Prediction of International Stock Market Movements Using a Statistical Time Series Analysis Method

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Submitted to the
Institute of Graduate Studies and Research
in partial fulfillment of the requirements for the Degree of

Master of Science
in
Computer Engineering

Eastern Mediterranean University
September 2013
Gazimağusa, North Cyprus

Approval of the Institute of Graduate Studies and Research

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ABSTRACT

The thesis has used econometric time series models to model and forecast the development in closing prices of main international stock markets. These are New York, London, Tokyo and Shanghai stock market. The time series data set includes the trading days from 1st January, 2008 to 31st December, 2012 i.e. (5 years).

After pre-processing the data to substitute the missing values using interpolation method and convert all closing prices to USD currency, the first attempt of this thesis employs the Auto Regressive Moving Average (ARMA) framework, which has been used to model a time series data set. It is found that the model can be used to fit the data in the estimation period. The Root Mean Square Error (RMSE) is used to find an estimating order of the parameter in ARMA model i.e. *r, m* proper values.

The forecasting process is constructed based on the ARMA model to forecast the future value for the data indices in the period (2010-2012) in New York, London, Tokyo, and Shanghai stock market. The idea of forecasting in this work is predicting two-days-ahead closing price based on previous two years closing price for each two days. The forecasting is very important in the analysis of economic and industrial time series, and in sailing and buying movement. The money was invested in these stock markets and the results made it clear that the investment in London stock market is the best investment.

Keywords: Time series analysis**,** ARMA, RMSE, Forecastıng and Investment.

ÖZ

Bu tez uluslararası hisse senedi pazarlarında ekonomik zaman serisi modeli kullanarak kapanış fiyatı öngörüsü yapma yöntemini incelemektedir. Yöntem New York, London, Tokyo and Shanghai hisse senedi pazarlarından elde edilen Ocak 2008 ile Aralık 1012 arasındaki 5 yıllık zaman serisi verilerine uygulanmıştır.

Verilerin ön işleme aşamasında eksik değerleri tamamlanmış ve günlük kazanç oranına çevrilerek ARMA modelinde en düşük karekök-ortalama-kare-hatası (RMSE) veren yapısal parametreleri *r* ve *m* belirlenmiştir.

Öngörüş ARMA modeli kullanılarak NewYork, Londra, Tokyo ve Şankay hisse senedi pazarlarında daha ileri tarihlerdeki fiyatları öngörmek üzere kurulmuştur. ARMA model ile 2008 başından 2010 sonuna kadar üç yıl boyunca her gün için daha önceki iki yıllık veri kullanılarak iki gün sonrasının kapanış fiyatı tahmin edilmiştir. Elde edilen tahmine göre sabit miktardaki kapital dört pazardan en iyi getiri beklenene yatırılma yönünde hisse alım ve satımı kararları oluşturulmuştur. Benzeşimsel yatırım etkinliği sonucu dört hisse senedi pazarı arasında yalnızca Londra'da yatırım yapmak, kapitali dört pazarın en iyisine yatırmaktan daha fazla getiri sağlamıştır.

Anahtar Kelimeler: Zaman seriya analizi, ARMA, RMSE, Fiyat tahmini, Yatırım.

ACKNOWLEDGMENT

In the name of God and all trust in God

First of all, I would like to express my gratitude to the Department of Computer Engineering at Eastern Mediterranean University. Especially thanks to my Supervisor Asst. Prof. Dr. Mehmet Bodur, who stated directions of this work without him I could have not completed this work.

I would like to say thanks to all my friends for cooperating with me during my studies. My special thanks go to my family for their prayers and support at each step of my life.

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LIST OF ABBREVIATIONS

ACF	Autocorrelation function
AIC	Akaike Information Criteria
AR	Autoregressive
ARMA	Autoregressive moving average
BIC	Bayesian Information Criterion
CNY	Chinese Yuan
GBP	British Pound
Invp b	Invested capital to buy the shares
Invp s	Return capital by selling the shares
JPY	Japanese Yen
LD	London stock market in code
LSE	London Stock Exchange
M b	The market of buying
M s	The market of selling
MA	Moving Average
MAE	Mean absolute error
Matlab	A software package for matrix operations, Math Works, Inc., R2012a
NY A p	New York actual price
NY p p	New York prediction price
NY p r	New York prediction return
NY	New York stock market in code
NYSE	New York Stock Exchange
PACF	Partial Autocorrelation function

RMSE	Root Mean Square Error
SH	Shanghai stock market in code
Shr b	The amount of shares in buying state
Shr s	The amount of shares in selling state
SSE	Shanghai Stock Exchange
TK	Tokyo stock market in code
TSE	Tokyo Stock Exchange
USD	United States Dollar

LIST OF SYMBOLS

y_b	Next point value
y_a	Previous point value
$\bar{x}$	Sample mean
x_b	The next point
x_a	The previous point
y	The target point
c	Constant
e	The prediction error
$e_{k,h}$	h-day-ahead prediction error
h	Lag of period
$\hat{x}_{k+2}$	Prediction value of two-days-ahead
$\hat{x}_{k+h}$	The forecasting return value
m	Order of the moving average part
n	The number of data points in x i.e. the sample size
p_{t}	Closing price observation at time t
r	Order of the autoregressive part
r_t	Daily return series
x_{k+2}	Actual value of two-days-ahead
x_{k+h}	The actual return value
x_t	Current value of the series
X_t	Time series of data
x_{t-h}	Time series started from t-h

x_{t-k}	Past values of observation
θ	Moving average parameter
μ	Expectation of X_t
Φ	Autoregressive parameter
ε_τ	White noise

Chapter 1

INTRODUCTION

1.1 Time Series Data Set and Prediction

A time series is a set or sequence of observed data arranged in consecutive order and in an equally spaced time intervals such as daily or hourly air temperature. Time series data sets are used in many fields such as finance and economy, engineering, and science.

A time series data is called "*univariate*" if it consists of only values collected from a single scalar observation at regular periodic time intervals, such as, the temperature measurements taken from one thermometer, or the flow rate measurements taken from a point of a stream. An univariate dataset X is typically a sequence values $X=\{x_1, ... x_n\}$ of the same variable x.

If the mean, variance, and autocorrelation of a univariate time series is not changing over the time such data sets are called *stationary*. Many analysis methods apply to only stationary data sets. There are several methods to convert a data set to stationary such as transforming it to difference data, or removing the slope from the data set.

The analysis of time series has got a wide application in areas like population studies, economic forecasting, process control, marketing, biomedical science [1]. Time series analysis uses systematic approaches to extract information and understand the characteristics of a physical system that creates the time series. There are a number of different approaches to deal with time series analysis including dynamic model building and performing correlations [2].

Methods for analysis of stock market consists of mainly two elemental modelling philosophies; Fundamental and Technical approaches. In Fundamental approach, stock market price movements are believed to depend on information about the security, such as the politics, relations to other companies, history and plans, and carried projects, etc. Fundamentalists use numeric information such as earnings, ratios, and management effectiveness to determine future forecasts. In the technical approach, it is believed that all external effects and inner dynamics of the financial object are summarized in the observations of that object. Technicians utilize charts and modelling techniques to identify the dynamics of the object from the trends in price and volume observations. They rely on historical data in order to predict future outcomes and use statistical analysis methods on time series data sets [3].

There are various statistical analysis methods to process a time series data. They can be applied to estimate the future level (expected value), the trend of observations, or the variability of the estimation and observations. As an example, time series regression is used to find out the expected value of time series data, the trend of the data set, and also the confidence level of the expected value. More advanced statistical linear estimation methods such as Auto-Regressive Moving-Average (ARMA) were developed more than 20 years ago, and they are still in use for accurate estimations [4].

The ARMA model is a statistical time series analysis technique based on discrete time dynamic modelling of the observations by using the weighted sum of previous *r* observations to predict the expected next observation, building an autoregressive model. Moreover, the expectation error is considered to represent the external effects to the dynamics of this autoregressive model, and the weighted average of *m* of past error terms is used to drive the model parallel to the observations. The weighted sum of the past observations builds the Auto Regressive model, and, the weighted average of errors is called the Moving Average part of ARMA [5].

Other than the statistical tools there are non-statistical methods to estimate the expected future value of observations. The field of time series analysis and forecasting methods has significantly changed in the last decade due to the influence of new knowledge in non-linear dynamics. Artificial neural networks are new methods changed traditional approaches which usually were suitable for linear models [6].

ARMA model is commonly used as a prediction model [5] [7] [8] [9] [10] [11] [12] [13]. It gives the researchers the opportunity to forecast the future value of time series data set. J., A., M. and A. [7] applied an ARMA model to forecast the hourly average wind speed in Navarre (Spain) and the result has been proven that the ARMA model is work well for forecasting the future, especially in the longer-term forecasting.

1.2 Globalization of the World Stock Market

The globalization of the stock markets is a part of development that has occurred during the recent decades. The following four factors are significantly contributed to this event; i) the advance of technology and increased demand for admission to global markets, ii) the actualization of new banking institutions offering finance casework, iii) trends of liberalization and the decrease of restrictions to adopt ownership, and iv) the movement appears bounded in connection to stock exchanges, allowance and settlements organizations. The globalization increased market efficiency, decreased its accident due to the achievability of diversification, and used arbitrage in an accordant way [14].

Development of internet tools has significant effect on the administration and decision tools for trading in the world's stock markets. The trading decisions are now spread all over the world markets, rather than in local stock exchange markets. In the last 5 years, the amount of investors who used internet applications has been grown rapidly. Also, there is a trend to access to distribute the investment and trading to the global stock markets [15].

1.3 Decision Making for Global Stock Market Investments

This thesis targets to answer the decision making problem for global investments when money can be transferred freely among the major global stock markets. In global perspective, there are a number of stock markets open for investment. The problem of decision making to invest this money to one of the stock markets requires the following steps: (a) a reliable estimate for the two-days-ahead future values of the available stock markets; (b) a decision making algorithm to select the market to invest; (c) the sell and buy operations to be carried in global markets based on the taken decision; (d) the transfer of the investment resources from one global market to another one within one day time.

Most stock traders nowadays depend on Intelligent Trading Systems which help them in predicting prices based on various situations and conditions, thereby helping them in making instantaneous investment decisions [16]. The prediction of the two-days-ahead future value requires time series prediction methods, and based on the literature we have decided to use ARMA method, because ARMA is described as a successful prediction method [7]. In addition, we optimized the orders of ARMA parameters to increase the accuracy of predictions. We collected the market data indices for the New York, London, Tokyo and Shanghai in the period 2008-2012. Considering the effect of the rapid improvements in banking and communication such as the internet banking, and the internet mass media technologies, we assumed that only the previous two years for each two days of this period of time series contain significant behaviour of market actors. Therefore the time series vector is restricted to only the previous two years for each two days when forecasting two-days-ahead market value. For example market value for Jan. 20, 2010 is predicted using time series from Jan. 18, 2008 to Jan. 18, 2010.

In common terms, investment means to deposit capital on financial assets such as bank deposits, bonds and shares for their future benefits. Investment also covers the production of new capital assets, such as education to get certified or to carry out an occupation. In this thesis, investment is used in narrow meaning of buying market shares for their return rate. Accordingly, the decision making algorithm is simply based on return rates of the stock markets. Basically, the highest predicted return determines the preferred market to invest whole capital effective from next day. The one-day period between the sell-of-shares in invested market and buy-of-shares in preferred market is considered as a necessary time lag due to international banking operations to transfer the capital from one country to another one. For the simplicity of the decision making process the variances and the stock trading volume are not considered to be a significant factor in the return rates. Both of them play an important role in theories of technical stock market analysis, as indicated by many researchers [17]. The proposed investment process is described in Chapter 6.

The proposed forecasting and decision making algorithm is tested by moving capital among four major global stock markets. The buying and selling actions are decided based on the predicted two-days-ahead market values applying ARMA model on the time series data set that contains only the observations for the last two years. ARMA model and decision making algorithm are applied on each market locally, and on four international markets globally to compare the effect of local and global investments.

1.4 The Main Steps and Techniques in this Thesis

The main steps and techniques have been reviewed in this thesis as following:

1- Interpolation methodology to pre-process the closing price of time series data set (fill missing values) for (2008-2012) period of four major international stock markets (New York (NY), London (LD), Tokyo (TK), and Shanghai (SH)). See Appendix E.1.

2- Converting all the closing price currency to the same currency; USD currency has been used. See Appendix E.1.

3- Converting all the data (closing price) to the return of closing price to induce the stationary time series data set. See appendix E.2

4- Checking 225 ARMA(r,m) models for each stock market to pick the best order r and m parameters with minimum values of RMSE. See appendix E.2.

5- Fitting the data according to the best ARMA(r,m) model detected in point (4) for each stock market. See appendix E.3.

6- Forecasting two days ahead along 3 years (2010, 2011 and 2012) based on its previous 2 years. See appendix E.3.

7- Investing the capital within 3 years (2010, 2011 and 2012) based on prediction values the initial capital was $100. It is Invested the capital in each stock market separately and also in stock market together at the same time have been covered. See appendix E.4.

1.5 Organization of this Document

This document is organized in the following Chapters: Chapter 1 contains a general introduction to explain the terminology and the nature of the stock market investment decision making problem. Chapter 2 gives historical information on the global stock markets, and describes the structure of the basic time series prediction method, ARMA, which is used in predicting the two-days-ahead return rates of the stock markets. The source and the pre-processing steps applied on the original daily closing price data sets for four global stock markets are described in Chapter 3. This Chapter also contains the daily price plots of the four markets. Chapter 4 is reserved to the determination of the best model parameters for each of the markets based on the minimization of the root-mean-square-error (RMSE) of the return rates. Chapter 5 contains the details of the forecasting process of the future market prices, and the plots of the predicted prices. Chapter 6 describes the investment process and exhibits the results of the investment which initialized only by a 100 dollars capital. Finally, Chapter 7 concludes on the overall algorithm of time series pre-processing, prediction, decision making, and comments on the possible future research topics related to this thesis.

Chapter 2

INTRODUCTION TO STOCK MARKETS AND STATISTICAL METHODS

2.1 Introduction to International Stock Markets

This thesis investigates the feasibility and opportunity of benefiting by investing a capital to global stock markets. The strongest global stock markets available for investment are: i) New York Stock Exchange (NYSE), in New York, United States established in 1792 [18]; ii) London Stock Exchange (LSE) in United Kingdom was founded initially as the Exchange in 1571 [19]; iii) and the Tokyo Stock Exchange (TSE) is a stock market in the middle of Tokyo, Japan, established in 1878 [20]; iv) the Shanghai Stock Exchange (SSE), which is a stock market that is based in Shanghai, China starting in the late 1860 [21].

The daily volume of a stock market is the amount of shares that are traded on any day. The average daily volume of exchange of NYSE, LSE and TS are around \$4 $x10^9$, \$1.05$x10^9$, and \$0.14$x10^9$ respectively. The Shanghai stock Exchange volume is missing because it is not announced in the internet, and not listed in Yahoo financial pages.

According to the daily volume of exchange, the significance of the markets are ordered as NYSE the highest, LSE following it closely, and TS is quite a small market than the first two markets. SSE is the smallest market volume, since its exchange volume is about \$0.01x10^9 [22]. It is expected that a market with larger volume to be less restricted to global investments, and thus the SSE has a question mark to be taken as a market open to global capital. Chapter 3 is dedicated to the time series data of these major international markets.

2.2 Theoretical Background for ARMA

The Auto-Regressive–Moving-Average (ARMA) model for prediction of the future value of a time series data set was proposed by Peter Whittle in 1951 [12], and further improved by George E. P. Box and Gwilym Jenkins in 1971 [13]. ARMA model contains two polynomial parts, one includes the past values of the target variable in an auto regressive structure (AR), and the other one includes the moving average of the prediction error as an input variable (MA). The notation AR(*r*) refers to the autoregressive model of order *r*. It is written:

$$x_t = c + \sum_{i=1}^{r} \phi_i x_{t-i} + \varepsilon_t \tag{2.1}$$

where ϕ_i are weighting parameters for autoregressive model, c is a constant, and the random variable ε_t is white noise.

The notation MA(*m*) refers to the moving average model of order *m*. It is set up by taking the average of sub orders. It is written:

$$x_t = \mu + \varepsilon_t + \sum_{i=1}^{m} \theta_i \varepsilon_{t-i} \tag{2.2}$$

where the $\theta_1 \dots \theta_m$ are the parameters of the model, μ is the expectation of X_t (often assumed to equal 0), and the$\mathcal{E}_t$, ε_{t-i} are again, white noise error terms.

The notation ARMA(*r*,*m*) refers to the model with *r* autoregressive terms and *m* moving-average terms:

$$x_t = \mu + \varepsilon_t + \sum_{i=1}^{r} \phi_i x_{t-i} + \sum_{i=1}^{m} \theta_i \varepsilon_{t-i} \tag{2.3}$$

The combined model, ARMA(*r*,*m*) provides two advantages; the autoregressive part (AR) predicts the next value of the time series by its dynamic model, while the moving average part (MA) predicts the effect of disturbances which appears as error in the auto regressive model.

Time series modelling required that the series is stationary. It is stationary if the statistical properties remain constant over time. ARMA model works well with stationary data [23] [24] [25] .

Chapter 3

THE STOCK MARKET DATA

3.1 The Time Series Data Sets of Markets

In this thesis, the two-day-ahead prediction of the market prices required time series daily closing prices of the four global stock markets; New York, London, Tokyo and Shanghai for the period starting from 1st January, 2008 to 31st December, 2012, for total 5 years. The data is collected from the financial data accessible on *finance.yahoo.com/* [26]. The original data set downloaded from *yahoo* contains missing days because stock markets are not opened every day of the year.

Missing vectors and values are an important problem in time series data sets when they are used for forecasting purposes, because the missing part distorts the features of the time series. The daily stock market data mainly consists of opening, high, low and closing prices, and the total volume of the transactions in the market for that day. The transactions mean an agreement and communication between buyer and a seller to exchange benefit of payment [27]. All the price currencies have been converted to corresponding United States Dollar (USD) currency [28]. Missing vector means no data available for a day, and missing value means that some of the values of a daily record are missing [29]. Mathematically, there are methods to construct missing data vectors within the range of a discrete data set, such as using previous day or next day values to complete the missing days. A commonly used method to fix missing data is method of linear interpolation, i.e. to complete missing values using the weighted average of the previous and next day values.

For example, in Table 3.1, the value of f for the 4^{th} k value is not available.

Table 3.1: Data with missing value

k	$f(k)$
0	0
1	0.8415
2	0.9093
3	?
4	-0.7568
5	-0.9589
6	-0.2794

Previous value method fills $f(3)$ by $f(2)$, which is available in the Table. Similarly next value method fills $f(3)=f(4)$. Interpolation method provides a means of estimating the function at intermediate points, from both previous and next values. In this case, $f(3)=(f(2)+f(4))/2$.

Linear interpolation finds the target y for a value of x using the previous (x_a, y_a) and the next (x_b, y_b) values as given by equation 3.1 [30].

$$y = y_a + (y_b - y_a)\frac{(x - x_a)}{(x_b - x_a)} \tag{3.1}$$

Table 3.2 contains sample of the original (raw) closing prices for New York stock market from 17/12/2012 to 31/12/2012 with missing values at 22/12/2012, 23/12/2012, 25 /12/2012.

Table 3.2: Raw data and date of NY stock market

Date	Closing price
17/12/2012	1430.36
18/12/2012	1446.79
19/12/2012	1435.81
20/12/2012	1443.69
21/12/2012	1430.15
24/12/2012	1426.66
26/12/2012	1419.83
27/12/2012	1418.1
28/12/2012	1402.43

After pre-processing the data, it shows in Table 3.3

Table 3.3: Sample of data after pre-processing

Date	Closing price
17/12/2012	1430.36
18/12/2012	1446.79
19/12/2012	1435.81
20/12/2012	1443.69
21/12/2012	1430.15
22/12/2012	**1428.99**
23/12/2012	**1427.82**
24/12/2012	1426.66
25/12/2012	**1423.25**
26/12/2012	1419.83
27/12/2012	1418.1
28/12/2012	1402.43

The value of mean absolute error (MAE) with the data (closing prices) which has missing values is approximately equal or higher than the value of MAE when the data without missing values. In NY stock market the value of MAE is approximately equal. In LD, TK and SH stock markets the MAE value is decreased by $30.048, $0.4 and $3.2 respectively.

The time series of the market prices have large movements in mean value, indicating that they are non-stationary in nature. They are unsuitable for ARMA method, which theoretically requires stationary time series data to predict the future values. The logarithm of daily rate of change in prices has zero mean in long term. That means, it is stationary and suitable for ARMA model. It is called *return rates*, *return series*, or shortly *returns*. The return series is stationary in nature. Let p_t and p_{t-1} denote the successive closing price observations at time *t*, corresponding transform the price series { p_t } into a daily return series { x_t } using [31]:

$$x_t = \log\left(\frac{p_t}{p_{t-1}}\right) = \log(p_t) - \log(p_{t-1}) \tag{3.2}$$

3.2 Daily Closing Price and Return of Stock Market

In a stock market the market price is a result of transactions (an agreement and communication between buyer and a seller to exchange benefit of payment) who have free access to all related information, and do not pay transaction costs, so that the prices change in time only in reaction to new information such as about the predictable return of the security, or about its riskiness, or because of a change in return of investors' risk preferences. A new piece of information establishes a new price level in the stock market, which tends to be continued until additional information warrants another price change.

A single transaction has no effect on market prices since there are many other investors ready to buy or sell small amounts of the security at value very close to that transactions price [32].

Figure 3.1 shows daily closing price of New York (NY) stock market. The random movement of the prices is clearly visible in the plot, where the prices starts from 1400 dollars at the start of the year 2008, makes a sharp bottom down to 700 dollars in 2009, marking the financial crisis, and recovers slowly in four years back to the 1400 dollars level. The shift of the prices in long period indicates the prices are non-stationary.

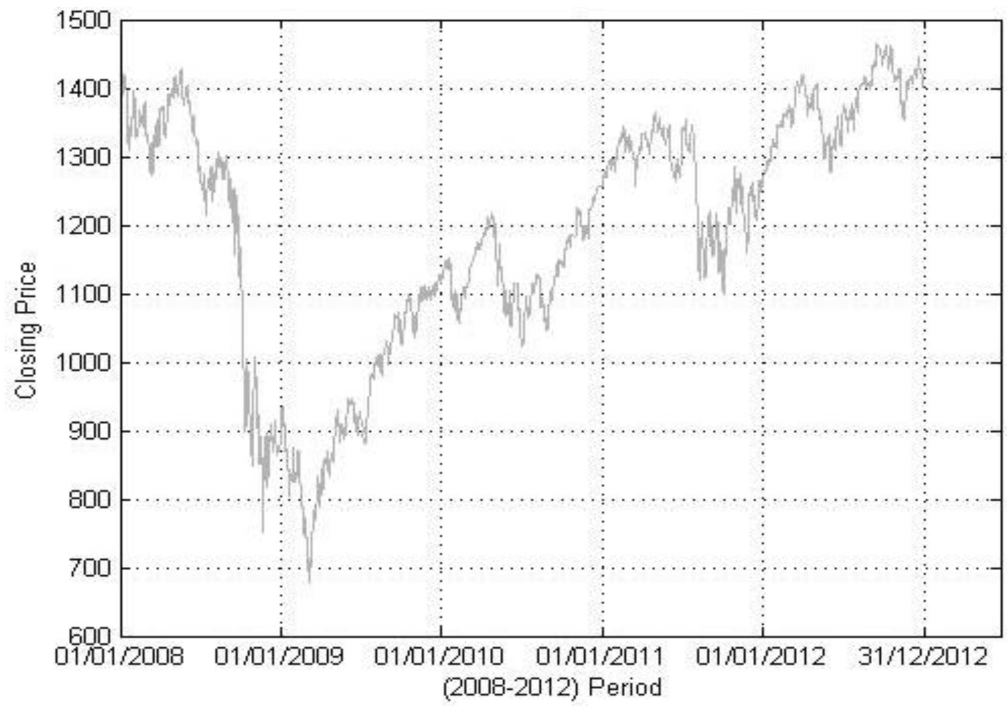


Figure 3.1: Closing price of NY stock market

The return value of the NY stock market for the same period (2008-2012) is shown in Figure 3.2, where the peaks of return occurs especially when the prices starts to raise or fall. The largest positive and negative return values are at the late 2008, during the sharp fall of the prices at the 2008-2009 financial crises, exceeding daily 10 percentages. As noted in literature, the return values have zero mean over the long period, verifying its stationary feature.

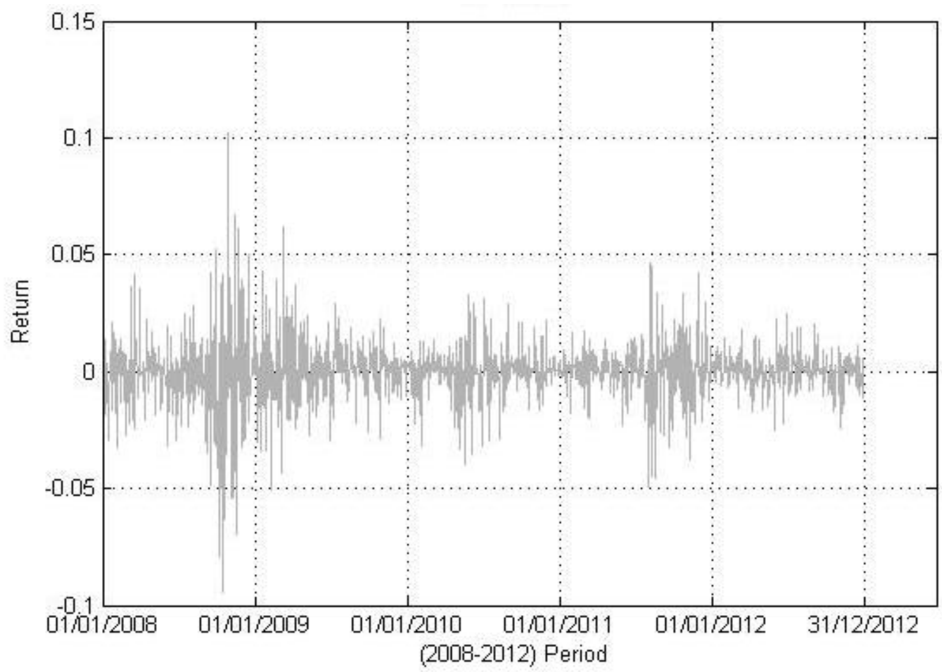


Figure 3.2: Return of NY stock market

London (LD) stock market daily closing prices are plotted in Figure 3.3. The prices initially around 11000 dollars, and resembles the patterns of NY market. However it does not recover fully after 4 years from the effects of the financial crisis. The similarities of the overall price patterns between NY and LD stock markets indicate that they have large number of common actors, and prove that these two markets are global in nature.

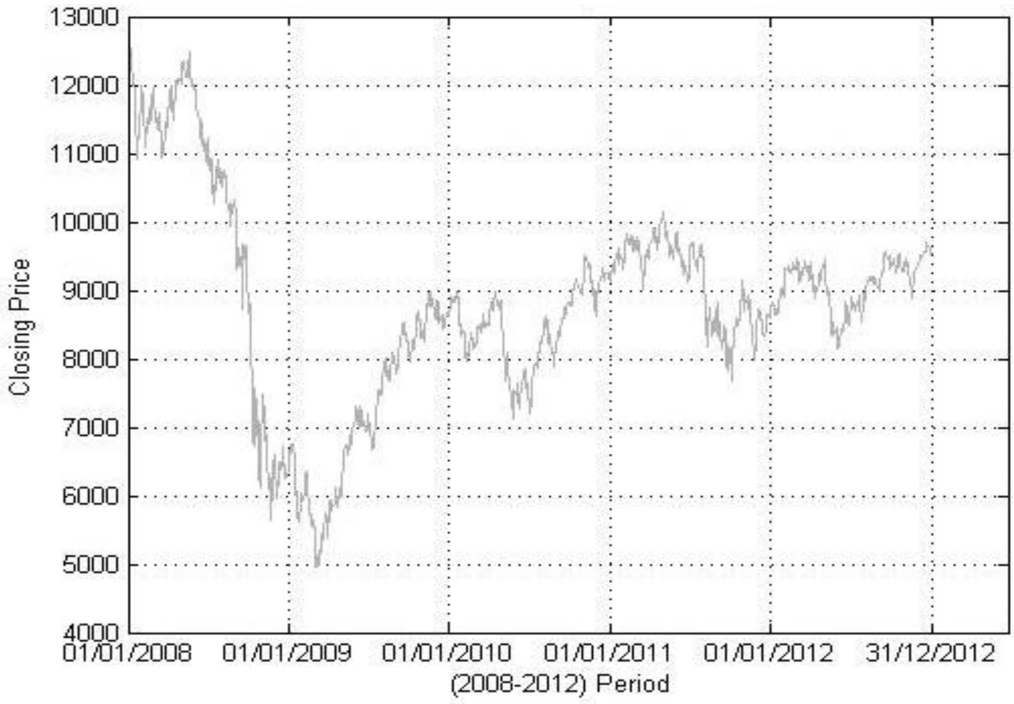


Figure 3.3: Closing price of LD stock market

The return value of the LD stock market for (2008-2012) is shown in Figure 3.4. It has very similar general pattern, even the peak positive and negative values are almost equal to each other, about daily 10 percentage.

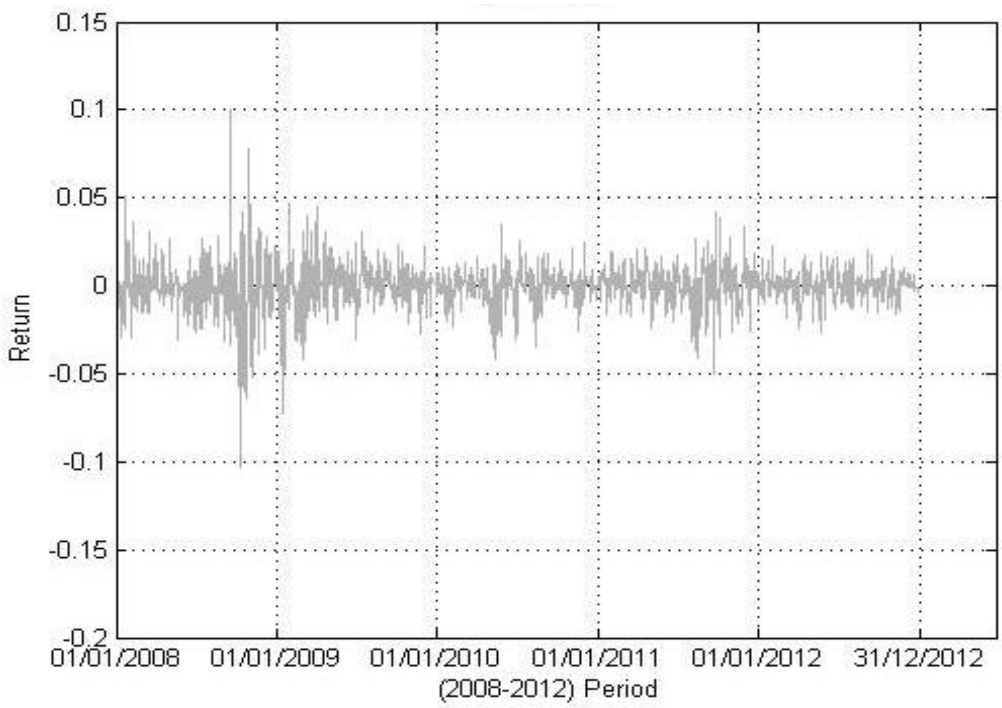


Figure 3.4: Return of LD stock market

For the period (2008-2012), the closing prices and corresponding return values of the Tokyo (TK) stock market is shown in Figure 3.5 and Figure 3.6. Again, similar general pattern is observed compared to both NY and LD markets. The return plot contains significant peaks that do not exist in NY and LD, which indicates the market has both global connections, and strong local actors as well.

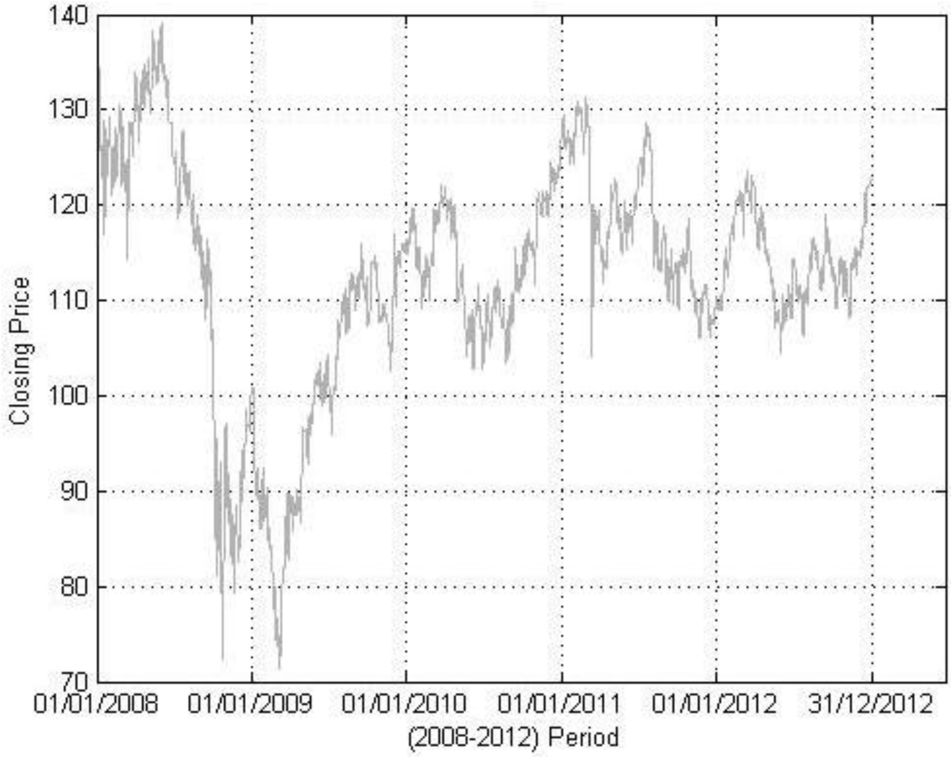


Figure 3.5: Closing price of TK stock market

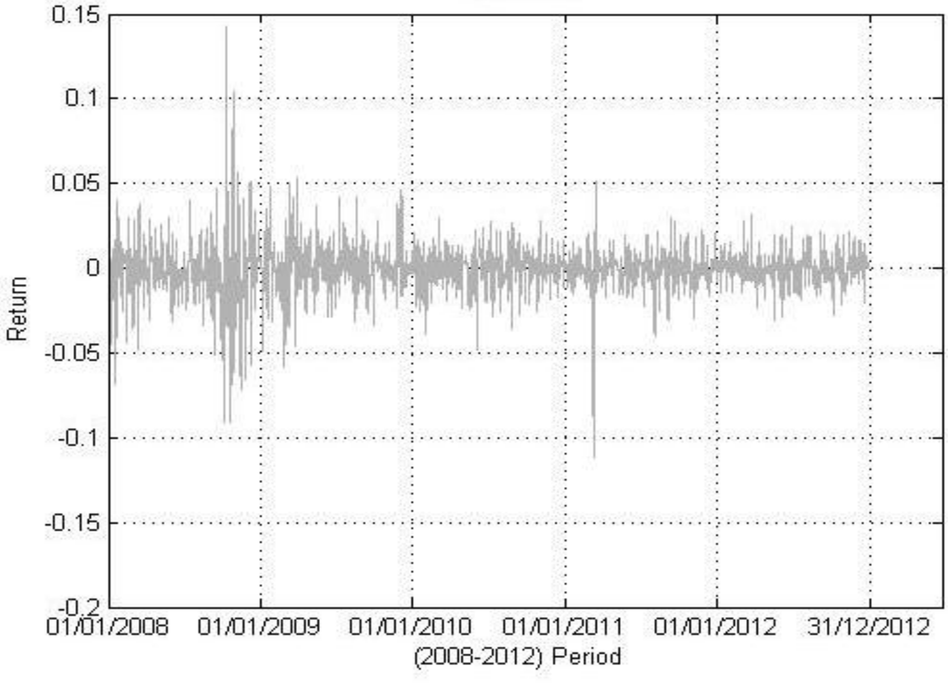


Figure 3.6: Return of TK stock market

The Shanghai (SH) stock market prices plot (Figure 3.7) shows less effect of the 2008-2009 financial crisis, and the peaks in the return plot of Shanghai, which are 2, 4, 6, and 8 percentage (Figure 3.8), are at different days than NY, LD, and TK market peaks. The general pattern of SH return differs from other three stock markets, meaning the market has less global connections, and strong local actors.

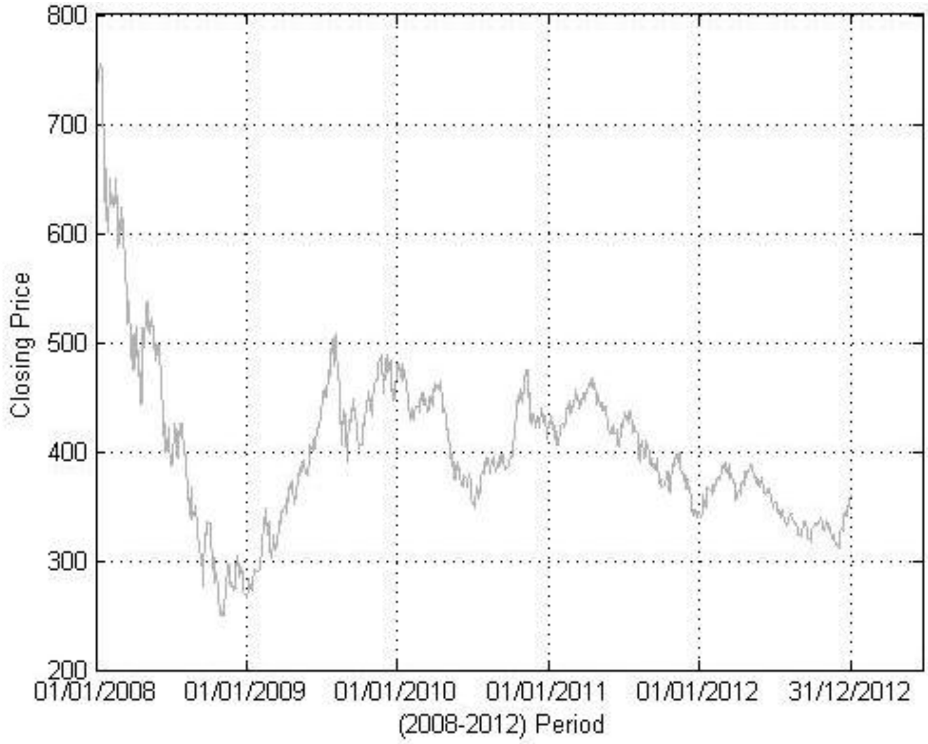


Figure 3.7: Closing price of SH stock market

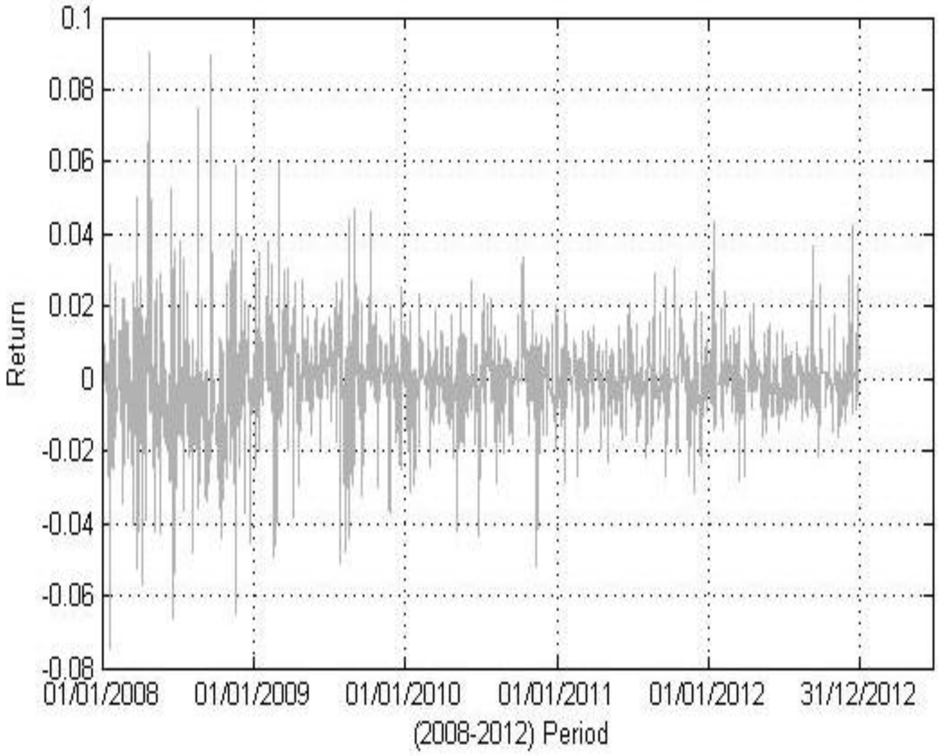


Figure 3.8: Return of SH stock market

In all of four return figures the volatility does not make large movements in short periods. Consequently, during a low volatility period next days are expected to have low volatility, and similarly in a high volatility period, short term expectation of the volatility is high. In all return figures the mean does not change in the long term, indicating that they have a stationary nature.

Chapter 4

PARAMETER ESTIMATION AND MODEL FITTING

4.1 Parameter Estimation and Performance Criteria

The aim of forecasting in this test is to predict the two-days-ahead return values $\hat{x}_{k+2}$ correctly. The performance of the ARMA model is measured by the smallness of the error of prediction, comparing the predicted value $\hat{x}_{k+2}$ by the actual return of two-days-later, i.e., $e_k = x_{k+2} - \hat{x}_{k+2}$. During the estimation of values for a long period of time, the error may change in positive and negative directions, and their sum $\Sigma_i\, e_{k-i}$ might stay nearly zero although the magnitude of error is much higher than the sum of errors. Therefore $\Sigma_i\, e_{k-i}$ is not a performance measure for the predicted values by an ARMA model. The mean of magnitudes of the error is obtained by the absolute value operation, $e_{\mathrm{MAE}} = (1/n)\ \Sigma_{i=1..n}\ |e_{k-i}|$, which is also called the *mean-absolute-error*. MAE punishes both of the positive and the negative errors, however, it punishes the error proportional to the magnitude of the error. In the most systems and small errors are tolerated to a degree, however, large errors are intolerable because they may result in unexpected hazards. Squaring the error, e_{k-i}, makes it positive, and also increases the effect of larger errors nonlinearly as desired in many cases. The mean of squared errors needs square rooted to make it compatible to the output. The resulting performance measure for n successive days of predictions using an ARMA model is:

$$e_{\mathrm{RMSE}} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e_{n-i+1}^{2}} \qquad (4.1)$$

It is called root-mean-square-error, and commonly used in estimation as a performance metrics [33]. The parameters of an ARMA(r, m) model may be trimmed to reduce e_{RMSE} of predicted return.

For practical considerations, ARMA model shall have the smallest order, which provides an acceptable low prediction error. The parameters r and m, which are the orders of AR and MA, are structural parameters of ARMA model, and in the literature, there are methods based on plotting the partial autocorrelation functions for an estimate of r, and m [34].

4.2 Determination of *r* and *m* by Autocorrelation

The autocorrelation function (ACF) measures the similarities of a series starting from x_t against another series starting from x_{t-h}. It is used for predictions. An auto correlated time series is predictable, probabilistically, because upcoming values rely upon present and previous values. The time series plot could be a tool for measurement the autocorrelation of a time series. Positive autocorrelation may show up a plot as remarkably long runs of many consecutive observations higher than or below the mean. Negative autocorrelation may show up as a curiously low incidence of such runs. For computing autocorrelation the relative a horizontal line planned at the sample mean is helpful in evaluating autocorrelation with the time series plot.

In addition, a partial autocorrelation (PACF) is defined to give the correlation between x_t and x_{t-h} after intermediate correlation has been removed. The PACF is obtained from the set of difference equations related to the ACF. Equation 4.2 shows the formula for the sample *lag-h* autocorrelation. For an observed series x_1, x_2,...,x_T

and the sample mean $\bar{x}$, the sample *lag-h* autocorrelation is given by [34] [35]:

$$lag\text{-}h = \frac{\sum_{t=h+1}^{T}(x_t - \bar{x})(x_{t-h} - \bar{x})}{\sum_{t=1}^{T}(x_t - \bar{x})^2} \tag{4.2}$$

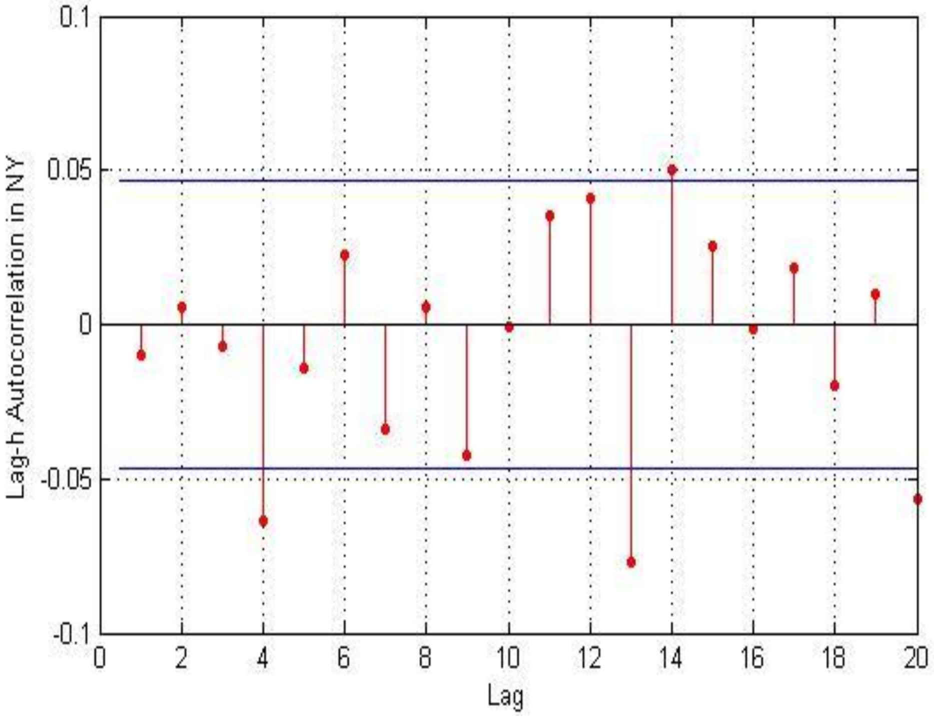


Figure 4.1: Autocorrelation of NY stock market

Figure 4.1 to Figure 4.8 show the *lag-h* autocorrelation (ACF) and *lag-h* partial autocorrelation (PACF) for New York, London, Tokyo, and Shanghai stock markets. These figures are obtained by MATLAB codes using autocorr and parcorr functions. See Appendix E.3.

For four stock markets it is not easily to identify the patterns for AR and MA models directly. For AR(r) model, the partial autocorrelation (PACF) will be close to zero at lags greater than r. For a MA(m) model the autocorrelation (ACF) be close to zero at lags greater than m. As a result, the expected m values according to ACF were

5,8,10, and 14 (Figure 4.1), and the *r* values according to PACF were 5,8,10, and14 in NY stock market (Figure 4.2).

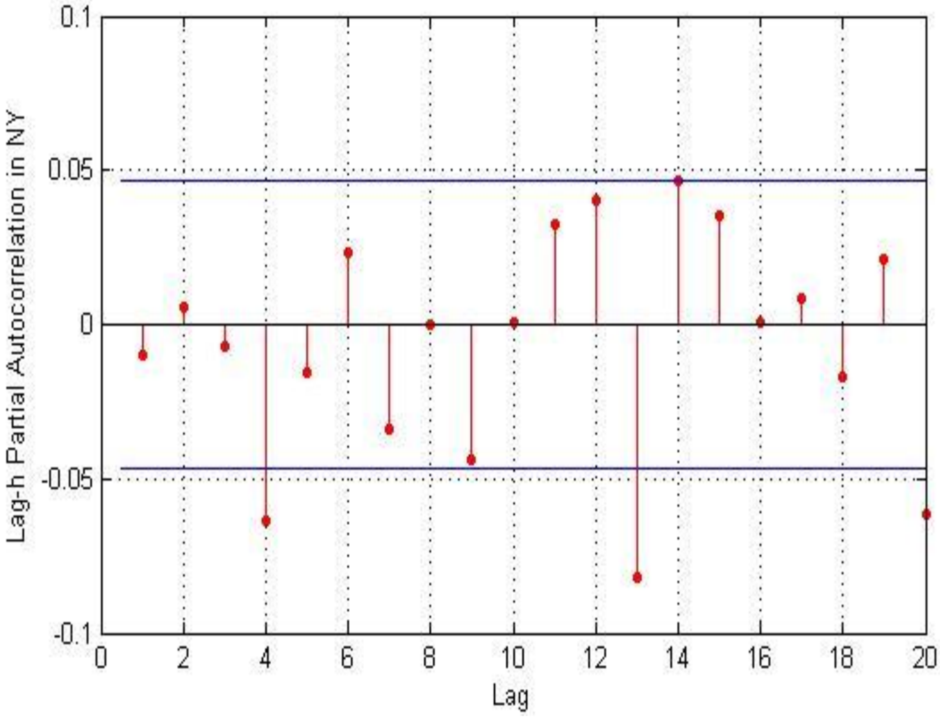


Figure 4.2: Partial autocorrelation of NY stock market

X-axis represents the order of m in ACF and order of r in PACF. Y-axis represents the lag-h of ACF and PACF for the time series data set. Similarly, the expected r and m values according to PACF and ACF are as in Table 4.1 for NY, LD, TK and SH stock markets (Figures 4.3 to 4.8).

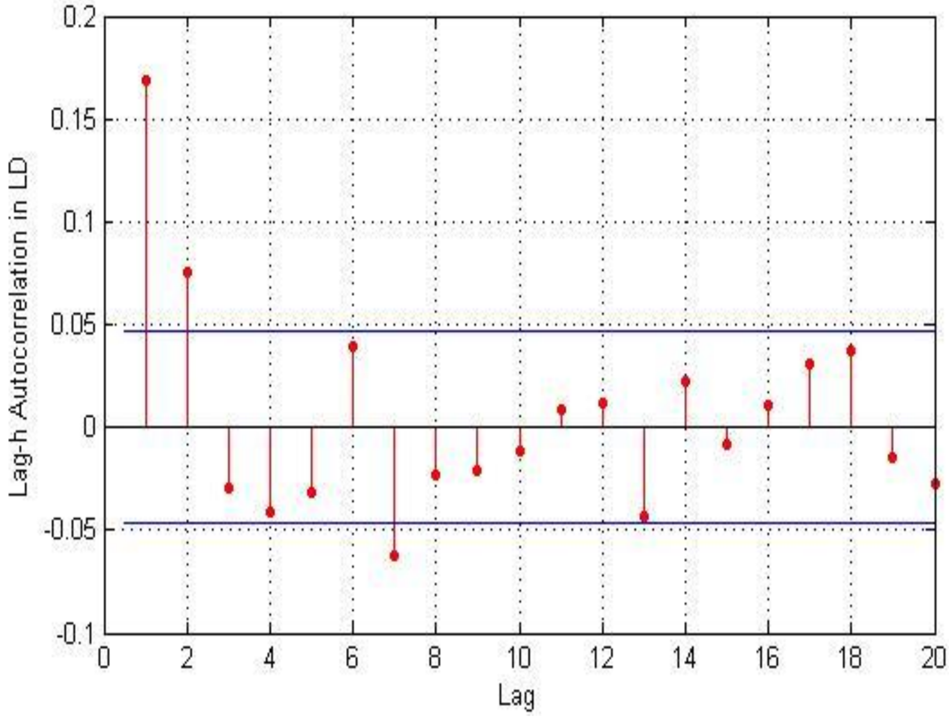


Figure 4.3: Autocorrelation of LD stock market

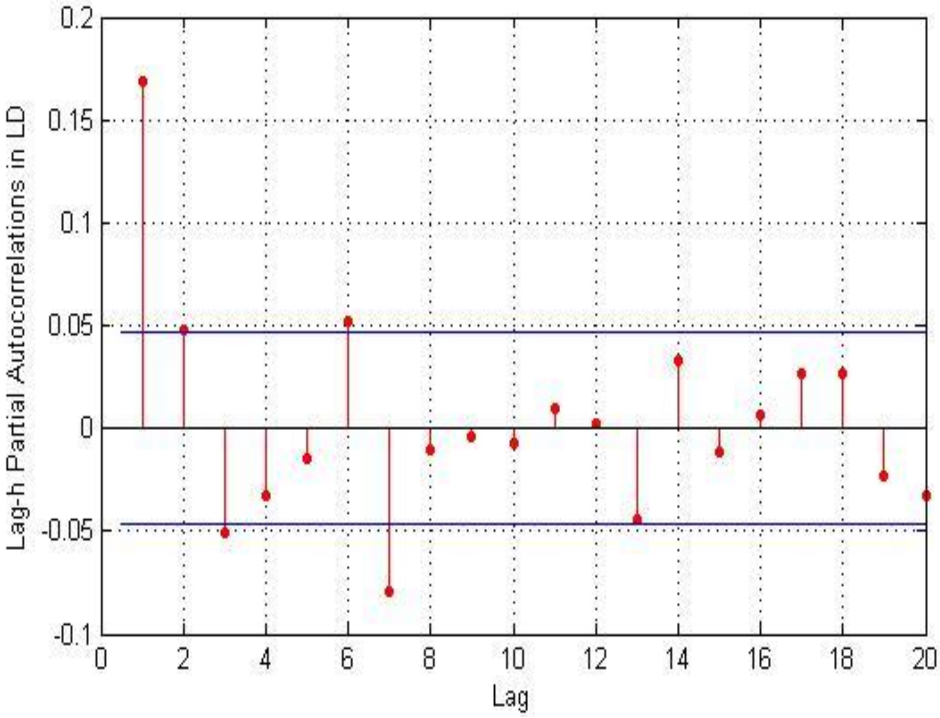


Figure 4.4: Partial autocorrelation of LD stock market

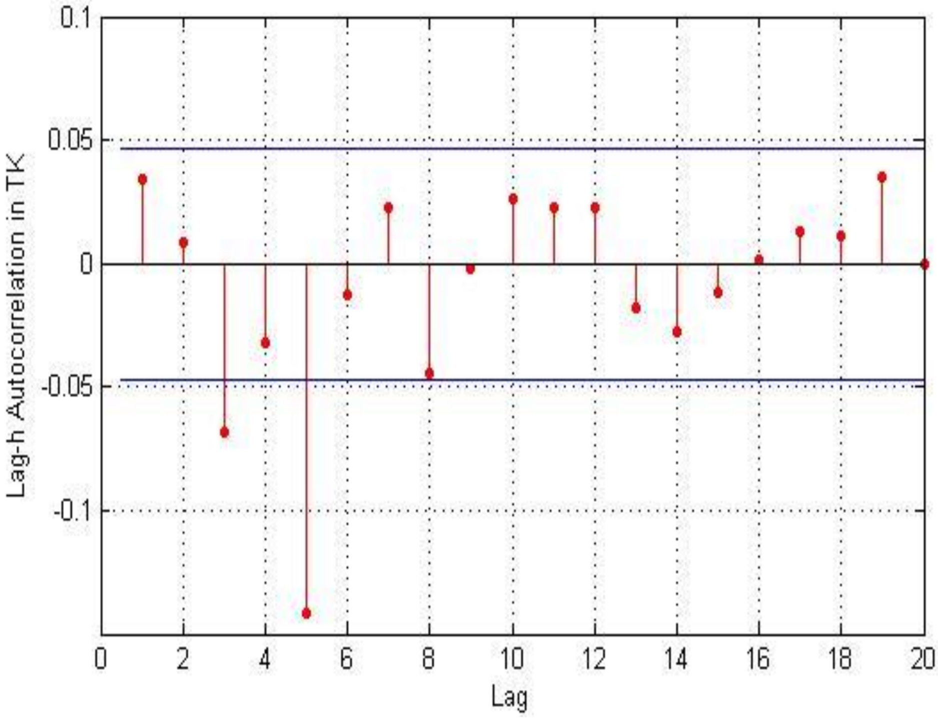


Figure 4.5: Autocorrelation of TK stock market

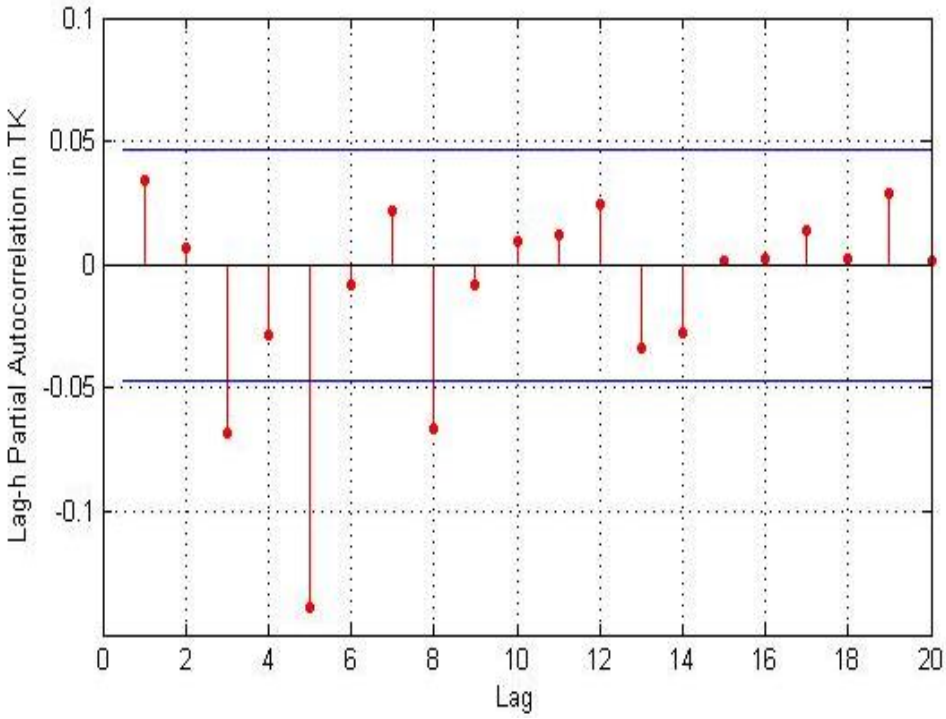


Figure 4.6: Partial autocorrelation of TK stock market

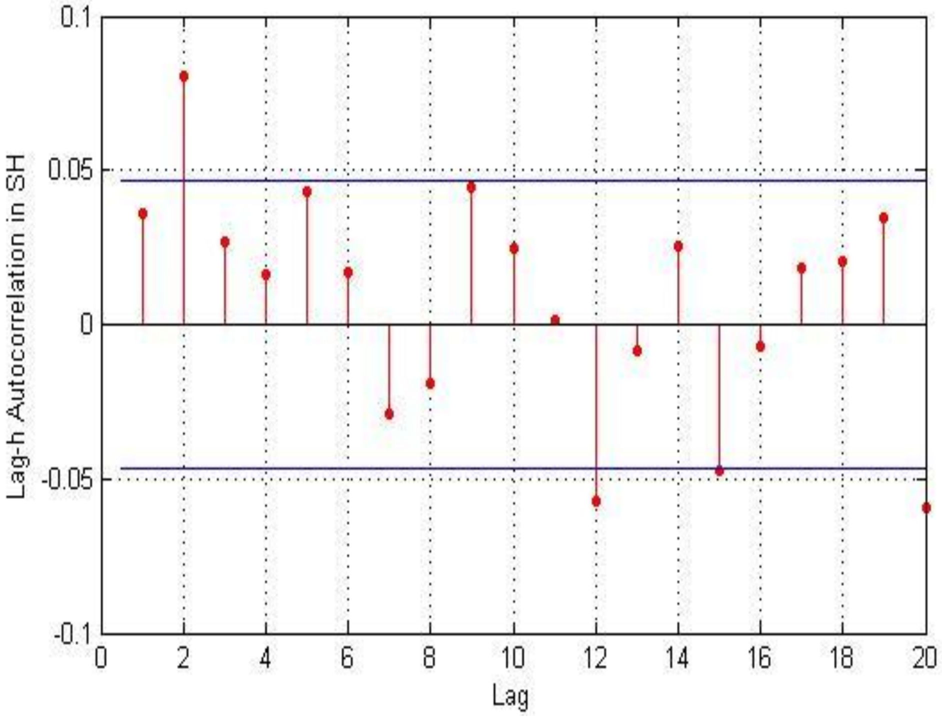


Figure 4.7: Autocorrelation of SH stock market

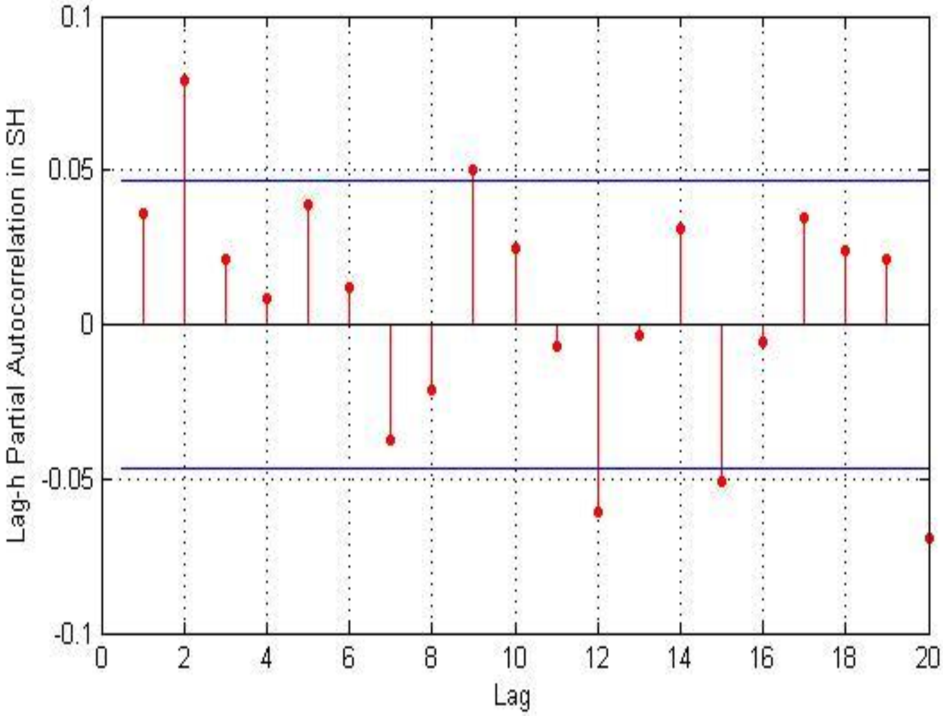


Figure 4.8: Partial autocorrelation of SH stock market

Table 4.1: The *r* and *m* order values according to PACF and ACF

Stock market	*r*	*m*
NY	5,8,10, and 14	5,8,10, and 14
LD	4,8,and 14	8 and 14
TK	4,6,and 9	4,6, and 9
SH	13 and 16	13 and 16

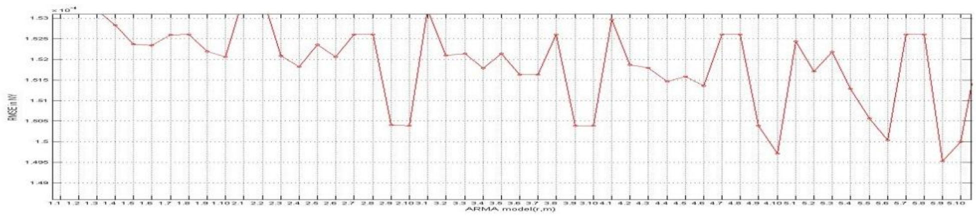
Additionally, there are widely used information criteria which are the Akaike Information Criteria (AIC) [36] and Bayesian information criterion (BIC) [37]. The idea behind both is simple select the model which has the lowest value of the criteria.

4.3 Optimum Structural Parameters of ARMA Models

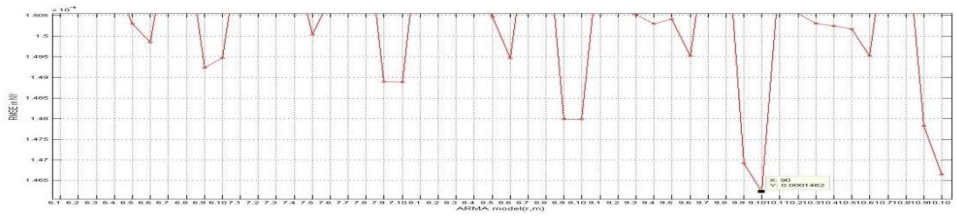
The time series data sets of four stock markets were pre-processed to complete the missing days, and to convert all prices to dollars, to make them ready for forecasting using ARMA (*r*, *m*) models. The parameters *r* and *m* are called structural parameters to distinguish them from the autoregressive parameters ϕ_i and moving average parameters θ_i in the ARMA (*r*, *m*). The best forecasting ARMA (*r, m*) model is obtained by two steps. The *r* and *m* values that give the lowest estimation error are determined for each market data using the root mean square error (RMSE) of two days ahead forecasting over the previous two years data values. The search of structural parameters for the minimum error of prediction provides validation of the determination of structural parameters by autocorrelation model. The ultimate goal of the forecasting is to have sufficiently small error of prediction with less structural order so that sufficiently accurate prediction is obtained by an ARMA model with the minimum possible order.

In the search tests for NY market (Figure 4.9), the correlation of the (*r*, *m*) values determined by autocorrelation functions and the RMSE plots are clearly observable. For example in the partial auto correlation function (Figure 4.2), the 5^{th}, 8^{th}, 10^{th}, and

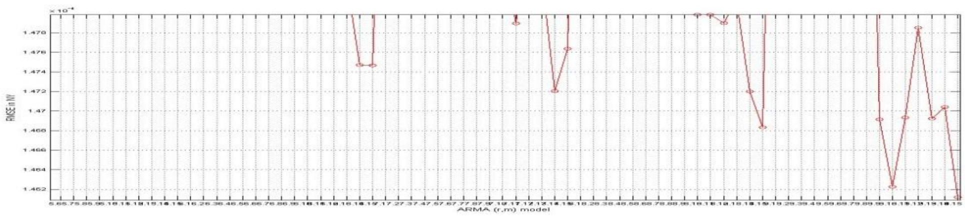
14th terms (counting them as zero-term+Lag) have significant high values. The RMSE plots for NY indicates clearly minimums at (r, m)=(5,9), (8,9), (9,10), (10,10), (5,14), (8,14), (6,15), and (9,15). Figure 4.9 also clears the testing (225) ARMA(r,m) models in NY stock market for all possible values in the range ARMA (1,1) to ARMA(15,15) as parts because of the difficulty of showing all the models in the same plot, in each part x-axis represent the order of ARMA(r,m) models and y-axis represents the corresponding RMSE value. All these ARMA(r,m) models and related figures are obtained by MATLAB codes. See Appendix E.2.



X-axis represents ARMA from (1, 1) to (5, 10) models.
Y-axis represents RMSE value for these models.



X-axis represents ARMA model from (6, 1) to (10,10)



X-axis represents ARMA model from (7, 11) to (9,15)

Figure 4.9: The ARMA(r,m) model and RMSE value for NY stock market.

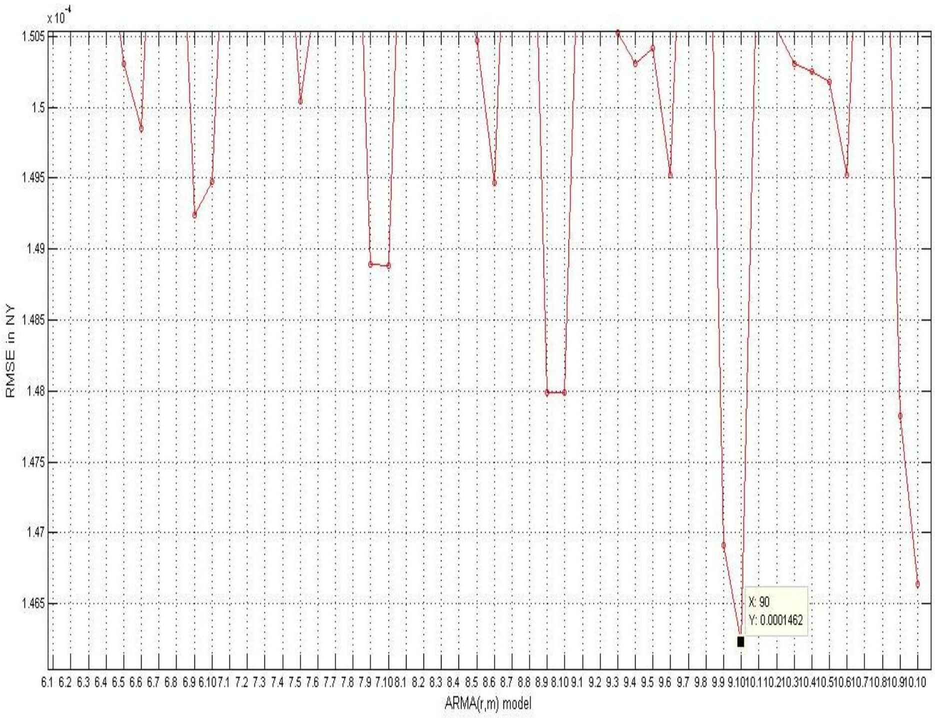


Figure 4.10: Best ARMA(r,m) model and RMSE value for NY stock market

In this work, the best ARMA(*r,m*) model corresponding to minimum RMSE value in NY stock market is ARMA(9,10) with $1.46x10^{-4}$ RMSE value (Figure 4.10).

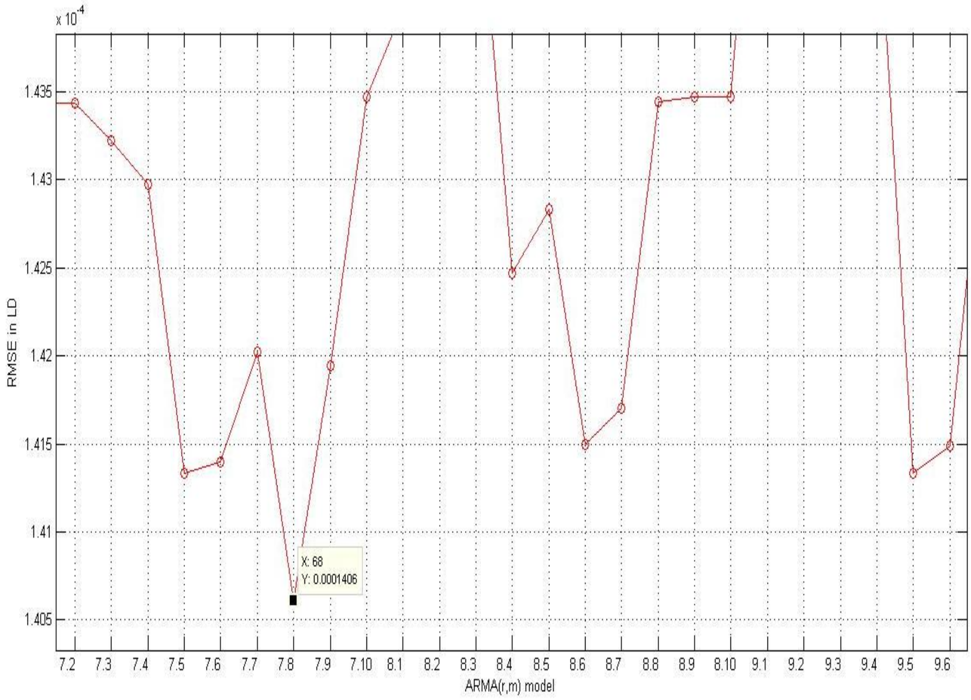


Figure 4.11: Best ARMA(r,m) model and RMSE value for LD stock market

The two days ahead prediction RMSE plots for LD, TK, and SH markets for all possible (r,m) values are showing in Figures (4.11 to 4.13), where the value of structural parameters for minimum RMSE are detected accordingly. Consequently, for the LD, TK, and SH stock markets, the optimum (r, m) and return-RMSE values are (7,8), $1.4\text{x}10^{-4}$ for LD,(10,10) , $2\text{x}10^{-4}$ for TK, and (9,10), $1.6\text{x}10^{-4}$ for SH.

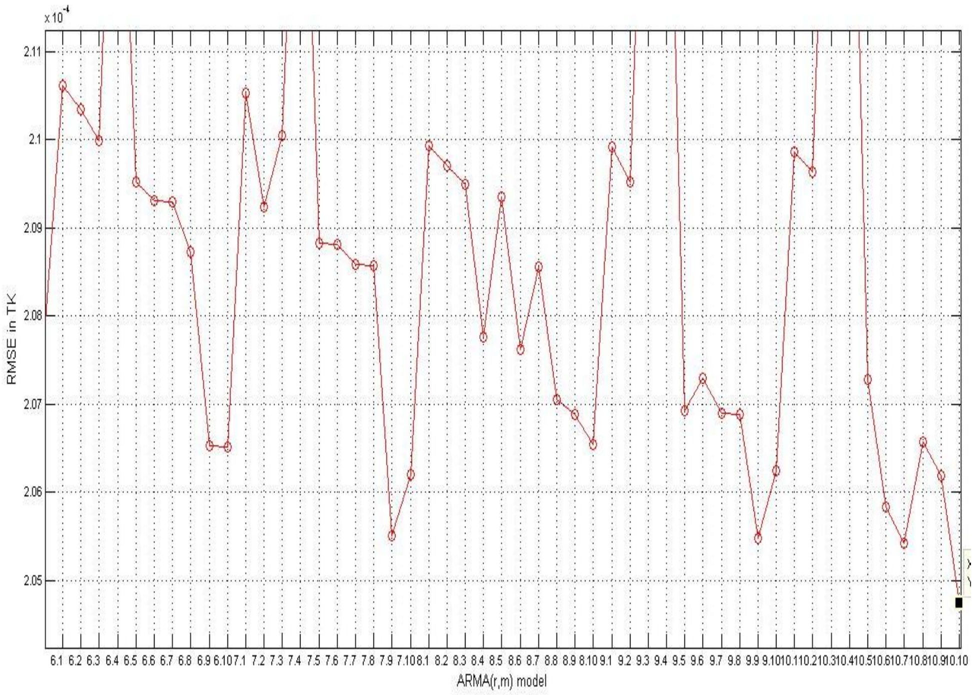


Figure 4.12: Best ARMA(r,m) model and RMSE value for TK stock market

Particularly, the favorite model is the model which has a minimum number of parameters to avoid the large number of computational steps as much as possible, abolish increase the percentage of error and to get close to the prediction accuracy.

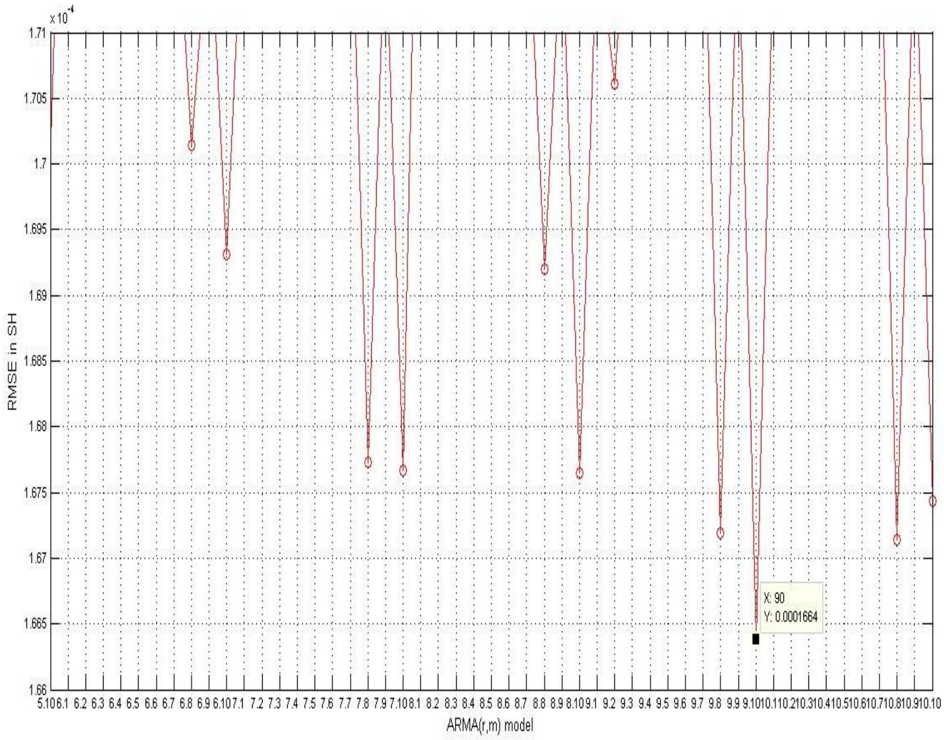


Figure 4.13: Best ARMA(r,m) model and RMSE value for SH stock market

The next Chapter explains the results of the forecasted closing prices using the ARMA(r,m) model with the best r and m values.

Chapter 5

THE FORECASTING

5.1 Forecasting

Estimation of the future using the trend and patterns in a set of available observations means forecasting. In the finance sector, forecasting is used by actors to allocate their resources for a future period of time. The forecasting of economic and industrial time series is important as a tool of analysis for the business decisions such as selling or buying in the markets [24] [38]. As a scientific technique, forecasting helps organizations for decision making in the state of uncertainty.

Model based forecasting assumes that the system changes states by the inputs, and the states are reflected to outputs by the inner dynamics of the system. Once the model parameters are correctly estimated, the trend and future value is easily forecasted by using the model.

For this thesis, the objective period consists of the years from 2008, to 2012, inclusive. The observations are collected as the time series of closing prices of New York, London, Tokyo and Shanghai stock markets, all converted to dollars after filling the missing values by method of interpolation, and then converted to daily return values to obtain a stationary time series, which is suitable for an ARMA model.

The h-day-ahead prediction error is $e_{k,h} = x_{k+h} - \hat{x}_{k+h}$, where x_{k+h} is the actual return value at the end of h-days and $\hat{x}_{k+h}$ is the forecasted return value by ARMA model.

5.2 Dependence of Future Market Value to the Past

There are scholars, who claim that markets are illogical and influenced by psychological factors [39]. Taleb (2008) argues harshly against the idea that someone is able to forecast the future [40]. In his book the "Black Swan" he argues that financial markets are simply impossible to predict before they happen.

Rational expectation theory is against this view. Valid assumptions in the rational expectation theory states that: a) the random disturbances are normally distributed; b) Certainty equivalents exist for the variables to be predicted; c) the equations of the system, including the expectations formulas, are linear. Starting with these assumptions, the rational expectation theory states that expectation of the future value is significant, even with restricted economic information. Moreover, any speculative action after prediction of the future values reduces the variance of the market prices, and improves its predictability [41]. Accordingly, the linear dynamic ARMA model, which processes the time series data, is suitable for prediction of the future values with a sufficiently low variance.

5.3 The Results of Forecasting Using ARMA Model

In this thesis, the forecasting of two-days-ahead return is obtained by training the ARMA(r,m) model for each forecasted day by using its previous two years stock market data. Although the parameter values of each day's ARMA model are similar to each other they are calculated particular for that day. Once the estimation errors e_{t-i} are calculated from the previous actual values and their estimated by $e_{t-i} = x_{t-i} - \hat{x}_{t-i}$, the future values $\hat{e}_{xt}$ is predicted by ARMA model with an error e_t by the relation:

$$\hat{x}_t = x_t - \varepsilon_t = \mu + \sum_{i=1}^{r} \phi_i x_{t-i} + \sum_{i=1}^{m} \theta_i \varepsilon_{t-i} \,, \qquad (5.1)$$

where *r* and *m* were determined for NYSE as *r*=9 and *m*=10.

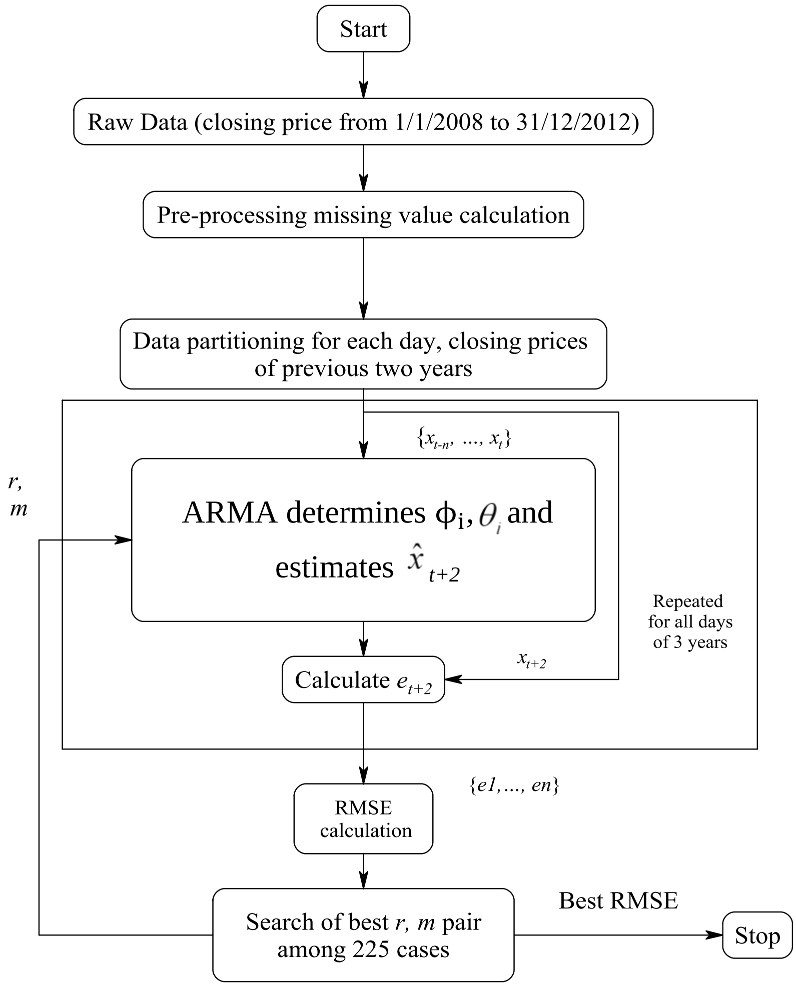


Figure 5.1: Block diagram for selecting the best ARMA(*r*,*m*)model

As shown in the Figure 5.1 two years data is prepared for predicting each day of three years period and the ARMA parameters are calculated for each day. We obtain for each day a different set of values for ϕ_i, θ_i coefficients; however they are mostly very close to each other. As an example of typical parameter values for NYSE to forecast the day (31/12/2012), μ is 1177.362, Table 5.1 shows the values of ϕ_i for i=1...9, θ_i for i=1...10 and also contains the values of $\varepsilon_{t\text{-i}}$ and $x_{t\text{-}i}$ for the same forecasting day. The estimation expression by ARMA(9,10) model is written using these parameters as following:

$$\hat{x}_t = 1177.362 - 0.7809x_{t-1} + 0.7303x_{t-2} + 0.5239x_{t-3} - 0.1764x_{t-4} - 0.1941x_{t-5} - 0.0348\,x_{t-6} - 0.0165\,x_{t-7} - 0.0081\,x_{t-8} - 0.009\,x_{t-9} + 1.9754\varepsilon_{t-1} + 1.5\varepsilon_{t-2} + \varepsilon_{t-3} + 0.9997\varepsilon_{t-4} + \varepsilon_{t-5} + \varepsilon_{t-6} + \varepsilon_{t-7} + \varepsilon_{t-8} + 0.8476\varepsilon_{t-9} + 0.3306\varepsilon_{t-10}.$$

Table 5.1: The x_{t-i}, ϕ_{i}, $\varepsilon_{\tau-\iota}$, and θ_{i} values

Time lag (days)	Closing price and estimation error		ARMA parameters	
i	$x_{t\text{-i}}$	$\varepsilon_{\text{t-i}}$	ϕ_{i}	θ_{i}
1	1427.823	4.823691	-0.7809	1.9754
2	1426.66	19.01754	0.7303	1.5
3	1423.245	12.05432	0.5239	1
4	1419.83	16.55566	-0.1764	0.9997
5	1418.1	15.04184	-0.1941	1
6	1402.43	14.94693	-0.0348	1
7	1410.35	21.99996	-0.0165	1
8	1418.27	3.343305	-0.0081	1
9	1426.19	28.26329	-0.009	0.8476
10		25.79558		0.3306

Figure 5.2 shows the two-days-ahead forecasting prices for NY stock market from (1/1/2010) to (31/12/2012). Figure 5.3 shows the closing price absolute prediction error in NY stock market for the same three years, x-axis denoted to the period from 2010 to 2012 and y-axis represents the error value in NY closing price. The mean absolute error was less than 15.5 while closing prices were around \$1200 in NY; 103.4 while closing prices were \$9000 in LD; in TK the mean absolute error was less than 0.5 while closing prices were \$120; in SH was less than 1.6 while closing prices were \$500. Figure 5.3 shows the closing prices and two-days-ahead forecasting prices using ARMA (9, 10) model for NY stock market on the same plot, in this

figure x-axis represents the period in range (2008-2012) and y-axis refers to closing price by USD. However, because the forecasting prices are very close to the actual prices, the difference is not distinguishable on the plot. See appendix E.3.

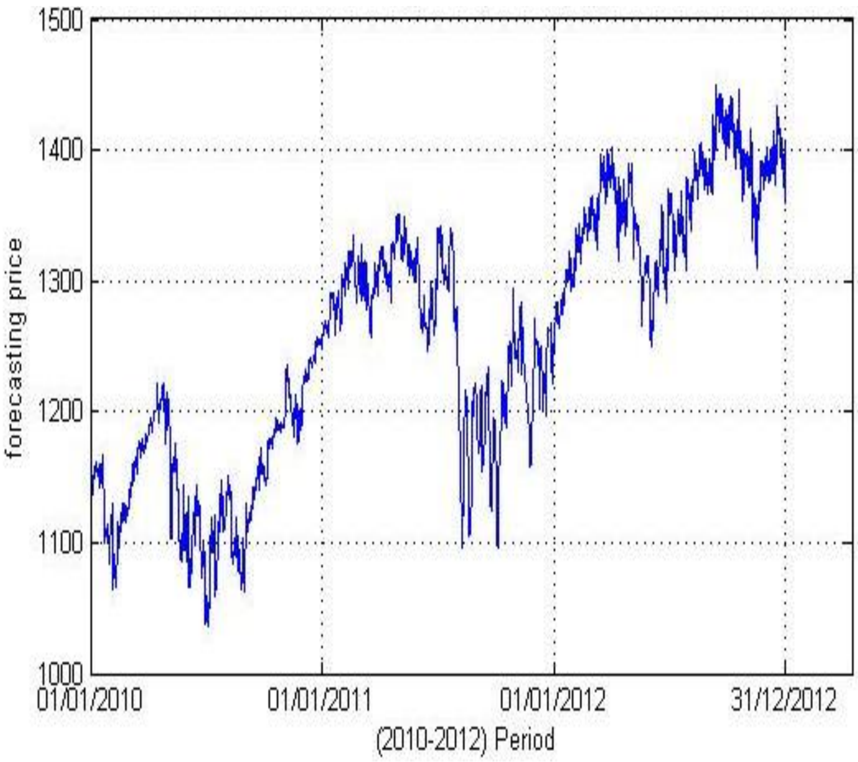


Figure 5.2: ARMA (9,10) forecasting price for 3 years of NY stock market

For NY stock market as an example of the forecasting value and absolute prediction error of forecasting, on (03/01/2010), the actual closing price was =1128.5175 and forecasted price was =1138.8, the absolute prediction error is $e_{p,t} = | p_t - \hat{p}_t | = 10.2$.

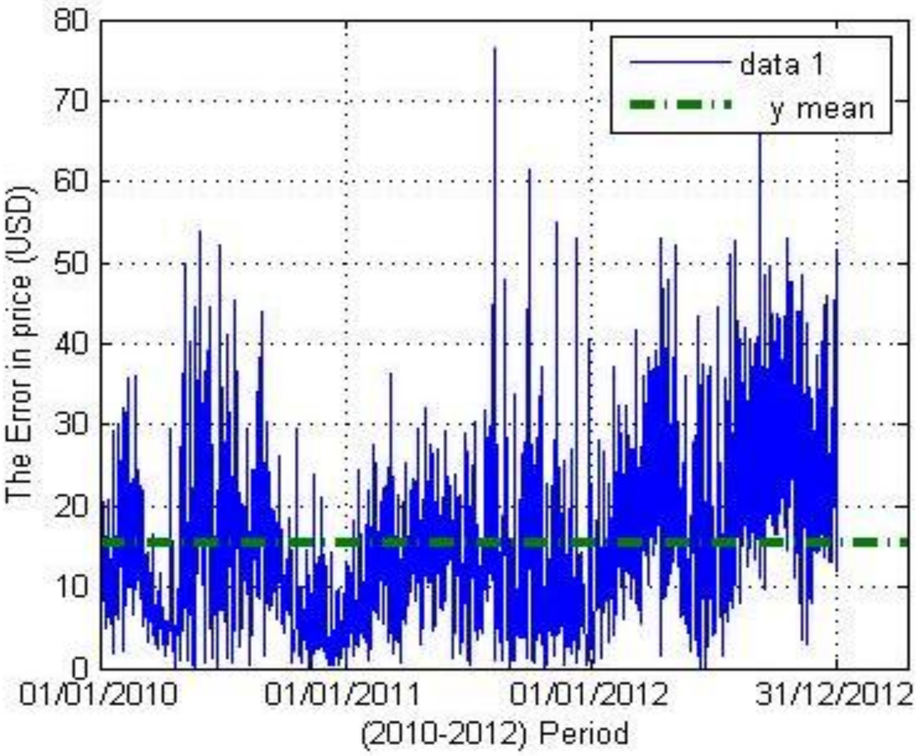


Figure 5.3: The closing price absolute prediction error in NY stock market

Trying to forecast the future closing price in these stock market depending on the raw data which has missing values, the mean absolute error was 15 in NY stock market; 133.448 in LD; in TK the mean absolute error was 0.933 and in SH was 3.745 (Table 5.2). It is clear that the MAE is approximately equal or higher than the values of mean absolute error in the data without missing values, so the forecasting process in this work depending on the data set after pre-processing it i.e. the data without missing values.

Table 5.2: MAE for the data sets with and without missing values

	NYSE	LSE	TSE	SSE
MAE with missing values.	15	133.448	0.933	3.745
MAE without missing val.	15.5	103.4	0.5	1.6

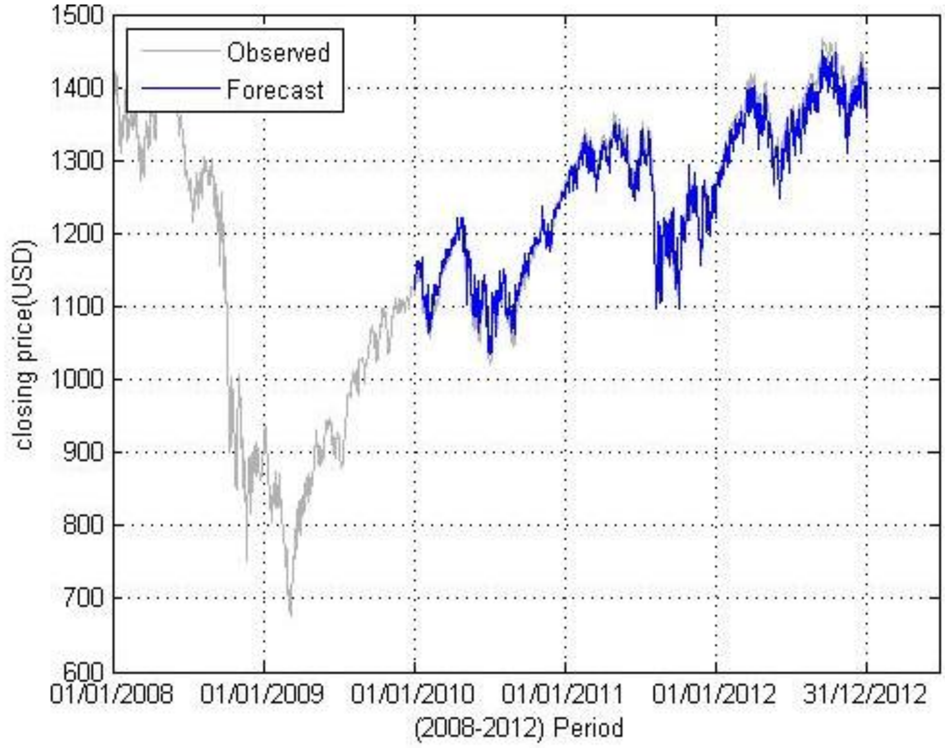


Figure 5.4: ARMA (9, 10) closing and forecasting price for 3 years in NY

Figures (5.4, 5.5, 5.7, and 5.9) show the five years closing prices and the three year forecasted prices of NY, LD, TK, and SH stock markets on the same plots. The forecasting process started from 2010 because each two days are forecasted depending on the previous 2 years belonging to these two days. See appendix C.

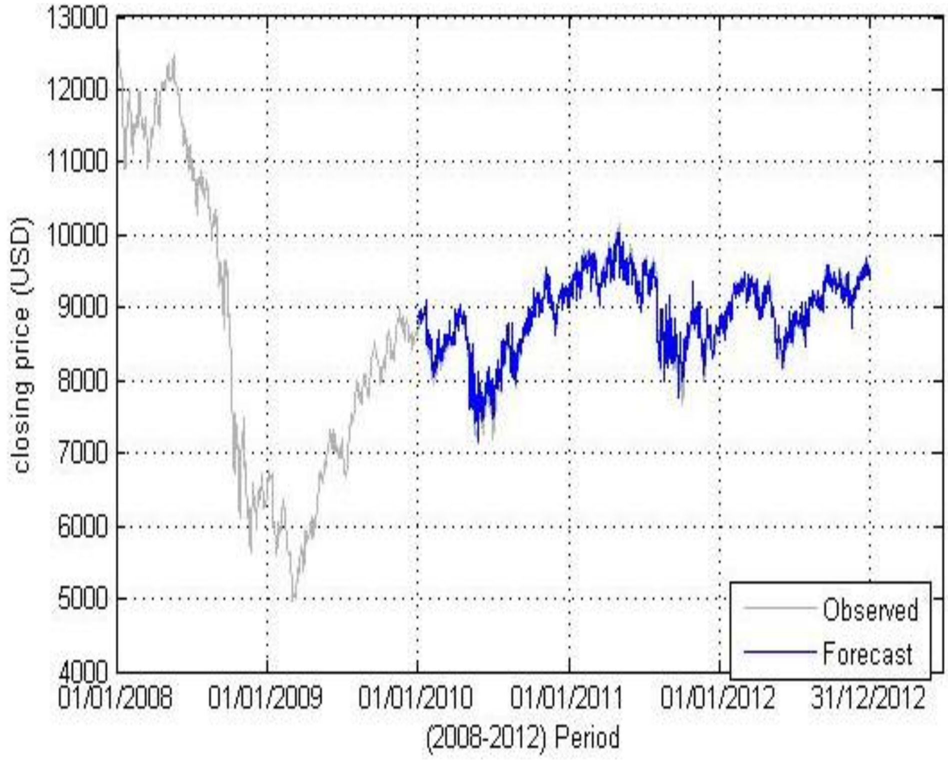


Figure 5.5: ARMA (7, 8) closing and forecasting price for 3 years in LD

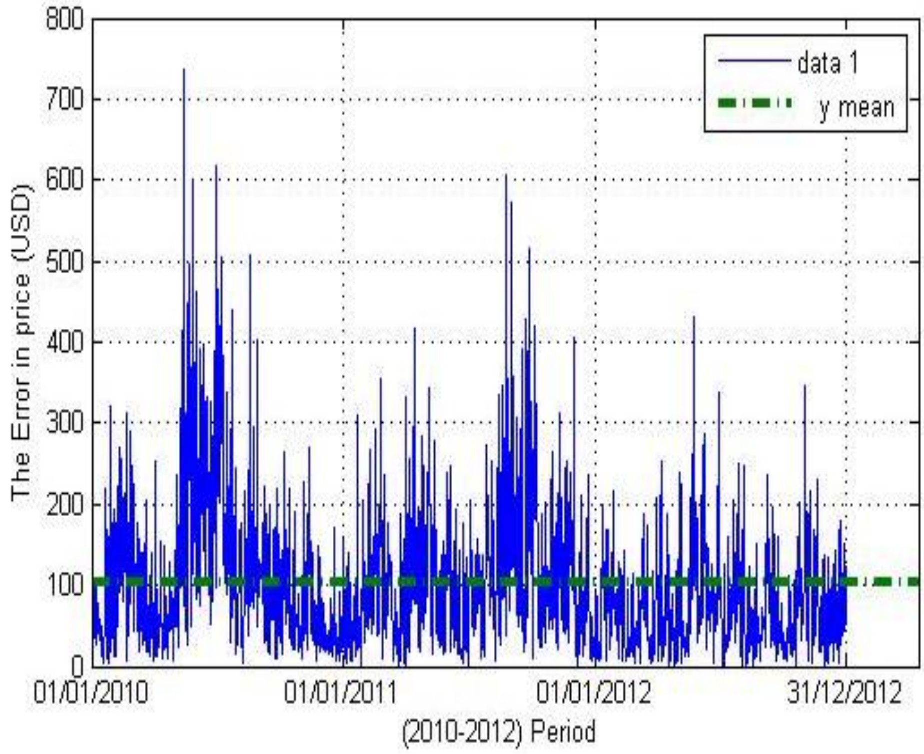


Figure 5.6: The closing price absolute prediction error in LD stock market

Figures (5.6, 5.8, and 5.10) refer to the absolute prediction error of LD, TK, and SH stock markets.

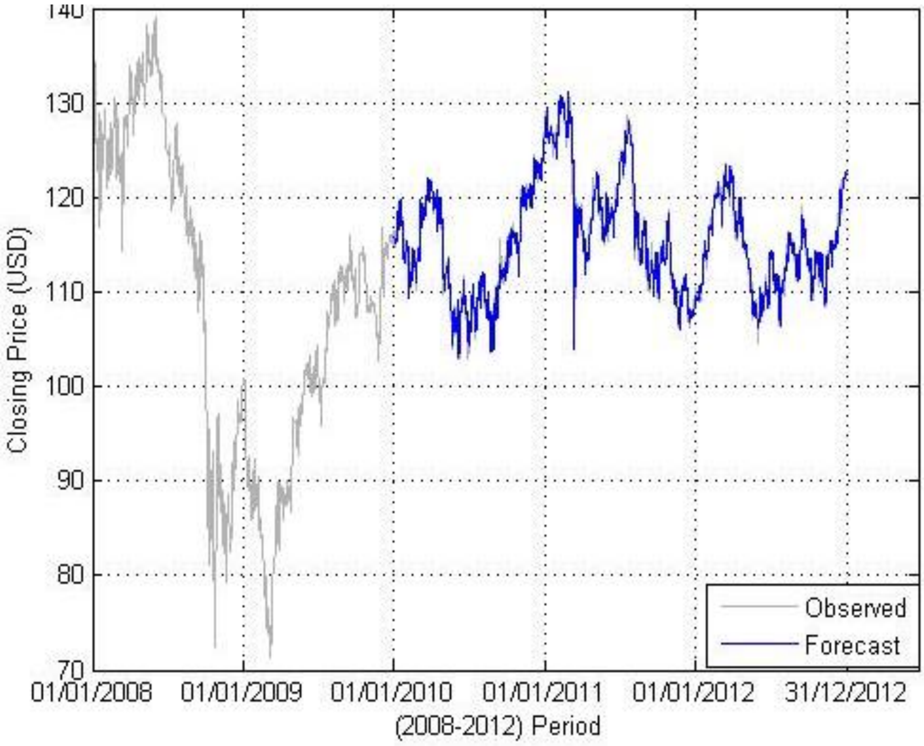


Figure 5.7: ARMA (10, 10) closing and forecasting price for 3 years in TK

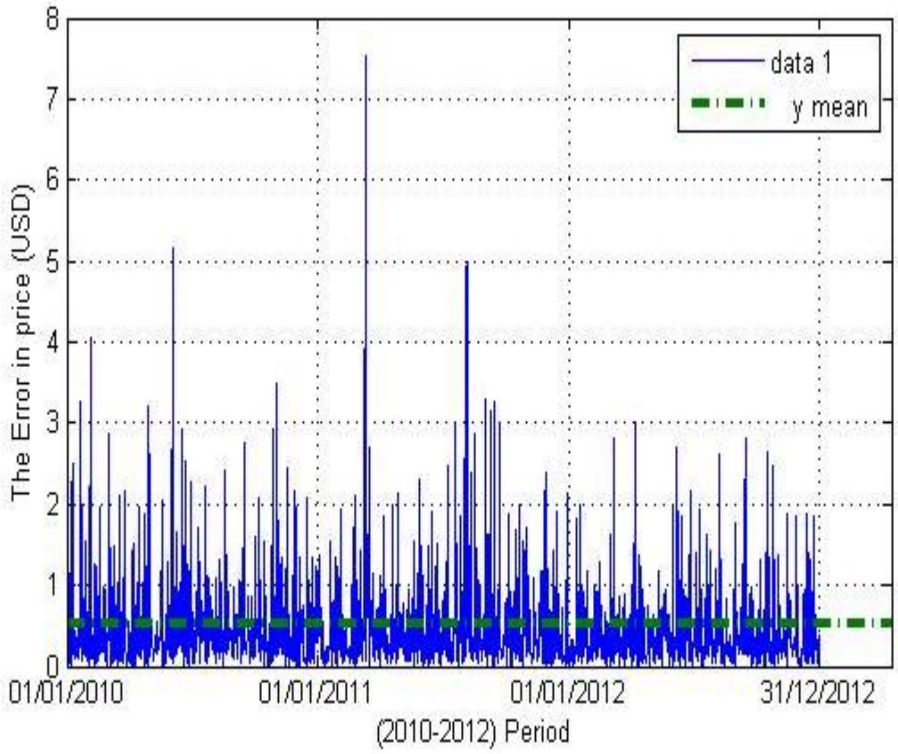


Figure 5.8: The closing price absolute prediction error in TK stock mark

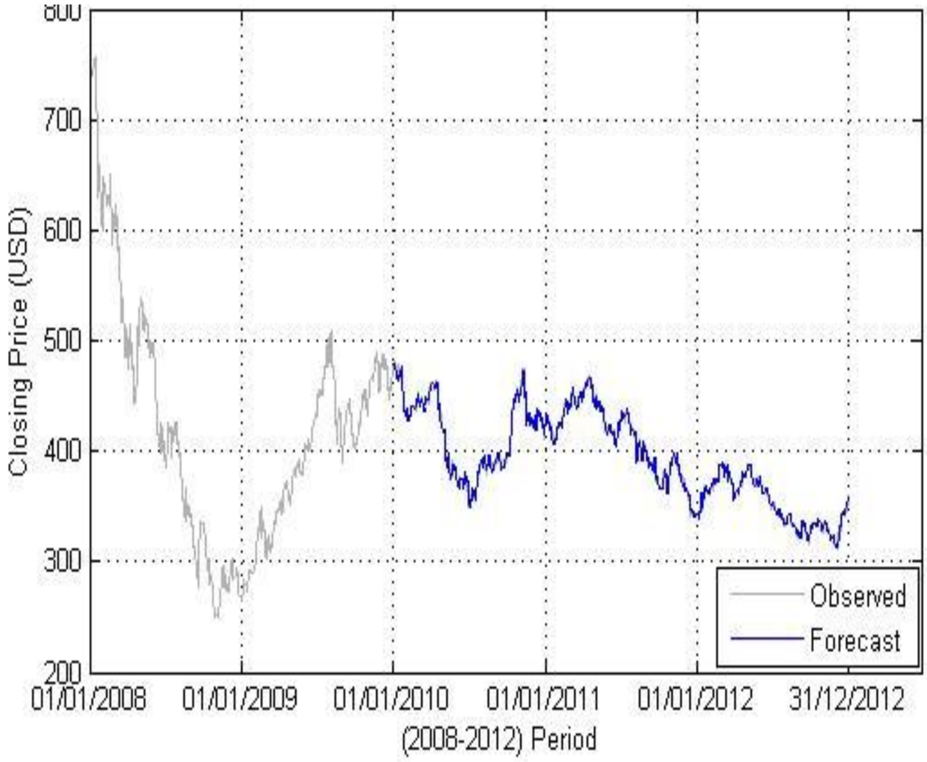


Figure 5.9: ARMA (9, 10) closing and forecasting price for 3 years in SH

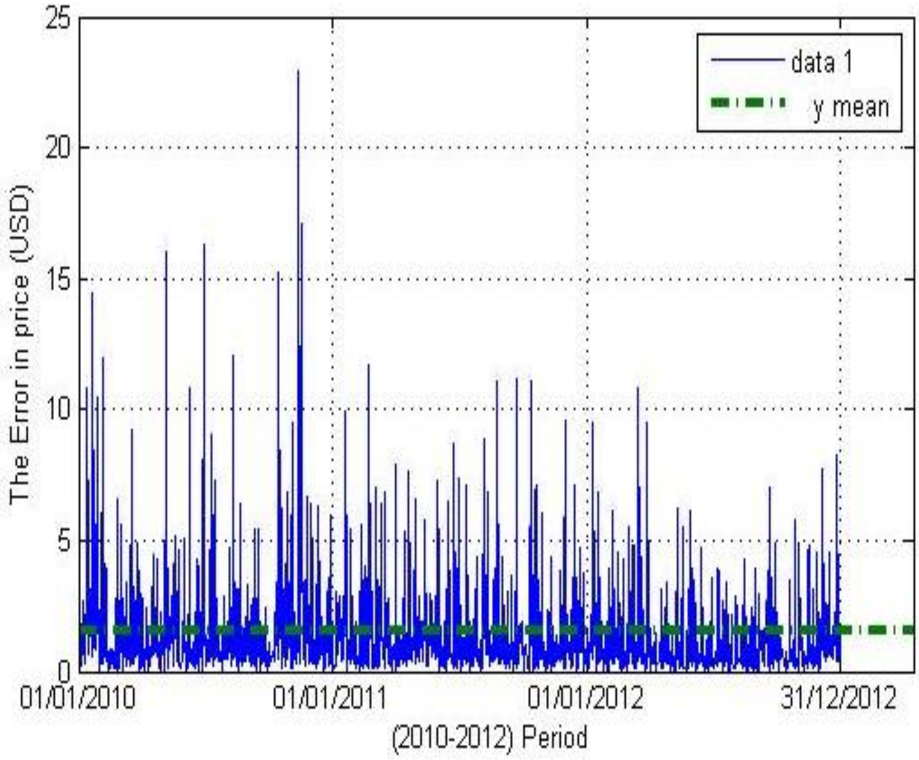


Figure 5.10: The closing price absolute prediction error in SH stock market

Table 5.3 contain sample of the Actual and prediction values by ARMA in all these stock markets.

Table 5.3: Actual and forecasting closing price

London stock market		
Date	Actual price	ARMA forecasting
01/01/2010	8686.322	8789.0537
02/01/2010	8752.0595	8874.3535
03/01/2010	8817.797	8909.1603
04/01/2010	8883.5345	8934.1211
05/01/2010	8906.688	8943.095
06/01/2010	8879.521	8984.8868
07/01/2010	8837.7459	8811.6198
08/01/2010	8834.7968	8778.8938
09/01/2010	8847.2099	8894.0669
10/01/2010	8859.623	8937.9644
New York stock market		
Date	Actual price	ARMA forecasting
01/01/2010	1119.5725	1125.2987
02/01/2010	1124.045	1132.7034
03/01/2010	1128.5175	1138.759
04/01/2010	1132.99	1150.9653
05/01/2010	1136.52	1146.3868
06/01/2010	1137.14	1157.6491
07/01/2010	1141.69	1150.1642
08/01/2010	1144.98	1161.2432
09/01/2010	1145.6466	1155.459
10/01/2010	1146.3133	1161.6652
Tokyo stock market		
Date	Actual price	ARMA forecasting
01/01/2010	115.7897	115.7822
02/01/2010	115.4686	115.5897
03/01/2010	115.1476	115.1093
04/01/2010	114.8265	115.218
05/01/2010	115.3619	115.4228
06/01/2010	117.7016	115.4295
07/01/2010	118.5826	118.5984
08/01/2010	115.9383	118.4435
09/01/2010	116.4044	116.2426
10/01/2010	116.8706	116.0371
Shanghai stock market		
Date	Actual price	ARMA forecasting
01/01/2010	478.3058	478.7511
02/01/2010	477.166	478.2803
03/01/2010	476.0262	475.7647
04/01/2010	474.8864	475.2196
05/01/2010	480.1829	479.1875
06/01/2010	476.0923	478.7534
07/01/2010	466.7844	466.1087
08/01/2010	467.2552	465.0677
09/01/2010	468.2856	468.0196

The investment and other idea will refer to it in the next Chapter.

Chapter 6

THE INVESTMENT

6.1 Investment in Economic

In economics, Investment is complex in abounding areas, such as business administration and accounts whether for households, companies, or governments.

In finance, investment is putting money into somewhat with the hesitation of gain. This may or may not be backed by analysis. Most or all forms of investment absorb some structure of risk, such as investment in stock and property. In adverse putting money into somewhat with an achievement of concise gain, with or after absolute analysis, is banking or assumption. Under the capable market hypothesis, all investments with according accident should accept the accepted amount of return but that does not anticipate one from advance in unreliable assets in the achievement of benefiting from this trade-off [42]

6.2 Investment of Money among Stock Markets

The stock markets have absolutely correlation with the corporate investment, both of them depends on the time series. Keynes (1936) argues that stock prices contain an important element of irrationality [43].

This attempt is based on the values that have been predicted by ARMA(r,m) model during 3 years (2010, 2011 and 2012). The objective of investment process between the stock markets (New York, London, Tokyo, and Shanghai) is to find out which stock market is better for investment. Firstly, $100 is invested in each stock market separately seeking to calculate the value of investment at the end of 3 years. Figure 6.1 shows the steps of investment in each stock market.

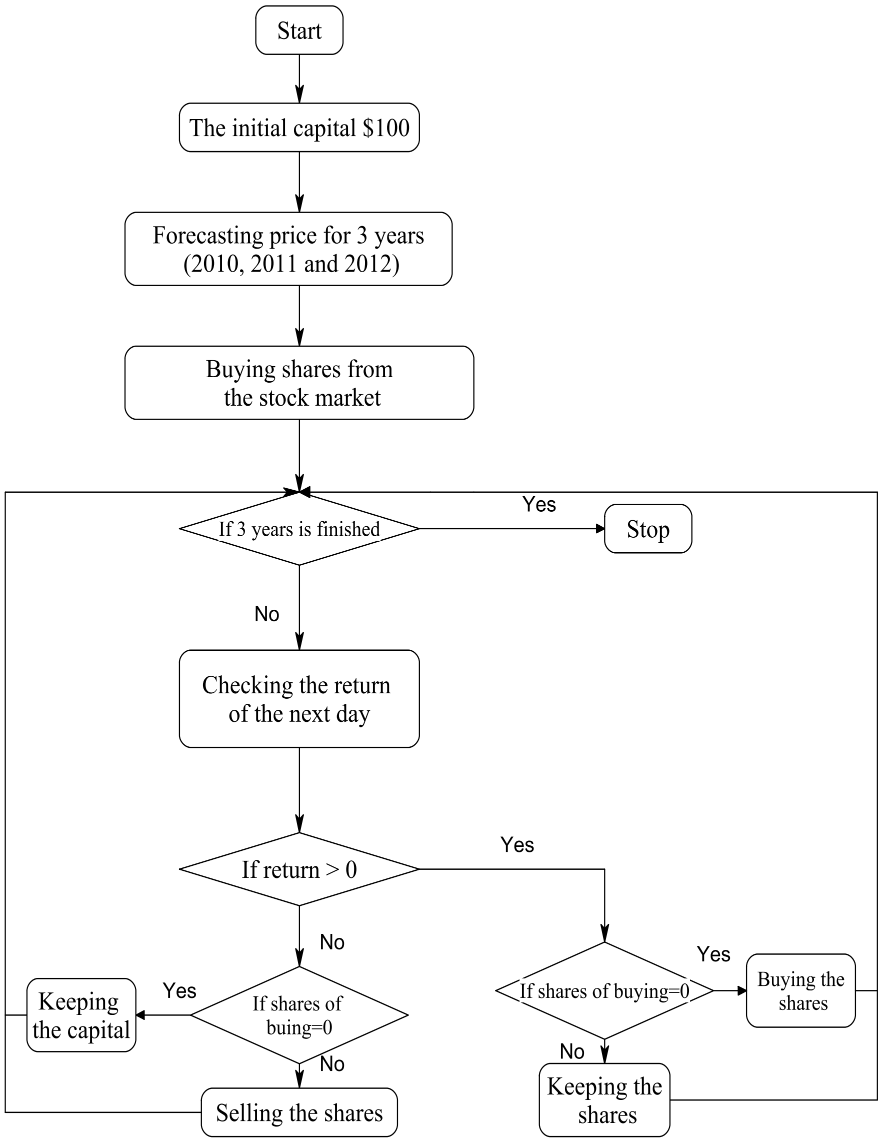


Figure 6.1: Block diagram of investment in each stock market

Secondly, testing the outcomes of the investment process in the abovementioned stock markets to identify the value of investment in the same period. In each stock market the value of the return determines the decision of buying or selling in the day. In the process of investment among the stock markets, shares are purchased from and sold in the best stock market which has the highest return daily during 3 years. The transfer of the invested money from one global market to another one takes one day due to banking operations. Figure 6.2 shows the steps of investment in all stock markets at the same time.

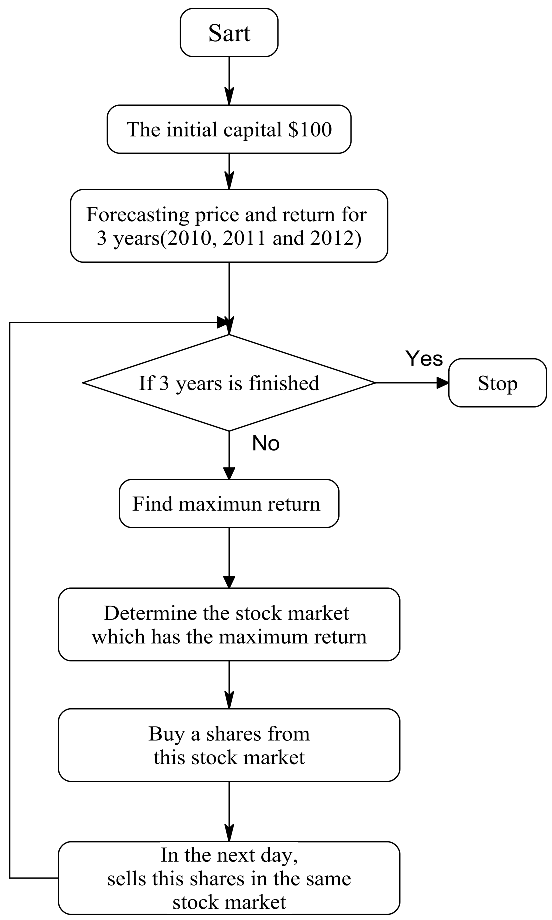


Figure 6.2: Block diagram of investment in all stock markets at the same time

Table 6.1 to Table 6.4 contains sample of investment date, price, shares, and return value of investment for each stock market, for example in NY stock market on 05/01/2010 the prediction price (NY p p =$1146.387), the return (NY p r= 0.0097), number of bought shares (shrb=0.087) with initial amount $100, in the next day (06/01/2010), the prediction price was $1157.649, as the return value was less than zero (-0.0064) ,it is recommended to sell the shares, so the shares are sold with

$100.9824. One day later, the capital increased by one dollar approximately. On 20/12/2011 the prediction price was $1246.005, the return was 0.0061, number of bought shares was 1.292 with amount $1611.067, in the next day (21/12/2011) the prediction price was $1253.694, as the return value was more than zero (0.0067). It is recommended to keep the shares rather than selling them. See appendix E.4.

Table 6.1: Investment date, stock market, and value in NYSE

Date	NY A p	NY p p	NYp r	Shr b	Shr s	Invp b	Invp s
05/01/2010	1136.52	1146.387	0.0097	0.087	0	100	0
06/01/2010	1137.14	1157.649	-0.0064	0	0.0872	0	100.9824
07/01/2010	1141.69	1150.164	0.0095	0.087	0	100.982	0
08/01/2010	1144.98	1161.243	-0.0049	0	0.0878	0	101.9551
09/01/2010	1146.98	1155.459	0.0054	0.088	0	101.9551	0
12/01/2010	1136.22	1156.94	-0.0044	0	0.0882	0	102.0857
13/01/2010	1145.68	1151.909	0.0065	0.088	0	102.0857	0
14/01/2010	1148.46	1159.461	-0.0156	0	0.0886	0	102.7551
15/01/2010	1136.03	1141.503	0.005	0.09	0	102.7551	0
16/01/2010	1150.23	1147.245	0.0015	0.09	0	102.7551	0
2011							
Date	NY A p	NY p p	NYp r	Shr b	Shr s	Invp b	Invp s
16/12/2011	1205.35	1224.848	0.0034	1.3107	0	1575.367	0
17/12/2011	1241.3	1229.093	-0.0255	0	1.3107	0	1611.067
20/12/2011	1243.72	1246.005	0.0061	1.292	0	1611.067	0
21/12/2011	1254	1253.694	0.0067	1.292	0	1611.067	0
22/12/2011	1265.33	1262.206	0.002	1.292	0	1611.067	0
23/12/2011	1265.43	1264.766	-0.0011	0	1.292	0	1635.324
24/12/2011	1249.64	1263.319	-0.0012	0	1.292	0	1635.324
29/12/2011	1263.02	1222.403	0.024	1.337	0	1635.324	0
30/12/2011	1257.6	1252.172	-0.005	0	1.337	0	1675.148
2012							
Date	NY A p	NY p p	NYp r	Shr b	Shr s	Invp b	Invp s
14/12/2012	1435.81	1394.778	-0.0156	0	6.6662	0	9431.54
15/12/2012	1443.69	1373.165	0.0259	6.8684	0	9431.54	0
18/12/2012	1430.15	1433.655	-0.0142	0	6.8685	0	9847.0119
19/12/2012	1426.66	1413.4	0.0084	6.9669	0	9847.0118	0
20/12/2012	1419.83	1425.326	-0.0149	0	6.9669	0	9930.1008
21/12/2012	1418.1	1404.207	0.0082	7.0717	0	9930.1008	0
22/12/2012	1402.43	1415.769	-0.0142	0	7.0717	0	10011.8619
25/12/2012	1426.19	1386.916	0.0066	7.2188	0	10011.8619	0
28/12/2012	1402.43	1382.087	-0.017	0	7.2187	0	9977.0023

Table 6.2: Investment date, stock market, and value in LSE

Date	LD A p	LD p p	LD p r	Shr b	Shr s	Invp b	Ivnp s
05/01/2010	8906.688	8943.095	0.0047	0.0112	0	100	0
06/01/2010	8879.521	8984.887	-0.01947	0	0.0112	0	100.4673
07/01/2010	8837.74597	8811.62	-0.0037	0	0.0112	0	100.4673
08/01/2010	8834.79688	8778.894	0.013	0.0114	0	100.467	0
09/01/2010	8872.0362	8894.067	0.0049	0.0114	0	100.467	0
12/01/2010	8860.60518	8917.91	-0.0083	0	0.0114	0	102.05824
13/01/2010	8825.4714	8843.481	0.0029	0.0115	0	102.0582	0
14/01/2010	8919.72986	8869.615	0.0039	0.0115	0	102.0582	0
15/01/2010	8889.5743	8904.704	0.0012	0.0115	0	102.0582	0
16/01/2010	8932.24608	8915.027	-0.0002	0	0.0115	0	102.8839
2011							
Date	LD A p	LD p p	LD p r	Shr b	Shr s	Invp b	Ivnp s
16/12/2011	8340.61786	8306.523	0.0163	0.5476	0	4548.6162	0
17/12/2011	8336.6735	8442.906	-0.0016	0	0.5475	0	4623.2991
20/12/2011	8404.17372	8458.106	0.0208	0.5466	0	4623.2991	0
21/12/2011	8399.30848	8636.226	0.0012	0.5466	0	4623.2991	0
22/12/2011	8558.7588	8646.05	0.0142	0.5466	0	4623.2991	0
23/12/2011	8641.70852	8769.677	-0.0141	0	0.5466	0	4793.6072
24/12/2011	8615.22582	8646.901	0.005	0.5543	0	4793.6072	0
29/12/2011	8690.88816	8706.916	-0.0105	0	0.5543	0	4826.8777
30/12/2011	8595.82998	8616.435	0.005	0.5602	0	4826.8777	0
2012							
Date	LD A p	LD p p	LD p r	Shr b	Shr s	Invp b	Ivnp s
14/12/2012	9551.27122	9502.816	-0.00976	0	1.9894	0	19013.0782
15/12/2012	9561.04812	9410.549	0.0121	2.0204	0	19013.0781	0
18/12/2012	9607.84774	9586.822	-0.0089	0	2.0204	0	19369.220
19/12/2012	9669.11904	9501.744	0.0194	2.0385	0	19369.22	0
20/12/2012	9692.96244	9688.506	-0.008	0	2.0384	0	19749.9325
21/12/2012	9659.034	9611.312	-0.0046	0	2.0384	0	19749.9325
22/12/2012	9625.55972	9566.505	-0.0114	0	2.0384	0	19749.9325
25/12/2012	9604.88133	9505.303	0.0039	2.0778	0	19749.9325	0
28/12/2012	9561.81798	9519.222	-0.0096	0	2.0777	0	19778.8538

Table 6.3: Investment date, stock market, and value in TSE

Date	TK A p	TK p p	TK p r	Shr b	Shr s	Invp b	Invp s
05/01/2010	114.8265	115.4228	0.00005	0.8664	0	100	0
06/01/2010	115.3619	115.4295	0.027083	0.8664	0	100	0
07/01/2010	117.7017	118.5984	-0.00131	0	0.8664	0	102.7513
08/01/2010	118.5826	118.4436	-0.01876	0	0.8664	0	102.7513
09/01/2010	115.9383	116.2426	-0.00177	0	0.8664	0	102.7513
12/01/2010	117.8029	117.5399	0.017045	0.8742	0	102.7513	0
13/01/2010	119.7049	119.5605	-0.00238	0	0.8742	0	104.5177
14/01/2010	119.4059	119.2765	-0.01802	0	0.8742	0	104.5177
15/01/2010	117.3374	117.1463	-0.00083	0	0.8742	0	104.5177
16/01/2010	118.1127	115.4228	0.006059	0.8929	0	104.5177	0
2011							
Date	TK A p	TK p p	TK p r	Shr b	Shr s	Invp b	Invp s
16/12/2011	108.9187	106.927	0.003805	6.4033	0	684.6869	0
17/12/2011	110.0632	107.3346	0.005748	0	6.4033	0	687.4528
20/12/2011	111.1575	107.359	-0.00029	6.4052	0	687.4528	0
21/12/2011	112.3158	107.3277	0.005377	0	6.4052	0	691.1591
22/12/2011	113.9627	107.9063	-0.00088	0	6.4052	0	691.1591
23/12/2011	113.9567	107.8112	-0.00046	6.41377	0	691.1591	0
24/12/2011	114.2093	107.7618	3.97E-05	6.41377	0	691.1591	0
29/12/2011	115.488	108.2896	0.010864	0	6.4138	0	702.1314
30/12/2011	115.0431	109.4725	-0.00158	0	6.4138	0	702.1314
2012							
Date	TK A p	TK p p	TK p r	Shr b	Shr s	Invp b	Invp s
14/12/2012	121.8112	119.0257	-0.00293	0	13.3411	0	1587.9374
15/12/2012	122.6631	118.6773	0.018016	13.3803	0	1587.9374	0
18/12/2012	122.6631	119.1056	-0.00183	0	13.3803	0	1593.6682
19/12/2012	121.9248	118.8875	0.008488	13.4048	0	1593.6681	0
20/12/2012	119.468	119.9009	-7.28E-05	0	13.4048	0	1607.2526
21/12/2012	118.2867	119.8922	0.016871	13.4058	0	1607.2526	0
22/12/2012	121.7413	121.9321	-0.00095	0	13.4058	0	1634.5985
25/12/2012	121.8112	121.1701	0.012903	13.4901	0	1634.5986	0
28/12/2012	122.6631	122.4198	-0.00122	0	13.4901	0	1651.457

Table 6.4: Investment date, stock market, and value in SSE

Date	SH A p	SH p p	SH p r	Shr b	Shr s	Invp b	Invp s
05/01/2010	474.886464	479.1875	-0.00091	0.2086	0	100	0
06/01/2010	480.182934	478.7534	-0.02677	0	0.2087	0	99.9094
07/01/2010	476.092386	466.1087	-0.00224	0	0.2087	0	99.9094
08/01/2010	466.784436	465.0677	0.006327	0.2148	0	99.9094	0
09/01/2010	467.2552	468.0197	-0.00169	0	0.2148	0	100.5436
12/01/2010	476.092386	468.1921	-0.00716	0	0.2148	0	100.5436
13/01/2010	480.182934	464.852	-0.00363	0	0.2148	0	100.5436
14/01/2010	476.092386	463.1671	0.016362	0.2171	0	100.5436	0
15/01/2010	466.784436	470.8079	-0.00061	0	0.2171	0	102.2022
16/01/2010	467.2552	470.5205	0.003231	0.2172	0	102.2022	0
2011							
Date	SH A p	SH p p	SH p r	Shr b	Shr s	Invp b	Invp s
16/12/2011	366.083787	346.5929	0.000213	1.4576	0	505.1772	0
17/12/2011	365.865643	346.6666	-0.00054	0	1.45755	0	505.2845
20/12/2011	367.404975	347.7359	0.001086	1.4531	0	505.2845	0
21/12/2011	366.94665	348.1138	-0.01625	0	1.45307	0	505.8337
22/12/2011	364.655025	342.502	-0.00157	0	1.45307	0	505.8337
23/12/2011	359.77178	341.9637	0.011464	1.4792	0	505.8337	0
24/12/2011	352.129194	345.9065	0.000213	1.4792	0	505.8337	0
29/12/2011	348.540528	341.0527	0.012587	1.4792	0	505.8337	0
30/12/2011	340.65658	345.3725	0.001814	1.4792	0	505.8337	0
2012							
Date	SH A p	SH p p	SH p r	Shr b	Shr s	Invp b	Invp s
14/12/2012	343.579936	342.574	0.004845	2.8113	0	886.2499	0
15/12/2012	344.76765	344.2379	-0.00238	0	2.8113	0	967.7702
18/12/2012	341.945628	344.6355	0.002026	2.8081	0	967.7702	0
19/12/2012	341.99352	345.3343	-0.0031	0	2.8081	0	969.7325
20/12/2012	350.857185	344.2669	-0.00015	0	2.8081	0	969.7325
21/12/2012	351.732105	344.2163	-0.00362	0	2.8081	0	969.7325
22/12/2012	349.63515	342.9717	1.54E-03	2.8274	0	969.7325	0
25/12/2012	353.523475	342.5775	0.028793	2.8274	0	969.7325	0
28/12/2012	353.523475	353.9074	0.002493	2.8274 0	0	969.7325	0

Tables from 6.5 to 6.7 show the investment among the stock market at the same time, for example on 19/01/2010 the best return value was in NY stock market (NY r=0.008777), the prediction price was (NY p p= 1156.966), the amount of money was (invp b= 103.272), and number of bought shares was (shr b =0.089261). In the next day (20/01/2010) the shares sold in the said market (shr s=0.089261), therefore, the capital became 104.5944 (it increased by 1.2184 approximately). See appendix D.

On 21/01/2010, the best return value was in TK stock market (0.001926), the prediction price was 115.3501, and the invested amount was 104.1824, the number of shares bought from this stock market was (shr b= 0.903184). On 22/01/2010, the amount of money obtained from the sale of shares in TK stock market was (invp s =104.3833), so it increased by $0.2. Additionally, on 23/01/2010, the best return value was in LD stock market (LD r=0.011975), the prediction price was (LD p p= 8572.58), the amount of money was (invp b= 104.3833), and the number of bought shares was (shr b= 0.012176). On 26/01/2010, the amount of money that was gained due to the sold the shares in LD stock market was (invp s =103.3762). So, it decreased by (-$1.0071). The best return was in LD stock market on 27/01/2010. One day later, on 28/01/2010, (shr s=0.012107) shares were sold in LD stock market for (invp s =$104.5944), so there was an increase by $1.2182. See appendix E.4.

Regarding the transactions in each stock market as mentions in Table 6.8, the number of transactions is calculated based on the frequency of sales transactions in each stock market.

Table 6.5: Investment date, stock market, and value in 2010

Date	LD A p	NY A p	SH AP	TK A p	LD p p	NY p p	SH p p	TK p p	LD r	NY r	SH r	TK r	M b	M s	Shr b	Shr s	Invp b	Invp s
05/01/2010	8906.688	1136.52	474.8864	114.8265	8943.095	1146.387	479.1875	115.4228	0.004662	0.0097	-0.00091	5.82E-05	NY		0.087231	0	100	0
06/01/2010	8879.521	1137.14	480.1829	115.3619	8984.887	1157.649	478.7534	115.4295	-0.01947	-0.00649	-0.02677	0.027083		NY	0	0.087231	0	100.9824
07/01/2010	8837.74597	1141.69	476.0923	117.7017	8811.62	1150.164	466.1087	118.5984	-0.00372	0.009586	-0.00224	-0.00131	NY		0.087798	0	100.9824	0
08/01/2010	8834.79688	1144.98	466.7844	118.5826	8778.894	1161.243	465.0677	118.4436	0.013034	-0.00499	0.006327	-0.01876		NY	0	0.087798	0	101.9551
09/01/2010	8872.0362	1146.98	467.2552	115.9383	8894.067	1155.459	468.0197	116.2426	0.004923	0.005357	-0.00169	-0.00177	NY		0.088238	0	101.9551	0
12/01/2010	8860.60518	1136.22	476.0923	117.8029	8917.91	1156.94	468.1921	117.5399	-0.00838	-0.00436	-0.00716	0.017045		NY	0	0.088238	0	102.0858
13/01/2010	8825.4714	1145.68	480.1829	119.7049	8843.481	1151.909	464.852	119.5605	0.002951	0.006535	-0.00363	-0.00238	NY		0.088623	0	102.0858	0
14/01/2010	8919.72986	1148.46	476.0923	119.4059	8869.615	1159.461	463.1671	119.2765	0.003948	-0.01561	0.016362	-0.01802		NY	0	0.088623	0	102.7551
15/01/2010	8889.5743	1136.03	466.7844	117.3374	8904.704	1141.503	470.8079	117.1463	0.001159	0.005018	-0.00061	-0.00083	NY		0.090017	0	102.7551	0
16/01/2010	8932.24608	1150.23	467.2552	118.1127	8915.027	1147.245	470.5205	117.0495	-0.00017	0.00153	0.003231	0.006059		NY	0	0.090017	0	103.272
19/01/2010	8992.41741	1138.04	470.3466	119.5525	9060.534	1156.966	475.6205	119.7989	0.00397	0.008777	-0.00018	-0.00034	NY		0.089261	0	103.272	0
20/01/2010	8878.72832	1116.48	478.9818	116.4961	9096.572	1167.165	475.5363	119.7582	-0.05915	-0.04275	-0.03429	-0.0375		NY	0	0.089261	0	104.1824
21/01/2010	8696.74651	1091.76	464.1601	115.6396	8574.087	1118.32	459.5071	115.3501	-0.0075	-0.00029	-0.00106	0.001926	TK		0.903184	0	104.1824	0
22/01/2010	8605.7084	1096.78	470.4349	113.5781	8509.995	1117.999	459.0213	115.5725	0.007327	-0.01618	-0.00878	-0.01736		TK	0	0.903184	0	104.3833
23/01/2010	8472.76521	1092.17	471.6931	113.7981	8572.58	1100.06	455.0099	113.5833	0.011975	0.009336	-0.00342	0.005569	LD		0.012176	0	104.3833	0
26/01/2010	8523.77657	1097.5	474.2351	115.5986	8489.874	1114.2	452.2312	114.0694	0.005727	-0.00957	-0.03776	-0.00646		LD	0	0.012176	0	103.3762
27/01/2010	8445.56725	1084.53	475.0171	113.1982	8538.638	1103.593	435.4727	113.3344	0.011715	0.010036	-0.00399	-0.00139	LD		0.012107	0	103.3762	0
28/01/2010	8317.5094	1073.87	461.1156	114.2962	8639.258	1114.724	433.7396	113.177	-0.02365	-0.02736	0.006883	0.019178		LD	0	0.012107	0	104.5944
29/01/2010	8401.2192	1089.19	462.1412	115.1191	8437.344	1084.639	436.7353	115.3684	0.0109	0.0183	-0.0034	-0.0025	NY		0.0964	0	104.5944	0
30/01/2010	8384.29572	1103.32	457.399858	115.4881	8530.002	1104.674	435.2398	115.0784	-0.00456	-0.00445	-0.0077	0.003493		NY	0	0.0964	0	106.5264
02/02/2010	8418.41022	1097.28	453.021624	114.9514	8431.515	1116.023	429.0406	114.5555	0.014451	0.012	-0.00353	0.001037	LD		0.012634	0	106.5264	0
03/02/2010	8378.6945	1063.11	441.736757	110.628	8554.24	1129.496	427.5274	114.6744	-0.03958	-0.05863	0.023883	-0.04732		LD	0	0.012634	0	108.077
04/02/2010	8209.51782	1066.19	436.941043	109.47	8222.277	1065.177	437.8608	109.3743	-0.00289	0.004463	-0.0011	0.00328	NY		0.101464	0	108.077	0
05/02/2010	8023.04477	1056.74	438.042682	109.2619	8198.511	1069.941	437.3792	109.7336	-0.01741	0.00425	-0.01939	0.005729		NY	0	0.101464	0	108.5604
06/02/2010	7973.01411	1070.52	437.333127	111.5967	8057.006	1074.498	428.9815	110.3641	0.019041	0.015488	0.00034	0.004965	LD		0.013474	0	108.5604	0

Table 6.6: Investment date, stock market, and value in 2011

Date	LD A p	NY A p	SH AP	TK A p	LD p p	NY p p	SH p p	TK p p	LND r	NY r	SH r	TK r	M b	M s	Shr b	Shr s	Invp b	Invp s
05/01/2011	9394.03377	1276.56	433.482252	127.7409	9270.778	1263.862	431.9564	129.1168	0.008319	0.004817	-0.0127	-0.01767		LD	0	0.085201	0	789.8798
06/01/2011	9360.3225	1273.85	427.938	126.2915	9348.22	1269.966	426.506	126.8552	-0.00432	-0.00314	-0.00098	-0.00126	SH		1.851978	0	789.8798	0
07/01/2011	9275.665	1271.5	434.454856	124.3217	9307.884	1265.983	426.0864	126.6953	-0.01335	0.000332	0.000123	-0.00312		SH	0	1.851978	0	789.1028
08/01/2011	9255.49457	1269.75	431.685	126.2103	9184.428	1266.403	426.1389	126.3003	-0.00672	-0.00514	-0.00103	0.001982	TK		6.24783	0	789.1028	0
11/01/2011	9345.756	1274.48	428.854044	126.6195	9242.453	1256.373	419.1231	126.8823	0.020307	0.02215	0.015934	-0.00815		TK	0	6.24783	0	792.7391
12/01/2011	9422.15004	1285.96	425.841032	125.863	9432.055	1284.513	425.8551	125.8525	0.001196	-0.00331	-0.00074	-0.00045	LD		0.084047	0	792.7391	0
13/01/2011	9429.81306	1283.76	419.71326	126.7918	9443.346	1280.263	425.542	125.7955	0.002333	0.007885	-0.00923	0.012127		LD	0	0.084047	0	793.6881
14/01/2011	9472.51422	1293.24	411.588294	125.3601	9465.402	1290.398	421.6328	127.3303	-0.00767	-0.00307	-0.00109	-0.00103	TK		6.233299	0	793.6881	0
15/01/2011	9495.71448	1295.02	414.655571	124.9026	9393.113	1286.445	421.1728	127.1991	0.006608	0.003704	-0.01683	0.002446		TK	0	6.233299	0	792.8701
18/01/2011	9615.14064	1281.92	416.143156	124.3215	9603.71	1290.551	409.0298	125.9447	-0.00269	-0.0047	-0.002	-0.00075	TK		6.295382	0	792.8701	0
19/01/2011	9535.82485	1280.26	424.581696	127.5798	9577.892	1284.497	408.2143	125.8498	-0.03453	-0.01156	-0.00426	0.003159		TK	0	6.295382	0	792.2727
20/01/2011	9383.94568	1283.35	424.581696	127.2626	9252.826	1269.732	406.4797	126.248	-0.01762	-0.0086	-0.00195	0.000526	TK		6.275525	0	792.2727	0
21/01/2011	9399.88146	1290.84	432.176475	128.6309	9091.233	1258.86	405.6863	126.3144	0.039247	0.017386	0.012858	0.000382		TK	0	6.275525	0	792.6893
22/01/2011	9506.07927	1291.18	428.910949	130.2821	9455.13	1280.938	410.9362	126.3626	0.004121	-0.00725	0.000297	0.000651	LD		0.083837	0	792.6893	0
25/01/2011	9451.75044	1296.63	427.58388	129.759	9356.698	1280.113	407.3242	125.4463	0.013948	0.00977	0.009549	-0.00028		LD	0	0.083837	0	784.4372
26/01/2011	9485.65572	1299.54	427.80716	129.5375	9488.123	1292.682	411.2322	125.4107	-0.00071	-0.00272	0.000958	-0.0014	SH		1.907529	0	784.4372	0
27/01/2011	9448.12189	1276.34	420.725767	129.3889	9481.401	1289.17	411.6264	125.2351	-0.02368	-0.01385	0.01312	-0.00739		SH	0	1.907529	0	785.1892
28/01/2011	9364.36508	1286.12	422.009525	130.8516	9259.554	1271.44	417.0624	124.3129	-0.01361	-0.00678	-0.00017	-0.0011	SH		1.882666	0	785.1892	0
29/01/2011	9291.52392	1307.59	425.45204	130.0347	9134.409	1262.845	416.993	124.1764	0.013775	0.009454	0.012253	0.020174		SH	0	1.882666	0	785.0585

Table 6.7: Investment date, stock market, and value in 2012

Date	LD A p	NY A p	SH AP	TK A p	LD p p	NY p p	SH p p	TK p p	LD r	NY r	SH r	TK r	M b	M s	Shr b	Shr s	Invp b	Invp s
04/12/2012	9422.6795	1418.07	323.055008	122.6631	9417.619	1391.06	315.241	116.4419	-0.00454	-0.01163	0.001608	6.63E-05		NY	0	9.031513	0	12563.38
05/12/2012	9487.45942	1418.55	328.030789	122.6631	9374.941	1374.979	315.7484	116.4496	0.010622	0.017655	0.024731	-0.00254	SH		39.7892	0	12563.38	0
06/12/2012	9502.43428	1427.84	330.902676	122.6631	9475.056	1399.47	323.6544	116.1545	-0.0059	-0.01565	0.000839	-0.00017		SH	0	39.7892	0	12877.95
07/12/2012	9514.49528	1428.48	329.8773	121.9248	9419.33	1377.74	323.9259	116.1346	0.005609	0.017244	0.019394	-0.00743	SH		39.75585	0	12877.95	0
08/12/2012	9498.2464	1419.45	330.945797	119.468	9472.31	1401.703	330.2695	115.2751	-0.00722	-0.01434	0.0036	6.14E-05		SH	0	39.75585	0	13130.14
11/12/2012	9507.255	1413.58	327.77532	118.2867	9381.942	1382.928	331.2747	116.0006	0.018504	0.022812	0.00062	0.015257	NY		9.494453	0	13130.14	0
12/12/2012	9565.16933	1430.36	341.94858	119.9534	9557.164	1414.838	331.4803	117.784	-0.00469	-0.01269	0.00266	-0.00225		NY	0	9.494453	0	13433.11
13/12/2012	9562.07296	1446.79	343.278026	121.7413	9512.412	1397.002	332.3631	117.519	-0.00101	-0.00159	0.03026	0.012739	SH		40.41699	0	13433.11	0
14/12/2012	9551.27122	1435.81	343.579936	121.8112	9502.816	1394.778	342.574	119.0257	-0.00976	-0.01562	0.004845	-0.00293		SH	0	40.41699	0	13845.81
15/12/2012	9561.04812	1443.69	344.76765	122.6631	9410.549	1373.165	344.2379	118.6773	0.012085	0.025933	-0.00238	0.018016	NY		10.08314	0	13845.81	0
18/12/2012	9607.84774	1430.15	341.945628	122.6631	9586.822	1433.655	344.6355	119.1056	-0.00891	-0.01423	0.002026	-0.00183		NY	0	10.08314	0	14455.74
19/12/2012	9669.11904	1426.66	341.99352	121.9248	9501.744	1413.4	345.3343	118.8875	0.019465	0.008403	-0.0031	0.008488	LD		1.521377	0	14455.74	0
20/12/2012	9692.96244	1419.83	350.857185	119.468	9688.506	1425.326	344.2669	119.9009	-0.008	-0.01493	-0.00015	-7.28E-05		LD	0	1.521377	0	14739.87
21/12/2012	9659.034	1418.1	351.732105	118.2867	9611.312	1404.207	344.2163	119.8922	-0.00467	0.0082	-0.00362	0.016871	TK		122.9427	0	14739.87	0
22/12/2012	9625.55972	1402.43	349.63515	121.7413	9566.505	1415.769	342.9717	121.9321	-0.01149	-0.01427	1.54E-03	-0.00095		TK	0	122.9427	0	14990.66
25/12/2012	9604.88133	1426.19	353.523475	121.8112	9505.303	1386.916	342.5775	121.1701	0.003981	0.0066	0.028793	0.012903	SH		43.75844	0	14990.66	0
28/12/2012	9561.81798	1402.43	353.523475	122.6631	9519.222	1382.087	353.9074	122.4198	-0.00968	-0.01706	0.002493	-0.00122		SH	0	43.75844	0	15486.43
29/12/2012	9526.71634	1426.19	359.203279	122.6631	9427.537	1358.714	354.7909	122.2709	0.006158	0.03441	0.008607	0.002541	NY		11.39786	0	15486.43	0

As a result, the capital in New York stock market increased from $100 to $9977, in London stock market the $100 became $19779, in Tokyo stock market, the invested capital increased to $1651 and, in Shanghai, it increased to $970. However, when testing the investment of $100 among these four stock markets at the same time, the initial value of investing $100 gave $15486 in return. The investment in London stock market had the highest rate of profit and the highest number of transactions in comparison with the other stock markets.

Table 6.8: Number of transaction in the stock markets

Stock market	Number of transactions
NY	86
LD	153
TK	68
SH	70

Chapter 7

CONCLUSIONS AND FUTURE WORK

7.1 Conclusions

This research has applied Auto-Regressive Moving Average (ARMA) model on the indexes of global stock market (New York, London, Tokyo and Shanghai) from 2008 to 2012 with the aim to predict the closing price and the feasibility of investment.

For each stock market, the best structural parameter set (r, m) of ARMA(r,m) model is searched among 225 cases: {ARMA(1,1), ... ARMA(15,15)}. The structural parameters r and m which are obtained by searching the minimum RMSE case has been overlapped with the parameters determined using the autocorrelation function (ACF) and the partial autocorrelation function (PACF) graphs. Searching the parameters with minimum RMSE is time consuming; however, it provides indication of prediction error, which cannot be obtained by the ACF and PACF method.

The closing values for the missing days of each time series data set are completed to further reduce the RMSE error. The predicted prices after completing the missing values provided extra reduction of RMSE. The mean absolute prediction errors in data set (closing price) after filling the missing values were approximately equal or less than the mean absolute prediction errors in the data with missing values. The prediction model with the preprocessing provides more accurate future values for investment.

The best ARMA(r,m) model which predicts the two-days-ahead future values with minimum RMSE error has been used to determine the market in which the capital shall be invested until that market gives negative future two-days-ahead return. The hypothesis of investing in multiple markets make higher profit compared to investing in a single market is tested by investing an initial $100 capital to each market, and to the highest returning market of all four markets.

In conclusion, the investment in London stock market gave the best result by raising the capital almost 200 times relative to the initial capital. However, the capital has increased only about 150 times when the capital has been invested in the highest returning market of the four global stock markets.

7.2 Future Work

This study shows that application of ARMA(r,m) model based on RMSE value predicts the future closing price of stock market accurately. However, the method may be improved further using other linear and non-linear models such as GARCH (Generalized Autoregressive Conditional Heteroskedasticity), NGARCH (Nonlinear GARCH), and IGARCH(Integrated GARCH). External factors that have an effect on stock market prices could not be neglected completely. Detection and analysis of external factors should be studied in the future to improve the accuracy of the prediction method.

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APPENDICES

Appendix A: Time Series Data Set

A.1: NY stock market data in 2008-2012(1)

Date	Open	High	Low	Close
15/01/2008	1411.88	1411.88	1380.6	1380.95
16/01/2008	1377.41	1391.99	1364.27	1373.2
17/01/2008	1374.79	1377.72	1330.67	1333.25
18/01/2008	1333.9	1350.28	1312.51	1325.19
22/01/2008	1312.94	1322.09	1274.29	1310.5
23/01/2008	1310.41	1339.09	1270.05	1338.6
24/01/2008	1340.13	1355.15	1334.31	1352.07
25/01/2008	1357.32	1368.56	1327.5	1330.61
28/01/2008	1330.7	1353.97	1322.26	1353.96
29/01/2008	1355.94	1364.93	1350.19	1362.3
2009				
15/01/2009	841.99	851.59	817.04	843.74
16/01/2009	844.45	858.13	830.66	850.12
20/01/2009	849.64	849.64	804.47	805.22
21/01/2009	806.77	841.72	804.3	840.24
22/01/2009	839.74	839.74	811.29	827.5
23/01/2009	822.16	838.61	806.07	831.95
26/01/2009	832.5	852.53	827.69	836.57
27/01/2009	837.3	850.45	835.4	845.71
28/01/2009	845.73	877.86	845.73	874.09
29/01/2009	868.89	868.89	844.15	845.14
2010				
15/01/2010	1147.72	1147.77	1131.39	1136.03
19/01/2010	1136.03	1150.45	1135.77	1150.23
20/01/2010	1147.95	1147.95	1129.25	1138.04
21/01/2010	1138.68	1141.58	1114.84	1116.48
22/01/2010	1115.49	1115.49	1090.18	1091.76
25/01/2010	1092.4	1102.97	1092.4	1096.78
26/01/2010	1095.8	1103.69	1089.86	1092.17
27/01/2010	1091.94	1099.51	1083.11	1097.5
28/01/2010	1096.93	1100.22	1078.46	1084.53
29/01/2010	1087.61	1096.45	1071.59	1073.87
2011				
14/01/2011	1282.9	1293.24	1281.24	1293.24
18/01/2011	1293.22	1296.06	1290.16	1295.02
19/01/2011	1294.52	1294.6	1278.92	1281.92
20/01/2011	1280.85	1283.35	1271.26	1280.26
21/01/2011	1283.63	1291.21	1282.07	1283.35

A.1: NY stock market data in 2008-2012(2)

Date	Open	High	Low	Close
24/01/2011	1283.29	1291.93	1282.47	1290.84
25/01/2011	1288.17	1291.26	1281.07	1291.18
26/01/2011	1291.97	1299.74	1291.97	1296.63
27/01/2011	1297.51	1301.29	1294.41	1299.54
28/01/2011	1299.63	1302.67	1275.1	1276.34
2012				
17/12/2012	1413.54	1430.67	1413.54	1430.36
18/12/2012	1430.47	1448	1430.47	1446.79
19/12/2012	1446.79	1447.75	1435.8	1435.81
20/12/2012	1435.81	1443.7	1432.82	1443.69
21/12/2012	1443.67	1443.67	1422.58	1430.15
24/12/2012	1430.15	1430.15	1424.66	1426.66
26/12/2012	1426.66	1429.42	1416.43	1419.83
27/12/2012	1419.83	1422.8	1401.8	1418.1
28/12/2012	1418.1	1418.1	1401.58	1402.43
31/12/2012	1402.43	1426.74	1398.11	1426.19

A.2: Sample of data in target stock market (1)

Date	LD	TK	SH
10/01/2008	12238.80636	128.548196	750.27425
11/01/2008	12143.516	122.891041	753.595032
12/01/2008	12149.55687	123.8702158	754.3837713
13/01/2008	12155.59775	124.8493905	755.1725107
14/01/2008	12161.63862	125.8285653	755.96125
15/01/2008	11807.1632	126.80774	749.609883
16/01/2008	11654.62119	128.909997	729.575119
17/01/2008	11575.78688	125.263836	710.92494
18/01/2008	11605.69305	116.929365	713.874278
19/01/2008	11372.06976	118.1506313	701.8166107
2009			
09/01/2009	6727.46655	91.979105	278.300046
10/01/2009	6721.09394	91.249014	278.080409
11/01/2009	6714.72133	90.518923	277.860772
12/01/2009	6708.34872	89.788832	277.641135
13/01/2009	6594.69081	89.058741	272.238357
14/01/2009	6131.68602	92.17768	281.807907
15/01/2009	6001.14582	92.47672	280.542681
16/01/2009	6055.18071	90.336848	285.543684
17/01/2009	6055.01929	89.19724533	287.0470627
18/01/2009	6054.85787	88.05764267	288.5504413
19/01/2009	6054.69645	86.91804	290.05382
2010			
15/01/2010	8889.5743	117.33741	471.693145
16/01/2010	8903.798227	117.5958467	472.54048
17/01/2010	8918.022153	117.8542833	473.387815
18/01/2010	8932.24608	118.11272	474.23515
19/01/2010	8992.41741	119.55251	475.017081
20/01/2010	8878.72832	116.49605	461.115655
21/01/2010	8696.74651	115.63959	462.141218
22/01/2010	8605.7084	113.57808	457.399858
23/01/2010	8561.394003	113.651416	455.9404467
24/01/2010	8517.079607	113.724752	454.4810353
2011			
15/01/2011	9480.24764	127.4334087	418.324068
16/01/2011	9487.98106	127.5871593	414.597528
17/01/2011	9495.71448	127.74091	410.870988
18/01/2011	9615.14064	126.291451	410.41047
19/01/2011	9535.82485	124.321692	418.12796

A.2: Sample of data in target stock market (2)

Date	LD	TK	SH
21/01/2011	9399.88146	126.619482	411.637964
22/01/2011	9435.28073	126.367318	410.9185987
23/01/2011	9470.68	126.115154	410.1992333
24/01/2011	9506.07927	125.86299	409.479868
2012			
22/12/2012	9647.875907	121.758754	341.961592
23/12/2012	9636.717813	121.776224	341.977556
24/12/2012	9625.55972	121.793694	341.99352
25/12/2012	9618.666923	121.811164	350.857185
26/12/2012	9611.774127	122.663124	351.732105
27/12/2012	9604.88133	122.663124	349.63515
28/12/2012	9561.81798	122.663124	353.523475
29/12/2012	9550.117433	122.663124	355.416743
30/12/2012	9538.416887	122.663124	357.310011
31/12/2012	9526.71634	122.663124	359.203279

Appendix B: The currency rate with dollar 2008-2012 (1)

Date	CNY/USD	GBP/USD	JPY/USD
15/01/2008	0.1377	1.9595	0.0092
16/01/2008	0.1379	1.9611	0.0093
17/01/2008	0.138	1.9612	0.0094
18/01/2008	0.1378	1.9665	0.0093
19/01/2008	0.1379	1.9643	0.0093
20/01/2008	0.1379	1.9549	0.0094
21/01/2008	0.1379	1.9549	0.0094
22/01/2008	0.1379	1.9503	0.0094
23/01/2008	0.138	1.9475	0.0094
24/01/2008	0.1381	1.9561	0.0094
2009			
13/01/2009	0.1461	1.4991	0.0111
14/01/2009	0.1461	1.4667	0.0112
15/01/2009	0.1461	1.4562	0.0112
16/01/2009	0.1461	1.4601	0.0112
17/01/2009	0.1461	1.4792	0.0111
18/01/2009	0.146	1.4728	0.011
19/01/2009	0.146	1.4737	0.011
20/01/2009	0.1461	1.4695	0.011
21/01/2009	0.146	1.412	0.0111
22/01/2009	0.146	1.3841	0.0112
2010			
15/01/2010	0.1463	1.6295	0.0109
16/01/2010	0.1463	1.63	0.011
17/01/2010	0.1465	1.6259	0.011
18/01/2010	0.1465	1.6257	0.011
19/01/2010	0.1463	1.6311	0.011
20/01/2010	0.1463	1.6379	0.011
21/01/2010	0.1463	1.6301	0.011
22/01/2010	0.1462	1.6228	0.011
23/01/2010	0.1463	1.618	0.0111
24/01/2010	0.1463	1.61	0.0111
2011			
15/01/2011	0.1515	1.5848	0.0121
16/01/2011	0.1517	1.5864	0.0121
17/01/2011	0.1518	1.5864	0.0121
18/01/2011	0.1515	1.5876	0.0121
19/01/2011	0.1516	1.5955	0.0121
20/01/2011	0.1517	1.5992	0.0122
21/01/2011	0.1516	1.5942	0.0121
22/01/2011	0.1517	1.5936	0.0121

Appendix B: The currency rate with dollar 2008-2012 (2)

Date	CNY/USD	GBP/USD	JPY/USD	Date
23/01/2011	0.1517	1.5995	0.0121	23/01/2011
24/01/2011	0.1519	1.5993	0.0121	24/01/2011
2012				
22/12/2012	0.1586	1.6229	0.0119	22/12/2012
23/12/2012	0.1584	1.6167	0.0119	23/12/2012
24/12/2012	0.1584	1.6166	0.0119	24/12/2012
25/12/2012	0.1585	1.6159	0.0118	25/12/2012
26/12/2012	0.1585	1.6132	0.0118	26/12/2012
27/12/2012	0.1585	1.6131	0.0118	27/12/2012
28/12/2012	0.1583	1.6137	0.0117	28/12/2012
29/12/2012	0.1584	1.6122	0.0116	29/12/2012
30/12/2012	0.1583	1.6153	0.0116	30/12/2012

Appendix C: Sample of data forecasting by ARMA(r,m) model (1)

Date	LD p	NY p	SHp	TKp	LNr	NY r	SH r	TK r
01/01/2010	8789.054	1125.299	478.7512	115.7822	0.009658	0.006559	-0.00098	-0.00166
02/01/2010	8874.354	1132.703	478.2804	115.5897	0.003915	0.005332	-0.00527	-0.00416
03/01/2010	8909.16	1138.759	475.7648	115.1093	0.002798	0.010662	-0.00115	0.000944
04/01/2010	8934.121	1150.965	475.2196	115.2181	0.001004	-0.00399	0.008315	0.001776
05/01/2010	8943.095	1146.387	479.1875	115.4228	0.004662	0.009776	-0.00091	5.82E-05
06/01/2010	8984.887	1157.649	478.7534	115.4295	-0.01947	-0.00649	-0.02677	0.027083
07/01/2010	8811.62	1150.164	466.1087	118.5984	-0.00372	0.009586	-0.00224	-0.00131
08/01/2010	8778.894	1161.243	465.0677	118.4436	0.013034	-0.00499	0.006327	-0.01876
09/01/2010	8894.067	1155.459	468.0197	116.2426	0.004923	0.005357	-0.00169	-0.00177
10/01/2010	8937.964	1161.665	467.228	116.0372	-0.00361	-0.00847	0.003278	0.01028
11/01/2010	8905.737	1151.873	468.7621	117.2362	0.001366	0.004389	-0.00122	0.002587
12/01/2010	8917.91	1156.94	468.1921	117.5399	-0.00838	-0.00436	-0.00716	0.017045
13/01/2010	8843.481	1151.909	464.852	119.5605	0.002951	0.006535	-0.00363	-0.00238
14/01/2010	8869.615	1159.461	463.1671	119.2765	0.003948	-0.01561	0.016362	-0.01802
15/01/2010	8904.704	1141.503	470.8079	117.1463	0.001159	0.005018	-0.00061	-0.00083
16/01/2010	8915.027	1147.245	470.5205	117.0495	-0.00017	0.00153	0.003231	0.006059
17/01/2010	8913.534	1149.002	472.0431	117.7609	0.001159	0.009663	-0.00196	-0.00072
18/01/2010	8923.87	1160.158	471.1188	117.676	0.015198	-0.00275	0.00951	0.017879
19/01/2010	9060.534	1156.966	475.6205	119.7989	0.00397	0.008777	-0.00018	-0.00034
2011								
01/01/2011	9021.2	1249.268	424.2196	126.8941	0.026413	0.013675	0.00046	0.00858
02/01/2011	9262.655	1266.469	424.4145	127.9875	0.003463	-0.00211	0.00066	-0.00332
03/01/2011	9294.787	1263.8	424.6945	127.5635	0.003404	0.003022	0.017877	0.014422
04/01/2011	9326.48	1267.626	432.3551	129.4166	-0.00599	-0.00297	-0.00092	-0.00232
05/01/2011	9270.778	1263.862	431.9564	129.1168	0.008319	0.004817	-0.0127	-0.01767
06/01/2011	9348.22	1269.966	426.506	126.8552	-0.00432	-0.00314	-0.00098	-0.00126
07/01/2011	9307.884	1265.983	426.0864	126.6953	-0.01335	0.000332	0.000123	-0.00312
08/01/2011	9184.428	1266.403	426.1389	126.3003	-0.00672	-0.00514	-0.00103	0.001982
09/01/2011	9122.936	1259.914	425.7022	126.5508	0.015162	0.003315	-0.01457	0.002551
10/01/2011	9262.312	1264.098	419.5434	126.8741	-0.00215	-0.00613	-0.001	6.48E-05
11/01/2011	9242.453	1256.373	419.1231	126.8823	0.020307	0.02215	0.015934	-0.00815
12/01/2011	9432.055	1284.513	425.8551	125.8525	0.001196	-0.00331	-0.00074	-0.00045
13/01/2011	9443.346	1280.263	425.542	125.7955	0.002333	0.007885	-0.00923	0.012127
14/01/2011	9465.402	1290.398	421.6328	127.3303	-0.00767	-0.00307	-0.00109	-0.00103
15/01/2011	9393.113	1286.445	421.1728	127.1991	0.006608	0.003704	-0.01683	0.002446
16/01/2011	9455.392	1291.219	414.1431	127.5106	-0.00681	-0.0039	-0.00174	-0.00074
17/01/2011	9391.176	1286.195	413.4236	127.4168	0.022379	0.003381	-0.01068	-0.01162
18/01/2011	9603.71	1290.551	409.0298	125.9447	-0.00269	-0.0047	-0.002	-0.00075
19/01/2011	9577.892	1284.497	408.2143	125.8498	-0.03453	-0.01156	-0.00426	0.003159
2012								
11/12/2012	9381.942	1382.928	331.2747	116.0006	0.018504	0.022812	0.00062	0.015257

Appendix C: Sample of data forecasting by ARMA(r,m) model (2)

Date	LD p	NY p	SHp	TKp	LNr	NY r	SH r	TK r
12/12/2012	9557.164	1414.838	331.4803	117.784	-0.00469	-0.01269	0.00266	-0.00225
13/12/2012	9512.412	1397.002	332.3631	117.519	-0.00101	-0.00159	0.03026	0.012739
14/12/2012	9502.816	1394.778	342.574	119.0257	-0.00976	-0.01562	0.004845	-0.00293
15/12/2012	9410.549	1373.165	344.2379	118.6773	0.012085	0.025933	-0.00238	0.018016
16/12/2012	9524.966	1409.242	343.4203	120.8348	-0.00577	-0.01377	0.001257	-0.00183
17/12/2012	9470.165	1389.973	343.8523	120.6142	0.012243	0.030942	0.002275	-0.01259
18/12/2012	9586.822	1433.655	344.6355	119.1056	-0.00891	-0.01423	0.002026	-0.00183
19/12/2012	9501.744	1413.4	345.3343	118.8875	0.019465	0.008403	-0.0031	0.008488
20/12/2012	9688.506	1425.326	344.2669	119.9009	-0.008	-0.01493	-0.00015	-0.00007
21/12/2012	9611.312	1404.207	344.2163	119.8922	-0.00467	0.0082	-0.00362	0.016871
22/12/2012	9566.505	1415.769	342.9717	121.9321	-0.01149	-0.01427	1.54E-03	-0.00095
23/12/2012	9457.225	1395.706	343.5016	121.8157	0.01367	0.008914	-0.003	-0.00312
24/12/2012	9587.393	1408.203	342.4057	121.4361	-0.0086	-0.01523	5.02E-04	-0.00219
25/12/2012	9505.303	1386.916	342.5775	121.1701	0.003981	0.0066	0.028793	0.012903
26/12/2012	9543.223	1396.1	352.5849	122.7437	-0.00931	-0.01682	0.001678	-0.00091
27/12/2012	9454.772	1372.808	353.177	122.6325	0.006794	0.006736	0.002066	-0.00174
28/12/2012	9519.222	1382.087	353.9074	122.4198	-0.00968	-0.01706	0.002493	-0.00122
29/12/2012	9427.537	1358.714	354.7909	122.2709	0.006158	0.03441	0.008607	0.002541

Appendix D: Investment date, stock market, and value

Date	LD p	NY p	SHp	TKp	LNr	NY r	SHr	TKr	M b	M s	shr b	shr s	inv b	Inv s
15/01/2010	8904.704	1141.503	470.8079	117.1463	0.001159	0.005018	-0.00061	-0.00083	NY		0.09	0	102.7551	0
16/01/2010	8915.027	1147.245	470.5205	117.0495	-0.00017	0.00153	0.003231	0.006059		NY	0	0.090017	0	103.272
19/01/2010	9060.534	1156.966	475.6205	119.7989	0.00397	0.008777	-0.00018	-0.00034	NY		0.089	0	103.272	0
20/01/2010	9096.572	1167.165	475.5363	119.7582	-0.05915	-0.04275	-0.03429	-0.0375		NY	0	0.089261	0	104.1824
21/01/2010	8574.087	1118.32	459.5071	115.3501	-0.0075	-0.00029	-0.00106	0.001926	TK		0.9	0	104.1824	0
22/01/2010	8509.995	1117.999	459.0213	115.5725	0.007327	-0.01618	-0.00878	-0.01736		TK	0	0.903184	0	104.3833
23/01/2010	8572.58	1100.06	455.0099	113.5833	0.011975	0.009336	-0.00342	0.005569	LD		0.012176	0	104.3833	0
26/01/2010	8489.874	1114.2	452.2312	114.0694	0.005727	-0.00957	-0.03776	-0.00646		LD	0	0.012176	0	103.3762
27/01/2010	8538.638	1103.593	435.4727	113.3344	0.011715	0.010036	-0.00399	-0.00139	LD		0.012107	0	103.3762	0
28/01/2010	8639.258	1114.724	433.7396	113.177	-0.02365	-0.02736	0.006883	0.019178		LD	0	0.012107	0	104.5944
2011														
14/12/2011	8346.927	1208.683	347.8211	107.7365	0.003334	-0.00566	-0.00011	0.000522	LD		0.257438	0	2148.817	0
15/12/2011	8374.805	1201.857	347.7817	107.7928	-8.19E-03	0.018948	-0.00342	-0.00806		LD	0	0.257438	0	2155.994
16/12/2011	8306.523	1224.848	346.5929	106.927	0.016285	0.00346	0.000213	0.003805	LD		0.259554	0	2155.994	0
17/12/2011	8442.906	1229.093	346.6666	107.3346	-0.00156	-0.02552	-0.00054	0.005748		LD	0	0.259554	0	2191.393
20/12/2011	8458.106	1246.005	347.7359	107.359	0.02084	0.006151	0.001086	-0.00029	LD		0.259088	0	2191.393	0
21/12/2011	8636.226	1253.694	348.1138	107.3277	0.001137	0.006767	-0.01625	0.005377		LD	0	0.259088	0	2237.542
22/12/2011	8646.05	1262.206	342.502	107.9063	0.014197	0.002026	-0.00157	-0.00088	LD		0.258794	0	2237.542	0
23/12/2011	8769.677	1264.766	341.9637	107.8112	-0.0141	-0.00114	0.011464	-0.00046		LD	0	0.258794	0	2269.536
24/12/2011	8646.901	1263.319	345.9065	107.7618	0.00497	-1.23E-03	0.000213	3.97E-05	LD		0.262468	0	2269.536	0
29/12/2011	8706.916	1222.403	341.0527	108.2896	-0.01045	0.024061	0.012587	0.010864		LD	0	0.262468	0	2285.288
2012														
14/12/2012	9502.816	1394.778	342.574	119.0257	-0.00976	-0.01562	0.004845	-0.00293		SH	0	40.41699	0	13845.81
15/12/2012	9410.549	1373.165	344.2379	118.6773	0.012085	0.025933	-0.00238	0.018016	NY		10.08314	0	13845.81	0
18/12/2012	9586.822	1433.655	344.6355	119.1056	-0.00891	-0.01423	0.002026	-0.00183		NY	0	10.08314	0	14455.74
19/12/2012	9501.744	1413.4	345.3343	118.8875	0.019465	0.008403	-0.0031	0.008488	LD		1.521377	0	14455.74	0
20/12/2012	9688.506	1425.326	344.2669	119.9009	-0.008	-0.01493	-0.00015	-7.28E-05		LD	0	1.521377	0	14739.87
21/12/2012	9611.312	1404.207	344.2163	119.8922	-0.00467	0.0082	-0.00362	0.016871	TK		122.9427	0	14739.87	0
22/12/2012	9566.505	1415.769	342.9717	121.9321	-0.01149	-0.01427	1.54E-03	-0.00095		TK	0	122.9427	0	14990.66
25/12/2012	9505.303	1386.916	342.5775	121.1701	0.003981	0.0066	0.028793	0.012903	SH		43.75844	0	14990.66	0
28/12/2012	9519.222	1382.087	353.9074	122.4198	-0.00968	-0.01706	0.002493	-0.00122		SH	0	43.75844	0	15486.43
29/12/2012	9427.537	1358.714	354.7909	122.2709	0.006158	0.03441	0.008607	0.002541	NY		11.39786	0	15486.43	0

Appendix E: The source code of this work

E.1:Pre-processing closing price

The following MATLAB codes contain pre-processing operations on the raw data of LD stock market as an example from four stock markets.

```
clc; clear all;
% London pre-processing closing price
% London preprocess.xlsx :it is excel file contain Date
%for 5 years from(1/1/2008-31/12/2012),Date of stock
%market working day and Closing price with missing values

[num,txt,raw] = xlsread('london preprocess.xlsx');
N=length(raw);
Close1(N,1)=0;
for i=2:N
date_all=cell2mat(raw(i,1));
for j=2:N
date2=cell2mat(raw(j,2));
if date_all==date2
Close1(i-1,1)=num(j-1,1);
end
end
end
% Converting the closing price currency from (GBP) to USD
currency
%lndcurrency.m : if file in matlab contain the rate of
GBP currenct to USD

load lndcurrency.m;
lndusd=lndcurrency;
Close1=Close1.*lndusd;

% Using the value of next cell in each cell have value
=0, Closenext(i) are
% new Close value after substitute next cell value
for i=1:N
if Close1(i,1)==0 &&(i+1~=N+1)&& ( Close1(i+1,1)~=0)
Closenext(i,1)=Close1(i+1,1);
else if Close1(i,1)==0 &&(i+1~=N+1)&& (Close1(i+1,1)==0)
Closenext(i,1)=Closenext(i-1,1);
else Closenext(i,1)=Close1(i,1);
end
end
end
%use the value of previous cell in each cell have value
=0,Closepre(i)are
```

```
%new close values after substitute previous cell value
for i=1:N
if Close1(i,1)==0 &&(i-1~=0)&& (Close1(i-1,1)~=0)
Closepre(i,1)=Close1(i-1,1);
else if Close1(i,1)==0 &&(i-1~=0)&& (Close1(i-1,1)==0)
Closepre(i,1)=Closepre(i-1,1);
else Closepre(i,1)=Close1(i,1);
end
end
end
%using the interpolation method in the cell with value=0

Y = Close1(:,1);
Xi =1:length(Y);
errors = Y == 0;
X = Xi(~errors);
Y = Y(~errors);
Yi = interp1(X, Y, Xi);
% Yiv is closing price vector after interpolation method
Yiv=Yi';
Closeinter=Yiv(:,1);
```

E.2: Selection of best ARMA(*r*,*m*) model

The following MATLAB codes show the operation of selection the best ARMA(*r*,*m*) model in the range ARMA(1,1) to ARMA(15,15) in LD stock market

```
%% LONDON ARMA(r,m) SELECTION

clc; clear all;
% load LD closing price after pre-processing it by
interpolation method
load LNDclsint.m;
y=LNDclsint;       % y is closing price
r = price2ret(y); %r is the return of clocing price
N=length(r);
k=0;

for i=1:15
for j=1:15
% specifying the model
model = arima(i,0,j);
%fitting reteurn according to model specified by previous
step
fit = estimate(model,r);
%spesifying the AR and MA coefficient values
model = arima('AR',fit.AR,'MA',fit.MA,...
'Constant',fit.Constant ,'Variance',fit.Variance);
```

```
c=1;
for d=1:2:1096  %counter for 3 years

% fitting two years data set
fit = estimate(model,r(d:730+d));
%RMSE is prediction root mean square error for each model
%Yf is the forecasting 3 years return price

[Yf(c:c+1,j+k)  RMSE(c:c+1,j+k)] =
forecast(fit,2,'Y0',r(d:730+d));

c=c+2;

end

if i>=100 && j>=100
f(j+k,:)=[num2str(i),'.',num2str(j)];
elseif
((((i>=10)&&(i<100))&&(j>=100))||((i>=100)&&((j>=10)&&(j<
100))))
f(j+k,:)=[num2str(i),'.',num2str(j),' '];
elseif((i<10&&j>=100)||((i>=10&&i<100)&&(j>=10&&j<100))||
...
(i>=100&&j<10))
f(j+k,:)=[num2str(i),'.',num2str(j),'  '];
elseif ((i<10 && (j>=10&&
j<100))||((i>=10&&(i<100))&&j<10))
f(j+k,:)=[num2str(i),'.',num2str(j),'   '];
elseif i<10 && j<10
f(j+k,:)=[num2str(i),'.',num2str(j),'    '];
end

end
k=k+j;

end

% plotting the models, x-axis contain the ARMA(1,1)to
ARMA(15,15)
%y-axis contain values of RMASE for corresponding to the
models
figure();
plot(1:k,min(RMSE),'r-o');
set(gca,'XTick',1:k);
set(gca,'XTickLabel',{f});
xlabel('ARMA(r,m) model');
ylabel(' RMSE in LD');

grid on;

save('LNDarmaselection2');
```

E.3: Forecasting two –days-ahead using best ARMA(*r*,*m*) model

The codes below show the forecasting of two –days-ahead according to minimum RMSE value

```
%ARMA FITTING of LONDON STOCK MARKET
% load LD closing price after pre-processing it by
interpolation method
load LNDclsint.m;
y=LNDclsint;
r=price2ret(y);
N=length(y);
autocorr(r); %Autocorrelation function
parcorr(r);  %partial autocorrelation function
%The best ARMA model is ARMA(7,8)in LD stock market
model = arima(7,0,8);
%fitting the closing price using ARMA(7,8) model
fit = estimate(model,y);

%specifying the AR value and MA value

model = arima('AR',fit.AR,'MA',fit.MA,...
'Constant',fit.Constant ,'Variance',fit.Variance);

c=1;
for d=1:2:1096  % counter to forecast 3 years
fit = estimate(model,y(d:730+d)); % fitting two years
data set
%forcasting 2 days ahead based on its previous 2 years
closing price,
%RMSE is root mean squar error
%Yf is the forecasting 3 years closing price
% E is the error ralated with MA part

[Yf(c:c+1,1)  RMSE(c:c+1,1)] =
forecast(fit,2,'Y0',y(d:730+d));
[E,V] = infer(model,y(d:730+d));

c=c+2;
end
% plotting the actual and forecasting price
figure();
[num,txt,raw] = xlsread('dateto2012.xlsx');
h1 =plot(y,'Color',[.7,.7,.7]);
hold on
h2 = plot(731:N,Yf,'b','LineWidth',1);
set(gca,'XTick',[1:365:N]);
set(gca,'XTickLabel',cell2mat(raw(1:365:N)));
```

```
xlabel('(2008-2012) Period ');
ylabel('Closing Price');
legend([h1
h2],'Observed','Forecast','Location','NorthWest');
hold off;
grid on;

%obtaining the absolute prediction error in London stock
market

abser=abs(y(731:end,1)-Yf(:,1));

%m contain the mean of absolute prediction error

m=mean(abser);

% plotting the absolute prediction error in London stock
market
plot(abser);
set(gca,'XTick',[1:365:1096]);
set(gca,'XTickLabel',cell2mat(raw(1:365:1096)));
grid on;
xlabel('(2010-2012) Period ');
ylabel('The Error in price (USD)');

save('LND ARMA(7,8)fitting');
```

E.4: Investment operation in stock markets

The following codes show the investment operation in each stock market separately and investment money among stock markets in the same time. Initial value of investment is $100.

```
%The Investment: The selling and buying operation among
stock markets.

%STOCKFORECAST3yprmis.xlsx: excel file contain price and
return of
%stock market for (2010-2012) period

[num,txt,raw] = xlsread('STOCKFORECAST3yprmis.xlsx');
buy(1,1)=100; sel(1,1)=0;
b=1;
s=2;
for i=1:2:755
mxr(i,1)=max(num(i,5:8)); %find MAX return
```

```
[ro,co] = find(num==max(num(i,5:8)));
if co==5
bsname(b,:)='LD'; % LD=London stock market
elseif co==6
bsname(b,:)='NY'; % NY= New York
elseif co==7
bsname(b,:)='SH'; %SH=Shanghai
else
bsname(b,:)='TK'; %TK= Tokyo
end
% sharb(b,1)=the number of shares that have been
purchased
% bsname(b,:)= contains the stock market where purchased
%ssname(s,:)= contains the stock market name where
selling
sharb(b,1)=buy(b,1)/num(i,((co(1,1))-4));
ssname(s,:) =bsname(b,:);

%sel(s,1)=amount of yield from selling of shares
% shars(s,1)=the number of shares that have been sold
sel(s,1)=sharb(b,1)*num(i+1,((co(1,1))-4));
shars(s,1)=sharb(b,1);
b=b+2;
%buy(b,1)=the amount of mony to purchase shares
buy(b,1)=sel(s,1);
s=s+2;
end

save('invtbtweenstock');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Invest in LD stock market
%STOCKFORECAST3yprmis.xlsx: excel file contain price and
return of
%stock market for (2010-2012)priod
[num,txt,raw] = xlsread('STOCKFORECAST3yprmis.xlsx');
buy(1,1)=100;sharb(1,1)=buy(1,1)/num(1,1);shars(1,1)=0;
sel(1,1)=0; b=2;
s=2;
for i=2:754
if num(i,5)> 0 && sharb(b-1,1)==0    %%buying process

sharb(b,1)=sel(s-1,1)/num(i,1);
sel(s,1)=0;
shars(s,1)=0;
buy(b,1)=sel(s-1,1);
else
if  num(i,5)> 0 && sharb(b-1,1)~=0
sharb(b,1)=sharb(b-1,1);
buy(b,1)=buy(b-1,1);
sel(s,1)=0;
shars(s,1)=0;
```

```
end
end

if num(i,5)<0 && sharb(b-1,1)~=0     %%selling process
sel(s,1)=sharb(b-1,1)*num(i,1);
shars(s,1)=sharb(b-1,1);
sharb(b,1)=0;
buy(b,1)=0;
else if num(i,5)<0 && sharb(b-1,1)==0
sel(s,1)=sel(s-1,1);
shars(s,1)=shars(s-1,1);
sharb(b,1)=0;
buy(b,1)=0;
end
end
b=b+1;
s=s+1;
end
% save the variable value in file name ‘invLD’
save('invtLD');
```