Projection Based Beamformer Algorithm for Adaptive Beamforming in Uniform Linear Array

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ABSTRACT

Adaptive beamforming is a spatial filtering technique for uniform linear array of sensors that has application in numerous fields of signal processing such as wireless communications, radar, sonar, seismology and radio astronomy. Classically the minimum-variance-distortionless-response (MVDR) beamformer provides an acceptable solution to the problem of recovering the signal-of-interest (SOI) in the array input while minimizing the array output power. A number of problems exist in practice with the MVDR beamformer due to a number of non-ideal conditions such as mismatch in the direction of arrival (DOA) of the SOI, array calibration errors, local scattering of the incident signal and the finite sample approximation of the array covariance matrix. Several adaptive beamforming techniques, which have robustness against the problems cited above, have been developed to overcome these difficulties. However, these techniques have in general high computational complexity, as they depend on the eigenvalue decomposition (EVD) of the array covariance matrix.

In this work we consider the application of the multiple signal classification (MUSIC) method to the solution of the beamforming problem. This involves the estimation of the unknown DOA of the SOI based on the MUSIC algorithm. The DOA of the SOI is estimated by minimizing a cost function in terms of the norm of an error vector, which is the difference between the presumed steering vector of the SOI, and the orthogonal projection of this vector on the signal subspace. Direct implementation of this approach, however, also comprises eigenvalue decomposition of the covariance matrix. We will investigate the possibility of performing the above-mentioned minimization without EVD by expressing the cost function in terms of a

parameterized estimate of the signal steering vector.

Keywords: Adaptive Beamforming, Minimum-Variance-Distortionless-Response, Mismatch, Direction Of Arrival, Signal Of Interest, Eigenvalue Decomposition

ÖZ

Uyarlanır demet oluşturma işaret işlemenin, telsiz haberleşme, radar, sonar, deprem dalga analizi, ve radyo astronomisi gibi çeşitli alanlarında uygulama bulan bir uzamsal işaret işleme yöntemidir. Geleneksel olarak en-az-değişkenlik-bozunumsuz-tepke (MVDR) demet oluşturucu, dizi girişinden, dizi çıkış gücünü en aza indirgeyerek istenen işaretin elde edilmesi sorununa kabul edilebilir bir çözüm sunmaktadır. Fakat uygulamada MVDR demet oluşturucunun, istenen işaretin geliş açısındaki uyumsuzluk, dizi ayar hataları, yerel dağılma ve dizi öz-ilinti matrisinin sınırlı örneklenmesi gibi ideal olmayan durumlardan kaynaklanan sorunları vardır. Sözkonusu sorunları gidermek için birtakım dayanıklı uyarlanır demet oluşturma yöntemleri geliştirilmiştir. Fakat bu yöntemler, öz-ilinti matrisinin özdeğer ayrışımını kullanmalarından kaynaklanan yüksek hesaplama karmaşıklıkları vardır.

Bu çalışmada, MUSIC yönteminin demet oluşturma probleminin çözümü için uygulanması amaçlanmaktadır. Bu amaca ulaşmak için, istenen işaretin bilinmeyen geliş açısının MUSIC algoritması kullanılarak kestirimi hedeflenmektedir. İstenen işaretin geliş açısı, varsayılan dizi yönlendirme vektörü ile bu vektörün işaret altuzayına olan dik izdüşümü arasındaki farktan oluşan bir hata vektörünün büyüklüğünden oluşan bir maliyet işlevinin enazlanması yoluyla kestirilmektedir. Bu yaklaşımın doğrudan uygulanması öz-ilinti matrisinin özdeğer ayrışımını (EVD) gerektirir. Dolayısiyle, yukarıda bahsedilen enazlama EVD kullanılmadan ve maliyet işlevinin işaret yönlendirme vektörünün parametrik bir kestirimi cinsinden ifade edilmesi suretiyle yapılmaya çalışılmıştır.

Anahtar Kelimeler: Uyarlanır Demet Oluşturmada, Tekdüze Doğrusal Dizge,

Uyumsuzluk, Karışım, İlinti Matrisi, Genelliştirilmiş Yükleme Matrisi

Dedicated to

My dear parents

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c	Light velocity
C(n,m)	General loading matrix
d	Interelement spacing
$E\{\cdot\}$	Expected value
E_s	Signal-plus-interference subspace
E_n	Noise subspace
F_c	Carrier frequency
$H(F_c)$	Directional response
$\mathrm{H}_d(\theta_d)$	Directional response of filter
I	Identity matrix
$\mathrm{i}(k)$	Interference
J_a	Cost function
N	Number of sensors (elements)
$\mathrm{n}(k)$	Thermal noise
P	Rank of interference
P_y	Output power of beamformer
Q	Noise covariance matrix
$Q_m(\phi)$	Eigen-beam
q_m	Eigen-vector
R	Correlation matrix
$\hat{\mathrm{R}}$	Sample covariance matrix

R_{i+n}	Interference plus noise correlation matrix
$s(k)$	Desired signal
T	Transpose
$v(\phi)$	Array steering vector
w	Weight vector
$w(n)$	Thermal noise
$W_q(\phi)$	Quiescent response
$x(k)$	Array steering vector
$y(k)$	Output for narrowband beamformer
$\wp\{.\}$	Operator to calculate Eigen-vectors
α & β	Shrinkage parameters
$\gamma(\theta_d)$	Weight function
Δ	Hermitian error matrix
ε	Uncertainty level
ς	Diagonal loading factor
η	Arbitrary constant
θ_c	Cut of angle
λ	Wave length
λ_m	Eigen-value
Λ	Diagonal matrix
σ_s	Power of signal
σ_w	Power of noise
$\rho(\varphi)$	Normalized angular power density
τ	Delay time

ϕ	Direction of impinging signal
AU-IRCB	Adaptive Uncertainty Iterative Robust Capon Beamformer
CMT	Covariance Matrix Taper
DOA	Direction Of Arrival
DOF	Degree Of Freedom
DRS	Directional Response Shape
ESB	Eigen-Based Beamformer
FIR	Finite Impulse Response
FU-IRCB	Fixed Uncertainty Iterative Robust Capon Beamformer
GSC	Generalized Sidelobe Canceller
INR	Interference to Noise Ratio
IRMVB	Iterative Robust Minimum Variance Beamformer
LCMV	Linearly Constrained Minimum Variance
MUSIC	Multiple Signal Classification
MVDR	Minimum Variance Distortionless Response
RCB	Robust Capon Beamformer
SCB	Standard Capon Beamformer
SMI	Sample Matrix Inversion
SOI	Signal Of Interest
ULA	Uniform Linear Array

Chapter 1

INTRODUCTION

1.1 Statement of the Problem

The topic of array processing is related to extraction of data from signals gathered using array of sensors. These signals spread spatially over a material, like air or water, and the outcome of the wavefront is sampled by the sensor array. The desired information in the signal might be either the content of the signal which is considered in communications or the particular situation of the source or reflection that generate the signal, like in radar and sonar applications. In all the cases, the sensor array information must be planned to draw out proper information. For linear arrays, the sensors are located in patterns and organized along a direct line. The data contained in a spatially broadcasting signal also gives the place of its origin or the amount of the signal itself. If we are interested in getting this data we commonly must face with the presence of other undesired signals. Much as a frequency selective filter emphasizes signals at a certain frequency, we can choose to focus on signals from a particular direction. Obviously this process can be performed by employing a single sensor, provided that it can spatially discriminate to pass transmitted signals from a specified direction and reject those from other directions. The array contains a series of elements placed on a straight line with identical inter-element distance. This kind of array is known as a uniform linear array (ULA).

1.2 Literature Survey

There are various approaches to establish robust adaptive beamformers. Plenty of methods have been provided for the particular condition of signal look direction mismatches. Among such methods we can mention the linearly constrained minimum variance (LCMV) beamformer [1], signal blocking based algorithms [2], [3], and Bayesian beamformer [4]. Although all these approaches establish high robustness versus the signal look direction mismatch, they are not robust versus other types of mismatches caused by low array calibration, unknown sensor mutual coupling, near-far wave-front mismodeling, signal wave-front distortions, source spreading, and coherent/incoherent local steering, as well as other effects[5].

Some other methods are known to propose a modified robustness against more general types of mismatches, between which we can mention the methods that use the diagonal loading of the sample covariance matrix [6], [7], the eigenspace-based beamformer [8], [9], and the covariance matrix taper (CMT) approach [10], [11]. However a main problem of the diagonal loading approach is that there is no dependable way to determine the diagonal loading factor. If this factor is specified incorrectly, the robustness of the diagonal loading method may be inappropriate.
The eigenspace-based beamforming technique is basically limited in its efficiency at low signal-to-noise ratio (SNRs) and when the dimension of the signal-plus-interference subspace is high [12]. Moreover, this dimension must be known in the latter method [8]. The CMT method is used to establish a good robustness in cases with nonstationary interferers [8]. But its robustness against mismatches of desired signal array response is improper.

Recently a suitable method has been provided that models an arbitrary mismatch in the desired signal array response and apply worst-case optimization implementation to modify the robustness of the minimum-variance-distortionless-response (MVDR) beamformer [12] [13]. This method is based on a convex optimization formulation using second-order cone programming (SOCP).

A main deficiency of the robust techniques cited above is that they have been generally provided for the point signal source model (we denote a point source as a source that contributes a rank-one component to the covariance matrix) and majority of them cannot be developed in a direct way to handle the scenario of a higher-than-one rank of the signal model. Another approach is covariance matrix taper (CMT) which is known to provide excellent robustness when the interference is non-stationary [14], but robustness against mismatches for desired signal array response is acceptable. Another method is robust adaptive beamforming using worst-case performance optimization [15]. The performance of this method is fairly close to the simple algorithm which is known as diagonal loading of the sample matrix inversion (LSMI) algorithm. The Generalized Sidelobe Canceller (GSC) [16] is a technique that modifies its blocking matrix in order to extend the sharp nulls [17].

1.3 Thesis Objective

Adaptive beamforming is a spatial filtering technique for uniform linear array of sensors that has application in numerous fields of signal processing such as wireless communications, radar, sonar, seismology and radio astronomy. Classically the minimum-variance-distortionless-response (MVDR) beamformer provides an acceptable solution to the problem of recovering the signal-of-interest (SOI) in the array input while minimizing the array output power. A number of problems exist in practice with the MVDR beamformer due to a number of non-ideal conditions such as mismatch in the direction of arrival (DOA) of the SOI, array calibration errors, local scattering of the incident signal and the finite sample approximation of the array covariance matrix. Several adaptive beamforming techniques, which have robustness against the problems cited above, have been developed to overcome these difficulties. However, these techniques have in general high computational complexity, as they depend on the eigenvalue decomposition (EVD) of the array covariance matrix.

In this work we consider the application of the multiple signal classification (MUSIC) method to the solution of the beamforming problem. This involves the estimation of the unknown DOA of the SOI based on the MUSIC algorithm. The DOA of the SOI is estimated by minimizing a cost function in terms of the norm of an error vector, which is the difference between the presumed steering vector of the SOI, and the orthogonal projection of this vector on the signal subspace. Direct implementation of this approach, however, also comprises eigenvalue decomposition of the covariance matrix. We will investigate the possibility of performing the above-mentioned minimization without EVD by expressing the cost function in terms of a parameterized estimate of the signal steering vector.

1.4 Organization

Chapter 2 provides details about beamforming in uniform linear array. This is followed by an explanation of the beamforming methods, such as diagonal loading, robust Capon and Eigen-based beamformers, which take in to the part in chapter 3. Next in chapter 4, the proposed method is presented and then discussed in varied situations in chapter 5. Finally, chapter 6 makes some conclusions and purveys our future work.

Chapter 2

BEAMFORMING IN UNIFORM LINEAR ARRAYS (ULA's)

2.1 Introduction

In most of the applications, the proper information to be drawn out from an array of sensors is the content of a spatially broadcasting signal from a specific direction. The content may be a message contained in the signal, such as in communication applications, or just the being of the signal, as in radar and sonar. Because of this, we need to linearly combine outputs from all sensors in a way, that is with an appropriate weighting, so as to extract signals coming from a particular angle. This process is known as beamforming because the weighting operation confirms signal from a specific direction while decreasing those from other directions, and can be thought of as casting or forming a beam.

The performance of an adaptive beamforming technique is known to decrease considerably if there are mismatches between the true and assumed array steering vector responses to the desired signal [2],[8],[18]. Such a case may often happen in different conditions like violation of fundamental assumption on the surrounding, look direction errors, sensor array or environment being non-stationary. Some cases of degradation can happen when the signal array response is known exactly but the training size is small [8],[19],[6],[20]. Accordingly, robust methods for adaptive beamforming emerge to be of main importance in these cases [2],[8],[7],[5].

Adaptive beamforming has applications in sonar, radar, seismology, microphone array speech processing [21],[22] and even in wireless communications [23], [24].

2.2 Uniform Linear Array

Uniform linear array (ULA) is an antenna array including beam elements with uniform spacing between the elements and can be utilized to generate a directional radiation array. Antenna arrays may have different geometrical structures, the most common being linear arrays. Some antennas (such as diploes, loops and broad side) exhibit omnidirectional patterns. In radio communication, an omnidirectional antenna is a class of antenna which radiates radio wave power uniformly in all directions in one plane. In this work we intend to use this kind of antenna. Every single element antenna has beam-patterns that are wide and they have small directivity that is not suitable for high space communications. Arrays commonly employ identical antenna elements. The beam pattern of the array depends on the shape, the spacing between the sensors, the amplitude and phase excitation of the elements, and also the radiation pattern of every sensor. Figure 2.1 shows a beampattern. Associated with the pattern of an antenna is a parameter designated as beam width.

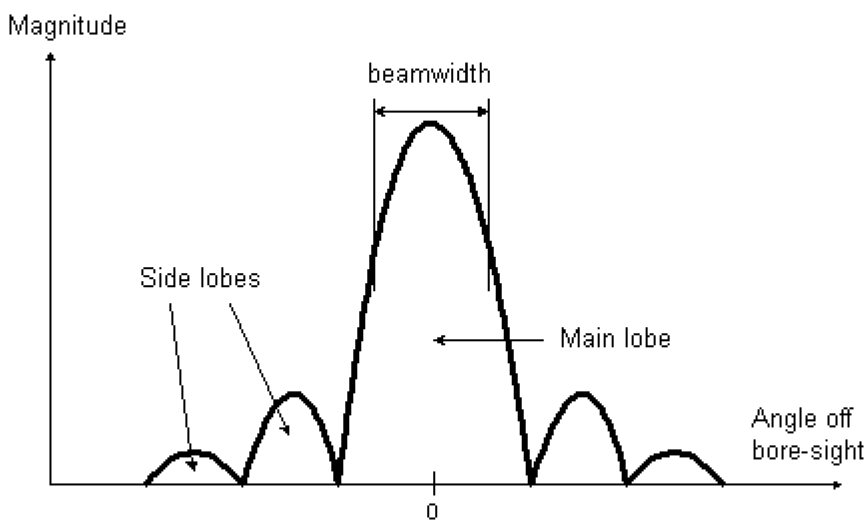

Figure 2.1: Beam Pattern

The beam width of a pattern is defined as the angular separation between two identical points on opposite side of the pattern maximum. In antenna pattern, there are a number of beam width. In antenna's radiation pattern, the mainlobe or main beam is the lobe containing the maximum power. This is the lobe that exhibits the greatest field strength. The other lobes are called sidelobes, and usually represent unwanted radiation in undesired directions. In antenna engineering sidelobes are the lobes of the far field radiation pattern that are not the mainlobe. In receiving antenna, sidelobes may pick up interfering signals and increase the noise level in the receiver, so we want to cancel the sidelobe. Figure 2.2 shows the ULA, where interelement spacing is defined by d and single propagating signal impinges on the ULA from angle ϕ.

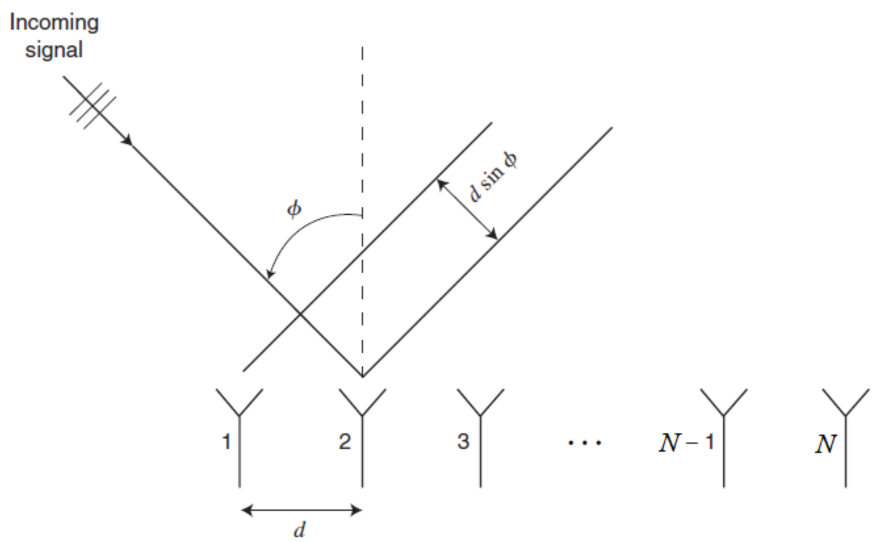

Figure 2.2: Impinging Signal on Uniform Linear Array [25]

For providing a model for a single spatial signal in interference and noise received by ULA, we presume a signal with angle ϕ which is discrete signal and contain the individual sensor signals

$$\boldsymbol{x}(n) = [x_1(n)\, x_2(n)\, \dots\, x_N(n)]^T \qquad (2.1)$$

where N is the total number of sensors. A signal measurement of this vector is denoted as an array snapshot. With respect to (2.1) full array discrete time signal is configured for every signal of interest (SOI) which is attracted by individual sensors

$$x(n) = v(\phi)s(n) + w(n) \tag{2.2}$$

where $w(n)$ is the thermal noise and $v(.\phi)$ is the array response vector and, $s(n) = H(F_c)\, s_0(n)$ is the impulse response of signal of interest (SOI) to n^{th} sensor, since $F_c = c\,/\,\lambda$. Where F_c is the carrier frequency and λ is the wavelength of the propagating. Sometimes in spatial filtering is related to receive a signal arriving from a determined point ϕ , and presume the signal is narrowband, a common choice for beamformer weight is the array response vector model as

$$v(\emptyset) = [1, e^{-j2\pi[(dsin\phi)/\lambda]}, e^{-j4\pi[(dsin\phi)/\lambda]} \ldots, e^{-j2\pi[(dsin\phi/\lambda](N-1)}]^T \tag{2.3}$$

Because all the sensors are uniformly spaced, the spatial signal has a difference in propagating distance between any two sequential sensors of d sin ϕ, that results in a time delay of

$$\tau\,(\phi) = \frac{d\,sin\,\phi}{c} \tag{2.4}$$

where c is the rate of propagation of the signal. Additionally, the delay to m^{th} sensor with respect to the first sensor in the array is

$$\tau_m(\phi) = (m-1)\,\frac{d\;sin\,\phi}{c} \tag{2.5}$$

It should be considered that full possible range of unambiguous angle is $-90^0 \leq \phi \leq 90^0$ and the spacing for sensors must be $d \leq \frac{\lambda}{2}$, to prevent spatial ambiguities.

For drawing out the desirable data from array of sensors which contains a spatially propagating signal from a particular direction, it is required to accomplish weighting that emphasizes signals from a specified angle, and attenuates other ones; this process is considered as forming a beam. Figure 3.1 shows the beam.

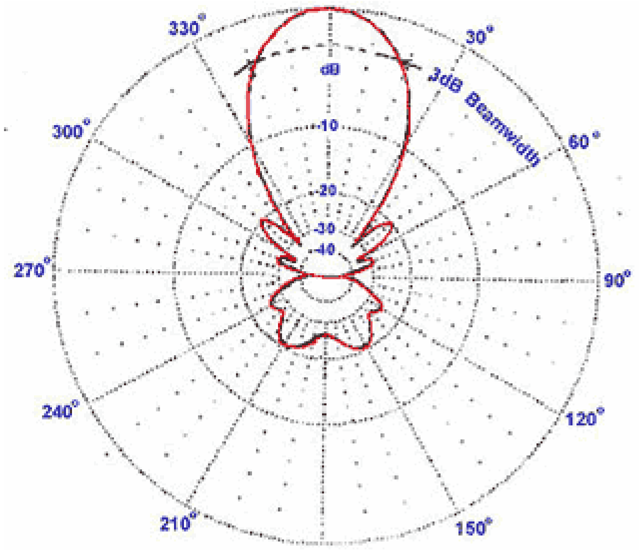

Figure 2.3: Beam

Beamforming is also arranged as data independent or statistically optimum, depending on how the weighs are selected. The weights in a data independent beamformer do not depend on the signal information and are selected to provide a determined response for all signal/interference scenarios. The weights in a statistically optimum beamformer are selected with respect to the statistics of the array information to optimize the array response. Commonly, the statistically optimum beamformer locates nulls in the direction of interfering sources in an effort to maximize the signal-to-interference-plus-noise ratio (SINR) at the beamformer output.[26] Figure 4.1 shows beamforming operation. An adaptive beamformer is a system that performs adaptive spatial signal processing with an array of transmitters or receivers.

Commonly, a beamformer generates its output by forming a weighted combination of signals (Data vector) from the N elements of sensor array

$$\mathrm{Y(n)} = w^{H}\mathrm{x(n)} \tag{2.6}$$

where

$$\mathrm{w} = [\mathrm{w}_1 \;\; \mathrm{w}_2 \;\; \dots \;\; \mathrm{w}_\mathrm{N}]^\mathrm{T} \tag{2.7}$$

is the weight vector of the beamformer.

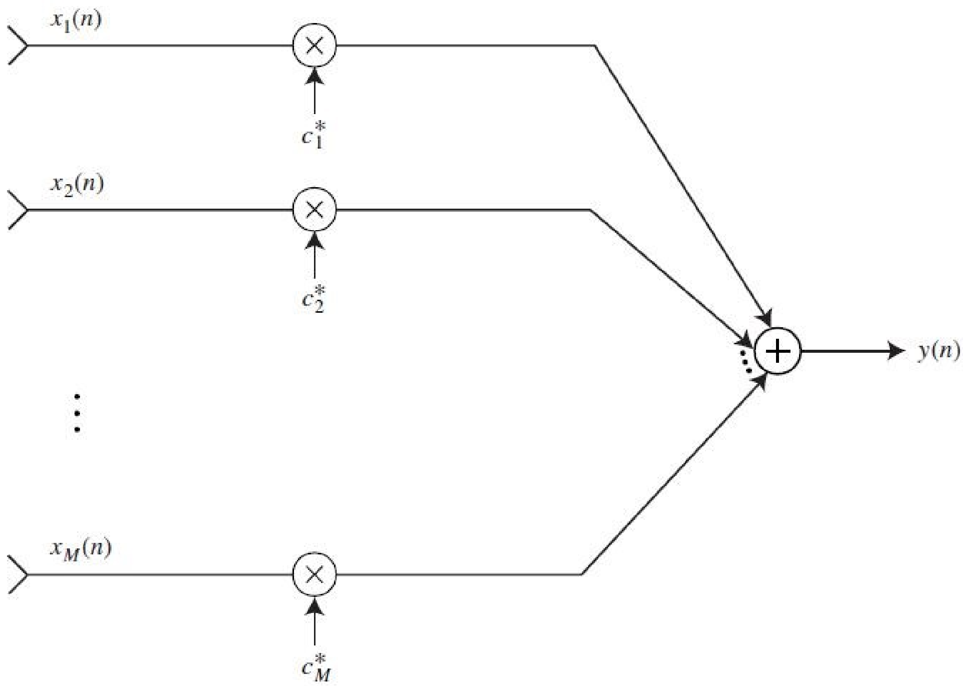

Figure 2.4: Beamforming Operation

A standard implement for analyzing the efficiency of a beamformer is the response for a given weight vector w as a function of ϕ, which is known as beam response. This angular response is denoted by applying the beamformer weight to a group of array response vectors from all conceivable angles, which is,

$$-90 \leq \phi \leq -90 \tag{2.8}$$

and

$$W(\phi) = w^{H}\, v(\phi) \tag{2.9}$$

The weight vector can be computed by maximizing the signal-to-interference plus noise ratio (SINR)

$$SINR = \frac{E\{|w^H\, s(n)\, v(n)|^2\}}{E\{|w^H\, x_{i+n}(n)|^2\}} = \frac{\sigma_s^2\, |w^H v(\phi)|^2}{w^H R_{i+n} w} \tag{2.10}$$

The optimal solution to (2.10) is computed by minimizing the cost function ($w^H R_{i+n} w$) while the beam response has unity gain ($w^H v(\phi) = 1$) at the angle-of-arrival of the SOI. Applying the Lagrange multiplier method we can write

$$J = w^H R_{i+n} w + \lambda(w^H v(\phi) - 1)$$

$$\frac{\delta J}{\delta w} = 2R_{i+n} + \lambda v(\phi) = 0 \Rightarrow w = -\frac{1}{2}\lambda R_{i+n}^{-1} v(\phi)$$

$$w^H v(\phi) = v^H(\phi) w = 1$$

$$\Rightarrow v^H(\phi) w = -\frac{1}{2}\lambda v^H(\phi) R_{i+n}^{-1} v(\phi) = 1$$

$$-\frac{1}{2}\lambda = \frac{1}{v(\phi)^H R_{i+n}^{-1} v(\phi)}$$

$$\Rightarrow w_{opt} = \frac{R_{i+n}^{-1}\, v(\phi)}{v(\phi)^H\, R_{i+n}^{-1}\, v(\phi)} \tag{2.11}$$

where R_{i+n} is interference-plus-noise correlation matrix.

Chapter 3

BEAMFORMING TECHNIQUES

3.1 Introduction

As we illustrated in prior chapters, an adaptive beamformer is an approach in which a spatial signal is adaptively processed by an array of sensors. The signals are gathered in a way which increases the strength of a signal in a pre-determined direction. Additionally, the purpose of this technique (beamforming) is to maximize the Signal-Interference-plus-Noise-Ratio (SINR) and diminish the effect of mismatches. In this Chapter, we will introduce some of the well-known methods such as diagonally Loaded Sample Matrix Inversion Beamformer (LSMI), Robust Capon Beamformer (RCB), Eigenspace-based beamformer, and the general-rank signal beamformer.

3.2 Loaded Sample Matrix Inversion Beamformer (LSMI)

Obviously, without appropriate sample size of the estimated covariance in the sample matrix inversion (SMI) adaptive beamformer, favorable sidelobe level and distortionless mainlobe of adaptive arrays will not be obtained. Additionally, in many applications a finite number of training information are at hand, so to attain this aim, in many cases it can be beneficial to consider the optimum beamformer with respect to the eigenvalues (λ_m) and eigenvectors (q_m) of the interference-plus-noise correlation matrix .

$$R_{i+n=} \sum_{m=1}^{N} \lambda_m \, q_m q_m^H \tag{3.1}$$

where the eigenvalues are $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$ and the rank of interference is P. we substitute (3.1) into the optimum beamformer weights

$$w_0 = \alpha R_{i+n}^{-1} v(\phi_s) \tag{3.2}$$

where

$$R_{i+n}^{-1} = \sum_{m=1}^{N} \frac{1}{\lambda_m} q_m q_m^H$$

and

$$\alpha = [v^H(\phi_s) R_{i+n}^{-1} v(\phi_s)]$$

Then we have

$$w_0(\phi) = \frac{\alpha}{\sigma_w^2}\{w_q(\phi) - \sum_{m=1}^{N} \frac{\lambda_m - \sigma_w^2}{\lambda_m}[q_m^H v(\phi_s) Q_m(\phi)]\} \tag{3.3}$$

where $w_q(\phi) = v^H(\phi_s)v(\phi)$ is the quiescent response of the optimum beamformer and $Q_m(\phi) = q_m^H v(\phi)$ is the beam response of the eigenvector (eigenbeam). This equation is existed when the optimum conditions are described and rank of interference is less than the number of sensors, and the smallest eigenvalues of R_{i+n} are eigenvalues which are equal to the thermal noise power $\lambda_m = \sigma_w^2$. If we consider (3.3) for the SMI adaptive beamformer, it will be

$$w_{smi}(\phi) = \frac{\alpha}{\hat{\lambda}_{min}}\{w_q(\phi) - \sum_{m=1}^{N} \frac{\hat{\lambda}_m - \hat{\lambda}_{min}}{\hat{\lambda}_m}[\hat{q}_m^H v(\phi)]\hat{Q}_m(\phi)\} \tag{3.4}$$

where $\hat{\lambda}_m$ is the eigenvalue and $\hat{q}_m$ is the eigenvector of $\hat{R}_{i+n}$, and, $w_q(\phi)$ and $\hat{Q}_m(\phi)$ are the beampatterns of the quiescent weight vector and the m_{th} eigenvector eigenbeam for SMI beamformer respectively. The summation part is weighted eigenbeams which locate nulls at angles of interferers. The weights for eigenbeams are characterized by the term $\frac{(\hat{\lambda}_m - \hat{\lambda}_{min})}{\hat{\lambda}_m}$ and the noise eigenvectors are chosen to fill the residue of the interference-plus-noise space that is not spanned by the interference. In the desired case, the noise eigenvectors should not affect the beam response since the eigenvalue for the true correlation matrix is $\lambda_m = \lambda_{min} = \sigma_w^2$. But, this expression does not consider for the SMI, because by adding the samples the noise power eigenvalues will have alteration. So, the eigenbeams have influence on the response by the deflection with the noise power. So because the eigenvalues are random variables that change according to the number of samples, response of the beam suffers from the increasing of casually weighted eigenbeams and as a result sidelobe level will be higher in the adaptive beampattern. Therefore, to decrease the variation of the eigenvalues, a weighted identity matrix is added to the sample correlation matrix [27].

$$\hat{R}_{dl=}\ \hat{R}_{i+n} + \zeta I \ and\ \zeta = \sigma_w^2 \tag{3.5}$$

where the ζ is loading factor. This method is named as diagonal loading. This approach adds the loading level to all eigenvalues of correlation matrix which generate a bias in eigenvalues toward reducing their alteration. The diagonally loaded SMI adaptive beamformer is given by

$$w_{LSMI} = \frac{\hat{R}_{dl}^{-1}\, v(\phi_{s})}{v^H(\phi_s)\hat{R}_{dl}^{-1} v(\phi_{s})} \tag{3.6}$$

It is obvious that the diagonal loading method increments variance of the white noise by parameter ζ and it can make better the performance of the SMI adaptive beamformer with random signal array response mismatch [28]. Convergence for LSMI beamformer will be more quick even while the number of snapshots is 2 times more than the number of sensors (2N) [17]. However, a major drawback of this approach is that there is no reliable way to choose a suitable quantity for the loading factor, because the optimal choice depends on the unknown signal and interference factors [29]. To solve the major disadvantage of the diagonal loading technique, in

[30] a method has been offered which attempts to resolve the problem by developing the General Linear Combination based (GLC) beamformer.

Actually, when the number of sample size N is small, the sample covariance matrix $\hat{R}$ is not an appropriate approximation of the true covariance matrix R. To attenuate this problem, in the GLC-based covariance matrix estimation, which is a shrinkage method [31], we consider a GLC of the sample covariance matrix $\widehat{R}$ and the identity matrix I to acquire a more precise R instead of $\hat{R}$:

$$\tilde{R} = \alpha I + \beta \hat{R} \; which \; \hat{R} \geq 0 \tag{3.7}$$

where $\tilde{R}$ is the improved estimation of R, α and β are the shrinkage parameters. To find the parameters, $\tilde{R}$ is minimized with respect to the $MSE(\tilde{R}) = E\{||\tilde{R} - \mathrm{R}||^2\}$ as proposed in [32]. Note that $\alpha \geq 0$ and ≥ 0 , because these assure that $\tilde{R} \geq 0$. By minimization of MSE for GLC the shrinkage parameters for M dimension (number of sensors) of array are computed as:

$$MSE(\tilde{R}) = \alpha^2 M - 2\alpha(1-\beta)tr(R)(1-\beta)^2||R||^2 + \beta^2 E\{||\tilde{R} - R||^2\} \tag{3.8}$$

So, the optimal value for β and α can be found as

$$\beta_0 = \frac{\gamma}{\rho + \gamma} \; where \; \gamma = ||\nu I - R||^2 \tag{3.9}$$

$$\alpha_{0=}\nu(1-\beta_0) = \nu \frac{\rho}{\rho + \gamma} \; where \; \rho \triangleq E\{||\tilde{R} - R||^2\}, \nu = tr(R)/M \tag{3.10}$$

It should be considered that $\beta_0 \epsilon [0,1]$ and $\alpha_0 \geq 0$. However α_0 and β_0 are completely dependent on the unknown covariance matrix R. Therefore these parameters should be estimated by approximating

$$\hat{\rho} = \frac{1}{N^2}\sum_{n=1}^{N} ||X(n)||^4 - \frac{1}{N}||\hat{R}||^2 \tag{3.11}$$

As a result, the estimated α_0 and β_0 are obtained which guarantee that the estimate of β_0 is not negative [32]:

$$\hat{\alpha}_0 = min[\hat{v}\frac{\hat{\rho}}{||\hat{R} - \hat{v}I||^2}, \hat{v}] \; where \; \hat{v} = tr(\hat{R})/M \tag{3.12}$$

$$\hat{\beta}_0 = 1 - \frac{\hat{\alpha}_0}{v} \tag{3.13}$$

Now, diagonally loaded estimate of covariance matrix can be denoted as

$$\hat{R}_{GLC} = \hat{\alpha}_0 + \hat{\beta}_0\hat{R} \tag{3.14}$$

Using the above relation instead of R in the standard Capon Beamformer, the GLC based robust adaptive beamformer will be achieved

$$w_{GLC} \frac{\hat{R}_{GLC}^{-1}\, v(\phi_s)}{v^H(\phi_s)\, \hat{R}_{GLC}^{-1}\, v(\phi_s)} \tag{3.15}$$

By rewriting (3.15) for enhanced GLC based weight vector becomes

$$\hat{w}_{GLC} = \frac{[\frac{\hat{\alpha}_0}{\hat{\beta}_0}\, I + \hat{R}]^{-1}\, v(\phi_s)}{v^H(\phi_s)\, [\frac{\hat{\alpha}_0}{\hat{\beta}_0}\, I + \hat{R}]^{-1}\, v(\phi_s)} \tag{3.16}$$

It is obvious that the GLC based robust adaptive beamformer is a type of Diagonal Loading approach with loading factor ($\frac{\hat{\alpha}_0}{\hat{\beta}_0}$) which is automatically obtained from the data samples $\{X(n)\}_{n=1}^{N}$.

3.3 Robust Capon beamformer

3.3.1 Introduction

The Capon beamformer has superior resolution and much better interference rejection ability in comparison with the standard (data-independent) beamformer, provided that the array steering vector corresponding to the signal of interest (SOI) is accurately known.

But whenever the information of the SOI steering vector is ambiguous (most of the time this case happens in practice), the efficiency of the Capon beamformer may go worse when compared with the standard beamformer. Diagonal loading (including its extended versions) has been a common method to modify the robustness of the Capon beamformer. The standard Capon beamformer (SCB) [30] can be an optimal spatial filter if both the exact covariance matrix and the array steering vector are known. In this case, the array signal-to-interference-plus-noise ratio (SINR) output is maximized and interferences are better rejected. Nevertheless, usually the covariance matrix can be wrongly approximated due to the limited number of data samples, and the knowledge for array steering vector can be imprecise because of view direction mismatch or differences between assumed signal incoming angle and the actual arrival angle [33]. Whenever these mismatches exist there is performance degradation of SCB. This degradation becomes more pronounced if the signal-of-interest (SOI) is present in the estimated covariance matrix. Therefore adaptive beamforming encounters small sample size problems and array steering vector errors. Also, if the information of signal-of-interest is ambiguous, the efficiency for the Capon beamformer will be worse than the standard Capon beamformer.

3.3.2 Extension of the Capon Beamformer

Now we discuss the development of the Capon Beamformer when the steering vectors are uncertain [31]. Consider an array including M sensors, and let the covariance matrix of the array output vector be R. We consider R that has the following form:

$$R = \sigma_s^2 v(\phi) v^H(\phi_s) + \sum_{p=1}^{p} \sigma_p^2\, v(\phi_p) v^H(\phi_p) + Q \tag{3.17}$$

where σ_s^2 and σ_p^2 are powers of signals impinging on the array; ϕ_s and ϕ_p are the parameters for the positions of sources which emit the signals. $v(.)$ is the array steering vector and Q is the noise covariance matrix given by $Q = \sigma^2$ I (the covariance matrix has full rank despite the rest of the terms, each of which having rank one). With respect to this description, the first term of R is related to the SOI and remaining terms correspond to the P interferences. To simplify notation, let $v(\phi_s) = v_s$.

This method is aimed to extend the Capon Beamformer to specify the power of signal-of-interest even when just uncertain knowledge of its steering vector v_s is available. Specifically, consider that only knowledge about v_s is available which belongs to the uncertainty ellipsoid:

$$[v_s - \bar{v}]^H c^{-1} [v_s - \bar{v}] \leq 1 \tag{3.18}$$

where $\bar{v}$ is given. When the general formulation for beamforming is utilized for the SCB, it is going to determine the weight vector w_0(M×1) by the linearly constrained quadratic problem:

$$Min\ w^H R w \ \ subject\ to\ \ w^H v_s = 1 \tag{3.19}$$

It gives the solution as

$$w_0 = \frac{R^{-1}v_s}{v_s^H R^{-1} v_s} \tag{3.20}$$

And estimation of σ_s^2 by $w_0^H R w_0$ gives

$$\tilde{\sigma}_s^2 = \frac{1}{v_s^H R^{-1} v_s} \tag{3.21}$$

The latest RCB methods in [15] when there is uncertainty in v_s, the constraint on $w^H v_s$ In (3.19) is replaced by any vector ν in the uncertainty set. Then acquired w is utilized in $w^H R w$ to estimate the $\sigma_s^2$of SCB. However, in the new method, the Capon beamformer problem in [31] is formulated in a simple form when the uncertainty set is included. By continuing in this manner, a robust estimation of σ_s^2 is acquired without any prior computation for weight vector w [31].

In [31] it is proved that $\tilde{\sigma}_s^2 = \hat{\sigma}_s^2$ with respect to the problem

$$Min w^H R^{-1} w \text{ subject to } [v_s - \bar{v}]^H C^{-1} [v_s - \bar{v}] \leq 1 \tag{3.22}$$

Now if the matrix C is decomposed ($C \succ 0$) and put in (3.22), it will change to a quadratic problem with a quadratic equality constraint [27]:

$$Min\, w^H R^{-1} w \text{ subject to } ||v - \bar{v}||^2 = \varepsilon \tag{3.23}$$

where ε is defined as the uncertainty level. The solution to the RCB formulation in (3.23) can be acquired by the Lagrange multiplier method:

$$f = v^H R^{-1} v + (||v - \overline{v}||^2 - \varepsilon) \tag{3.24}$$

By solving this optimization problem, $\hat{v}_s$ is obtained as

$$\hat{v}_s = \hat{v} - (I + \lambda R)^{-1} \bar{v} \tag{3.25}$$

The Lagrange multiplier λ is obtained by solving the equation g(λ) = $||(I +$

$\lambda R)^{-1}\bar{v}||^2$=ε and then lower and upper bounds of λ are imposed.

To sum up, $\hat{v}_s$ is determined by using (3.25), and $\hat{\sigma}_s^2$ is computed by using (3.21) where v_s is replaced with $\hat{v}_s$. Therefore, the main computational complexity of the RCB method arises from the Hermitian matrix eigen-decomposition. So, the computational complexity of RCB is acceptable compared with the SCB [29]. Once there is estimation for signal-of-interest steering vector, the estimated weight vector can be attained

$$\widehat{w}_0 = \frac{\widehat{R}^{-1}\,\widehat{v}_s}{\widehat{v}_s^H \widehat{R}^{-1}\,\widehat{v}_s} = \frac{\left(R + \frac{1}{\lambda}I\right)^{-1}\bar{v}}{\bar{v}^H \left(R + \frac{1}{\lambda}I\right)^{-1} R \left(R + \frac{1}{\lambda}I\right)^{-1}\bar{v}} \tag{3.26}$$

Obviously, robust Capon beamformer weight vector is in the form of diagonal loading.

Robust Capon Beamformer will not support some problems where the uncertainty set of desired array steering vector applied to achieve robustness against steering vector mismatches. Specifically, when large steering vector mismatches are present, the uncertainty set must expand to account for the increased error of the desired array steering vector. This decreases the output signal-to-interference-plus-noise ratio (SINRs) of these beamformers since their interference-plus-noise suppression capabilities are attenuated.

To overcome this problem, a technique has been offered by [32] which uses a small uncertainty sphere to look iteratively for the desired array steering vector. In this method, the ability of the beamformer in interference-plus-noise suppression can be kept by maintaining its degrees of freedom (DOFs) also by using the modified desired array steering vector. The Iterative Robust Minimum Variance Beamformer (IRMVB) method yields greater output for SINR. By employing a stopping criterion the steering vector computed by the IRMVB method is not allowed to converge to the steering vectors of the interferences [32].

The concept of the IRMVB (with spherical uncertainty set) is shown in Figure (3.1), when there is a mismatch in steering direction, where the desired array steering vector s_0 (corresponding to the desired signal direction θ_0) and the assumed array steering vector $\bar{s}_0$ (corresponding to the assumed desired signal direction $\overline{\theta_0}$) do not coincide. If the errors are big then the size of the uncertainty sphere ε_1, used in (3.23) has to be bigger [27]. Hence, the ability of the beamformer to suppress the interference will be weakened due to the increasing of the DOFs. To solve this problem, the IRMVB uses a small uncertainty sphere which is smaller than $\varepsilon_1(\varepsilon_2 \leq \varepsilon_1)$ to regulate the steering vector form $\bar{s}_0$ to approach s_0.

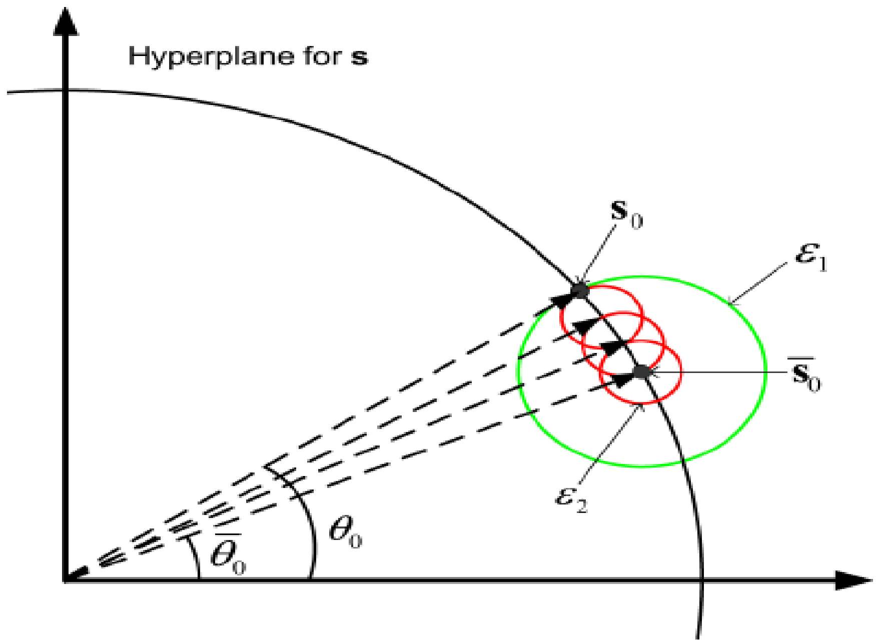

Figure 3.1: Concept of the IRMVB Method [31]

It can be done by using the constraint (3.23) (with ε_2 in place of ε_1) centered at the assumed desired array steering vector $\bar{s}_0$. At the first iteration $||s_0 - \bar{s}_0||^2 = \varepsilon_2$ and the RCB is solved for the modified desired array steering vector. After every iteration, the computed steering vector by IRMVB approach is scaled. Again, the spherical constraint is exerted centered at the calculated steering vector of the prior iteration of IRMVB to solve for the following steering vector. This process is continued until the desired array steering vector is obtained. This can be attained by using a stopping criteria. Then, IRMVB weight vector can be calculated by using the converged steering vector by (3.20).

Recently, in some reports [32] for the RCB, it has been mentioned that whenever large steering vector mismatch arises, degradation exists in signal-to-interference-plus-noise ratio (SINR). The reason is that the ability to suppress the interference is sacrificed whenever the radius for the uncertainty sphere is increased (for instance in IRMVB) to have sufficient uncertainty level. So, the degradation of SINR becomes considerable when the interferences are dominant. Therefore, having a robust adaptive beamformer method, a sphere is desirable which can maintain its interference suppression capability in the large mismatch case without increasing the radius of the uncertainty.

The authors in [32] refer to the Iterative RCB with a small fixed uncertainty level as the Fixed Uncertainty Iterative Robust Capon Beamformer (FU-IRCB). Let ε_2 be the representative of the small fixed uncertainty. This technique calculates $\hat{v}$ by solving the RCB optimization iteratively in (3.23) when ε is replaced by ε_2. The vector $\hat{v}$ is a function of λ with respect to ε_2 which is obtained by solving $g(\lambda) = \varepsilon_2$. At each iteration, $\overline{v}$ is updated from $\hat{v}$ of the pervious iteration. The iteration continues until λ reaches a suitable small value. The convergence rate of the FU-IRCB depends on how fast λ converges to a small value. Since λ is dependent on the solution of $g(\lambda) = \varepsilon_2$, so it is directly related to the value of ε_2. Therefore, a larger value of ε_2 will make its value to reduce at a faster rate. On the other hand, with large ε_2, the interference suppression capability is sacrificed. The other defect of the FU-IRCB is that a *severe* stopping criterion is needed in order to avoid the convergence of the iteration to one of the strong interference steering vectors. This can be illustrated by the objective function of the RCB optimization in (3.23).

An approach has been offered by [34] which belongs to the iterative RCB family with adaptive uncertainty. At every iteration, the uncertainty level is readjusted and then the approximated steering vector is updated for the new uncertainty level. When the uncertainty level approximately becomes zero the iteration is converged. The new technique is based on the geometric estimated vector for the mismatch. The estimation is according to the concept that the mismatch vector can be decomposed into two types of subspaces, which are the signal-plus-interference subspace and the subspace for noise. The signal component is computed like a function of the projection of the assumed steering vector on signal subspace, while the noise component is calculated from its orthogonal projection.

3.4 Eigenspace Based Beamformer

One of the approaches for robust adaptive beamforming is the Eigenspace Based Beamformer. In this method the weight vector is computed by employing the subspace component for signal-plus-interference of the sample correlation matrix, which can alleviate the disturbed noise subspace. One common property of this method (ESB) for adaptive beamforming usually is the eigen-decomposition of the steering vector space into subspaces associated with the signal and the noise components. Moreover, the optimal weight vector with respect to the precise steering lies in the signal subspace. This beamformer needs to have previous knowledge about signal subspace component and the number of sources [35] that can be approximated by the method provided in [36]. If N samples are available, the covariance is obtained by applying

$$\hat{R} = \frac{1}{N} \sum_{n=1}^{N} x(n)\, x^H\, (\mathrm{n}) \qquad (3.27)$$

The presumed desired signal steering vector is defined as $\overline{v}$, v denotes the true steering vector of the desired signal and the estimated steering vector of the desired signal is defined as $\hat{v}$. In the eigenspace projection based robust adaptive beamforming, by using the presumed steering vector of desired signal, this method computes the projection of $\overline{v}$ onto the signal-plus-interference subspace, giving a modified estimation of the true desired signal steering vector. So eigendecomposition of $\hat{R}$ can be expressed as

$$\hat{\mathrm{R}} = E_s \Lambda_s E_s^H + E_n \Lambda_n E_n^H \tag{3.28}$$

where the $N \times (P+1)$ matrix E_s contains the signal-plus-interference subspace eigenvectors and $N \times (N-P-1)$ matrix E_n contains the noise subspace for $\hat{R}$. Also, the $(P+1) \times (P+1)$ matrix Λ_s includes the eigenvalues corresponding to E_s and Λ_n contains the eigenvalues for E_n respectively. P is the number of interfering signals.

The approximated true desired signal steering vector is defined by

$$\hat{v} = E_s E_s^H \overline{v} \tag{3.29}$$

where $E_s E_s^H$ is the projection matrix to the subspace of desired signal-plus-interference and the eigenspace based weight vector is given by

$$w_{ESB} = \hat{\mathrm{R}}^{-1} \hat{v} = \hat{\mathrm{R}}^{-1} E_s E_s^H \overline{v} = E_s \Lambda^{-1} E_s^H \overline{v} \tag{3.30}$$

Recently, application of the eigen-subspace idea has been expanded to deal with adaptive array beamforming where observation mismatch occurs [33], [37]. In the case of unsuitable observation, the optimal weight vector utilized by the eigen-subspace technique contains an undesired component existing on the noise subspace. For instance, degradation in efficiency of the array is frequently generated by this undesired component. To solve this problem, a robust technique is devised in [33] by taking the projection of the presumed steering vector onto the steering signal subspace to eliminate the undesired noise component. The technique is provided in [38] which adopts a linear combination of the eigenvectors of the signal subspace to resolve the undesired noise component. In a similar way, in the robust technique of [36] it is employed to find the orthogonal component of the correct steering vector to the noise subspace. Another robust approach for Eigenspace based adaptive beamforming presented in [39] aims to remove the undesired component by minimizing the power of array output in the signal subspace.

Nevertheless, there are major disadvantages to apply the ESB techniques for adaptive beamforming when steering errors are presented. It needs more complex computations to accomplish the eigendecomposition for determining the signal subspace. The second one is that, this technique is only appropriate for the point signal source case, so it is suitable to solve the small pointing errors. The third one is that the efficiency of this method is low when the signal-to-noise ratio (SNR) is low.

Chapter 4

THE APPROXIMATE PROJECTION – BASED BEAMFORMER

4.1 Introduction

Under ideal conditions, adaptive beamforming establishes a betterment in array output signal-to-interference-plus-noise ratio (SINR) in comparison with conventional beamforming [40]. This superiority may be widely decreased, although, if the steering vector characteristics does not match the actual signal environment, especially when the input signal level is high [41]. Mismatch between the presumed steering vector and its real fundamental value happens either as a result of calibration error (containing ambiguous information of the element locations) or because the true arrival angle differs from that presumed by the processor (pointing error). Steering vector mismatch because of both problems is considered as "the perturbation problem". Past work on the effects of perturbation error has contained pointing error [42], the more common problem of small phase errors at each sensor [43], and the influence of concurrent gain and phase errors [44]. In all the recent work, the method chosen to protect the beamformer from error was to use linearly-constrained beamforming with additional constraints. Here another approach is employed to dominate the disordered effects of error due to perturbation. This method is suggested that employ adaptive modification of presumed steering vector. In this approach which is known as the projection method, the presumed steering vector is easily replaced by its projection onto the signal-plus-interference subspace.

4.2 Mathematical Development

The M × M autocorrelation matrix for $x(n)$ is a sum of an autocorrelation matrix due to desired signal, R_s, the noise, R_n, and the interference, R_i.

$$R_x = R_s + R_n + R_i \tag{4.1}$$

The theoretical covariance matrix of the received signal is given by

$$\mathrm{R} = \sigma_n^2 \mathrm{I} + \sigma_s^2 a_0 a_0^H + \sum_{l=1}^{L} \sigma_{iL}^2 \, a_{iL} a_{iL}^H \tag{4.2}$$

where σ_n^2 is the broadband noise power. It is assumed that there are L interfering signals incident from directions with corresponding steering vectors a_{iL}, with powers σ_{iL}^2 , l = 1,…,L , and that the SOI and the interfering signals are not incoherently scattered. It is further assumed that the interference steering vectors are linearly independent. Equation (4.2) can also be written in the form

$$\mathrm{R} = \sigma_n^2 \mathrm{I} + \sigma_s^2 a_0 a_0^H + A_i \textstyle\sum_i A_i \tag{4.3}$$

where

$$A_i = [a_{i1} a_{i2} \dots a_{iL}]$$

and

$$\textstyle\sum_i = \mathrm{diag}\{\sigma_{i1}^2, \sigma_{i2}^2, \dots, \sigma_{iL}^2\}$$

The covariance matrix of the received signal vector in practice is calculated using the finite sample approximation

$$\hat{R} = \frac{1}{M} \sum_{k=1}^{M} x(k) x^H \,(\mathrm{k}) \tag{4.4}$$

Let the eigenvalue decomposition (EVD) of $\hat{R}$ be

$$\hat{R} = \sum_{j=1}^{N} \hat{q}_j \, \hat{e}_j \hat{e}_j^H = \hat{E}\hat{Q}\hat{E}^H \tag{4.5}$$

Where $\hat{q}_j$ and $\hat{e}_j$, $\mathrm{j} = 1, \dots, \mathrm{N}$ are the eigenvalues and eigenvectors of $\hat{R}$, respectively, $\hat{E} = [\hat{e}_1 \dots \hat{e}_N]$ and $\hat{Q} = \mathrm{diag}\{\hat{q}_1, \dots, \hat{q}_N\}$. Let's assume that the first J eigenvalues correspond only to the white noise in the received signal, and the rest correspond to the SOI and the interference signals. It is further assumed that following orthogonal projection of the presumed signal steering vector $\bar{a}$ on the signal subspace as the estimate of the true steering vector

$$c_p = \sum_{l=J+1}^{N} (\hat{e}_l^H \, \bar{a})\hat{e}_\mathrm{l} \tag{4.6}$$

Assuming that the desired and interference signal steering vectors are linearly independent, then the orthogonal projection can also be written as

$$c_p = \alpha_0 a_0 + \sum_{ll=1}^{L} \alpha_l \, a_{iL} = A_s \mathrm{\alpha} \tag{4.7}$$

where

$$A_s = [a_0 \; A_i] \text{ And } A_i = [a_{i1} \dots a_{iL}]$$

Are the desired signal and interference steering vectors.

Then, c_p can be written as

$$c_p = A_s(A_s^H \, A_s)^{-1} A_s^H \bar{a} \tag{4.8}$$

It can be shown that (see Appendix A)

$$A_s(A_s^H \, A_s)^{-1} A_s^H = P_i + \frac{1}{a_0^H (I - P_i) a_0} (\mathrm{I} \; - \; P_i) a_0 a_0^H \, (\mathrm{I} - P_i) \tag{4.9}$$

where

$$P_i = A_i(A_i^H \, A_i)^{-1} A_i^H$$

Using (4.8), (4.9) can be written as

$$c_p = P_i\bar{a} + \frac{a_0^H(I-P)\bar{a}}{a_0^H(I-P)a_0}(\mathrm{I}-P_i)a_0 = \eta_0 a_0 + P_i(\bar{a}-\eta_0 a_0) \tag{4.10}$$

where

$$\eta_0 = \frac{a_0^H(I-P_i)\bar{a}}{a_0^H(I-P_i)a_0}$$

Note that in (4.10) as $\bar{a} \to a_0$, $c_p \to a_0$. The question at this stage is whether an SOI steering vector estimate can be found that may approximate (4.10), and can be computed using available data. For this, we consider the following estimate of the SOI steering vector obtained in [27]

$$\mathrm{c}(\lambda) = \bar{a} - (I+\lambda R)^{-1}\bar{a} \tag{4.11}$$

Substituting (4.3) in (4.11), it can be shown that the estimate $\mathrm{c}(\lambda)$ can be written as (see Appendix B)

$$\mathrm{c}(\lambda) = \frac{1}{1+\lambda\sigma_n^2}[\mathrm{P}\bar{a} + \lambda\sigma_n^2\bar{a} + \eta(\lambda)(\mathrm{I}-\mathrm{P})a_0] \tag{4.12}$$

where

$$\eta(\lambda) = \frac{a_0^H\,(I-P)\bar{a}}{\mu(\lambda) + a_0^H(I-P)a_0} \tag{4.13}$$

and

$$\mu(\lambda) = \frac{1+\lambda\sigma_N^2}{\lambda\,\sigma_s^2}$$

The estimate (4.12) can be modified so that it is in the same form as the projection-based estimate (4.9) as follows

$$\hat{c}(\lambda) = \mathrm{c}(\lambda) - \frac{\lambda\,\sigma_n^2}{1+\lambda\sigma_n^2}\bar{a} = \frac{1}{1+\lambda\sigma_n^2}[\mathrm{P}\bar{a} + \eta(\lambda)(\mathrm{I}-\mathrm{P})a_0]$$

$$= \frac{1}{1+\lambda\sigma_n^2}[\eta(\lambda)a_0 + \mathrm{P}(\bar{a} - \eta(\lambda)a_0)] \tag{4.14}$$

When (4.14) is compared with (4.10), it seems possible to make $\hat{c}(\lambda)$ approximate c_p (except for the scalar term) by making $\eta(\lambda)$ as close to η_0 as possible. This can be achieved by maximizing the numerator of $\eta(\lambda)$ and minimizing $\mu(\lambda)$ in its denominator. The first requires that the direction-of-arrival of the SOI is estimated with sufficient accuracy such that $\bar{a}(\hat{\theta}_0)$ becomes a good estimate of the desired signal steering vector. The second is possible by choosing λ such that $\lambda\sigma_n^2 \gg 1$, in which case we also have

$$\widehat{c}(\lambda) \cong c(\lambda) - \bar{a} \tag{4.15}$$

In the MUSIC method for the estimation of the DOAs of coherent signals impinging on a ULA, the following cost function is minimized with respect to the angle θ,

$$J_{MUSIC}(\theta) = ||a(\theta) - \sum_{n=1}^{k}(s_n^H a(\theta))s_n||^2 = a^H(\theta)GG^H a(\theta) \tag{4.16}$$

Where $a(\theta)$ is given by [27] , $\{\mathbf{s}_n\}_{n=1}^{K}$, are the signal subspace eigenvectors (k is the number of coherent signals), and $GG^H = I - SS^H$ where S is the matrix with columns which are the signal subspace eigenvectors. A similar approach can be applied to estimate the DOA of the SOI by minimizing the following cost function in the vicinity of the presumed DOA,

$$J(\theta) = ||\bar{a}(\theta) - c_p(\theta)||^2 \tag{4.17}$$

Where $c_p(\theta)$ is the orthogonal projection of $\bar{a}(\theta)$ onto the signal –plus-interference subspace. In the preceding section, it was shown that this projection can be approximated by the vector $(1+\lambda\sigma_n^2)\hat{c}(\lambda)$.

Hence, using this vector instead of $c_p(\theta)$ in (4.17) we get

$$J(\theta) = ||\bar{a}(\theta) - (1+\lambda\sigma_n^2)\hat{c}(\lambda)||^2 = (1+\lambda\sigma_n^2)^2||\bar{a}(\theta) - c(\lambda,\theta)||^2 \tag{4.18}$$

Where the desired signal steering vector estimate of (4.11) is parameterized in the DOA θ as

$$\mathrm{c}(\lambda,\Theta) = \bar{a}(\Theta) - (I+\lambda R)^{-1}\bar{a}(\Theta) \tag{4.19}$$

Therefore, the DOA of the desired signal can be estimated by minimizing

$$\mathrm{J}(\Theta) = \left||\Delta|\right|^2 = ||\bar{a}(\theta) - c(\lambda,\theta)||^2 \tag{4.20}$$

With respect to Θ where || • || is the Euclidean norm, as proved in the following proposition:

Proposition 1: The cost $J(\theta)$ is approximately convex in the angle error $\Delta\theta = \theta - \theta_0$, where $\theta_0$is the true DOA of the SOI.

Proof: From(4.12), Δ can be written as

$$\Delta = \frac{1}{1+\lambda\sigma_n^2}\,(I-P)[\bar{a}(\theta) - \eta(\lambda,\theta)a_0] \tag{4.21}$$

where

$$\eta(\lambda,\theta) = \frac{a_0^H(I-P)\bar{a}(\theta)}{\mu(\lambda) + a_0^H(I-P)a_0} \tag{4.22}$$

Defining$\delta(\theta) \triangleq \bar{a}(\theta) - \eta(\lambda,\theta)a_0$, the cost can be expressed as

$$||\Delta||^2 = \frac{1}{(1+\lambda\sigma_n^2)^2}||(I-P)\delta(\theta)||^2 \tag{4.23}$$

Provided that the vector $\delta(\boldsymbol{\theta})$ is not in the subspace spanned by interference steering vectors, we can write

$$||\Delta||^2 \leq \frac{1}{(1+\lambda\sigma_n^2)^2}||\delta(\theta)||^2 \tag{4.24}$$

Letting

$$\bar{a}(\theta) = [1\ e^{-j\theta}\ \ldots\ e^{-j(N-1)\theta}]^T$$

And $a_0 = \bar{a}(\theta_0)$, then $\bar{a}(\theta)$ can be decomposed

$$\bar{a}(\theta) = ra_0 + \bar{a}_\perp \tag{4.25}$$

Where $\bar{a}_\perp$is the component of $\bar{a}(\theta)$ orthogonal to a_0 and r is given by

$$r = \frac{a_0^H\bar{a}(\theta)}{||a_0||^2} = \frac{a_0^H\bar{a}(\theta)}{N} \tag{4.26}$$

Letting $\Delta\theta = \theta$ - θ_0 and ϕ be the angle between the vectors $\bar{a}(\theta)$ and a_0, the following can be easily verified

$$r = \frac{1}{N}e^{j(N-1)^{\Delta\theta}/_2}\frac{\sin({}^{N\Delta\theta}/_2)}{\sin({}^{\Delta\theta}/_2)}$$

$$|r|^2 = cos^2(\phi) = \left(1-\frac{1}{24}N(\Delta\theta)^2\right)^2$$

$$||\bar{a}_\perp||^2 = Nsin^2(\phi) = \frac{1}{2}N^2(\Delta\theta)^2$$

With the definition $p_0 = a_0^H(I-P)a_0$, (4.18) and (4.25) yield

$$\eta(\lambda,\theta) = \frac{rp_0 - a_0^H P\bar{a}_\perp}{\mu(\lambda)+p_0} \tag{4.27}$$

The error vector $\delta(\theta)$ can be written as

$$\delta(\theta) = (\frac{rp_0 + a_0^H P\bar{a}_\perp}{\mu + p_0})a_0 + \bar{a}_\perp \tag{4.28}$$

Where λ-dependencies have been dropped for simplicity of notation. Let

$$a_0^H P\bar{a}_\perp = |a_0^H P\bar{a}_\perp| e^{ja} = ||Pa_0||.||\bar{a}_\perp||cos(\Psi)e^{ja} \tag{4.29}$$

Where $\Psi$is the angle between the vectors Pa_0 and $\bar{a}_\perp$. Then, the norm of $\delta(\theta)$ can be evaluated to yield

$$||\delta(\theta)||^2 \simeq \frac{N}{(\mu + p_0)^2}\{\mu^2 \frac{1}{12}N[(2\mu + p_0)p_{0+}N\left||Pa_0|\right|^2 cos^2(\Psi)](\Delta\theta)^2 + \frac{1}{\sqrt{3}}N[\mu\left||Pa_0|\right| \cos(\Psi)\cos\left(\alpha - \frac{1}{2}(N-1)\Delta\theta\right)]\Delta\theta\} \tag{4.30}$$

The $\Delta\theta$-dependencies of the cosine terms within the square brackets would yield higher-order terms in $\Delta\theta$, hence can be neglected. Therefore, the cost is a convex function of the angle error around the minimum of $||\delta(\theta)||^2$, which occurs at

$$(\Delta\theta)_{min} \simeq -\frac{2\sqrt{3}\mu\left||Pa_0|\right| \cos(\Psi)\cos(\alpha)}{(2\mu + p_0)p_0 + N\left||Pa_0|\right|^2 cos^2(\Psi)} \tag{4.31}$$

Equation (4.31) implies that there is an inherent error in the estimation of the DOA, which depends on the relationship of the SOI steering vector $a_0$to the interference steering vectors. Equation (4.31) can further be simplified by noting that

$$||Pa_0||^2 = a_0^H P^2 a_0 = a_0^H P a_0 = ||a_0||^2 - a_0^H(I - P)a_0 = N - P_0 \tag{4.32}$$

Substitution of (4.32) in (4.31) yields

$$(\Delta\theta)_{min} \simeq -\frac{2\sqrt{3}\sqrt{N-P_0}\cos(\Psi)\cos(\alpha)}{(2\mu+P_0)P_0+N(N-P_0)cos^2(\Psi)} \tag{4.33}$$

The best case occurs when a_0 is orthogonal to the subspace spanned by the interference steering vectors, in which case $p_0 = N \Longrightarrow (\Delta\theta)_{min} = 0$. On the other hand, as the orthogonal projection of a_0 on the interference subspace increases, P_0 decreases. In this case, however, the angle Ψ tends to π/2 and hence $cos(\Psi)$ approaches zero. Without an explicit knowledge of the interference steering vectors, it is impossible to estimate the worst case error in the estimation of the DOA. However, the DOA of the SOI and those of the interference signals are generally sufficiently separated so that P_0 remains close to N.

4.3 Discussions

Several adaptive beamforming techniques, which have robustness against the problems such as mismatch in the direction-of-arrival of the signal-of-interest, array calibration errors, local scattering of the incident signal and finite sample approximation have been developed to overcome these difficulties but these techniques have in general high computational complexity, as they depend on the eigenvalue decomposition (EVD) of the array covariance matrix.

Performance of adaptive beamformers is known to degrade due to calibration and other perturbation errors as well as under conditions in which the data covariance estimate is in error. One technique based on the use of the presumed steering vector in to the signal-plus-interference subspace has been proposed. In this work we investigated the possibility of performing the minimization without EVD by expressing the cost function in terms of a parameterized estimate of signal steering vector.

Chapter 5

SIMULATIONS AND DISCUSSIONS

5.1 Introduction

In this part with considering the experiential methodology we presume three methods (SMI, RCB, and IRMVB) for computing output SINR versus the number of snapshots, output SINR versus the different SNR, and normalized magnitude response versus the directional arrival. Finally the method will be compared with these approaches and results by applying Tables and Figures.

5.2 Simulation Approach

In simulation part we appraise our technique by applying Monte Carlo simulations. In all cases, we consider a uniform linear array (ULA) with $N = 10$ Omni-directional sensors separated by half-wavelength. For every result, to achieve each simulated point, the middle of 100 simulation runs is utilized. In the all scenarios, we presume that there is one desired and two interfering sources. The desired signal is always existent in the training data samples, and the interference-to-noise ratio (INR) is assumed to be 30 dB for all conditions. The diagonal loading parameter is selected to be $\lambda = 80$ for the LSMI method in all conditions, but the IRMVB method which is chosen to be $\lambda = 0$. The efficiency of the all approaches is compared with respect to the output SINR. We have to mention that all of simulations have been performed under the same conditions. The desired signal and interferers are plane waves which impinge on the sensors from 5°, 20^{o} and 30^{o}, whereas whiles the path of assumed signal is 0^{o} .So we have a 5° look direction mismatch.

5.3 Simulations

The approaches that have been appraised in all simulations are 1) benchmark SMI algorithm 2) RCB algorithm 3) IRMVB (Li's) method 4) proposed method, DRS algorithm.

First table is measure of the efficiency of the method versus training data samples (snapshots) $M = 20:500$.

Table 5.1: efficiency of methods by different training data samples

Number of Snapshots	20	60	100	140	180	220	260	300	340	380	420	460	500
LSMI	4.39	5.35	5.57	5.67	5.69	5.74	5.76	5.79	5.8	5.81	5.83	5.84	5.85
RCB	4.10	5.26	5.4	5.56	5.58	5.62	5.66	5.67	5.68	5.71	5.72	5.73	5.73
IRMVB(Li's)	5.8	7.86	8.4	8.69	8.77	8.88	8.96	8.97	8.98	9.03	9.04	9.05	9.05
Proposed Method	7	8.63	9.03	9.20	9.25	9.39	9.43	9.51	9.54	9.56	9.59	9.60	9.62
optimal	9.79	9.79	9.79	9.79	9.79	9.79	9.79	9.79	9.79	9.79	9.79	9.79	9.79

The efficiency of the methods versus the SNR for training data samples taking the values 100,200,300,400 are displayed in Figure 5.2 up to 5.6.

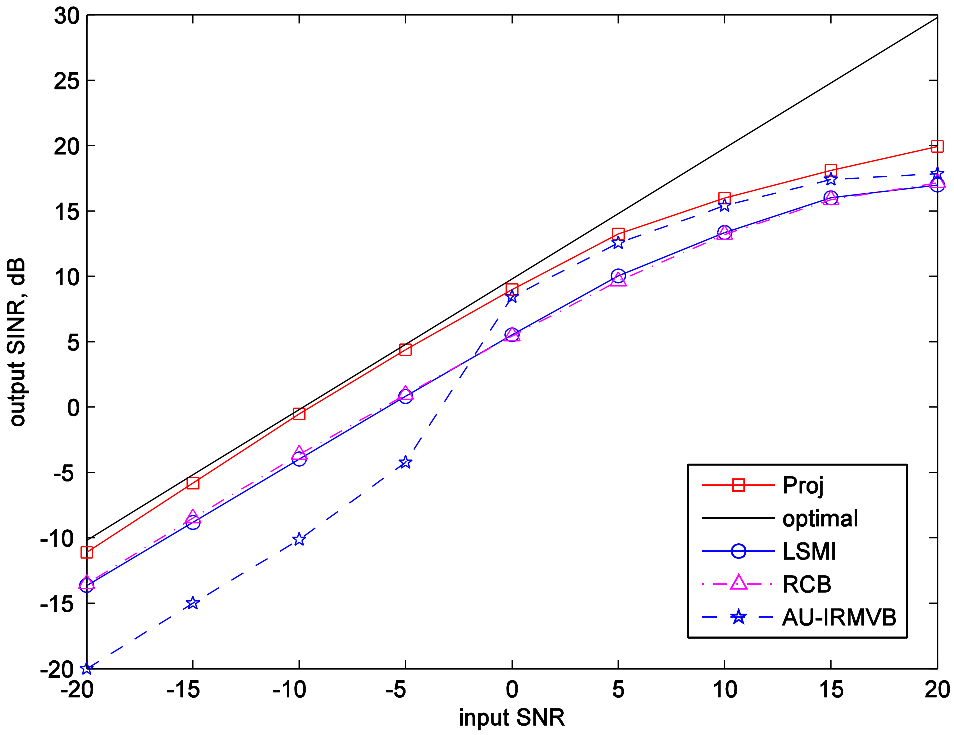

Figure 5.1: Output SINR versus SNR with Training Data Sample=100

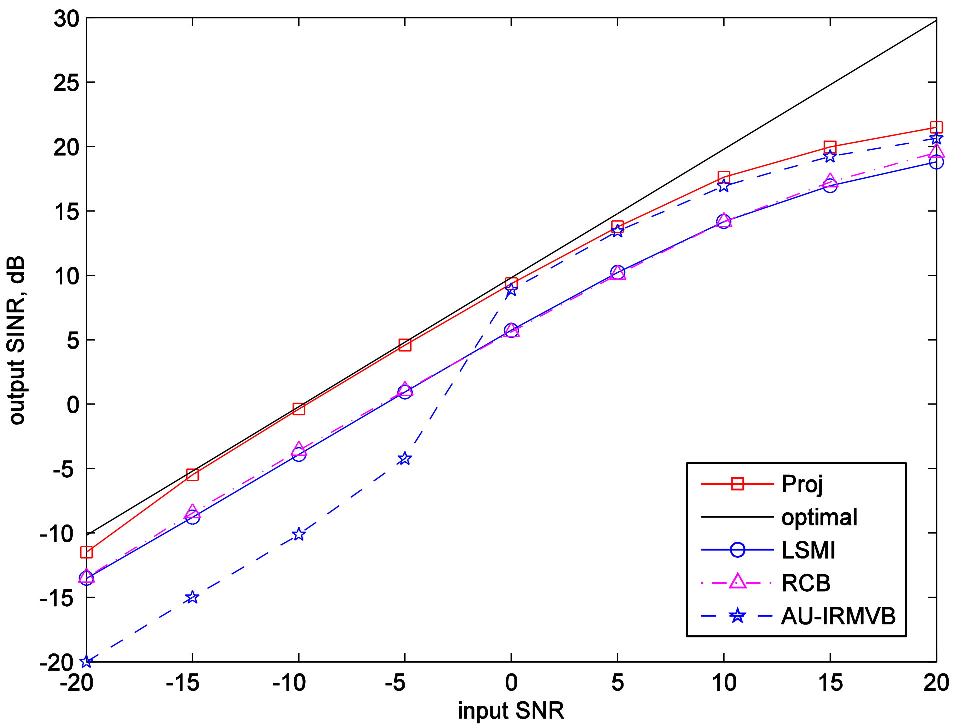

Figure 5.2: Output SINR versus SNR with Training Data Sample=200

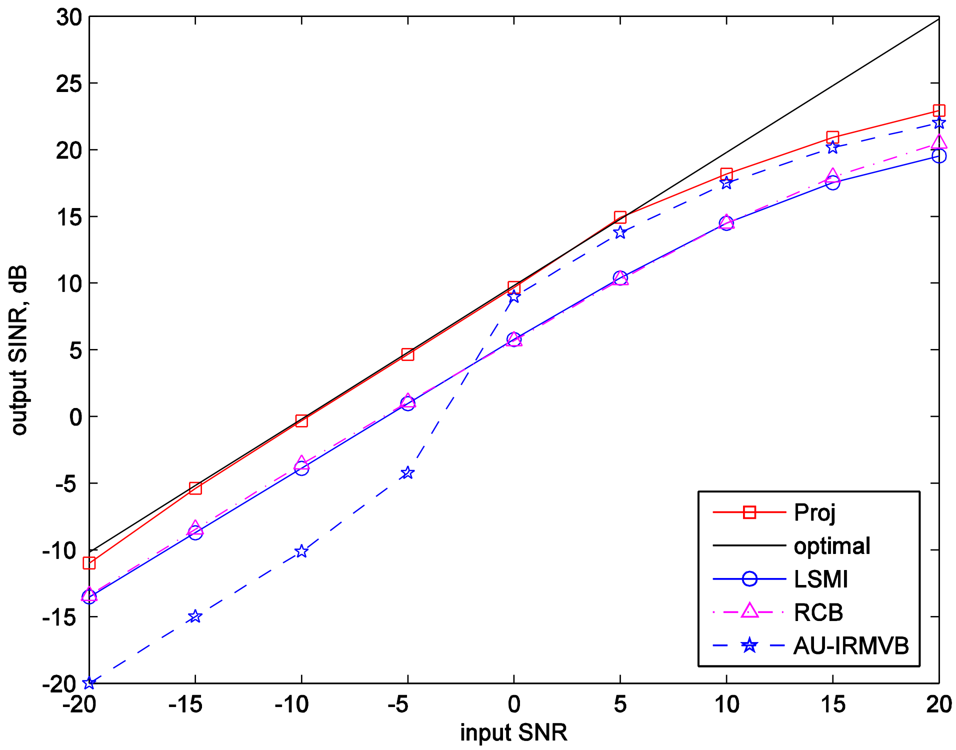

Figure 5.3: Output SINR versus SNR with Training Data Sample=300

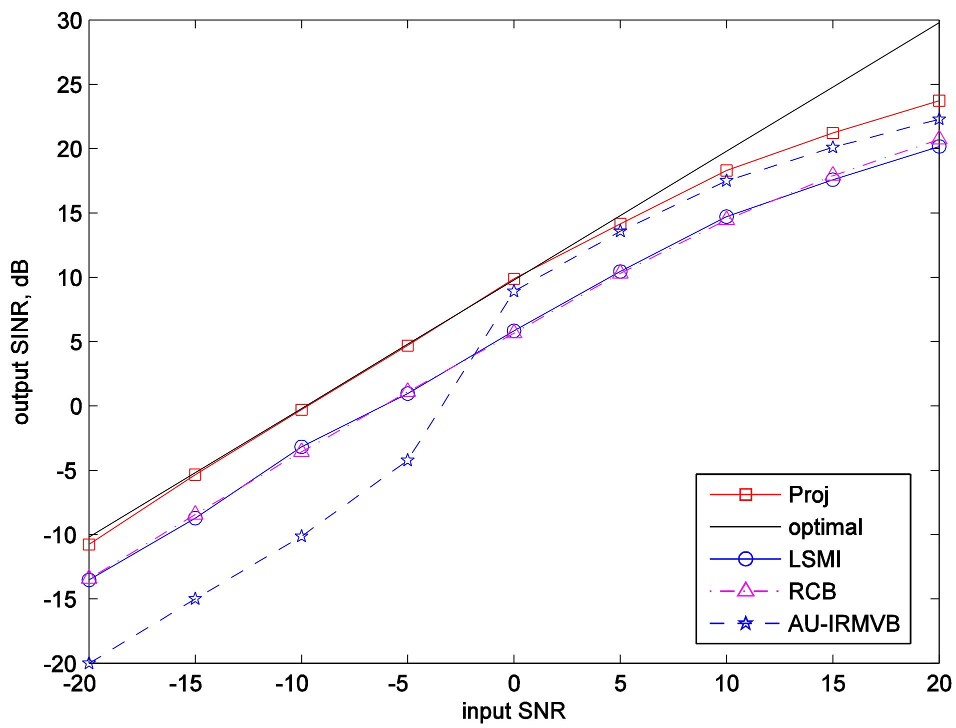

Figure 5.4: Output SINR versus SNR with Training Data Sample=400

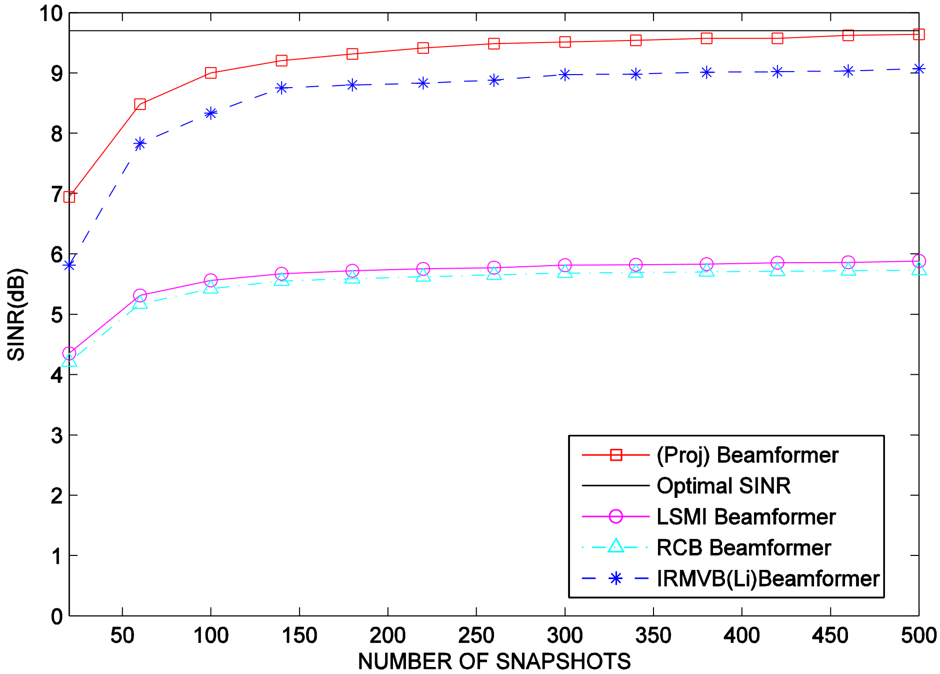

Figure 5.5: Output SINR versus the number of snapshots

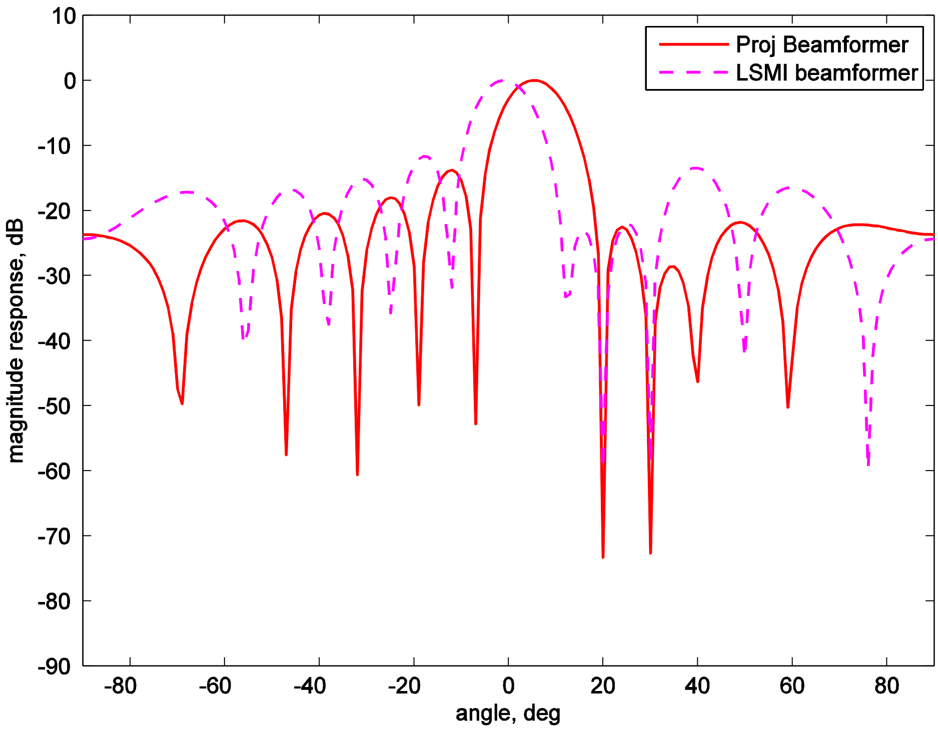

Figure 5.6: Output magnitude response of proposed method and SMI Beamformer

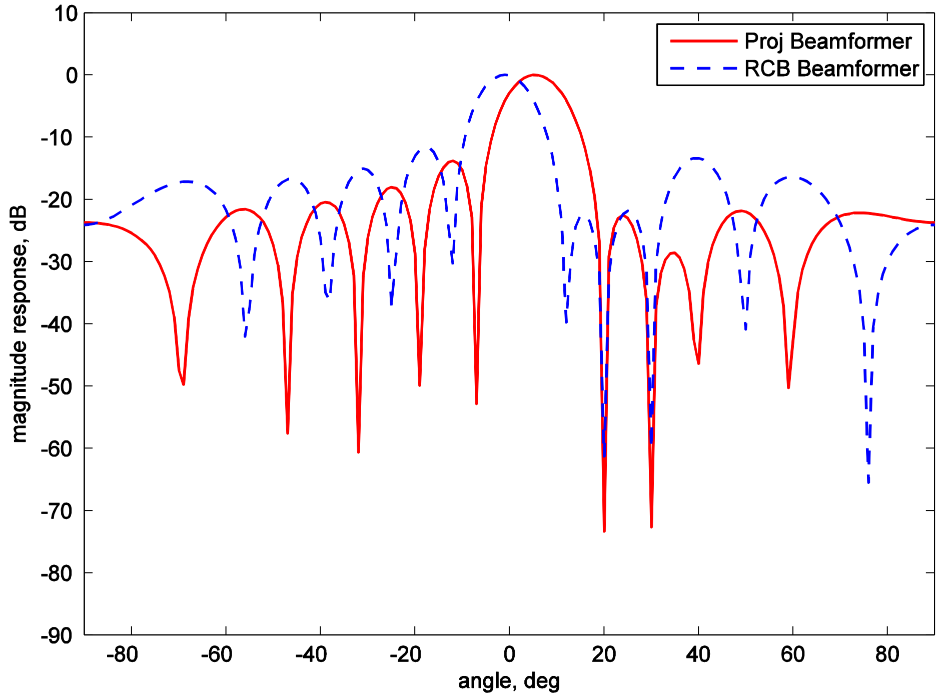

Figure 5.7: Output magnitude response of proposed method and RCB Beamformer

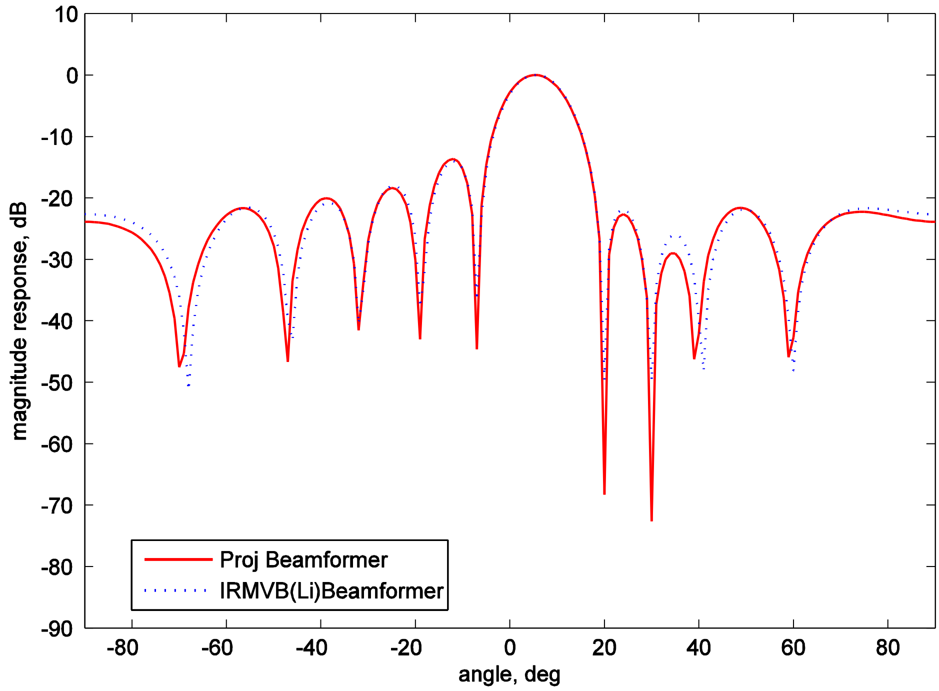

Figure 5.8: Output magnitude response of proposed method and IRMVB(LI) Beamformer

5.4 Discussion

Figures 5-1 to 5-8 obviously demonstrate that in all cases, the proposed approach gives superior efficiency among the compared methods. The SINR values for the offered method are close to the optimal values in a wide range of N, SNR and mismatches for direction of arrival (DOA).

To appraise the convergence of the proposed method, power of SOI is constant at 1 dB and simulation is replicated up to 400. The amount of convergence of presented approach is shown by calculating the average output SINR at every repeat. Fig. 5.1 displays the output SINR versus the number of snapshots (data training samples) =400. Expressly it is observed that the output SINR of the Proj beamformer is considerably better than the proposed techniques whiles the IRMVB (Li's) beamformer keeps its level to be modified. However, the LSMI technique nears to the proposed beamformer slightly for all N snapshots.

The purpose of the simulations in (Fig 5.1 - 5.4) is to compare the efficiency of output SINR versus the input SNR diverse from −20dB to 20dB. The resulting output SINR for every input SNR is averaged over 100 perception. It can be noticed that the proposed method performs better than the other compared approaches at all SNRs. It can be considered that, although, by increasing the SNR, all presented beamformers go to to optimal value of SINR but, our technique‘s result is better than the others, whiles, the IRMVB beamformer is acceptable just for SNR>0.

Chapter 6

CONCLUSION AND FUTURE WORK

6.1 Conclusion

In this work, we offered a modified method to robust adaptive beamforming based on estimating signal steering vector not depend on the eigenvalue decomposition (EVD) of the array covariance matrix. This approach shows the capability of our method in the presence of direction-of-arrival (DOA) mismatches of desired signal by forming the directional response of the adaptive ULA.

The performance of presented beamformer is shown to decrease errors between the real and assumed array steering vectors of the desired signal. To achieve this goal the DOA of the SOI is estimated by minimizing a cost function in terms of the norm of an error vector, which is the difference between the presumed steering vector of the SOI, and the orthogonal projection of this vector onto the signal subspace. Additionally, effective implementations of our technique for the processing condition have been expanded. Moreover, numerical examples in terms of SINR and data training illustrate that the presented method is robust to sample covariance matrix errors. Also, to appraise the approaches the various simulations are run in terms of SINR and normalized magnitude response. It shows that by forming the directional response, our beamformer can improve the mismatches in array steering vector for desired signal.

The performance of this approach is illuminated when desired signal are attended by noise and interferences. The algorithm repress the interferences with deeper nulls in the directional of interferences by forming the directional response of desired signal. Furthermore, the simulation SINR against the SNR displays that the presented beamformer holds its convergence with respect to the number of snapshots to most favorable and it is qualified to eliminate the noises with less noise level (low SNR).

At the end, in conditions with various types of desired signal errors, our method is displayed to stably profit a considerably improved efficiency and quicker convergence rate in compared with proposed adaptive beamforming techniques.

6.2 Future Work

Extensive simulation studies have indicated that the proposed beamforming method performs better or as good as the existing methods in various scenarios. Theoretical analysis of the proposed beamformer should be performed to verify the results achieved from simulation. Such an analysis will enable the choice of the parameters used in the algorithm.

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APPENDICES

Appendix A: Substantiation of Equation (4.9)

The following inverse of a partitioned matrix can be used to evaluate the inverse on the left-hand-side of (4.9)

$$\begin{bmatrix} A & C \\ D & B \end{bmatrix}^{-1} = \begin{pmatrix} 0 \\ I \end{pmatrix} B^{-1}(0\ I) + \begin{pmatrix} I \\ -B^{-1}D \end{pmatrix} (A - CB^{-1}D)^{-1}[I - CB^{-1}]$$

(Apx-1-Eq 1)

Where A∈ $\mathbb{C}^{m\times m}$, B∈ $\mathbb{C}^{m\times m}$, C∈ $\mathbb{C}^{m\times m}$, D∈ $\mathbb{C}^{m\times m}$. The matrix to be inverted is

$$(A_s^H A_s)^{-1} = \begin{bmatrix} ||a_0||^2 & a_0^H A_i \\ A_i^H a_0 & A_i^H A_i \end{bmatrix} \qquad \text{(Apx-1-Eq 2)}$$

With the appropriate associations, on the obtains

$$(A_s^H A_s)^{-1} = \begin{pmatrix} 0 \\ I \end{pmatrix} (A_i^H A_i)^{-1}[0\ I]$$

$$+ \frac{1}{\left||a_0|\right|^2 - a_0^H A_i (A_i^H A_i)^{-1} A_i^H a_0} \begin{pmatrix} 1 \\ -(A_i^H A_i)^{-1} A_i^H a_0 \end{pmatrix} \qquad \text{(Apx-1-Eq 3)}$$

$$[1 - a_0^H A_i (A_i^H A_i)^{-1}]$$

Multiplying (4.36) by A_s from the left, by A_s^H from the right and rearranging gives (4.8).

Appendix B: Substantiation of Equation (4.12)

Using (4.2) the matrix (I+λR) can be written as

$$I + \lambda R = (1 + \lambda\sigma_n^2)I + \lambda\sigma_s^2 a_0 a_0^H + \lambda A_i \Sigma_i A_i^H = R_{n\lambda} + \lambda\sigma_s^2 a_0 a_0^H$$

(Apx-2-Eq 1)

Where $R_{n\lambda} = (1 + \lambda\sigma_n^2)I + \lambda A_i \Sigma_i A_i^H$. Using the well-known matrix inversion lemma, the inverse becomes

$$(R_{n\lambda} + \lambda\sigma_s^2 a_0 a_0^H)^{-1} = R_{n\lambda}^{-1} - R_{n\lambda}^{-1}\left(\frac{\lambda\sigma_s^2 a_0 a_0^H}{1 + \lambda\sigma_s^2 a_0^H R_{n\lambda}^{-1} a_0}\right)R_{n\lambda}^{-1}$$

(Apx-2-Eq 2)

Using the same lemma the inverse of $R_{n\lambda}$ can be written as,

$$R_{n\lambda}^{-1} = \frac{1}{1 + \lambda\sigma_n^2}[I - A_i((\lambda_d + \sigma_n^2)\Sigma_i^{-1} + A_i^H A_i)^{-1} A_i^H) = \frac{1}{1 + \lambda\sigma_n^2}(I - P)$$

(Apx-2-Eq 3)

Where P is the second term within the square brackets and$\lambda_d = \frac{1}{\lambda}$. Substituting (4.39) in (4.38) and simplifying gives,

$$(I + \lambda R)^{-1} = \frac{1}{1 + \lambda\sigma_n^2}[(I - P) - \frac{1}{\beta(\lambda)}(I - P)a_0 a_0^H (I - P)]$$

(Apx-2-Eq 4)

Where

$$\beta(\lambda) = \mu(\lambda) + a_0^H (I - P) a_0$$

Substitution of (4.40) in (4.11) results in(4.12). Note that P becomes approximately equal to P_i for sufficiently large λ and noise power much less than the interference signal powers.