Optimization of the Production Planning and Supplier-Material Selection Problems in Carton Box Production Industries

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ABSTRACT

This study deals with the problem of supplier-material selection as well as production planning in carton box production industries. The initial step in solving these problems is to deal with the cutting problem which refers to the problem of dividing a usually large piece of the rectangular raw materials into smaller pieces for producing various products. Dealing with this problem is at the stake of dealing with the uncertainties related to the environment of the problem in most cases. A critical problem arises when size, amount, and suppliers of the raw materials have an uncertain nature from the price point of view. In such cases, selecting the correct size and quantity of the raw material as well as right suppliers are the crucial elements for a competent production. In this research, a model reflecting the nature of the problem is proposed and a new solution approach is employed for solving it. Moreover, the problem's related uncertainties are incorporated to the original problem through utilizing the fuzzy variables. The new problem then is solved with different fuzzy methods. The mentioned approaches are verified through implementaion on a newly established company which produces carton boxes for various manufacturers. These carton boxes must meet a very accurate specification related to their material type and dimension congruous with their purchaser's requirements. The objective is to optimizing the production procedure and defining an efficient production system as one of the essential requirements for sustainability of the production within this company. In the following chapters, all of the steps are discussed in detail.

Keywords: Production Planning and Scheduling, Supplier selection, Material selection, Carton box production industries.

ÖZ

Bu çalışmada karton kutu üretim sanayisi için tedarikçi-malzeme seçimi problemi ve üretim planlama çalışmaları ele alınmıştır. Bu problemlerin çözümündeki ilk aşama genellikle büyük boyutlu dikdörtgen şeklindeki hammaddeleri çeşitli ürünlerin üretilmesi için küçük parçalar halinde kesme probleminin çözülmesidir. Bu problemin çözümü çoğu durumda problemle ilgili belirsizliklerle uğraşmayı gerektirmektedir. Ölçü, miktar ve hammade tedarikçilerinin belirsizlik ortamında olması fiyatla ilgili kritik bir problemin ortaya çıkmasına sebep olur. Böyle durumlarda, hammaddeyle iligi doğru ölçü, mikrat ve de tedarikçi seçimi rekabetçi üretimin önemli etkenleridir. Bu çalışmada, problemin bir modeli önerilmiş ve yeni bir çözüm yöntemi uygulanmıştır. Dahası, problemle ilgili belirsizlikler bulanık değişkenler kullanarak orijinal probleme dahil edilmiştir. Bu yeni problem farklı bulanık yöntemler kullanılarak çözülmüştür. Bahsi geçen yaklaşımlar daha sonra çeşitli imalatçılar için karton kutu üreten yeni bir şirket için uygulanarak doğrulanmıştır. Bu karton kutular müşterilerin gereksinimlerine uygun olarak hammade tipi ve boyutlarla ilgili çok kesin özellikleri sağlamalıdır. Amaç, bu şirket için üretimin devamlılığyala ilgili temel ihtiyaçlardan biri olarak üretim işlemini en iyilemek ve etkili bir üretim sistemi tanımlamaktır. İzleyen bölümlerde tüm aşamalar detaylı bir şekilde anlatılmıştır.

Anahtar kelimeler: Üretim planlama ve çizelgeleme, Tedarikçi seçimi, Hammade seçimi, Karton kutu üretim sanayii.

To My Mother and My Sister

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INTRODUCTION

1.1 The Essential Objectives of a Production System

In all industries, the goal of production system is producing and delivering the products through manufacturing processes. The material conversation of transforming the raw material into the products takes place at this stage. In a competitive enterprise, the material conversation must simultaneously meet the following objectives;

1. Superb quality of the product (equal to / better than the other competitors).
2. Lower production cost in comparison with the other competitors.
3. Consistency in time for customer delivery.

Accomplishing the mentioned objectives, the decision makers of the organization must make proper decisions. Decision types in a production system are divided into Long, Medium and short-term decisions over the planning horizon. Long term planning, also known as strategic planning, covers up to several years into the future. These decisions must be consistent with the long term organizational goals. Medium planning horizon covers any period from one month up to one year. Decisions made in this time frame are oriented towards achieving the annual goals of the company. Short planning horizon covers a period from one day to one month, it is also known as operational decisions and concerned with meeting the predetermined monthly production plan. A disciplined management system, particularly a well-established production planning and control (PPC) system, is required for conducting these decisions. The PPC function integrates material flow using the information system through a common database (see Figure 1). In this figure, “MPS” stands for material planning system.

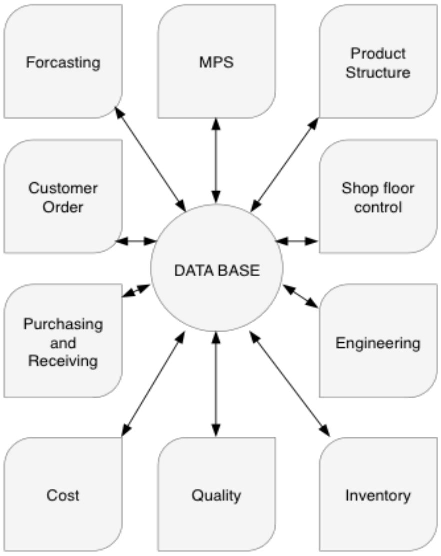

Figure 1. A generic Production Information System

1.1.1 Material Conversation

In several industrial units including carton box industries, dividing the raw materials into the required measures through cutting with the smallest amount of wastage is the essential factor of a successful production. In industries such as wood, glass and carton boxes, large sheets of raw materials must be divided to smaller rectangular components through cutting for producing different goods. By a similar logic, in the tasks such as newspapers paging, the articles and advertisements are the rectangular components that should be packed into the larger rectangular pages. In all of these cases, in order to accomplish the tasks a category of cutting problem must be solved which is referred as two-dimensional packing problems (2DCP) in the literature.

In this research, the 2DCP in carton box industries is investigated. This problem belongs to the production floor. For this purpose, a typical real life company in this industry is considered and investigated. The employed concepts, techniques and procedures are applicable for all other similar industries with minor tailoring. As a typical procedure, the company buys the required raw materials in form of carton planes (sheets/objects). These planes can only be ordered in limited dimensions due to the supplier's technical constraints. The company cuts these raw sheets to smaller planes. In some cases, these sheets are used directly, while in most of the cases they are fold to produce three dimensional boxes.

Some of the most important decision variables of this problem are the choice of sheets according to the customer requirements, selecting the appropriate ordering dimension for the raw sheets, determining the production patterns, supplier selection and pricing. These procedures will be discussed in the following chapters in detail.

1.1.2 The Significance of Suppliers

As previously mentioned, another key element for remaining competitive is the existence of a proper relation with the customers and suppliers. Extending production planning and control to suppliers and customers is known as supply chain management in the literature. The relation between the production system and suppliers and customers is illustrated in figure 2.

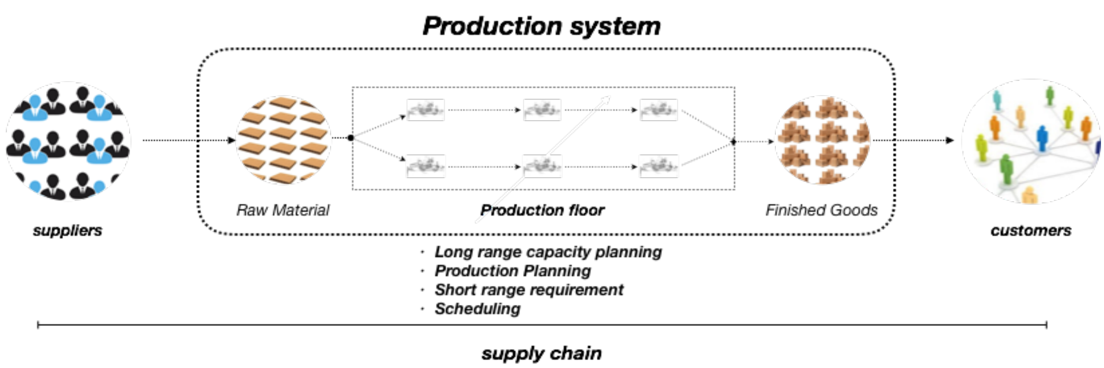

Figure 2. Elements of a supply chain

The most important objectives of the box production industries are normally the followings:

1. Standardizing and improving the general procedures of the company.
2. Defining an effective production system to reduce the production costs, waste and incompatibilities.
3. Creating an optimized supply chain for facilitating the production procedure and its profitability according to company's requirements.
4. utilizing the material handling systems.

These targets are mainly related to inventory management, improving the purchasing and the production system. Refining the mentioned areas are the main concerns in this study. The current production system of the company is visualized in the following figure:

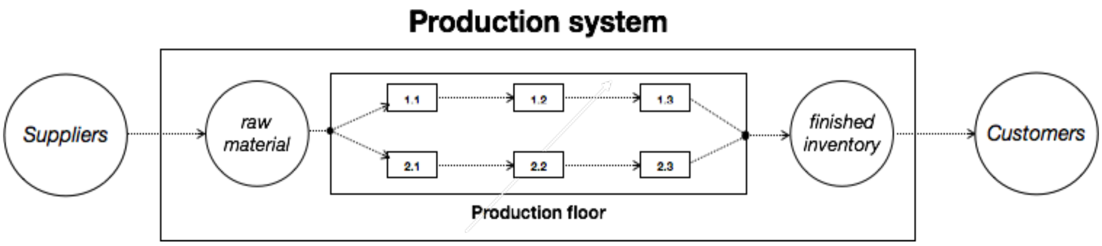

Figure 3. The Production System of the Case of Study

1.1.3 Uncertainty

In the real life, most of the components of a supplier-material selection and production planning problems include a high level of ambiguity. For determining a good solution for these problems, it is essential to consider this ambiguity in the problem modeling phase and determine a solution to deal with it. In the 3rd chapter of this study, the associated uncertainties in this problem are discussed and dealt.

1.3 The Company's Production Technology

As it is shown, the company has two production lines utilizing a flow shop technology due to the high-volume production of standardized products. Each line contains three machines which are potentially capable of handling all required operations for producing a part provided that the required tools are installed on their tool magazines. The production duration of a task is different on each machine depending on the product types. The other component of this production system is the inventory of raw material. The same inventory storage is employed to feed both lines. Providing the proper production schedules for each category of products is important for maximizing the profitability of the company. In the next section, the essential preliminaries of this industry are explained and discussed.

1.4 Preliminaries

The company which is discussed in this research produces over two hundred different types of products including carton boxes and divider planes. The main difference amongst the products is associated with their dimensions and their material combination. The technical details and definitions are as follows:

1. **Correct sheet type for production:** Deciding on the material combination of the carton sheets of the products takes place according to the customer's order; however, the companies normally provide a counselling service for the customers to facilitate their decision-making procedure.
2. **Object:** The raw materials of the company for producing the items are produced by the suppliers of the company with different material combination in different measures. Each variant of these raw materials is called an object which is considered as a separate raw material.

3. **Item:** Each of the company's product which is purchasable by a customer is an item.
4. **Cutting pattern:** it is a style for dividing the objects into one or more items. The items might be from one or several types.
5. **The object's material types:** The main objects utilized in the company are 3-Layer and 5-Layer objects. These sheets are produced at the suppliers by combining several layers of carton papers and one or more (depend on the number of plane's layers) corrugated medium between the papers which is called the Flute Layer (*fl*). There are two major types of carton papers; the Craft denoted by (C) which is a fresh production of paper from wood (virgin paper) and Liner (Li) which is recycled paper. While the papers in the outer layer of an object may have any material, the material type of the corrugated medium and the paper in middle layers are usually liners. The different combinations of the paper types and medium layers provides a total number of 6 different objects utilized in the company; Five Layers & Double Craft (C2-5), Five Layers & Single Craft (C1-5), Five Layers & Liner (Li-5), Three Layers & Double Craft (C2-3), Three Layers & Single Craft (C1-3), and Three Layers & Liner (Li-3).
6. **Dimension of the objects:** The *fl* has a wavy shape along the length of an object which is considered as the direction of the *fl (across flute)*. Hence, the direction of *fl* is a correct indicator to determine the length (L) and the width (W) of an object; the side of a sheet which is alongside the *fl* waves is always the length and the other side is always the width. The objects are normally represented by their material type and their length (L) and width (W) in the form of "$X/L \times W$" (i.e. C1-3/90×120). In Figure 4 the composition of the

objects as well as its length and width are illustrated. In this figure, the longer side of the sheet is length and the shorter side is the width of the object. The outer layer of the object might be both liner, both craft, or one liner and one craft.

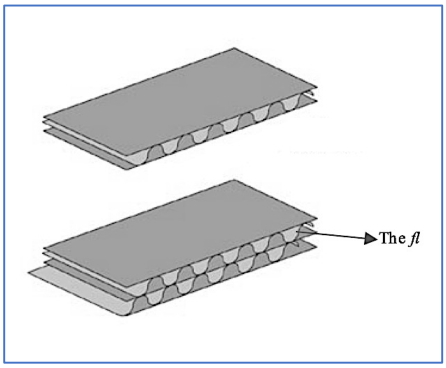

Figure 4. 3-Layers and 5-Layers Objects

7. **Dimension of the items:** As previously mentioned, the products of the company are boxes and divider planes. The measure of a box is normally represented by its length, width, and height respectively as; $(a \times b \times c)$. Since the planes are two dimensional, their measure is simply represented by length×width as $(a \times b)$.
8. **The spread dimension:** Represented by $l \times w$, The spread dimension of an item is the dimension of the carton sheet which is required for producing that product. The spread dimension for boxes are calculated according to the following formula:

$$l = [(a + b) \times 2)] + 4 \quad (1.1)$$

$$w = b + c \quad (1.2)$$

For the two-dimensional products, this procedure is much simpler; the required dimension of the carton sheet for producing a plane is equal to the dimension of the product itself. In a simpler word, the spread dimension of a product is equal to the minimum dimension of the raw carton sheet capable of producing it.

9. **The strength of the items:** The strength of an item is dependent on two factors; the material composition and the direction of the *fl*. According to a general rule, a carton sheet with more craft layer in its structure represents a higher strength. However, utilizing more craft layers is associated with a higher production cost and more expensive products. On the other hand, the direction of the *fl* has a crucial role in the strength of the boxes. An item acquires the minimum necessary strength if and only if, the direction of the *fl* is vertical against the weight that it carries. Therefore, the items are designed such that the weight is applied on them in the orthogonal direction of their composing object's *fl*. Consequently, the length and width of objects and items have a solid definition and cannot be altered.
10. **The validity rule:** Considering the mentioned facts, the length and width of an item must be extracted from the length and width of an object respectively. Hence, rotating an object for producing an item which is not matched with this definition is not possible.
11. **Cluster of the products:** Before proceeding to the solution approach, the production data of the problem must be marshalled and categorized. In this regard, initially the data related with the item types are collected and the products with the identical material combination are placed in the same category. Based on this classification, 6 different clusters of products are

defined; C1-5, C2-5, C1-3, C2-3, Li-5, and Li-3. It is notable that for producing the items in each cluster, the material combination of the objects must be identical to the material combination of the cluster. however, several sizes of the objects can be used. Selecting the proper object(s) for each cluster is one of the objectives of this problem.

12. **The constraints related with the suppliers:** As previously discussed, the aim of this study is to determine the proper dimension of the raw materials. One of the associated constraints with this problem is the supplier restrictions in delivering the requested measures. Due to the technical issues, the suppliers are not able to cut the raw sheets in any desirable measures; the available lengths of a sheet in at the suppliers may vary between 45 to 200 based on five centimeter increments (i.e. 45,50, 55, ..., 200). Moreover, the stocks can only be cut according to the following predefined widths; 90, 100, 110, 120, 140, 150, 160, and 200. The next table represents the possible measures of lengths and widths as the dimension of an object.

Table 1. Possible cutting measures for lengths and widths

Possible Widths							
90	100	110	120	140	150	160	180
Possible length							
45	50	55	60	65	70	75	80
85	90	95	100	105	110	115	120
125	130	135	140	145	150	155	160
165	170	175	180	185	190	195	200

1.5 The Structure of the Study

This study is divided to 4 chapters; chapter 1: Introduction, Chapter 2: A Literature review, Chapter 3: The cutting problem, Chapter 4: Supplier-Material selection under uncertainty, and Chapter 5: Conclusion.

1.5.1 The Cutting Problem

The company's challenge is determining an appropriate dimension for the raw sheets in each cluster of products such that it satisfies the following two conditions:

1- All the products of that cluster are produced

2- The waste of material is minimized

In chapter 2, the cutting problem associated with the pointed desires is discussed. As mentioned formerly, these problems refer to the problem of dividing a predominantly large piece of the rectangular raw materials into the smaller pieces for producing various products. Noting that the cutting problems are NP-hard problems, offering good solutions for these problems has been the subject of a numerous researches over the past few years. In the present study, a model reflecting the nature of the problem is proposed and a new column generating solution approach is suggested to solve it.

1.5.2 The Supplier-Material Selection Under Uncertainty

Simultaneous selection of supplier and material plays a critical role for managers specially in carton box manufacturing industries which means, the customers who order the raw material must be extremely determined in choosing the raw material and the supplier from which they buy. Not all suppliers are able to deliver all variations of the raw material. Moreover, the suppliers may offer different scheme of pricing with respect to their own policies. Therefore, the environment of the problem becomes tainted with a certain level of uncertainty. In chapter 3, the problem of "simultaneous selection of suppliers and material" is incorporated to the original problem. Therefore, the problem is reformulated utilizing a multi-objective modeling and optimization approach which is a suitable approach for dealing with such problems as per (Kovács *et al.*, 2002; Franco *et al.*, 2009; Jablonsky, 2014). Taking an enterprise point of view in the new formulation, the three different objectives of the problem are minimizing the wastage amount of raw material, raw material cost and product surplus respectively. Additionally, the uncertain nature of some parameters of the problem must be reflected in the model (Fullér and Majlender, 2004; Fullér et al., 2012; Salahshour and Allahviranloo, 2013; Moloudzadeh et al., 2013; Wang et al., 2014; Salmasnia et al., 2015). Hence, the formulation is extended to an uncertain environment with uncertain costs and demands to be more realistic. These uncertainties may be dealt with various approaches. In this study, "Fuzzy possibilistic multi-criteria approach" and "Fuzzy chance constrained multi objective necessity approach" are utilized to deal with the ambiguity of the cost and demand.

Chapter 2

LITERATURE REVIEW AND CHAPTER'S INTRODUCTION

2.1 The Significance of Cutting Problem

An important factor in keeping an industry profitable is to have a well-designed production plan. One of the most important subjects to be solved in the production planning stage in the industrial applications such as wood, paper or glass industries, is to divide the rectangular raw materials into smaller rectangle pieces through cutting with specific measures such that the wastage is minimized (Mosallaeipour *et al.*, 2017; Russo *et al.*, 2014). Up to the present, plentiful investigations are conducted to examine the best method for cutting the raw materials. The resulting optimization problems are addressed as bin packing problems, two-dimensional cutting problems (2DCP) or two-dimensional strip packing problems (2DSP) in the literature. Most of the investigations about these problems are dedicated to the cases where a set of rectangular products are to be allocated into a minimum set of rectangular sheets of raw material. An identical representation of the problem is dividing the raw sheets into smaller pieces such that the maximum number of items are deliverable with minimum wastage. Without loss of the generality, it is assumed that all input data are positive integers and the dimension of the items are always less than or equal to the objects. This problem is strongly NP-hard (Lodi *et al.*, 2002). (Gilmore & Gomory, 1965) are the first people who attempted to formulate the 2DCPs. Based on the enumeration of all subsets of items, they utilized a column generation approach to pack several items into a single object. (Beasley, 1985b), combined the idea of turnover for each item which is to be packed and aimed to maximize the profit through packing the most profitable items into a single object. (Christofides & Hadjiconstantinou, 1995) proposed a similar model for this problem benefitting the Lagrangian relaxation and sub-gradient optimization method. (Scheithauer & Terno, 1996) presented the raster points which can be used in an exact dynamic programming algorithm without losing the optimality (Beasley, 1985a). (Cintra *et al*, 2008) proposed an exact dynamic programming procedure that simplifies the computation of the knapsack function and provides an efficient procedures for the computation of the discretization points built on Beasley's researches. (Kang & Yoon, 2011) proposed a branch and bound algorithm for unconstrained DCPs (UDCP) which is amongst the best algorithms proposed for this category of problems. Furthermore, they implemented a pre-processing procedure to reduce the number of valid pieces for entering the process. Lately, (Birgin *et al.*, 2012) proposed a two-phase heuristic method for solving the problems related with the non-guillotine case of U2DCP. This method solves the guillotine variant of the problem in the first phase in two steps: a fast heuristic step based algorithm proposed by Gilmore & Gomory and an exact dynamic programming step proposed by (Russo *et al*, 2013). Furthermore, they employed the reduction of the discretization points method proposed by (Cintra *et al.*, 2008) and pre-processing method proposed by (Birgin *et al.*, 2012) in their algorithm. This algorithm is one of the most effective exact dynamic programming algorithms proposed for solving the U2DCPs.

Box production is one of the most famous production industries which is required to deal with the cutting problems (Russo et al., 2014), the choice of the supplier and material is equally important for profitability of the business (Mosallaeipour et al., 2017).

In this study, both production planning (dealing with cutting problem) and selecting the supplier and material problems are investigated. In the third chapter, a model reflecting the objectives of the problem for maximizing the profit is proposed. The objective of this research is to maximize the profit of the mentioned industry through minimizing the amount of wastage and surpluses generated during the production procedure. A modified algorithm comprised of a production planning–column generating approach is proposed to serve the mentioned objective. This algorithm determines the proper dimension of the raw material required for the production such that all products of the company are producible with minimum wastage. The quantity of surpluses and procurement cost are reduced through determining the best combination and quantity of the raw materials.

In chapter 4, the problem of selectin the material and suppliers as well as the uncertain nature of these problems' key variables such as demand and procurement of the raw material is considered and discussed.

2.2 The Uncertainty and its Role in Supplier-Material Selection

In the complex world of today where the market is filled with strong competitors, having an effective procurement system is amongst the essential keys for a successful business (Tan and Alp, 2015). Therefore, finding the appropriate suppliers which are able to provide the required raw materials for the companies becomes an important problem for the decision makers (Batuhan and Selcuk, 2015). This problem becomes even more complicated if there exist no supplier who is capable to supply all requests. This category of problems are in fact the procedure of finding the correct suppliers with the right price, at the appropriate time, with the right capacities and excellences (Ayhan, 2013). In the literature, these problems are mainly referred as the supplier selection problems.

Conferring from the statistics, materials acquiring cost covers almost 60% of the total sales of the enterprises in production industries (Krajewski and Ritzman, 2001). This cost can become as large as 70% of total income in automotive industries to even 80% in high tech industries (Weber *et al.*, 1991). Selecting the appropriate supplier results in substantial cost reduction complemented with significant raise in profitability of the enterprises. Moreover, it positively contributes to improvement of the products quality, competitiveness capabilities and responsiveness to the customers' needs in an indirect manner (Abdollahi *et al.*, 2015). The decision criteria of this problem are ordinarily determining the suitable contractors and appropriate quantity of the procurement.

In the real-life market, what is unneglectable is the uncertainty factor of the demand. In these regards, supplying the material should be accomplished such that the uncertainty of the demand is considered. Since the required items or services of the manufacturer can be supplied by a finite number of suppliers, it is important for the manufacturer to decide on utilizing the right sources at the right scope (Tan and Alp, 2015). In presence of demand uncertainty, the ideal suppler selection requires an integrated method, using all available cost parameters, capacity constraints and price information of the alternative suppliers simultaneously. A good supplier selection takes place when the punctuality in delivery, good quality of products and effective strategic partnerships is considered properly (Tan and Alp, 2015). Achieving this objectives, the enterprises have to improve their supplier selection technique in order to stay competitive and able to satisfy the high expectations of the customers (Ozgen and Gulsun, 2014). During the last decade, a numerous researches are conducted, investigating the improvement chances of this problem. An analytical model is developed by (Cakravastia *et al.* 2002). Their purpose was minimizing the level of customer dissatisfaction considering two factors; price and delivery lead time. Alp and Tan (2008) and Tan and Alp (2009) investigated a problem in a multi-period, with two supply options, having fixed procurement cost. Alp *et al.* (2013) considered another version of that problem having a linear cost function, identical suppliers in an infinite horizon and fixed components. Awasthi *et al.* (2009) deliberated a situation with several suppliers, with minimum order quantity and/or a maximum supply capacity neglecting the associated procurement fixed costs. They proved that the problem is NP-hard and introduced a heuristic algorithm for solving the general version of the problem. Hazra and Mahadevan (2009) investigated an environment where the buyer reserves a certain capacity from a set of suppliers, utilizing a contracting mechanism before observing the random demand. This capacity is assigned homogeneously to the nominated suppliers. However, if this capacity is not enough, shortage arises which will be satisfied through a spot market with higher unit price.

As mentioned formerly, one of the most important concerns in box production industries (similar to any other industries) is the profitability of the business. On one hand in this industry, optimality of the production is highly dependent to optimally solving the cutting problem. On the other hand, a proper choice of material (selecting the suitable material type), proper detection of the raw materials' dimensions and proper supplier selection are the main concerns of the decision maker. Therefore, both problems must be handled as good as possible. In majority of the real-world problems, the material type is specified by the customers therefore there is not much flexibility in selecting the type of material. However, selecting the right dimension for the raw materials and buying it from the right supplier under the correct condition is a problem with a lot of different solutions. Depending on the preferences of the decision maker and the level of associated uncertainty with the parameters of the problem, there are various method for solving these problems optimally.

The fourth chapter of the present study is conducted based on the data obtained from a carton box production company. The study aims to maximize the profit through both factors; minimizing waste and surplus amount through finding the optimal solution for our cutting problem and minimizing our procurement costs through wise selection of our supplier who provide the firm with raw materials. For this purpose, the concepts of multi-objective modeling and optimization is used (Kovács *etal.*, 2002; Franco *et al.*, 2009; Jablonsky, 2014). The problem is also modeled in an uncertain environment (Fullér and Majlender, 2004; Fullér *et al.*, 2012; Salahshour and Allahviranloo, 2013; Moloudzadeh *et al.*, 2013; Wang *et al.*, 2014; Salmasnia *et al.*, 2015; Semwal *et al.*, 2015; Semwal *et al.*, 2016; Singha *et al.*, 2016) where some parameters *e.g.* demand and raw material price are uncertain. These uncertainties are reflected utilizing two different approaches;

1. Fuzzy numbers and possibilistic uncertainty concept which are suitable for the uncertain environment of the carton box production company. To cope with uncertainty of the introduced mathematical formulation, a possibilistic approach is applied to convert the fuzzy formulation to a crisp model. To tackle the multi-criteria crisp formulation, a new multi-objective solution approach is

proposed to solve the problem in comparison to four multi-objective optimization approaches such as LH, TH, So, and ABS methods (Lai and Hwang, 1993; Selim and Ozkarahan, 2008; Torabi and Hassini, 2008; Alavidoost *et al.*, 2016) of the literature. Computational experiments and sensitivity analysis which performed on real numerical data given by study case, shows the superior performance of the proposed approach comparing to the others.

2. Fuzzy variables and necessity chance-constrained modeling approach which is another suitable match for dealing with the ambiguities of the problem. Considering the mentioned factors, the problem is modeled using fuzzy variables incorporated in a chance-constrained multi-objective formulation for which the pareto optimal solution is determined.

In the next chapters, all mentioned cases are discussed and investigated.

Chapter 3

CUTTING PROBLEM AND ITS SPECIFICATIONS

In this study, the production planning and supplier-material selection as two of the most important problems in box production industries are investigated. The key element for solving the problems is to deal with a cutting problem which refers to the problem of dividing a usually large piece of the rectangular raw materials into smaller pieces for producing various products. The cutting problems are NP-hard problems which means they are difficult to solve in large scales. Therefore, over the past few years a numerous researches are conducted offering good solutions for these problems. In the present study, considering the complexity of the problem and the problem's environment, a model reflecting the nature of the problem is proposed and a new production planning-column generating algorithm is suggested to solve it. Utilizing the proposed solution approach significantly reduces the material wastage and surplus items. Furthermore, to evaluate the efficiency and usefulness of the proposed method, a specific application of it is tested through a case study.

3.1 Problem Description

In this research, the products are carton boxes of various sizes according to the customer's demands. These carton boxes must meet accurate specifications regarding to their material types and dimensions congruent with the customer's requested specifications. The carton boxes are produced from raw sheets of carton provided by the suppliers of the companies in various predefined sizes. The suppliers can supply the raw sheets in specific standardized sizes. Dealing with this problem, a mathematical formulation is proposed representing the characteristics of the problem and a solution is suggested built upon a modified pattern generating approach. More details about the problem are as follows:

1. In each planning horizon, the customer orders a specific number of boxes;
2. Several sizes of the raw materials are available at each supplier which is known for the companies.
3. The number of deliverable products is easily determinable for the company, if and only if, a specific raw material is assigned to produce a specific product.
4. There exists more than one suitable candidate raw material for producing one or more products.
5. The raw material procured by the companies are distinguished and separated based on their dimension and the combination of the material which is used for building them.
6. Each specific size of the raw material on which a cutting patterns is applied, generates a certain amount of waste. This wastage is dependent on the employed production strategy for assigning the products to the raw material.
7. Each company may have its own individual policies for selecting the measures of the purchased raw materials.

Resembling any other industry, the profitability of the business is the most important concern of the companies. Therefore, nearly all companies in this business are interested in achieving the following objectives;

a) **Reducing the wasted material and their related costs.**
b) **Satisfying the received demand with efficient use of raw material:** In these industries, a high variant of raw material is confusing, therefore, determining the

accurate dimension for the raw materials is crucial due to the wastage minimization purpose. Since almost all companies have a huge variety of products, utilizing dedicated raw materials with correct dimensions for producing a product corresponds to the minimum possible waste theoretically. However, this one to one approach is almost impossible in practice for the following reasons; firstly, supplying the raw material is restricted to some limited specified dimensions and secondly, it corresponds to procuring a vast variety of raw materials in different quantities which is not possible due to inventory related restrictions. Hence, the companies need to reduce the size of their problem to have a standard manufacturing system with minimum incompatibilities. The solution is limiting the variety raw material such that the production capabilities are not reduced.

c) **Reducing the amount of inventory and surplus products:** Fundamentally, there exist two types of inventories at the companies; the finished products and raw materials. The extra inventory of the finished products (surplus) are quite likely to remain useless for a long period of time due to the uncertain ordering style of the customers. In addition, the inventories are too fragile against shrinkage, fire burn, and similar hazards in this industry which leaves the companies always in at the risk of inventory loss. On the other hand, taking the required measures to encounter these risks are extremely costly. Consequently, the companies choose to lower their risks by keeping their inventory at the lowest possible level.

Determining the raw material's appropriate dimensions, correct purchasing quantity, and assigning them to the products properly are the most important fundamentals to fulfill the objectives of the companies. The mentioned requirements are the decision variables of a production planning problem and a subcategory of 2DCPs addresses as bin packing problem / strip packing problem in the literature. The proposed algorithm of this study is designed to deal with these problems. The method is extendable to any other box production company as well as similar industries with minor tailoring. In order to evaluating the efficiency of the proposed method, it is implemented in a specific box production company as a case study. In the next section, the specification of the case study is discussed.

The existing material selection approach- As formerly mentioned, the total available variant of the objects is 256 (8 predetermined measure of widths times 32 various measures of lengths). In Addition, no specific limitation is associated with the purchasing quantity of each object. Traditionally, the company utilizes a cutting software to determine the useable objects and their quantity to satisfy the demands. The company's software presents a production scheme for producing the required number of items of each cluster for a certain planning horizon. The software's input is the list of all applicable objects, the list of items to be produced and the demands of the items. A sample of the software inputs for cluster C1-3 is provided in tables 2 and 3. The output proposal is a suitable production plan in which the valid objects, the pattern as per which the objects should be cut and their required usage frequency is illustrated. Using the software's given data the company may decide which raw material to buy, at what quantity and for which items. This method is easy to use and effective. However, it is not the best applicable production method. Solving the problem by the traditional method corresponds to large surplus items as well as a none optimal material consumption.

Table 2. A sample of applicable objects for cluster C1-3

#	Label	L	W
1	C1-3/45*90	45	90
2	C1-3/45*100	45	100
3	C1-3/45*110	45	110
⋮	⋮	⋮	⋮
254	C1-3/200*150	200	150
255	C1-3/200*160	200	160
256	C1-3/200*180	200	180

Table 3. Average of yearly demand for items in cluster C1-3

Items	A.Y.D*	Items	A.Y.D	Items	A.Y.D	Items	A.Y.D	Items	A.Y.D
G1	21067	G8	17707	G15	46582	G22	49693	G29	73472
G2	37447	G9	37322	G16	67454	G23	35836	G30	54195
G3	86251	G10	41298	G17	81630	G24	6968	G31	71252
G4	19975	G11	17313	G18	20762	G25	47915	G32	74498
G5	21930	G12	59716	G19	16161	G26	63872	G33	18882
G6	35814	G13	54023	G20	23776	G27	33588	G34	80607
G7	55257	G14	51508	G21	60578	G28	34823	G35	50558

* Average Yearly Demand.

Associated with each combination of the pattern applied on the object (*which will be addressed simply as patterns shortly*) is a material cost in terms of consumed material. Using our proposed method, the aim of this study is to minimize this material usage. To have a measure for evaluating the performance of our proposed method, a comparison is made between the outcome of the company's traditional method and our proposed method. To make this comparison, the average of yearly demand for the last 7 years of the company is calculated and used as the demand of the items. All calculations belong to cluster of C1-3. In the following, the deterministic formulation of the problem, the alternative form of the problem, and the steps of this procedure are illustrated.

3.2 Deterministic Formulation

The company's problem is comprised of two parts; the production planning problem and a material selection problem. In this section, a deterministic representation of the problem is proposed and explained. The mathematical formulation of the above-mentioned problem uses the following notations:

i: index for the products

j: index for the available objects

m: the number of the items in each cluster

n: the number of the available objects

M: a large positive value

d_i: demand for item i in a cluster

$\boldsymbol{\mathcal{P}}$: The set patterns satisfying the minimum acceptable waste condition, $\boldsymbol{\mathcal{P}} = \{1,2,\dots,p\}$

g_{ipj}: The number of extractable item i from object j if pattern p is applied

c_{jp}: Unit cost of object j having pattern p applied on it

x_{pj}: The frequency that pattern p is applied on object j

z_j: Decision variable for using object j

The following model is proposed:

$$Obj.function\ 1: min \sum_{j=1}^{n} z_j \tag{3.1}$$

$$Obj.function\ 2: min \sum_{j=1}^{n} \sum_{p=1}^{P} c_{jp} x_{pj} \tag{3.2}$$

$Subject\ to$:

$$\forall i: \sum_{j=1}^{n} \sum_{p=1}^{P} g_{ipj} x_{pj} \geq d_i \quad (3.3)$$

$$\forall p: x_{pj} \leq M z_j \quad (3.4)$$

$$x_{pj} \geq 0, integer \quad (3.5)$$

$$z_j = 1: if\ raw\ material\ j\ is\ used, 0: otherwise \quad (3.6)$$

The description of the model is as follows; the objective function (3.1) minimizes the variety of objects (i.e. variety of the raw materials) that should be used in the production procedure. Objective function (3.2) minimizes the procurement cost through optimizing the usage frequency of the object-pattern combination. At the same time, objective function (3.2) minimizes the surplus amount through justifying the purchased material at the required level. Constraint (3.3) guarantees that the production quantity satisfies the demand for each item. Constraint (3.4) denotes that there is no limitation with providing the required number of objects. Finally, the constraints (3.5) and (3.6) define the nature of the variables.

The alternative problem- Depending on the problem's environment, various methodologies and solution approaches might be applicable. Reducing the complexity of the problem in this study, an alternative point of view is employed to convert the mentioned problem into a simpler cutting stock optimization problem. To solve the new version of the problem, a modified pattern generating methodology is utilized considering the problem owner's objectives. This method determines appropriate objects, the right purchase quantity of them, and the suitable patterns by which they should be cut. The procedure proposed by this method is applied on each cluster of products and guarantees that all items are produced in that cluster. The first step in approaching this problem using the alternative approach is categorizing the products.

As mentioned previously, the products are divided into 6 different clusters: C2-5, C1-5, Li-5, C2-3, C1-3 and Li-3 based on their material types. Therefore, each cluster is a set of products sharing the same property for the raw material, however with different dimensions.

3.3 The Proposed Algorithm

Solving the alternative problem proposed in this research corresponds to the procedure of finding the appropriate practice for dividing the objects into item(s). The key element for being successful at this task is to find the appropriate combination of the object and cutting pattern such that the cost and wastage of the object as well as surplus items are minimized. The proposed method for solving the problem in this study determines the most suitable cutting patterns to be applied on the objects to satisfy the mentioned objectives. The algorithm is comprised of the following steps:

a) **Eliminating the cumbering objects:** As previously mentioned, 8 possible widths and 32 predetermined lengths are purchasable for the objects (totally 256 variant of objects). Recalling the definition of the length and width in an object, it becomes evident that if an object is to be used for producing an item, its length (L) and width (W) must be larger than the spread length (l) and width (w) of the item respectively. This restriction disqualifies all object which are not coincident with this requirement (i.e. for an item with l= 50 an object with the dimension of 50*90 is allowed but 45*90 is not allowed). For each cluster, the objects which are not capable of producing at least one item should be taken out.

b) **Determining the cutting patterns:** In this step, an initial number of at least 1000 applicable patterns is required to be applied on the available objects for producing the items in each cluster. Hence, different cutting style is investigated for all remaining valid objects. After eliminating the invalid patterns – respecting the direction of the length and width from which the items should be extracted – the outcomes are neat patterns each each of which capable of producing either a single item or multiple items. These patterns are patched into the objects and create a separate production material. In the following, the combination of the object-pattern is simply referred as pattern as mentioned previously.

c) **The cost minimization problem:** The objective of this step is to solve a production planning "cost minimization problem" for determining the best usage frequency of the patterns (*the result of this step should be compared with the cost of the traditional method*). Since the object and patterns are now a merged concept the following modifications must be applied before proceeding to the cost minimization problem:

i: index for the number of items in a cluster

j: index for the number of available patterns

A_j: the vector of a pattern j

a_{ij}: the number of item i in the pattern j

c_j: the material consumption (i.e. cost) associated with pattern j if utilized

x_j: the usage frequency of pattern j

d_i: the demand of item i

The mathematical model is the following:

$$min \sum_j c_j x_j \qquad (3.7)$$

$$s.t.$$

$$\sum_j a_{ij} x_j \geq d_i \qquad (3.8)$$

$$x_j \geq 0 \qquad (3.9)$$

$$x_j = integer \qquad (3.10)$$

The Relaxed LP model (3.7)–(3.9)

The objective function (3.7) minimizes the material usage associated with a pattern. The constraint (3.8) guarantees that the demand of item i is satisfied. Constraint (3.9) and (3.10) are the technical constraint proportional with the nature of the problem. After sensitivity analysis of this problem, the dual variables of the relaxed LP associated with each item will be determined for the column generating step.

d) **The Column generating problem**

In this step, the dual variables obtained from the relaxed LP model are utilized to generate the improving patterns through improving the old correspondent patterns:

$\underline{U}$: The vector of the obtained dual variables (shadow prices)

$\underline{y}$: The vector of the improving pattern (*i.e.* the new column)

y_{ii}: The number of product i in the improving pattern

s_i: The area of item i

c_y: The area of pattern $\underline{y}$

The following condition must be satisfied for each $\underline{y}$:

$\underline{U^T y} > c_y$

$\underline{s.t.}$

$\sum_i s_i\, y_i \leq c_y$

$\underline{y} \geq \underline{0}\,, integer$

The given problem can be formulated as the following maximization problem for each $\underline{y}$:

$$max \sum_i u_i y_i \qquad (3.11)$$

Subject to

$$\sum_i s_i\, y_i \leq \mathrm{c}y \qquad (3.12)$$

$$y_i \geq 0 \text{ and integer} \qquad (3.13)$$

Solving the maximization problem provides a new cutting pattern. However, before this pattern can be released it must be validated. The validation procedure means that it should be determined whether the obtained pattern can produce the items to which are assigned or not. The following algorithm is designed for this purpose:

i. Put L_i and W_i as the length and the width of pattern *i* respectively;

ii. $\mathrm{L}^{\mathrm{arrang}}$ = The total sum of the length of the items to be extracted from pattern *i*;

iii. $\mathrm{W}^{\mathrm{arrang}}$ = The total sum of the widths of the items to be extracted from pattern *I*;

iv. If $\mathrm{L}^{\mathrm{arrang}} \leq L$ & $\mathrm{W}^{\mathrm{arrang}} \leq W$, then the pattern is valid; otherwise it is invalid.

If the pattern becomes valid, then it is releasable and should be added to the main production planning problem.

Stopping criteria- steps (i) to (iv) must be applied constantly until column generating approach fails to produce a new unique column or the cost minimization problem represents an acceptable level of improvement in comparison with the traditional method.

3.4 Case Study Implementation

In this section, a case study is conducted to evaluate the performance of the proposed method. Before proceeding to the solution, some initializations are required as formerly mentioned. The first step of the initialization is categorizing the product's data into the mentioned clusters. Tables 4 and 5 represent a sample of uncategorized and categorized data, respectively.

Table 4. sample of uncategorized data

Id Code	Dimension			Id Code	Dimension		
G1	49.5	24.5	11	G36	42	8.6	0
G143	7.5	3.5	10.9	G37	98.5	15	0
G144	5	4	58	G38	79.5	15	0
G145	6.7	6	66.5	G58	135	10	0
G2	50.5	25.4	22.8	G59	28.5	10	0
G216	50	7	0	G60	113	11	0
G217	25	7	0	G71	39	11	0
G218	80	7.5	0	G72	101	11	0
G3	61	33.7	25	G73	14.4	13.5	9.5

Table 5. sample of categorized data (3 items / cluster are shown)

	Id Code	Dimension			spread dimension	
					Length	Width
C1-3	G1	49.5	24.5	11	152	35.5
	G2	50.5	25.4	22.8	155.8	48.2
	G3	61	33.7	25	193.4	58.7
C2-3	G36	42	8.6	0	42	8.6
	G37	98.5	15	0	98.5	15
	G38	79.5	15	0	79.5	15
C1-5	G58	135	10	0	135	10
	G59	28.5	10	0	28.5	10
	G60	113	11	0	113	11
C2-5	G71	39	11	0	39	11
	G72	101	11	0	101	11
	G73	14.4	13.5	9.5	62.8	23
Li-3	G143	7.5	3.5	10.9	26	14.4
	G144	5	4	58	22	62
	G145	6.7	6	66.5	29.4	72.5
Li-5	G216	50	7	0	50	7
	G217	25	7	0	25	7
	G218	80	7.5	0	80	7.5

Having the products categorized into their related cluster, the next step is to solve the production planning and material selection problem for each cluster. In this research, cluster C1-3 is investigated. The key elements to be determined for solving the mentioned problems, is determining the suitable objects, suitable pattern to cut the objects, and the right usage frequency of the patterns (i.e. right quantity of raw material) such that the demand of all items in the cluster is satisfied. In the following, the steps of the algorithm are applied on the case study and the results are illustrated.

3.4.1 Eliminating the Cumbering Objects

For this step, the following algorithm must run in each cluster:

1- Determine all possible objects (totally 256 variant)
2- Determine the length and width of each object respecting the direction of the flute layer (i.e. L & W)

3- Determine the spread length (l) and width (w) of all items in a cluster according to equations (1) and (2).

4- For each item: if $\boldsymbol{L} \geq \boldsymbol{l}$ & $\boldsymbol{W} \geq \boldsymbol{w} \rightarrow$ the object is valid, otherwise, it is invalid

5- Form the matrix of usable objects:

Table 6. The matrix of useable objects

		Objects (L, W)		
		(45,90)	...	***(200,180)***
Items (l, w)	***(152,35.5)***	0	...	1
	⋮	⋮	⋱	⋮
	(77.4,26.5)	0	...	1
	Total	***4***	***...***	***35***

For C1-3, all objects are usable according to table 6. The next step is to determine the cutting patterns for the valid objects.

3.4.2 Determining the Cutting Patterns

Various methods can be used to determine the cutting patterns for the objects. In this study, the cutting software of the company is used to produce the initial patterns. After eliminating the repetitive patterns, the most important issue regarding the patterns is their validity. The validity of the patterns is coincident with the validity rule mentioned in preliminaries. In this study after validating the patterns, 933 neat patterns are remained which are used in the cost minimization problem.

Table 7. The list of obtained patterns

		Patterns(P_j)				
		P_1	P_2	...	P_{932}	P_{933}
Items (G_i)	G_1	5	0	...	0	0
	G_2	0	3		0	0
	⋮	⋮	⋮	⋱	⋮	⋮
	G_{34}	0	0	...	0	0
	G_{35}	0	0	...	0	0
Total producible items by P_j		***5***	***3***	***...***	***3***	***2***

3.4.3 Solving the Production Planning Problem

For this step, the surface of the objects which are used for producing the items, their usage frequency and their items productivity must be determined. These measures are used in the cost minimization problem. Solving this problem apprehend the cost associated with the applied production scheme. At this point, it should be noted that the relaxed version of the problem must be solved in order to obtain the shadow prices which are used for the column generating part.

Table 8. The solved Production planning problem

		Patterns(Pi)						
		P_1	P_2	...	P_{932}	P_{933}	***Demand***	***Shadow Price***
Items G_j	G_1	5	0	...	0	0	*21067*	207,7381
	G_2	0	3		0	0	*37447*	625
	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋮
	G_{34}	0	0	...	0	0	*80607*	7125
	G_{35}	0	0	...	0	0	*50558*	2266.66
	x_j	***0***	***0***	***...***	***0***	***0***	***Cost***	***5721880416,823***

3.4.4 Column Generating

In this step using the concept of the shadow prices obtained from the previous step, the improving patterns are determined. similar to step 2, these patterns must be validated through the same procedure. The outcome of this procedure is 144 new patterns out of which 8 are valid. The result is illustrated in table 9.

Table 9. The table of improving patterns

		Improving Patterns													
		P_{934}	**…**	**P_{961}**	**P_{962}**	**…**	**P_{962}**	**P_{962}**	**P_{962}**	**…**	**P_{1042}**	**P_{1043}**	**P_{1044}**	**…**	**P_{1048}**
Items G_j	G_1	0	⋮	0	0	⋮	0	0	0	⋮	0	0	0	⋮	0
	G_2	0	⋮	0	0	⋮	0	0	0	⋮	0	0	0	⋮	0
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	G_{34}	0	⋮	0	0	⋮	0	0	0	⋮	0	0	0	⋮	0
	G_{35}	1	⋮	0	0	⋮	2	0	0	⋮	0	0	0	⋮	1
Productivity		9	…	2	2	…	2	2	2	…	1	1	1	…	6
Validity		N	N	Y	Y	N	Y	Y	Y	N	Y	Y	Y	N	N

Stopping criteria- The stopping criteria of this case occurred when the 8 final valid patterns were added to the "production planning problem". After this step, the pattern generating procedure failed to produce new patterns. Hence, the answer of the last step of the solved production planning problem was considered as the final result which is **5661438840** Persian Rials (which is almost equal to **$**1581408). The total cost of the plan apprehended by the company's existing (traditional) method was **9129546536** Perisan Rials (≅ **$**2550153). Comparing these two values represents a meaningful improvement in cost reduction. It should be noted that quite expectedly, the impovement in production cost was not evenly distributed among all iterations of the runs. The largest improvement occurred in the initial itterations and steadily continued until the procedure was stopped. Since the amount of production is equal to the demand, applying the proposed algorithm results ineliminating the surplus amount for C1-3.

Chapter 4

SUPPLIER-MATERIAL SELECTION UNDER UNCERTAINTY

A critical problem in carton box production industries arises when size, amount and suppliers of the raw materials are affected by an uncertain competitive environment from price point of view. In such cases, selecting the correct size and quantity of the raw material as well as right suppliers are the crucial elements for a competent production. This chapter of this study introduces a multi-criteria mixed integer formulation to select the most efficient size, amount and supplier of the raw material to minimize the cost, wastage, and surplus of the production simultaneously. Demand of the boxes and price of raw sheets are considered as fuzzy numbers reflecting the uncertainty of the market. Nevertheless, using the fuzzy variables are one of the most appropriate method for reflecting the uncertain nature of this problem; to cope with a fuzzy model is not an easy task and requires special technics. In this research two different approaches are employed for solving the fuzzy model;

1- A possibilistic approach which converts the fuzzy formulation to a crisp model for solving which a new multi-objective solution approach is proposed. The solution is then compared to LH, TH, So, and ABS methods which are multi-objective optimization approaches.

2- Using the concepts of necessity-based chance-constrained modelling approach to convert the fuzzy version of the problem to a crisp form. Then a new hybrid form of the fuzzy programming approach is proposed to solve the obtained multi-objective crisp problem effectively.

In both cases, computational experiments and sensitivity analysis performed on the real numerical data of the case study reveals the superior performance of the proposed approach comparing to the other methods in the literature.

4.1 Problem Characteristics Under Uncertainty

As described previously, the case study in this research, produces carton boxes in various dimensions proportional to the its customer's desires. The required raw materials for producing these boxes are raw sheets of the carton which are supplied by the suppliers of the company in requested dimensions. Purchasing of the raw materials occurs in specific planning horizons and all sources can supply all sizes of the raw sheets. The updated detail about the problem including the uncertain parameters are as follows:

1. The demand of the customer in each planning scheme arrives in a specific amount.
2. The suppliers are competitive; they offer competitive prices for their raw materials.
3. Each size of the raw sheet can produce a known quantity of a box type.
4. For producing a certain box, at least one candidate of raw material must be available.
5. Producing a carton box from a raw sheet corresponds to a certain amount of generated wastage. This amount – which dependent to the "raw sheet – carton box" assignment manner – is known and varies for each case.

6. There is a competition amongst the suppliers for supplying the raw sheets to the company. The suppliers, therefore, offer a discount for each size of the raw sheets based on the received order quantity. The supplier who offers a better price is more preferred.
7. The suppliers' discount policy is based on quantities. It means if the order quantity exceeds a certain level the supplier considers a price discount, otherwise, there would be no discount.
8. Minimizing the variation of the raw materials in terms of dimensions and increasing the volume of each type's purchase, positively contributes to profit maximization of the company (supplier discount policy).

Considering the characteristics of the problem, the objectives of the company are as follows:

a) **Minimizing the waste of material-** The wastage can be minimized through determining a proper dimension for the raw material and assigning them to a proper set of products in the production phase.

b) **Minimizing the cost of raw materials-** The company must decide on a proper purchasing quantity such that the total payment for the ordered raw sheets is the minimum amount in each planning scheme. The suppliers' quantity discounts play a critical role in this decision.

c) **Minimizing the Surplus quantity of the boxes-** The produced quantity of each item must be equal to or greater than the demand of that item, having a certain amount of surplus is inevitable. In this problem, the extra products are deposited in the inventory to be used in the next scheduling horizon. These extra boxes are considered as surplus. Due to the technical issues, the company desires to minimize the total surpluses of all types of boxes.

The price of the raw sheets, break points of the discounts and demand of the boxes are not controllable by the company due to having a high degree of uncertainty.

4.2 Reflecting the Uncertainty of the Problem Using Fuzzy Sets

As a general principle, the uncertainty of mathematical models is presented by, (1) flexibility of objective function and/or flexibility of constraints, and/or, (2) uncertain data. Flexibility occurs when targets of objective function and constraints are flexible towards the changes. In such cases, utilizing the fuzzy sets are an appropriate way of reflecting the uncertainty of the model (Dubois *et al.* (2003)).

In this study, the ambiguous nature of these factors is imitated by trapezoidal fuzzy values. The value of the stated parameters may adopt any value from the domain of its corresponding fuzzy variable considering its membership function. The problem is modeled as a fuzzy multi-objective functions with fuzzy constraints. On the other hand, uncertainty of data in a problem may be of two types: (1) data with random values which means the values of data possesses a random nature. In this case the stochastic programming techniques is a suitable solution approach for the problem (Listes and Dekker, 2005; El-Sayed *et al.*, 2010). (2) epistemic uncertainty which arises when there is no enough knowledge about the values of parameters. As mentioned formerly, there are numerous applicable methods for solving such problems. In the next sections, the two methods which are employed for solving the problem in this research are introduced and discussed.

4.3 Fuzzy Mathematical Formulation

In this section, a fuzzy mathematical formulation is presented based on the characteristics and assumptions of the problem. Please note that in the notations of the model, the symbols with a tilde indicate the uncertainty of the parameter:

Indices:

- i index used for types of box
- j index used for sizes of raw sheet
- k index used for suppliers

Parameters:

- I The number of box types to be produced in a planning horizon
- J The number of sizes of raw sheet presented by suppliers
- K The number of suppliers
- M A large positive value
- $\widetilde{d_i}$ The demand of box type *i* shown by unit of quantity
- a_{ij} The number of box type *i* that can be cut from raw sheet size *j*
- w_{ij} The waste amount remained after cutting box type *i* from raw sheet *j*
- $\widetilde{g_{jk}}$ The break point for ordering raw sheet size *j* offered by supplier *k*. For orders, more than this amount discounted price will be applied for the order (all unit discount)
- $\widetilde{c_{jk}^1}$ The normal unit prices for raw sheet size *j* by supplier *k*
- $\widetilde{c_{jk}^2}$ The discounted unit prices for raw sheet size *j* by supplier *k*

Decision variables

- T_{jk}^1, T_{jk}^2 binary variables indicating that whether discount is applied for sheet type *j* by supplier *k*. Normal price is applied if $T_{jk}^1 = 1$ and discounted price is applied if $T_{jk}^2 = 1$.

Y_{ijk} number of raw sheets size *j* which is ordered to supplier *k* for producing box type *i*,

Z_{jk}^1, Z_{jk}^2 non-negative continuous values to be used instead of non-linear terms $T_{jk}^1\left(\sum_{i=1}^{I} Y_{ijk}\right), T_{jk}^2\left(\sum_{i=1}^{I} Y_{ijk}\right)$ respectively.

The non-linear model:

$$min\ OF_1 = \sum_{i=1}^{I}\sum_{j=1}^{j}\sum_{k=1}^{k}(w_{ij}\, Y_{ijk}) \tag{4.1}$$

$$min\ OF_2 = \sum_{j=1}^{J}\sum_{k=1}^{K}\left(\widetilde{c_{jk}^1} T_{jk}^1 \sum_{i=1}^{I} Y_{ijk} + \widetilde{c_{jk}^2} T_{jk}^2 \sum_{i=1}^{I} Y_{ijk}\right) \tag{4.2}$$

$$min\ OF_3 = \sum_{i=1}^{I}\left(\left(\sum_{j=1}^{J}\sum_{k=1}^{K} a_{ij} Y_{ijk}\right) - \widetilde{d_i}\right) \tag{4.3}$$

$Subject\ to$

$$\sum_{j=1}^{J}\sum_{k=1}^{K} a_{ij} Y_{ijk} \geq \widetilde{d_i} \qquad \forall i \tag{4.4}$$

$$T_{jk}^1 + T_{jk}^2 \leq 1 \qquad \forall i,k \tag{4.5}$$

$$\sum_{i=1}^{I} Y_{ijk} \geq T_{jk}^1 \qquad \forall i,k \tag{4.6}$$

$$\sum_{i=1}^{I} Y_{ijk} \leq \widetilde{g_{jk}} T_{jk}^1 + M T_{jk}^2 \qquad \forall i,k \tag{4.7}$$

$$\sum_{i=1}^{I} Y_{ijk} \geq (\widetilde{g_{jk}} + 1) T_{jk}^2 \qquad \forall i,k \tag{4.8}$$

$$T_{jk}^1, T_{jk}^2 \in \{0, 1\} \qquad \forall i,k \tag{4.9}$$

$$Y_{ijk} \geq 0, integer \qquad \forall i,k \tag{4.10}$$

The detailed information about the model is as follows;

The first objective function minimizes the total raw sheet waste which is generated in production process. The second objective function computes and minimizes the total material cost, considering the price of the raw sheets. The all unit discount policy introduced in Section 2 is deliberated in this objective function such that at most one of the nonlinear terms $\widetilde{c_{jk}^{1}} T_{jk}^{1} \sum_{i=1}^{I} Y_{ijk}$ and $\widetilde{c_{jk}^{2}} T_{jk}^{2} \sum_{i=1}^{I} Y_{ijk}$ take positive value. The third objective function tends to minimize the surplus. (suppose the demand is 100 and 10 raw sheets are selected for production, each raw sheet can produce at most 11 boxes, the surplus in this case is 10 (11) – 100 = 10). Constraint set (4.4) fulfils the demand of the product type, constraint (4.5) to (4.8) assure that for ordering size *j* of the raw material from contractor *k*, only one of the following may occur;

a) $\sum_{i=1}^{I} Y_{ijk} \leq \widetilde{g_{jk}}$ and T_{jk}^{1} = 1, where the normal price in objective function (2) is considered by supplier *k*.
b) $\sum_{i=1}^{I} Y_{ijk} \geq \widetilde{g_{jk}}$ +1 and T_{jk}^{2} = 1, where the discounted price in objective function (2) is considered by supplier *k*.
c) $\sum_{i=1}^{I} Y_{ijk} = 0$, T_{jk}^{1} = 1 and T_{jk}^{2} = 0, where the raw sheet *j* is not bought from supplier *k*.

Lastly, constraint (4.9) and (4.10) define the nature of the variables.

In the proposed model, T_{jk}^{1} $(\sum_{i=1}^{I} Y_{ijk})$ and T_{jk}^{2} $(\sum_{i=1}^{I} Y_{ijk})$ are nonlinear terms which make the whole model nonlinear. In order to cope with the nonlinearity of the model, the mentioned variablels are replaced by Z_{jk}^{1}, Z_{jk}^{2}. The following constraints are added to the model to guarantee that Z_{jk}^{1} and Z_{jk}^{2} are equivalent to the nonlinear value T_{jk}^{1} $(\sum_{i=1}^{I} Y_{ijk})$ and T_{jk}^{2} $(\sum_{i=1}^{I} Y_{ijk})$.

$$Z_{jk}^{1} \leq \mathrm{M}T_{jk}^{1} \qquad \forall i,k \qquad (4.11)$$

$$Z_{jk}^{1} \leq \sum_{i=1}^{I} Y_{ijk} \qquad \forall i,k \qquad (4.12)$$

$$Z_{jk}^{1} \geq \left(\sum_{i=1}^{I} Y_{ijk}\right) - M(1 - T_{jk}^{1}) \qquad \forall i,k \qquad (4.13)$$

$$Z_{jk}^{1} \geq 0 \qquad \forall i,k \qquad (4.14)$$

Therefore, the nonlinear model (4.1) - (4.10) is linearized as the given mixed integer linear model (MILP) as follows;

$$min\, OF_1 = \sum_{i=1}^{I} \sum_{j=1}^{j} \sum_{k=1}^{k} \left(w_{ij} Y_{ijk}\right) \qquad (4.15)$$

$$min\, OF_2 = \sum_{j=1}^{J} \sum_{k=1}^{K} \left(\widetilde{c_{jk}^{1}} Z_{jk}^{1} + \widetilde{c_{jk}^{2}} Z_{jk}^{2}\right) \qquad (4.16)$$

$$min\, OF_3 = \sum_{i=1}^{I} \left(\left(\sum_{j=1}^{J} \sum_{k=1}^{K} a_{ij} Y_{ijk}\right) - \widetilde{d_j}\right) \qquad (4.17)$$

$Subject\ to$

$$\sum_{j=1}^{J} \sum_{k=1}^{K} a_{ij} Y_{ijk} \geq \widetilde{d_j} \qquad \forall i \qquad (4.18)$$

$$T_{jk}^{1} + T_{jk}^{2} \leq 1 \qquad \forall i,k \qquad (4.19)$$

$$\textstyle\sum_{i=1}^{I} Y_{ijk} \geq T_{jk}^{1} \qquad \forall i,k \qquad (4.20)$$

$$\sum_{i=1}^{I} Y_{ijk} \leq \widetilde{g_{jk}} T_{jk}^{1} + MT_{jk}^{2} \qquad \forall i,k \qquad (4.21)$$

$$\sum_{i=1}^{I} Y_{ijk} \geq (\widetilde{g_{jk}} + 1)T_{jk}^2 \qquad \forall i,k \qquad (4.22)$$

$$Z_{jk}^1 \leq \mathrm{M}T_{jk}^1 \qquad \forall i,k \qquad (4.23)$$

$$Z_{jk}^1 \leq \sum_{i=1}^{I} Y_{ijk} \qquad \forall i,k \qquad (4.24)$$

$$Z_{jk}^1 \geq \left(\sum_{i=1}^{I} Y_{ijk}\right) - M(1 - T_{jk}^1) \qquad \forall i,k \qquad (4.25)$$

$$Z_{jk}^2 \leq \mathrm{M}T_{jk}^2 \qquad \forall i,k \qquad (4.26)$$

$$Z_{jk}^2 \leq \sum_{i=1}^{I} Y_{ijk} \qquad \forall i,k \qquad (4.27)$$

$$Z_{jk}^2 \geq \left(\sum_{i=1}^{I} Y_{ijk}\right) - M(1 - T_{jk}^2) \qquad \forall i,k \qquad (4.28)$$

$$T_{jk}^1, T_{jk}^2 \in \{0, 1\} \qquad \forall i,k \qquad (4.29)$$

$$Y_{ijk} \geq 0, integer \qquad \forall i,k \qquad (4.30)$$

$$Z_{jk}^1, Z_{jk}^2 \geq 0 \qquad \forall i,k \qquad (4.31)$$

4.4 The Possibilistic Programming Approach

In possibilistic programming approach each parameter with epistemic uncertainty has a possibility distribution. It means that each uncertain data value occurs according to a possibility degree which is determined by knowledge of experts objectively. According to (Mula *et al.*, 2006), the main advantages of possibilistic programming approach is the followings: (1) it is easy to compute, (2) it can employ both triangular and trapezoidal fuzzy numbers for uncertain data, (3) decision makers can obtain the optimal solutions using different feasibility degrees of fuzzy constraints, and (4) it uses the strong mathematical concepts like expected interval and expected value of fuzzy members.

As in the proposed fuzzy formulation (4.15) - (4.31) the parameters have epistemic uncertainty in their data, it is suitable to use possibilistic programming to cope with the uncertainty of the model. The model has two characteristics to be considered when introducing the solution approach: (1) its fuzziness, and (2) the multi-objectivity of the model. Therefore, the solution approach contains two phases. First, converting the model to a crisp equivalent. Then, obtaining a good Pareto optimal solution for the crisp version of the model. To perform the first phase, the fuzzy model is converted in to an equivalent auxiliary crisp model applying an efficient possibilistic method through hybridizing the novel methods of Jimenez *et al.* (2007) and Parra *et al.* (2005). Then, the second phase is done using some effective multi-objective solution approaches of the literature of multi-objective optimization. These stages are explained in detail in the following sub-sections.

4.4.1 Stage 1: The Equivalent Auxiliary Crisp Model

In this section, first, the concepts and definition of possibilistic method is mentioned and then, the method is used to convert the fuzzy formulation (4.15) - (4.31) to a crisp optimization model.

Let $\tilde{c} = (c^1, c^2, c^3, c^4)$ be a trapezoidal fuzzy number whose membership function is defined as follows;

$$\mu_c(x) = \begin{cases} \dfrac{x - c^1}{c^2 - c^1} & c^1 \leq x \leq c^2 \\ 1 & c^2 \leq x \leq c^3 \\ \dfrac{c^4 - x}{c^4 - c^3} & c^3 \leq x \leq c^4 \\ 0 & Otherwise \end{cases} \quad (4.32)$$

The expected interval (EI) and expected value (EV) of trapezoidal fuzzy number $\tilde{c} = (c^1, c^2, c^3, c^4)$ can be defined as follow (Jimenez *et al.*, 2007):

$$EI(\tilde{c}) = [\, E_1^c, E_2^c \,] = \left[\frac{c^1 + c^2}{2}, \frac{c^3 + c^4}{2}\right] \quad (4.33)$$

$$EV(\tilde{c}) = \frac{E_1^c + E_2^c}{2} = \frac{c^1 + c^2 + c^3 + c^4}{2} \quad (4.34)$$

Definition 1 (Jimenez, 1996). For any pair of fuzzy numbers c and d, the degree which show how c is bigger than d is defined as follow;

$$\mu_M(\tilde{c}, \tilde{d}) = \begin{cases} 0 & E_2^c - E_1^d < 0 \\ \dfrac{E_2^c - E_1^d}{E_2^c - E_1^d - (E_1^c - E_2^d)} & 0 \in [E_1^c - E_2^d, E2_1^c - E_1^d] \\ 1 & E_1^c - E_2^d > 0 \end{cases} \quad (4.35)$$

For the cases that, $\mu_M(\tilde{c}, \tilde{d}) \geq \alpha$, it is said that $\tilde{c}$ is bigger than or equal to $\tilde{d}$ at least in degree of α. This relation is represented by $\tilde{c} \geq_\alpha \tilde{d}$.

Definition 2 (Parra *et al.*, 2005). For a pair of fuzzy numbers like c and d, the numbers are equal in degree of α if the following relationship holds:

$$\frac{\alpha}{2} \leq \mu_M(\tilde{c}, \tilde{d}) \leq 1 - \frac{\alpha}{2} \qquad (4.36)$$

Now, to explain the possibilistic method of converting a fuzzy model to a crisp one, in the next page a general mathematical model with trapezoidal fuzzy parameters is considered.

$$\begin{aligned}
& min\ \tilde{c}^T x \\
& Subject\ to \\
& \widetilde{a_i} x \geq \widetilde{b_i} \qquad i = 1,2,\ldots,l \\
& \widetilde{a_i} x = \widetilde{b_i} \qquad i = l+1,\ldots,m \\
& x \geq 0
\end{aligned} \qquad (4.37)$$

A decision vector $x \in \mathbb{R}^n$ is feasible in degree of α if $min\{\mu_M(\widetilde{a_i}x, \widetilde{b_i}) = \alpha\ , i = 1, \ldots, m\}$ (Jimenez *et al.*, 2007). Therefore, α is considered as feasibility degree of the model which is determined by decision maker (DM). Assigning higher values to α causes a smaller feasible solution space and consequently the optimal solution becomes worse. In the cases that there is more than one conflicting objective functions, DM tries to balance the objective function values in order to find a compromised solution over different levels of α.

Now, According to (35) and (36), the relation $\tilde{a_i} x \geq \tilde{b_i}$ and $\tilde{a_i} x = \tilde{b_i}$ are equivalent to the following equations.

$$\frac{E_2^{a_i x} - E_1^{b_i}}{E_2^{a_i x} - E_1^{b_i} - (E_1^{a_i x} - E_2^{b_i})} \geq \alpha \qquad i = 1,2, \dots, l \qquad (4.38)$$

$$\frac{\alpha}{2} \leq \frac{E_2^{a_i x} - E_1^{b_i}}{E_2^{a_i x} - E_1^{b_i} - \left(E_1^{a_i x} - E_2^{b_i}\right)} \leq 1 - \frac{\alpha}{2} \qquad i = l+1, \dots, m \qquad (4.39)$$

These equations can be written as follows:

$$\left((1-\alpha)E_2^{a_i} + \alpha E_1^{a_i}\right)x \geq \alpha E_2^{b_i} + (1-\alpha)E_1^{b_i} \qquad i = 1,2, \dots, l \qquad (4.40)$$

$$\left((1-\frac{\alpha}{2})E_2^{a_i} + \frac{\alpha}{2}E_1^{a_i}\right)x \geq \frac{\alpha}{2}E_2^{b_i} + (1-\frac{\alpha}{2})E_1^{b_i} \qquad i = l+1, \dots, m \qquad (4.41)$$

$$\left(\frac{\alpha}{2}E_2^{a_i} + (1-\frac{\alpha}{2})E_1^{a_i}\right)x \leq (1-\frac{\alpha}{2})E_2^{b_i} + (\frac{\alpha}{2})E_1^{b_i} \qquad i = l+1, \dots, m \qquad (4.42)$$

Consequently, using the definition of expected interval and expected value of a fuzzy number which explained by equations (4.33) and (4.34), the equivalent crisp parametric model of the model (4.37) is constructed as follows. It is notable that in the objective function the expected value of the fuzzy parameters is to be minimized.

$$min\ [EV(c^T)]x$$

$$Subject\ to$$

$$\left((1-\alpha)E_2^{a_i} + \alpha E_1^{a_i}\right)x \geq \alpha E_2^{b_i} + (1-\alpha)E_1^{b_i} \qquad i = 1,2, \dots, l \qquad (4.43)$$

$$\left((1-\frac{\alpha}{2})E_2^{a_i} + \frac{\alpha}{2}E_1^{a_i}\right)x \geq \frac{\alpha}{2}E_2^{b_i} + (1-\frac{\alpha}{2})E_1^{b_i} \qquad i = l+1, \dots, m$$

$$\left(\frac{\alpha}{2}E_2^{a_i} + (1-\frac{\alpha}{2})E_1^{a_i}\right)x \le (1-\frac{\alpha}{2})E_2^{b_i} + (\frac{\alpha}{2})E_1^{b_i} \qquad i = l+1,\dots,m$$

$$x \ge 0$$

Based on the above-mentioned definitions and formulations, the equivalent auxiliary crisp model of the proposed fuzzy formulation (4.15) - (4.31) using the possibilistic approach is formulated in the following;

$$min\ OF_1 = \sum_{i=1}^{I}\sum_{j=1}^{j}\sum_{k=1}^{k}(w_{ij}\, Y_{ijk}) \tag{4.44}$$

$$min\ OF_2 =$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K}\left(\frac{c^{1.1} + c^{1.2} + c^{1.3} + c^{1.4}}{4} z_{jk}^{1} + \frac{c^{2.1} + c^{2.2} + c^{2.3} + c^{2.4}}{4} z_{jk}^{2}\right) \tag{4.45}$$

$$min\ OF_3 = \sum_{i=1}^{I}\left(\left(\sum_{j=1}^{J}\sum_{k=1}^{K} a_{ij}Y_{ijk}\right) - \frac{d^1 + d^2 + d^3 + d^4}{4}\right) \tag{4.46}$$

$$Subject\ to$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K} a_{ij}Y_{ijk} \ge \alpha\left(\frac{d_i^3 + d_i^4}{2}\right) + (1-\alpha)\left(\frac{d_i^1 + d_i^2}{2}\right) \qquad \forall i \tag{4.47}$$

$$T_{jk}^1 + T_{jk}^2 \le 1 \qquad \forall i,k \tag{4.48}$$

$$\sum_{i=1}^{I} Y_{ijk} \ge T_{jk}^1 \qquad \forall i,k \tag{4.49}$$

$$\sum_{i=1}^{I} Y_{ijk} \le (\alpha\left(\frac{g_i^1 + g_i^2}{2}\right) + (1-\alpha)\left(\frac{g_i^3 + g_i^4}{2}\right))T_{jk}^1 + MT_{jk}^2 \qquad \forall i,k \tag{4.50}$$

$$\sum_{i=1}^{I} Y_{ijk} \geq (\alpha\left(\frac{g_i^3 + g_i^4}{2}\right) + (1-\alpha)\left(\frac{g_i^1 + g_i^2}{2}\right) + 1)T_{jk}^2 \qquad \forall i,k \qquad (4.51)$$

$$Z_{jk}^1 \leq MT_{jk}^1 \qquad \forall i,k \qquad (4.52)$$

$$Z_{jk}^1 \leq \sum_{i=1}^{I} Y_{ijk} \qquad \forall i,k \qquad (4.53)$$

$$Z_{jk}^1 \geq \left(\sum_{i=1}^{I} Y_{ijk}\right) - M(1 - T_{jk}^1) \qquad \forall i,k \qquad (4.54)$$

$$Z_{jk}^2 \leq \mathrm{M}T_{jk}^2 \qquad \forall i,k \qquad (4.55)$$

$$Z_{jk}^2 \leq \sum_{i=1}^{I} Y_{ijk} \qquad \forall i,k \qquad (4.56)$$

$$Z_{jk}^2 \geq \left(\sum_{i=1}^{I} Y_{ijk}\right) - M(1 - T_{jk}^2) \qquad \forall i,k \qquad (4.57)$$

$$T_{jk}^1, T_{jk}^2 \in \{0, 1\} \qquad \forall i,k \qquad (4.58)$$

$$Y_{ijk} \geq 0, integer \qquad \forall i,k \qquad (4.59)$$

$$Z_{jk}^1, Z_{jk}^2 \geq 0 \qquad \forall i,k \qquad (4.60)$$

4.4.2 Stage 2: Multi-Objective Solution Approaches

As the crisp formulation (4.44) - (4.60) is a multi-objective optimization model, its efficient Pareto optimal solutions should be obtained via its single-objective form (Jablonsky, 2007; Hadi-Vencheh *et al.*, 2014; Hadi-Vencheh and Mohamadghasemi, 2015). There are many well-known solution approaches in the literature of multi-objective optimization which can be used for this aim. The most utilized solution approach is max-min approach (Zimmermann, 1978). However, this solution sometimes is not able to introduce efficient solutions for a multi-objective model (Pishvaee and Torabi, 2010). To improve the max-min method and in order to generate efficient solutions for multi-objective models, there are many modifications *e.g.* the methods of Lai and Hwang (1993), Selim and Ozkarahan (2008), Torabi and Hassini (2008), and Alavidoost *et al.* (2016). In this study in addition to all these four methods, a new method is also proposed to solve the crisp formulation (4.44) - (4.60).

4.4.3 The Solution Scheme Using the Previous Approaches

The methods in the literature follow approximately similar structure to convert a multi-objective formulation to a single-objective formulation. For the crisp formulation (4.44)-(4.60), the structure is summarized in the following steps.

Step 1: For each objective function determine an α feasibility degree, and find α – positive and α – negative ideal solutions (${OF_i}^{\alpha-PIS}$ and ${OF_i}^{\alpha-NIS}$). For objective function $i \in \{1,2,3\}$, the ${OF_i}^{\alpha-PIS(NIS)}$ can be obtained by:

$$\begin{aligned} & min(max)OF_i \\ & Subject\ to \qquad\qquad\qquad (4.61) \\ & Constraints\ (47) - (60) \end{aligned}$$

Step 2: Using the results of Step 1 for an objective function *i,* a membership function is determined in which $\mu_i(x)$, calculates the satisfaction level (degree) of i^{th} objective function as follows:

$$\mu_M(x) = \begin{cases} 0 & OF_i < OF_i^{\alpha-PIS} \\ \dfrac{OF_i^{\alpha-NIS} - OF_i}{OF_i^{\alpha-NIS} - OF_i^{\alpha-PIS}} & OF_i^{\alpha-PIS} \le OF_i \le OF_i^{\alpha-NIS} \\ 1 & OF_i > OF_i^{\alpha-NIS} \end{cases} \quad (4.62)$$

Step 3: Using the membership functions obtained from Step 2, each of the above-mentioned multi-objective optimization methods uses the following single-objective models.

Selim and Ozkarahan (2008) (SO):

$$\begin{aligned} & max\, \lambda(x) = \gamma\lambda_0 + (1-\gamma)\sum_{i=1}^{3} \theta_i\, \lambda_i \\ & Subject\ to \\ & Constraints\ (47) - (60) \qquad\qquad (4.63) \\ & \lambda_0 + \lambda_i \le \mu_i(x) \qquad i \in \{1,2,3\} \\ & \lambda_0, \lambda_i \in [0,1] \qquad i \in \{1,2,3\} \end{aligned}$$

Where λ_0 the value of minimum satisfaction level, γ and θ_i are the weights determined by decision maker such that $\gamma \in [0,1]$, $\theta_i \in [0,1]$, and $\sum_i^3 \theta_i$=1. Among the objective functions, higher value for θ_i implies more importance for that objective function. In the model (4.63), γ controls the compromise level of the objective functions in order to reach the minimum satisfaction level for the objectives.

Lai and Hwang (1993) (LH):

$$\begin{aligned}
&max\,\lambda(x) \\
&= \lambda_0 + \delta \sum_{i=1}^{3} \theta_i\, \mu_i(x) \\
&Subject\ to \\
&Constraints\ (47) - (60) \\
&\lambda_0 \leq\ \mu_i(x) \qquad\qquad i \in \{1,2,3\} \\
&\lambda_0 \in [0,1]
\end{aligned} \tag{4.64}$$

Where δ is a positive small value.

Torabi and Hassini (2008) (TH):

$$\begin{aligned}
&max\,\lambda(x) = \gamma\lambda_0 + (1-\gamma) \sum_{i=1}^{3} \theta_i\, \mu_i(x) \\
&Subject\ to \\
&Constraints\ (47) - (60) \\
&\lambda_0 \leq\ \mu_i(x) \qquad\qquad i \in \{1,2,3\} \\
&\lambda_0 \in [0,1]
\end{aligned} \tag{4.65}$$

Alavidoost, Babazadeh and Sayyari (2016) (ABS):

$$\begin{aligned}
&max\,\lambda(x) = \lambda_0 + \delta \sum_{i=1}^{3} \theta_i\, \lambda_i \\
&Subject\ to \\
&Constraints\ (47) - (60)
\end{aligned} \tag{4.66}$$

$$\theta_i \lambda_0 + \lambda_i \leq \mu_i(x) \qquad i \in \{1,2,3\}$$

$$\lambda_0, \lambda_i \in [0,1] \qquad i \in \{1,2,3\}$$

Following steps 1 to 3, efficient Pareto optimal solutions for the multi-objective crisp formulation (4.44)-(4.60) can be obtained.

4.4.4 The Solution Scheme Using the Proposed Approach

This method follows approximately similar structure comparing to the scheme of previous section. The only difference is with Step 3 where a new model for converting a multi-objective formulation to a single-objective formulation is proposed. For the crisp formulation (4.44)-(4.60), this scheme is summarized in the following steps. It is notable to mention that this proposed approach can be used for any multi-objective problem with a set of objective functions and constraints in any field of science and technology.

Step 1: The same as Step 1 of the previous section.

Step 2: The same as Step 2 of the previous section.

Step 3: Using the membership functions obtained from Step 2, each of the above-mentioned multi-objective optimization methods uses the following single-objective models.

$$\begin{aligned}
& max\ \lambda(x) = \sum_{i=1}^{3} \theta_i\, \mu_i(x) \\
& Subject\ to \\
& Constraints\ (47) - (60) \qquad\qquad (4.67) \\
& \theta_i \lambda_0 \leq \mu_i(x) \qquad i \in \{1,2,3\} \\
& \lambda_0 \in [0,1]
\end{aligned}$$

Thus, the efficient Pareto optimal solutions for the multi-objective crisp formulation (4.44)-(4.60) will be obtained by following steps 1 to 3.

As the most important step of the above-mentioned schemes is the single-objective model step (Step 3), some advantages of the single-objective model of the proposed approach (formulation (4.67)) comparing to the approaches of the literature (formulations (4.63)-(4.66)) are detailed here,

- The optimization procedure of the single-objective model is done in one phase.
- Obtaining unique or efficient solution is obvious.
- The varying weights of the objective function are eliminated.
- Only membership function values are used in the Formulation.

4.4.5 Overall Solution Procedure

In order to solve the multi-objective fuzzy formulation (4.15)-(4.31) the stages 1 and 2 which were detailed in the previous sections, have to be integrated. First, the model should be converted to the multi-objective crisp formulation (4.44)-(4.60). Then, as mentioned in Stage 2, the model (4.44)-(4.60) is to be solved by four mentioned multi-objective optimization methods of the literature and also the proposed approach of this study. It is necessary to mention that the coefficients *e.g.* α, γ, δ and θ_i should be tuned according to DM in order to obtain a satisfactory solution. The overall procedure of the proposed solution approach is summarized in the flowchart of Figure 1.

4.4.6 Computational Experiments on the Studied Case

The fuzzy formulation (4.15)-(4.31) and the developed solution approaches of Section 4 is numerically studied in this section. The models are solved using CPLEX solver of GAMS. It is run on a computer with an Intel Core 2 Duo 2.53 GHz processor and 4.00 GB RAM. A set of data for one planning horizon of the company which was introduced in Section 2 is obtained from the production planning department for performing the computations. The data obtained from the company include: (1) the fuzzy demand of 15 different types of carton box, (2) 6 supplier, each one supplying 20 different sizes of row sheet, (3) the amount of wastage to be remained after cutting each type of raw sheet, (4) the fuzzy discount break point for each raw sheet size, and (5) Normal and discounted prices of each size of raw sheet. The items (4) and (5) are determined by the suppliers. For instance, some numerical values of the parameters are presented by Table 1 and Table 2. Notably, as the full data of the case study require large tables to be shown, the values of paper box type 1 and supplier 1 is represented by Table 2. The solution procedure summarized in Figure 1 is implemented for the data of the studied case. All four explained multi-objective optimization methods and the proposed one were used to solve the single objective form of the crisp model (4.44)-(4.60). For this aim, some parameters of the methods should be determined in advance. Although the solutions are sensitive to the values used for the parameters, just some well-known values (particularly γ and δ) from the literature is considered for them (see Lai and Hwang, 1993; Selim and Ozkarahan, 2008; Torabi and Hassini, 2008; Alavidoost *et al.*, 2016). The parameters and their values are reported by Table 12.

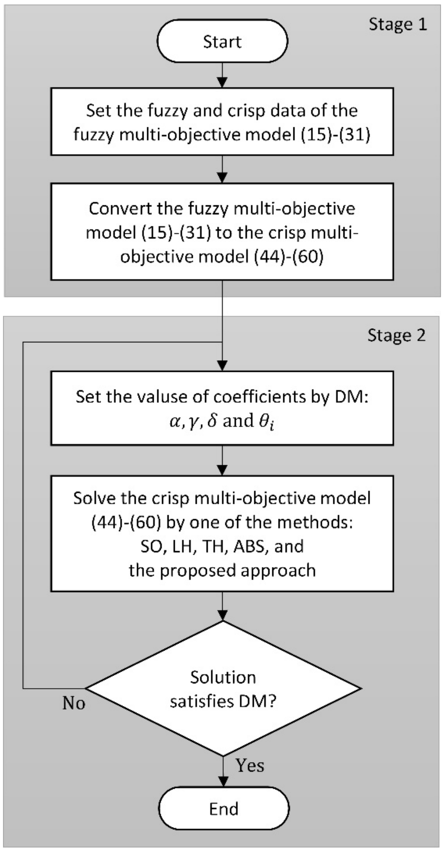

Figure 5. A flowchart covering the proposed solution approach and previous approaches of the literature.

Table 10. Trapezoidal fuzzy values for demand of different types of paper box for one planning horizon.

Box no.	Demand	Box no.	Demand	Box no.	Demand
1	(15000, 17000, 20000, 22000)	6	(15000, 17500, 19500, 21000)	11	(18500, 21500, 23500, 25000)
2	(17500, 19500, 22000, 25000)	7	(48000, 51500, 54500, 56500)	12	(54000, 57000, 60500, 63500)
3	(30000, 32500, 35500, 37500)	8	(42000, 44000, 46500, 48500)	13	(46500, 49500, 52000, 55000)
4	(52500, 55000, 57000, 59500)	9	(17500, 20500, 22500, 25500)	14	(32000, 36000, 38500, 42000)
5	(12500, 15500, 17000, 19500)	10	(13500, 16000, 18500, 20500)	15	(63000, 66000, 69500, 72500)

The set of values for feasibility degree change from 0.6 to 1 ($\alpha \in \{0.6, 0.7, 0.8, 0.9, 1\}$). For each level of α, first the $OF_i^{\alpha-PIS}$ and $OF_i^{\alpha-NIS}$ values are obtained and then the value of $\mu_i(x)$ is calculated. Considering the same level of α

and the value of parameters mentioned in Table 12, Pareto optimal solutions using the methods SO, ABS, LH, TH, and the proposed approach are obtained. Logically, if the solutions do not satisfy DM, the level of α and other parameters are changed until a satisfactory solution is obtained. In the computations of this study, the values of Table 12 are fixed while the value of α changes over the above-mentioned set of values. The results of this computational study is given in Table 13, Table 14, and Table 15.

Table 11. Fuzzy and crisp numerical values of all raw sheet sizes for box type 1 given by supplier 1.

Raw sheet (j)	a_{1j}	w_{1j}	g_{j1} g_{j1}	c_{j1}^{1}	c_{j1}^{2}
1	65	804.3	(2050, 2350, 2650, 2950)	(700, 900, 1100, 1300)	(600, 800, 1000, 1200)
2	75	808.8	(2100, 2300, 2600, 2900)	(850, 1150, 1350, 1550)	(800, 1000, 1200, 1400)
3	80	966.0	(1500, 1700, 2000, 2300)	(850, 1050, 1250, 1450)	(800, 1000, 1200, 1400)
4	90	970.5	(1400, 1700, 2000, 2300)	(1050, 1250, 1450, 1650)	(1000, 1200, 1400, 1600)
5	105	1132.3	(1100, 1400, 1600, 1900)	(1000, 1300, 1600, 1900)	(950, 1200, 1550, 1800)
6	115	1136.8	(800, 950, 1200, 1400)	(1200, 1500, 1850, 2050)	(1100, 1400, 1700, 2000)
7	120	1294.0	(850, 1100, 1400, 1700)	(1350, 1600, 1800, 2000)	(1300, 1600, 1800, 2000)
8	135	1455.8	(1000, 1300, 1300, 1900)	(1800, 2100, 2400, 2700)	(1800, 2000, 2200, 2400)
9	78	939.3	(1100, 1450, 1750, 2000)	(1050, 1250, 1450, 1650)	(1000, 1200, 1400, 1600)
10	90	958.8	(1200, 1500, 1700, 1900)	(1100, 1300, 1500, 1700)	(1100, 1250, 1400, 1600)
11	96	1131.0	(1100, 1450, 1650, 1900)	(1000, 1250, 1450, 1600)	(1000, 1200, 1400, 1600)
12	108	1150.5	(550, 750, 900, 1100)	(1150, 1450, 1700, 2000)	(1100, 1400, 1700, 2000)
13	126	1342.3	(600, 800, 1000, 1200)	(1400, 1700, 2000, 2300)	(1400, 1650, 1900, 2200)
14	138	1361.8	(850, 1000, 1250, 1450)	(1800, 2000, 2200, 2400)	(1800, 2000, 2150, 2350)
15	144	1534.0	(350, 550, 750, 950)	(1750, 2050, 2250, 2550)	(1700, 2000, 2200, 2500)

Raw sheet (*j*)	a_{1j}	w_{1j}	g_{j1} g_{j1}	c_{j1}^{1}	c_{j1}^{2}
16	162	1725.8	(550, 700, 900, 1100)	(2000, 2300, 2600, 2900)	(2000, 2200, 2500, 2800)
17	91	1074.3	(1000, 1200, 1400, 1600)	(1000, 1250, 1500, 1750)	(950, 1200, 1450, 1700)
18	105	1108.8	(500, 700, 900, 1150)	(1100, 1350, 1500, 1800)	(1100, 1300, 1500, 1700)
19	112	1296.0	(750, 900, 1150, 1300)	(1300, 1500, 1700, 1900)	(1250, 1450, 1650, 1800)
20	126	1330.5	(700, 900, 1150, 1300)	(1400, 1700, 2000, 2300)	(1400, 1700, 1900, 2200)

Table 12. The values used for the parameters of the proposed solution approaches

Parameter	Method				
	SO	ABS	LH	TH	The proposed approach
$(\theta_1, \theta_2, \theta_3)$	(0.3, 0.4, 0.3)	(0.3, 0.4, 0.3)	(0.3, 0.4, 0.3)	(0.3, 0.4, 0.3)	(0.3, 0.4, 0.3)
γ	0.4	-	-	0.4	-
δ	-	0.01	0.01	-	-

Focusing on the results of Table 13, Table 14, and Table 15, performance of the methods can be more analyzed and compared with each other. Obviously, the performance of LH, TH, and the proposed approach is better than SO and ABS methods. Interestingly, LH, TH, and the proposed approach provides solutions very close to the $\alpha -$positive ideal solutions ($OF_i^{\alpha-PIS}$) in all α feasibility level for all objective functions (see figures 2-4 where for being more clear, logarithmic values of the objective function values are considered on $y-$ axis). Opposite to these methods, the approaches SO and ABS, for each of three objective functions provide a solution between its $OF_i^{\alpha-PIS}$ and $OF_i^{\alpha-NIS}$ values but more close to its $OF_i^{\alpha-PIS}$ value for all α feasibility levels. The performance superiority of LH, TH, and the proposed approach comparing to SO and ABS methods can be seen in figures 2-4.

As a more detailed analysis of the results given by Table 13, Table 14, and Table 15, the followings can be concluded.

- When takes value of 0.6, the best value of first and second objective functions are obtained by the proposed approach, where, in the case of third objective function, LH, TH, and the proposed approach perform equally.
- When takes value of 0.7, the best value of first objective function is obtained by LH, TH, and the proposed approach. For second objective function, the best value is obtained by LH, and TH methods, where, in the case of third objective function, the proposed approach performs better than the others.
- When takes value of 0.8, in the case of first and second objective functions, the proposed approach performs better than the others.
- When takes value of 0.9, the best value of first objective function is obtained by LH, and TH methods, where, in the case of second and third objective functions, the proposed approach performs better than the others.
- When takes value of 1, in the case of first and second objective functions, the proposed approach performs better than the others.

The superiority of the proposed approach of this study can be proved by the above analysis of the obtained results, where, in most of cases, it performs better than the other methods.

In the rest of this section sensitivity analysis of the proposed approach and the approaches of the literature which were used in this study, is studied separately. First we focus on the sensitivity analysis of the approaches of literature and then the proposed approach is considered. To study the sensitivity of the solution approaches

LH, TH, SO, and ABS to their parameters (the parameters that do not exist in the proposed approach) the followings is considered;

- Parameter θ_i in all the approaches LH, TH, SO, and ABS is fixed to the values of Table 3 12where α is set to be 0.7.
- In the methods LH and ABS, the sensitivity on parameter δ is studied over its set value {0.01,0.02,0.03,0.04,0.05,0.07,0.09,0.1,0.2,0.3,0.4,0.5,0.7,0.9}
- In the methods TH and SO, the sensitivity on parameter γ is studied to be changed on the set value {0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1}.

Table 13. The results of the different approaches when $\alpha = 0.6$ and $\alpha = 0.7$

Approach	$\alpha = 0.6$			$\alpha = 0.7$		
	OF_1	OF_2	OF_3	OF_1	OF_2	OF_3
SO	4.44630E+11	1.49024E+11	2.96747E+08	4.35944E+11	1.47258E+11	2.92812E+08
ABS	4.52563E+11	1.44014E+11	4.05613E+08	3.95083E+11	1.33280E+11	3.30606E+08
LH	4.21457E+07	1.19631E+08	**7.96325E+03**	**4.28949E+07**	**1.210290E+08**	1.567830E+04
TH	4.21457E+07	1.19319E+08	**7.96325E+03**	**4.28949E+07**	**1.210290E+08**	1.567830E+04
The proposed	**4.214567E+07**	**1.193187E+08**	**7.96325E+03**	**4.28949E+07**	1.210291E+08	**1.567825E+04**

Table 14. The results of the different approaches when $\alpha = 0.8$ and $\alpha = 0.9$

Approach	$\alpha = 0.8$			$\alpha = 0.9$		
	OF_1	OF_2	OF_3	OF_1	OF_2	OF_3
SO	4.47460E+11	1.50894E+11	2.97796E+08	4.41756E+11	1.49032E+11	2.96783E+08
ABS	3.88583E+11	1.29907E+11	3.08630E+08	4.53140E+11	1.42102E+11	4.60601E+08
LH	4.364950E+07	1.228580E+08	**2.348225E+04**	**4.440020E+07**	1.247240E+08	3.117730E+04
TH	4.364950E+07	1.227470E+08	**2.348225E+04**	**4.440020E+07**	1.245690E+08	3.117730E+04
The proposed	**4.364948E+07**	**1.227469E+08**	**2.348225E+04**	4.440022E+07	**1.244551E+08**	**3.117725E+04**

Table 15. The results of the different approaches when $\alpha = 1$

Approach	$\alpha = 1$		
	OF_1	OF_2	OF_3
SO	4.29236E+11	1.49029E+11	3.54695E+08
ABS	4.35921E+11	1.37229E+11	1.19959E+09
LH	4.514480E+07	1.262780E+08	**3.885525E+04**
TH	4.514480E+07	1.262780E+08	**3.885525E+04**
The proposed	**4.514476E+07**	**1.261617E+08**	**3.885525E+04**

The studied case is solved by LH and ABS methods considering the above-mentioned conditions and the results is reported by Table 16. The results illustrate that the objective function values obtained by LH method are not sensitive to the changes made in the value of parameter δ. On the other hand, there is an unstable trend in the objective function values obtained by ABS method while the value of parameter δ is changed. Notably, ABS method performs better when uses smaller values of δ.

In order to study the effect of different values of γ, the studied case is solved by TH and SO methods considering the above-mentioned conditions and the results is shown in Table 17. The results illustrate that the objective function values obtained by TH method are not sensitive to the changes made in the value of parameter γ except in the case of $\gamma = 1$ which gives different value for OF_2 comparing to what obtained by other γ values. On the other hand, there is an unstable trend in the objective function values obtained by SO method while the value of parameter γ is changed. Interestingly, SO method performs better when $\gamma = 0$ or $\gamma = 1$. Particularly, for the case where $\gamma = 1$ the results of SO method is the same as the results obtained by TH method.

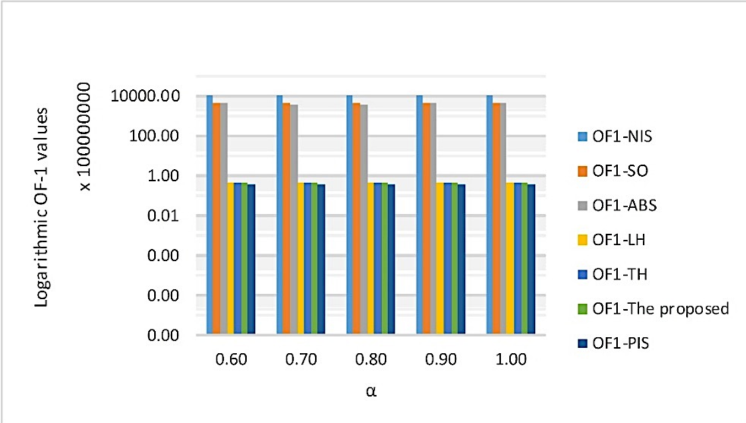

Figure 6. Logarithmic OF_1 values obtained by the different methods and ideal solutions.

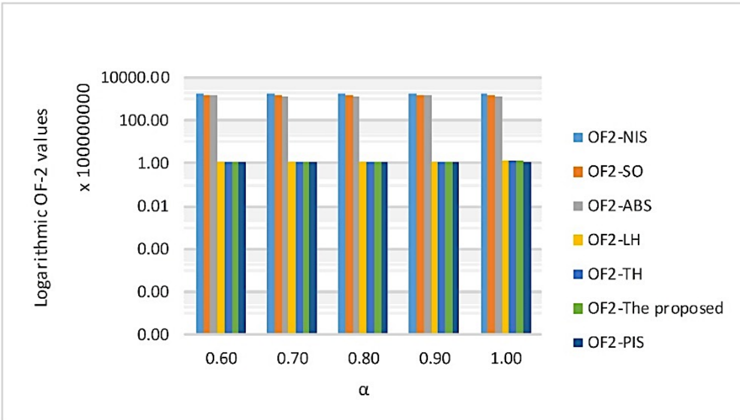

Figure 7. Logarithmic OF_2 values obtained by the different methods and ideal solutions.

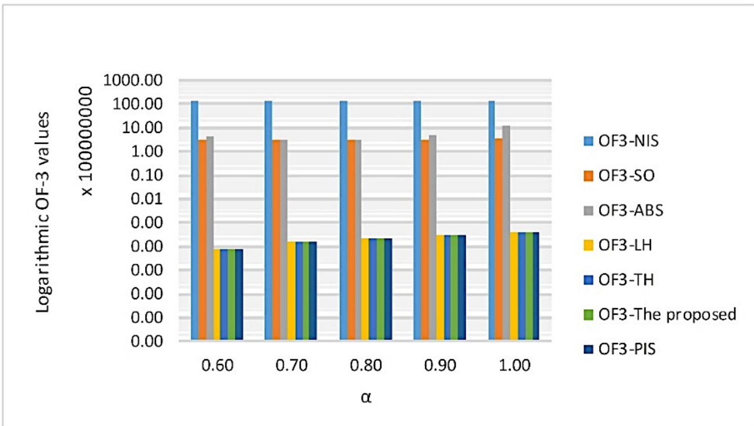

Figure 8. Logarithmic OF_3 values obtained by the different methods and ideal solutions.

Table 16. Performance of LH and ABS methods over different levels of δ .

δ	LH method			ABS method		
	OF_1	OF_2	OF_3	OF_1	OF_2	OF_3
0.01	4.2895E+07	1.2103E+08	1.5678E+04	3.9508E+11	1.3328E+11	3.3061E+08
0.02	4.2895E+07	1.2103E+08	1.5678E+04	3.9508E+11	1.3328E+11	3.3061E+08
0.03	4.2895E+07	1.2103E+08	1.5678E+04	1.2126E+10	1.9213E+10	4.2392E+08
0.04	4.2895E+07	1.2103E+08	1.5678E+04	4.0702E+11	1.4663E+11	3.8261E+08
0.05	4.2895E+07	1.2103E+08	1.5678E+04	4.5091E+11	1.5098E+11	8.9169E+08
0.07	4.2895E+07	1.2103E+08	1.5678E+04	4.6252E+11	1.5285E+11	8.9768E+08
0.09	4.2895E+07	1.2103E+08	1.5678E+04	4.4964E+11	1.5098E+11	8.9169E+08
0.1	4.2895E+07	1.2103E+08	1.5678E+04	1.9588E+11	7.1110E+10	7.2060E+08
0.2	4.2895E+07	1.2103E+08	1.5678E+04	2.9047E+11	9.7716E+10	8.0961E+08
0.3	4.2895E+07	1.2103E+08	1.5678E+04	1.1543E+11	3.2224E+10	2.3066E+08
0.4	4.2895E+07	1.2103E+08	1.5678E+04	4.5088E+11	1.5097E+11	8.9169E+08
0.5	4.2895E+07	1.2103E+08	1.5678E+04	4.6252E+11	1.5284E+11	8.9768E+08
0.7	4.2895E+07	1.2103E+08	1.5678E+04	4.6252E+11	1.5286E+11	8.9768E+08
0.9	4.2895E+07	1.2103E+08	1.5678E+04	4.6252E+11	1.5285E+11	8.9768E+08

Table 17. Performance of TH and SO methods over different levels of δ.

γ	TH method			SO method		
	OF_1	OF_2	OF_3	OF_1	OF_2	OF_3
0	4.2895E+07	1.2103E+08	1.5678E+04	5.9860E+07	1.2734E+08	1.5678E+04
0.1	4.2895E+07	1.2103E+08	1.5678E+04	3.8642E+11	1.4902E+11	2.0387E+09
0.2	4.2895E+07	1.2103E+08	1.5678E+04	3.8642E+11	1.4902E+11	2.0387E+09
0.3	4.2895E+07	1.2103E+08	1.5678E+04	4.4157E+11	1.4903E+11	2.9477E+08
0.4	4.2895E+07	1.2103E+08	1.5678E+04	4.3594E+11	1.4726E+11	2.9281E+08
0.5	4.2895E+07	1.2103E+08	1.5678E+04	4.3394E+11	1.3731E+11	4.5559E+08
0.6	4.2895E+07	1.2103E+08	1.5678E+04	4.4145E+11	1.3876E+11	4.5760E+08
0.7	4.2895E+07	1.2103E+08	1.5678E+04	4.2638E+11	1.3368E+11	4.1770E+08
0.8	4.2895E+07	1.2103E+08	1.5678E+04	4.3341E+11	1.3862E+11	4.2961E+08
0.9	4.2895E+07	1.2103E+08	1.5678E+04	4.3341E+11	1.3862E+11	4.2961E+08
1	4.2895E+07	1.2441E+08	1.5678E+04	4.2895E+07	1.2441E+08	1.5678E+04

Now the sensitivity of the proposed solution approach to its parameters is studied. Considering the overall formulation (44)-(60) and the formulation (67), the proposed approach is sensitive to the value of α and also to the weight values θ_i. Therefore, for this analysis the followings are considered;

- The set of values for feasibility degree change from 0.6 to 1 ($\alpha \in \{0.6,0.7,0.8,0.9,1\}$).
- The weight combinations of Table 18 is considered.

Table 18. The weight combinations used for the sensitivity analysis of the proposed approach.

Combination of weights	The weight values		
	θ_1	θ_2	θ_3
W1	0.1	0.4	0.5
W2	0.2	0.5	0.3
W3	0.3	0.4	0.3
W4	0.4	0.2	0.4
W5	0.5	0.3	0.2
W6	0.5	0.1	0.4
W7	0.4	0.5	0.1

The results obtained by using all weight combinations of Table 18 and the above-mentioned values of α are represented in Table 19, Table 20, and Table 21. Some effects of the changes in parameter values can be concluded from these tables. Interestingly, when the value of α is increased, the value of the objective functions are also increased for all of the weight combinations. For the case of $\alpha = 0.6$, the value of OF_1 and OF_2 is not sensitive to the changes in the combinations of weights while the value of OF_2 has an unstable trend of changes over the changes in the combinations of weights. A similar effect happens for other cases of α, where, in any α level, the value of OF_1 and OF_3 is not sensitive to the changes in the combinations of weights while the value of OF_2 has an unstable trend of changes over the changes in the combinations of weights.

Table 19. The results of the proposed approach for all combinations of weights when $\alpha = 0.6$ and $\alpha = 0.7$

Combination of weights	$\alpha = 0.6$			$\alpha = 0.7$		
	OF_1	OF_2	OF_3	OF_1	OF_2	OF_3
W1	4.214567E+07	1.193187E+08	7.96325E+03	4.28949E+07	1.213780E+08	1.567825E+04
W2	4.214567E+07	1.194247E+08	7.96325E+03	4.28949E+07	1.213491E+08	1.567825E+04
W3	4.214567E+07	1.193187E+08	7.96325E+03	4.28949E+07	1.210291E+08	1.567825E+04
W4	4.214567E+07	1.193187E+08	7.96325E+03	4.28949E+07	1.211376E+08	1.567825E+04
W5	4.214567E+07	1.193187E+08	7.96325E+03	4.28949E+07	1.213491E+08	1.567825E+04
W6	4.214567E+07	1.193187E+08	7.96325E+03	4.28949E+07	1.210291E+08	1.567825E+04
W7	4.214567E+07	1.194247E+08	7.96325E+03	4.28949E+07	1.210291E+08	1.567825E+04

Table 20. The results of the proposed approach for all combinations of weights when $\alpha = 0.8$ and $\alpha = 0.9$

Combination of weights	$\alpha = 0.8$			$\alpha = 0.9$		
	OF_1	OF_2	OF_3	OF_1	OF_2	OF_3
W1	4.364948E+07	1.227469E+08	2.348225E+04	4.440022E+07	1.244551E+08	3.117725E+04
W2	4.364948E+07	1.241447E+08	2.348225E+04	4.440022E+07	1.244551E+08	3.117725E+04
W3	4.364948E+07	1.227469E+08	2.348225E+04	4.440022E+07	1.244551E+08	3.117725E+04
W4	4.364948E+07	1.227469E+08	2.348225E+04	4.440022E+07	1.258709E+08	3.117725E+04
W5	4.364948E+07	1.228579E+08	2.348225E+04	4.440022E+07	1.258709E+08	3.117725E+04
W6	4.364948E+07	1.228579E+08	2.348225E+04	4.440022E+07	1.258709E+08	3.117725E+04
W7	4.364948E+07	1.230743E+08	2.348225E+04	4.440022E+07	1.262056E+08	3.117725E+04

Table 21. The results of the proposed approach for all combinations of weights when $\alpha = 1$

Combination of weights	$\alpha = 1$		
	OF_1	OF_2	OF_3
W1	4.514476E+07	1.261617E+08	3.885525E+04
W2	4.514476E+07	1.262775E+08	3.885525E+04
W3	4.514476E+07	1.261617E+08	3.885525E+04
W4	4.514476E+07	1.261617E+08	3.885525E+04
W5	4.514476E+07	1.261617E+08	3.885525E+04
W6	4.514476E+07	1.261617E+08	3.885525E+04
W7	4.514476E+07	1.261617E+08	3.885525E+04

4.5 Necessity Chance-Constraint Programming Approach

Another technique for dealing with the uncertainty involved with fuzzy sets is the chance-constrained modeling approach. This technique, converts a fuzzy constraint into a crisp equivalent using three different methods; possibility-constrained modeling, credibility constrained modeling, and necessity constrained modeling. Possibility-constrained modeling converts the fuzzy constraint into the crisp taking an optimistic point of view. The necessity-constrained modeling converts the fuzzy constraint to a crisp taking a pessimistic point of view. Finally, credibility-constrained modeling considers the average of the above-mentioned two models for converting fuzzy constraints to a crisp form. In the fuzzy multi-objective supplier-material selection problem (FMSMSP) of this study, the most important objective from company's point of view is satisfying the fuzzy constraint related with the demand satisfaction (4.18). In this respect, the necessity-constrained modeling is the more suitable approach for converting the FSMSP to its crisp form. This conversion together with its required basic information is presented in the following sub-sections.

4.5.1 Necessity-Constrained Modelling

As mentioned above, the necessity-constrained modeling converts the fuzzy constraints to their crisp equivalent from a pessimistic point of view. Some definitions and relations, required for converting a fuzzy constraint or fuzzy objective function to its crisp form using this approach are presented in this section.

If a trapezoidal fuzzy number ξ is shown by $\xi = (\xi^1, \xi^2, \xi^3, \xi^4)$, its necessity measures will calculated as follows;

$$Nec\{\tilde{\xi} \leq s\} = \begin{cases} 0 & s \in (-\infty, \xi^1] \\ \dfrac{s - \xi^1}{2(\xi^2 - \xi^1)} & s \in (\xi^1, \xi^2] \\ \dfrac{1}{2} & s \in (\xi^2, \xi^3] \\ \dfrac{s - 2\xi^3 - \xi^4}{2(\xi^4 - \xi^3)} & s \in (\xi^3, \xi^4] \\ 1 & s \in (\xi^4, +\infty) \end{cases} \quad (4.68)$$

$$Nec\{\tilde{\xi} \geq s\} = \begin{cases} 1 & s \in (-\infty, \xi^1] \\ \dfrac{2\xi^2 - \xi^1 - s}{2(\xi^2 - \xi^1)} & s \in (\xi^1, \xi^2] \\ \dfrac{1}{2} & s \in (\xi^2, \xi^3] \\ \dfrac{\xi^4 - s}{2(\xi^4 - \xi^3)} & s \in (\xi^3, \xi^4] \\ 0 & s \in (\xi^4, +\infty) \end{cases} \quad (4.69)$$

Then the following equations are used to find the crisp values of the fuzzy constraints (which are called fuzzy chance constraints) $Nec\{\tilde{\xi} \leq s\} \geq \varphi$ and $Nec\{\tilde{\xi} \geq s\} \geq \varphi$, where $\varphi > 0$:

$$Nec\{\tilde{\xi} \leq s\} \geq \alpha \iff s \geq (1-\alpha)\xi^1 + \alpha\xi^2 \quad (4.70)$$

$$Nec\{\tilde{\xi} \geq s\} \geq \alpha \iff s \leq \alpha\xi^1 + (1-\alpha)\xi^2 \quad (4.71)$$

4.5.2 Crisp Version of the FMSMSP Using Necessity-Constrained Modeling

The FMSMSP introduced in the previous section, contains a fuzzy type uncertain data in the objective functions (4.16) and (4.17) in addition to constraints (4.18), (4.21), and (4.22). Thus, the following formulation using necessity-constrained modeling is suggested. Please note that the second and third objective functions are initially moved to the constraints and then are bounded by the dummy variables f_2 and f_3 per their optimization direction.

$$min\, OF_1 = \sum_{i=1}^{I}\sum_{j=1}^{j}\sum_{k=1}^{k}(w_{ij}\, Y_{ijk}) \qquad (4.72)$$

$$f_2 \qquad (4.73)$$

$$f_3 \qquad (4.74)$$

$Subject\ to$

$$Nec\left\{\sum_{j=1}^{J}\sum_{k=1}^{K}(\widetilde{c_{jk}^1}\, z_{jk}^1 + \widetilde{c_{jk}^2} z_{jk}^2) \le f_2\right\} \ge \varphi \qquad (4.75)$$

$$Nec\left\{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} a_{ij}\, Y_{ijk} - \sum_{i=1}^{I}\widetilde{d}_i \le f_3\right\} \ge \beta \qquad (4.76)$$

$$Nec\left\{\sum_{j=1}^{J}\sum_{k=1}^{K} a_{ij}\, Y_{ijk} \ge \widetilde{d}_i\right\} \le \gamma_i \qquad \forall i \qquad (4.77)$$

$$T_{jk}^1 + T_{jk}^2 \le 1 \qquad \forall j,k \qquad (4.78)$$

$$\sum_{i=1}^{I} Y_{ijk} \ge T_{jk}^1 \qquad \forall j,k \qquad (4.79)$$

$$Nec\left\{\sum_{i=1}^{I} Y_{ijk} \le \tilde{g_{jk}} T_{jk}^1 + MT_{jk}^2\right\} \ge \psi_{jk} \qquad \forall j,k \qquad (4.80)$$

$$Nec\left\{\sum_{i=1}^{I} Y_{ijk} \ge \tilde{g_{jk}} T_{jk}^2\right\} \ge \delta_{jk} \qquad \forall j,k \qquad (4.81)$$

$$Z_{jk}^1 \le MT_{jk}^1 \qquad \forall j,k \qquad (4.82)$$

$$Z_{jk}^1 \le \sum_{i=1}^{I} Y_{ijk} \qquad \forall j,k \qquad (4.83)$$

$$Z_{jk}^1 \ge \left(\sum_{i=1}^{I} Y_{ijk}\right) - M(1 - T_{jk}^1) \qquad \forall j,k \qquad (4.84)$$

$$Z_{jk}^2 \le MT_{jk}^2 \qquad \forall j,k \qquad (4.85)$$

$$Z_{jk}^2 \leq \sum_{i=1}^{I} Y_{ijk} \qquad \forall j,k \quad (4.86)$$

$$Z_{jk}^2 \geq \left(\sum_{i=1}^{I} Y_{ijk}\right) - M(1 - T_{jk}^2) \qquad \forall j,k \quad (4.87)$$

$$T_{jk}^1, T_{jk}^1 \in \{0,1\} \qquad \forall j,k \quad (4.88)$$

$$Y_{ijk} \geq 0 \; and \; integer \qquad \forall i,j,k \quad (4.89)$$

$$Z_{jk}^1, Z_{jk}^2 \geq 0 \qquad \forall j,k \quad (4.90)$$

According to Eqs. (4.70) and (4.71), the above-mentioned necessity-constrained programming model is converted into the following crisp multi-objective supplier-material selection problem (CMSMSP). Note that the objective functions in which, were moved to the constraints (Eqs. (4.75) and (4.76)), have returned as the objective functions of the model after being converted to their crisp form. The CMSMSP is as follows:

$$min\, OF_1 = \sum_{i=1}^{I}\sum_{j=1}^{j}\sum_{k=1}^{k}(w_{ij}\, Y_{ijk}) \qquad (4.91)$$

$$min\, OF_2 = (1-\varphi)\sum_{j=1}^{J}\sum_{k=1}^{K}\left(c_{jk}^{1,3} z_{jk}^1 + c_{jk}^{2,3} z_{jk}^2\right) + \varphi\sum_{j=1}^{J}\sum_{k=1}^{K}\left(c_{jk}^{1,4} z_{jk}^1 + c_{jk}^{2,4} z_{jk}^2\right) \qquad (4.92)$$

$$min\, OF_3 = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} a_{ij} Y_{ijk} - \beta\sum_{i=1}^{I} d_i^1 - (1-\beta)\sum_{i=1}^{I} d_i^2 \qquad (4.93)$$

$Subject\ to$

$$\sum_{j=1}^{J}\sum_{k=1}^{K} a_{ij} Y_{ijk} \geq (1-\gamma_i) d_i^3 + \gamma_i d_i^4 \qquad \forall i \qquad (4.94)$$

$$T_{jk}^1 + T_{jk}^2 \leq 1 \qquad \forall j,k \qquad (4.95)$$

$$\sum_{i=1}^{I} Y_{ijk} \geq T_{jk}^{1} \qquad \forall j,k \qquad (4.96)$$

$$\sum_{i=1}^{I} Y_{ijk} \leq \psi_{jk} g_{jk}^{1} T_{jk}^{1} + (1-\psi_{jk}) g_{jk}^{2} T_{jk}^{2} + M T_{jk}^{2} \qquad \forall j,k \qquad (4.97)$$

$$\sum_{i=1}^{I} Y_{ijk} \geq (1-\delta_{jk}) g_{jk}^{3} T_{jk}^{2} + \delta_{jk} g_{jk}^{4} T_{jk}^{2} \qquad \forall j,k \qquad (4.98)$$

$$Z_{jk}^{1} \leq M T_{jk}^{1} \qquad \forall j,k \qquad (4.99)$$

$$Z_{jk}^{1} \leq \sum_{i=1}^{I} Y_{ijk} \qquad \forall j,k \qquad (4.100)$$

$$Z_{jk}^{1} \geq \left(\sum_{i=1}^{I} Y_{ijk}\right) - M(1 - T_{jk}^{1}) \qquad \forall j,k \qquad (4.101)$$

$$Z_{jk}^{2} \leq M T_{jk}^{2} \qquad \forall j,k \qquad (4.102)$$

$$Z_{jk}^{2} \leq \sum_{i=1}^{I} Y_{ijk} \qquad \forall j,k \qquad (4.103)$$

$$Z_{jk}^{2} \geq \left(\sum_{i=1}^{I} Y_{ijk}\right) - M(1 - T_{jk}^{2}) \qquad \forall j,k \qquad (4.104)$$

$$T_{jk}^{1}, T_{jk}^{1} \in \{0,1\} \qquad \forall j,k \qquad (4.105)$$

$$Y_{ijk} \geq 0 \textit{ and integer} \qquad \forall i,j,k \qquad (4.106)$$

$$Z_{jk}^{1}, Z_{jk}^{2} \geq 0 \qquad \forall j,k \qquad (4.107)$$

Additionally, it should note that in the above-mentioned formulation, it is assumed that the chance constraints should be satisfied with the confidence level greater than 0.5 (*i.e.* $\varphi, \beta, \gamma_i, \psi_{jk}, \delta_{jk} > 0.5$). The represented CMSMSP should be solved as a multi-objective problem. This aim is followed by the next section of the paper.

4.5.3 The Proposed Necessity Based Solution Approach

In this section, an effective solution approach is proposed to tackle the CMSMSP for finding a good Pareto-optimal solution. In the literature of multi-objective optimization, various approaches such as goal programming, ε-constraint approach, fuzzy programming approach, etc. are proposed and applied. Zimmermann (1978) for the first time applied a fuzzy programming approach (max-min operator) to solve a multi-objective model. Unfortunately, his solution approach may not give an efficient (Pareto-optimal) solution in some cases (Alavidoost *et al.*, 2016). This weakness of fuzzy programming approach later was focused by the studies that introduced the hybrid versions of fuzzy programming method. SO (Selim and Ozkarahan, 2008), LH (Li and Hu, 2007), DY (Demirli and Yimer, 2008), and ABS (Alavidoost *et al.*, 2016) are some of these proposed methods. In this section, a new hybrid version of fuzzy programming approach is proposed to solve the CMSMSP. The method is explained in the next sub-section and in continuation, its efficiency is proved.

4.5.4 Description of The Approach

The proposed solution approach of this study is a new hybrid version of fuzzy programming method to produce a competent solution for the CMSMSP. The procedure is applied through the following steps.

Step 1. Set the confidence level value for the chance constraints. i.e. determine the values of $\varphi, \beta, \gamma_i, \psi_{jk}, \delta_{jk}$.

Step 2. Solve the following sub-models to obtain the positive ideal solution (PIS) and negative ideal solution (NIS) of each objective function individually.

$$OF_1^{PIS} = min \sum_{i=1}^{I} \sum_{j=1}^{j} \sum_{k=1}^{k} (w_{ij}\, Y_{ijk}) \tag{4.108}$$

$Subject\ to$

Constraints (58) - (71)

$$OF_1^{NIS} = max \sum_{i=1}^{I} \sum_{j=1}^{j} \sum_{k=1}^{k} (w_{ij}\, Y_{ijk}) \tag{4.109}$$

$Subject\ to$

Constraints (58) - (71)

$$OF_2^{PIS} = min\left((1-\varphi) \sum_{j=1}^{J} \sum_{k=1}^{K} (c_{jk}^{1,3} z_{jk}^1 + c_{jk}^{2,3} z_{jk}^2) + \varphi \sum_{j=1}^{J} \sum_{k=1}^{K} (c_{jk}^{1,4} z_{jk}^1 + c_{jk}^{2,4} z_{jk}^2) \right) \tag{4.110}$$

$Subject\ to$

Constraints (58) - (71)

$$OF_2^{NIS} = max\left((1-\varphi) \sum_{j=1}^{J} \sum_{k=1}^{K} (c_{jk}^{1,3} z_{jk}^1 + c_{jk}^{2,3} z_{jk}^2) + \varphi \sum_{j=1}^{J} \sum_{k=1}^{K} (c_{jk}^{1,4} z_{jk}^1 + c_{jk}^{2,4} z_{jk}^2) \right) \tag{4.111}$$

$Subject\ to$

Constraints (58) - (71)

$$OF_3^{PIS} = min\left(\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} a_{ij} Y_{ijk} - \beta \sum_{i=1}^{I} d_i^1 - (1-\beta) \sum_{i=1}^{I} d_i^2 \right) \tag{4.112}$$

$Subject\ to$

Constraints (58) - (71)

$$OF_3^{NIS} = max\left(\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} a_{ij}Y_{ijk} - \beta\sum_{i=1}^{I} d_i^1 - (1-\beta)\sum_{i=1}^{I} d_i^2\right) \quad (4.113)$$

Subject to
Constraints (58) - (71)

Step 3. As each objective function, can be related to a fuzzy membership function (MF), the MFs of the objective functions are calculated through the following relationships (see also Figure 1);

$$\mu_r(x) = \begin{cases} 1 & OF_r < {OF_r}^{PIS} \\ \dfrac{{OF_r}^{NIS} - OF_r}{{OF_r}^{NIS} - {OF_r}^{PIS}} & {OF_r}^{PIS} \leq OF_r \leq {OF_r}^{NIS} \\ 0 & OF_r > {OF_r}^{NIS} \end{cases} \quad (4.114)$$

where $\mu_r(x)$ for $r \in \{1,2,...,R\}$ (in this paper $R = 3$) is the linear MF of the objective function OF_r.

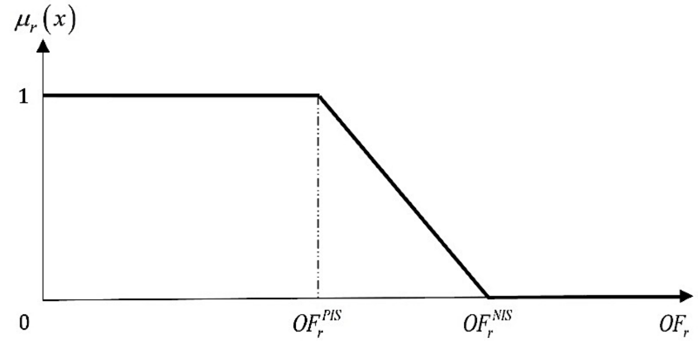

Figure 9. Schematic representation of the fuzzy membership functions of the objective functions.

Step 4. (single-objective model step) Convert the CMSMSP to the following proposed single objective formulation.

$$
\begin{aligned}
& max \sum_{r=1}^{R} \theta_r \mu_r (x) + \lambda_0 \\
& Subject\ to \\
& \theta_r \lambda_0 \leq \mu_r(x) \qquad \forall r \in \{1,2,\dots,R\} \\
& \lambda_0, \lambda_r \in [0,1] \qquad \forall r \in \{1,2,\dots,R\} \\
& \text{Constraints (58) - (71)}
\end{aligned}
\tag{4.115}
$$

In the formulation (4.115), the positive value θ_r is the importance weight of r^{th} objective function with the condition of $\sum_{r=1}^{R} \theta_r = 1$. The continuous and non-negative variables λ_0 and λ_r are used to control the minimum satisfaction level of the objective functions as well as their compromise degrees.

Step 5. Solve the single-objective model (4.115) with a given set of values for weights of the objective functions (θ_r). If the obtained solution is satisfactory for the decision maker, stop. Otherwise, do one of the following changes and repeat the steps 1 to 4 until a satisfactory solution is obtained;

- Increase the NIS value for maximization type objective functions.
- Decrease the PIS value for minimization type objective functions.
- Change the given set of values for weights of the objective functions (θ_r).

4.5.5 The Single-Objective Model (4.115) Step of the Proposed Approach

The single-objective model of a fuzzy programming approach is the most important step of any hybrid version of fuzzy programing. The Advantages of the single-objective model of the proposed approach in this study (formulation (4.115), are the followings;

- The optimization procedure of the single-objective model is done in one step.
- Obtaining a unique or an efficient solution is obvious.
- The minimum satisfaction level of the objective functions as well as their compromise degrees are controlled by just one variable.
- Membership function values are used in the objective functions.
- The goals are partially prioritized in the objective function and constraints. The weights of the objective functions are used in the single objective function and constraints simultaneously.

The feasibility and efficiency of (4.115) is described by the Theorem 1.

Theorem 1. Formulation (4.115) has solution and its solution is efficient to the CMSMSP.

Proof. Let's first define the following formulation which is a part of formulation (4.115).

$$
\begin{aligned}
& max\,\lambda_0 \\
& Subject\ to \\
& \lambda_0 \leq \mu_r(x) \qquad \forall r \in \{1,2,\ldots,R\} \qquad (4.116) \\
& \lambda_0 \in [0,1] \qquad \forall r \in \{1,2,\ldots,R\} \\
& \text{Constraints (58) - (71)}
\end{aligned}
$$

Clearly, considering the sign of constraints and type of the objective function, problem (4.116) has an optimal solution (say x^0). Considering the value of θ_r between zero and one, x^0 and $\lambda_r = 0$, $r \in \{1,2, \dots, R\}$ together is a feasible solution to the problem (4.115). Therefore, the feasible region of the problem (4.115) is not empty.

The efficiency of the solution of model (4.115) is proved by a contradiction. Suppose that x^* is an optimal solution of model (4.115) which is inefficient solution to the problem (4.91) - (4.107). Therefore, there should be an efficient solution like x^{**} to the problem (4.91)-(4.107) which is obtained by the model (4.115) satisfying the conditions;

i. $f_r(x^{**}) \leq f_r(x^*)$ $(\forall r \in \{1,2, \dots, R\}$ and $\exists i \in [0,1]$: $f_i(x^{**}) \leq f_i(x^*)$

ii. $\mu_r(x^{**}) \leq \mu_r(x^*)$ $(\forall r \in \{1,2, \dots, R\}$ and $\exists i \in [0,1]$: $\mu_i(x^{**}) \leq \mu_i(x^*)$

Respecting the minimum satisfaction level of the objectives of x^* and x^{**}, the condition $\lambda_0^{**} \geq \lambda_0^*$ must be true. Considering the objective functions of the two solutions in formulation (4.115), the following inequality is obtained;

$$\left\{\sum_{r=1}^{R}(\theta_r\, \mu_r(x^*) + \lambda_0^*) = \sum_{r=1}^{R}(\theta_r\, \mu_r(x^*) + R\lambda_0^*) = \sum_{\substack{r=1\\ r\neq i}}^{R} \theta_r\, \mu_r(x^*) + \theta_i \mu_i(x^*) + R\lambda_0^*\right\}$$
$$< \left\{\sum_{r=1}^{R}(\theta_r\, \mu_r(x^{**}) + \lambda_0^{**}) = \sum_{r=1}^{R}(\theta_r\, \mu_r(x^{**}) + R\lambda_0^{**}) = \sum_{\substack{r=1\\ r\neq i}}^{R} \theta_r\, \mu_r(x^{**}) + \theta_i \mu_i(x^{**}) + R\lambda_0^{**}\right\} \quad (4.117)$$

Consequently, x^* cannot be the optimal solution of the problem (4.115) which is contradictory to the initial assumption for x^* and the theorem is proved.

4.5.6 Comparison Metrics

In order to compare the performance of the proposed approach of this study with the other mentioned methods of the literature, the following distance measure formula is used (Alavidoost *et al.*, 2016);

$$D_p(\theta, R) = \sqrt[1/p]{\sum_{r=1}^{R} \theta_r^p \left(1 - \mu_r(x)\right)^p} \quad \forall p \geq 1 \; and \; integer \qquad (4.118)$$

Some well-known distance measures obtained from formula (4.118) are defined below;

Manhattan distance ($p = 1$): This distance is in fact the weighted sum of distance from the goal that takes the value of one here. The value of this distance has an inverse relation with the value of MF as follow;

$$D_1(\theta, R) = 1 - \sum_{r=1}^{R} \theta_r \mu_r(x) \qquad (4.119)$$

Euclidean distance ($p = 2$): This distance plays the same role as Manhattan distance except, the quality of membership function values is evaluated. It means, the closer MF values give less distance in the case of equal solutions.

$$D_2(\theta, R) = \sqrt{\sum_{r=1}^{R} \theta_r^2 \, (1 - \mu_r(x))^2} \qquad (4.120)$$

Tchebycheff distance ($p = \infty$): This is the shortest distance comparing to the above two distances. When this distance (also other distances with $p > 1$) is calculated, more penalty is given to the smaller MF values. Therefore, the solutions who have a close MFs will get less distance value when this distance is considered.

$$\{D_\infty(\theta, R) = max_R\{\theta_r\big(1 - \mu_r(x)\big)\} \qquad (4.121)$$

4.5.7 Computational Experiments on the Real Case

In this section, the previously proposed CMSMSP together with the developed solution method which was discussed in Section 5 is deliberated numerically. The required data in which, are collected from the production planning department of the company are the followings; (1) the fuzzy demand of 15 different box types, (2) 6 contractor, each one supplying 20 different sizes of raw sheets, (3) the boundary remaining wastage after cutting each type of raw sheet, (4) the fuzzy discount break point for each raw sheet size, and finally, (5) discounted and non-discounted prices of each size of raw sheet. Please note that (4) and (5) are controlled by the contractors. The problem is solved by GAMS solver, ran on a computer with an Intel Core 2 Duo 2.53 GHz processor and 4.00 GB RAM.

For providing a better illustration, some of the parameters of the problem are shown in Tables 22 and 23. Since representing the full data of the case study needs large tables, in Table 23, only the values of one paper box type and supplier #1 is depicted.

Table 22. Trapezoidal fuzzy values for demand of different types of paper box for one planning horizon

Box no.	Demand	Box no.	Demand	Box no.	Demand
1	(15000, 17000, 20000, 22000)	6	(15000, 17500, 19500, 21000)	11	(18500, 21500, 23500, 25000)
2	(17500, 19500, 22000, 25000)	7	(48000, 51500, 54500, 56500)	12	(54000, 57000, 60500, 63500)
3	(30000, 32500, 35500, 37500)	8	(42000, 44000, 46500, 48500)	13	(46500, 49500, 52000, 55000)
4	(52500, 55000, 57000, 59500)	9	(17500, 20500, 22500, 25500)	14	(32000, 36000, 38500, 42000)
5	(12500, 15500, 17000, 19500)	10	(13500, 16000, 18500, 20500)	15	(63000, 66000, 69500, 72500)

In continuation, a comparison is made among the proposed solution approach for the CMSMSP in this study and the existing methods of the literature such as SO, LH, DY, and ABS. This performance assessment provides a better understanding of the method's capabilities. In this investigation, the following assumptions are considered;

- A set of values are considered as the chance constraints' confidence level. These values are the same when the indices of their related constraint change; ($\varphi, \beta, \gamma_i, \psi_{jk}, \delta_{jk} = \Omega$ where $\Omega \in \{0.6, 0.7, 0.8, 0.9\}$).
- Using different combinations of weights (θ_r), the single-objective model of Step 4 of the proposed solution approach is tested. The results are shown in Table 24.
- Some of the existing methods for solving this category of problems, require another weight component (sayλ). In such cases, $\lambda = 0.4$ is introduced as the best value in the literature. Accordingly, in this research the same value is used.
- In some of the proposed methods in the literature, a coefficient (say δ) is required. In such cases, in the previous researches, (δ=0.01) is used as the best value. Correspondingly, in this research the same value is used.

Considering the mentioned assumptions, the CMSMSP is solved by all proposed approaches, both in this study and the literature. The results are represented in terms of the values of objective function and distance metrics in Tables 25 to 28. The bold values represent the best obtained values. The further analysis of the obtained results, can be investigating whether the values are from the objective function values point of view or distances point of view.

Table 23. Fuzzy and crisp numerical values of all raw sheet sizes for box type 1 given by supplier 1

Raw sheet (*j*)	a_{1j}	w_{1j}	g_{j1}	c_{j1}^{1}	c_{j1}^{2}
1	65	804.3	(2050, 2350, 2650, 2950)	(700, 900, 1100, 1300)	(600, 800, 1000, 1200)
2	75	808.8	(2100, 2300, 2600, 2900)	(850, 1150, 1350, 1550)	(800, 1000, 1200, 1400)
3	80	966.0	(1500, 1700, 2000, 2300)	(850, 1050, 1250, 1450)	(800, 1000, 1200, 1400)
4	90	970.5	(1400, 1700, 2000, 2300)	(1050, 1250, 1450, 1650)	(1000, 1200, 1400, 1600)
5	105	1132.3	(1100, 1400, 1600, 1900)	(1000, 1300, 1600, 1900)	(950, 1200, 1550, 1800)
6	115	1136.8	(800, 950, 1200, 1400)	(1200, 1500, 1850, 2050)	(1100, 1400, 1700, 2000)
7	120	1294.0	(850, 1100, 1400, 1700)	(1350, 1600, 1800, 2000)	(1300, 1600, 1800, 2000)
8	135	1455.8	(1000, 1300, 1300, 1900)	(1800, 2100, 2400, 2700)	(1800, 2000, 2200, 2400)
9	78	939.3	(1100, 1450, 1750, 2000)	(1050, 1250, 1450, 1650)	(1000, 1200, 1400, 1600)
10	90	958.8	(1200, 1500, 1700, 1900)	(1100, 1300, 1500, 1700)	(1100, 1250, 1400, 1600)
11	96	1131.0	(1100, 1450, 1650, 1900)	(1000, 1250, 1450, 1600)	(1000, 1200, 1400, 1600)
12	108	1150.5	(550, 750, 900, 1100)	(1150, 1450, 1700, 2000)	(1100, 1400, 1700, 2000)
13	126	1342.3	(600, 800, 1000, 1200)	(1400, 1700, 2000, 2300)	(1400, 1650, 1900, 2200)
14	138	1361.8	(850, 1000, 1250, 1450)	(1800, 2000, 2200, 2400)	(1800, 2000, 2150, 2350)
15	144	1534.0	(350, 550, 750, 950)	(1750, 2050, 2250, 2550)	(1700, 2000, 2200, 2500)
16	162	1725.8	(550, 700, 900, 1100)	(2000, 2300, 2600, 2900)	(2000, 2200, 2500, 2800)
17	91	1074.3	(1000, 1200, 1400, 1600)	(1000, 1250, 1500, 1750)	(950, 1200, 1450, 1700)
18	105	1108.8	(500, 700, 900, 1150)	(1100, 1350, 1500, 1800)	(1100, 1300, 1500, 1700)
19	112	1296.0	(750, 900, 1150, 1300)	(1300, 1500, 1700, 1900)	(1250, 1450, 1650, 1800)

Table 24. Different combinations of weights used to run the proposed method and the methods of literature

Combination	θ_1	θ_2	θ_3
C1	0.5	0.3	0.2
C2	0.4	0.2	0.4
C3	0.3	0.2	0.5
C4	0.4	0.1	0.5
C5	0.6	0.1	0.3
C6	0.6	0.3	0.1
C7	0.3	0.1	0.6

In terms of the obtained values of the objective functions, once the parameters of the chance constraints of the CMSMSP is set to 0.6 ($\varphi, \beta, \gamma_i, \psi_{jk}, \delta_{jk} = 0.6$), in 6 out of 7 weights combinations, the proposed approach gives a better OF_1 and OF_2 values (only in C4 weights combination such performance is not observed). For the case of ($\varphi, \beta, \gamma_i, \psi_{jk}, \delta_{jk} = 0.7$), in 4 out of 7 weights combinations, the proposed method delivers a better OF_1 and OF_2 values. However, in C3, C4, and C5, such performance is not observed and the proposed approach performs better in solely one objective function). In the case where($\varphi, \beta, \gamma_i, \psi_{jk}, \delta_{jk} = 0.8$), a superior performance is obtained by the proposed approach in 6 out of 7 weights combinations (except for C7). It gives the best values for all objective functions in comparison to the other methods. The performance of the proposed method in the case of C7 is equal to LH method. Finally, once ($\varphi, \beta, \gamma_i, \psi_{jk}, \delta_{jk} = 0.9$), again a better performance is obtained by the proposed approach in 5 out of 7 weights combinations (except for C4 and C7). The proposed approach delivers the best values for all objective functions compared with the other approaches. In the case of C4 and C7, the proposed approach gives a better objective function value in two objective functions comparing with the other approaches.

The performances of the applied approaches can be comprehended using the determined distances values as well. The smaller value for a distance correspond to a better performance. In this regards, for the case where ($\varphi, \beta, \gamma_i, \psi_{jk}, \delta_{jk} = 0.6$), the proposed approach delivers a better performance strictly in all combinations of weights except for C4. When the parameters are set to $\varphi, \beta, \gamma_i, \psi_{jk}, \delta_{jk} = 0.7$, approximately the same performance is repeated in 2 out of 7 combinations by the proposed approach having a precise performance supremacy comparing with the other approaches. The dominance of the proposed approach in comparison with the other methods in the literature is correspondingly proved once $\varphi, \beta, \gamma_i, \psi_{jk}, \delta_{jk} = \Omega \in \{0.8, 0.9\}$. In these cases, the proposed approach in all combinations of weights (except for C7), delivers a superior performance in terms of all distances values in comparison with the other methods.

Table 25. The obtained objective functions by the proposed approach and the methods of literature with different weight combinations and Ω ∈ {0.6,0.7}

Weights	Objective function	$\varphi,\beta,\gamma_i,\psi_{jk},\delta_{jk} = o.6$					$\varphi,\beta,\gamma_i,\psi_{jk},\delta_{jk} = 0.7$				
		ABS	DY	LH	SO	Proposed	ABS	DY	LH	SO	Proposed
C1	OF_1	1.18E+08	1.89E+08	1.19E+09	1.29E+08	**4.83E+07**	6.43E+07	1.94E+08	4.59E+07	4.99E+10	**4.83E+07**
	OF_2	1.57E+08	1.62E+08	4.13E+08	1.61E+08	**1.49E+08**	1.55E+08	1.65E+08	1.48E+08	2.50E+10	**1.49E+08**
	OF_3	**8.56E+04**	**8.56E+04**	4.54E+05	**8.56E+04**	1.01E+05	**9.34E+04**	9.37E+04	9.37E+04	2.40E+08	1.01E+05
C2	OF_1	1.19E+08	4.78E+10	1.20E+09	1.29E+08	**4.83E+07**	9.27E+07	4.78E+10	4.63E+07	4.99E+10	**4.83E+07**
	OF_2	1.57E+08	1.61E+10	4.15E+08	1.61E+08	**1.49E+08**	1.57E+08	1.63E+10	1.50E+08	2.50E+10	**1.49E+08**
	OF_3	**8.56E+04**	1.45E+08	4.64E+05	**8.56E+04**	1.01E+05	**9.36E+04**	1.45E+08	1.04E+05	2.40E+08	1.01E+05
C3	OF_1	2.11E+08	4.78E+10	1.20E+09	1.29E+08	**4.83E+07**	4.55E+08	4.78E+10	**4.59E+07**	4.99E+10	4.83E+07
	OF_2	1.65E+08	1.61E+10	4.15E+08	1.61E+08	**1.50E+08**	1.97E+08	1.63E+10	1.52E+08	2.50E+10	**1.50E+08**
	OF_3	**8.56E+04**	1.45E+08	4.64E+05	**8.56E+04**	1.01E+05	**9.36E+04**	1.45E+08	9.37E+04	2.40E+08	1.01E+05
C4	OF_1	2.11E+08	5.20E+07	**4.78E+07**	5.20E+07	4.80E+07	4.87E+07	4.78E+10	**4.59E+07**	4.99E+10	4.87E+07
	OF_2	1.65E+08	1.48E+08	**1.46E+08**	1.48E+08	**1.46E+08**	**1.48E+08**	1.63E+10	1.52E+08	2.50E+10	**1.48E+08**
	OF_3	8.56E+04	9.07E+04	**8.55E+04**	9.07E+04	1.01E+05	1.02E+05	1.45E+08	**9.37E+04**	2.40E+08	1.02E+05
C5	OF_1	2.10E+08	2.06E+08	1.19E+09	1.29E+08	**4.57E+07**	6.43E+07	2.12E+08	4.59E+07	4.99E+10	**4.83E+07**
	OF_2	1.65E+08	1.62E+08	4.10E+08	1.61E+08	**1.50E+08**	1.55E+08	1.65E+08	**1.48E+08**	2.50E+10	1.49E+08
	OF_3	**8.56E+04**	**8.56E+04**	4.53E+05	**8.56E+04**	1.01E+05	**9.34E+04**	9.37E+04	9.37E+04	2.40E+08	1.01E+05
C6	OF_1	2.10E+08	2.06E+08	1.19E+09	1.29E+08	**4.87E+07**	6.43E+07	2.12E+08	2.12E+08	4.99E+10	**4.87E+07**
	OF_2	1.65E+08	1.62E+08	4.13E+08	1.61E+08	**1.49E+08**	1.55E+08	1.65E+08	1.65E+08	2.50E+10	**1.51E+08**
	OF_3	**8.56E+04**	**8.56E+04**	4.54E+05	**8.56E+04**	1.01E+05	**9.34E+04**	9.37E+04	9.37E+04	2.40E+08	1.02E+05
C7	OF_1	2.11E+08	4.78E+10	1.56E+09	1.29E+08	**5.79E+07**	4.55E+08	4.78E+10	6.43E+07	4.99E+10	**4.87E+07**
	OF_2	1.65E+08	1.61E+10	2.20E+09	1.61E+08	**1.57E+08**	1.97E+08	1.63E+10	1.51E+08	2.50E+10	**1.50E+08**
	OF_3	**8.56E+04**	1.45E+08	3.60E+07	**8.56E+04**	1.01E+05	**9.36E+04**	1.45E+08	**9.36E+04**	2.40E+08	1.02E+05

Table 26. The obtained objective functions by the proposed approach and the methods of literature with different weight combinations and $\Omega \in \{0.8,0.9\}$

Weights	Objective function	$\varphi, \beta, \gamma_i, \psi_{jk}, \delta_{jk} = 0.8$					$\varphi, \beta, \gamma_i, \psi_{jk}, \delta_{jk} = 0.9$				
		ABS	DY	LH	SO	Proposed	ABS	DY	LH	SO	Proposed
C1	OF_1	1.59E+08	1.99E+08	5.61E+10	4.99E+10	**4.86E+07**	1.69E+08	1.09E+08	3.99E+09	2.98E+08	**4.90E+07**
	OF_2	1.68E+08	1.68E+08	2.21E+10	2.53E+10	**1.52E+08**	1.72E+08	1.62E+08	9.72E+09	1.87E+08	**1.55E+08**
	OF_3	1.68E+08	**1.01E+05**	3.86E+08	2.40E+08	**1.01E+05**	**1.09E+05**	**1.09E+05**	1.98E+08	**1.09E+05**	**1.09E+05**
C2	OF_1	1.59E+08	1.99E+08	3.29E+09	4.99E+10	**4.86E+07**	1.69E+08	1.09E+08	2.71E+09	2.98E+08	**4.90E+07**
	OF_2	1.68E+08	1.68E+08	4.48E+09	2.53E+10	**1.51E+08**	1.72E+08	1.62E+08	4.26E+09	1.87E+08	**1.55E+08**
	OF_3	**1.01E+05**	**1.01E+05**	1.78E+08	2.40E+08	**1.01E+05**	**1.09E+05**	**1.09E+05**	1.45E+08	**1.09E+05**	**1.09E+05**
C3	OF_1	2.91E+08	2.55E+08	1.78E+08	4.99E+10	**4.86E+07**	2.99E+08	1.05E+08	2.71E+09	2.98E+08	**4.90E+07**
	OF_2	1.83E+08	1.71E+08	1.69E+08	2.53E+10	**1.52E+08**	1.87E+08	1.62E+08	4.15E+09	1.87E+08	**1.55E+08**
	OF_3	**1.01E+05**	**1.01E+05**	2.71E+05	2.40E+08	**1.01E+05**	**1.09E+05**	**1.09E+05**	1.45E+08	**1.09E+05**	**1.09E+05**
C4	OF_1	2.91E+08	2.55E+08	4.86E+07	4.99E+10	**4.86E+07**	2.99E+08	1.05E+08	**4.73E+07**	2.98E+08	4.90E+07
	OF_2	1.83E+08	1.71E+08	1.51E+08	2.53E+10	**1.50E+08**	1.87E+08	1.62E+08	1.56E+08	1.87E+08	**1.53E+08**
	OF_3	**1.01E+05**	**1.01E+05**	**1.01E+05**	2.40E+08	**1.01E+05**	**1.09E+05**	**1.09E+05**	1.27E+05	**1.09E+05**	**1.09E+05**
C5	OF_1	1.65E+08	2.19E+08	4.78E+10	4.99E+10	**4.63E+07**	1.75E+08	1.09E+08	7.07E+07	2.98E+08	**4.90E+07**
	OF_2	1.69E+08	1.69E+08	1.79E+10	2.53E+10	**1.55E+08**	1.73E+08	1.62E+08	1.66E+08	1.87E+08	**1.54E+08**
	OF_3	**1.01E+05**	**1.01E+05**	2.31E+08	2.40E+08	**1.01E+05**	**1.09E+05**	**1.09E+05**	3.00E+05	**1.09E+05**	**1.09E+05**
C6	OF_1	1.65E+08	2.19E+08	**4.86E+07**	4.99E+10	**4.86E+07**	1.75E+08	1.09E+08	5.76E+10	2.98E+08	**4.90E+07**
	OF_2	1.69E+08	1.69E+08	**1.51E+08**	2.53E+10	**1.51E+08**	1.73E+08	1.62E+08	2.22E+10	1.87E+08	**1.54E+08**
	OF_3	**1.01E+05**	**1.01E+05**	**1.01E+05**	2.40E+08	**1.01E+05**	**1.09E+05**	**1.09E+05**	2.17E+08	**1.09E+05**	**1.09E+05**
C7	OF_1	2.91E+08	2.55E+08	4.86E+07	4.99E+10	**4.63E+07**	2.99E+08	1.05E+08	**4.73E+07**	2.98E+08	4.90E+07
	OF_2	1.83E+08	1.71E+08	**1.51E+08**	2.53E+10	1.55E+08	1.87E+08	1.62E+08	1.56E+08	1.87E+08	**1.54E+08**
	OF_3	**1.01E+05**	**1.01E+05**	**1.01E+05**	2.40E+08	**1.01E+05**	**1.09E+05**	**1.09E+05**	1.27E+05	**1.09E+05**	**1.09E+05**

Table 27. The distances of the obtained objective functions from ideal solutions for the proposed approach and the methods of literature with different weight combinations and $\Omega \in \{0.6,0.7\}$

Weights	Distance	$\varphi,\beta,\gamma_i,\psi_{jk},\delta_{jk}=0.6$					$\varphi,\beta,\gamma_i,\psi_{jk},\delta_{jk}=0.7$				
		ABS	DY	LH	SO	Proposed	ABS	DY	LH	SO	Proposed
C1	D_1	6.13E-05	1.01E-04	9.41E-04	7.23E-05	**1.77E-05**	8.47E-05	1.58E-04	**6.56E-05**	6.32E-02	6.85E-05
	D_2	4.40E-05	7.56E-05	6.67E-04	5.16E-05	**1.41E-05**	2.18E-05	7.73E-05	**8.72E-06**	4.32E-02	1.07E-05
	D_∞	3.00E-01	3.00E-01	3.01E-01	3.00E-01	**3.00E-01**	1.93E-05	6.97E-05	**8.54E-06**	3.65E-02	1.03E-05
C2	D_1	4.63E-05	3.78E-02	7.11E-04	5.37E-05	**1.24E-05**	1.44E-04	3.79E-02	1.20E-04	5.02E-02	**1.20E-04**
	D_2	3.38E-05	2.40E-02	5.06E-04	3.89E-05	**9.30E-06**	2.37E-05	2.40E-02	7.82E-06	3.13E-02	**6.94E-06**
	D_∞	2.00E-01	2.16E-01	2.00E-01	2.00E-01	**2.00E-01**	1.86E-05	1.75E-02	7.66E-06	2.43E-02	**6.54E-06**
C3	D_1	7.24E-05	3.46E-02	6.07E-04	4.54E-05	**1.28E-05**	3.06E-04	3.47E-02	1.48E-04	4.74E-02	**1.48E-04**
	D_2	5.34E-05	2.13E-02	4.20E-04	3.23E-05	**1.01E-05**	1.26E-04	2.13E-02	9.26E-06	2.94E-02	**7.81E-06**
	D_∞	2.00E-01	2.16E-01	2.00E-01	2.00E-01	**2.00E-01**	1.14E-04	1.58E-02	9.20E-06	2.43E-02	**7.61E-06**
C4	D_1	7.54E-05	8.81E-06	**5.74E-06**	8.81E-06	6.63E-06	1.44E-04	3.12E-02	1.44E-04	3.99E-02	**1.44E-04**
	D_2	6.40E-05	6.09E-06	**4.06E-06**	6.09E-06	4.30E-06	3.65E-06	2.00E-02	4.81E-06	2.39E-02	**3.65E-06**
	D_∞	1.00E-01	1.00E-01	**1.00E-01**	1.00E-01	1.00E-01	2.72E-06	1.75E-02	4.60E-06	1.83E-02	**2.72E-06**
C5	D_1	1.06E-04	1.02E-04	7.77E-04	5.97E-05	**8.49E-06**	1.02E-04	1.88E-04	**8.79E-05**	4.53E-02	8.99E-05
	D_2	9.44E-05	9.20E-05	6.49E-04	5.04E-05	**5.86E-06**	1.38E-05	9.44E-05	**3.54E-06**	3.06E-02	4.83E-06
	D_∞	1.00E-01	1.00E-01	1.00E-01	1.00E-01	**1.00E-01**	1.22E-05	9.38E-05	**2.85E-06**	2.75E-02	3.42E-06
C6	D_1	1.31E-04	1.24E-04	1.04E-03	8.05E-05	**1.78E-05**	5.91E-05	1.55E-04	1.55E-04	6.59E-02	**4.41E-05**
	D_2	1.01E-04	9.70E-05	7.54E-04	5.84E-05	**1.37E-05**	2.28E-05	9.97E-05	9.97E-05	4.57E-02	**1.33E-05**
	D_∞	3.00E-01	3.00E-01	3.01E-01	3.00E-01	**3.00E-01**	1.93E-05	9.38E-05	9.38E-05	3.65E-02	**1.28E-05**
C7	D_1	5.97E-05	2.78E-02	3.10E-03	3.51E-05	**1.40E-05**	3.06E-04	2.79E-02	1.76E-04	3.72E-02	**1.72E-04**
	D_2	4.88E-05	1.68E-02	2.00E-03	2.68E-05	**9.70E-06**	1.17E-04	1.67E-02	7.51E-06	2.14E-02	**4.38E-06**
	D_∞	1.00E-01	1.08E-01	1.01E-01	1.00E-01	**1.00E-01**	1.14E-04	1.32E-02	6.11E-06	1.37E-02	**3.97E-06**

Table 28. The distances of the obtained objective functions from ideal solutions for the proposed approach and the methods of literature with different weight combinations and $\Omega \in \{0.8, 0.9\}$

Weights	Distance	$\varphi, \beta, \gamma_i, \psi_{jk}, \delta_{jk} = 0.8$					$\varphi, \beta, \gamma_i, \psi_{jk}, \delta_{jk} = 0.9$				
		ABS	DY	LH	SO	Proposed	ABS	DY	LH	SO	Proposed
C1	D_1	8.91E-05	1.08E-04	6.37E-02	6.31E-02	**1.48E-05**	9.45E-05	5.22E-05	1.86E-02	1.75E-04	**1.42E-05**
	D_2	6.46E-05	8.07E-05	4.15E-02	4.33E-02	**1.16E-05**	6.84E-05	3.73E-05	1.42E-02	1.31E-04	**1.20E-05**
	D_∞	5.46E-05	7.28E-05	3.20E-02	3.65E-02	**1.10E-05**	5.77E-05	2.99E-05	1.37E-02	1.17E-04	**1.18E-05**
C2	D_1	6.67E-05	8.15E-05	1.09E-02	5.01E-02	**9.40E-06**	7.07E-05	3.88E-05	9.39E-03	1.32E-04	**1.01E-05**
	D_2	4.94E-05	6.27E-05	7.02E-03	3.14E-02	**7.05E-06**	5.23E-05	2.82E-05	6.04E-03	1.01E-04	**8.37E-06**
	D_∞	4.37E-05	5.82E-05	5.49E-03	2.44E-02	**6.36E-06**	4.61E-05	2.40E-05	4.47E-03	9.34E-05	**8.15E-06**
C3	D_1	1.07E-04	8.47E-05	6.85E-05	4.73E-02	**9.67E-06**	1.10E-04	3.15E-05	1.02E-02	1.09E-04	**9.60E-06**
	D_2	7.87E-05	6.45E-05	4.54E-05	2.94E-02	**7.73E-06**	8.06E-05	2.23E-05	6.82E-03	8.02E-05	**8.30E-06**
	D_∞	6.90E-05	5.92E-05	3.78E-05	2.44E-02	**7.38E-06**	7.04E-05	1.69E-05	5.59E-03	7.00E-05	**8.17E-06**
C4	D_1	1.11E-04	9.17E-05	6.20E-06	3.97E-02	**5.91E-06**	1.14E-04	2.98E-05	6.71E-06	1.13E-04	**5.23E-06**
	D_2	9.39E-05	8.00E-05	4.38E-06	2.39E-02	**4.17E-06**	9.59E-05	2.37E-05	4.95E-06	9.54E-05	**3.82E-06**
	D_∞	9.19E-05	7.90E-05	3.15E-06	1.83E-02	**3.04E-06**	9.39E-05	2.25E-05	4.73E-06	9.34E-05	**3.31E-06**
C5	D_1	8.08E-05	1.10E-04	4.03E-02	4.52E-02	**8.58E-06**	8.53E-05	4.33E-05	2.88E-05	1.60E-04	**6.44E-06**
	D_2	6.99E-05	9.91E-05	2.82E-02	3.06E-02	**6.24E-06**	7.36E-05	3.67E-05	1.82E-05	1.41E-04	**4.58E-06**
	D_∞	6.88E-05	9.84E-05	2.63E-02	2.75E-02	**5.32E-06**	7.25E-05	3.59E-05	1.48E-05	1.40E-04	**3.58E-06**
C6	D_1	1.05E-04	1.34E-04	**1.43E-05**	6.59E-02	**1.43E-05**	1.11E-04	5.81E-05	6.50E-02	1.99E-04	**1.45E-05**
	D_2	7.76E-05	1.05E-04	**1.08E-05**	4.57E-02	**1.08E-05**	8.20E-05	4.22E-05	4.48E-02	1.52E-04	**1.20E-05**
	D_∞	6.88E-05	9.84E-05	**9.74E-06**	3.65E-02	**9.74E-06**	7.25E-05	3.59E-05	3.17E-02	1.40E-04	**1.16E-05**
C7	D_1	8.80E-05	7.20E-05	**5.72E-06**	3.70E-02	6.96E-06	9.01E-05	2.42E-05	6.52E-06	8.96E-05	**5.02E-06**
	D_2	7.15E-05	6.06E-05	**4.12E-06**	2.14E-02	5.56E-06	7.31E-05	1.84E-05	4.89E-06	7.27E-05	**3.86E-06**
	D_∞	6.90E-05	5.92E-05	**3.43E-06**	1.37E-02	5.32E-06	7.04E-05	1.69E-05	4.73E-06	7.00E-05	**3.58E-06**

Chapter 5

CONCLUSION

5.1 Cutting Problem

In this study, a category of cutting problem which is highly in use in box production companies was investigated. The problem was formulated according to the objectives of the problem owners and a modified column generating algorithm was introduced to solve it efficiently. The man principles of the approach were minimizing the production waste, the production costs and maximizing the efficiency of the material selection for production.

One of the discussable issues within this tudy is a huge amount of improvement after running the proposed algorithm. The main reason behind this improvement could be the very fact that in the tarditional method of the company, it is more fashionable to use dedicated objects for producing the items whereas in the proposed method in this study, the obejct are utilized for produing multiple products. the other important factor is that in company's cutting software, the minimum number of required patterns for satisfying the demand are considered while in our study the most effective set of apllicable patterns are determined and employed. This method is easy to implicate, returns a very good answer and is applicable for a wide range of the similar problems in this industry with minor adaptation. In this study, the competition among the supplier for providing the raw material and the demand's uncertainty was not considered. Hence, the price competitive suppliers, the uncertainty of the demands and a different method for the material selection (such as maximizing the useable leftovers) seems to be a very interesting to be considered in future development of the study.

5.2 Fuzzy Possibilistic Modelling Approach

An important application of soft computing and operations research was studied in this paper. A new uncertain multi-criteria problem in carton box production systems is modeled and solved. A mixed integer linear formulation was proposed to select suppliers and raw materials of a carton box company for a planning horizon where the parameters like demand and raw material costs were uncertain. A fuzzy possibilistic approach was proposed to respect the uncertainties in the formulation in order to minimize three objective functions of wastage amount of raw material, raw material cost and product surplus simultaneously. A new multi-objective solution approach was proposed to solve the problem in comparison to four multi-objective optimization approaches such as LH, TH, So, and ABS methods of the literature. Computational experiments and sensitivity analysis which performed on real numerical data given by study case, showed the superior performance of the proposed approach comparing to the others.

As future study, the uncertainty of the problem can be further tackled using other methods like robust optimization and stochastic programming. As a combinatorial optimization problem, the deterministic version of the model can be attractive for operational researchers to solve its larger sizes applying any heuristic and meta-heuristic approach.

5.3 Necessity Chance-Constraint Programming Approach

An important application of soft computing and operations research was studied in this paper. A new uncertain multi-objective problem in carton box production systems is modeled and solved. A mixed integer linear formulation was proposed to select suppliers and raw materials of a carton box company for a planning horizon where the parameters like demand and raw material costs were uncertain. A necessity-based fuzzy chance-constrained approach was proposed to respect the uncertainties in the formulation in order to minimize three objective functions of wastage amount of raw material, raw material cost and product surplus simultaneously. A new multi-objective solution approach was proposed to solve the problem in comparison to four multi-objective optimization approaches such as LH, DY, SO, and ABS methods of the literature. Computational experiments performed on real numerical data given by study case showed the superior performance of the proposed approach comparing to the others.

As future study, the uncertainty of the problem can be further tackled using other methods like robust optimization and stochastic programming. As a combinatorial optimization problem, the deterministic version of the model can be attractive for operational researchers to solve its larger sizes applying any heuristic and meta-heuristic approach.

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