Ranking All Units in DEA

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ABSTRACT

Data Envelopment Analysis (DEA) is a methodology to compare efficiency of Decision Making Units (DMUs). DEA is an extension of Charnes, Cooper and Rhodes work by introducing CCR model in 1978. Ranking DMUs is one of the main purposes of DEA in management and engineering. DEA evaluates some DMUs with efficiency score one as efficient DMUs and we therefore need to produce a reliable method for fully ranking DMUs. Some methods have been proposed in this concept and newly Khodabakhshi and Aryavash (2012) ranked DMUs relative to their combined maximum and minimum efficiency scores where efficiency is defined as ratio of weighted sum of outputs to weighted sum of inputs. Due to some obtained weights (multipliers) in DEA may be zero, previous methods have low ability in ranking DMUs because of eliminating the effect of corresponding input and outputs on DEA evaluations. We expand their method by assigning lower bounds on multipliers using facet analysis and then we propose an equitable and precise method for ranking all DMUs based on the modified CCR.

Keywords: Data envelopment analysis, decision making unit, rank.

ÖZ

Veri zarflama analizi, karar alma birimlerinin etkinlik lerini karşilaştirmaya yarayan. Veri zarflama analizi Charnes, Cooper ve Rhodes un 1978 de ki CCR model adinda ki çalışmalarinin geliştilirmiş halidir. Veri zarflama analizin karar alma birimlerinin sıralaması, mühendisilik ve işletme alanlarında ki esas konulardandir. Çoğu zaman veri zarflama analizi birden fazla etkili Kara Alma Birimi tanimlamasi için onlarin arasinda güvenilir ve bütünsel bir sıralama zorunluluğu söz konusu olabilir. Bu nedenle bazı methodlar Veri Zarflama Analizi modellerini, esasında karar alma birimlenin sıralama amacı ile tanimlanmışdir. Yakın zamanda Khodabakhshi ve Aryavash (2012) karar alma birimlerinin en yüksek ve en düşük verim puanlamalarının kombinasyon esasında sıralamayı başarmışlardır. Tanımlanan ışlemde etki, toplam ağırlıklı çıktıların, toplam ağırlıklı girdilere oranı ile ifade edilir. Ama bazen Veri Zarflama Analizi sonuçlarında bazı cıktıları ve girdilerinin ağırlıklarının sıfır olması bundan önceki methotların özellikle Khodabakhshi ve Aryavash methodunun düşük performansina sebeb olmaktadır. bundandolayı bu tür verilerin tesiri sıralama sonuçlarında ihmal ediliyor. Bu tezde Khodabakhshi ve Aryavash metodunun genişletilip, bunu yüzey analizi aracılığıyla, modelde yeralan ağırlıklara aşağıdan sınırlamak prensibiyle, sonraki aşamada değiştirilmiş CCR modeline dayanarak kesin bir karar alma birimini sıralama methotu sunuyoruz.

Anahtar Kelimeler: Veri zarflama analizi, karar alma birimi, sıralaması.

To My Family

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LIST OF ABBREVIATIONS

DEA..Data Envelopment Analysis

DMU..Decision Making Unit

PPS..Production Possibility Set

LP..Linear Programming

FP..Fractional Programming

RTS..Return to Scale

Chapter 1

INTRODUCTION

1.1 Preamble

Nowadays, change and competition are the main characteristics in this world and only organizations can achieve their objectives which are able to allocate their available resources effectively in these complex and dynamic conditions. Using modern technologies and determination of opportunities and restrictions depend on identification of present status. In this regard, performance evaluation plays the significant role and it can be used to identify strengths and weaknesses in organizations. One of the most important techniques in evaluating performance is Data Envelopment Analysis (DEA). This technique has been used extensively and successfully to improve efficiency in a wide variety of organizations.

Since the objective of all the evaluation systems is obviously to perform a precise and exact evaluation, DEA is being increasingly recognized as a key evaluation tool. Efficiency measurement of different sectors of an organization, performance comparison of organizations, and major capabilities determination of the sectors can reflect in productivity improvement and DEA as a strategic management tactic and operations research-based methodology can be used to assess the performance of comparable Decision Making Units (DMUs like organizations and systems) in different parts consisting management, education, finance, industry, even strategies and policies. DEA aids managers establish where to look to improve efficiency and the extent of improvement in which is likely to be achieved.

1.2 Problem Description

DEA employs mathematical models for evaluating and these models do not indicate sensible and valid results in some cases. Many papers have proposed the methods for improving these models and erasing their difficulties, but these methods have their limitations and one of the difficulties is shortage of discrimination in DEA uses, specifically when the number of DMUs is not enough or the number of DMUs is too small in comparison with the number of inputs and outputs and DEA cannot produce a full ranking of the efficient units, particularly if three times of the total number of outputs and inputs be greater than the number of DMUs, DEA will evaluate most of them as efficient DMUs.

DEA ranks DMUS in efficient and inefficient groups and it does not provide full ranking. DEA does not need any supposition of the input-output functions and DMUs assess their efficiencies by themselves with the input and output weights which are only most satisfactory. The dynamic system in the choice of input and output weights frequently allows some DMUs to be evaluated as DEA efficient, causing them unable to be completely classified. In other words, the usage of variable weights prevents various DMUs being fully ranked and basically compared. It is important to be considered some criterions for ranking these DMUs while analysts and managers are often interested in a full ranking besides the dichotomous sorting to enable them evaluates all DMUs. The research reported in this thesis details a new approach for ranking all DMUs.

This study introduces facet analysis in the basic DEA models. These models are used to assess the efficiency of the observed DMU in comparison with the efficient frontier which envelope all of DMUS that form Production Possibility Set (PPS). A section of this frontier may be contained the weak efficient parts including the weak efficient DMUs. Under the focus of hyperplanes of the efficient and the weak efficient frontiers, we try to modify the CCR models for eliminating the effect of the weak efficient frontier (the weak frontier) in evaluation using facet analysis. Regarding this subject, we consider input and output weighs as the normal vector of hyperplanes which envelop the PPS in the efficient DMUs located on the efficient frontier. Using facet analysis, we improve the CCR models through replacing the admissible hyperplanes with the weak hyperplanes.

Additionally, non-Archimedean element epsilons are applied as lower bound on input and output weights (multipliers) in the CCR model for discriminating the weak efficient and the strong efficient DMUs. Introducing epsilons in the CCR model move the weak frontier hyperplanes of PPS and this movement revises their efficiency scores. In a similar manner, these non-Archimedean elements change the efficiency scores of DMUs which compared with the weak frontier in the CCR model but this improvement cannot provide precise evaluation for the weak efficient DMUs and DMUs compared with weak efficient DMUs. Using facet analysis, we try to determine the lower bounds on each weight and modify the CCR model in a way that the efficiency scores of the weak efficient DMUs and DMUs compared with efficient DMUs, have been evaluated correctly. To validate the modified CCR model, numerical examples are implemented in the next sections.

1.3 Assumptions

In DEA literature, the same inputs and outputs are used for all DMUs and we assume that all data are positive. The data (choice of DMUs and their outputs/inputs) must express a manager's or an analyst's interest in a manner that will come into the efficiency evaluation of the DMUs. In general, higher outputs and lower input amounts are preferred and the efficiencies values should reveal these effects. Moreover, the measuring units of different inputs and outputs are not necessarily same.

There are some orientations for evaluating efficiency values of the observed DMUs in the DEA models. For instance, input-oriented model attempts to minimize the input amounts by whilst satisfying given outputs and output orientation tries to increase output amounts with keeping given input levels. During the research, we deal with the input orientations of the DEA models and output-oriented models can be developed for future studies. Furthermore, all computations are done using the GAMS, LINGO, and WINQSB and the obtained results are supplied in Appendix.

1.4 Structure of Thesis

We organize this thesis as follows. The next chapter starts with a short presentation of DEA literature. Then, facet analysis is illustrated in Chapter 3 and modification of the CCR model, which is one of the most basic models in DEA, has been discussed in Chapter 4. We propose a new ranking method based on the CCR model using facet analysis in Chapter 5 and finally Chapter 6 concludes with a summary and suggestions. Figure 1.1 depicts structure of the main sections of this thesis in the next page.

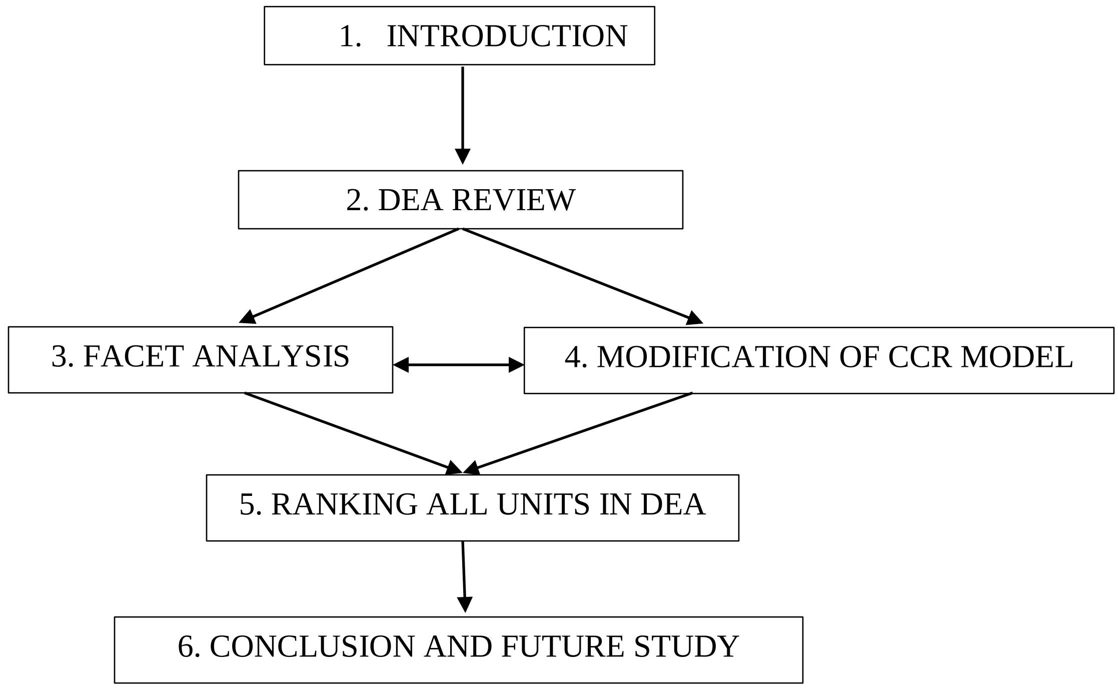

Figure 1.1: Structures of Main Sections of Thesis

Chapter 2

DEA REVIEW

2.1 Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA) is an approach to assess the efficiency of a set of entities named Decision Making Units (DMUs) that produce various outputs using various inputs. Decision making is one of the most prominent subjects which the people always deal with it, even in their normal life. We may meet with diverse alternatives in which we should choose the best action.

In recent years a large amount of DEA applications have used for assessing performance of various types of entities involved in various places and activities. DEA evaluates the performances of different forms of DMUs such as organizations, schools, courts, business companies, industrial firms, department stores, bank branches, production centers, purchasing offices, powerhouses, refineries, cities, countries, regions, and so on. DEA is most effective where the organization uses multiple types of inputs (resources) to produce multiple outputs (services or products) and where production function is not well defined or well known. Since DEA does not need lots of functional assumptions between inputs and outputs for DMUs, it has been applied to other methods as well. DEA has been developed to provide new ideas for entities which have been assessed by other approaches. For example, DEA application have acknowledged various resources of efficiency in benchmarking studies of the successful companies and it has supplied a tool for specifying the best benchmarks in numerous useful areas. The power of DEA is extremely enhanced. For example, financial service businesses have regularly identified ways to decrease operating costs by 20% to over 30% without decreasing their service levels by means of DEA.

DEA is frequently used in industrial and research applications and it has proved highly in improving manufacturing productivity as effective as productivity improvement of service operations. DEA determines the improved standards and it can be used to identify the best practice of manufacturing operations. Modern manufacturing companies which use production systems, integrated manufacturing, Just in Time (JIT), and customized manufacturing use DEA to consider the multiple dimensions of the manufacturing process and develop their standards as much as possible. In various industries, DEA helps managers to see the advantages and disadvantages of new technologies designed to improve their performance. This record should supply enough information for managers and researchers to seriously see the sights of the DEA potential in the engineering and management areas.

2.2 How Does DEA Work?

DEA employs mathematical models for observational data to identify relations such as the efficient frontiers and the production functions which are basic concepts of engineering and economics.

DEA measures efficiencies of DMUs with given inputs and outputs by organizing a practically based best-performance. The DEA results show the hyperplanes which define an envelope surface or efficient frontier. Efficient DMUs are DMUs which rest on the surface efficient and other DMUs as inefficient DMUs which are not laid on the efficient surface. DEA measures the distance between DMUs to the envelope surface or efficient frontier as their relative efficiencies. Since DEA attempts to determine relative efficiencies and this frontier or surface "envelops" inefficient DMUs which are below the efficient frontier; this method is named Data Envelopment Analysis (DEA).

DMUs define Production Possibility Set (PPS) and some elements of this set are boundary DMUs that form efficient surface or frontier and DEA tries to estimate the relative position of each DMU relative to efficient surface in PPS. Generally, DEA determines the following objectives:

1. Best performance or most productive group of DMUs (efficient DMUs);
2. Inefficient or less productive DMUs in comparison with efficient DMUs;
3. Excess levels of inputs used by inefficient DMUs;
4. Capacity of output levels to be considered for inefficient DMUs;
5. Best performance DMUs which signify that excess resources are being used by the inefficient DMUs;
6. The efficiency of merger and break up of DMUs;

This information implies that DMUs productivity can be improved and the amount of resource savings and output increases which the inefficient DMUs expect to meet the efficiency levels of the efficient DMUs.

DEA is a technique focused on frontiers rather than central tendencies. In contrast with DEA, statistical regression tries to fit a hyperplane over center of entities and this property has been proved DEA highly in discovering relations which are being unseen from other methods.

Commonly, relative efficiency is obtained by dividing total weighted of outputs by total weighted of inputs and this definition of efficiency often called "technical efficiency". Full efficiency (100%) is attained for the observational DMU when its outputs or inputs cannot be improved any more by improving some of its other inputs or outputs in DEA concept.

DEA is a nonparametric method which does not need any functional forms of the production between inputs and outputs whereas other methods for estimating production functions which necessarily assume several limitations which are meaningless in a case for estimating a parametric form.

The superiority of DEA over other techniques results from the fact that other techniques are not designed to manage productivity or are less well-matched to the types of organizations in which are used. Another reason may be attributed which is useful in conjunction with DEA and there are certain situations where DEA either cannot be used or is not the most appropriate technique for productivity management. The distinct benefits of DEA take account in particular where there are restrictions and limitations of some common types of analyses like standards, profitability analysis, and ratios.

2.3 Sensitivity Analysis in DEA

Effect of input or output changes should be verified for determining efficiency scores in various economic conditions. In DEA, this is achieved through sensitivity analysis. Sensitivity analysis demonstrates how DEA models are responsive to data changes and deals with methods for studying effects on evaluation when data are varied. Sensitivity analysis further examines the effects when inputs or outputs are added or omitted or when DMUs are added or removed.

2.4 DEA Background

The definitions of performance relation with effective relations lead to generate a function as a production function that aims to produce maximum possible outputs using inputs. Obviously, estimation of this function is very difficult and impossible in some cases. DEA is an extension of Farrell work [2] in introducing first non-parametric approach for estimating production function. He determined Production Possibility Set (PPS) and estimated production function as a part of this set named efficient frontier and defined efficient DMUs which lie on this frontier.

Charnes, Cooper, and Rhodes [1] presented initial DEA model in 1987 based on the prior work of Farrell. This model was formulated in thesis work of Rhodes at Carnegie Mellon University in USA. Under the supervision of Cooper, this method was motivated to assess educational programs for disadvantaged students in a large number of studies in U.S. universities. Then Charnes joined them in which guaranteed to complete study effectively. It was verified that a fractional programming (FP) model change into a linear programming (LP) one to evaluate efficiency. Using the previous study of Charnes and Cooper, which had based on the fractional modeling, enabled Charnes to replace the dual linear programming models developed by Cooper and Rhodes with the equivalent format and it produced a reference for expanding applications and uses of DEA with former works to the performance evaluation applied in large areas of studies such as management and engineering.

After DEA was originated in its initial form in 1978, such fast growth and extensive acceptance of DEA is evidence to its powerfulness. Investigators have rapidly realized that DEA is a superior method for analyzing production processes, and its empirical base without needing any suppositions of functional relations has led to its use in a large series of studies such as production functions and efficient production frontiers.

2.5 CCR Model

This section introduces CCR model which is one of the principal DEA models. The name CCR model was originated by acronym of Charnes, Cooper and Rohdes [1]. Suppose that the number of DMUs is n and each DMU uses m inputs to produce s outputs. Let x_{ij} and y_{rj} ($i = 1 \dots m, j = 1 \dots n, r = 1 \dots s$), which are assumed to be non-negative for all DMUs, be inputs and outputs of DMU_j, respectively. Let DMU_o $(o = 1 \dots n)$ be DMU under study and consider the input weights (v_i) and the output weights (u_r). Then the CCR model evaluates the efficiency of DMU_o to obtain input and output weights in the following programming model. Since the number of DMUs is n, this model should be run n times for optimizing all DMUs.

$$Max\ \theta = \sum_{r=1}^{s} u_r y_{ro} / \sum_{i=1}^{m} v_i x_{io}$$

$$subject\ to\ (s.\ t.): \quad \sum_{r=1}^{s} u_r y_{rj} / \sum_{i=1}^{m} v_i x_{ij} \leq 1 \qquad j = 1 \dots n$$

$$v_i, u_r \geq 0 \qquad i = 1 \dots m,\ r = 1 \dots s$$

The second constraints show that value of relative efficiency should not exceed one for every DMU.

This model is a fractional programing (FP) problem and the linear programming (LP) is given by,

$$
\begin{aligned}
Max \quad & \theta = \sum_{r=1}^{s} u_r y_{ro} \qquad\qquad (2.1)\\
s.t \quad & \sum_{i=1}^{m} v_i x_{io} = 1\\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad j = 1,2,...,n\\
& v_i \ge 0 \quad i = 1,2,...,m\\
& u_r \ge 0 \quad i = 1,2,...,s
\end{aligned}
$$

This model refers to input orientation of the CCR model and it tries to minimize input amounts without increasing output amounts. There are other types of the CCR model for evaluating efficiency values of the observed DMUs in DEA. For example, output-oriented model that aims to increase output levels whereas satisfying at most the present input levels.

The above linear programming model can be expressed as a vector form (multiplier form) as follow:

$$
\begin{aligned}
Max \quad & \theta = UY_o \qquad\qquad (2.2)\\
s.t \quad & VX_o = 1\\
& UY_j - VX_j \le 0 \quad j = 1,2,...,n\\
& U \ge 0\\
& V \ge 0
\end{aligned}
$$

Definition 2.1

A. If $\theta^* = 1$ and $V^* > 0$ and $U^* > 0$ be optimal solutions of the CCR model for DMU under evaluation, this DMU is said to be CCR-efficient.

B. If $\theta^* = 1$ and $V^* \ge 0$, $U^* \ge 0$ be optimal solutions of the CCR model for DMU under evaluation and there is at least one V^* or U^* with zero values, this DMU is said to be CCR-weak efficient.

C. Otherwise, DMU under evaluation is CCR-inefficient, that is $\theta^* \neq 1$.

Less efficient DMUs can be improved through sending them to the efficient frontier. Efficiency of one DMU can be improved by minimizing inputs proportionally in input orientations while output-oriented models try to increase output levels.

As mentioned earlier, all data presumed to be equal or greater than zero. We now assume that there is at least one input and one output with positive values for all DMUs. This property is called semipositive assumption and a pair of such semipositive inputs $X \in R^m$ and outputs $Y \in R^n$ define an activity and denote it by *(X, Y).*

We now define a set of feasible activities as Production Possibility Set (PPS) and denote it by S_C . We express the following properties of S_C:

(C1) All observational activities (X_j, Y_j) include in S_C .*(j = 1 … n)*

(C2) If activity *(tX, tY)* is included in S_C for any positive scalar *t*, we refer this property as constant Return to Scale (RTS) assumption.

(C3) For all activities *(X, Y)* included in S_C, any activity $(\bar{X}, \bar{Y})$ with $\bar{X} \geq X$ and $\bar{Y} \leq Y$ belongs to S_C.

(C4) All linear combinations of activities in S_C are included in S_C.

According to above assumptions, S_C can be defined as:

$$S_C = \{(X,Y) \mid X \geq \sum_{j=1}^{n} \lambda_j X_j,\ Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \lambda_j \geq 0, \forall j\} \qquad (2.3)$$

Now based on the constraints of model 1.2, the dual model of this linear programming model is given by the real variable θ and the nonnegative variables $\lambda_j \geq 0 \quad j = 1 \ldots n$ as follows:

$$
\begin{aligned}
& Min \quad \theta \qquad\qquad\qquad (2.4) \\
& s.t \quad -\sum_{j=1}^{n} \lambda_j X_j + \theta X_0 \ge 0 \\
& \qquad \sum_{j=1}^{n} \lambda_j Y_j \ge Y_0 \\
& \qquad \lambda_j \ge 0,\ j = 1, \ldots, n. \\
& \qquad \theta \ free
\end{aligned}
$$

We next can write the above model as a vector form (envelopment form) where $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ for *j=1, 2… n* as follows:

$$
\begin{aligned}
& Min \quad \theta \qquad\qquad\qquad (2.5) \\
& s.t \quad \theta X_o - X_j \lambda \ge 0 \\
& \qquad Y_j \lambda - Y_o \ge 0 \\
& \qquad \lambda \ge 0 \\
& \qquad \theta \quad free
\end{aligned}
$$

Table 2.1 depicts the relations between primal-dual variables and constraints in the CCR model.

Table 2.1: Primal and Dual Relations in CCR Model

Constraints of (2.2)	Variables of (2.5)	Constraints of (2.5)	Variables of (2.2)
$VX_o = 1$	θ	$\theta X_o - \lambda X_j \ge 0$	$V \ge 0$
$-VX_j + UY_j \le 0$	$\lambda_j \ge 0$	$\lambda Y_j - Y_o \ge 0$	$U \ge 0$

Since there is a feasible solution $\theta = 1, \quad \lambda_o = 1, \quad \lambda_j = 0\,(j \ne o)$ of model 2.5, the optimal θ (θ^*) may not exceed one. We also assumed that all data to be nonnegative in model 2.5, thus λ cannot be zero because $Y_o \ge 0$ and $Y_o \ne 0$. We can see that θ^* should be less than 1 and greater than zero ($0 \le \theta^* \le 1$).

Notice that model 2.5 shows activity $(\theta X_o, Y_o)$ belonging to S_C so as to minimize θ while the input vector X_o is reduced to θX_o in S_C that is:

$$
\begin{aligned}
&Min \quad \theta && (2.6)\\
&s.t \quad (\theta X_o, Y_o) \in S_C\\
&\qquad \theta \quad free
\end{aligned}
$$

Here we determine the slack variables $s^- \in R^m$, $s^+ \in R^s$ such that $s^- \geq 0$ and $s^+ \geq 0$ for any feasible solution (θ, λ) of model 2.5:

$$
\begin{aligned}
s^- &= \theta X_o - \lambda X_j\\
s^+ &= \lambda Y_j - Y_o
\end{aligned}
$$

Thus, we rewrite model (2.5) for all *j=1, 2… n* as follows:

$$
\begin{aligned}
&Min \quad \theta && (2.7)\\
&s.t \quad \theta X_o - X_j\lambda - s^- = 0\\
&\qquad Y_j\lambda - Y_o + s^+ = 0\\
&\qquad \lambda \geq 0\\
&\qquad \theta \quad free\\
&\qquad s^- \geq 0\\
&\qquad s^+ \geq 0
\end{aligned}
$$

Definition 2.2

A. If an optimal solution $(\theta^*, \lambda^*, s^{-*}, s^{+*})$ of model 2.7 satisfies $\theta^* = 1$ for all slack variables with zero values, DMU_o is CCR-efficient. On the other hand, if DMU_o has no output shortfalls and input excesses, it is CCR-efficient.

B. If for above optimal solution $\theta^* = 1$and all slack variables are not equal to zero, that is $s^- \geq 0$ and $s^+ \geq 0$, the DMU_o called CCR-weak efficient.

C. If $\theta^* \neq 1$ then DMU_o is CCR-inefficient.

2.6 BCC Model

Different models have been developed built on the initial CCR model. Banker, Charnes and Cooper introduced a variable Return to Scale (RTS) version of the CCR model, namely the BCC model [3] in 1984. They defined a new PPS denoted by S_B including following properties:

(B1) All observational activities (X_j, Y_j) include in S_B. *(j = 1 … n)*

(B2) If the activities (X_j, Y_j) belongs to S_B and then the convex combination of these activities $(\sum_{j=1}^{n}\lambda_j X_j, \sum_{j=1}^{n}\lambda_j Y_j), \sum_{j=1}^{n}\lambda_j = 1,\ \lambda_j \geq 0 \quad j = 1,2,...,n$ also belongs to S_B.

(B3) For all activities *(X, Y)* included in S_B, any activity $(\bar{X}, \bar{Y})$ with $\bar{X} \geq X$ and $\bar{Y} \leq Y$ belongs to S_B.

(B4) All linear combinations of activities in S_B are included in S_B.

This PPS can be shown as:

$$S_B = \{(X,Y) \mid X \geq \sum_{j=1}^{n}\lambda_j X_j,\ Y \leq \sum_{j=1}^{n}\lambda_j Y_j, \sum_{j=1}^{n}\lambda_j = 1, \lambda_j \geq 0, \forall j\} \qquad (2.8)$$

Regarding this subject, the BCC model can be formulated in the following model:

$$\begin{aligned} &Min \quad \theta && (2.9) \\ &s.t \quad (\theta X_o, Y_o) \in S_B \\ &\qquad \theta \quad free \end{aligned}$$

According to 2.8, the above model can be replaced as follows:

$$Min \quad \theta \qquad (2.10)$$

$$s.t \quad -\sum_{j=1}^{n}\lambda_j X_j + \theta X_0 \ge 0$$

$$\sum_{j=1}^{n}\lambda_j Y_j \ge Y_0$$

$$\sum_{j=1}^{n}\lambda_j = 1$$

$$\lambda_j \ge 0, \ j = 1,\ldots,n.$$

$$\theta \ \ free$$

We now rewrite the above model as a vector form:

$$Min \quad \theta \qquad (2.11)$$

$$s.t \quad \theta X_o - X\lambda \ge 0$$

$$Y\lambda - Y_o \ge 0$$

$$1\lambda = 0$$

$$\lambda \ge 0$$

$$\theta \quad free$$

The dual model of this problem (multiplier side) is given by:

$$Max \quad \theta = \sum_{r=1}^{s} u_r y_{ro} + u_o \qquad (2.12)$$

$$s.t \quad \sum_{i=1}^{m} v_i x_{io} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_o \le 0 \quad j = 1,2,\ldots,n$$

$$v_i \ge 0 \quad i = 1,2,\ldots,m$$

$$u_r \ge 0 \quad i = 1,2,\ldots,s$$

$$u_o \quad free$$

Hence model 2.12 can be replaced as a vector form:

$$Max \quad \theta = UY_o + u_o \qquad (2.13)$$

$$s.t \quad VX_o = 1$$

$$UY_j - VX_j + u_o \le 0 \quad j = 1,2,\ldots,n$$

$$U \ge 0$$

$$V \ge 0$$

$$u_o \ \ free$$

Table 2.2 depicts the relations between primal-dual variables and constraints in the BBC model.

The important consideration is that the difference between BCC models and CCR models results from variable u_o and this variable is associated with the convexity constraint $1\lambda = 1$ in PPS as its dual variable.

In a similar manner, slack variables can be added to model 2.11 as follows:

$$
\begin{aligned}
&Min \quad \theta && (2.14)\\
&s.t \quad \theta X_o - X_j\lambda - s^- = 0\\
&\qquad Y_j\lambda - Y_o + s^+ = 0\\
&\qquad 1\lambda = 1\\
&\qquad \lambda \geq 0\\
&\qquad \theta \quad free\\
&\qquad s^- \geq 0\\
&\qquad s^+ \geq 0
\end{aligned}
$$

Table 2.2: Primal and Dual Relations in BCC Model

Constraints of (2.14)	Variables of (2.11)	Constraints of (2.11)	Variables of (2.14)
$VX_o = 1$	θ	$\theta X_o - \lambda X_j \geq 0$	$V \geq 0$
$-VX_j + UY_j + u_o \leq 0$	$\lambda_j \geq 0$	$\lambda Y_j - Y_o \geq 0$	$U \geq 0$
		$1\lambda = 1$	u_o

Definition 2.4

A. If an optimal solution $(\theta^*, \lambda^*, s^{-*}, s^{+*})$ of model 2.14 satisfies $\theta^* = 1$ and $s^{-*} = 0, s^{+*} = 0$, DMU under evaluation is said to be BCC-efficient.

B. If an optimal solution $(\theta^{*}, \lambda^{*}, s^{-*}, s^{+*})$ of model 2.14 satisfies $\theta^{*} = 1$ and $s^{-*} \neq 0, s^{+*} \neq 0$, DMU under evaluation is said to be BCC-weak efficient.

C. If $\theta^{*} \neq 1$, then DMU_o is BCC-inefficient.

2.7 Non-Archimedean Element Epsilon

One difficulty that has been discussed in DEA concept is evaluating some less efficient DMUs as efficient DMUs when some of their input and output weights are equal to zero. Since some of input and output weights are equal to zero, corresponding input and output cannot reflect in evaluating the efficiency. In this regard, we try to specify non-Archimedean element ε as lower bound on weights to eliminate this difficulty. Epsilon is usual non-Archimedean infinitesimal element referred to a small positive value. Introducing these non-Archimedean elements as minimum weight restriction in the basic DEA models impose the positivity on input or output weights. Inappropriate determinations of epsilon values often lead to infeasibility in the multiplier side and unboundedness in the envelopment side. Therefore, estimating the appropriate value of epsilon is one of the most important topics in DEA.

As an illustration, we consider Example 2.1 with two DMUs, two inputs and one output and Table 2.3 shows the data of these DMUs.

Table 2.3: Data of Example 2.1

DMU	***1***	***2***
Input X_1	2	2
Input X_2	5	6
Output Y	1	1

Now applying the classic CCR model for these DMUs, we have:

$$\begin{aligned} &Max \quad u \\ &s.t \quad 2v_1 + 6v_2 = 1 \\ &\qquad u - 2v_1 - 6v_2 \leq 0 \\ &\qquad u - 2v_1 - 5v_2 \leq 0 \\ &\qquad u \geq 0 \\ &\qquad v_1 \leq 0 \\ &\qquad v_2 \leq 0 \end{aligned} \qquad \begin{aligned} &Max \quad u \\ &s.t \quad 2v_1 + 5v_2 = 1 \\ &\qquad u - 2v_1 + 6v_2 \leq 0 \\ &\qquad u - 2v_1 - 5v_2 \leq 0 \\ &\qquad u \geq 0 \\ &\qquad v_1 \geq 0 \\ &\qquad v_2 \geq 0 \end{aligned}$$

Both these linear programming problems have the same optimal solution of $u^* = 1, \quad v_1^* = 0.5,\ v_2^* = 0$. The associated objective values for both of these problems are equal to one and so DMU_1 and DMU_2 are efficient. Since output values of these two DMUs are the same and DMU_2 requires more inputs than DMU_1, this shows that DMU_2 is inefficient. This results from $v_2^* = 0$ and then linear programming models show them as efficient DMUs because the second input weight of DMU_2 is not affected by efficiency evaluation in DEA.

In order to eliminate this problem, Charnes, Cooper and Rohdes used the non-Archimedean element in DEA concept [4] through imposing non-negativity constraints in the CCR model in which $v_i \geq 0$, $i=1,2,\ldots,m$ and $u_r \geq 0$, $r=1,2,\ldots,r$ replace with $v_i \geq \varepsilon$, $i=1,2,\ldots,m$ and $u_r \geq \varepsilon$, $r=1,2,\ldots,s$ and the CCR mode is revised by introducing new intervals for weights. Hence the effect of weak efficient DMUs is eliminated using non-Archimedean element in the CCR model.

Hence the CCR model is restructured by inserting epsilons as weight minimum as follows:

$$
\begin{aligned}
Max \quad & \sum_{r=1}^{s} u_r y_{ro} \qquad\qquad (2.15) \\
s.t \quad & \sum_{i=1}^{m} v_i x_{io} = 1 \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \geq 0 \quad j = 1,2,...,n \\
& u_r \geq \varepsilon \quad r = 1,2,...,s \\
& v_i \geq \varepsilon \quad i = 1,2,...,m
\end{aligned}
$$

The dual format of the above model is given by:

$$
\begin{aligned}
Min \quad & \theta - \varepsilon[\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+] \qquad (2.16) \\
s.t \quad & \sum\nolimits_{j=1}^{n} \lambda_j X_j + s_i^- = \theta X_0 \quad i = 1,,...,m \\
& \sum\nolimits_{j=1}^{n} \lambda_j Y_j - s_r^+ = Y_0 \quad r = 1,,...,s \\
& \lambda_j \geq 0 \quad j = 1,...,n. \\
& s_i^- \geq 0 \quad i = 1,,...,m \\
& s_r^+ \geq 0 \quad r = 1,,...,s \\
& \theta \ free
\end{aligned}
$$

2.8 Ranking Methods Review

All efficiency values of DMUs are obtained between zero and one using DEA methodologies. In this regard, DEA usually evaluate more than one DMU with unity values as efficient DMU. Hence researchers are motivated to find new methods to distinguish efficient DMUs.

Efficiency evaluation which enables decision makers to fully rank DMUs is one of the most important purposes of DEA. There are many approaches for ranking DMUs and we introduce some ranking methods in this section.

Sexton et al. [5] submitted the cross efficiency method for ranking DMUs in DEA concept. Doyle and Green suggested the cross evaluation matrix regarding the fact that analysts cannot usually have a realistic procedure for selecting assurance regions to sort DMUs. In this methodology, efficiency value of each DMU is computed n times obtaining optimal weights by n linear problems and all obtained efficiency values are summarized in a cross-efficiency matrix to compare all DMUs. This method seems to be reasonable, but it has some drawbacks particularly when there are some alternative solutions in the linear problems of DEA.

Wang et al. carried out a ranking by assigning a suitable minimum weight restriction for all inputs and outputs. But, their method has some problems. One of the major problems is high calculations and comparisons, particularly when there are large numbers of efficient units. The second problem may occur when reassessing the efficiencies remain some DMUs with efficiency score one. It means that partial ranking of all units by this method cannot produce full ranking of all DMUs to distinguish between DEA efficient DMUs.

Anderson and Petersen introduced a model (AP model) [6] to rank efficient DMUs through eliminating them from PPS and then a method was developed to improve the AP model by Mehrabian et al. [7] namely the MAJ model. since the values of some inputs of some DMUs are comparatively too small, infeasibility and instability may occur in the AP model and the MAJ model because these models are too sensitive to small values in data and eliminating some of DMUs obtain very large θ which may not be ranked properly. Avoiding this problem, Seiford et al. suggested characteristic bounds on weights of a super-efficiency ranking model. Moreover, Saati et al. [8] improved the MAJ model to remove infeasibility and the type of data normalization was changed by Jahanshahloo et al. [9] in the AP and the MAJ model in order to provide equitable outcomes.

To overcome the problems of AP and MAJ models, some researchers introduce some methods to rank efficient DMUs using especial norms such as Jahanshahloo et al. [10]. Amirteimoori et al. [11] have utilized norm to obtain the distance between efficient DMUs and inefficient DMUs. Gradient line and ellipsoid norms were also applied by Jahanshahloo et al [12] for discriminating efficient DMUs.

Torgersen et al. proposed a method to sort efficient units by determining their significances as reference for inefficient units. In addition, there are other methods and techniques that reflect in ranking DMUs in DEA concept. For example, Thompson et al [13] introduced assurance regions in DEA models and the efficiency scores of some efficient DMUs have been reduced using this method. Since precise determination of weights is difficult, this method does not signify reasonable results. Adler et al. [14] tried to minimize the number of inputs and outputs with component analysis resulting in reductions of the efficiency scores of some efficient DMUs. Ganley and Cubbin considered common weights for all DMUs through increasing their efficiency totals. Liu and Peng [15] determined a set of indexes for common weights using common weights analysis (CWA) to discriminate efficient units. Using this method, they try to reduce the efficiency values of efficient DMUs into the values smaller than one. Cooper and Tone [16] attempted to present a ranking approach using scalar measures of inefficiency based on the slack variables. Moreover, a series of various approaches have been also presented in the fuzzy environment (see, for example, [17-19]). Newly, a ranking method for assigning a common fixed cost or revenue among units has been suggested by Khodabakhshi and Aryavash [20] in DEA.

All these methods have some limitations and weaknesses for ranking DMUs and none of them provides enough information to completely rank efficient and inefficient DMUs in DEA concept.

Chapter 3

FACET ANALYSIS

3.1 Facet Analysis Use in DEA

This chapter illustrates the basic concept of “facet analysis” in the CCR models. Generally, facet analysis focuses on hyperplanes which pass through the efficient frontier. It is demonstrated that the efficient frontier estimates production function in input-output space. Employing DEA models, the efficient frontier is generated by hyperplanes, which envelope Production Possibility Set (PPS) at efficient DMUs. In addition, the hyperplanes which form the weak frontier will be moved while satisfying the PPS properties in order to improve efficiency scores of weak efficient DMUs.

Facet analysis initially was originated by Bessent et al. [21] for use in the DEA models. Facet is defined as a face with *n-1* freedom degree or a face with *n-1*dimensional for a polyhedral in *n* dimensions space. Notice that facets are hyperplanes for a polyhedral with linear structures in *n* dimensions case. These facets are used as the reference to categorize DMUs in DEA. Facet analysis shows relation between algebraic and geometric view of points in DEA and depicts the relation between feasible region of constraints in CCR model for DMU under evaluation and the corresponding PPS under the focus of the Returns to Scale (RTS). The concept of the RTS is illustrated in details in the next section. Furthermore, it can be shown that the feasible solutions of CCR model (model 2.2) are the normal vectors for the corresponding supporting hyperplanes of PPS for a specific DMU.

3.2 Importance of Facet

Facet is an important subject of concern in order to evaluate efficiency in DEA. The efficiency measure enables analysts to realize whether can convert fewer inputs into given output or increase outputs using present inputs. As a result, only part of the efficient frontier is concerned in evaluating efficiency score. This part is said to be facet.

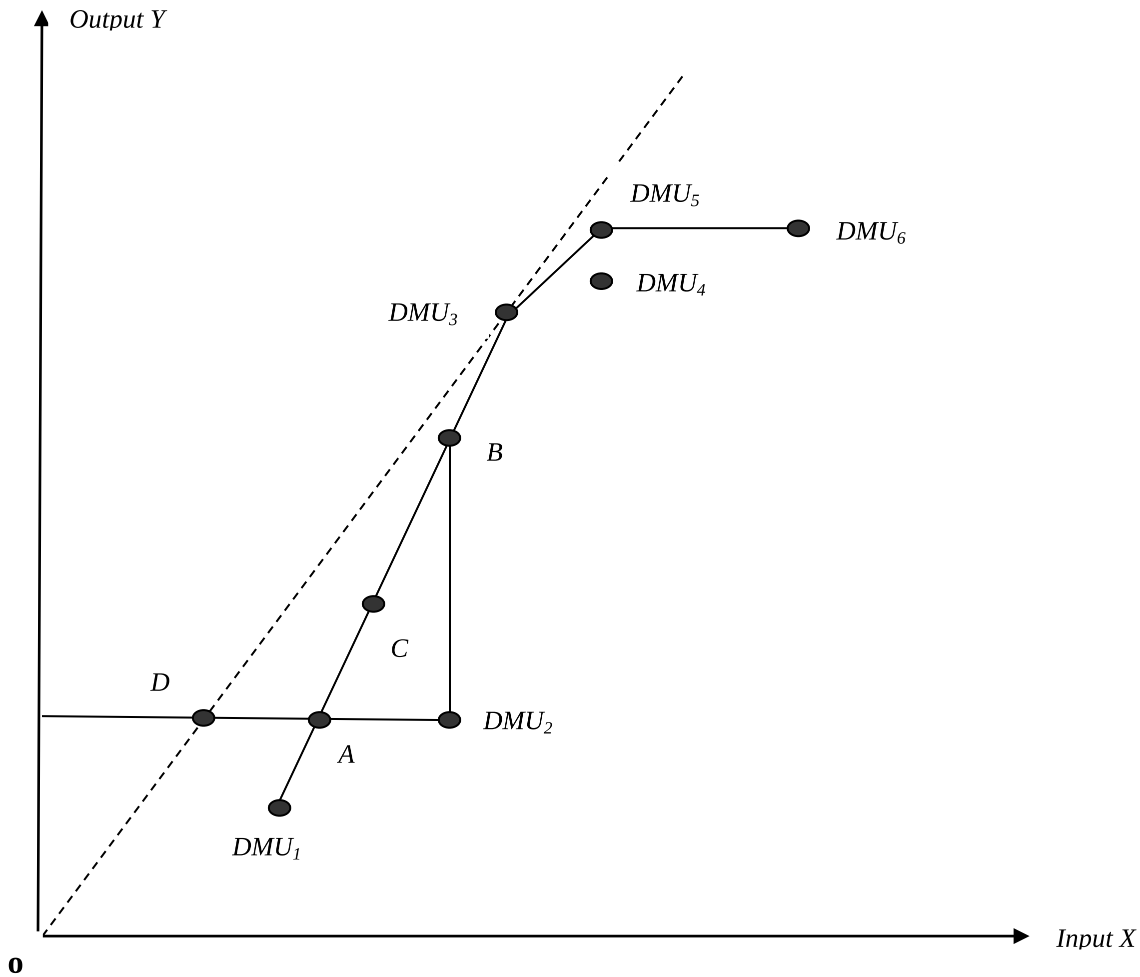

Figure 3.1: Efficient Frontier

For instance, in Figure 3.1, only the facet from DMU_1 to DMU_3 is concerned for evaluating the efficiency of the DMU denoted by DMU_2. In a similar manner, the facet from DMU_3 to DMU_5 is considered to evaluate DMU_4.

The applications of facets aid managers or analysts to identify the inefficient DMUs and find ways so that the inefficient DMUs may improve their efficiencies by comparing with the efficient DMUs. For instance, in Figure 3.1, efficiency of DMU_2 can be improved through moving to some points on the facet DMU_1 to DMU_3. Especially, this DMU can move to *A* by requiring less input, to *B* by increasing output or both increasing output and decreasing input.

3.3 Return to Scale (RTS)

We begin with theoretical formulations in which we apply Figure 3.2 to comprehend the concept of Return to Scale (RTS). Function $y=f(x)$ in Figure 3.2 as production function aims to maximize y using value of x and this production function forms the frontier to evaluate relative efficiency in DEA. For this reason, we consider only points located on the frontier as desirable points and hence points such as *s*, which places inside PPS, are not favorable in the idea that we are currently expanding.

Figure 3.2 also depicts manners of average ($a.p=y/x$) and marginal ($m.p=dy/dx$) of production function, where y/x relates to the slope of the ray from the origin to y and dy/dx is the derivative of $f(x)$ at this point.

The slopes of rays (average productivity) rise up to x_o while x is increasing, in this case we say RTS is increasing, and then slopes of the relevant rays start reducing, here we say RTS is decreasing, and for x_o RTS is constant in Figure 3.2. In a Similar way, the marginal productivity start increasing till relevant point and then the marginal productivity reduces. As can be seen in Figure 3.2, the marginal productivity places below the average productivity in the right of x_o and above the average productivity for left side. This shows that input is increasing relatively slower than output for right of x_o whereas this situation happens conversely in right side of x_o.

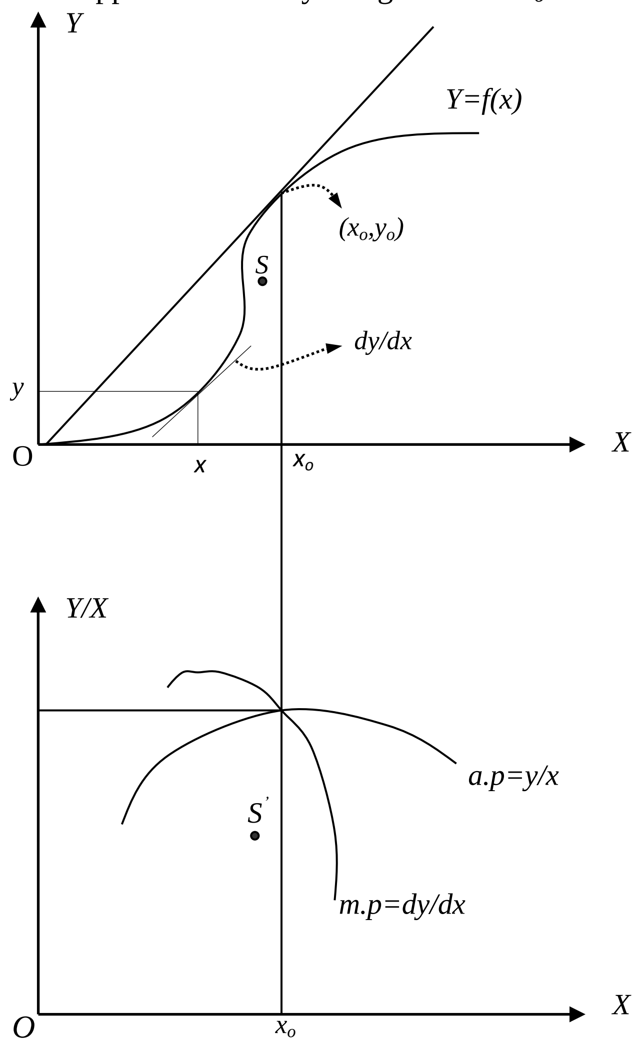

Figure 3.2: Returns to Scale

Economic contexts have usually defined RTS for single output case. The development of the conception of RTS can be attributed for multiple output cases. In multiple inputs and outputs case, RTS defines as effect of product factors changes on production. Mathematically, for multiple inputs and outputs RTS is defined as follows:

Definition 3.1 Suppose that $(X_o, Y_o) \in T$ (S_C or S_B) and $\beta > 0$ is fixed scalar, let

$\beta(a) = Max\{\beta \mid (\alpha X_o, \beta Y_o) \in S\}$

And

$$\gamma = Lim_{\beta \to 1} \frac{\beta(a) - 1}{a - 1}$$

If $\gamma = 1$, RTS is constant for (X_o, Y_o)

If $\gamma > 1$, RTS is increasing for (X_o, Y_o)

If $\gamma < 1$, RTS is decreasing for (X_o, Y_o)

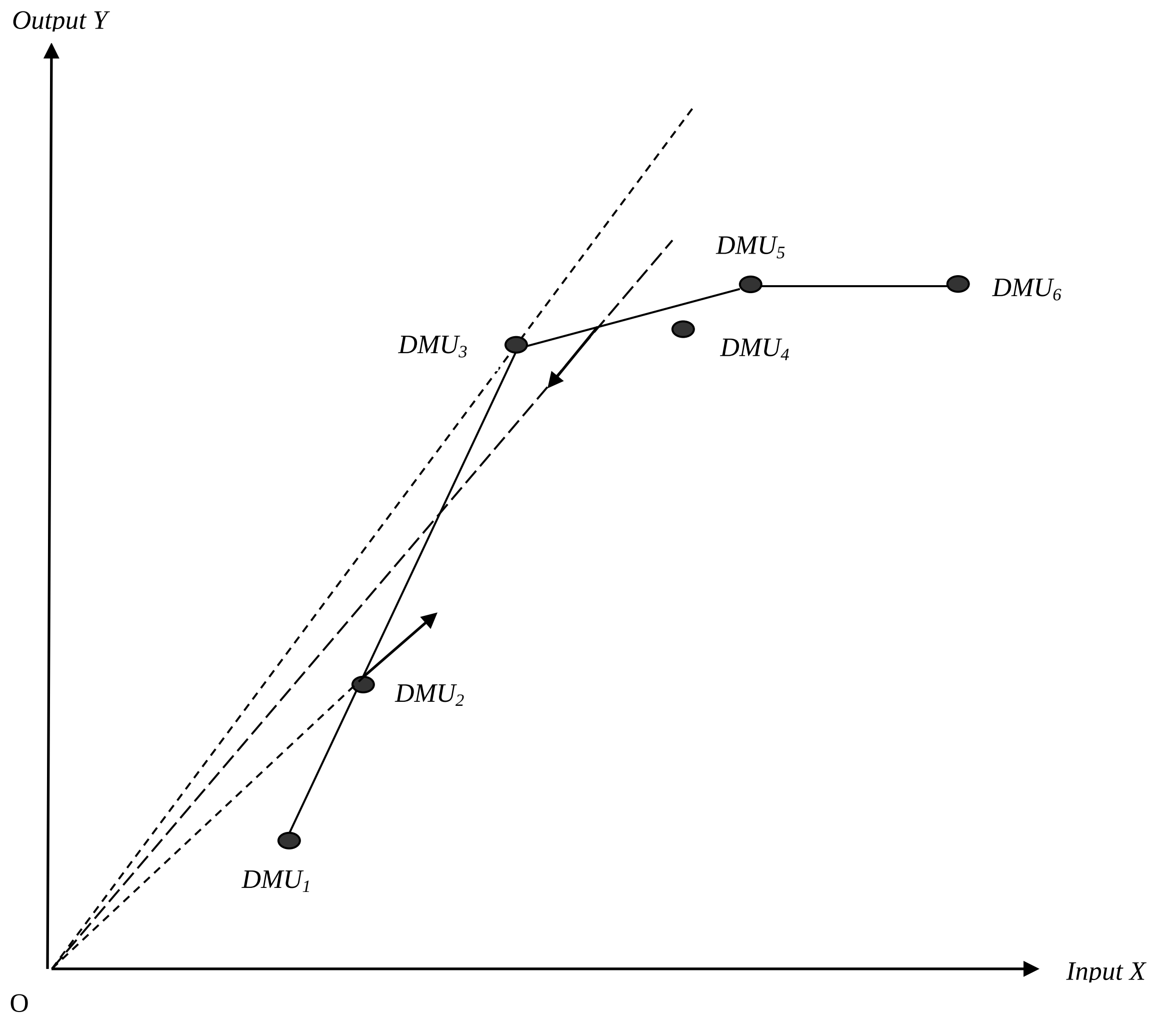

Figure 3.3: An Illustration of Return to Scale (RTS)

Despite evaluating relative efficiency, DEA produce information about scale efficiency in Production Possibly Set (PPS) since the measure of scale efficiency differs from one model to another model in DEA and so it should be the center of attention. To this effect, consider the facet from DMU_1 to DMU_3 in Figure 3.3. For DMUs lied on this facet, RTS is increasing because of increasing relationally their output and input remains them in PPS. A proportional decrease in their output and input cannot occur because it may place them outside of PPS. This is demonstrated by passing a ray from the origin through DMU_1 to DMU_3 at DMU_2.

DMUs rested on the facet from DMU_1 to DMU_3 depicts decreasing Returns to Scale (RTS) since a decrease relative to their output and input move them inside the PPS and a rise relative to their output and input place them outside of the PPS.

RTS is constant for a DMU if all decrease or increase relative to their output and input move the DMU either above or along the PPS. For instance, in Figure 3.3, DMU_3 implies constant Returns to Scale (RTS) due to the fact that proportional decrease and increase might move it outside of the PPS.

Due to facets are formed by efficient DMUs, the scale efficiency of them is identified by the properties of their relevant facet and scale efficiency of inefficient DMUs are specified by their relevant reference facets, respectively. Therefore, RTS is decreasing for DMU_4 and RTS is increasing for DMU_2.

3.4 Facet Analysis in General Case

The conception of a facet (plane) for the production function in Figure 3.2 can be developed for a general situation in supporting hyperplane in multiple inputs and outputs case as follows. A hyperplane H_o in an $m + s$ dimensional space (where m and s are the numbers of inputs and outputs respectively) which passes through the point represented by the vectors (X_o, Y_o) can be shown by this equation:

$$H_o: \quad U(Y - Y_o) - V(X - X_o) = 0 \quad (3.1)$$

Where $U \in R^s$ and $V \in R^m$ are coefficients in this equation considered as normal vectors. Let u_o has following value:

$$u_o = VX_o - UY_o \quad (3.2)$$

Thus, the hyperplane presented by 3.1 can be formulated in the followingequation:

$$UY - VX + u_o = 0 \quad (3.3)$$

A hyperplane divides a space into the two halfspaces. We define hyperplane H_o as the supporting hyperplane of PPS, if it envelops the PPS in one of these two halfspaces at the point (X_o, Y_o).

For any DMU related to any (X, Y) belonging to PPS, we have:

$$UY - VX + u_o \le 0$$

Since input and output multipliers are supppsed to be positive in the CCR model and according to the above, for supporting hyperplanes of PPS can be shown that:

$$U > 0, \quad V > 0$$

Moreover, we consider the following constraint as a normalization constraint:

$$VX_o = 1 \quad (3.4)$$

PPS is generated by the observed DMUs and hyperplanes defined by $UY - VX - u_o = 0$ and the relation between $U \in R^s$ and $V \in R^m$ as factor weights in the CCR model and set of observed DMUs in PPS can be considered from this point of view.

Now for an efficient DMU in the CCR model say (X_o, Y_o) from (3.2), (3.3), (3.4) and

$UY_o = 1$

We can see that $u_o = 0$ and so we conclude that hyperplane $UY - VX = 0$ is a supporting hyperplane of PPS at (X_o, Y_o), with $(-V, U)$ as normal vector which passes through the origin.

3.5 Facet Analysis in CCR Model

Let (X_o, Y_o) be the CCR-efficient DMU and for the optimal weight V^*, U^* of model 2.1, we obtain:

$$\sum_{r=1}^{s} u_r^* y_{ro} = 1 = \sum_{i=1}^{m} v_i^* x_{io}$$

$$\sum_{r=1}^{s} u_r^* y_{ro} - \sum_{i=1}^{m} v_o^* x_{io} = 0$$

As mentioned earlier, the hyperplane $\sum_{r=1}^{s} u_r y_r - \sum_{i=1}^{m} v_i x_i = 0$ in input-output space is a hyperplane which passes through the origin and support T_C in (X_o, Y_o) with $(-V^*, U^*)$ as the normal vector. Since (X_o, Y_o) is the CCR-efficient DMU, optimal solution of model 2.1 (CCR model) equals to one ($\theta^* = 1$). Now, if for all $i = 1,2,\ldots,m$ and $r = 1,2,\ldots,s$ we have $v_i^* \neq 0$, $u_r^* \neq 0$, and consequently complementary slackness theorem shows that $s_i^- = 0$, $s_r^+ = 0$ for all $i = 1,2,\ldots,m$ and $r=1,\ldots,s$ in model 2.7 as a dual linear programming of multiplier side. But if any i exists such that $v_i^* = 0$ or any r exists such that $u_r^* = 0$ and so corresponding slack variable s_i^- or s_r^+ can be nonzero based on the complementary slackness theorem. In this case, the DMU under evaluation is the CCR-weak efficient. Hence there is at least one component with zero value in its normal vector $(-V^*, U^*)$ for the hyperplane passing through the weak efficient DMU.

Figure 3.4 shows that the hyperplane with zero components in normal vector is parallel with the axes whose corresponding weight is equal to zero. Therefore, these hyperplanes of weak frontier are parallel with at least one of the input or output axes.

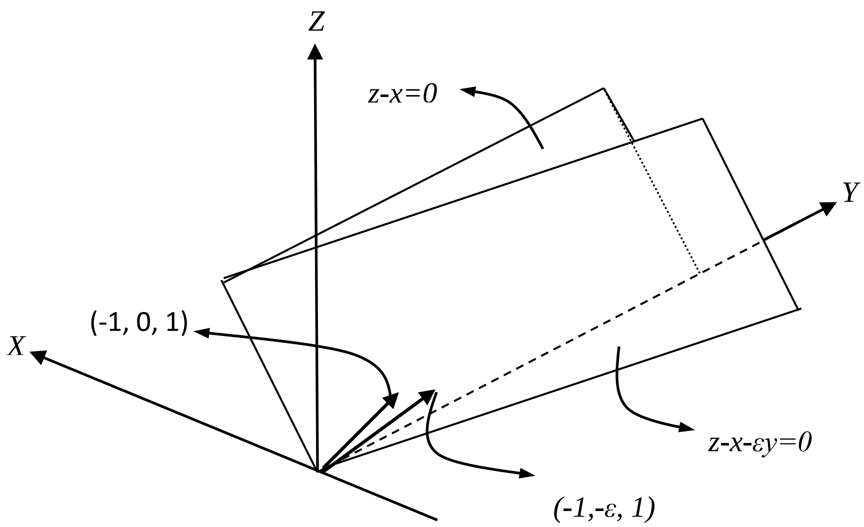

Figure 3.4: Plane with Zero Components in Its Normal Vector

It is illustrated that epsilon ε is used as a minimum weight restriction to differentiate between the weak efficient DMUs and efficient ones where epsilon is usual non-Archimedean infinitesimal element referred to a small positive value ($\varepsilon > 0$). This lower bound forces input or output weights to be nonzero and then corresponding weights reflect in evaluating efficiency in DEA. In fact, determination of weight minimum moves the normal vector preventing the hyperplanes of weak frontier to be formed. Depending on the ε value, efficiency scores of the weak efficient DMUs and the DMUs which are compared with them are changed. Figure 3.5 portrays the situation geometrically.

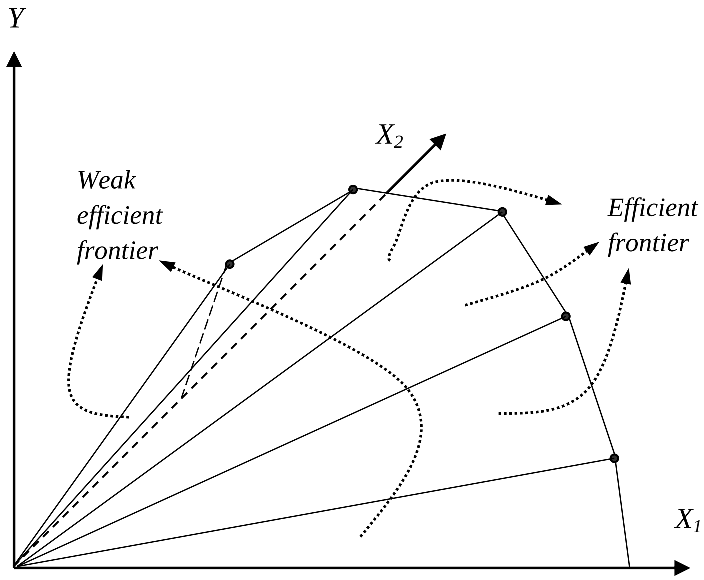

Figure 3.5: PPS in Two Inputs and One Output Case

Figure 3.3 depicts PPS for one output and two inputs case and relevant frontiers.

3.6 Determining Admissible Hyperplanes

It is shown that the weights vector *(-V, U)* can be considered as normal vector of a supporting hyperplane of PPS (S_C). The CCR model generally evaluates these weights for observed DMUs resulting as the normal vectors of supporting hyperplanes in PPS for each efficient DMU. Here we try to determine appropriate non-Archimedean epsilons as lower bounds on components of normal vectors considered for each efficient DMU while satisfying the properties of PPS. These intervals are also used to obtain the most appropriate hyperplanes as admissible hyperplanes which can be replaced with hyperplanes of weak frontier.

Let (X_o, Y_o) be the efficient DMU, for all $r = 1,2,\ldots,s$ and $i = 1,2,\ldots,m$. We consider the following problems:

$$\begin{aligned} &Max \quad u_r && (3.5) \\ &s.t \quad UY_o = 1 \\ &\quad UY_j - VX_j \le 0 \;\; for \;\; j = 1,\ldots,n \\ &\quad VX_o = 1 \\ &\quad U \ge 0, V \ge 0 \end{aligned} \qquad \begin{aligned} &Min \quad u_r && (3.6) \\ &s.t \quad UY_o = 1 \\ &\quad UY_j - VX_j \le 0 \;\; for \;\; j = 1,\ldots,n \\ &\quad VX_o = 1 \\ &\quad U \ge 0, V \ge 0 \end{aligned}$$

And

$$\begin{aligned} &Max \quad v_i && (3.7) \\ &s.t \quad UY_o = 1 \\ &\quad UY_j - VX_j \le 0 \;\; for \;\; j = 1,\ldots,n \\ &\quad VX_o = 1 \\ &\quad U \ge 0, V \ge 0 \end{aligned} \qquad \begin{aligned} &Min \quad v_i && (3.8) \\ &s.t \quad UY_o = 1 \\ &\quad UY_j - VX_j \le 0 \;\; for \;\; j = 1,\ldots,n \\ &\quad VX_o = 1 \\ &\quad U \ge 0, V \ge 0 \end{aligned}$$

Suppose that optimal solutions of model 3.5 and model 3.6 be u_r^+ and u_r^-. Additionally, optimal solutions of model 3.7 and model 3.8 assumed to be v_i^+ and v_i^-, respectively. We now determine intervals for epsilon while place the DMUs inside the PPS which satisfy the properties of PPS as follows:

$$\varepsilon_r^+ = Min\{u_r^+ \text{ for efficient DMU}\}$$

$$\varepsilon_r^- = Max\{u_r^- \text{ for efficient DMUs}\}$$

$$\varepsilon_i^+ = Min\{v_i^+ \text{ for efficient DMUs}\}$$

$$\varepsilon_i^- = Max\{v_i^- \text{ for efficient DMUs}\}$$

Definition 3.2

Let (X_o, Y_o) be CCR efficient DMU and $(-V^*, U^*)$ be relevant weights considered as normal vector of the hyperplane, which satisfied following inequalities is the admissible supporting hyperplane for S_C:

$$\varepsilon_r^- \leq u_r^* \leq \varepsilon_r^+ \quad \forall r = 1,2,...,s$$

$$\varepsilon_i^- \leq v_i^* \leq \varepsilon_i^+ \quad \forall i = 1,2,...,m$$

Chapter 4

MODIFICATION OF CCR MODEL

4.1 Introduction

This chapter represents a modification of the CCR model using facet analysis. As already described, when the CCR model is employed to specifics DMUs without assigning minimum weight restriction, efficiency evaluation is not affected by DMUs located on weak frontier and DMUs which are compared with this frontier. The non-Archimedean element epsilons are used as lower bounds on weights to remove this difficulty through preventing weights to be zero. Introducing a unique epsilon to intervals for minimum weights of the CCR model cannot produce the precise and exact efficiency scores for weak efficient DMUs and DMUs which are related to them for evaliation. Here we modify CCR model to improve efficiency measures of weak efficient DMUs. We organize this chapter such that the next section provides a problem definition and the Section 4.3 exhibits a modification of the CCR model using facet analysis. The modified CCR model and the classical CCR models are compared in Section 4.4 via an example.

4.2 Problem Definition

Charnes, Cooper and Rohdes [4] introduced non-Archimedean epsilons in DEA models which have been used as lower bounds on factor weights to show the inefficiency of the weak efficient DMUs in the CCR model. Afterwards, many approaches are presented for estimating ε value. Majority try to find epsilon while preventing infeasibility and unboundedness in multiplier and envelopment orientations, respectively. These methods could not obtain interesting results for some real problems. Finally, Mehrabian, Jahanshahloo and Alirezai determined the assurance intervals and showed that each number within these intervals can be used as non-Archimedean number ε. All of these approaches try to reduce efficiency values of weak efficient DMUs and DMUs compared with weak efficient ones. It is verified that less efficiency scores of these DMUs while properties of PPS are satisfied, result in more exact and precise evaluation of efficiency scores. We want to show that a unique choice of ε value as lower bound on all multipliers (weights) may not obtain true efficiency scores of weak efficient DMUs and DMUs compared with weak efficient ones. Hence we aim to determine lower bound on each multiplier. These lower bounds are used to revise CCR model.

4.3 Modification of CCR Model Using Facet Analysis

When a unique value of ε is assigned as lower bound on all input and output multipliers, zero components of normal vectors in weak frontier hyperplanes are changed by same value. In this case, depending on evaluated ε, the hyperplanes of the weak frontier move while preserving properties of PPS.

Returning to the previous chapter, there is at least one component with zero value in normal vectors of weak frontier hyperplanes, that is an r or i exists such that $u_r = 0$ or $v_i = 0$. Then normal vectors of these hyperplanes are also moved by determining intervals for ε. This changes move the weak frontier hyperplanes because epsilons force the normal vectors of the weak frontier hyperplanes to have non-zero components.

The optimal values of multipliers in the multiplier direction of CCR model are relevant non-zero slacks. Based on complementary slackness theorem either any multiplier is greater than zero then relevant slack should be zero and reversely or both of them can be zero as well. Therefore, complementary slackness theorem signifies that corresponding dual variable of these zero components (s_i^- and s_r^+) can be non-zero because based on envelopment side of the CCR model (the following model), if for the optimal solution $\theta^* = 1$, for $s^- \geq 0$ DMU under evaluation consumes more inputs than others and in a case $s^+ \geq 0$, DMU under evaluation produces less outputs than other DMUs. Thus, efficiency value of DMU under evaluation is not really equal to unity and this DMU is referred to a weak efficient DMU. Consequently, if we specify the appropriate minimum value for each multiplier, then we can suppress the non-zero slacks by the complementary slackness theorem condition to improve the efficiency of the weak efficient DMUs.

$$
\begin{aligned}
&Min \quad \theta && (2.7)\\
&s.t \quad \theta X_o - X_j\lambda - s^- = 0 &&\\
&\qquad Y_j\lambda - Y_o + s^+ = 0 &&\\
&\qquad \lambda \geq 0 &&\\
&\qquad \theta \quad free &&\\
&\qquad s^- \geq 0 &&\\
&\qquad s^+ \geq 0 &&
\end{aligned}
$$

According to the CCR model in order to move the weak frontier hyperplanes, we consider the DMUs which are located on the region formed by the intersection of the efficient and weak efficient frontiers. Figure 4.1 shows some of these DMUs for two inputs/one output and one input/two outputs cases. We therefore discriminate these DMUs from observed DMUs for modifying the CCR model.

To this effect, we consider the following model for CCR-efficient DMUs based on definition of CCR-efficiency.

$$
\begin{aligned}
Max \quad & \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ && (4.4)\\
s.t \quad & -x_{io} + \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = 0 \quad i = 1,2,\ldots,m \\
& -y_{ro} + \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^- = 0 \quad r = 1,2,\ldots,s \\
& \lambda_j \geq 0 \quad j = 1,2,\ldots,n \\
& s_i^- \geq 0 \quad i = 1,2,\ldots,m \\
& s_r^+ \geq 0 \quad r = 1,2,\ldots,r
\end{aligned}
$$

The above linear programming model can be considered for all observed DMUs but it is infeasible for inefficient DMUs. This model identifies CCR-efficient DMU placed on the intersection of efficient frontier and weak efficient frontier hyperplanes with positive value in its optimal solution. Let *Z* be the set of these DMUs (see Figure 4.1).

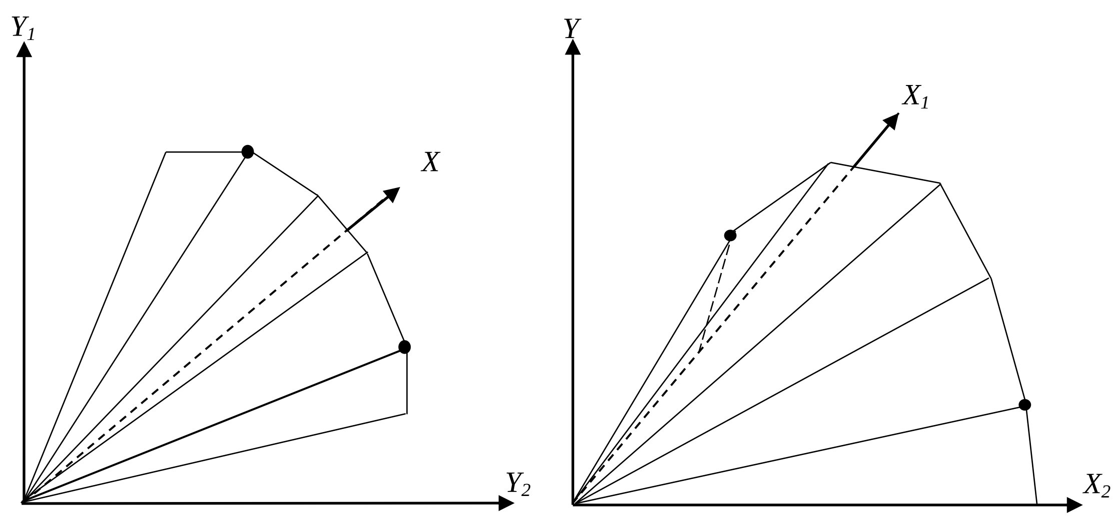

Figure 4.1: Elements of Set *Z* for S_C

Let DMU_w be DMU belongs to set Z. Now for DMUs belonging to Z denoted by (X_w, Y_w), we have the following problems where v_{iw} and u_{rw} are input weights and output weights for DMU_w, respectively.

$$\begin{aligned}
&Max \quad v_{iw} \qquad (4.5)\\
&s.t \quad \sum_{i=1}^{m} v_{iw} x_{iw} = 1\\
&\sum_{r=1}^{s} u_{rw} y_{rw} = 1\\
&\sum_{r=1}^{s} u_{rw} y_{rj} - \sum_{i=1}^{m} v_{iw} x_{ij} \le 0 \quad j = 1,2,...,n\\
&\quad v_{iw} \ge 0 \quad i = 1,2,...,m\\
&\quad u_{rw} \ge 0 \quad r = 1,2,...,s
\end{aligned}$$

And

$$\begin{aligned}
&Max \quad u_{rw} \qquad (4.6)\\
&s.t \quad \sum_{i=1}^{m} v_{iw} x_{iw} = 1\\
&\sum_{r=1}^{s} u_{rw} y_{rw} = 1\\
&\sum_{r=1}^{s} u_{rw} y_{rj} - \sum_{i=1}^{m} v_{iw} x_{ij} \le 0 \quad j = 1,2,...,n\\
&\quad v_{iw} \ge 0 \quad i = 1,2,...,m\\
&\quad u_{rw} \ge 0 \quad r = 1,2,....,s
\end{aligned}$$

Assume that v_{iw}^{+} and u_{rw}^{+} are the optimal values for model 4.5 and model 4.6, respectively. To reduce the number of problems, problem 4.5 and 4.6 can be solved for only v_i s and u_r s when we have w $s_i^- > 0$ and $s_r^+ > 0$ in the optimal solution of problem 4.4. Finally we determine epsilons as follows.

$$\varepsilon_r = Min\{u_{rw}^{+} \ne 0 \mid DMU \in \mathrm{Z}\} \quad \forall r = 1,2,..,s \quad (4.7)$$

$$\varepsilon_i = Min\{v_{iw}^{+} \ne 0 \mid DMU \in \mathrm{Z}\} \quad \forall i = 1,2,...,m \quad (4.8)$$

Based on models 4.7 and 4.8, the CCR model is modified as follow. Notice that we determine the above intervals for epsilon while satisfying the properties of PPS. In other words, we try to locate all DMUs inside PPS while moving the hyperplanes of weak efficient DMUs.

$$
\begin{aligned}
Max \quad & \sum_{r=1}^{s} u_r y_{ro} \qquad\qquad (4.9) \\
& \sum_{i=1}^{m} v_i x_{io} = 1 \\
s.t \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad j = 1,2,\ldots,n \\
& v_i \ge \varepsilon_i \quad i = 1,2,\ldots,m \\
& u_r \ge \varepsilon_r \quad r = 1,2,\ldots,s
\end{aligned}
$$

In accordance with values of $\varepsilon_r,\quad r = 1,2,\ldots,s$ and $\varepsilon_i \quad i = 1,2,\ldots,m$, we assign them as lower bound on each multiplier in CCR model to produce admissible hyperplanes. These hyperplanes are replaced with hyperplanes of weak frontier. This replacement satisfies the feasibility of multiple sides in modified CCR model.

In next section, the modified CCR model is illustrated via a numerical example. Then the results have been compared with the classical CCR and CCR models with fixed epsilon.

4.4 A Numerical Example

Table 4.1 shows data of Example 4.1 for eight DMUs with one input and two outputs and results of the CCR models are summarized in Table 4.4. Efficiency values of the classical CCR model are obtained in the second column of Table 4.4. DMUs *C*, *F*, *G* and *H* are CCR-efficient and DMU *A* is a CCR weak efficient and others are inefficient. Using model 4.4 for all efficient DMUs, optimal solutions are given by Table 4.2.

According to definition of set Z, DMU C and DMU H are those belong to set Z. Now models 4.5 and 4.6 are applied for these two DMUs and Table 4.3 summarizes the results.

Table 4.1: Data of Example 4.1

DMUs	***A***	***B***	***C***	***D***	***E***	***F***	***G***	***H***
Input x	1	1	1	1	1	1	1	1
Output1 y_1	1	1	2	3	4	4	5	6
Output2 y_2	7	5	7	4	3	6	5	2

Table 4.2: Optimal Values of Model 4.4 for Efficient DMUs

DMUs	***C***	***F***	***G***	***H***
Optimal value of (4.4)	0.875	0	0	0.9

Table 4.3: Optimal Values of Model 4.5 and Model 4.6

DMUs	v_1^+	u_1^+	u_2^+
C	1	0.0625	0.1429
H	1	0.1667	0.05

Thus, from (4.7) and (4.8), we have:

$$\varepsilon_1^v = Min\{1,1\} = 1$$
$$\varepsilon_1^u = Min\{0.0625, 0.15\} = 0.0625$$
$$\varepsilon_2^u = Min\{0.125, 0.05\} = 0.05$$

Therefore, the CCR model for the observed DMUs can be modified as follow:

$$
\begin{aligned}
& Max \quad u_1 y_{1o} + u_2 y_{2o} \\
& \qquad\quad v_1 x_{1o} = 1 \\
& s.t \qquad u_1 y_{1j} + u_2 y_{2j} - v_1 x_{1j} \le 0 \quad j = 1,2,...,9 \\
& \qquad\quad v_1 \ge 1 \\
& \qquad\quad u_1 \ge 0.0625 \\
& \qquad\quad u_r \ge 0.05
\end{aligned}
$$

Table 4.4 summarizes the efficiency values using modified CCR model, the classical CCR model and CCR model with ε = 0.05 (model 2.16).

Table 4.4: Results of CCR Models

DMUs	Classical CCR	CCR with ε = 0.05	Modified CCR
A	1	0.95	0.9375
B	0.7143	0.6929	0.6875
C	1	1	1
D	0.7	0.7	0.7
E	0.75	0.75	0.75
F	1	1	1
G	1	1	1
H	1	1	1

As can be seen in Table 4.4 and Figure 4.1 and Figure 4.2, DMU *A* is weak efficient and DMU *B* is one which has been compared with the weak frontier. DMU *A* is efficient in the classical CCR model, because it is spotted on the weak efficiency frontier. Using the CCR/ε model, this DMU is considered by an admissible hyperplane with the corresponding ε equal to 0.05. The normal vector of this hyperplane is (-1, 0.05, 0.1286) and the efficiency score of this DMU is reduced to 0.95 and the efficiency score of this

DMU is equal to 0.9375 in the modified CCR model because DMU *A* is compared by an admissible hyperplane with vector (-1, 0.0625, 0.125) as its normal vector. Now consider DMU *B* which is compared with the weak frontier. To this effect, efficiency score of DMU *B* is equal to 0.714. This efficiency score is reduced to 0.929 when DMU *B* compared with the admissible hyperplane with vector (-1, 0.05, 0.1286) as a normal vector in the CCR/0.05 model. Finally, in the modified CCR model, DMU *B* compared with admissible hyperplane with vector (-1, 0.0625, 0.125) as normal vector and its efficiency score is improved to 0.6875. Figure 4.2 shows PPS (S_c) and Figure 4.3 depicts intersection of this set with plane $x=1$. Figure 4.4 illustrates the situation geometrically after modification and Figure 4.5 depicts intersection of new S_c with plane $x = 1$.

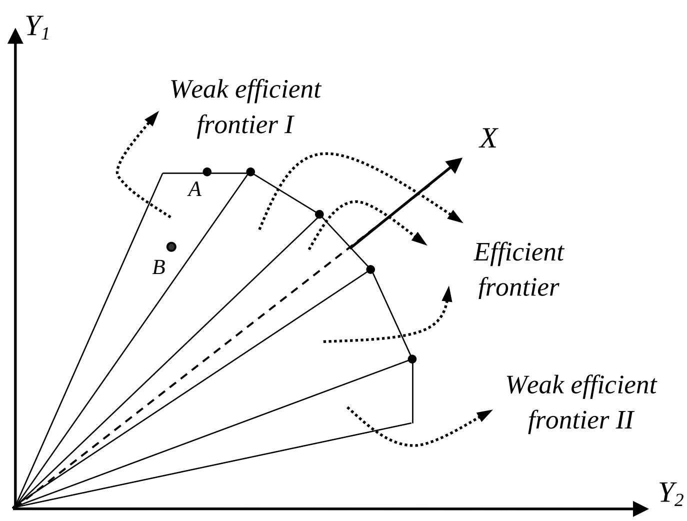

Figure 4.2: Efficient and Weak Efficient Frontiers in S_C

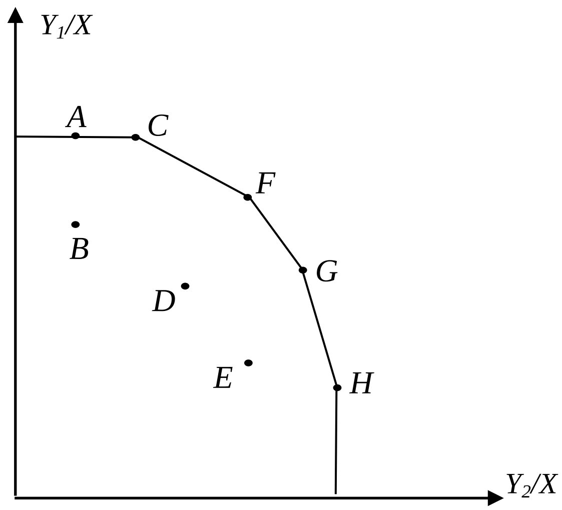

Figure 4.3: Intersection of S_C and Plane *x=1*

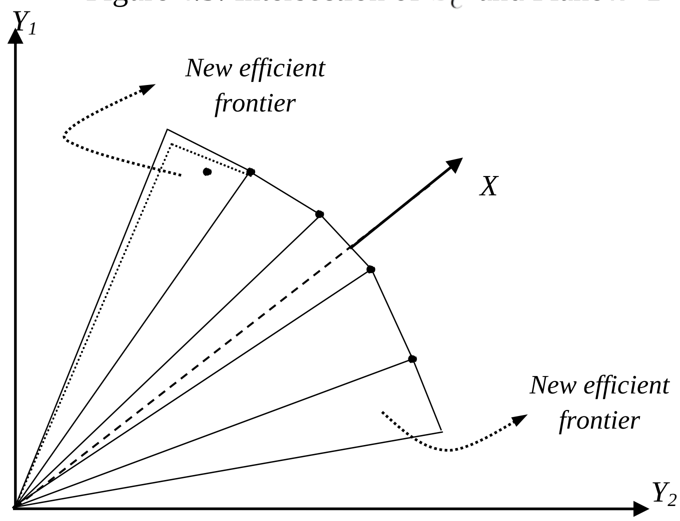

Figure 4.4: Efficient and Weak Efficient Frontiers in Modified S_C

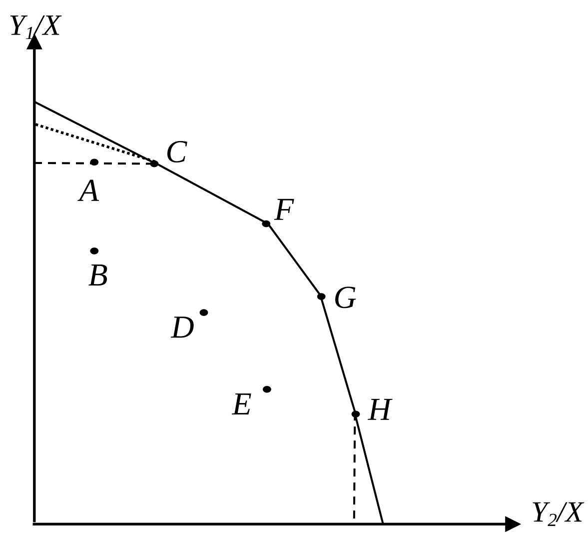

Figure 4.5: Intersection of Modified S_C and Plane *x=1*

As can be seen in this example, assigning assurance interval for non-Archimedean elements as lower bounds on each factor weight by using facet analysis, we modify the CCR model while the properties of PPS (T_C) are satisfied. This model clearly depicts more exact and precise results than the CCR/ε model, which impose a unique lower bound for all factor weights.

Chapter 5

RANKING ALL UNITS IN DEA

5.1 An Introduction to a Ranking Method

In this section, we introduce a ranking method presented by Khodabakhshi and Aryavash [22]. Their idea was motivated to remove difficulties of ranking DMUs in DEA literature. In this regard, firstly, they suppose that total efficiencies be equal to one and then they determine maximum and minimum efficiency scores of DMUs. Finally, DMUs are ranked relative to their combined maximum and minimum efficiency scores. Here we illustrate this method in details as follows.

Let the number of DMUs is n while DMU_j $(j = 1 \ldots n)$ produce s outputs y_{rj} $(r = 1 \ldots s)$ using m inputs x_{ij} $(i = 1 \ldots m)$. Assume that DMU_o is a specific DMU to be evaluated and all data (inputs and outputs) is equal or greater than zero. They try to measure the single efficiency score of DMU_o (θ_o) under focus of total efficiencies is one $(\sum_{j=1}^{n} \theta j=1)$. In general, the efficiency of a specific DMU is obtained by dividing total weighted of outputs by total weighted of inputs as follows where we define v_i $(i = 1 \ldots m)$ as input weights and u_r $(r = 1 \ldots s)$ as output weights.

$$\theta_j = \sum_{r=1}^{s} y_{rj}\ u_r \ / \sum_{i=1}^{m} x_{ij}\ v_i \qquad j = 1 \ldots n \qquad (5.1)$$

$$\sum_{j=1}^{n} \theta_j = 1$$

Equations 5.1 are not used to estimate the unique scores θ_j , but we apply them to compute their maximum and minimum scores in the following model.

$$Min \text{ and } Max\ \theta_o \tag{5.2}$$

s.t. $\theta_j = \sum_{r=1}^{s} y_{rj}\ u_r / \sum_{i=1}^{m} x_{ij}\ v_i \qquad j = 1 \ldots n$

$\sum_{j=1}^{n} \theta_j = 1$

$v_i \geq 0 \quad i = 1 \ldots m$

$u_r \geq 0 \quad r = 1 \ldots s$

$\theta_j \geq 0 \quad j = 1 \ldots n$

This problem is needed to evaluate DMUs twice. The maximum and minimum values of θ_o are obtained by maximizing and minimizing the objective function of model 5.2, respectively. Notice that the above model is a fractional programming and hence we convert this model into a linear programming and we further re-equate this nonlinear program 5.2 as follows:

$$Min \text{ and } Max\ \theta_o = \sum_{r=1}^{s} y_{ro}\ u_r \tag{5.3}$$

$s.t. \quad \sum_{i=1}^{m} x_{io}\ v_i = 1$

$\sum_{i=1}^{m} x_{ij}\ v_i \theta_j - \sum_{r=1}^{s} y_{rj}\ u_r = 0 \qquad j = 1 \ldots n$

$\sum_{j=1}^{n} \theta_j = 1$

$v_i \geq 0 \quad i = 1 \ldots m$

$u_r \geq 0 \quad r = 1 \ldots s$

$\theta_j \geq 0 \quad j = 1 \ldots n$

Using the transformation $h_{ij} = v_i\, \theta_j$, model 5.3 is substituted as follows:

$$Min \text{ and } Max\ \theta_o = \sum_{r=1}^{s} y_{ro}\ u_r \tag{5.4}$$

$s.t \quad \sum_{i=1}^{m} x_{io}\ v_i = 1$

$\sum_{i=1}^{m} x_{ij}\, h_{ij} - \sum_{r=1}^{s} y_{rj}\ u_r = 0 \qquad j = 1 \ldots n$

$\sum_{j=1}^{n} h_{ij} = v_i \quad i = 1 \dots m$

$v_i \geq 0 \quad i = 1 \dots m$

$u_r \geq 0 \quad r = 1 \dots s$

$h_{ij} \geq 0 \quad i = 1 \dots m \quad j = 1 \dots n$

Model 5.4 identifies the maximum and minimum scores of θ_j and then we can determine interval of efficiency scores as follows:

$$\theta_j^{min} \leq \theta_j \leq \theta_j^{max}, \quad j = 1, \dots, n \tag{5.5}$$

We next write the above ranges as convex combinations of the maximum and minimum scores of θ_j :

$$\theta_j = \theta_j^{min} \lambda_j + \theta_j^{max} (1- \lambda_j), \ \forall \lambda_j, 0 \leq \lambda_j \leq 1, \quad j = 1, \dots, n \tag{5.6}$$

To attain the single efficiency scores in an equitable way, all θ_j $(j = 1 \dots n)$ should be evaluated relative to their ranges. Consequently, λ_j $(j = 1 \dots n)$ should be identically determined, that is $\lambda = \lambda_1 = \cdot \cdot \cdot = \lambda_n$. Notice that we assumed that$\sum_{j=1}^{n} \theta j=1$ and so the θ_j is obtained from the associated equations as follows:

$$\theta_j = \theta_j^{min} \lambda + \theta_j^{max}(1- \lambda), \ 0 \leq \lambda \leq 1, \quad j = 1, \dots, n \tag{5.7}$$

$$\sum_{j=1}^{n} \theta_j = 1$$

λ can be simply obtained by submitting convex combination of the maximum and minimum scores of θ_j in our assumption as follows:

$$1 = \sum_{j=1}^{n} \theta_j = \sum_{j=1}^{n} (\theta_j^{min}\lambda + \theta_j^{max}(1-\lambda)) = \lambda \sum_{j=1}^{n} (\theta_j^{min} - \theta_j^{max}) + \sum_{j=1}^{n} \theta_j^{max} \tag{5.8}$$

And we therefore have

$$\lambda = (1 - \sum_{j=1}^{n} \theta_j^{max}) \, / \sum_{j=1}^{n}(\theta_j^{min} - \theta_j^{max}) \tag{5.9}$$

Finally, values of θ_j $(j = 1, 2 \ldots n)$ are determined by Equation 5.6 Using obtained λ. Now, in accordance with efficiency score (θ_j), the DMUs can be ranked. On the other hand, DMUs are sorted in a decreasing arrangement of efficiency scores. In addition, we can rank DMUs from distance attitudes based on comparisons between efficiency scores where distance from DMU_i to DMU_j can be defined by $d(i,j) = |\theta_i - \theta_j|$.

5.2 A Proposed Approach for Ranking All DMUs

We now present a new ranking method through introducing non-Archimedean element epsilons as lower bound on each input and output weight to the model presented by Khodabakhshi and Aryavash as follows. On the other hand, we expand their method by determining appropriate minimum weight for each input and output.

$$\text{Min and max } \theta_o = \sum_{r=1}^{s} y_{ro}\, u_r \qquad (5.10)$$

$$s.t. \quad \sum_{i=1}^{m} x_{io}\, v_i = 1$$

$$\sum_{i=1}^{m} x_{ij}\, h_{ij} - \sum_{r=1}^{s} y_{rj}\, u_r = 0 \qquad j = 1 \ldots n$$

$$\sum_{j=1}^{n} h_{ij} = v_i \qquad i = 1 \ldots m$$

$$v_i \geq \varepsilon_i \qquad i = 1 \ldots m$$

$$u_r \geq \varepsilon_r \qquad r = 1 \ldots s$$

$$h_{ij} \geq 0 \qquad j = 1 \ldots n$$

It is demonstrated that it happens that some inefficient DMUs evaluated as efficient DMUs in efficiency evaluation using DEA techniques when some input and/or output weights of weak efficient DMUs are equal to zero because the corresponding input and/or output cannot reflect in evaluating efficiency scores of these DMUs. We use non-Archimedean element epsilon as lower bound on multipliers to remove the difficulty for discriminating efficient and inefficient DMUs so that this non-Archimedean element forces input or output weights to be non-zero in the proposed model and evaluation will be effected to obtain true and exact efficiency scores. Usage of facet analysis shows that there is at least one component with zero value in normal vector $(-V^*, U^*)$ as input and output weights of hyperplane passing by weak efficient DMU or weak efficient hyperplane. Epsilons impose positivity on weights and move normal vectors of weak hyperplanes causing their efficiency to be measured correctly. Simultaneously, epsilons change efficiency scores of DMUs compared with the weak efficient hyperplanes. To be notice that hyperplanes of weak frontier move depending on epsilon value, and this movement must be in a manner that the properties of PPS are preserved by considering epsilons as lower bound of weights in order to remain feasibility of the model. Since we attempt to reduce efficiency scores of these DMUs while holding properties of PPS, assignment of a unique value of ε as lower bound on all multiplers cannot obtain real results in efficiency evaluation because zero components in normal vectors of the weak frontier hyperplanes are changed by the same value. Using facet analysis, we try to determine lower bounds on each weight to modify the introduced model such that efficiency scores of weak efficient DMUs and DMUs compared with weak efficient DMUs, has been evaluated correctly.

Furthermore, our approach provides a full and complete ranking for all efficient and inefficient DMUs using non-Archimedean element epsilons as lower bound on input and output weights based on the modified CCR model. In the next section, we illustrate how to compute these for non-Archimedean elements for lower bounds of weights in the improved model.

5.3 Determining Minimum Weight Restrictions

In this section, we determine assurance intervals for non-Archimedean element ε based on modified CCR model. We first determine the efficiency scores from the input-oriented version of the CCR model using this problem:

$$
\begin{aligned}
& Max \quad \theta = \sum_{r=1}^{s} u_r y_{ro} && (2.1) \\
& s.t \quad \sum_{i=1}^{m} v_i x_{io} = 1 \\
& \qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad j = 1,2,\ldots,n \\
& \qquad v_i \ge 0 \quad i = 1,2,\ldots,m \\
& \qquad u_r \ge 0 \quad i = 1,2,\ldots,s
\end{aligned}
$$

Then we select the efficient DMUs and obtain their optimal values of the following model to decrease the number of problems for solving in the next steps.

$$
\begin{aligned}
& Max \quad \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ && (4.4) \\
& s.t \quad x_{io} - \sum_{j=1}^{n} \lambda_j x_{ij} - s_i^- = 0 \quad i = 1,2,\ldots,m \\
& \qquad y_{ro} - \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^- = 0 \quad r = 1,2,\ldots,s \\
& \qquad \lambda_j \ge 0 \quad j = 1,2,\ldots,n \\
& \qquad s_i^- \ge 0 \quad i = 1,2,\ldots,m \\
& \qquad s_r^+ \ge 0 \quad r = 1,2,\ldots,r
\end{aligned}
$$

Let Z be set of DMUs with the positive optimal solutions of the above model and DMU_w be DMU belongs to set Z. Then we determine the maximum values of input and output multipliers for DMUs belonging to set Z by solving the following problems where v_{iw} and u_{rw} are input weights and output weights for DMU_w, respectively.

$$
\begin{aligned}
&Max \quad v_{iw} && (4.5)\\
&s.t \quad \sum_{i=1}^{m} v_{iw}x_{iw} = 1 \\
&\qquad \sum_{r=1}^{s} u_{rw}y_{rj} - \sum_{i=1}^{m} v_{iw}x_{ij} \le 0 \quad j = 1,2,\ldots,n \\
&\qquad v_{iw} \ge 0 \qquad i = 1,2,\ldots,m \\
&\qquad u_{rw} \ge 0 \qquad r = 1,2,\ldots,s
\end{aligned}
$$

$$
\begin{aligned}
&Max \quad u_{rw} && (4.6)\\
&s.t \quad \sum_{i=1}^{m} v_{iw}x_{iw} = 1 \\
&\qquad \sum_{r=1}^{s} u_{rw}y_{rw} - \sum_{i=1}^{m} v_{iw}x_{ij} \le 0 \quad j = 1,2,\ldots,n \\
&\qquad v_{iw} \ge 0 \qquad i = 1,2,\ldots,m \\
&\qquad u_{rw} \ge 0 \qquad r = 1,2,\ldots.,s
\end{aligned}
$$

Assume that v_{iw}^{+} and u_{rw}^{+} are the optimal values for model 4.5 and model 4.6, respectively and finally we obtain epsilons from (4.7) and (4.8) as follows:

$$\varepsilon_r = Min\{u_{rw}^{+} \neq 0 \mid DMU \in Z\} \quad \forall r = 1,2,..,s \quad (4.7)$$
$$\varepsilon_i = Min\{v_{iw}^{+} \neq 0 \mid DMU \in Z\} \quad \forall i = 1,2,\ldots,m \quad (4.8)$$

Notice that we determine epsilons while satisfying properties of PPS especially convexity property.

5.4 A Numerical Example

Here new method is demonstrated via Example 5.1. There are 8 DMUs including 1 input and 2 outputs in this example listed in Table 5.1. Firstly, we deduce epsilons by following steps. The efficiency scores of CCR models are determined in Table 5.2 and then we represent optimal values of model 4.4 for the efficient DMUs in Table 5.3. Notice that DMU_3 and DMU_8 belong to set Z because model 4.4 identifies their optimal solutions with positive values. Hence we obtain maximum values of input and output weights for DMU_3 and DMU_8 in

Table 5.4 by solving model 4.5 and model 4.6. Finally epsilons are obtained from (4.7) and (4.8) as follows:

$$\varepsilon_1^v = Min\{1,1\} = 1$$
$$\varepsilon_1^u = Min\{0.0625, 0.1500\} = 0.0625$$
$$\varepsilon_2^u = Min\{0.1250, 0.0500\} = 0.0500$$

Table 5.1: Data of Example 5.1

DMUs	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***
Input x	1	1	1	1	1	1	1	1
Output1 y_1	1	1	2	3	4	4	5	6
Output2 y_2	7	5	7	4	3	6	5	2

Table 5.2: Optimal Solutions of CCR Model

DMUs	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***
CCR Results	1	0.7143	1	0.7	0.75	1	1	1

Table 5.3: Optimal Values of Model 4.4 for Efficient DMUs

DMUs	***3***	***6***	***7***	***8***
Optimal value of (4.4)	0.8750	0	0	0.9000

Table 5.4: Optimal Values of Model 4.5 and Model 4.6

DMUs	v_1^+	u_1^+	u_2^+
3	1	0.0625	0.1429
8	1	0.1667	0.0500

Then maximum and minimum scores of a specific DMU (DMU_o) are united into a single value by solving model 5.4 as follows:

$Min\ and\ Max\ \theta_o = y_{1o}u_1 + y_{2o}u_2$

$s.t. \quad x_{1o}v_1 = 1$

$x_{11}h_{11} - (y_{11}u_1 + y_{21}u_2) = 0$

$x_{12}h_{12} - (y_{12}u_1 + y_{22}u_2) = 0$

$x_{13}h_{13} - (y_{13}u_1 + y_{23}u_2) = 0$

$x_{14}h_{14} - (y_{14}u_1 + y_{24}u_2) = 0$

$x_{15}h_{15} - (y_{15}u_1 + y_{25}u_2) = 0$

$x_{16}h_{16} - (y_{16}u_1 + y_{26}u_2) = 0$

$x_{17}h_{17} - (y_{17}u_1 + y_{27}u_2) = 0$

$x_{18}h_{18} - (y_{18}u_1 + y_{28}u_2) = 0$

$h_{11} + h_{12} + h_{13} + h_{14} + h_{15} + h_{16} + h_{17} + h_{18} = v_1$

$v_1 \geq 1$

$u_1 \geq 0.0625$

$u_2 \geq 0.0500$

$h_{11}\ h_{12}\ h_{13}\ h_{14}\ h_{15}\ h_{16}\ h_{17}\ h_{18} \geq 0$

(where *o* varies from 1 to *n*).

For instance, we write the new model for evaluating θ_1 of first DMU as follows:

$Min\ and\ Max\ \theta_o = 1u_1 + 7u_2$

$s.t. \quad 1v_1 = 1$

$1h_{11} - (1u_1 + 1u_2) = 0$

$1h_{12} - (1u_1 + 5u_2) = 0$

$1h_{13} - (2u_1 + 7u_2) = 0$

$1h_{14} - (3u_1 + 4u_2) = 0$

$1h_{15} - (4u_1 + 3u_2) = 0$

$1h_{16} - (4u_1 + 6u_2) = 0$

$1h_{17} - (5u_1 + 5u_2) = 0$

$1h_{18} - (6u_1 + 2u_2) = 0$

$h_{11} + h_{12} + h_{13} + h_{14} + h_{15} + h_{16} + h_{17} + h_{18} = v_1$

$v_1 \geq 1$

$u_1 \geq 0.0625$

$u_2 \geq 0.0500$

$h_{11}\ h_{12}\ h_{13}\ h_{14}\ h_{15}\ h_{16}\ h_{17}\ h_{18} \geq 0$

The obtained θ_j^{min} and θ_j^{max} are listed in the fourth column of Table 5.5. After computing θ_j^{min} and θ_j^{max} for all DMUs, the value of λ can be determined from 5.9 as follow:

$\lambda = (1 - \sum_{j=1}^{n} \theta_j^{max}) / \sum_{j=1}^{n}(\theta_j^{min} - \theta_j^{max}) = 0.6717$

Then integrated scores for all DMUs are summarized in Table 5.5 using $\theta_j = \theta_j^{min} \lambda + \theta_j^{max}(1- \lambda)$ and we rank DMUs based on the obtained efficiency scores. For instance, the minimum and maximum values of DMU_1 are $\theta_1^{min} = 0.0400$ and

$\theta_1^{max} = 0.2188$, respectively. Thus, the efficiency score of DMU_1 must be in the interval [0.0400, 0.2188]. Moreover, θ_1 is an integrated value of θ_1^{min} and θ_1^{max}. Notice that computations are obtained from WinQSB and GAMS and the results are provided in Appendix.

We can rank DMUs from distance point of view as well. For instance, DMU_6, DMU_3 and DMU_4 as the second, third and fourth positions of new ranking, are compared from a

Table 5.5: A Full Ranking of DMUs

DMU	CCR Results	Khodabakhshi&Aryavash Results	$[\theta_j^{min}, \theta_j^{max}]$	θ_j	Rank
1	1	0.1090 (6)	[0.0400, 0.2188]	0.0987	7
2	0.714	0.0834 (8)	[0.0400, 0.1562]	0.0781	8
3	1	0.1282 (4)	[0.0800, 0.2188]	0.1256	3
4	0.688	0.1090 (7)	[0.1200, 0.1250]	0.1216	4
5	0.750	0.1154 (5)	[0.0938, 0.1600]	0.1155	6
6	1	0.1538 (2)	[0.1600, 0.1875]	0.1690	2
7	1	0.1603 (1)	[0.1562, 0.2000]	0.1706	1
8	1	0.1411 (3)	[0.0625, 0.2400]	0.1208	5

Distance viewpoint. We have $d(6,3) = 0.0434$ and $d(3,4) = 0.004$ and so $d(6,3) = 10.85(3,4)$. Thus, the distance from DMU_6 to DMU_3 is approximately 10 times more than the distance from DMU_3 to DMU_4.

In addition, the optimal solutions of CCR model and Khodabakhshi and Aryavash model are shown in Table 5.5, respectively. Obviously, these values differ from our ranking in some cases. For example, the rank of DMU_3 is third position in our method whereas the third position of Khodabakhshi and Aryavash ranking belongs to DMU_8. We produce a complete ranking approach for all DMUs in Data Envelopment Analysis (DEA). The results indicate that new approach is more reasonable and precise than other approaches. In this approach, weak efficient DMUs are concerned with improvement their efficiency values and a combination of both pessimistic and optimistic attitude is applied to specify the scores. Subsequently, strengths of DMU play significant role in identifying θ_o^{max}, and weaknesses of DMU_o play significant role in identifying θ_o^{min}. Thus both θ_o^{min} and θ_o^{max} are used to determine the rank of DMU_o.

Chapter 6

CONCLUSION AND FUTURE STUDY

6.1 Conclusion

This thesis has introduced Data Envelopment Analysis (DEA) and its application and importance. Using output-to-input measures and performance evaluation, DEA aims to improve productivity of management as well as efficiency measures related to business, economics, and engineering. Chapter 2 covered a complete reference in treating this subject. We also illustrated interpretations and uses of facet analysis in Chapter 3. We expand usage of facet analysis to modify the CCR model in Chapter 4. Efficiency evaluation which enables researchers to rank DMUs is one of the most critical objectives of DEA and we represent a new ranking method for all DMUs using facet analysis in Chapter 5.

In DEA, some of inputs and/or outputs cannot reflect in evaluating the efficiency of DMUs when DEA evaluates some of their corresponding input and/or output weights with zero values. Optimal values of weights (multipliers) in the multiplier side for the CCR model (model 2.2) are relevant slacks for DMUs under evaluation in the envelopment side (model 2.7) and hence the complementary slackness theorem indicates that corresponding dual variables s_i^- (input excesses) and s_r^+ (output shortfalls) of input and/or output weights with zero values, can be non-zero in the envelopment side. If there are input excesses and output shortfalls for an efficient DMU, this DMU is referred to the weak efficient DMU because it has some positive slack variables in the envelopment side and relevant zero multipliers of multiplier side simultaneously.

In this regard, we determined non-Archimedean element epsilons as lower bound on each weight to eliminate this difficulty where epsilon is usual non-Archimedean infinitesimal element referred to a small positive value. To be noticed in introducing epsilon as a minimum weight restriction in DEA is the fact that this element imposes the positivity on input or output weights. This lower bound forces input or output weight to be nonzero and then corresponding weights reflect in evaluating efficiency in DEA. Consequently, we specified the appropriate lower bound on each multiplier, and so we can suppress the non-zero slacks through the complementary slackness theorem condition in order to correct efficiency scores of weak efficient DMUs. We modified the CCR model, which is one of the basic DEA models, by introducing lower bounds on input and output multipliers using facet analysis. Generally, we presented a method for determining lower bounds of the non-Archimedean in DEA models.

It is demonstrated that the feasible solutions of the CCR model (weights) are the normal vectors for the corresponding supporting hyperplanes of PPS using facet analysis and for the efficient DMUs; these hyperplanes pass through the origin. Thus, there is at least one component with zero value in normal vectors $(-V^*, U^*)$ of weak efficient hyperplanes. Imposing epsilons as weight minimum move the normal vectors and do not allow the hyperplanes of weak frontier to be generated. Depending on the evaluated ε value, efficiency measures of weak efficient DMUs and DMUs, which are related to them for evaluation, are revised. We determine an appropriate non-Archimedean element epsilon as lower bound on components of normal vectors considered for each efficient DMU while satisfying the properties of PPS. On the other hand, we find them such that feasibility and boundedness are validated in the multiplier and envelopment sides. This results in the most appropriate hyperplane as admissible hyperplanes which can be replaced with hyperplanes of weak frontier.

When a unique value of ε is assigned as lower bound on all factor weights, the zero components in normal vectors of the weak frontier hyperplanes are changed by the same value. In this case, depending on evaluated ε, the hyperplanes of the weak frontier move while preserving properties of PPS. Assigning a unique epsilon as minimum restriction on all input and output weights of CCR model cannot produce the precise and exact efficiency scores for weak efficient DMUs and DMUs which are related to them for evaluation. Hence we modify the CCR model so as to improve the efficiency scores of these DMUs. We aim to reduce efficiency scores of these DMUs while satisfying the properties of PPS. Using facet analysis, we determined non-Archimedean element as lower bound on each multiplier and then we improved the CCR model such that efficiency scores of the weak efficient DMUs and the DMUs which are compared with them, has been evaluated correctly. In fact, we considered factor weights as normal vectors of hyperplanes, which envelop PPS at efficient frontier which is generated by efficient DMUs and we improved the CCR models by replacing admissible hyperplanes with the weak frontier hyperplanes.

Facet analysis was discussed in some details in Chapter 3 which has been used in modification of the CCR model. The efficiency evaluation in DEA enables decision maker to show either a DMU can increase outputs without requiring any more input amounts or convert fewer input amounts into the present output amounts. Subsequently, only section of efficient frontier is associated while a specific DMU is being evaluated in DEA. This section is called facet and usage of facets helps managers or analysts to specify the inefficient DMUs in order to identify ways in a way that the inefficient DMUs improve their efficiencies through comparing with the efficient DMUs. Due to facets are produced by efficient DMUs, the Return to Scale (RTS) is recognized by the properties of their relevant facet and the scale efficiency of inefficient DMUs are specified by their relevant facets, respectively.

We focus on hyperplanes (facets) which pass through the efficient frontier using facet analysis. In DEA, the efficient frontier is generated by hyperplanes, which envelope Production Possibility Set (PPS) at efficient DMUs. In addition, the hyperplanes which form the weak frontier are moved while satisfying the properties of PPS so as to improve efficiency scores of weak efficient DMUs.

Khodabakhshi and Aryavash [22] ranked DMUs in 2012 relative to combinations of their maximum and minimum efficiency scores. Then we expand their method through introducing epsilons as minimum weight restriction for each DMU in their model and we suggest a reliable and precise method to rank all DMUs using facet analysis.

Another difficulty in DEA conception is shortage of discrimination in DEA uses, specifically when the number of DMUs is not enough or the number of DMUs is too small in comparison with the number of inputs and outputs and DEA cannot produce a full ranking of efficient DMUs. This difficulty is eliminated from our approach. Due to some of weight values of inputs or outputs of DMUs equal to zero, former methods have low ability in ranking DMUs and they have some limitations and drawbacks. This research has developed an equitable and precise approach for ranking all DMUs based on the modified CCR model using facet analysis and validation of the study has been achieved. Using this method, we observe a full ranking for both efficient and inefficient

DMUs. Two numerical examples have been studied using the new approach to examine its capability in fully ranking DMUs and all the DMUs have been completely ranked and successfully discriminated. This implies the power of new method to distinguish units, especially efficient DMUs. In this thesis, we supplied examples to show how new results can be validated when modification is applied to use.

Infeasibility and instability that happen in the super-efficiency models (like AP model) because of being extreme sensitive to small values in data of these models and eliminating some of DMUs obtain very large θ are removed in this approach.

Moreover, for modification of the CCR model, the number of problems for determining the lower bounds on each factor weight is great. Using the complementary slackness theorem, we reduce the number of problems for solving and the computational burden is considerably decreased. The new approach presents a precise ranking with fewer computations.

Our approach is concerned form both optimistic and pessimistic points of view in DEA and it could be more reasonable than other methods which considered only one of these views and a full ranking is developed using this method. Furthermore, this approach can compare DMUs in accord with distance attitude.

6.2 Suggestions for Future Work

The importance of ranking subject in DEA concept has shown that future study in this direction is necessary and our work opens up several research directions. A study on combining our work in DEA models is recommended as a significant future expansion. It is verified that the applications of facet analysis and non-Archimedean elements with other DEA methodologies can be developed to rank DMUs. Another direction for research is provided by ranking DMUs with imprecise and vague data. Moreover, determination of λ can be expanded in our method. In this thesis, we focused on modeling input orientations and our study can be easily extended to output-oriented models.

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APPENDICES

Appendix A: Coding Example 5.1 Using WinQSB

A.1 Optimal Solutions of CCR Models Summarized in Table 5.2

Optimal solution of the CCR model (model 2.1) for DMU_1:

	12:59:17		Thursday	May	02	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0	1.0000	0	0	at bound	-M	1.0000
3	X3	0.1429	7.0000	1.0000	0	basic	7.0000	M
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	1.0000	0	M
2	C2	0.0000	<=	0	0	1.0000	-1.0000	0
3	C3	-0.2857	<=	0	0.2857	0	-0.2857	M
4	C4	0.0000	<=	0	0	0	0	M
5	C5	-0.4286	<=	0	0.4286	0	-0.4286	M
6	C6	-0.5714	<=	0	0.5714	0	-0.5714	M
7	C7	-0.1429	<=	0	0.1429	0	-0.1429	M
8	C8	-0.2857	<=	0	0.2857	0	-0.2857	M
9	C9	-0.7143	<=	0	0.7143	0	-0.7143	M

Optimal solution of the CCR model (model 2.1) for DMU_2:

	13:00:29		Thursday	May	02	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0	1.0000	0	0	basic	0.7143	1.4286
3	X3	0.1429	5.0000	0.7143	0	basic	3.5000	7.0000
	Objective	Function	(Max.) =	0.7143				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.7143	0	M
2	C2	0.0000	<=	0	0	0.4286	-0.0625	0
3	C3	-0.2857	<=	0	0.2857	0	-0.2857	M
4	C4	0.0000	<=	0	0	0.2857	0	0.0455
5	C5	-0.4286	<=	0	0.4286	0	-0.4286	M
6	C6	-0.5714	<=	0	0.5714	0	-0.5714	M
7	C7	-0.1429	<=	0	0.1429	0	-0.1429	M
8	C8	-0.2857	<=	0	0.2857	0	-0.2857	M
9	C9	-0.7143	<=	0	0.7143	0	-0.7143	M

Optimal solution of the CCR model (model 2.1) for DMU_3:

	13:01:25		Thursday	May	02	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0	2.0000	0	0	basic	1.0000	2.0000
3	X3	0.1429	7.0000	1.0000	0	basic	7.0000	14.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	1.0000	0	M
2	C2	0.0000	<=	0	0	0	-0.0625	0
3	C3	-0.2857	<=	0	0.2857	0	-0.2857	M
4	C4	0.0000	<=	0	0	1.0000	0	0.0455
5	C5	-0.4286	<=	0	0.4286	0	-0.4286	M
6	C6	-0.5714	<=	0	0.5714	0	-0.5714	M
7	C7	-0.1429	<=	0	0.1429	0	-0.1429	M
8	C8	-0.2857	<=	0	0.2857	0	-0.2857	M
9	C9	-0.7143	<=	0	0.7143	0	-0.7143	M

Optimal solution of the CCR model (model 2.1) for DMU_4:

	13:02:14		Thursday	May	02	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.1000	3.0000	0.3000	0	basic	2.6667	4.0000
3	X3	0.1000	4.0000	0.4000	0	basic	3.0000	4.5000
	Objective	Function	(Max.) =	0.7000				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.7000	0	M
2	C2	-0.2000	<=	0	0.2000	0	-0.2000	M
3	C3	-0.4000	<=	0	0.4000	0	-0.4000	M
4	C4	-0.1000	<=	0	0.1000	0	-0.1000	M
5	C5	-0.3000	<=	0	0.3000	0	-0.3000	M
6	C6	-0.3000	<=	0	0.3000	0	-0.3000	M
7	C7	0.0000	<=	0	0	0.5000	-0.1000	0.0400
8	C8	0.0000	<=	0	0	0.2000	-0.0625	0.0714
9	C9	-0.2000	<=	0	0.2000	0	-0.2000	M

Optimal solution of the CCR model (model 2.1) for DMU_5:

	13:03:27		Thursday	May	02	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.1500	4.0000	0.6000	0	basic	3.0000	9.0000
3	X3	0.0500	3.0000	0.1500	0	basic	1.3333	4.0000
	Objective	Function	(Max.) =	0.7500				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.7500	0	M
2	C2	-0.5000	<=	0	0.5000	0	-0.5000	M
3	C3	-0.6000	<=	0	0.6000	0	-0.6000	M
4	C4	-0.3500	<=	0	0.3500	0	-0.3500	M
5	C5	-0.3500	<=	0	0.3500	0	-0.3500	M
6	C6	-0.2500	<=	0	0.2500	0	-0.2500	M
7	C7	-0.1000	<=	0	0.1000	0	-0.1000	M
8	C8	0.0000	<=	0	0	0.5000	-0.1667	0.0714
9	C9	0.0000	<=	0	0	0.2500	-0.2000	0.2000

Optimal solution of the CCR model (model 2.1) for DMU_6:

	13:04:27		Thursday	May	02	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.0625	4.0000	0.2500	0	basic	1.7143	4.0000
3	X3	0.1250	6.0000	0.7500	0	basic	6.0000	14.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	1.0000	0	M
2	C2	-0.0625	<=	0	0.0625	0	-0.0625	M
3	C3	-0.3125	<=	0	0.3125	0	-0.3125	M
4	C4	0	<=	0	0	0	-0.1000	0.0455
5	C5	-0.3125	<=	0	0.3125	0	-0.3125	M
6	C6	-0.3750	<=	0	0.3750	0	-0.3750	M
7	C7	0	<=	0	0	1.0000	-0.1429	0.0400
8	C8	-0.0625	<=	0	0.0625	0	-0.0625	M
9	C9	-0.3750	<=	0	0.3750	0	-0.3750	M

Optimal solution of the CCR model (model 2.1) for DMU_7:

	13:05:17		Thursday	May	02	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.1500	5.0000	0.7500	0	basic	5.0000	15.0000
3	X3	0.0500	5.0000	0.2500	0	basic	1.6667	5.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	1.0000	0	M
2	C2	-0.5000	<=	0	0.5000	0	-0.5000	M
3	C3	-0.6000	<=	0	0.6000	0	-0.6000	M
4	C4	-0.3500	<=	0	0.3500	0	-0.3500	M
5	C5	-0.3500	<=	0	0.3500	0	-0.3500	M
6	C6	-0.2500	<=	0	0.2500	0	-0.2500	M
7	C7	-0.1000	<=	0	0.1000	0	-0.1000	M
8	C8	0.0000	<=	0	0	1.0000	-0.1667	0.0714
9	C9	0.0000	<=	0	0	0	-0.2000	0.2000

Optimal solution of the CCR model (model 2.1) for DMU_8:

	13:06:19		Thursday	May	02	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.1667	6.0000	1.0000	0	basic	6.0000	M
3	X3	0	2.0000	0	0	at bound	-M	2.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	1.0000	0	M
2	C2	-0.8333	<=	0	0.8333	0	-0.8333	M
3	C3	-0.8333	<=	0	0.8333	0	-0.8333	M
4	C4	-0.6667	<=	0	0.6667	0	-0.6667	M
5	C5	-0.5000	<=	0	0.5000	0	-0.5000	M
6	C6	-0.3333	<=	0	0.3333	0	-0.3333	M
7	C7	-0.3333	<=	0	0.3333	0	-0.3333	M
8	C8	-0.1667	<=	0	0.1667	0	-0.1667	M
9	C9	0.0000	<=	0	0	1.0000	-1.0000	0.2000

Optimal solution of the CCR model (model 2.16) with $\varepsilon = 0.05$ for DMU_1:

	13:20:00		Thursday	May	02	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.0500	1.0000	0.0500	-1.0000	at bound	-M	2.0000
3	X3	0.1286	7.0000	0.9000	0	basic	3.5000	M
	Objective	Function	(Max.) =	0.9500				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	1.0000	0.8000	M
2	C2	-0.0500	<=	0	0.0500	0	-0.0500	M
3	C3	-0.3071	<=	0	0.3071	0	-0.3071	M
4	C4	0.0000	<=	0	0	1.0000	-0.5500	0.0333
5	C5	-0.3357	<=	0	0.3357	0	-0.3357	M
6	C6	-0.4143	<=	0	0.4143	0	-0.4143	M
7	C7	-0.0286	<=	0	0.0286	0	-0.0286	M
8	C8	-0.1071	<=	0	0.1071	0	-0.1071	M
9	C9	-0.4429	<=	0	0.4429	0	-0.4429	M

Optimal solution of the CCR model (model 2.16) with $\varepsilon = 0.05$ for DMU_2:

	13:19:08		Thursday	May	02	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.0500	1.0000	0.0500	-0.4286	at bound	-M	1.4286
3	X3	0.1286	5.0000	0.6429	0	basic	3.5000	M
	Objective	Function	(Max.) =	0.6929				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.7143	0.8000	M
2	C2	-0.0500	<=	0	0.0500	0	-0.0500	M
3	C3	-0.3071	<=	0	0.3071	0	-0.3071	M
4	C4	0.0000	<=	0	0	0.7143	-0.5500	0.0333
5	C5	-0.3357	<=	0	0.3357	0	-0.3357	M
6	C6	-0.4143	<=	0	0.4143	0	-0.4143	M
7	C7	-0.0286	<=	0	0.0286	0	-0.0286	M
8	C8	-0.1071	<=	0	0.1071	0	-0.1071	M
9	C9	-0.4429	<=	0	0.4429	0	-0.4429	M

Optimal solution of the CCR model (model 2.16) with $\varepsilon = 0.05$ for DMU_3:

	13:14:36		Thursday	May	02	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.0625	2.0000	0.1250	0	basic	2.0000	4.6667
3	X3	0.1250	7.0000	0.8750	0	basic	3.0000	7.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	1.0000	0.8000	M
2	C2	-0.0625	<=	0	0.0625	0	-0.0625	M
3	C3	-0.3125	<=	0	0.3125	0	-0.3125	M
4	C4	0	<=	0	0	1.0000	-0.1000	0.0333
5	C5	-0.3125	<=	0	0.3125	0	-0.3125	M
6	C6	-0.3750	<=	0	0.3750	0	-0.3750	M
7	C7	0	<=	0	0	0	-0.0286	0.0400
8	C8	-0.0625	<=	0	0.0625	0	-0.0625	M
9	C9	-0.3750	<=	0	0.3750	0	-0.3750	M

Optimal solution of the CCR model (model 2.16) with $\varepsilon = 0.05$ for DMU_4:

	13:13:33		Thursday	May	02	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.1000	3.0000	0.3000	0	basic	2.6667	4.0000
3	X3	0.1000	4.0000	0.4000	0	basic	3.0000	4.5000
	Objective	Function	(Max.) =	0.7000				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.7000	0.5000	M
2	C2	-0.2000	<=	0	0.2000	0	-0.2000	M
3	C3	-0.4000	<=	0	0.4000	0	-0.4000	M
4	C4	-0.1000	<=	0	0.1000	0	-0.1000	M
5	C5	-0.3000	<=	0	0.3000	0	-0.3000	M
6	C6	-0.3000	<=	0	0.3000	0	-0.3000	M
7	C7	0.0000	<=	0	0	0.5000	-0.1000	0.0400
8	C8	0.0000	<=	0	0	0.2000	-0.0625	0.0714
9	C9	-0.2000	<=	0	0.2000	0	-0.2000	M

Optimal solution of the CCR model (model 2.16) with $\varepsilon = 0.05$ for DMU_5:

	13:12:32		Thursday	May	02	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.1500	4.0000	0.6000	0	basic	3.0000	M
3	X3	0.0500	3.0000	0.1500	-1.0000	at bound	-M	4.0000
	Objective	Function	(Max.) =	0.7500				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.8000	0.5000	1.0000
2	C2	-0.5000	<=	0	0.5000	0	-0.5000	M
3	C3	-0.6000	<=	0	0.6000	0	-0.6000	M
4	C4	-0.3500	<=	0	0.3500	0	-0.3500	M
5	C5	-0.3500	<=	0	0.3500	0	-0.3500	M
6	C6	-0.2500	<=	0	0.2500	0	-0.2500	M
7	C7	-0.1000	<=	0	0.1000	0	-0.1000	M
8	C8	0.0000	<=	0	0	0.8000	-0.5000	0
9	C9	0.0000	<=	0	0	0	0	M

Optimal solution of the CCR model (model 2.16) with $\varepsilon = 0.05$ for DMU_6:

	13:11:32		Thursday	May	02	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.1000	4.0000	0.4000	0	basic	4.0000	6.0000
3	X3	0.1000	6.0000	0.6000	0	basic	4.0000	6.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	1.0000	0.5000	M
2	C2	-0.2000	<=	0	0.2000	0	-0.2000	M
3	C3	-0.4000	<=	0	0.4000	0	-0.4000	M
4	C4	-0.1000	<=	0	0.1000	0	-0.1000	M
5	C5	-0.3000	<=	0	0.3000	0	-0.3000	M
6	C6	-0.3000	<=	0	0.3000	0	-0.3000	M
7	C7	0.0000	<=	0	0	1.0000	-0.1000	0.0400
8	C8	0.0000	<=	0	0	0	-0.0625	0.0714
9	C9	-0.2000	<=	0	0.2000	0	-0.2000	M

Optimal solution of the CCR model (model 2.16) with $\varepsilon = 0.05$ for DMU_7:

	13:09:45		Thursday	May	02	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.1500	5.0000	0.7500	0	basic	5.0000	M
3	X3	0.0500	5.0000	0.2500	0	at bound	-M	5.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	1.0000	0.5000	1.0000
2	C2	-0.5000	<=	0	0.5000	0	-0.5000	M
3	C3	-0.6000	<=	0	0.6000	0	-0.6000	M
4	C4	-0.3500	<=	0	0.3500	0	-0.3500	M
5	C5	-0.3500	<=	0	0.3500	0	-0.3500	M
6	C6	-0.2500	<=	0	0.2500	0	-0.2500	M
7	C7	-0.1000	<=	0	0.1000	0	-0.1000	M
8	C8	0.0000	<=	0	0	1.0000	-0.5000	0
9	C9	0.0000	<=	0	0	0	0	M

Optimal solution of the CCR model (model 2.16) with $\varepsilon = 0.05$ for DMU_8:

	13:08:52		Thursday	May	02	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.1500	6.0000	0.9000	0	basic	2.0000	M
3	X3	0.0500	2.0000	0.1000	-4.0000	at bound	-M	6.0000
	Objective	Function	(Max.) =	1.0000				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	1.2000	0.5000	1.0000
2	C2	-0.5000	<=	0	0.5000	0	-0.5000	M
3	C3	-0.6000	<=	0	0.6000	0	-0.6000	M
4	C4	-0.3500	<=	0	0.3500	0	-0.3500	M
5	C5	-0.3500	<=	0	0.3500	0	-0.3500	M
6	C6	-0.2500	<=	0	0.2500	0	-0.2500	M
7	C7	-0.1000	<=	0	0.1000	0	-0.1000	M
8	C8	0.0000	<=	0	0	1.2000	-0.5000	0
9	C9	0.0000	<=	0	0	0	0	M

Optimal solution of the modified CCR model (model 4.9) for DMU_1:

	14:35:11		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.0625	1.0000	0.0625	-1.0000	at bound	-M	2.0000
3	X3	0.1250	7.0000	0.8750	0	basic	3.5000	M
	Objective	Function	(Max.) =	0.9375				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	1.0000	1.0000	M
2	C2	-0.0625	<=	0	0.0625	0	-0.0625	M
3	C3	-0.3125	<=	0	0.3125	0	-0.3125	M
4	C4	0	<=	0	0	1.0000	-0.5250	0
5	C5	-0.3125	<=	0	0.3125	0	-0.3125	M
6	C6	-0.3750	<=	0	0.3750	0	-0.3750	M
7	C7	0	<=	0	0	0	0	M
8	C8	-0.0625	<=	0	0.0625	0	-0.0625	M
9	C9	-0.3750	<=	0	0.3750	0	-0.3750	M

Optimal solution of the modified CCR model (model 4.9) for DMU_2:

	14:36:09		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.0625	1.0000	0.0625	-0.4286	at bound	-M	1.4286
3	X3	0.1250	5.0000	0.6250	0	basic	3.5000	M
	Objective	Function	(Max.) =	0.6875				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.7143	1.0000	M
2	C2	-0.0625	<=	0	0.0625	0	-0.0625	M
3	C3	-0.3125	<=	0	0.3125	0	-0.3125	M
4	C4	0	<=	0	0	0.7143	-0.5250	0
5	C5	-0.3125	<=	0	0.3125	0	-0.3125	M
6	C6	-0.3750	<=	0	0.3750	0	-0.3750	M
7	C7	0	<=	0	0	0	0	M
8	C8	-0.0625	<=	0	0.0625	0	-0.0625	M
9	C9	-0.3750	<=	0	0.3750	0	-0.3750	M

Optimal solution of the modified CCR model (model 4.9) for DMU_3:

	14:36:55		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.0625	2.0000	0.1250	0	at bound	-M	2.0000
3	X3	0.1250	7.0000	0.8750	0	basic	7.0000	M
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	1.0000	1.0000	M
2	C2	-0.0625	<=	0	0.0625	0	-0.0625	M
3	C3	-0.3125	<=	0	0.3125	0	-0.3125	M
4	C4	0	<=	0	0	1.0000	-0.5250	0
5	C5	-0.3125	<=	0	0.3125	0	-0.3125	M
6	C6	-0.3750	<=	0	0.3750	0	-0.3750	M
7	C7	0	<=	0	0	0	0	M
8	C8	-0.0625	<=	0	0.0625	0	-0.0625	M
9	C9	-0.3750	<=	0	0.3750	0	-0.3750	M

Optimal solution of the modified CCR model (model 4.9) for DMU_4:

	14:37:43		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.1000	3.0000	0.3000	0	basic	2.6667	4.0000
3	X3	0.1000	4.0000	0.4000	0	basic	3.0000	4.5000
	Objective	Function	(Max.) =	0.7000				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.7000	1.0000	M
2	C2	-0.2000	<=	0	0.2000	0	-0.2000	M
3	C3	-0.4000	<=	0	0.4000	0	-0.4000	M
4	C4	-0.1000	<=	0	0.1000	0	-0.1000	M
5	C5	-0.3000	<=	0	0.3000	0	-0.3000	M
6	C6	-0.3000	<=	0	0.3000	0	-0.3000	M
7	C7	0	<=	0	0	0.5000	-0.1000	0.0400
8	C8	0.0000	<=	0	0	0.2000	-0.0625	0.0714
9	C9	-0.2000	<=	0	0.2000	0	-0.2000	M

Optimal solution of the modified CCR model (model 4.9) for DMU_5:

	14:38:41		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.1500	4.0000	0.6000	0	basic	3.0000	M
3	X3	0.0500	3.0000	0.1500	-1.0000	at bound	-M	4.0000
	Objective	Function	(Max.) =	0.7500				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.8000	1.0000	1.0000
2	C2	-0.5000	<=	0	0.5000	0	-0.5000	M
3	C3	-0.6000	<=	0	0.6000	0	-0.6000	M
4	C4	-0.3500	<=	0	0.3500	0	-0.3500	M
5	C5	-0.3500	<=	0	0.3500	0	-0.3500	M
6	C6	-0.2500	<=	0	0.2500	0	-0.2500	M
7	C7	-0.1000	<=	0	0.1000	0	-0.1000	M
8	C8	0.0000	<=	0	0	0.8000	-0.4375	0
9	C9	0.0000	<=	0	0	0	0	M

Optimal solution of the modified CCR model (model 4.9) for DMU_6:

	14:39:52		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.0625	4.0000	0.2500	0	basic	1.7143	4.0000
3	X3	0.1250	6.0000	0.7500	0	basic	6.0000	14.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	1.0000	1.0000	M
2	C2	-0.0625	<=	0	0.0625	0	-0.0625	M
3	C3	-0.3125	<=	0	0.3125	0	-0.3125	M
4	C4	0	<=	0	0	0	-0.1000	0
5	C5	-0.3125	<=	0	0.3125	0	-0.3125	M
6	C6	-0.3750	<=	0	0.3750	0	-0.3750	M
7	C7	0	<=	0	0	1.0000	0	0.0400
8	C8	-0.0625	<=	0	0.0625	0	-0.0625	M
9	C9	-0.3750	<=	0	0.3750	0	-0.3750	M

Optimal solution of the modified CCR model (model 4.9) for DMU_7:

	14:40:29		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.1500	5.0000	0.7500	0	basic	5.0000	M
3	X3	0.0500	5.0000	0.2500	0	at bound	-M	5.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	1.0000	1.0000	1.0000
2	C2	-0.5000	<=	0	0.5000	0	-0.5000	M
3	C3	-0.6000	<=	0	0.6000	0	-0.6000	M
4	C4	-0.3500	<=	0	0.3500	0	-0.3500	M
5	C5	-0.3500	<=	0	0.3500	0	-0.3500	M
6	C6	-0.2500	<=	0	0.2500	0	-0.2500	M
7	C7	-0.1000	<=	0	0.1000	0	-0.1000	M
8	C8	0.0000	<=	0	0	1.0000	-0.4375	0
9	C9	0.0000	<=	0	0	0	0	M

Optimal solution of the modified CCR model (model 4.9) for DMU_8:

	14:42:12		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.1500	6.0000	0.9000	0	basic	2.0000	M
3	X3	0.0500	2.0000	0.1000	-4.0000	at bound	-M	6.0000
	Objective	Function	(Max.) =	1.0000				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	1.2000	1.0000	1.0000
2	C2	-0.5000	<=	0	0.5000	0	-0.5000	M
3	C3	-0.6000	<=	0	0.6000	0	-0.6000	M
4	C4	-0.3500	<=	0	0.3500	0	-0.3500	M
5	C5	-0.3500	<=	0	0.3500	0	-0.3500	M
6	C6	-0.2500	<=	0	0.2500	0	-0.2500	M
7	C7	-0.1000	<=	0	0.1000	0	-0.1000	M
8	C8	0.0000	<=	0	0	1.2000	-0.4375	0
9	C9	0.0000	<=	0	0	0	0	M

A.2 Optimal Values of Models (4.5) and (4.6) Summarized in Table 5.4

Optimal v_1 of model 4.5 for DMU_3:

	14:17:42		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	1.0000	1.0000	0	basic	-M	M
2	X2	0	0	0	0	at bound	-M	0
3	X3	0.1429	0	0	0	basic	0	M
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	1.0000	1.0000	M
2	C2	0.0000	<=	0	0	0	0	M
3	C3	-0.2857	<=	0	0.2857	0	-0.2857	M
4	C4	0.0000	<=	0	0	0	0	M
5	C5	-0.4286	<=	0	0.4286	0	-0.4286	M
6	C6	-0.5714	<=	0	0.5714	0	-0.5714	M
7	C7	-0.1429	<=	0	0.1429	0	-0.1429	M
8	C8	-0.2857	<=	0	0.2857	0	-0.2857	M
9	C9	-0.7143	<=	0	0.7143	0	-0.7143	M
10	C10	1.0000	=	1.0000	0	0	0	1.0000

Optimal u_1 of model 4.6 for DMU_3:

	14:19:18		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.0625	1.0000	0.0625	0	basic	0	M
3	X3	0.1250	0	0	0	basic	-M	3.5000
	Objective	Function	(Max.) =	0.0625				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.4375	1.0000	1.1111
2	C2	-0.0625	<=	0	0.0625	0	-0.0625	M
3	C3	-0.3125	<=	0	0.3125	0	-0.3125	M
4	C4	0	<=	0	0	0	0	M
5	C5	-0.3125	<=	0	0.3125	0	-0.3125	M
6	C6	-0.3750	<=	0	0.3750	0	-0.3750	M
7	C7	0	<=	0	0	0.4375	-0.1429	0.0400
8	C8	-0.0625	<=	0	0.0625	0	-0.0625	M
9	C9	-0.3750	<=	0	0.3750	0	-0.3750	M
10	C10	1.0000	=	1.0000	0	-0.3750	0.9000	1.0000

Optimal u_2 of model 4.6 for DMU_3:

	14:20:32		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0	0	0	-0.2857	at bound	-M	0.2857
3	X3	0.1429	1.0000	0.1429	0	basic	0	M
	Objective	Function	(Max.) =	0.1429				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0	1.0000	M
2	C2	0.0000	<=	0	0	0	0	M
3	C3	-0.2857	<=	0	0.2857	0	-0.2857	M
4	C4	0.0000	<=	0	0	0	0	M
5	C5	-0.4286	<=	0	0.4286	0	-0.4286	M
6	C6	-0.5714	<=	0	0.5714	0	-0.5714	M
7	C7	-0.1429	<=	0	0.1429	0	-0.1429	M
8	C8	-0.2857	<=	0	0.2857	0	-0.2857	M
9	C9	-0.7143	<=	0	0.7143	0	-0.7143	M
10	C10	1.0000	=	1.0000	0	0.1429	0	1.0000

Optimal v_1 of model 4.5 for DMU_8:

	14:22:31		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	1.0000	1.0000	0	basic	-M	M
2	X2	0.1500	0	0	0	basic	-M	0
3	X3	0.0500	0	0	0	basic	0	M
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	1.0000	1.0000	1.2500
2	C2	-0.5000	<=	0	0.5000	0	-0.5000	M
3	C3	-0.6000	<=	0	0.6000	0	-0.6000	M
4	C4	-0.3500	<=	0	0.3500	0	-0.3500	M
5	C5	-0.3500	<=	0	0.3500	0	-0.3500	M
6	C6	-0.2500	<=	0	0.2500	0	-0.2500	M
7	C7	-0.1000	<=	0	0.1000	0	-0.1000	M
8	C8	0.0000	<=	0	0	0	-0.1667	0.0714
9	C9	0.0000	<=	0	0	0	0	M
10	C10	1.0000	=	1.0000	0	0	0.8000	1.0000

Optimal u_1 of model 4.6 for DMU_8:

	14:25:10		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.1667	1.0000	0.1667	0	basic	0	M
3	X3	0	0	0	-0.3333	at bound	-M	0.3333
	Objective	Function	(Max.) =	0.1667				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0	1.0000	M
2	C2	-0.8333	<=	0	0.8333	0	-0.8333	M
3	C3	-0.8333	<=	0	0.8333	0	-0.8333	M
4	C4	-0.6667	<=	0	0.6667	0	-0.6667	M
5	C5	-0.5000	<=	0	0.5000	0	-0.5000	M
6	C6	-0.3333	<=	0	0.3333	0	-0.3333	M
7	C7	-0.3333	<=	0	0.3333	0	-0.3333	M
8	C8	-0.1667	<=	0	0.1667	0	-0.1667	M
9	C9	0.0000	<=	0	0	0	0	M
10	C10	1.0000	=	1.0000	0	0.1667	0	1.0000

Optimal u_2 of model 4.6 for DMU_8:

	14:23:54		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.1500	0	0	0	basic	-M	3.0000
3	X3	0.0500	1.0000	0.0500	0	basic	0	M
	Objective	Function	(Max.) =	0.0500				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.3000	1.0000	1.2500
2	C2	-0.5000	<=	0	0.5000	0	-0.5000	M
3	C3	-0.6000	<=	0	0.6000	0	-0.6000	M
4	C4	-0.3500	<=	0	0.3500	0	-0.3500	M
5	C5	-0.3500	<=	0	0.3500	0	-0.3500	M
6	C6	-0.2500	<=	0	0.2500	0	-0.2500	M
7	C7	-0.1000	<=	0	0.1000	0	-0.1000	M
8	C8	0.0000	<=	0	0	0.3000	-0.1667	0.0714
9	C9	0.0000	<=	0	0	0	0	M
10	C10	1.0000	=	1.0000	0	-0.2500	0.8000	1.0000

A.3 Optimal Values of Models (5.4) Presented by Khodabakhshi and Aryavash

Maximum value of model (5.4) for DMU_1:

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0	1.0000	0	-3.6667	at bound	-M	4.6667
3	X3	0.0256	7.0000	0.1795	0	basic	1.5000	M
4	X4	0.1795	0	0	0	basic	-1.0000	M
5	X5	0.1282	0	0	0	basic	-1.5714	M
6	X6	0.1795	0	0	0	basic	-1.3750	M
7	X7	0.1026	0	0	0	basic	-M	11.0000
8	X8	0.0769	0	0	0	basic	-M	1.8333
9	X9	0.1538	0	0	0	basic	-M	M
10	X10	0.1282	0	0	0	basic	-M	2.2000
11	X11	0.0513	0	0	0	basic	-M	0.7857
	Objective	Function	(Max.) =	0.1795				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.1795	0	M
2	C2	0	=	0	0	-0.1795	-0.2188	1.0000
3	C3	0	=	0	0	-0.1795	-0.1471	1.0000
4	C4	0	=	0	0	-0.1795	-0.2188	1.0000
5	C5	0	=	0	0	-0.1795	-0.1143	1.0000
6	C6	0	=	0	0	-0.1795	-0.0833	1.0000
7	C7	0	=	0	0	-0.1795	-0.1818	1.0000
8	C8	0	=	0	0	-0.1795	-0.1471	1.0000
9	C9	0	=	0	0	-0.1795	-0.0541	1.0000
10	C10	0.0000	=	0	0	0.1795	-1.0000	M

Maximum value of model (5.4) for DMU_2:

	14:55:42		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0	1.0000	0	-2.3333	at bound	-M	3.3333
3	X3	0.0256	5.0000	0.1282	0	basic	1.5000	M
4	X4	0.1795	0	0	0	basic	-0.6364	M
5	X5	0.1282	0	0	0	basic	-1.0000	M
6	X6	0.1795	0	0	0	basic	-0.8750	M
7	X7	0.1026	0	0	0	basic	-M	7.0000
8	X8	0.0769	0	0	0	basic	-M	1.1667
9	X9	0.1538	0	0	0	basic	-M	M
10	X10	0.1282	0	0	0	basic	-M	1.4000
11	X11	0.0513	0	0	0	basic	-M	0.5000
	Objective	Function	(Max.) =	0.1282				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.1282	0	M
2	C2	0	=	0	0	-0.1282	-0.2188	1.0000
3	C3	0	=	0	0	-0.1282	-0.1471	1.0000
4	C4	0	=	0	0	-0.1282	-0.2188	1.0000
5	C5	0	=	0	0	-0.1282	-0.1143	1.0000
6	C6	0	=	0	0	-0.1282	-0.0833	1.0000
7	C7	0	=	0	0	-0.1282	-0.1818	1.0000
8	C8	0	=	0	0	-0.1282	-0.1471	1.0000
9	C9	0	=	0	0	-0.1282	-0.0541	1.0000
10	C10	0.0000	=	0	0	0.1282	-1.0000	M

Maximum value of model (5.4) for DMU_3:

	14:56:49		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0	2.0000	0	-2.6667	at bound	-M	4.6667
3	X3	0.0256	7.0000	0.1795	0	basic	3.0000	M
4	X4	0.1795	0	0	0	basic	-0.7273	M
5	X5	0.1282	0	0	0	basic	-1.1429	M
6	X6	0.1795	0	0	0	basic	-1.0000	M
7	X7	0.1026	0	0	0	basic	-M	8.0000
8	X8	0.0769	0	0	0	basic	-M	1.3333
9	X9	0.1538	0	0	0	basic	-M	M
10	X10	0.1282	0	0	0	basic	-M	1.6000
11	X11	0.0513	0	0	0	basic	-M	0.5714
	Objective	Function	(Max.) =	0.1795				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.1795	0	M
2	C2	0	=	0	0	-0.1795	-0.2188	1.0000
3	C3	0	=	0	0	-0.1795	-0.1471	1.0000
4	C4	0	=	0	0	-0.1795	-0.2188	1.0000
5	C5	0	=	0	0	-0.1795	-0.1143	1.0000
6	C6	0	=	0	0	-0.1795	-0.0833	1.0000
7	C7	0	=	0	0	-0.1795	-0.1818	1.0000
8	C8	0	=	0	0	-0.1795	-0.1471	1.0000
9	C9	0	=	0	0	-0.1795	-0.0541	1.0000
10	C10	0.0000	=	0	0	0.1795	-1.0000	M

Maximum value of model (5.4) for DMU_4:

	14:58:09		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.0385	3.0000	0.1154	0	basic	2.6667	M
3	X3	0	4.0000	0	-0.5000	at bound	-M	4.5000
4	X4	0.0385	0	0	0	basic	-M	0.0909
5	X5	0.0385	0	0	0	basic	-M	0.1429
6	X6	0.0769	0	0	0	basic	-M	0.1250
7	X7	0.1154	0	0	0	basic	-1.0000	M
8	X8	0.1538	0	0	0	basic	-0.1667	M
9	X9	0.1538	0	0	0	basic	-M	M
10	X10	0.1923	0	0	0	basic	-0.2000	M
11	X11	0.2308	0	0	0	basic	-0.0714	M
	Objective	Function	(Max.) =	0.1154				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.1154	0	M
2	C2	0	=	0	0	-0.1154	-0.0400	1.0000
3	C3	0	=	0	0	-0.1154	-0.0400	1.0000
4	C4	0	=	0	0	-0.1154	-0.0833	1.0000
5	C5	0.0000	=	0	0	-0.1154	-0.1304	1.0000
6	C6	0	=	0	0	-0.1154	-0.1818	1.0000
7	C7	0	=	0	0	-0.1154	-0.1818	1.0000
8	C8	0	=	0	0	-0.1154	-0.2381	1.0000
9	C9	0.0000	=	0	0	-0.1154	-0.3000	1.0000
10	C10	0.0000	=	0	0	0.1154	-1.0000	M

Maximum value of model (5.4) for DMU_5:

	15:03:27		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.0385	4.0000	0.1538	0	basic	2.0000	M
3	X3	0	3.0000	0	-3.0000	at bound	-M	6.0000
4	X4	0.0385	0	0	0	basic	-M	0.5455
5	X5	0.0385	0	0	0	basic	-M	0.8571
6	X6	0.0769	0	0	0	basic	-M	0.7500
7	X7	0.1154	0	0	0	basic	-6.0000	M
8	X8	0.1538	0	0	0	basic	-1.0000	M
9	X9	0.1538	0	0	0	basic	-M	M
10	X10	0.1923	0	0	0	basic	-1.2000	M
11	X11	0.2308	0	0	0	basic	-0.4286	M
	Objective	Function	(Max.) =	0.1538				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.1538	0	M
2	C2	0	=	0	0	-0.1538	-0.0400	1.0000
3	C3	0	=	0	0	-0.1538	-0.0400	1.0000
4	C4	0	=	0	0	-0.1538	-0.0833	1.0000
5	C5	0.0000	=	0	0	-0.1538	-0.1304	1.0000
6	C6	0	=	0	0	-0.1538	-0.1818	1.0000
7	C7	0	=	0	0	-0.1538	-0.1818	1.0000
8	C8	0	=	0	0	-0.1538	-0.2381	1.0000
9	C9	0.0000	=	0	0	-0.1538	-0.3000	1.0000
10	C10	0.0000	=	0	0	0.1538	-1.0000	M

Maximum value of model (5.4) for DMU_6:

	15:04:02		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0	4.0000	0	0	at bound	-M	4.0000
3	X3	0.0256	6.0000	0.1538	0	basic	6.0000	M
4	X4	0.1795	0	0	0	basic	0	M
5	X5	0.1282	0	0	0	basic	0	M
6	X6	0.1795	0	0	0	basic	0	M
7	X7	0.1026	0	0	0	basic	-M	0
8	X8	0.0769	0	0	0	basic	-M	0
9	X9	0.1538	0	0	0	basic	-M	M
10	X10	0.1282	0	0	0	basic	-M	0
11	X11	0.0513	0	0	0	basic	-M	0
	Objective	Function	(Max.) =	0.1538	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.1538	0	M
2	C2	0	=	0	0	-0.1538	-0.2188	1.0000
3	C3	0	=	0	0	-0.1538	-0.1471	1.0000
4	C4	0	=	0	0	-0.1538	-0.2188	1.0000
5	C5	0	=	0	0	-0.1538	-0.1143	1.0000
6	C6	0	=	0	0	-0.1538	-0.0833	1.0000
7	C7	0	=	0	0	-0.1538	-0.1818	1.0000
8	C8	0	=	0	0	-0.1538	-0.1471	1.0000
9	C9	0	=	0	0	-0.1538	-0.0541	1.0000
10	C10	0.0000	=	0	0	0.1538	-1.0000	M

Maximum value of model (5.4) for DMU_7:

	15:05:16		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.0385	5.0000	0.1923	0	basic	3.3333	M
3	X3	0	5.0000	0	-2.5000	at bound	-M	7.5000
4	X4	0.0385	0	0	0	basic	-M	0.4545
5	X5	0.0385	0	0	0	basic	-M	0.7143
6	X6	0.0769	0	0	0	basic	-M	0.6250
7	X7	0.1154	0	0	0	basic	-5.0000	M
8	X8	0.1538	0	0	0	basic	-0.8333	M
9	X9	0.1538	0	0	0	basic	-M	M
10	X10	0.1923	0	0	0	basic	-1.0000	M
11	X11	0.2308	0	0	0	basic	-0.3571	M
	Objective	Function	(Max.) =	0.1923				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.1923	0	M
2	C2	0	=	0	0	-0.1923	-0.0400	1.0000
3	C3	0	=	0	0	-0.1923	-0.0400	1.0000
4	C4	0	=	0	0	-0.1923	-0.0833	1.0000
5	C5	0.0000	=	0	0	-0.1923	-0.1304	1.0000
6	C6	0	=	0	0	-0.1923	-0.1818	1.0000
7	C7	0	=	0	0	-0.1923	-0.1818	1.0000
8	C8	0	=	0	0	-0.1923	-0.2381	1.0000
9	C9	0.0000	=	0	0	-0.1923	-0.3000	1.0000
10	C10	0.0000	=	0	0	0.1923	-1.0000	M

Maximum value of model (5.4) for DMU_8:

	15:06:17		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.0385	6.0000	0.2308	0	basic	1.3333	M
3	X3	0	2.0000	0	-7.0000	at bound	-M	9.0000
4	X4	0.0385	0	0	0	basic	-M	1.2727
5	X5	0.0385	0	0	0	basic	-M	2.0000
6	X6	0.0769	0	0	0	basic	-M	1.7500
7	X7	0.1154	0	0	0	basic	-14.0000	M
8	X8	0.1538	0	0	0	basic	-2.3333	M
9	X9	0.1538	0	0	0	basic	-M	M
10	X10	0.1923	0	0	0	basic	-2.8000	M
11	X11	0.2308	0	0	0	basic	-1.0000	M
	Objective	Function	(Max.) =	0.2308				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.2308	0	M
2	C2	0	=	0	0	-0.2308	-0.0400	1.0000
3	C3	0	=	0	0	-0.2308	-0.0400	1.0000
4	C4	0	=	0	0	-0.2308	-0.0833	1.0000
5	C5	0.0000	=	0	0	-0.2308	-0.1304	1.0000
6	C6	0	=	0	0	-0.2308	-0.1818	1.0000
7	C7	0	=	0	0	-0.2308	-0.1818	1.0000
8	C8	0	=	0	0	-0.2308	-0.2381	1.0000
9	C9	0.0000	=	0	0	-0.2308	-0.3000	1.0000
10	C10	0.0000	=	0	0	0.2308	-1.0000	M

Minimum value of model (5.4) for DMU_1:

	15:24:16		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.0385	1.0000	0.0385	0	basic	-M	4.6667
3	X3	0	7.0000	0	5.5000	at bound	1.5000	M
4	X4	0.0385	0	0	0	basic	-1.0000	M
5	X5	0.0385	0	0	0	basic	-1.5714	M
6	X6	0.0769	0	0	0	basic	-1.3750	M
7	X7	0.1154	0	0	0	basic	-M	11.0000
8	X8	0.1538	0	0	0	basic	-M	1.8333
9	X9	0.1538	0	0	0	basic	-M	M
10	X10	0.1923	0	0	0	basic	-M	2.2000
11	X11	0.2308	0	0	0	basic	-M	0.7857
	Objective	Function	(Min.) =	0.0385				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.0385	0	M
2	C2	0	=	0	0	-0.0385	-0.0400	1.0000
3	C3	0	=	0	0	-0.0385	-0.0400	1.0000
4	C4	0	=	0	0	-0.0385	-0.0833	1.0000
5	C5	0.0000	=	0	0	-0.0385	-0.1304	1.0000
6	C6	0	=	0	0	-0.0385	-0.1818	1.0000
7	C7	0	=	0	0	-0.0385	-0.1818	1.0000
8	C8	0	=	0	0	-0.0385	-0.2381	1.0000
9	C9	0.0000	=	0	0	-0.0385	-0.3000	1.0000
10	C10	0.0000	=	0	0	0.0385	-1.0000	M

Minimum value of model (5.4) for DMU_2:

	15:25:36		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.0385	1.0000	0.0385	0	basic	-M	3.3333
3	X3	0	5.0000	0	3.5000	at bound	1.5000	M
4	X4	0.0385	0	0	0	basic	-0.6364	M
5	X5	0.0385	0	0	0	basic	-1.0000	M
6	X6	0.0769	0	0	0	basic	-0.8750	M
7	X7	0.1154	0	0	0	basic	-M	7.0000
8	X8	0.1538	0	0	0	basic	-M	1.1667
9	X9	0.1538	0	0	0	basic	-M	M
10	X10	0.1923	0	0	0	basic	-M	1.4000
11	X11	0.2308	0	0	0	basic	-M	0.5000
	Objective	Function	(Min.) =	0.0385				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.0385	0	M
2	C2	0	=	0	0	-0.0385	-0.0400	1.0000
3	C3	0	=	0	0	-0.0385	-0.0400	1.0000
4	C4	0	=	0	0	-0.0385	-0.0833	1.0000
5	C5	0.0000	=	0	0	-0.0385	-0.1304	1.0000
6	C6	0	=	0	0	-0.0385	-0.1818	1.0000
7	C7	0	=	0	0	-0.0385	-0.1818	1.0000
8	C8	0	=	0	0	-0.0385	-0.2381	1.0000
9	C9	0.0000	=	0	0	-0.0385	-0.3000	1.0000
10	C10	0.0000	=	0	0	0.0385	-1.0000	M

Minimum value of model (5.4) for DMU_3:

	15:26:30		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0.0385	2.0000	0.0769	0	basic	-M	4.6667
3	X3	0	7.0000	0	4.0000	at bound	3.0000	M
4	X4	0.0385	0	0	0	basic	-0.7273	M
5	X5	0.0385	0	0	0	basic	-1.1429	M
6	X6	0.0769	0	0	0	basic	-1.0000	M
7	X7	0.1154	0	0	0	basic	-M	8.0000
8	X8	0.1538	0	0	0	basic	-M	1.3333
9	X9	0.1538	0	0	0	basic	-M	M
10	X10	0.1923	0	0	0	basic	-M	1.6000
11	X11	0.2308	0	0	0	basic	-M	0.5714
	Objective	Function	(Min.) =	0.0769				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.0769	0	M
2	C2	0	=	0	0	-0.0769	-0.0400	1.0000
3	C3	0	=	0	0	-0.0769	-0.0400	1.0000
4	C4	0	=	0	0	-0.0769	-0.0833	1.0000
5	C5	0.0000	=	0	0	-0.0769	-0.1304	1.0000
6	C6	0	=	0	0	-0.0769	-0.1818	1.0000
7	C7	0	=	0	0	-0.0769	-0.1818	1.0000
8	C8	0	=	0	0	-0.0769	-0.2381	1.0000
9	C9	0.0000	=	0	0	-0.0769	-0.3000	1.0000
10	C10	0.0000	=	0	0	0.0769	-1.0000	M

Minimum value of model (5.4) for DMU_4:

	15:27:16		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0	3.0000	0	0.3333	at bound	2.6667	M
3	X3	0.0256	4.0000	0.1026	0	basic	-M	4.5000
4	X4	0.1795	0	0	0	basic	-M	0.0909
5	X5	0.1282	0	0	0	basic	-M	0.1429
6	X6	0.1795	0	0	0	basic	-M	0.1250
7	X7	0.1026	0	0	0	basic	-1.0000	M
8	X8	0.0769	0	0	0	basic	-0.1667	M
9	X9	0.1538	0	0	0	basic	-M	M
10	X10	0.1282	0	0	0	basic	-0.2000	M
11	X11	0.0513	0	0	0	basic	-0.0714	M
	Objective	Function	(Min.) =	0.1026				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.1026	0	M
2	C2	0	=	0	0	-0.1026	-0.2188	1.0000
3	C3	0	=	0	0	-0.1026	-0.1471	1.0000
4	C4	0	=	0	0	-0.1026	-0.2188	1.0000
5	C5	0	=	0	0	-0.1026	-0.1143	1.0000
6	C6	0	=	0	0	-0.1026	-0.0833	1.0000
7	C7	0	=	0	0	-0.1026	-0.1818	1.0000
8	C8	0	=	0	0	-0.1026	-0.1471	1.0000
9	C9	0	=	0	0	-0.1026	-0.0541	1.0000
10	C10	0.0000	=	0	0	0.1026	-1.0000	M

Minimum value of model (5.4) for DMU_5:

	15:28:04		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0	4.0000	0	2.0000	at bound	2.0000	M
3	X3	0.0256	3.0000	0.0769	0	basic	-M	6.0000
4	X4	0.1795	0	0	0	basic	-M	0.5455
5	X5	0.1282	0	0	0	basic	-M	0.8571
6	X6	0.1795	0	0	0	basic	-M	0.7500
7	X7	0.1026	0	0	0	basic	-6.0000	M
8	X8	0.0769	0	0	0	basic	-1.0000	M
9	X9	0.1538	0	0	0	basic	-M	M
10	X10	0.1282	0	0	0	basic	-1.2000	M
11	X11	0.0513	0	0	0	basic	-0.4286	M
	Objective	Function	(Min.) =	0.0769				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.0769	0	M
2	C2	0	=	0	0	-0.0769	-0.2188	1.0000
3	C3	0	=	0	0	-0.0769	-0.1471	1.0000
4	C4	0	=	0	0	-0.0769	-0.2188	1.0000
5	C5	0	=	0	0	-0.0769	-0.1143	1.0000
6	C6	0	=	0	0	-0.0769	-0.0833	1.0000
7	C7	0	=	0	0	-0.0769	-0.1818	1.0000
8	C8	0	=	0	0	-0.0769	-0.1471	1.0000
9	C9	0	=	0	0	-0.0769	-0.0541	1.0000
10	C10	0.0000	=	0	0	0.0769	-1.0000	M

Minimum value of model (5.4) for DMU_6:

	15:29:15		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0	4.0000	0	0	at bound	4.0000	M
3	X3	0.0256	6.0000	0.1538	0	basic	-M	6.0000
4	X4	0.1795	0	0	0	basic	-M	0
5	X5	0.1282	0	0	0	basic	-M	0
6	X6	0.1795	0	0	0	basic	-M	0
7	X7	0.1026	0	0	0	basic	0	M
8	X8	0.0769	0	0	0	basic	0	M
9	X9	0.1538	0	0	0	basic	-M	M
10	X10	0.1282	0	0	0	basic	0	M
11	X11	0.0513	0	0	0	basic	0	M
	Objective	Function	(Min.) =	0.1538	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.1538	0	M
2	C2	0	=	0	0	-0.1538	-0.2188	1.0000
3	C3	0	=	0	0	-0.1538	-0.1471	1.0000
4	C4	0	=	0	0	-0.1538	-0.2188	1.0000
5	C5	0	=	0	0	-0.1538	-0.1143	1.0000
6	C6	0	=	0	0	-0.1538	-0.0833	1.0000
7	C7	0	=	0	0	-0.1538	-0.1818	1.0000
8	C8	0	=	0	0	-0.1538	-0.1471	1.0000
9	C9	0	=	0	0	-0.1538	-0.0541	1.0000
10	C10	0.0000	=	0	0	0.1538	-1.0000	M

Minimum value of model (5.4) for DMU_7:

	15:30:35		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0	5.0000	0	1.6667	at bound	3.3333	M
3	X3	0.0256	5.0000	0.1282	0	basic	-M	7.5000
4	X4	0.1795	0	0	0	basic	-M	0.4545
5	X5	0.1282	0	0	0	basic	-M	0.7143
6	X6	0.1795	0	0	0	basic	-M	0.6250
7	X7	0.1026	0	0	0	basic	-5.0000	M
8	X8	0.0769	0	0	0	basic	-0.8333	M
9	X9	0.1538	0	0	0	basic	-M	M
10	X10	0.1282	0	0	0	basic	-1.0000	M
11	X11	0.0513	0	0	0	basic	-0.3571	M
	Objective	Function	(Min.) =	0.1282				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.1282	0	M
2	C2	0	=	0	0	-0.1282	-0.2188	1.0000
3	C3	0	=	0	0	-0.1282	-0.1471	1.0000
4	C4	0	=	0	0	-0.1282	-0.2188	1.0000
5	C5	0	=	0	0	-0.1282	-0.1143	1.0000
6	C6	0	=	0	0	-0.1282	-0.0833	1.0000
7	C7	0	=	0	0	-0.1282	-0.1818	1.0000
8	C8	0	=	0	0	-0.1282	-0.1471	1.0000
9	C9	0	=	0	0	-0.1282	-0.0541	1.0000
10	C10	0.0000	=	0	0	0.1282	-1.0000	M

Minimum value of model (5.4) for DMU_8:

	15:31:51		Tuesday	April	09	2013		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.0000	0	0	0	basic	-M	M
2	X2	0	6.0000	0	4.6667	at bound	1.3333	M
3	X3	0.0256	2.0000	0.0513	0	basic	-M	9.0000
4	X4	0.1795	0	0	0	basic	-M	1.2727
5	X5	0.1282	0	0	0	basic	-M	2.0000
6	X6	0.1795	0	0	0	basic	-M	1.7500
7	X7	0.1026	0	0	0	basic	-14.0000	M
8	X8	0.0769	0	0	0	basic	-2.3333	M
9	X9	0.1538	0	0	0	basic	-M	M
10	X10	0.1282	0	0	0	basic	-2.8000	M
11	X11	0.0513	0	0	0	basic	-1.0000	M
	Objective	Function	(Min.) =	0.0513				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	1.0000	=	1.0000	0	0.0513	0	M
2	C2	0	=	0	0	-0.0513	-0.2188	1.0000
3	C3	0	=	0	0	-0.0513	-0.1471	1.0000
4	C4	0	=	0	0	-0.0513	-0.2188	1.0000
5	C5	0	=	0	0	-0.0513	-0.1143	1.0000
6	C6	0	=	0	0	-0.0513	-0.0833	1.0000
7	C7	0	=	0	0	-0.0513	-0.1818	1.0000
8	C8	0	=	0	0	-0.0513	-0.1471	1.0000
9	C9	0	=	0	0	-0.0513	-0.0541	1.0000
10	C10	0.0000	=	0	0	0.0513	-1.0000	M

Appendix B: Coding Example 5.10 Using GAMS

B.1 Maximum Solutions of Proposed Model 5.10 Summarized in Table 5.5

Maximum solution of model 5.10 for DMU_1:

```
S O L V E      S U M M A R Y

MODEL   Mehrdad           OBJECTIVE  z
TYPE    LP                DIRECTION  MAXIMIZE
SOLVER  CPLEX             FROM LINE  29

**** SOLVER STATUS     1 Normal Completion
**** MODEL STATUS      1 Optimal
**** OBJECTIVE VALUE             0.2188
```

Maximum solution of model 5.10 for DMU_2:

```
S O L V E      S U M M A R Y

MODEL   Mehrdad           OBJECTIVE  z
TYPE    LP                DIRECTION  MAXIMIZE
SOLVER  CPLEX             FROM LINE  29

**** SOLVER STATUS     1 Normal Completion
**** MODEL STATUS      1 Optimal
**** OBJECTIVE VALUE             0.1562
```

Maximum solution of model 5.10 for DMU_3:

S O L V E   S U M M A R Y

MODEL   Mehrdad   OBJECTIVE z

TYPE   LP   DIRECTION MAXIMIZE

SOLVER CPLEX   FROM LINE 29

**** SOLVER STATUS   1 Normal Completion

**** MODEL STATUS   1 Optimal

**** OBJECTIVE VALUE   0.2188

Maximum solution of model 5.10 for DMU_4:

S O L V E   S U M M A R Y

MODEL   Mehrdad   OBJECTIVE z

TYPE   LP   DIRECTION MAXIMIZE

SOLVER CPLEX   FROM LINE 29

**** SOLVER STATUS   1 Normal Completion

**** MODEL STATUS   1 Optimal

**** OBJECTIVE VALUE   0.1250

Maximum solution of model 5.10 for DMU_5:

S O L V E   S U M M A R Y

MODEL   Mehrdad   OBJECTIVE z

TYPE   LP   DIRECTION MAXIMIZE

SOLVER CPLEX   FROM LINE 29

**** SOLVER STATUS   1 Normal Completion

**** MODEL STATUS   1 Optimal

**** OBJECTIVE VALUE   0.1600

Maximum solution of model 5.10 for DMU_6:

S O L V E   S U M M A R Y

MODEL   Mehrdad   OBJECTIVE z

TYPE   LP   DIRECTION MAXIMIZE

SOLVER CPLEX   FROM LINE 29

**** SOLVER STATUS   1 Normal Completion

**** MODEL STATUS   1 Optimal

**** OBJECTIVE VALUE   0.1875

Maximum solution of model 5.10 for DMU_7:

```
S O L V E     S U M M A R Y

MODEL   Mehrdad            OBJECTIVE  z

TYPE    LP                 DIRECTION  MAXIMIZE

SOLVER  CPLEX              FROM LINE  29

**** SOLVER STATUS     1 Normal Completion

**** MODEL STATUS      1 Optimal

**** OBJECTIVE VALUE              0.2000
```

Maximum solution of model 5.10 for DMU_8:

```
S O L V E     S U M M A R Y

MODEL   Mehrdad            OBJECTIVE  z

TYPE    LP                 DIRECTION  MAXIMIZE

SOLVER  CPLEX              FROM LINE  29

**** SOLVER STATUS     1 Normal Completion

**** MODEL STATUS      1 Optimal

**** OBJECTIVE VALUE              0.2400
```

B.2 Minimum Solutions of Proposed Model 5.10 Summarized in Table 5.5

Minimum solution of model 5.10 for DMU_1:

S O L V E   S U M M A R Y

MODEL   Mehrdad   OBJECTIVE z

TYPE   LP   DIRECTION MINIMIZE

SOLVER CPLEX   FROM LINE 29

**** SOLVER STATUS   1 Normal Completion

**** MODEL STATUS   1 Optimal

**** OBJECTIVE VALUE   0.0400

Minimum solution of model 5.10 for DMU_2:

S O L V E   S U M M A R Y

MODEL   Mehrdad   OBJECTIVE z

TYPE   LP   DIRECTION MINIMIZE

SOLVER CPLEX   FROM LINE 29

**** SOLVER STATUS   1 Normal Completion

**** MODEL STATUS   1 Optimal

**** OBJECTIVE VALUE   0.0400

**** MODEL STATUS   1 Optimal

**** OBJECTIVE VALUE   0.0400

Minimum solution of model 5.10 for DMU_3:

S O L V E S U M M A R Y

MODEL Mehrdad OBJECTIVE z

TYPE LP DIRECTION MINIMIZE

SOLVER CPLEX FROM LINE 29

**** SOLVER STATUS 1 Normal Completion

**** MODEL STATUS 1 Optimal

**** OBJECTIVE VALUE 0.0800

Minimum solution of model 5.10 for DMU_4:

S O L V E S U M M A R Y

MODEL Mehrdad OBJECTIVE z

TYPE LP DIRECTION MINIMIZE

SOLVER CPLEX FROM LINE 29

**** SOLVER STATUS 1 Normal Completion

**** MODEL STATUS 1 Optimal

**** OBJECTIVE VALUE 0.1200

Minimum solution of model 5.10 for DMU_5:

S O L V E   S U M M A R Y

MODEL   Mehrdad   OBJECTIVE z

TYPE   LP   DIRECTION MINIMIZE

SOLVER CPLEX   FROM LINE 29

**** SOLVER STATUS   1 Normal Completion

**** MODEL STATUS   1 Optimal

**** OBJECTIVE VALUE   0.0938

Minimum solution of model 5.10 for DMU_6:

S O L V E   S U M M A R Y

MODEL   Mehrdad   OBJECTIVE z

TYPE   LP   DIRECTION MINIMIZE

SOLVER CPLEX   FROM LINE 29

**** SOLVER STATUS   1 Normal Completion

**** MODEL STATUS   1 Optimal

**** OBJECTIVE VALUE   0.1600

Minimum solution of model 5.10 for DMU_7:

S O L V E   S U M M A R Y

MODEL   Mehrdad   OBJECTIVE z

TYPE   LP   DIRECTION MINIMIZE

SOLVER CPLEX   FROM LINE 29

**** SOLVER STATUS   1 Normal Completion

**** MODEL STATUS   1 Optimal

**** OBJECTIVE VALUE   0.1562

Minimum solution of model 5.10 for DMU_8:

S O L V E   S U M M A R Y

MODEL   Mehrdad   OBJECTIVE z

TYPE   LP   DIRECTION MINIMIZE

SOLVER CPLEX   FROM LINE 29

**** SOLVER STATUS   1 Normal Completion

**** MODEL STATUS   1 Optimal

**** OBJECTIVE VALUE   0.0625

USER: GAMS Development Corporation, Washington, DC   G871201/0000CA-ANY