in probabilistic appearance; therefore the

main task of combination step is done. To derive the final conclusion, the maximum element of each row is determined. The places of these amounts present the recognized people in those situations. Table 6.4 demonstrates the result of combination by this method.

6.6 Product Rule

The Two events A and B in pervious section are independent if the occurrence of one does not change the probability of the other occurring. The probability that A and B will occur is the product of the probability of each event.

$$P A \text{ and } B = P A .P(B) \quad (5.3)$$

The expression is called Product Rule. The Euclidean Distance matrices which are belong to recognition stage for parts are independent therefore to calculate the probability of occurring all these matrices, the Product rule is employed. The elements in a matrix product by same located elements in another matrix, this procedure will be continued for all matrices and then the new matrix is constructed. The final result is produced by figuring out the highest element in each row, by the other word, the most probable element in each row. Table 6.5 illustrates the extracted final results by exerting this method.

6.7 Example

Now all these methods will be shown in the case of example. If all 5 processes are wanted to be surveyed;

- 10-Fold Cross Validation selects second sample of each person as testing dataset and trains remain sample to the system as training dataset Figure 6.1.

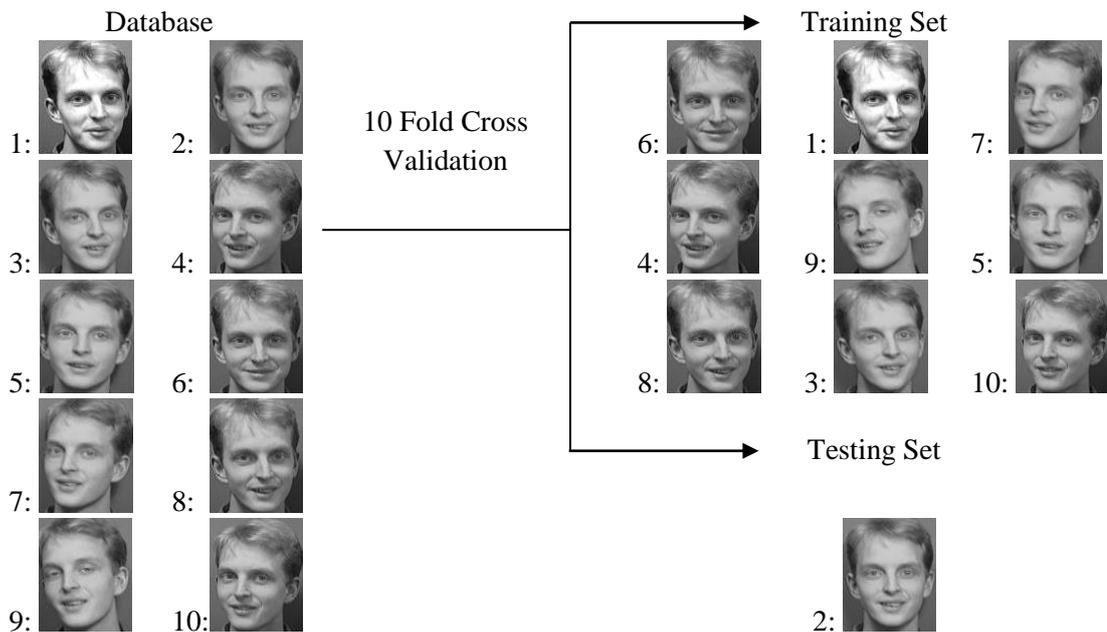


Figure 6.1: Subdivide the images into testing and training group by 10 Fold Cross Validation

- Viola-Jones detector finds faces and features of each face as shown in Figure (6.2).

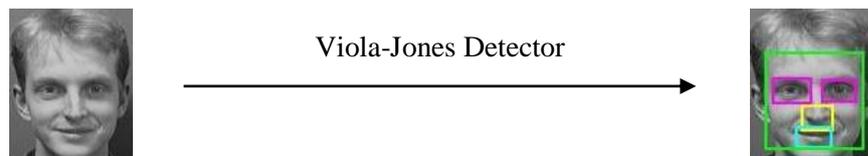


Figure 6.2: Detecting face and facial features by Viola –Jones Detector

- Harris Corner and Hugh Circular detector determine special points on features and cropping program cut the rectangular around these points and prepares them in same size.

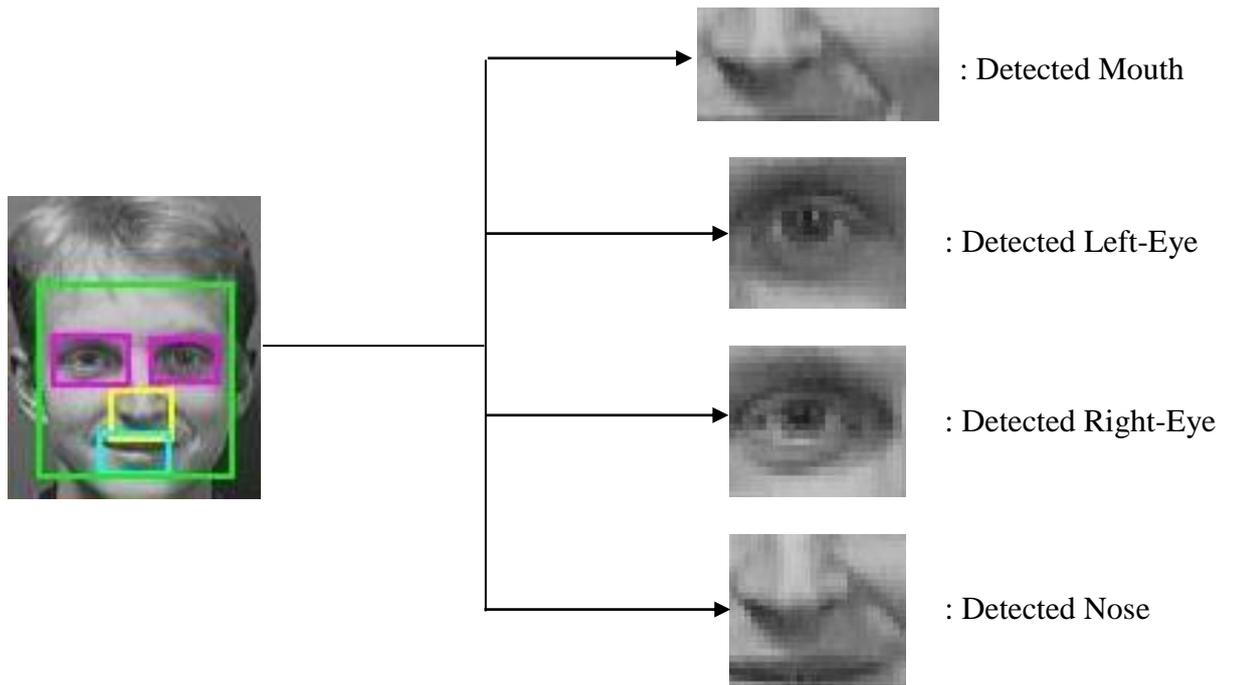


Figure 6.3: Refining extracted features in same size

- PCA method recognizes testing samples and products recognized result sets.
- Finally, the combination methods exert the recognized sets and represent the combined result set.

In Table 6.1, different classifiers are combined by Max Rule where the class with maximum probability is reported to be the combination result. This process is done once for combining face and facial feature classifiers(Combination (1)), next for combining facial feature classifiers only (Combination (2)).

Table 6.1: Recognition results by Max Probability

Recognized Sets :	
Face :	$\left\{ \begin{array}{l} 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \\ 21 \ 22 \ 23 \ 15 \ 25 \ 26 \ 27 \ 28 \ 29 \ 30 \ 31 \ 32 \ 33 \ 33 \ 35 \ 36 \ 37 \ 38 \ 39 \ 40 \end{array} \right\}$
Mouth :	$\left\{ \begin{array}{l} 37 \ 32 \ 28 \ 9 \ 12 \ 6 \ 40 \ 10 \ 1 \ 10 \ 11 \ 12 \ 13 \ 19 \ 15 \ 17 \ 17 \ 40 \ 14 \ 26 \\ 21 \ 22 \ 23 \ 5 \ 35 \ 35 \ 19 \ 19 \ 29 \ 33 \ 20 \ 20 \ 30 \ 30 \ 35 \ 36 \ 39 \ 32 \ 39 \ 38 \end{array} \right\}$
Left-Eye :	$\left\{ \begin{array}{l} 24 \ 2 \ 3 \ 4 \ 9 \ 37 \ 19 \ 8 \ 9 \ 10 \ 3 \ 30 \ 4 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 15 \\ 21 \ 22 \ 23 \ 24 \ 25 \ 25 \ 27 \ 28 \ 29 \ 29 \ 29 \ 29 \ 33 \ 33 \ 22 \ 36 \ 27 \ 23 \ 30 \ 40 \end{array} \right\}$
Right-Eye :	$\left\{ \begin{array}{l} 32 \ 2 \ 2 \ 21 \ 23 \ 6 \ 7 \ 8 \ 9 \ 10 \ 40 \ 12 \ 13 \ 14 \ 15 \ 16 \ 16 \ 18 \ 19 \ 20 \\ 21 \ 22 \ 23 \ 29 \ 25 \ 25 \ 27 \ 28 \ 29 \ 30 \ 30 \ 30 \ 33 \ 33 \ 35 \ 6 \ 37 \ 38 \ 39 \ 40 \end{array} \right\}$
Nose :	$\left\{ \begin{array}{l} 35 \ 2 \ 3 \ 26 \ 23 \ 6 \ 7 \ 8 \ 9 \ 18 \ 11 \ 12 \ 13 \ 33 \ 15 \ 8 \ 17 \ 30 \ 19 \ 20 \\ 21 \ 22 \ 23 \ 25 \ 25 \ 25 \ 27 \ 11 \ 29 \ 30 \ 31 \ 31 \ 33 \ 33 \ 37 \ 36 \ 37 \ 38 \ 39 \ 40 \end{array} \right\}$
Combined Result Set (1) :	
$\left\{ \begin{array}{l} 1 \ 2 \ 28 \ 4 \ 5 \ 6 \ 40 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \\ 21 \ 22 \ 23 \ 25 \ 25 \ 25 \ 27 \ 28 \ 29 \ 30 \ 30 \ 30 \ 33 \ 34 \ 35 \ 36 \ 37 \ 38 \ 39 \ 38 \end{array} \right\}$	
Combined Result Set (2) :	
$\left\{ \begin{array}{l} 24 \ 2 \ 28 \ 4 \ 12 \ 6 \ 40 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 17 \ 17 \ 18 \ 19 \ 20 \\ 21 \ 22 \ 23 \ 25 \ 25 \ 25 \ 27 \ 28 \ 29 \ 30 \ 30 \ 30 \ 33 \ 33 \ 35 \ 36 \ 37 \ 38 \ 39 \ 38 \end{array} \right\}$	

In Table 6.2, different classifiers are combined by Sum Rule where the probability of same class classifier add up and the most probable class is reported as combination result. This process is done once for combining face and facial feature classifiers (Combination(1)), next for combining facial feature classifiers only (Combination(2)).

Table 6.2: Recognition results by Sum Rule

Recognized Sets :	
Face :	$\left\{ \begin{array}{cccccccccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 15 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 33 & 35 & 36 & 37 & 38 & 39 & 40 \end{array} \right\}$
Mouth :	$\left\{ \begin{array}{cccccccccccccccccccc} 37 & 32 & 28 & 9 & 12 & 6 & 40 & 10 & 1 & 10 & 11 & 12 & 13 & 19 & 15 & 17 & 17 & 40 & 14 & 26 \\ 21 & 22 & 23 & 5 & 35 & 35 & 19 & 19 & 29 & 33 & 20 & 20 & 30 & 30 & 35 & 36 & 39 & 32 & 39 & 38 \end{array} \right\}$
Left-Eye :	$\left\{ \begin{array}{cccccccccccccccccccc} 24 & 2 & 3 & 4 & 9 & 37 & 19 & 8 & 9 & 10 & 3 & 30 & 4 & 14 & 15 & 16 & 17 & 18 & 19 & 15 \\ 21 & 22 & 23 & 24 & 25 & 25 & 27 & 28 & 29 & 29 & 29 & 29 & 33 & 33 & 22 & 36 & 27 & 23 & 30 & 40 \end{array} \right\}$
Right-Eye :	$\left\{ \begin{array}{cccccccccccccccccccc} 32 & 2 & 2 & 21 & 23 & 6 & 7 & 8 & 9 & 10 & 40 & 12 & 13 & 14 & 15 & 16 & 16 & 18 & 19 & 20 \\ 21 & 22 & 23 & 29 & 25 & 25 & 27 & 28 & 29 & 30 & 30 & 30 & 33 & 33 & 35 & 6 & 37 & 38 & 39 & 40 \end{array} \right\}$
Nose :	$\left\{ \begin{array}{cccccccccccccccccccc} 35 & 2 & 3 & 26 & 23 & 6 & 7 & 8 & 9 & 18 & 11 & 12 & 13 & 33 & 15 & 8 & 17 & 30 & 19 & 20 \\ 21 & 22 & 23 & 25 & 25 & 25 & 27 & 11 & 29 & 30 & 31 & 31 & 33 & 33 & 37 & 36 & 37 & 38 & 39 & 40 \end{array} \right\}$
Combined Result Set (1) :	
$\left\{ \begin{array}{cccccccccccccccccccc} 16 & 2 & 3 & 4 & 40 & 6 & 7 & 8 & 9 & 10 & 28 & 12 & 13 & 14 & 15 & 16 & 17 & 12 & 19 & 20 \\ 21 & 22 & 23 & 24 & 5 & 25 & 28 & 28 & 29 & 29 & 31 & 37 & 33 & 31 & 35 & 36 & 37 & 38 & 39 & 40 \end{array} \right\}$	
Combined Result Set (1) :	
$\left\{ \begin{array}{cccccccccccccccccccc} 24 & 2 & 3 & 40 & 40 & 6 & 7 & 23 & 9 & 10 & 28 & 9 & 13 & 38 & 15 & 16 & 17 & 38 & 19 & 21 \\ 21 & 22 & 23 & 35 & 5 & 5 & 28 & 28 & 29 & 29 & 29 & 29 & 33 & 33 & 35 & 40 & 27 & 18 & 33 & 3 \end{array} \right\}$	

In Table 6.3, different classifiers are combined by Product Rule where the probability of same class classifier multiply by each other and the most probable class is reported as combination result. This process is done once for combining face and facial feature classifiers (Combination (1)), next for combining facial feature classifiers only (Combination (2)).

Table 6.3: Recognition results by Product Rule

Recognized Sets :	
Face :	$\left\{ \begin{array}{l} 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \\ 21 \ 22 \ 23 \ 15 \ 25 \ 26 \ 27 \ 28 \ 29 \ 30 \ 31 \ 32 \ 33 \ 33 \ 35 \ 36 \ 37 \ 38 \ 39 \ 40 \end{array} \right\}$
Mouth :	$\left\{ \begin{array}{l} 37 \ 32 \ 28 \ 9 \ 12 \ 6 \ 40 \ 10 \ 1 \ 10 \ 11 \ 12 \ 13 \ 19 \ 15 \ 17 \ 17 \ 40 \ 14 \ 26 \\ 21 \ 22 \ 23 \ 5 \ 35 \ 35 \ 19 \ 19 \ 29 \ 33 \ 20 \ 20 \ 30 \ 30 \ 35 \ 36 \ 39 \ 32 \ 39 \ 38 \end{array} \right\}$
Left-Eye :	$\left\{ \begin{array}{l} 24 \ 2 \ 3 \ 4 \ 9 \ 37 \ 19 \ 8 \ 9 \ 10 \ 3 \ 30 \ 4 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 15 \\ 21 \ 22 \ 23 \ 24 \ 25 \ 25 \ 27 \ 28 \ 29 \ 29 \ 29 \ 29 \ 33 \ 33 \ 22 \ 36 \ 27 \ 23 \ 30 \ 40 \end{array} \right\}$
Right-Eye :	$\left\{ \begin{array}{l} 32 \ 2 \ 2 \ 21 \ 23 \ 6 \ 7 \ 8 \ 9 \ 10 \ 40 \ 12 \ 13 \ 14 \ 15 \ 16 \ 16 \ 18 \ 19 \ 20 \\ 21 \ 22 \ 23 \ 29 \ 25 \ 25 \ 27 \ 28 \ 29 \ 30 \ 30 \ 30 \ 33 \ 33 \ 35 \ 6 \ 37 \ 38 \ 39 \ 40 \end{array} \right\}$
Nose :	$\left\{ \begin{array}{l} 35 \ 2 \ 3 \ 26 \ 23 \ 6 \ 7 \ 8 \ 9 \ 18 \ 11 \ 12 \ 13 \ 33 \ 15 \ 8 \ 17 \ 30 \ 19 \ 20 \\ 21 \ 22 \ 23 \ 25 \ 25 \ 25 \ 27 \ 11 \ 29 \ 30 \ 31 \ 31 \ 33 \ 33 \ 37 \ 36 \ 37 \ 38 \ 39 \ 40 \end{array} \right\}$
Combined Result Set (1) :	
$\left\{ \begin{array}{l} 16 \ 2 \ 3 \ 4 \ 40 \ 6 \ 7 \ 8 \ 9 \ 10 \ 28 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 12 \ 19 \ 21 \\ 21 \ 22 \ 23 \ 24 \ 5 \ 25 \ 28 \ 28 \ 29 \ 29 \ 31 \ 37 \ 33 \ 31 \ 35 \ 36 \ 37 \ 38 \ 39 \ 40 \end{array} \right\}$	
Combined Result Set (2) :	
$\left\{ \begin{array}{l} 24 \ 2 \ 3 \ 40 \ 40 \ 6 \ 7 \ 23 \ 9 \ 10 \ 28 \ 9 \ 13 \ 38 \ 15 \ 16 \ 17 \ 38 \ 19 \ 28 \\ 21 \ 22 \ 23 \ 35 \ 5 \ 5 \ 28 \ 28 \ 29 \ 29 \ 29 \ 29 \ 33 \ 33 \ 35 \ 40 \ 27 \ 18 \ 33 \ 3 \end{array} \right\}$	

In Table 6.4, different classifiers are combined using Majority Voting where the class with maximum votes is reported as the combination result. The special cases in this method are; the same vote case or 5 different votes. To handel this problem, two solution are used; choosing randomly and choseing by obtaining Minimum Distance method. This process is done once for combining face and facial feature classifiers (Combination (1)), next for combining facial feature classifiers only(Combination (2)).

Table 6.4: Recognition results by Majority Voting method

Recognized sets:	
Face:	$\left\{ \begin{array}{cccccccccccccccccccc} 32 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 & 26 & 26 & 28 & 29 & 30 & 31 & 32 & 33 & 34 & 35 & 17 & 37 & 38 & 39 & 40 \end{array} \right\}$
Mouth:	$\left\{ \begin{array}{cccccccccccccccccccc} 37 & 2 & 2 & 14 & 30 & 35 & 24 & 24 & 21 & 12 & 12 & 21 & 29 & 15 & 14 & 16 & 17 & 18 & 36 & 31 \\ 5 & 31 & 18 & 24 & 24 & 38 & 21 & 18 & 13 & 31 & 39 & 32 & 30 & 30 & 30 & 36 & 19 & 20 & 11 & 5 \end{array} \right\}$
Left-Eye:	$\left\{ \begin{array}{cccccccccccccccccccc} 35 & 2 & 2 & 31 & 25 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 25 & 16 & 17 & 18 & 19 & 30 \\ 21 & 22 & 23 & 24 & 24 & 25 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 33 & 33 & 36 & 37 & 38 & 39 & 40 \end{array} \right\}$
Right-Eye:	$\left\{ \begin{array}{cccccccccccccccccccc} 25 & 2 & 21 & 37 & 25 & 6 & 7 & 35 & 9 & 10 & 11 & 12 & 25 & 14 & 15 & 29 & 29 & 23 & 19 & 20 \\ 21 & 22 & 23 & 24 & 24 & 26 & 27 & 28 & 29 & 30 & 30 & 32 & 33 & 33 & 33 & 16 & 37 & 38 & 39 & 40 \end{array} \right\}$
Nose:	$\left\{ \begin{array}{cccccccccccccccccccc} 29 & 2 & 2 & 4 & 5 & 23 & 7 & 8 & 9 & 3 & 11 & 9 & 9 & 33 & 24 & 1 & 17 & 26 & 19 & 33 \\ 21 & 22 & 23 & 24 & 24 & 13 & 38 & 25 & 29 & 30 & 31 & 25 & 33 & 33 & 33 & 36 & 37 & 38 & 39 & 40 \end{array} \right\}$
Combination in Random Form (1) :	
$\left\{ \begin{array}{cccccccccccccccccccc} 29 & 2 & 2 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 24 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 33 & 33 & 36 & 37 & 38 & 39 & 40 \end{array} \right\}$	
Combination in Random Form (2) :	
$\left\{ \begin{array}{cccccccccccccccccccc} 25 & 2 & 2 & 31 & 25 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 9 & 14 & 14 & 16 & 17 & 18 & 19 & 33 \\ 21 & 22 & 23 & 24 & 24 & 38 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 33 & 33 & 36 & 37 & 38 & 39 & 40 \end{array} \right\}$	
Combination in Minimum Distance Form (1) :	
$\left\{ \begin{array}{cccccccccccccccccccc} 25 & 2 & 2 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 24 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 33 & 33 & 36 & 37 & 38 & 39 & 40 \end{array} \right\}$	
Combination in Minimum Distance Form (2) :	
$\left\{ \begin{array}{cccccccccccccccccccc} 25 & 2 & 2 & 4 & 25 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 24 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 33 & 33 & 36 & 37 & 38 & 39 & 40 \end{array} \right\}$	

In Table 6.5, different classifiers are combined by Minimum Distancem where the classifier with minimum distance for same placed component on each classifier is

presented as combination result. This process is done once for combining face and facial feature classifiers (Combination (1)), next for combining facial feature classifiers only (Combination (2)).

Table 6.5 : Recognition results by Minimum Distance

Recognized Sets :	
Face :	$\left\{ \begin{array}{cccccccccccccccccccc} 32 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 & 26 & 26 & 28 & 29 & 30 & 31 & 32 & 33 & 34 & 35 & 17 & 37 & 38 & 39 & 40 \end{array} \right\}$
Mouth :	$\left\{ \begin{array}{cccccccccccccccccccc} 37 & 2 & 2 & 14 & 30 & 35 & 24 & 24 & 21 & 12 & 12 & 21 & 29 & 15 & 14 & 16 & 17 & 18 & 36 & 31 \\ 5 & 31 & 18 & 24 & 24 & 38 & 21 & 18 & 13 & 31 & 39 & 32 & 30 & 30 & 30 & 36 & 19 & 20 & 11 & 5 \end{array} \right\}$
Left-Eye :	$\left\{ \begin{array}{cccccccccccccccccccc} 35 & 2 & 2 & 31 & 25 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 25 & 16 & 17 & 18 & 19 & 30 \\ 21 & 22 & 23 & 24 & 24 & 25 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 33 & 33 & 36 & 37 & 38 & 39 & 40 \end{array} \right\}$
Right-Eye :	$\left\{ \begin{array}{cccccccccccccccccccc} 25 & 2 & 21 & 37 & 25 & 6 & 7 & 35 & 9 & 10 & 11 & 12 & 25 & 14 & 15 & 29 & 29 & 23 & 19 & 20 \\ 21 & 22 & 23 & 24 & 24 & 26 & 27 & 28 & 29 & 30 & 30 & 32 & 33 & 33 & 33 & 16 & 37 & 38 & 39 & 40 \end{array} \right\}$
Nose :	$\left\{ \begin{array}{cccccccccccccccccccc} 29 & 2 & 2 & 4 & 5 & 23 & 7 & 8 & 9 & 3 & 11 & 9 & 9 & 33 & 24 & 1 & 17 & 26 & 19 & 33 \\ 21 & 22 & 23 & 24 & 24 & 13 & 38 & 25 & 29 & 30 & 31 & 25 & 33 & 33 & 33 & 36 & 37 & 38 & 39 & 40 \end{array} \right\}$
Combined Result Set (1) :	
$\left\{ \begin{array}{cccccccccccccccccccc} 25 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 34 & 35 & 36 & 37 & 38 & 39 & 40 \end{array} \right\}$	
Combined Result Set (2) :	
$\left\{ \begin{array}{cccccccccccccccccccc} 25 & 2 & 2 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 24 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 33 & 33 & 36 & 37 & 38 & 39 & 40 \end{array} \right\}$	

Chapter 7

RESULTS AND DISCUSSIONS

7.1 Introduction

In this chapter an recognition performances of the proposed classifier ensembles are reported. The classifiers are combined with different data fusion techniques and the final results are discussed.

7.2 Performance Analysis

The results which are extracted statistically always contain more and exact information about the system and used database. These results can be taken by repeating the testing process several times. In this case, as this thesis deals with image processing subject, the chosen standard metric parameter for recognition procedure is very special and important. The performance accuracy with corresponding formula has been defined in chapter 4. Now, the achieved results in 10 repetitions for six introduced combination methods (Minimum Distance, Majority Voting, Maximum Probability, Sum rule, Product Rule) are shown in following tables and related graphs in two groups as well , the first group involves the results just for recognized features, however the second group involves recognized faces as well.

In Tble 7.1, the accuracy of each classifier and Minimum Distance combination algorithm is shown. The recognition perocess is repeated 10 times and mean of accuracy in each iteration is calculated. The group one shows the combination of

facial features only, but group two gives the combination of facial features and face classifiers.

Table 7.1: Achieved results by Minimum Distance method

Recognized sets											Ave
Face	97.50	95.00	100.00	97.50	97.50	97.50	95.00	100.00	97.50	97.50	97.50
Mouth	35.00	17.50	27.50	35.00	27.50	35.00	17.50	27.50	25.00	35.00	28.25
R-Eye	70.00	65.00	60.00	70.00	65.00	57.50	65.00	60.00	72.50	70.00	65.50
L-Eye	57.50	75.00	50.00	57.50	62.50	50.00	75.00	50.00	65.00	57.50	60.00
Nose	67.50	55.00	65.00	67.50	52.50	60.00	55.00	65.00	67.50	67.50	62.25
Group One											
Results	87.50	87.50	90.00	87.50	77.50	82.50	87.50	90.00	95.00	87.50	87.25
Group Two											
Results	100.00	97.50	100.00	100.00	97.50	100.00	97.50	100.00	100.00	100.00	99.25

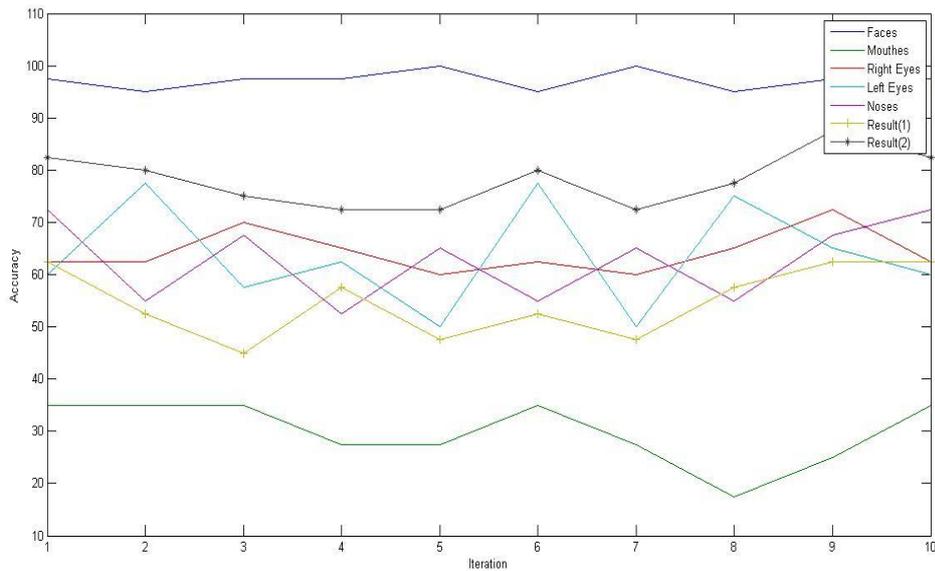


Figure 7.1: Graph of Minimum Distance performance

In Tble 7.2, the accuracy of each classifier and Majority Voting combination algorithm (Randomly Chose) is shown. The recognition perocess is repeated 10 times and mean of accuracy in each iteration is calculated. The group one shows the

combination of facial features only, but group two gives the combination of facial features and face classifiers.

Table 7.2: Achieved results by Majority Voting method (Randomly Chose)

Recognized sets											Ave
Face	95.00	95.00	100.00	95.00	100.00	97.50	97.50	100.00	97.50	97.50	97.50
Mouth	37.50	35.00	27.50	17.50	27.50	35.00	35.00	27.50	25.00	35.00	30.25
R-Eye	57.50	62.50	60.00	65.00	60.00	62.50	57.50	60.00	72.50	57.50	61.50
L-Eye	55.00	77.50	50.00	75.00	50.00	60.00	50.00	50.00	65.00	50.00	58.25
Nose	52.50	55.00	65.00	55.00	65.00	72.50	60.00	65.00	67.50	60.00	61.75
Group One											
Results	65.00	80.00	72.50	75.00	72.50	75.00	67.50	72.50	80.00	72.50	73.25
Group Two											
Results	80.00	90.00	90.00	85.00	85.00	90.00	80.00	87.50	92.50	82.50	86.25

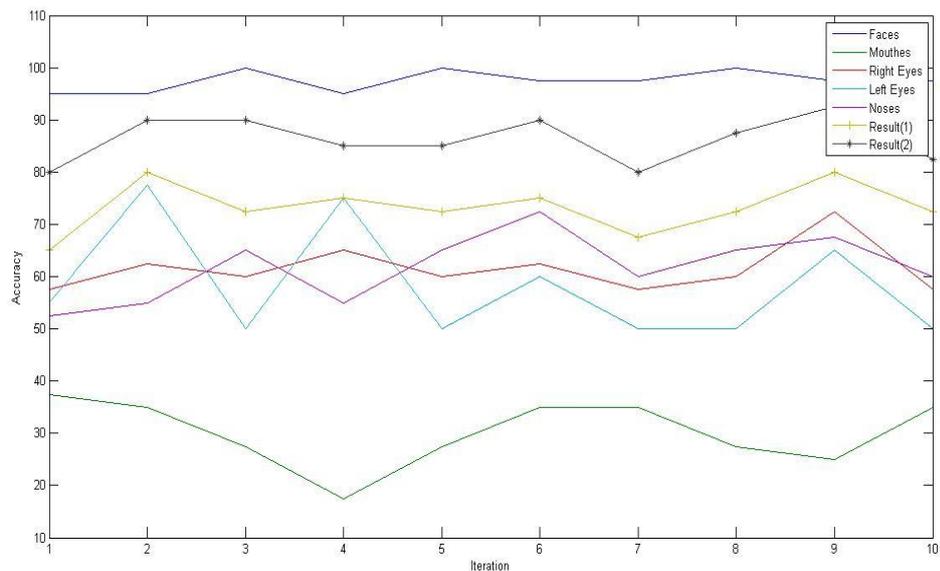


Figure 7.2: Graph of Majority Voting (Randomly Chose) performance

In Tble 7.3, the accuracy of each classifier and Majority Voting combination algorithm (Minimum Distance Chose) is shown. The recognition perocess is repeated 10 times and mean of accuracy in each iteration is calculated. The group one shows

the combination of facial features only, but group two gives the combination of facial features and face classifiers.

Table 7.3: Achieved results by Majority Voting method (Minimum Distance Chose)

Recognized sets											Ave
Face	92.50	97.50	100.00	97.50	92.50	100.00	97.50	95.00	97.50	95.00	96.50
Mouth	40.00	35.00	27.50	25.00	40.00	27.50	35.00	37.50	35.00	37.50	34.00
R-Eye	60.00	57.50	60.00	72.50	60.00	60.00	57.50	57.50	62.50	57.50	60.50
L-Eye	67.50	50.00	50.00	65.00	67.50	50.00	50.00	55.00	60.00	55.00	57.00
Nose	60.00	60.00	65.00	67.50	60.00	65.00	60.00	52.50	72.50	52.50	61.50
Group One											
Results	90.00	80.00	85.00	90.00	90.00	85.00	80.00	77.50	85.00	77.50	84.00
Group Two											
Results	97.50	92.50	92.50	95.00	97.50	92.50	92.50	80.00	90.00	80.00	91.00

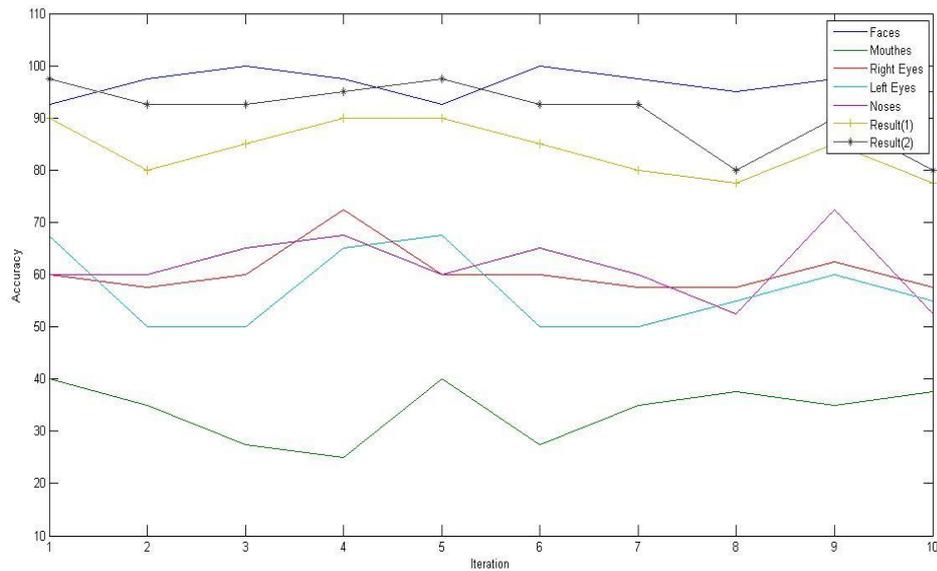


Figure 7.3: Graph of Majority Voting (Minimum Distance Chose) performance

In Tble 7.4, the accuracy of each classifier and Max Rule combination algorithm is shown. The recognition perocess is repeated 10 times and mean of accuracy in each iteration is calculated. The group one shows the combination of facial features only, but group two gives the combination of facial features and face classifiers.

Table 7.4: Achieved results by Maximum Probability method

Recognized sets											Ave
Face	97.50	95.00	95.00	92.50	95.00	97.50	97.50	97.50	97.50	95.00	96.00
Mouth	27.50	37.50	35.00	40.00	37.50	25.00	27.50	35.00	27.50	37.50	33.00
R-Eye	65.00	57.50	62.50	60.00	57.50	72.50	65.00	70.00	65.00	57.50	63.25
L-Eye	62.50	55.00	77.50	67.50	55.00	65.00	62.50	57.50	62.50	55.00	62.00
Nose	52.50	52.50	55.00	60.00	52.50	67.50	52.50	67.50	52.50	52.50	56.50
Group One											
Results	65.00	67.50	80.00	77.50	67.50	67.50	65.00	72.50	65.00	67.50	69.50
Group Two											
Results	67.50	72.50	82.50	85.00	72.50	72.50	67.50	82.50	67.50	72.50	74.25

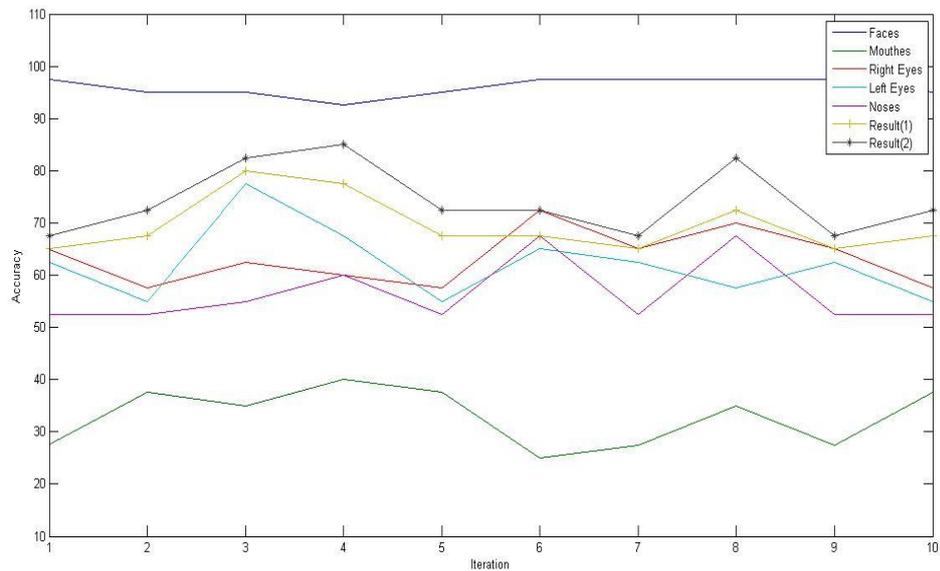


Figure 7.4 : Graph of Maximum Probability performance

In Tble 7.5, the accuracy of each classifier and Sum Rule combination algorithm is shown. The recognition perocess is repeated 10 times and mean of accuracy in each iteration is calculated. The group one shows the combination of facial features only, but group two gives the combination of facial features and face classifiers.

Table 7.5: Achieved results by Sum Rule method

Recognized sets											Ave
Face	97.50	95.00	97.50	97.50	100.00	95.00	100.00	95.00	97.50	97.50	97.25
Mouth	35.00	35.00	35.00	27.50	27.50	35.00	27.50	17.50	25.00	35.00	30.00
R-Eye	62.50	62.50	70.00	65.00	60.00	62.50	60.00	65.00	72.50	62.50	64.50
L-Eye	60.00	77.50	57.50	62.50	50.00	77.50	50.00	75.00	65.00	60.00	63.50
Nose	72.50	55.00	67.50	52.50	65.00	55.00	65.00	55.00	67.50	72.50	62.75
Group One											
Results	62.50	52.50	45.00	57.50	47.50	52.50	47.50	57.50	62.50	62.50	54.75
Group Two											
Results	82.50	80.00	75.00	72.50	72.50	80.00	72.50	77.50	87.50	82.50	78.25

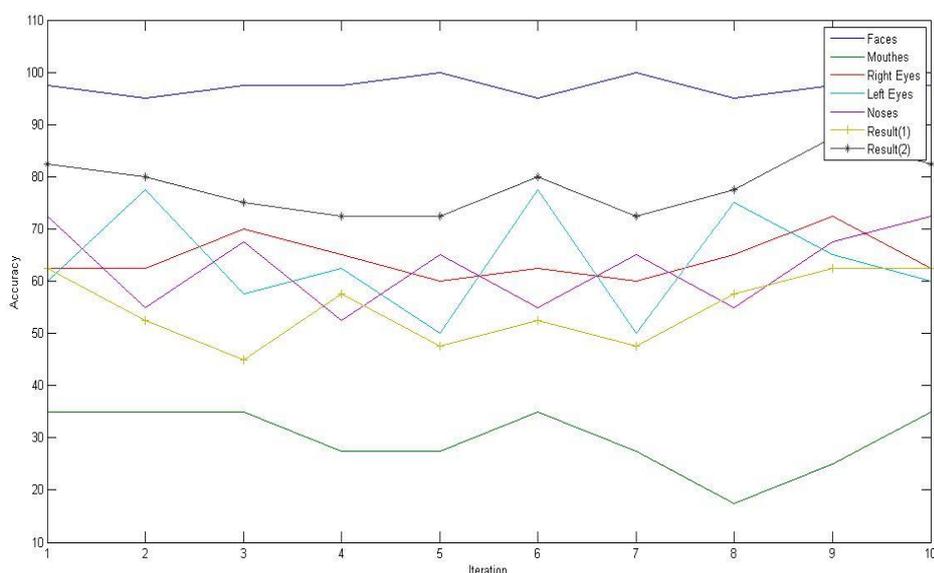


Figure 7.5: Graph of Sum Rule performance

In Tble 7.6, the accuracy of each classifier and Product Rule combination algorithm is shown. The recognition perocess is repeated 10 times and mean of accuracy in each iteration is calculated. The group one shows the combination of facial features only, but group two gives the combination of facial features and face classifiers.

Table 7.6: Achieved results by Product Rule method

Recognized sets											Ave
Face	95.00	95.00	97.50	97.50	97.50	97.50	95.00	100.00	97.50	95.00	96.75
Mouth	37.50	17.50	25.00	35.00	27.50	25.00	17.50	27.50	25.00	37.50	27.50
R-Eye	57.50	65.00	72.50	70.00	65.00	72.50	65.00	60.00	72.50	57.50	65.75
L-Eye	55.00	75.00	65.00	57.50	62.50	65.00	75.00	50.00	65.00	55.00	62.50
Nose	52.50	55.00	67.50	67.50	52.50	67.50	55.00	65.00	67.50	52.50	60.25
Group One											
Results	65.00	57.50	62.50	45.00	57.50	62.50	57.50	47.50	62.50	65.00	58.25
Group Two											
Results	75.00	75.00	87.50	72.50	72.50	87.50	75.00	70.00	87.50	75.00	77.75

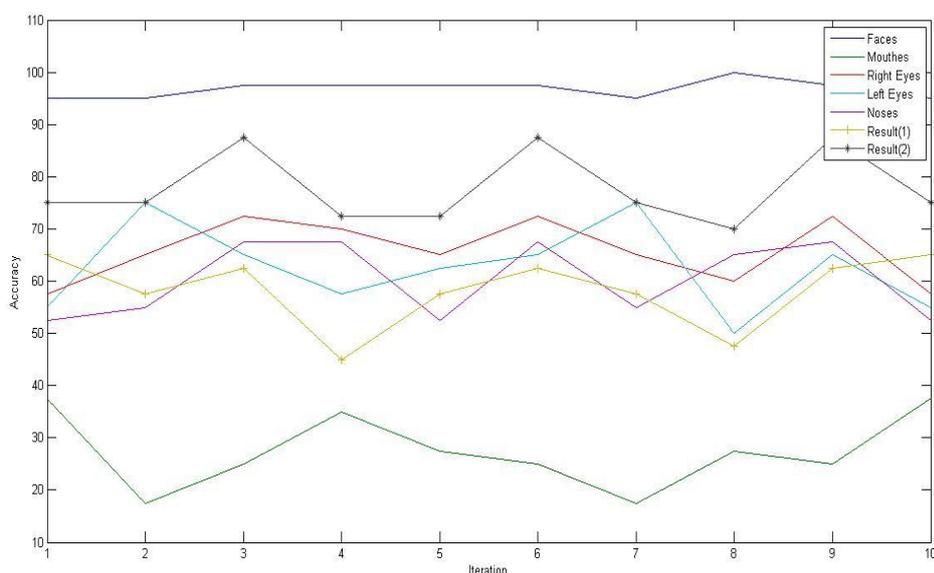


Figure 7.6: Graph of Product Rule performance

The conclusion of this work shows that the Minimum Distance data fusion technique improves the result of combination compare with other methods.

7.2 Two-Fold Cross Validation

As defined before, K-Fold Cross Validation method divides database into two types of K folds, K-1 folds are named as training folds and employed to learn the faces and features to the system and a single fold as testing fold is applied to test system to identify unknown face and features. In Two-Fold Cross Validation algorithm, there

are two folds only, one fold for training and the other for testing. Therefore the system is trained by half of database and test by other half. The recognition process is performed using Minimum Distance method, because of its higher performance compare with other methods.

Table 7.7: Achieved results by Minimum Distance method and 2-Fold Cross Validation technique

Recognized sets											Ave
Face	86.50	82.00	91.50	90.50	88.00	91.00	91.00	88.50	88.00	86.50	88.35
Mouth	25.00	25.00	21.00	24.00	24.50	25.50	24.50	26.50	24.00	23.50	24.35
R-Eye	53.50	52.50	53.00	60.00	56.50	58.00	54.50	50.50	56.00	56.00	55.25
L-Eye	52.50	51.50	52.00	58.50	51.50	54.00	58.50	52.00	58.00	53.50	54.20
Nose	50.50	47.50	44.00	51.50	50.50	50.00	52.00	52.50	42.50	51.50	49.25
Group One											
Results	79.50	81.50	81.00	86.00	80.00	82.00	83.00	80.50	84.00	83.50	82.10
Group Two											
Results	92.50	93.00	96.00	98.50	93.50	95.00	97.50	95.50	97.50	94.50	95.35

7.3 Accuracy Comparison with Previous Works

As face recognition techniques are become one of the most attractive topics in image processing field, so many researching are done in this case, therefore many different methods are submitted to present the highest performance to catch this aim. Some of the pervious works, which performs the similar processes to this thesis, are [40] and [44]. In these papers as our proposed method the face and features are detected and then by determining the special points, the usable data is prepared for recognition stage. The Gabor-based method represents features by using complex vectors [43], then the recognition has been performed in four form of classifiers such as; Nearest Neighbor (NN), Two-Layer Nearest Neighbor (TLNN), Nearest Feature Line (NFL) and Modular Nearest Feature Line (MNFL). In other paper the basic approach is extracting numbers of features according to different poses [44]. The experimental

results which compare these available researches with proposed method are going to be perused in following table.

Table 7.8: Comparing pervious works with the proposed method

Methods	Previous works					Proposed method
	NN [44]	TLNN [44]	NFL [44]	MNFL [44]	Multiple Features [43]	Minimum Distance
Experimental	92.3	94.55	93.3	95.15	94.05	99.25

Chapter 8

CONCLUSION

8.1 Conclusion

In this thesis, an automatic face recognition system based on detected facial features is represented. All facial features are detected automatically and significant improvements are made to achieve higher accuracy and efficiency of facial feature detection. To perform recognition process the collection of 400 gray-scale 112x92 images from ORL has been applied as database. The 10-Fold Cross Validation method was used at first to divide database into training and testing sets to start proposed algorithm, second, Viola-Jones detection method is utilized to determine each face and facial feature region on available data, other methodologies such as Harris Corner detector and Hough Circular detector are also employed to specify the special points on features to made the usable data on recognition step, PCA recognition method based on eigenfaces has been exerted to find individually testing face and features according to trained data. The recognition procedure has been ended by combining the achieved individual recognized results to get the higher performance, Minimum Distance method was exploited in this stage. Last stage faces with two forms of combination, the combination with considering recognized face and features and combination with just recognized features. The difference of these forms is observable on performance accuracy. Five different combination methods are also investigated to choose the best method of combination for improvement of available methods.

8.2 Future Work

In future work, the proposed recognition method will include alternative decomposition of the face data. Current work involves operations in the spatial domain, where noticeable face features are extracted for forming different classifiers. Other transform domain approaches can be developed for forming alternative classifiers. For example the images can be transformed into wavelet domain using discrete wavelet transform (DWT) to use different frequency/direction sub bands for training the alternative classifiers to improve the face recognition performance.

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