Prediction of International Stock Market Movement Using Technical Analysis Methods and TSK

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ABSTRACT

This research aimed to propose a method to improve forecasting accuracy of the technical analysis of future closing price using Takagi-Sugeno-Kang (TSK) fuzzy model to merge the forecasting of three technical prediction methods. The historical data available for London Stock Market is employed in this study to verify the performance of the proposed model compared to technical predictions.

Fuzzy data modelling emerges as an advanced technique in predicting future closing prices. In this study, the predictions of three technical analysis methods were modelled by Fuzzy Methods to enhance the predicted closing price. The fuzzy rules were extracted by using Fuzzy-C-Means (FCM) algorithm.

Data set from year 2008 to 2012 is dividing in two parts for training and verification purpose. The Fuzzy C-Means clustering (FCM) is applied on the six days Moving Average (SDMA), the Moving Average Convergence Divergence (MACD), and the Relative Strength Index (RSI) technical analysis to predict the future price, which is, target variable of the TSK fuzzy model. A prediction accuracy close to 94.7%, is achieved in predicting two days ahead closing prices of London Stock Market. The results are very encouraging and easy to implement in real-time trading system.

Keywords: Technical Forecasting, Fuzzy Modelling, TSK, FCM, clustering, moving average.

ÖZ

Bu araştırma Takagi Sugeno Kang (TSK) modeli kullanarak üç kapanış fiyatı teknik analiz yönteminin tahminlerini geliştirmeyi amaçlamıştır. Londra Hisse Senedi piyasası verileri önerilen yöntemin performansını sınamak üzere kullanılmıştır.

Bulanık mantıklı veri modellemesi piyasaların gelecekteki kapanış fiyatını tahminde başarılı bir yöntem olarak ortaya çıkmıştır. Bu çalışmada üç standart teknik analiz metodunun tahmin güçleri, kuralları FCM algoritması ile elde edilen bulanık modelleme yöntemleri ile birleştirilerek arttırılmıştır.

Kullanılan 2008 ile 2012 yılları arasındaki menkul değer piyasa kapanış fiyatları model oluşturma ve model sınama amaçlı iki bölüme ayrılmıştır. Altı-günlük ortalama, kayar ortalamalı yakınsama ıraksama, ve göreceli dayanım indeksi olmak üzere üç teknik analiz yöntemi model oluşturma verisi ile fiyat tahmini için kullanılmış, çıkan tahminler FCM ile gelecekteki fiyat hedeflenerek değerlendirilmiştir. Sınama verisini kullanarak yapılan karşılaştırmada Londra menkul değer piyasalarında 94.7% civarında başarıyla tahmin gerçekleşmiştir. Yöntemin uygulanmasının kolaylığı nedeniyle gerçek zamanlı yatırım uygulamalarında kullanımı açısından cesaret vericidir.

Anahtar Kelimeler: Teknik tahmin, Bulanık model, TSK, FCM, öbekleme, kayar ortalama.

DEDICATION

To my family

My Father & Mother

My wife

My siblings

My son

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LIST OF ABBREVIATIONS

AI	Artificial Intelligence
AR	Autoregressive
ARIMA	Autoregressive Moving Integrated Average
P	Closing Price
EMA	Exponential Moving Average.
FCM	Fuzzy C-Means.
GBP	British Pound
LD	London stock market
MA	Moving Average.
MACD	Moving Average Convergence Divergence.
Matlab	A software for matrix operations, Math Works, Inc., R2014a.
MF	Membership Function.
NN	Neural Networks.
RMSE	Root Mean Square Error.
RSI	Relative Strength Index.
SDMA	Six Days Moving Average
SMPM	Stock Market Prediction Model
TSK	Takagi-Sugeno-Kang.

LIST OF SYMBOLS

y	The target point
x_a	The previous point
$\bar{x}$	Sample mean
x_b	The next point
y_b	Next point value
y_a	Previous point value
$\in$	Set membership to the interval [0, 1].
A(x)	The membership function (MF) for the fuzzy set.
A, B	A fuzzy set in X.
A_1	Corresponding fuzzy set in TSK model
A_2	Corresponding fuzzy set in TSK model
b_0, b_1, b_2	Linear consequent parameters in TSK model
c	Cluster number.
z^*_i	Constant output value for each rule in zero-order Sugeno fuzzy model .
E	The prediction error
f	Function
x^*_k	Forecast value.
m	Fuzzification power of FCM.
n	The number of data points in x i.e. the time period.
N	Size of time series observations
P_k	Closing price observation at time $_k$

$P_{S,k+1}$	$SDMA_k$ next closing price.
v_i	The centre of cluster.
$v_{i,j}$	The degree to which element x_i belongs to cluster.
X	A collection of objects denoted normally by x.
x	Input variable
z	A crisp function in the consequence for the fuzzy set.
σ	Standard deviation.
α	Linear regression coefficient vector.
$SMA(U,n)$	Average of up closed days.
$SMA(D,n)$	Average of down closed days.
$E_{RMS,NT}$	Estimation error for training.
$E_{RMS,NV}$	Estimation error for verification.
X_T	Matrix of training inputs and outputs.
X_V	Matrix of verification inputs and outputs.

Chapter 1

INTRODUCTION

1.1 Time Series Data Set

A time series is a collection of observations collected consecutive in time, such as a particulate pollution measurements and temperature information. Most information in economic science and finance are time series data sets. Time series models in science of economics have mostly fixed intervals between the observations, such as days, weeks, months, etc., [1]. A time series is a set of N observations with observation periods T in historical ordering

$$Y = \{y_1, y_2, ..., y_k, ..., y_N\},$$

where k points the time of observations $t = kT$.

Many forecasting methods cannot use a time series if the statistical characteristic (mean, autocorrelation, and variance) changes over the time. A time series dataset with an almost constant mean and variance is called *stationary*. Transforming a non-stationary data set to difference data or removing its slope from the data set converts it to a stationary data set [2].

1.2 Financial time series estimation

Estimation of the future values for a time series is an essential task for business and decisions on financial investments. There are many analysis strategies for the estimation of the future value of a time series, such as Six day moving average (SDMA), Relative Strength Index (RSI) and 26 days Moving Average Convergence

Divergence (MACD) techniques, which are applied in this thesis. Other than these three techniques there are commonly used methods based on Artificial Neural Nets (ANN), Auto Regressive Integrated Moving Average (ARIMA), and Fuzzy Logic Modelling of time series data set. ARIMA discovers the dynamic behaviour of the system from the data set, and estimates the future effect of input error by the dynamic behaviour of the system. That means, if the estimated value diverges from the actual value, the future value is calculated according to the auto regressive moving average effect of the prediction error and the dynamic behaviour of the system. Stock market expectation is a very significant financial matter that has concerned researchers' consideration for many decades. Approaches of stock market prediction may involve assumptions of some relationships between the stock return and several variables of the observations that build the market data [3]. The columns of data set may include some variables such as, interest rates, exchange rates, growth rates, client value, financial gain statements and dividend yields. *Fundamental analysis* focuses on the overall economic indices and the success of the industry groups related to a business. Fundamental analysis is a precise active approach to estimate economic situations, but not essentially actual market prices. Financial time series analysis aims to discover the patterns of movements, unexpected changes on a non-stationary transform of time series data set.

The *technical approach* relies on the idea that the price depends on the psychological state of a mass ('the crowd') in action. The approach tries to connect the future value to the psychological condition of people, negatively such as alarm, fear, and doubt, or positively sureness, extreme assurance, and greed [4]. Stock price prediction has always an important matter for numerous financiers and skill analysts. Even so, looking for the most effective time to buy or sell has continued a troublesome task as a result of there are a factors which it guidance a stock price.

Technical analysis, as one of method for analysis, assumes that the market activities, related news, and psychological observations may affect stock prices, and they must be considered in forecasting future prices and market trends. Among the techniques, that builds this class of analysis, MA, and recent artificial intelligence techniques are gained more attention. The MA has smoothing effect on a dataset to explain the dataset trends. It is called moving because for every time step the latest period is added and the ancient period is dropped. The MA is based on historical prices.

Clustering is the method of distributing dataset into groups. As a result, the items in one group are similar as probable, and items in unlike group are different as probable. The measurements that used in clustering contain distance, connectivity, and intensity. A commonly algorithm used fuzzy clustering is the Fuzzy C-Mean (FCM) algorithm [5].

Fuzzy Takagi-Sugeno-Kang (TSK) model provides advanced forecasting methods based on extracted fuzzy rules from a given data set. The rules provide knowledge about behaviour of the modelled system. The fuzzy rules may be extracted directly from the data set, as well as, from the results of some reliable methods that summarize the characteristics of the data set [6] [7] [8] [9] [10].

The basic indication of fuzzy logic is to let not only the values *0* and *1*, matching to *false* and *true* but the whole interval [0, 1] as marks of truth.

The aim of this thesis is to predict two-day-ahead future price of London Stock Market by TSK fuzzy model, using MACD, SDMA and RSI. The prediction performance of the proposed TSK based method is compared against the performance of linear regression based prediction models using these three indices. The coefficients of the linear expressions are obtained from the first half of five years stock prices (2008-2012), and verified using the last half of the time series observations. The proposed method attempts to improve the prediction accuracy of the three indices using TSK model. This proposed model also provides hints of conditions to expect a rising or falling stock markets. The structure of a fuzzy rule of a first order TSK model with two input features x_1 and x_2, is:

$$\text{Rule: If } x_1 \text{ is}_r\ A_1\text{, and } x_2 \text{ is}_r\ A_2\text{, then } y^* = b_0 + b_1 x_1 + b_2 x_2 \ , \qquad (1)$$

where x_1 and x_2 are scalar linguistic variables, A_1 and A_2 are linguistic terms described by fuzzy sets and b_0, b_1, b_2 are coefficients of a liner expression that forms the consequent part of the rule. The inference for an input x is obtained by aggregating the y^* values of the rules using membership values of x in the terms of the premise [11], [12].

The performance of the model is measured by comparing the inferred y' values to the actual y values using Root Mean Square Error of the predicted values over the test period.

1.3 Summary of the Proposed Method

The important motivation for trying to predict the stock market prices is financial advantage. The skill to find a mathematical model that can reliably predict the direction of the future stock prices would make the owner of the model actual wealthy. Thus, academics, investors and investment professionals are always trying to find a stock market model that would profit them with higher returns.

This thesis proposes application of Takagi–Sugeno–Kang (TSK) method to merge three indices of closing prices, SDMA, MACD, RSI, and the current closing prices into the next day's predicted closing price. The TSK fuzzy model uses these indices as input variables to discover the rules that explain the dependence of the change of closing price on the change of indices by using the training data set. The consequents of the TSK rule base is a linear combination of the input variables. Once the rule base is extracted, TSK inference method predicts the next day's closing price from the current closing prices and available indices with a smaller prediction error than prediction of closing prices using each index alone.

1.4 Organization of the Thesis

In this Thesis Chapter 1 introduces Stock markets, source of data set, technical prediction methods, TSK Fuzzy Modelling and explains the problem. Chapter 2 describes the data set and theoretical of methods that used in this work which are as follows: the SDMA, the MACD and, the RSI technical analysis; the FCM and the TSK fuzzy model. Chapter 3 consists of the methodology for prediction model that proposed in this work. The performance of prediction of the proposed method is discussed in Chapter 4. Chapter 5 concludes the advantages of the proposed method, and comments on the future work.

Chapter 2

DATA SET AND EXISTSING METHODS OF PREDICTION

2.1 The Data Sets and Pre-processing

This thesis proposes a time series prediction method, and demonstrates the proposed method on five years closing prices (P) of LD stock market from 1-01-2008 to 1-01-2013. The dataset is achieved from *www.finance.yahoo.com* website [13].

The original dataset taken from *www.finance.yahoo.com* had missing days because the stock markets were closed during the weekends and holidays. The missing values of the time series data has negative effect in prediction of future prices because the skipped time steps in the data set corresponds to economic consumption during that period. There are mainly three methods to fill the missing records for the off days: the simplest method is filling either the next day or the previous day prices into the missing day's closing price. The third method fills the missing price of k^{th} day, P_k, by linear interpolation using both the previous and next day prices, P_{k-1} and P_{k+1}. Table 2.1 shows an example for this method; the value for the fourth day does not exist.

Table 2. 1 Data with missing value

day k	value P_k
0	0
1	0.3
2	0.5
3	0.1
4	Missing
5	0.7

Previous-value method fills P_4 by P_3. Next-value method fills P_4 by P_5. Linear interpolation method calculates P_k using the available previous and the next prices P_{k-n} and P_{k+m} by [14]

$$P_k = P_{k-n} + (P_{k+m} - P_{k-n})n/(m+n) \tag{2.1}$$

i.e., in Table 2.1, interpolation gives $P_4 = P_3 + (P_5 - P_3)(4-3)/(5-3)$.

2.2 Closing Prices of London Stock Market

The arbitrary movement of the closing prices for London stock market is shown in the Figure 2.1. The closing prices in London stock market in the beginning of 2008 are about 6000 GBP (Figure 2.1). It doesn't change well totally after 4 years from the impact of the financial crisis. The average of the closing prices over 5 years is 5365 GBP.

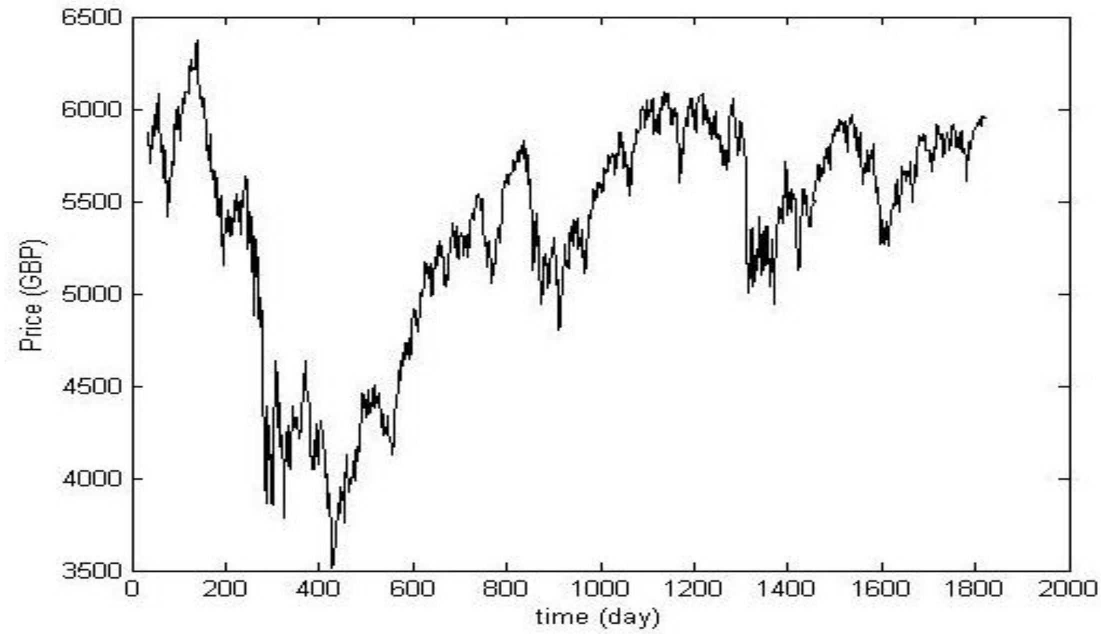


Figure 2.1: Closing price of London Stock Market

2.3 Moving Average Index of a Market

Moving Average (*MA*) smoothens out the noise on a data series, making it easy and reliable to compare to the latest value. It is named moving because for each time step the latest period is added into average, and the oldest period is dropped from the average value. In a stock market, it integrates the closing prices of the moving average period, and therefore, in technical analysis it has a lagging effect as an indicator. Simple Moving Average over *n* day is defined by

$$MA_k = \frac{1}{n}\sum_{i=0}^{n-1} P_{k-i} \tag{2.2}$$

In this thesis, 6-day Moving Average ($SDMA_k$) is used to indicate the short term closing prices with reduced effect of volatility based noise [15].

In general, $SDMA_k$ together with P_k may provide a prediction of next closing price P_{k+1} through a linear expression

$$P_{S,k+1} = \alpha_{S,1}\ P_k + \alpha_{S,2}\ SDMA_k \qquad (2.3)$$

The error in k^{th} prediction by *SDMA* is $E_{S,k} = P_k - P_{S,k}$. Over n observations the RMS error of the estimation by *SDMA* is calculated by

$$E_{RMS,S} = \sqrt{\frac{1}{n}\sum_{i=0}^{n-1}(P_{k-i} - P_{S,k-i})^2} \qquad (2.4)$$

MA method used to display when an investor can sell or buy in a particular financial market because it provides a measure of the momentum. Also, the investor can use the *MA* method to determine when the prices are likely to change its direction. Depending on historical commercialism ranges, the support and confrontation points are recognized where the price of a stock market reversed its upward or downward trend, within the past price, by this way the buy or sell decision are decides. The most common applications of *MA* are to identify the trend direction and to determine support and resistance levels.

2.4 Moving Average Convergence Divergence (MACD)

The difference between the long-term exponential moving average and the short-term exponential moving average is called Moving Average Convergence Divergence (*MACD*). It is a technical analysis indicator created by Gerald Appel in the late 1970s [16]. Thomas Aspray added a bar chart to the *MACD* in 1986, as a means to expect *MACD* crossovers, an indicator of necessary moves in the underlying security. It is used to spot changes within the strength, direction, momentum, and period of a trend during a stock's price. *MACD* is calculated in MatLab simply by the *macd*() function. *MACD* is calculated over the prices *Pk*, k=1...n using exponential moving average (*EMA*) [17]

$$EMA_{12,k} = 0.985 EMA_{12,k-1} + 0.015\ P_k \qquad (2.5)$$

and 26-day *EMA*

$$EMA_{26,k} = 0.926\, EMA_{26,k-1} + 0.074\, P_k \quad (2.6)$$

The difference of EMA_{12} and EMA_{26},

$$MACD_k = EMA_{12,k} - EMA_{26,k} \quad (2.7)$$

gives the short-term trend of the price. A 9-day *EMA* indicates the buying or selling day of the stock if the decision shall be based only on *MACD*.

The theory of *MACD* is that when two *MA*s cross, a major change of trend in the stock's price is more probably to occur. Like all indicators, the *MA* crossover has large uncertainty to consider it as an absolute truth in trading stocks [18].

In general, $MACD_k$ together with P_k may provide a prediction of next closing price P_{k+1} through a linear expression [19]

$$P_{M,k+1} = \alpha_{M,1}\ P_k + \alpha_{M,2}\, MACD_k \quad (2.8)$$

2.5 Relative Strength Index (RSI)

The relative strength index (RSI), one of the common technical indicators, RSI was first introduced by Welles Wilder [20]. A list of step-by-step commands to calculate and interpret the RSI is provided in Wilder's book, New Concepts in Technical Trading Systems. RSI is calculated in MatLab simply by the *RSindex()* function. It is based on the simple moving averages of the up steps ($SMA(U, n)$) and down steps ($SMA(D,n)$) in total n=14 day period.

$$RS = SMA(U,n)/SMA(D,n) \quad (2.9)$$

Consequently, *RS* is simply the ratio of average up closed days to the average of down closes days.

RSI is normalised value of *RS* to move between 0 and 100 by using the formula given below:

$$RSI_k = 100 - \frac{100}{(1 + RS_k)} \qquad (2.10)$$

RSI is a technical momentum indicator that compares the magnitude of recent gains to recent losses to detect overbought and oversold conditions of an asset. RSI may create false buy or sell signals. RSI is a valuable balance to other technical analysis methods.

Next closing price P_{k+1} is estimated using RSI_k and P_k through a linear expression

$$P_{R,k+1} = \alpha_{R,1} \; P_k + \alpha_{R,2} \; RSI_k \qquad (2.11)$$

2.6 Fuzzy System Modelling using an Observed Data Set

The input and output relation of a system can be obtained by many approaches. Zadeh's Fuzzy number and extension principle provides a solid ground to develop fuzzy modelling methods by a number of fuzzy rules that consists of two main parts: a premise and a consequent [21]. Several methods were proposed to transfer the experts' ideas or the characteristic relations in a set of observed inputs and consequent outputs into the fuzzy rules. Zadehs Singleton Method (SM), Kosko's Standard Additive Method (SAM), Mamdani's Center of Gravity Method (CoG), and Takagi-Sugeno's method (TS) are widely used and well known rule construction, representation, and inference methods [22].

2.6.1 Fuzzy Sets and Fuzzy Logic

In early seventies Zadeh introduced the fundamentals of fuzzy set and fuzzy logic [22]. Fuzzy logic provides several modelling methods with properties of a universal approximate, which means it can imprecise any continuous function in an interval to any degree of precision using the required number of fuzzy rules [24].

The idea behind Fuzzy logic is to use the whole interval [0, 1] as a measure of truth. In the 1920, J. Lukasiewicz introduced multivalued logic calculus, but its application was restricted until the introduction of computer technology in the late 1950 [21].

Fuzzy set extends the membership predicate "∈" to a membership value in the interval [0, 1]. This implies that a collection can contain points with a certain degree of membership. This degree of membership is often considered in several ways. Fuzzy set theory allows us to consider the uncertainty data attributes.

Let X is a set of objects denoted by x, a fuzzy set A in X is a set of well-ordered pairs:

$$A=\{(x, A(x) \mid x \in X, A(x) \in (0, 1) \}$$

where $A(x)$ is the membership function (MF) for the fuzzy set. The MF $A(x)$ maps each element of X to a membership value $A(x) \in (0, 1)$.

2.6.2 Fuzzy c-Means in Extracting Input-Output Relation

Bezdek's Fuzzy c-Means clustering method provides an easy approach to extract input-output relations from large observation data sets. Data clustering is the process of partitioning a data set into a number of classes or clusters. The aim of clustering is to have similar elements in the same class and dissimilar items in different classes. There are methods to cluster data according to distance, intensity, or property [25].

In fuzzy clustering, a data vector belongs to all clusters with some membership values in (0, 1). A vector belongs to one of the clusters dominantly if the membership value for that cluster is considerably higher than all other clusters. For the rule extraction purpose, FCM clustering algorithm is applied on input-output data vectors xy_k using a distance metrics $d(v_i, xy_k)$ which stands for the distance from xy_k to v_j,. Let the set of data contains N vectors. The iterative algorithm returns c cluster centres $\{v_1, v_2, ... v_c\}$ in a matrix form

$V=[v_1\ v_2\ ...\ v_c]^T$,

a partition matrix,

$U= [u_{j,k}]$, where $u_{j,k}$ is in the interval [0,1], for $j= 1,\dots,c$ and $k=1,\dots,N$.

where $u_{j,k}$ tells the degree to which element xy_k belongs to cluster of v_j. FCM aims to minimize the objective function for a data set $XY=[\ xy_1\ \ xy_2\ ...\ xy_k\ ...,xy_N]^T$:

$$J_m(U,V;XY) = \sum_{k=1}^{N} \sum_{j=1}^{c} u_{j,k}{}^{m}.d(xy_k, v_j\ ); \quad (2.12)$$

by calculating

$$u_{j,k} = \frac{1}{\sum_{i=1}^{c} \left(\frac{d(v_j, xy_k)}{d(v_i, xy_k)} \right)^{2/(m-1)}} \quad (2.13)$$

and

$$v_j = \frac{\sum_{k=1}^{N} u_{j,k}{}^{m} xy_k}{\sum_{k=1}^{N} u_{j,k}{}^{m}} \quad (2.14)$$

iteratively after each other.

The fuzzification constant m determines the level of cluster fuzziness. Any point xy has a set of coefficients giving the degree of being within the i^{th} cluster with the center v_i. With fuzzy c-means, the centre of a cluster is the fuzzy mean of all points, weighted by their degree of membership values in that cluster as expressed by (2.14)

The degree of belonging, $u_{j,k}$ depends inversely on the distance from xy to the cluster centre v_j and the fuzzification power m that controls how much weight is given to the closest centre [26].

2.6.3 TS and TSK Models

A *fuzzy model* is a mathematical model that uses fuzzy sets to describe the input output relationships in a data set. The models are based on fuzzy rules and inference methods. The fuzzy rules represent the relationship between the variables through linguistic terms.

The TS fuzzy model was proposed by Takagi and Sugeno as a novel method to express the input-output relation of a system by TS fuzzy rules that contains linear expression to compute the output as the consequent part of the rule. Takagi, Sugeno and Kang developed this approach using FCM to extract rules from a set of input-output vectors [26].

For two-input plus an output variable observations (x, z), where input x has two components (x_1, x_2), a fuzzy rule in a Sugeno fuzzy model has the following structure:

If (x_1 is_r A_1) and (x_2 is_r A_2) then $z = f(x_1, x_2)$,

where A_1 and A_2 are fuzzy sets in the antecedent, and $z = f(x_1, x_2)$ is an arithmetic expression in the consequence. $z = f(x_1, x_2)$ is a polynomial in the input variables x_1 and x_2. It can be any function as long as it approximates the input-output of the observations in a fuzzy region specified by the antecedent of the rule. When $f(x_1, x_2)$ is a first order polynomial, the resultant fuzzy inference system is named a first-order Sugeno fuzzy model [27].

For multiple fuzzy rules R_i , i= 1…c.,

R_i: if x_1 is $A_{i,1}$ and x_2 is $A_{i,2}$ then $z^*_I = f_i(x_1, x_2)$.

The model infers to z^*, the predicted value of z, by

$$z^*(x)= \sum_{i=1}^{c} \beta\left(A_{i,1}(x_1), A_{i,2}(x_2) \right) f_i(x_1,x_2) \Big/ \sum_{i=1}^{c} \beta\left(A_{i,1}(x_1), A_{i,2}(x_2) \right) \tag{2.15}$$

where $\beta(A_{i,1}(x_1), A_{i,2}(x_2))$ is the degree of fulfilment of input vector $x=(x_1, x_2)$ by the rule R_i. Zadeh's fuzzy singleton rule is formed from the Sugeno fuzzy model using zero-order z^*_i, a constant output value for each rule.

TS model originally proposed that experts shall set the premise part of the rules and the liner expressions in the consequents are then calculated using the available set of observations. TSK model introduced that the premise can be extracted by a clustering method that relates the inputs to the outputs. For each cluster obtained by FCM, the fuzzy sets of the premise of a fuzzy rule is obtained by processing the projections of the membership values of the observations, using convex points method that finds the envelope covering all projected membership values for a component of the inputs. The convex points are furthermore approximated by a simple membership function such as a triangular, trapezoidal, or Gaussian shape by determining the parameters of one of these shapes [11].

Chapter 3

PROPOSED PROCESS OF FORECASTING

This chapter presents the proposed forecasting method to build a Stock Market Prediction Model (SMPM), depending on TSK to predict the stock prices using well known forecasting methods in the input arguments. Closing prices of London Stock Market from 21/08/2009 to 2012, total 1200 days, were used as the time series data to verify the proposed method. A block diagram of all processes of proposed SMPM including its test processes is shown in Figure 3.1.where $E_{RMS,NT}$ = Estimation error for training, $E_{RMS,NV}$= Estimation error for verification.

3.1 Data Pre-processing

Data pre-processing is performed on the time series data in order to bridge the gap of the missing dates using the interpolation method. The comparison of the prediction error with and without missing data is carried on one-day and two-day ahead predictions using linear regression of the last two days to verify the effect of this process. After completing the missing values, the time series is divided in two parts: From almost 1800 days in time series data, the first 900 is used for training purpose in calculating the parameters of the prediction methods, and the last 900 is used for verification purpose to determine the performance of the prediction method.

3.2 Calculation of Stock Market Indices

Three technical indexes, SDMA, MADC, and RSI, are calculated on the pre-processed time series data using the procedures described in Chapter 2. From these indices, MADC and RSI show the trend of the prices, and SDMA gives an accurate short term delayed price status. The indices individually were used in predicting the one-day and two-day-ahead prices by linear regression to determine the level of information content in each of these indices by the following procedure.

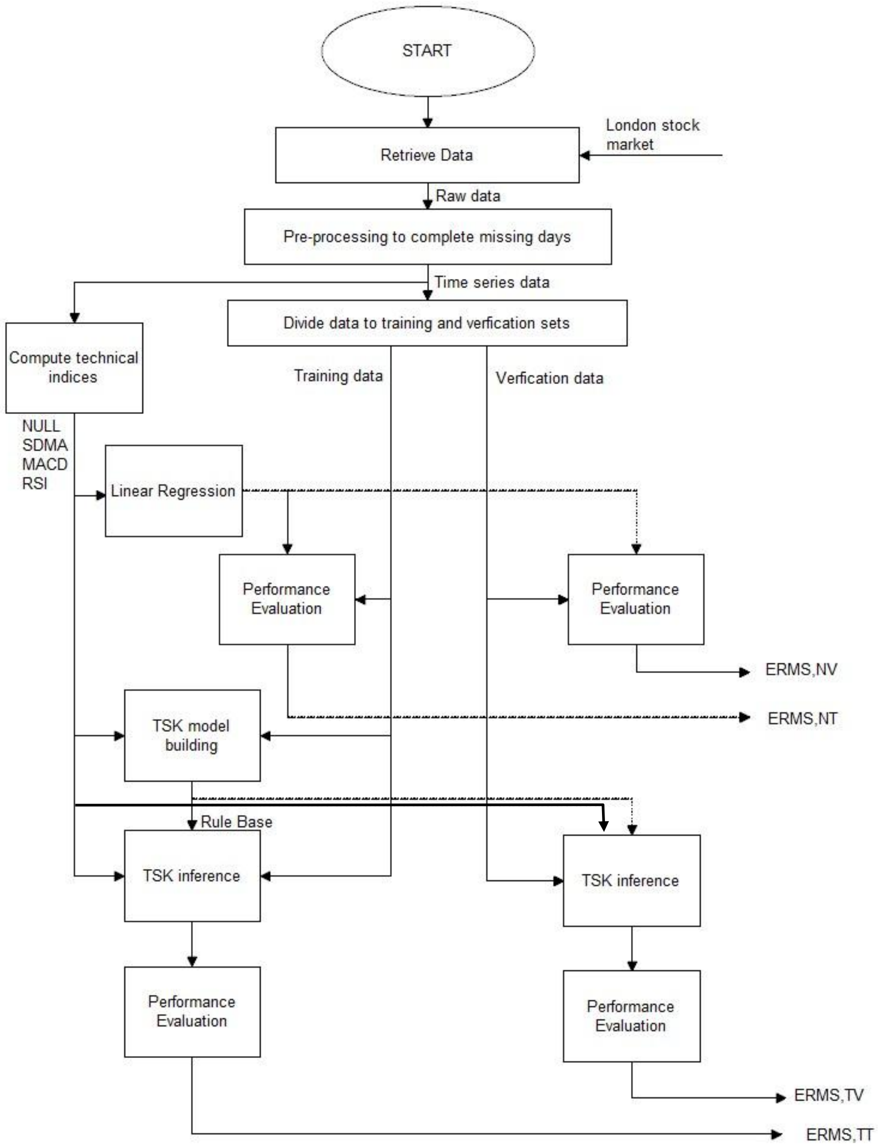


Figure 3.1 The structure of the proposed process including its testing.

3.3 Regression without Technical Indices

Regression without Technical Indices (Null Regression) is necessary to compare the prediction performance of an index in predicting the a-day-ahead future price $P_{N,k+a}$

with and without that index. A-day-ahead price regression is carried to determine the coefficients of the linear expression

$$P_{N,k+a} = \alpha_{N,1}\ P_k + \alpha_{N,2}\ P_{k-1} \tag{3.1}$$

The coefficients $\alpha_N = (\alpha_{N,1}, \alpha_{N,2})$ are obtained by forming matrices of training inputs and outputs

$$X_T = \begin{bmatrix} P_k & P_{k-1} \\ \dots & \dots \\ P_{N+k} & P_{N+k-1} \end{bmatrix};\ Y_T = \begin{bmatrix} P_{k+a} \\ \dots \\ P_{N+k+a} \end{bmatrix} \tag{3.2}$$

where a=1 provides 1-day ahead, and a=2 provides 2-day-ahead coefficients. Using X and Y the expression is written in the matrix form

$$Y = X\ \alpha_N , \tag{3.3}$$

and consequently α_N is calculated by

$$\alpha_N = (X^T X)^{-1} X^T\ Y . \tag{3.4}$$

Once α_N is determined, the RMS error for training and verification are

$$Y_{NT} = X_T\ \alpha_N , \text{ and } \quad Y_{NV} = X_V\ \alpha_N , \tag{3.5}$$

The estimation errors for training and verification are

$$E_{RMS,NT} = \sqrt{\frac{1}{N}(Y_{NT} - Y_T)^T (Y_{NT} - Y_T)} \tag{3.6}$$

$$E_{RMS,NV} = \sqrt{\frac{1}{N}(Y_{NV} - Y_V)^T (Y_{NV} - Y_V)} \tag{3.7}$$

3.4 SDMA index regression

SDMA is an indicator of the short term price with an approximate time lag of three days. A prediction of a-day ahead price by linear regression requires the last two days prices together with SDMA

$$SDMA_k = (P_k + P_{k-1} + \dots + P_{k-5})/6 \tag{3.8}$$

$$P_{S,k+a} = \alpha_{S,1} \; P_k + \alpha_{S,2} \; SDMA_k \tag{3.9}$$

The coefficients vector $\alpha_S = (\alpha_{S,1} + \alpha_{S,2})$ is obtained from regression using the training data set, and the training and verification errors $E_{RMS,ST}$ and $E_{RMS,SV}$ are computed by similar calculations given for the Null Regression.

3.5 MACD index regression

MACD is easily computed by MatLab function macd(), which calculates it as explained in Chapter 2. Once the $MACD_k$ value of the day is available, a linear expression predicts the future price

$$P_{M,k+a} = \alpha_{M,1} \; P_k + \alpha_{M,2} \; MACD_k \tag{3.10}$$

Similar to Null and SDMA cases, the parameters $\alpha_M = (\alpha_{M,1} \;\; \alpha_{M,2})$ are obtained by regression using training data, and the errors $E_{RMS,MT}$ and $E_{RMS,MV}$ are computed by similar calculations given for the Null Regression.

3.6 RSI index regression

RSI index needs counting the loss and gain days, as well as computing the total loss of loss days and the total gain of the gain days over the last 14 days period. The function RSindex() in MatLab calculates the index RSI_k as described in Chapter 2.

The RSI index is a score of trend, rather than the price value. A linear expression with RSI

$$P_{R,k+a} = \alpha_{R,1} \; P_k + \alpha_{R,2} \, RSI_k \tag{3.11}$$

gives the a-day-ahead price prediction after calculating the coefficients

$$\alpha_R = (\alpha_{R,1} \; \alpha_{R,2})$$

by regression using training data set.

The errors $E_{RMS,RT}$ and $E_{RMS,RV}$ are computed in similar way as described for Null Regression.

3.7 Linear Prediction of all Three Indices

The expression

$$P_{A,k+a} = \alpha_{A,1} P_k + \alpha_{A,2} SDMA_k + \alpha_{A,3} MACD_k + \alpha_{A,4} RSI_k \quad (3.12)$$

calculates the a-day-ahead future price from all three indices. The parameter vector

$$\alpha_A = (\alpha_{A,1} \ \ \alpha_{A,2} \ \ \alpha_{A,3} \ \ \alpha_{A,4}) \quad (3.13)$$

is obtained using training data set by linear regression method, while the future price is calculated by

$$P_{A,k+a} = (P_k \ \ SDMA_k \ \ MACD_k \ \ RSI_k)\, \alpha_A \quad (3.14)$$

The errors $E_{RMS,AT}$ and $E_{RMS,AV}$ are computed in similar way as described for Null Regression.

3.8 TSK modelling of future prices

The TSK fuzzy modelling has two main sections: the training section to build a model with sufficient fuzzy TS rules that describes the input-output relations in the data, and the inference section that infers the value of output for a given input vector. This thesis propose to use the stock market indices SDMA, MACD, RSI and the price movement in the last two days as the input vector, and the price movement from the present day to the future day two indexes are chosen through stepwise. The null regression arguments P_k, and P_{k-1} are also included to these selected indexes in form of price movements $P_k - P_{k-1}$ and $P_{k-1} - P_{k-2}$ to predict the a-day-ahead price difference $P_{k+a} – P_k$. An input vector of TSK prediction model consists of five features:

$$x_k = (P_k – P_{k-1} \ \ P_k – P_{k-2} \ SDMA_k \ MACD_k \ RSI_k) \quad (3.15)$$

3.8.1 Structural Parameters of the TSK Model

A TSK model is constructed for a number of fuzzy rules. The number of rules nr depends on the number of clusters c in FCM. Furthermore, the fuzzification power m of FCM plays an important role in clustering the input-output observation vectors by determining the extent of fuzziness of the clusters.

3.8.2 Obtaining the Fuzzy Sets of the Rule

Next phase in TSK modelling is curve-fitting on the membership value of each training observation after projecting the point on the plane of an input feature vs. membership value. At this phase, TSK modelling requires the choice for one of the possible membership functions such as triangular, trapezoidal, or Gaussian. This thesis prefers Gaussian MF for two major reasons. Gaussian MF is well defined and non-zero over the whole discourse of universe of the input feature, and it is a simple function that can be easily fit on the projected FCM membership values for each cluster. The Gaussian function fits on the projected membership values by

$$\sigma_{i,j} = \sum_{k=1}^{N} \frac{1}{N} \sqrt{-\frac{(x_{k,j} - v_{i,j})^2}{2\log(u_{i,k})}} \quad (3.16)$$

where $x_{k,j} \in R$ is the j^{th} feature of the k^{th} observation among N training observations; the $v_{i,j} \in R$ is the j^{th} feature of the i^{th} cluster center; $u_{i,k}$ is the FCM membership of k^{th} observation in i^{th} cluster. The computed $\sigma_{i,j}$ defines the Gaussian MF of the fuzzy set $A_{i,j}$, corresponding to the i^{th} rule, j^{th} input feature i on the plane of membership value vs. j^{th} feature of input vector, N is total number of observations.

3.9 Computing the Coefficients of Consequent Expressions

The linear output expression for the i^{th} rule consists of one constant and nx coefficients.

$$z^*_i = f_i(x_k) = b_{i,0} + b_{i,1}\, x_{k,1} + b_{i,2}\, x_{k,2} + \ldots + b_{i,nx}\, x_{k,nx}\ . \tag{3.17}$$

The constant and the coefficients can be obtained from the observations by forming the homogeneous input matrix Xi, output vector Z_i, and coefficient matrix B_i for the i^{th} rule using the i^{th} cluster membership values $u_{i,k}$ of the observation (x_k,z_k)

$$X_i=\begin{bmatrix} u_{i,1} & u_{i,1}x_1 \\ u_{i,2} & u_{i,2}x_2 \\ \ldots & \ldots \\ u_{i,nt} & u_{i,ny}x_k \end{bmatrix};\ Z_i=\begin{bmatrix} u_{i,1}z_1 \\ u_{i,2}z_2 \\ \ldots \\ u_{i,nt}z_{nt} \end{bmatrix};\text{ and } B_i=\begin{bmatrix} b_{i,0} \\ b_{i,1} \\ \ldots \\ b_{i,nx} \end{bmatrix}, \tag{3.18}$$

so that the observations are written

$$\mathrm{Z}=X\, B_i \tag{3.19}$$

Accordingly, the least squares error solution for the coefficients are obtained by

$$B_i = (X_i^{\mathrm{T}} X_i)^{-1} X_i^{\mathrm{T}}\, Z_i \quad i=1 \ldots \mathrm{c}\ . \tag{3.20}$$

3.10 Predicted Output by Inference of input vector

The inference of the TS model calculates the *degree of fulfilment* β_i of each rule i=1 … c for the input vector $\mathrm{x}=(\mathrm{x}_1, \mathrm{x}_2, \ldots \mathrm{x}_{nx})$ to be used for prediction.

$$\beta_i(x) = A_{i,1}(\mathrm{x}_1) \wedge A_{i,2}(\mathrm{x}_2) \wedge \ldots \wedge A_{i,2}(\mathrm{x}_{nx}) \tag{3.21}$$

The degree of fulfilments of the rules provides the predicted value of output z as a weighted average of the predictions of each rule.

$$z^*(x)=\frac{\sum_{i=1}^{c} \beta_i(x)\, z_i^*(x)}{\sum_{i=1}^{c} \beta_i(x)} \tag{3.22}$$

3.11 Evaluation of the Prediction Performance

The prediction performance of the model is evaluated by the RMS error of the verification data set, which is obtained using the last half of the time series data by

$$E_{\mathrm{RMS,FV}} = \sqrt{\frac{1}{nz}(Z^*_{FV} - Z_{\mathrm{FV}})^{\mathrm{T}}(Z^*_{FV} - Z_{\mathrm{FV}})}\ . \tag{3.23}$$

Where *nz* is number of verification vectors which are employed to get predictions Z_{FV}^{*}

3.12 Selection and Determination of Significance of Features

The input variables, which are called features of fuzzy model, are of prime importance for the success of the model to have sufficiently low RMS errors of predictions. The feature selection method used in this study is based on overall performance of the fuzzy model by testing one-missing and one-added features.

The proposed procedure in this thesis is based on testing all combinations of the input variables to determine the significance of the variables, as well as to simplify the model by eliminating the non-contributing inputs from the TSK model.

Chapter 4

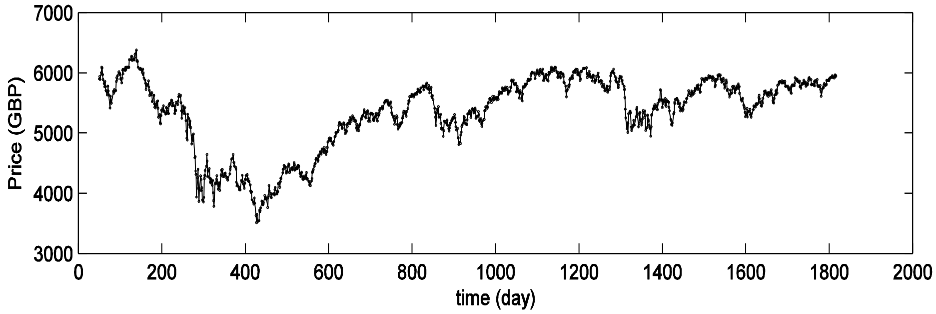
THE RESULTS OF FORECASTING USING THE PROPOSED MODEL

The proposed SMDM is applied on London Stock Market data set which is collected from finance section of *www.yahoo.com* web site and listed in Appendix. This Chapter focuses on i) performances of prediction by only stock market indices SDMA, MACD, and RSI, ii) performance improvement of pre-processing, iii) performance of fuzzy model compared to the stock market indices, iv) feature selection and feature significance determination, and finally v) the graphical representation of the model with the best RMS error.

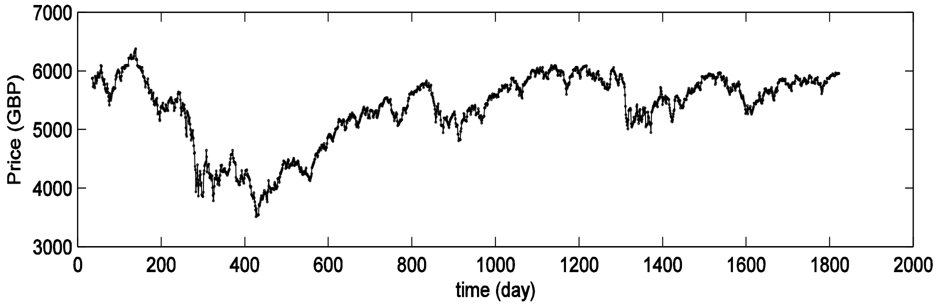
The training and verification time series data are over 900 points, which makes the point wise graphical representation impossible because of the large uncertainty in the data set. This thesis overcomes it by plotting a sample of training and test graphs instead of the full range plots. The sampled plots contain 100 days from 950 days training and 950 days verification data. Trading in a typical stock market through a bank takes almost one day, and therefore all predictions were carried over two-day-ahead prices instead of the usual one day ahead forecasting. Figure 4.1 a, b, c, d show the raw and pre-processed data sets, in complete range, and in the sampled period from the day-600 to the day-700. Table 4.1 (a) displays the parameters for linear prediction as a result of regression with training data set. The 5-year time series data has closing price properties (minimum=3512, maximum=6479, average=5365) GBP.

4.1 Forecasting by only Technical Indices SDMA, MACD, RSI

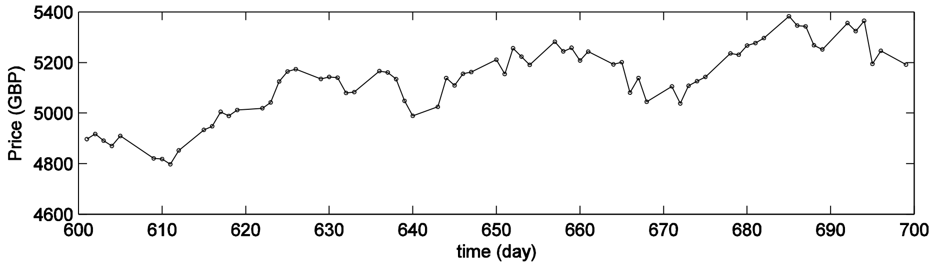
The errors of two-day-ahead forecasted prices using the indices SDMA, MACD, and RSI are shown in Table 4.1.



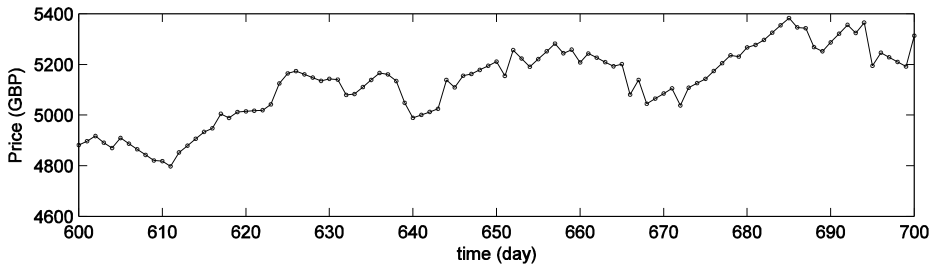
a)



b)



c)



d)

Figure 4.1 London Stock Market prices from 2-1-2008 to 29-12-2012
(a) raw data without completing the missing dates, (b) pre-processed data after completing the missing dates by interpolation. (c) Sample of raw data and from day-600 to day-700 (d) Sample of completed data from day-600 to day-700.

Table 4.1 RMS Errors of Linear Estimations using Technical Indices for raw data

	Indices	Regression Coefficients	RMSE training	RMSE verification
	None (by last two prices)	(0.8769 0.1224)	115.855	87.7402
	SDMA + last day price	(0.8727 0.1266)	115.676	**87.1570**
	MACD + last day price	(0.9993 -0.0075)	116.313	87.4836
	RSI + last day price	(1.0001 -0.0734)	116.301	87.4435
	All indices together+last day	(0.7746 0.2195 0.0143 0.4981)	**115.307**	87.5284

Table 4.2 RMS Errors using Technical Indices for pre-processed data

#	Indices	Regression Coefficients	RMSE training	RMSE verification
1	None (by last two prices)	(1.0710 -0.0715)	91.4520	**68.0317**
2	SDMA + last day price	(0.9615 0.0380)	91.5129	68.1659
3	MACD + last day price	(0.9995 -0.0045)	91.5625	68.1096
4	RSI + last day price	(0.9987 0.0715)	91.5391	68.2451
5	All indices together+last day	(0.8953 0.1014 -0.0102 0.2655)	**91.3384**	68.6755

4.2 Effect of the Missing Days on Prediction Performance

Table 4.1 and Table 4.2 displays an apparent benefit of the pre-processing. Accordingly, from this point on the computational efforts are focused on pre-processed data set. The prediction performance of linear model with all three indices after interpolating the missing days is improved from

$$\eta = \frac{5365 - 87.5284}{5365} = 98.37\ \% \qquad (4.1)$$

to

$$\eta = \frac{5365 - 68.6755}{5365} = 98.72\ \% \qquad (4.2)$$

with a percentage of 0.35%, which corresponds to 27.3% reduction in RMS error.

Table 4.2 also points that the last two day prices are as important as the technical indices in predicting the future prices. The over-fitting of model-5 to the crisis-period notch (days 200 to 600) in the training data set may be the reason of the considerably high verification RMS error.

Figure 4.2 and Figure 4.3 (b), (c) displays the calculated values of technical indices SDMA, MADC, and RSI along a sample of closing prices. SDMA (plot-a) has a smooth curve with a small amount of lag compared to the closing prices. MACD and RSI are indicators of trend rather than the price value.

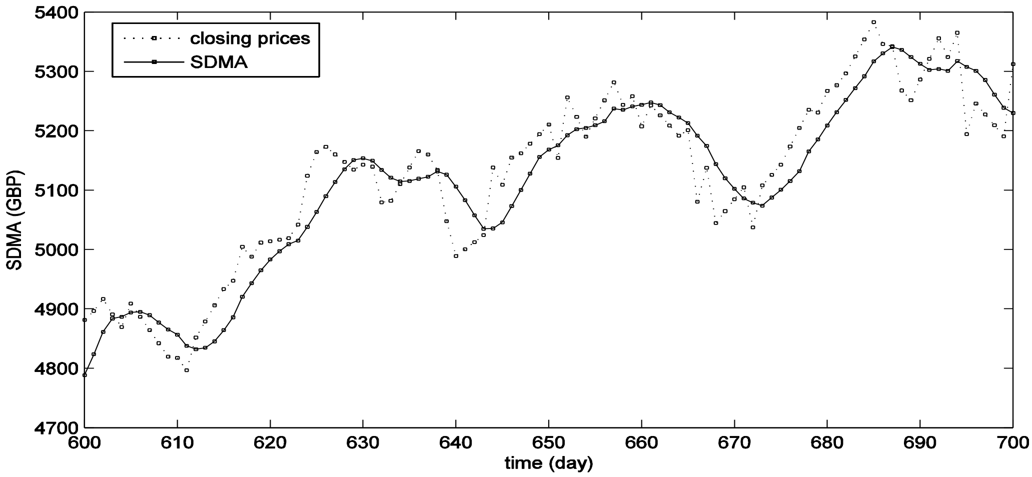
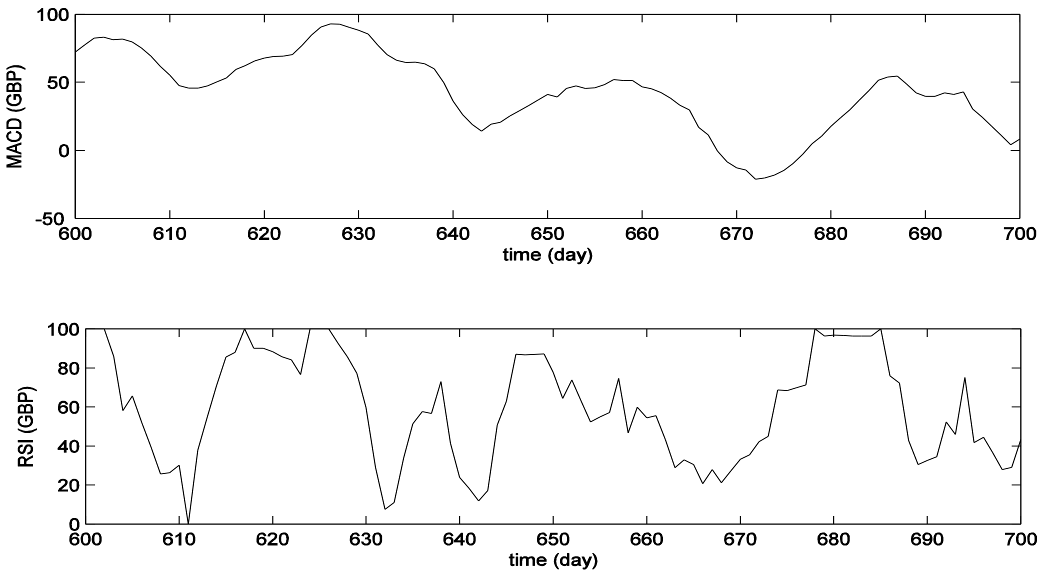


Figure 4.2 Plot of SDMA,



(a)

(b)

Figure 4.3 Plots of (a) MADC, (b) RSI values for a sample of data.

As seen in Figure 4.3, the Relative Strength Index is a percentage that changes between 0 and 100, and in usual practice an equity is preferred to buy when the index exceeds 80%.

4.3 Prediction Performance of TSK Model

This section compares RMS errors of prediction by TSK against the prediction error of technical indices (SDMA, MACD, RSI). TSK model is generated by using five input variables

$$x_k = [P_k – P_{k–1} \;\; P_k – P_{k–2} \;\; SDMA_k \;\; MACD_k \;\; RSI_k\,] \qquad (4.3)$$

and one output variable

$$\mathrm{yk} = P_{k+a} – P_k\,, \qquad (4.4)$$

where a=2 provides two-day-ahead prediction of TSK model.

Clustering a data set with the original ranges of observation variables may result in unexpected distribution of the cluster centres. Normalization of the input-output data vectors scales the vectors into a unit hyper-square, and similarity of the vectors are scored after this scaling. It makes the cluster centres distributed among all variables rather than along the input variable with the largest range, which is in this case the SDMA column.

For best performance, a set of TSK models with number of cluster from 2 to 12 were generated using MatLab Fuzzy Toolbox after correcting the rule-extraction section of the internal MatLab code. These models were built using the training partition of the 5-year time series data set with and without normalization.

RMS errors of TSK models without normalization of the data set are shown in Table 4.3. In the table, the six-rule TSK (c=6) gives the best model that has both training and verification error the lowest among the tested c values. The corresponding Fuzzy Rule Base for c=6 is plotted in Figure 4.4.

Table 4.3 RMSE values by TSK without normalization of features

Method	RMSE training	RMSE verification
All indices together	91.33847	68.67550
TS, c=2	90.888	68.8181
TS, c=3	91.0149	69.191
TS, c=4	90.9532	68.6699
TS, c=5	90.9881	68.0841
TS, c=6	**90.5881**	**68.257**
TS, c=7	90.7222	68.3209
TS, c=8	90.9628	68.6347
TS, c=9	91.0001	68.1932
TS, c=10	91.1865	68.311
TS, c=11	91.1867	68.4423
TS, c=12	91.479	68.5519

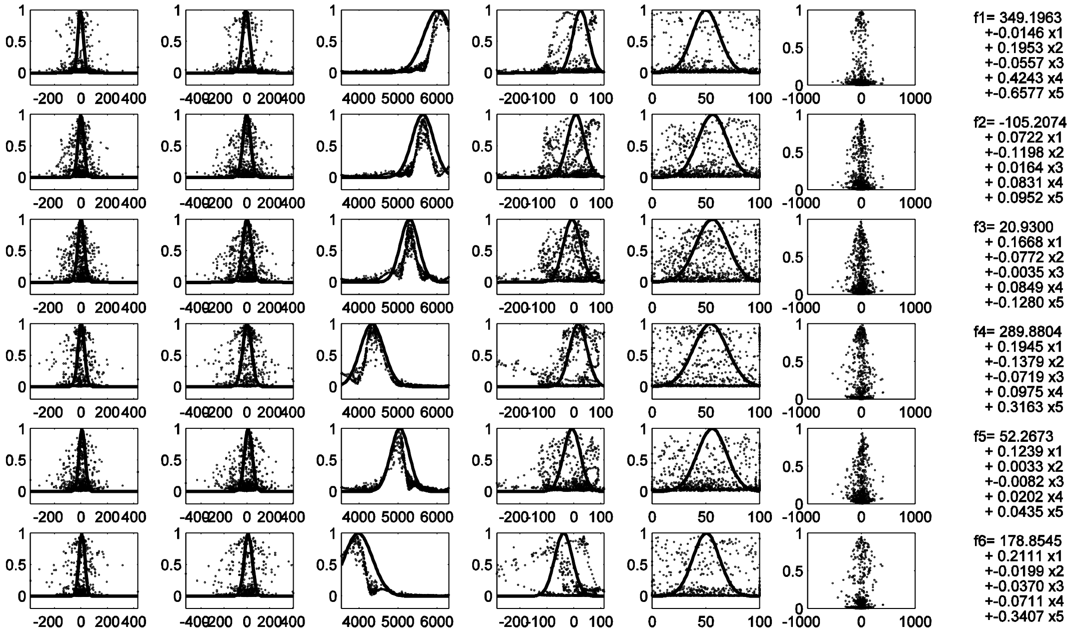


Figure 4.4 The rule base of TSK c=6 without normalization

Table 4. 4 TSK Rule Base Parameters of input fuzzy sets

rule:	1		2		3		4		5		6	
inp	s	c	s	c	s	c	s	c	s	c	s	c
1	20.11	-5.052	22.24	-2.176	24.45	-1.311	24.76	0.1829	22.30	6.399	22.25	5.463
2	30.57	-9.498	33.32	-3.085	37.05	-1.133	38.85	0.6142	34.34	10.42	34.33	9.300
3	363.0	6031.	276.9	5638.	233.4	5296.	300.5	4337.	235.9	5050.	364.1	3981.
4	28.61	24.74	31.38	7.14	31.94	-7.151	32.49	16.13	31.06	-7.474	32.99	-38.62
5	12.77	49.93	13.48	56.061	14.87	55.62	15.79	54.09	13.65	55.74	12.40	50.27

Table 4.5 TSK Rule Base Parameters of output expressions

Coeff: Rule# *i*	$b_{i,0}$	$b_{i,1}$	$b_{i,2}$	$b_{i,3}$	$b_{i,4}$	$b_{i,5}$
1	349.1963	-0.0146	0.1953	-0.0557	0.4243	-0.6577
2	-105.2074	0.0722	-0.1198	0.0164	0.0831	0.0952
3	20.93	0.1668	-0.0772	-0.0035	0.0849	-0.128
4	289.8804	0.1945	-0.1379	-0.0719	0.0975	0.3163
5	52.2673	0.1239	0.0033	-0.0082	0.0202	0.0435
6	178.8545	0.2111	-0.0199	-0.037	-0.0711	-0.3407

The normalization of data set is obtained by scaling and shifting each variable (x) using the minimum (x_{min}) and maximum (x_{max}) of that variable along all training observations.

$$x_n=(x-x_{min}) / (x_{max}-x_{min}). \qquad (4.5)$$

After the modeling, the predicted values are denormalized by

$$y= y_{min} + y_n (y_{max}-y_{min}). \qquad (4.6)$$

The training and verification RMS errors for the TSK Model with normalization are listed in Table 4.6. Considering that higher number of rules have higher risk of over fitting, the model with 6 rules appear to be the lowest error model with balanced training and verification error values.

Table 4.6 RMSE values in TSK with normalization

Method	RMSE train	RMSE verif
All indices together	91.33847	68.67550
TS, c=2	90.5448	**68.3445**
TS, c=3	90.4009	68.3849
TS, c=4	90.192	68.367
TS, c=5	90.1439	68.4931
TS, c=6	**90.1302**	**68.3439**
TS, c=7	90.1759	68.4035
TS, c=8	90.1289	68.4759
TS, c=9	90.0965	68.4681
TS, c=10	90.1063	68.5021
TS, c=11	**90.0579**	68.3967
TS, c=12	90.5206	68.4829

4.4 Significance of each input variable

Table 4.7 contains the training and verification RMS errors obtained for 6-cluster models with only one added feature, and with only one missing features. Tests with one-added-feature (1 2, 1 3, 1 4, 1 5) shows that, none of the technical indices dominate in predicting closing prices better than others, and the previous day price (input-2) is more informative than any individual of the three technical indices. However, the one-missing-feature tests (1345 1245 1235 1234) indicate that when all three technical indices are involved in prediction, the previous day price has disturbing effect on prediction.

Table 4.7 RMSE values in TSK with normalization

Method	RMSE train	RMSE verify
All indices together	91.33847	68.67550
TS, c=6, features 1 2 3 4 5	**90.5881**	**68.2570**
TS, c=6, features 1 2	93.4763	72.9736
TS, c=6, features 1 3	**90.7601**	**68.0068**
TS, c=6, features 1 4	95.9887	71.0612
TS, c=6, features 1 5	96.7880	73.5772
TS, c=6, features 1 2 3	90.5196	68.0668
TS, c=6, features 1 2 4	93.4646	71.3128
TS, c=6, features 1 2 5	93.7798	72.8904
TS, c=6, features 1 3 4	**91.0398**	**68.0678**
TS, c=6, features 1 3 5	90.8098	68.0882
TS, c=6, features 1 4 5	94.8400	71.9937
TS, c=6, features 1 2 3 4	90.7550	68.1503
TS, c=6, features 1 2 3 5	90.5549	68.4081
TS, c=6, features 1 2 4 5	92.9171	71.2290
TS, c=6, features 1 3 4 5	**91.0375**	**68.0801**

In conclusion, models with features (1, 3, 4, 5) and (1, 3, 4) provide highest reduction in verification error while keeping the training error low as well. RSI looks like the least significant among three technical indices, and SDMA is the most significant in reducing the prediction error.

Accordingly, the model with reduced features is obtained as shown in Figure 4.5, using the observation vectors and the RMS error of predicted prices

$$x_k = [P_k - P_{k-1} \;\; SDMA_k \;\; MACD_k \;] \qquad (4.7)$$

and one output variable

$$yk = P_{k+a} - P_k \,, \qquad (4.8)$$

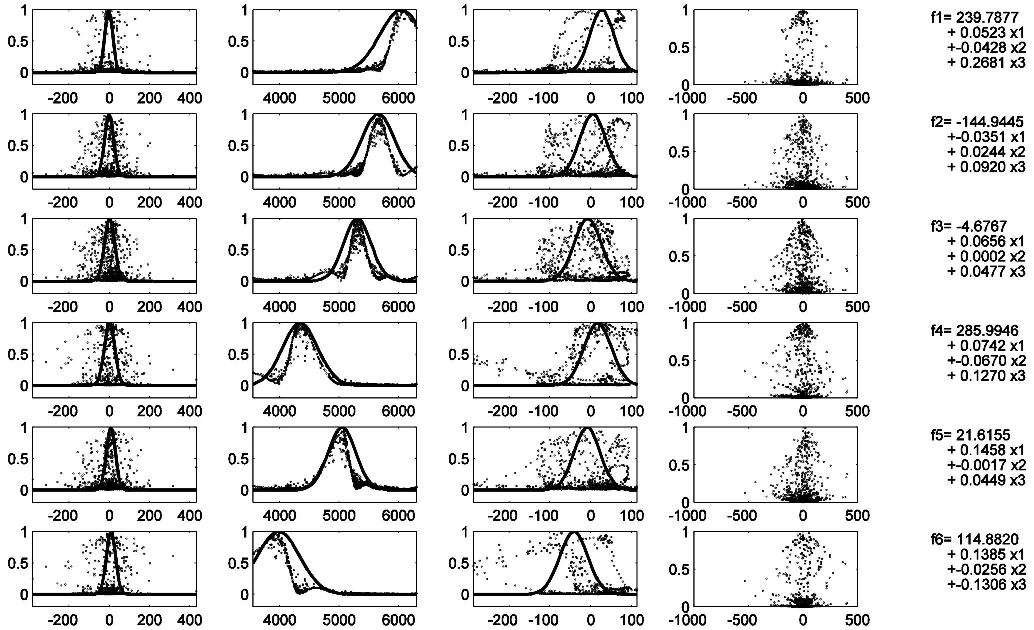


Figure 4.5 The rule base of TSK c=6 without normalization

The rules of the reduced model is shown in Figure 4.5, and the predicted 2-day-ahead prices for the training (from day-600 to day-700) and verification (from day-1500 to day-1600) sample periods are plotted in Figure 4.6 and Figure 4.7.

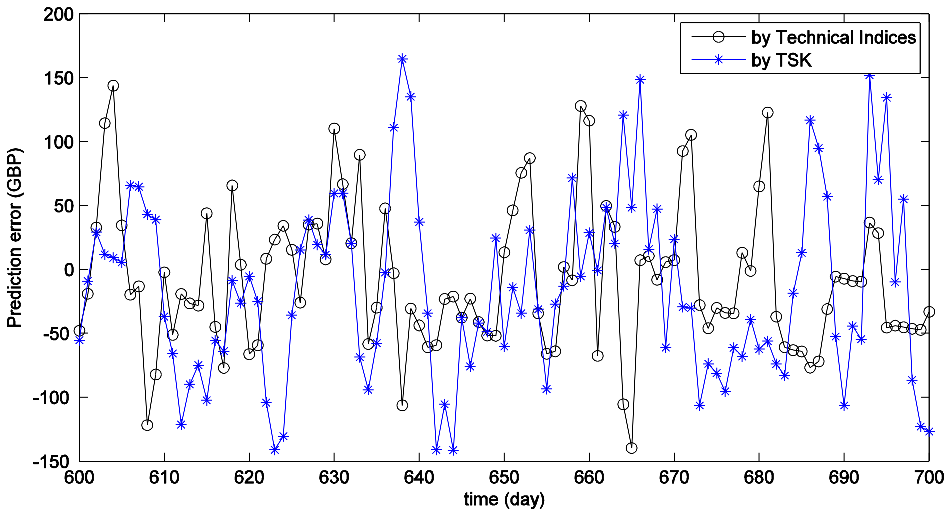


Figure 4.6 The prediction error in a sample of training data for reduced model

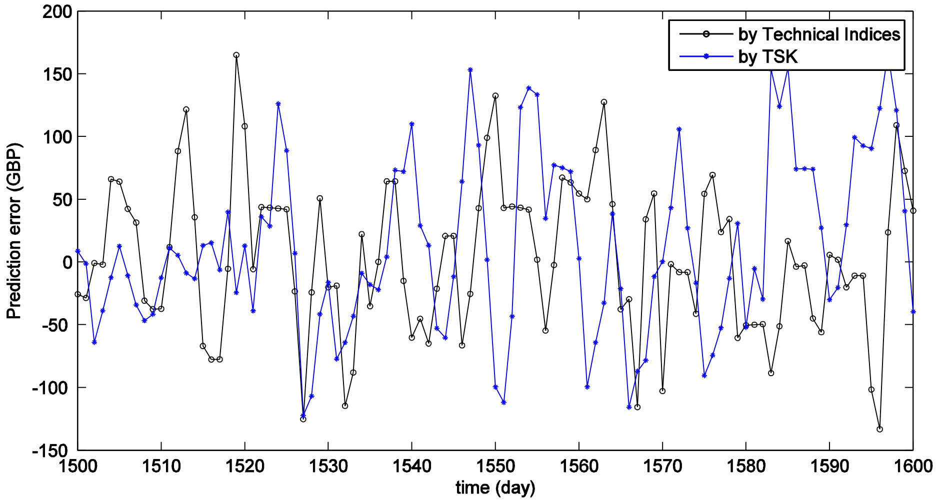


Figure 4.7 The prediction error in a sample of verification data for reduced model

The input membership function parameters, and the output expression coefficients of the rules are shown in Table 4.8 and 4.9.

Table 4.8 TSK Rule Base Parameters of input fuzzy sets of reduced model

rule:	1		2		3		4		5		6	
inp	s	c	s	c	s	c	s	c	s	c	s	c
1	20.00	-5.019	22.49	-3.001	25.52	-0.8958	26.14	-0.9429	22.48	5.988	22.90	6.308
3	357.8	6042.	273.3	5653	230.2	5303.	296.6	4339.	231.3	5044.	359.4	3974.
4	28.84	26.75	31.79	4.585	33.14	-7.777	33.68	16.70	31.66	-9.277	34.35	-41.84

Table 4.9 TSK Rule Base Parameters of output expressions of reduced model

Coeff: Rule# i	$b_{i,0}$	$b_{i,1}$	$b_{i,3}$	$b_{i,4}$
1	0.0523	-0.0428	0.2681	239.7877
2	-0.0351	0.0244	0.092	-144.945
3	0.0656	0.0002	0.0477	-4.6767
4	0.0742	-0.067	0.127	285.9946
5	0.1458	-0.0017	0.0449	21.6155
6	0.1385	-0.0256	-0.1306	114.882

The reduced model has verification RMS error 68.0678, which is 0.169 less than the verification error of linear regression with all technical indices (=68.257). It corresponds approximately to 0.2% reduction of the RMS error. With this reduction of the error, the approximate success of prediction becomes

$$\eta=(5385 - 68.0688)/5385= 98.74\% .$$

The improvement looks like small, but the positive contribution of the fuzzy modeling is apparent in Figure 4.5 as smaller error compared to the linear method.

Chapter 5

CONCLUSION

This study introduces a new method to improve forecasting accuracy of technical indices using a TSK fuzzy model. The proposed method is tested on London stock market time series data set from 2008 January to 2012 December.

According to the results of this research, the pre-processing of the time series data set to complete the closing prices of the missing dates significantly improved the prediction accuracy. The reduction of the RMSE error in verification data is around 25%.

Although the clustering of the normalized data set was expected to distribute the cluster centres along the ranges of all variables, the results indicated there is a very minor difference between the normalized and non-normalized models. In both cases, the lowest error figures were obtained at 6 clusters that gives a 6-rule prediction model.

Future work: This thesis accomplished an implementation of TSK fuzzy model, which can be expanded to include several additional features that may reduce the uncertainty. The techniques introduced here may be applied to other time series data sets. The stock market prices are almost randomly changing time series and prediction of this high uncertainty is beyond the capability of the TSK model. It is expected that markets with less uncertainty may give better performance increase by the TSK model, as well as other methods such as type-2 fuzzy may help to predict the variance of the future error together with the future price.

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APPENDICES

Appendix 1: The thesis code

```
function A = M0504
clc; clear('all'); format('compact'); sP=@sprintf ;
close('all'); CompleteP=1;
% read raw data
DP = LondonSM; nP=size(DP,1); % Raw Prices
% complete missing days rand
DC=completemissing(DP,nP);
if(CompleteP), DR=DC; grts=600;grvs=1500; grr=100;
else DR=DP; grts=416;grvs=1070; grr=69; end
grt=[grts:grts+grr]; grv=[grvs:grvs+grr];
% DP=alldata;
Dve=size(DR,1)-4;  %2 % with missing days
Dts=35; Dte=floor(Dts+(Dve-Dts)*0.5);  Dvs=Dte+1;
nkT=Dte-Dts+1; nkV=Dve-Dvs;
kA=[1:Dve+4]; kAA=[Dts-4:Dve+4];
kT=[Dts:Dte]; kV=[Dvs:Dve]; kTV=[kT,kV];
%=================================================
days=2; normalization=0; % gr options: kTV,kV,kT,grt,grv
gr=grt; gra='  EF RF'; gri=6; fig=1;% PL+ EA MA RA
%gra options PL SA MA RA EA EF
%=================================================
% x = -pi:pi/20:pi;
% plot(x,cos(x),'-ro',x,sin(x),'-.b')
% h = legend('cos_x','sin_x',2);
% set(h,'Interpreter','none')
D=DR(kA,2);
if(strf(gra,' PL')>0),figure(fig); fig=fig+1;
plot(DR(gr,1),D(gr),'-ok','Markersize',2);
```

```
ylabel('Price (GBP)'); xlabel('time (day)');
h=legend('closing prices','SDMA');
set(h,'Interpreter','none');
%set(gcf,'Position', [20  50  800  200] );
pause(0.5);  end
% Calculation of SDMA (six-day-simple-moving-average)
SA(:,1)= filter(ones(1,6)/6,[1],D(kA)); ST=SA(kT); SV=SA(kV);
% Calculation of MACD (Moving Av. Convergence Divergence)
if(strf(gra,' SA')>0),figure(fig); fig=fig+1;
plot(DR(gr,1),D(gr),':sk','Markersize',2); hold on;
plot(DR(gr,1),SA(gr),'-sk','Markersize',2);
ylabel('SDMA (GBP)'); xlabel('time (day)');
set(gcf,'Position', [20  50  800  400] );
pause(0.5);  end
MA= macd(D(kA)); MA(1:25)=0; MT=MA(kT); MV=MA(kV);
% Calculate 14 days RSI index
if(strf(gra,' MA')>0),figure(fig); fig=fig+1;
plot(DR(gr,1),MA(gr),'-k');
ylabel('MACD (GBP)'); xlabel('time (day)');
set(gcf,'Position', [20  50  800  400] );
pause(0.5); figure(fig); end
RA = rsindex(D(kA),6); RT=RA(kT); RV=RA(kV);
if(strf(gra,' RA')>0),figure(fig); fig=fig+1;
plot(DR(gr,1),RA(gr),'-k');
ylabel('RSI (GBP)'); xlabel('time (day)');
set(gcf,'Position', [20  50  800  400] );
pause(0.5); end
[aN,aS,aM,aR,aA,EN,ES,EM,ER] ...
=trainverifNMSR(D,SA,MA,RA,kT,kV,kA,days);
```

```
% TS modeling
disp('TSK RMS errors for training and verification.');
kA2=kA(10:end); kTS=kT-Dts+1; kVS=kV-Dts+1;
PM=[zeros(10,2) ; [D(kA2) MA(kA2)]]*aM;
PS=[zeros(10,2) ; [D(kA2) SA(kA2)]]*aS;
PR=[zeros(10,2) ; [D(kA2) RA(kA2)]]*aR;
PA=[zeros(10,4) ; [D(kA2) SA(kA2) MA(kA2) RA(kA2)]]*aA;
DTV =[D(kTV)-D(kTV-1) D(kTV)-D(kTV-2) ...
SA(kTV) MA(kTV) RA(kTV) D(kTV+days)-D(kTV) ];
DTT =DTV(1:nkT,:); DTV=DTV(nkT+1:nkT+1+nkV,:);
DDX= [ 0 0 0 0 0 0; 1 1 1 1 1 1]; % for w/o normalization
% 3 2 4 1 5
% Normalization of data set
[NTPar]= normalparam([DDX]);
if(normalization),[NTPar]= normalparam([DTT]); end
DNT=normalize(DTT,NTPar); DNV=normalize(DTV,NTPar);
% Feature selection
FS=[1 2 3 4 5];nOut=6; OS=[nOut]; %
%FS=[ 3 2 4 1 5 ];nOut=6; OS=[nOut]; %
disp([ sprintf('completed=%d #days=%d norm=%d',
CompleteP, days,normalization)]);
disp([ 'FS:' sprintf(' %d ',FS)]);
% TS Modeling
disp('  clusters    test     verif');
for nc=2:12
FMT =genfisTS(DNT(:,FS),DNT(:,OS), ...
'sugeno',nc,[2,100,0,0]);
RandStream.setDefaultStream(...
RandStream('mcg16807', 'Seed',0));
```

```
[Vv, Uu] = fcm( [DNT(:,FS) DNT(:,OS)], ...
nc, [2,100,0,0]);
[nR,nV]=size(Vv);
% sort cluster centers ascending output order
[V,Iv] = sortrows(Vv,nV); U=Uu; % Uu=zeros(lenDt,numC);
for ir=1:nR, U(ir,:) = Uu(Iv(ir),:); end
%PLOTC plots a complete Fuzzy Rule-Base
if(nc==gri)&&(strf(gra,' RF'))
figure(fig); fig=fig+1;
plotc( FMT, [DNT(:,FS) DNT(:,OS)], U(:,:))
dispruleparam(FMT);end
% get predicted prices for normalized prices
PNT=evalfis(DNT(:,FS),FMT); %3
% denormalization of prices and getting error
PFT =denormalize(PNT,NTPar(:,nOut))+D(kT);
EFT=PFT-D(kT+days);
ERFT=  sqrt(mean(EFT.^2));
% get error for validation
PNV=evalfis(DNV(:,FS),FMT);
% denormalization of prices and getting error
PFV=denormalize(PNV,NTPar(:,nOut))+D(kV);
EFV=PFV-D(kV+days);
ERFV=  sqrt(mean(EFV.^2));
EF=[EFT;EFV];
if( (strf(gra,' EF')>0) && (gri==nc) ),
figure(fig); fig=fig+1;
plot(DC(gr,1),EF(gr),'-ok'); hold on;
plot(DC(gr,1),PA(gr)-D(gr+days),'-*b'); hold on;
ylabel('Prediction error (GBP)');
```

```
xlabel('time (day)');
legend('by Technical Indices','by TSK');
set(gcf,'Position', [20  50  800  400] );
pause(0.5); end;
disp([ sP('%10d %10.4f %10.4f',nc, ERFT, ERFV) ]);
end
return
%=============================================
%=============================================
function [PN]=normalparam(X)
PN(1,:)=min(X); PN(2,:)= max(X); PN(3,:)=  PN(2,:)-PN(1,:)  ;
return
function [XN]=normalize(X,PN)
n=size(X,1); XN= (X-ones(n,1)*PN(1,:))./(ones(n,1)*PN(3,:)) ;
return
function [X]=denormalize(XN,PN)
n=size(XN,1);X= ones(n,1)*PN(1,:) +
XN.*(ones(n,1)*PN(3,:)) ;
return
function [DC] = completemissing(D,nP)
k=1;
for i=1:nP
while(k<D(i,1)),
DC(k,:)=D(i-1,:)+(D(i,:)-D(i-1,:))...
    *(k-D(i-1,1))/(D(i,1)-D(i-1,1));
k=k+1;end
if(k==D(i,1)), DC(k,:)=D(i,:); k=k+1;end
end
return
```

```
function [aN,aS,aM,aR,aA,EN,ES,EM,ER]= ...
trainverifNMSR(D,SA,MA,RA,kT,kV,kA,days)
kTV=[kT,kV];
sP=@sprintf ;
disp('====  train      verif      coeffs.  ');
% Null Test
aN=[ D(kT) D(kT-1)]\D(kT+days);
EN=[D(kTV) D(kTV-1)]*aN -D(kTV+days);
ENT=[D(kT) D(kT-1)]*aN -D(kT+days);
ERNT= sqrt(mean(ENT.^2));
ENV=[D(kV) D(kV-1)]*aN -D(kV+days);
ERNV= sqrt(mean(ENV.^2));
disp(['RMSE.N=' sP(' %10.5f ', ERNT, ERNV) ...
'  aN=' sP(' %10.4f', aN )  ]);
% Calculation of Coefficients for
%Pmacd(k)=[P(k-1 P(k-2)MACD(k-1)]*[A1 A2 A3] ;
aM=[ D(kT) MA(kT) ]\D(kT+days);
EM=[D(kTV) MA(kTV)]*aM -D(kTV+days);
EMT=[D(kT) MA(kT)]*aM -D(kT+days);
ERMT= sqrt(mean(EMT.^2));
EMV=[D(kV) MA(kV)]*aM -D(kV+days);
ERMV= sqrt(mean(EMV.^2));
disp(['RMSE.M='sP(' %10.5f ', ERMT, ERMV)  ...
'  aM=' sP(' %10.4f', aM )  ]);
% Predicting Prices using SDMA by a linear expression
% Psdma(k) = [P(k-1) P(k-2) SDMA(k-1)][a1 a2 a3];
aS=[ D(kT) SA(kT)]\D(kT+days);
ES=[D(kTV) SA(kTV)]*aS -D(kTV+days);
EST= [D(kT) SA(kT)]*aS -D(kT+days);
ERST= sqrt(mean(EST.^2));
```

```
ESV= [D(kV) SA(kV)]*aS -D(kV+days);
ERSV= sqrt(mean(ESV.^2));
disp(['RMSE.S= ' sP(' %10.5f', ERST, ERSV) ...
'  aS=' sP(' %10.4f', aS )  ]);
% predicting Prices using RSI by a linear expression
% Prsi(k) = [P(k-1) P(k-2) RSI(k-1)][a1 a2 a3];
aR=[ D(kT) RA(kT)]\D(kT+days);
ER=[D(kTV) RA(kTV)]*aR -D(kTV+days);
ERT= [ D(kT) RA(kT)]*aR-D(kT+days);
ERRT= sqrt(mean(ERT.^2));
ERV= [ D(kV) RA(kV)]*aR-D(kV+days);
ERRV= sqrt(mean(ERV.^2));
disp(['RMSE.R=' sP(' %10.5f ', ERRT, ERRV) ...
'  aR=' sP(' %10.4f', aR )  ]);
%  Predicting Prices using ALL by a linear expression
% Pall(k) = a1*P(k-1)+a2*P(k-2)+a3*S(k-1)+a4*M(k-1)+a5*R(k-1);
aA=[ D(kT) SA(kT) MA(kT) RA(kT)]\D(kT+days);
EAT= [ D(kT) SA(kT) MA(kT) RA(kT)]*aA-D(kT+days);
ERAT= sqrt(mean(EAT.^2));
EAV= [ D(kV) SA(kV) MA(kV) RA(kV)]*aA-D(kV+days);
ERAV= sqrt(mean(EAV.^2));
disp(['RMSE.A= ' sP(' %10.5f ', ERAT, ERAV) ...
'  aA=' sP(' %10.4f', aA ) ]);
return
function fismat = genfisTS( ...
X, Y, fistype, nC, fcmoptions)
%GENFIS3 Generates a FIS using FCM clustering
%FIS = GENFIS3(XIN, XOUT,TYPE,CLUSTER_N, FCMOPTIONS)
%  allows you to specify options for the FCM algorithm.
%   For TSK type may be 'sugeno' or 'mamdani'
```

```
if nargin < 4,
disp('X, Y, fistype, nC required.'); end
if nargin < 5, fcmoptions = []; end
mftype = 'gaussmf'; % only option
% Check fistype
fistype = lower(fistype);
if ~isequal(fistype, 'mamdani') ...
&& ~isequal(fistype, 'sugeno')
disp('Unknown fistype specified.'); end
RandStream.setDefaultStream(...
RandStream('mcg16807', 'Seed',0));
[Vv, Uu] = fcm([X Y], nC, fcmoptions);
[nR,nV]=size(Vv); U=Uu;
% sort cluster centers in ascending order of the output column
[V,Iv] = sortrows(Vv,nV); % Uu=zeros(lenDt,numC);
for ir=1:nR, U(ir,:) = Uu(Iv(ir),:); end
% Check Xin, Xout
numX = size(X,2); numY = size(Y,2);
% Initialize a FIS
theStr = sprintf('%s%g%g',fistype,numX,numY);
fismat = newfis(theStr, fistype);
% Loop through and add inputs
for i = 1:1:numX
fismat = addvar(fismat,'input', ...
['in' num2str(i)],minmax(X(:,i)'));
% Loop through and add mf's
for j = 1:1:nC
params = computemfparams(mftype, ...
X(:,i), U(j,:)', V(j,i));
```

```
fismat = addmf(fismat,'input', i, ...
['in' num2str(i) 'cluster' num2str(j)], ...
mftype, params); end;  end
switch lower(fistype)
case 'sugeno'
% Loop through and add outputs
for i=1:1:numY
fismat = addvar(fismat,'output', ...
['out' num2str(i)],minmax(Y(:,i)'));
% Loop through and add mf's
for j = 1:1:nC
%MB correction
%MB params = computemfparams('linear', ...
%MB     [Xin Xout(:,i)] );
params = computemfparams ('linear', ...
[X Y(:,i)],U(j,:)');
fismat = addmf(fismat,'output', i, ...
['out' num2str(i) 'cluster' num2str(j)], ...
'linear', params); end;  end
case 'mamdani'
% Loop through and add outputs
for i = 1:1:numOutp
fismat = addvar(fismat,'output', ...
  ['out' num2str(i)],minmax(Y(:,i)'));
% Loop through and add mf's
for j = 1:1:cluster_n
params = computemfparams (mftype,...
X(:,i), U(j,:)', V(j,numInp+i));
fismat = addmf(fismat,'output', i, ...
```

```
['out' num2str(i) 'cluster' num2str(j)],...
mftype, params); end; end
otherwise
error('unknownfistype', ...
'Unknown fistype specified'); end
% Create rules
ruleList = ones(nC, numX+numY+2);
for i = 2:1:nC, ruleList(i,1:numX+numY) = i; end
fismat = addrule(fismat, ruleList);
% Set the input variable ranges
minX = min(X); maxX = max(X); ranges = [minX ; maxX]';
for i=1:numX, fismat.input(i).range = ranges(i,:); end
% Set the output variable ranges
minY = min(Y); maxY = max(Y); ranges = [minY ; maxY]';
for i=1:numY, fismat.output(i).range = ranges(i,:); end
return
function mfparams = computemfparams(mf,x,m,c)
switch lower(mf)
case 'gaussmf'
sigma = invgaussmf4sigma (x, m, c);
mfparams = [sigma, c];
case 'linear'
[N, dims] = size(x);
%MB correction   xin = [x(:,1:dims-1) ones(N,1)];
%MB correction   xout = x(:, dims);
xin = [x(:,1:dims-1) ones(N,1)].*(m*ones(1,dims));
xout = x(:, dims).*m; b = xin \ xout;
mfparams = b';
otherwise
disp('Unknown type of membership function');
```

```
end
return
function pr=minmax(p)
if iscell(p), [m,n] = size(p); pr = cell(m,1);
for i=1:m, pr{i} = minmax([p{i,:}]); end
elseif isa(p,'double'), pr = [min(p,[],2) max(p,[],2)];
else disp('Argument has illegal type.')
end
return
function  plotc( R, Fd, U)
%PLOTC plots a complete Fuzzy Rule-Base
%  plotc( R,Fd,U)
%   R Fuzzy model struct
%   Fd, Training Data
%   U,  FCM Membership values of Training Data
%   para, Nr of parameters of the mdfn
nR= size(R.rule,2);
nV=size(R.rule(1,1).antecedent,2);
hold off;
NXDiv = 100;  %=nr of divisions on X axis
for ir=1:nR
for iv=1:nV
maxX=R.input(1,iv).range(1,2);
minX=R.input(1,iv).range(1,1);
ivv=(ir-1)*(nV+2)+iv;
subplot(nR,nV+2,ivv);
% plot the data points
x1=Fd(:,iv); y1=U(ir,:);
plot(x1,y1,'ko','MarkerSize',1); hold on;
```

```
% plot the curve
for i=1:NXDiv
x11(i) = minX+(i-1)*(maxX-minX)/(NXDiv-1);
y11(i) = gaussmf(x11(i), ...
R.input(1,iv).mf(1,ir).params(1,:));
end
plot(x11,y11,'k-','LineWidth',2); hold off;
xlim([minX maxX]); ylim([-0.2 1.0]); end
% select the box
subplot(nR,nV+2,ivv+1);
% plot the data points
x1=Fd(:,iv+1); y1=U(ir,:);
plot(x1,y1,'ko','MarkerSize',1); hold on;
subplot(nR,nV+2,ivv+2);
text(0.1, 0.9,sprintf('f%i= %7.4f',ir,...
R.output.mf(1,ir).params(1,iv+1)));
for iv=1:nV,
text(0.2,0.9-iv*0.2,sprintf('+%7.4f x%i',...
R.output.mf(1,ir).params(1,iv),iv));end
set(gcf,'Position', [20  50  1200  650] );
axis off;
end
pause(0.5);
return
function a=strf(b,c);
a=-1; if(strfind(b,c)>0) a=strfind(b,c);end; return
function dispruleparam(R)
sP=@sprintf;
ic= size(R.input(1,1).mf,2); ix= size(R.input,2);
```

```
disp('Rule Base Parameters');
for i=1:ic % for each rule
for j=1:ix
disp([sP('%d %d',i, j) sP('%9.4f ', ...
R.input(1,j).mf(1,i).params(1,:))]);
end
disp(R.output(1,1).mf(1,i).params(1,:));
end
return
```

Appendix 2: The Raw Data Set

Training Data Set		Verification Data Set	
date	closing price	date	closing price
02/01/2008	6416.7	02/07/2010	4838.1
03/01/2008	6479.4	05/07/2010	4823.5
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