Incorporating Quality and Operational Factors in Ranking of Production Lines Using Data Envelopment Analysis

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ABSTRACT

The competition in the Fast Moving Consumer Goods (FMCG) industry is high, especially in the perishable goods sector. Manufacturers need to compete for the market share as the demand is limited. For companies to have competitive advantage, they need to operate efficiently, and ranking their production lines will help identify the efficient and most important lines that contribute to their efficiency.

This study aims to evaluate efficiency and ranking of production lines by incorporating both operational and quality factors using Ranking models in Data Envelopment Analysis (DAE). A new Modified ranking model is proposed by comparing standard ranking model and modified DAE models. Standard ranking model and modified version are used on a data which collocated from a beverage producing company in Cyprus. It is shown that the modified model will help to identify the efficient production lines. Also the results of the study will help management in proper resources distribution for efficiency improvement and budget planning.

The study shows that can production line is the most efficient production line among the five production lines evaluated under standard and modified DAE models, and Pet-2 and Premix line are ranked among the highest by the modified ranking models. The analysis shows that, to improve the efficiency and rank of production lines, combination of operational and quality factor needs to be improved together.

Keywords: data envelopment analysis, production lines, quality factors, operational factors, modified BCC, super efficiency.

ÖZ

Hızlı Hareketli Tüketim Malları (HHTM) sektöründeki rekabet, özellikle bozulabilir mal sektöründe yüksektir. Üreticilerin talep sınırlı olduğundan pazar payı için rekabet etmeleri gerekir. Şirketler rekabet avantajı elde edebilmek için etkin bir şekilde çalışmalıdır ve şirketlerin verimliliklerine katkıda bulunacak olan verimli ve en önemli hatları belirlemek için üretim hatlarını sıralamak şirketlere yardımcı olacaktır. Bu çalışma, Veri Zarflamaları Analizinde (DAE) Sıralama modellerini kullanarak operasyonel ve kalite faktörlerini birleştirerek üretim hatlarının verimliliğini değerlendirmeyi amaçlamaktadır. Yeni Modified ranking modeli, standart sıralama modelini ve modifiye DAE modellerini karşılaştırarak önerilmektedir. Kıbrıs'taki bir içecek üreten şirketin bir araya getirdiği bir veri üzerinde standart sıralama modeli ve modifiye edilmiş versiyon kullanılır. Değiştirilen modelin verimli üretim hatlarının belirlenmesine yardımcı olacağı gösterilmiştir. Çalışmanın sonuçları, yönetimin etkinlik geliştirme ve bütçe planlaması için uygun kaynak dağılımında yardımcı olacaktır. Çalışma, hem standart hem de modifiye DAE modelleri altında değerlendirilen beş üretim hattı arasında teneke üretim hattının en verimli üretim olduğunu ve Pet-2 ve Premix hattının modifiye modeli ile en yüksek üretim seviyesine geldiğini göstermektedir. Analiz, diğer üretim hattının verimliliğini artırmak için operasyonel ve kalite faktörünün birlikte geliştirilmesi gerektiğini göstermektedir.

Anahtar Kelimeler: veri zarflama analizi, üretim hatları, kalite faktörleri, operasyonel faktörler, modifiye BCC, süper verimlilik.

To

My Family

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LIST OF ABBREVIATIONS

CRS	Constant Return to Scale
DEA	Data Envelopment Analysis
DMU	Decision Making Units
FMCGs	Fast Moving Consumer Goods
FMCP	Fast moving consumer product
PPS	Production Possibility Set
SE	Super-efficiency
VRS	Variable Return to Scale

Chapter 1

INTRODUCTION

1.1 Problem Description and Proposed Solutions

In the past decade it has been reviewed that Fast Moving Consumer Goods (FMCGs) are the biggest sectors of the economics business world. The global market involves a number of aspects such as buying, selling inventory stock record, competitors, low quality product, huge capital investment, shortage and waste of product, food poisoning as a result of bad storage system, pest and human damage of product, poor transportations of product to their designated region, lack of machineries. All of these components pose a serious problem to the large market companies in the world. FMCGs can be classified as the daily-based sales which could be essential product or non-essential products. Such products include food, soft and hard drinks, toiletries and disposable products. But the most interesting about all theses FMCGs is that they have some common features like they are sold quickly because of the short life cycle and they are relatively cheap to be bought at low prices by consumer. FMCGs has contributed greatly in the market of carbonated soft beverage drink. As it was reviewed by (Beverage Digest, 2014), the sales trends series for the present year has shown that the consumers' scales of preferences toward carbonated soft beverages drink have been decelerated downwards due to recent health issues. However, some American medical researchers have currently indicated that the increased numbers of health issues of most consumers is due to excess of consumption of carbonate beverage soft drink. Since these drinks contain high proportion of the acid, it may lead to the fact that children would be susceptible to obesity issues. As it is shown in this article, the carbonated soft drink sales in the United States of America, which is considered the biggest market, has dropped by 3%, 2.5%, 1.4%, 1% and 0.5% from 2011 to 2015 respectively as illustrated in Figure 1 below.

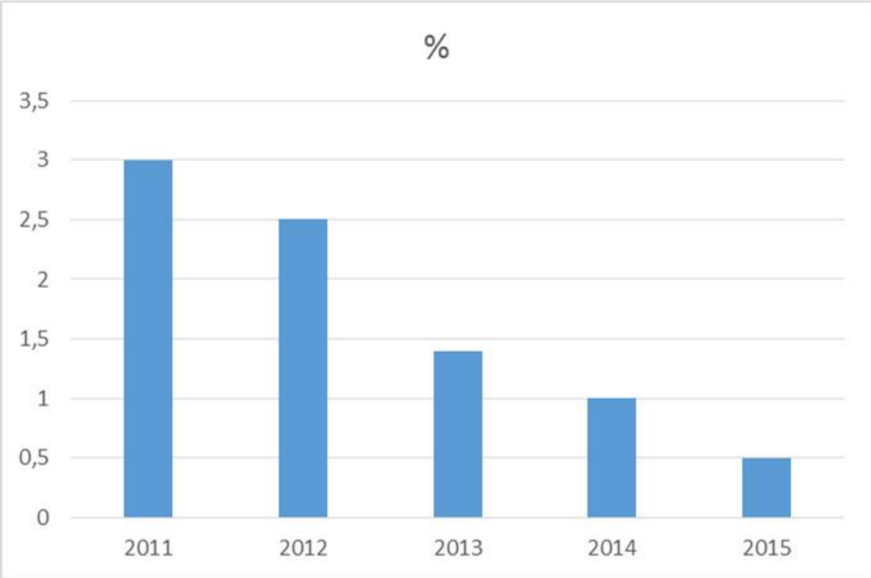

Figure 1: % Sales in USA Carbonated Soft Drink Market

Owing to the fact that consumers' awareness have risen, the danger of excess consumption of carbonated beverage by the medical practitioner that much of it contains a lot of chemicals such as methanol, phenol, tritium which, in turn, can affect some important tissues and organs in the body and that there should be more natural choice of food consumption instead of carbonated beverage drink, has caused resent to major companies and forced them to produce calories free product in order to satisfy consumers' desire or want. Even the Government, itself, has a major role to play in bringing an end solution towards a decrease in the amount of calories in manufacturing product. For instance, in 2010 Cypriot Ministry of Education had issued a warning notice to all institutes of learning that cafeterias of both primary, high school, colleges and universities stating the ban of sales of carbonated soft beverage drinks. But since then, only fresh foods, milk and water is allowed in the school cafeteria of most learning institutes in Turkey. All of these factors have a negative effect on the demand and sales of carbonated soft beverage drink. By referring to the situation in the FMCGs, a decrease in convexity of product and marketing of such product is getting rough in terms of human satisfaction and quality. However, some precautions are needed to be taken to cope with the increasing competition, which if not taken into consideration, may cause a serious shrinkage in the total consumption of FMCGs market economy.

Consequently, production lines need to be measured using Data Envelopment Analysis (DEA). The early description of Data Envelopment Analysis (DEA) was by Cooper and Rhodes (1978), as a mathematical programming model which employs observational data to provide a new way of obtaining empirical estimates of external relations toward possibly incorporating efficiency of production and quality of production which are the corner stones of modern economics. And also, Charnes et al., (1978), describe DEA as a technique that is used to evaluate the relative efficiency of comparable entities unit called Decision Making Units (DMUs). And also, they point out that DEA is used to identify the efficiency and inefficiency of input and output performance in relation to the production function**.** DEA models are capable of representing ranking of quality and operational factors based on their competencies. Two main models have been proposed in the literature which are the modified BCC and super-efficiency. In addition, the ideas of modifying the envelopment linear programming formulation is in relation to Banker (BCC model) that is an approach used for evaluating the efficiencies of Decision Making Units (DMUs) outcome, which implement DEA model efficiency score equal to one or less than one to inefficient DMUs. Anderson and Peterson (1993) have mainly proposed the DMUs ranking. The actual skills toward super-efficiency are to measure the efficiency of the qualitative units which is more than 1.

However, some proposed models were identified with modification to the BCC model such as: (Jahanshahloo, Lotfi, Shoja, Tohidi, & Razavyan, 2005) whose works deliberate on a new attempt in finding new stability region for efficient DMUs in Production Possibility Sets (PPSs), using supporting hyper planes of PPSs before and after elimination of the DMUs under evaluation from observed DMUs set. The use of facet analysis of Production Possibility Sets of modified BCC will help to develop all defining supporting hyper planes of efficient frontiers for BCC model. The modified BCC model is obtained from the classical BCC model when an upper bound is defined for its free variable u_o, using facet analysis.

All of these mentioned issues depend on the considered inputs and outputs in relation to quantitative and qualitative decision support system. In order to maintain this, it is important to evaluate and incorporate quality and operational factors when applied in fast dynamic products. This process could happen in a combination of the efficiency of mass-production line and flexibility of job market in order to produce a variety of work pieces on a group of machines. The basics idea in DEA is finding the best combination of prices and values to maximize the efficiency value of DMUs through taking the average weight of input and output factors. As a result, one way to reduce the cost of sales of beverages is to have an efficient operation. This could only be done by identifying and ranking of the production line. This would probably help in minimizing the financial aspect and maximizing the profit one.

1.2 The Purpose of the Study

Thus, this present study attempts to incorporate quality and operational factors in ranking of production lines, on their efficiency performance cycle, through finding the related criterions between quality and operational factors, using a specific model of DEA. These would be integrated as inputs and outputs of production lines so as to make a positive decision on the given data used, and to transform into a flexible manufacturing system designed to combine the efficiency of mass-production line and the flexibility of a job-Shop. On the basis of this, producing a variety of goods based on capability of a work station to respond quickly to the various requirements and expectations of the market would be more feasible. Additionally, this study aims to propose a new model for measuring super-efficiency of production lines by integrating ranking with modified BCC in order to tackle all methodological issues of the earlier version of BCC. This study has shed light on two fundamental problems for these models which are the unbounded solution and the large values retrieved which appear to be elusive. It is essential to note here that many research has been widely conducted in this area using ranking specifically with BCC but no research has applied ranking with the modified BCC. This could perhaps indicate a new trend for the investigation of production lines for FMCGs companies. And it may divert the decision makers' attention towards a different and promising perspectives in today's market.

1.3 Thesis Structure

Chapter 1 gives the problem definition in the FMCGs industry, followed by the statement of the study in a company. In Chapter 2 we will discuss the literature review of many relevant issues. The methodology which basically explores the efficiency and super-efficiency evaluation of the production lines is covered in chapter 3. Data collection procedure and efficiency and super-efficiency analyses are all presented in chapter 4, followed by discussion and conclusion in chapter 5.

Chapter 2

LITERATURE REVIEW

2.1 Fast Moving Consumer Goods (FMCGs)

Fast Dynamic Consumer Goods can be defined as goods that are relatively cheap in price and can be sold out quickly to the general public without any restriction, example are processed food soft drink and grocery (Ramanuji M. 2004).

Since we all the known that FMCGs are generally known for little life span, less expansive and the easily decay when exposed to free air such as grocery product and meat (Brierley, 2002). And the most unique aspect of Fast Dynamic Consumer Goods is that they are affected by some festival period and season, which makes the sales and demand higher for example meat which one need during festive period for his/her family. Though they are sold in large quantity, but their profit margin is relatively small to compare with other high capital goods.

Turkey is one of the most industrial target zone of business market of the world, and also one of largest economic growth in Asia in term of purchasing power. The Turkey FMCGs sector market size have exceed grow from 2013 to 2015 with US$11.1 billion plus, which is characterized with a design distribution network down to North Cyprus as island, where high demand for FMCGs is in high rate due to high growth population of students in the island. And the annual size of the island FMCGs market was estimated around $2.1 billion in 2010. Which sale account for about 5%

total goes for the grocery transaction, since most of the average in North Cyprus spends around 42% of his monthly income of fresh grocery good and 8% of the same class of student are the habitant who spend their monthly allowance on FMCGs personal care product, while the other remaining percentage goes to other kind of FMCGs. In which most of the product are exported from Turkey down to North Cyprus.

The general characteristics of FMCGs are:

- The all have little price cost.
- They are always needed or require for daily consumption by perspective consumers.
- They have low shelf life since the can easily decay or spoil in limited time.
- They require little or no effort to purchase then, since they are produced in high volume by the company and no shortage is involved.
- Low contribution margin to the company the cumulative total profit to the company account is substantial.

Various studies have focused on the efficiency operation of FMCGs companies. In investigating the effectiveness of FMCGs operation in Sri Lanka, Lakmal and Wickramarachchi (2011) focused on the relationship between factors that have an impact on the warehouse efficiency/effectiveness and on the total performance of their operation. Also, Paswan (2013) analyzed the financial performance using different measuring ratios with regard to solvency of specific FMCGs firms. By shedding light on marketing operations, Hezekiah et al (2016) studied advertising media efficiency of FMCGs companies using DEA in India. In addition, Testa et al (2016) measured the marketing performance of FMCGs companies. However, all of the above studies have focused specifically on efficiency of performance and less attention has been given to assess super-efficiency which is the primary purpose of this study.

2.1.1 Production Line

Over the past decade, production companies of most countries were nothing to write about since it is cumbersome to sustain high level of efficiency in production. As a result, many firms were not able to sustain such efficiency and due to their poor service level of supply has led them to close down. This, in turn, has resulted in the downfall of production with regard to quantity and quality of goods.

Production is said to be accomplished if it meets the desire and demand of the consumers. Nowadays, a lot of economic, social and global challenges are encountered such as increase in population, increase in competitiveness, lack of put through time, high operating cost, labor requirements, poor raw material adding to that the poor quality of the finished product. All of these factors may negatively affect any production system decision, application and professionalism in market price, quality and basically on the standard of goods and services. Based on what has been stated earlier, production lines can be defined as a set of sequential operations established in a factory whereby raw materials are put through a refining process or components are assembled to make a finished product.

Operation and quality are serious factors in the production line of most big companies such as food companies (Dotefoods.co), Pharmaceutical (Unilever) and chemical companies, which mostly deal with FMCGs. And such goods are produced fast with moderate prices index; such as, milk, soft drinks, toiletries, papers and beverages of different kinds.

In referring to the relative efficiency of production lines in relation to multiple inputs and multiple outputs, Liu et al. (2009) used DEA to evaluate thermal power plant operational performances where the efficiency is handled within an operational point of view. In total, operational performances of the thermal power plants were investigated between the years of 2004 to 2006. For the factory floor operations, DEA was utilized by Lin et al. (2009) to select a subset of potential product variants that can simultaneously minimize product proliferation and maintain market coverage. Efficient production lines and product variety selected with the results of the standard DEA model. Here, the product variations were under concern rather than production lines themselves in which they are utilized or bypassed according to the product mix. However, in the soft drink production plants which are under concern in this study, flow type production takes place and production line is constant throughout the process. In other words, it does not change with the alteration of the product mix.

2.1.2 Incorporation of Quality and Operational Factors

Quality is one of the most important tools which needs to be considered in most industry sectors in which one of these is perishable products that are highly competitive. And this could force us to check and improve quality measures every year to ensure a better production in the manufacturing of goods and service in the long run. As it is early indicated that the relevant measures of quality do not reside in the product, but it resides between the customer's ears. Since that even high technology operations factors won't get far in the market place if it is not produced with high quality. Apparently, quality and operational costs are serious factors in the production line of most FDCGs, especially the perishable products which was a secret described by business week as "America Built-In Quality of Perishable

Products". Many companies, however continue to define quality as relative to company service rather than to the customers' desire and which has been a huge problem in production line operation. This is because they want to make more profit by decreasing the cost of operation such as hiring more workers, sourcing more raw material, in adequate machines while others were neglected over the past decade resulting in inefficiency in production lines.

Nayar et al. (2008) utilized DEA approach to make a comparison on hospital efficiency and quality where specific quality measures are taken as output variables. In respect to the quality management aspect, Kuah et al. (2010) applied DEA to assess quality management efficiency where the steps for evaluating quality efficiency were described thoroughly, quality factors were introduced and improvement suggestions were given to the inefficient operations. Relative efficiency of an operation can be measured with DEA also with the contribution of the operational performances of each DMU. Subrahmanya et al. (2006) for example, studied the role of labour efficiency in promoting energy efficiency and economic performance with reference to small scale brick enterprises' cluster in Malur, Karnataka State, India. Önüt et al. (2006) used DEA to analyze energy use and efficiency in manufacturing sector where small and medium sized enterprises were studied for energy efficiency.

In review of the field, various versions of DEA models have been widely utilized for a variety of study areas, however, to the best of our knowledge it has not been used to evaluate production line efficiency of FMCGs manufacturing plant with the combination of operational and quality aspects of the process.

2.2 Data Envelopment Analysis

Date Envelopment Analysis (DEA) is a fundamental tool and methodology for improving operation functions in order to achieve a progressive long circle of competitiveness. DEA was first proposed and designed by (Charnes et al. 1978), was known as the CCR model and further implemented by (Banker et al, 1984) as BCC model, where both can be used to evaluate the efficiencies of Decision Making Units (DMUs). The variable of the criteria are selected based on the variables (Powers and McMullen, 2000). Mathematical models have been developed to evaluate the degree of performance criteria in relation to quality and flexibility (Nelson, 1986) and Data Envelopment Analysis were used in different cases as tools for analysis of companies advance technologies like flexible production and quality system (Ostadi and Rezaie, 2007). Which profound a mix kinetic programming model in production system using DEA to generate a simulation which comprises input data used to compute skills that will enhance output data (Sueyoshi and Shang, 1995). And the ideas of DEA were used for performance evaluation of flexibility in production system using several mathematical models to aid in decision making process (Sarkis, 1997).

Banker et al, (1984) have proposed what has been called the BCC model which is considered as an efficient tool for DEA analysis. The BCC model has been recognized to be a powerful tool in DEA analysis. It fully demonstrates how to alter a quotient linear measure of efficiency into a Linear programming function in align with the basic of multiple inputs and outputs so as to implement Decision Making Units (DMUs). Even when the production function was unknown and the non-parametric approach help toward solving the linear programming formulation per DMUs, the average weight given to each linear aggregate can result to the corresponding Linear programming. Additionally, the weight is chosen on the condition to be specific to the positive view of DMUs weight having more than 100 % efficient on the boundary enveloping input-output variable scale.

2.3 Modified BCC Model

Several works have been done towards checking the efficiency of decision in linear programming, which relate to estimating empirical production frontier in making decision unit by Charnes, Cooper and Rhodes (1978). DEA has also been used in comparing efficiency across firms by estimating the marginal productivity in production (Brockhoff, 1970). The application of DEA in distribution industries of electricity can be channel from one region of a particular place to another region, and can be finally spread all over the circuit within industry (Jamash, T. J., Pollitt, M.G.2001). How increase or decrease in efficiency of output level and input size can effect model specification and exclusion of variables on affects the results (Berg, 2010). DEA is also used to assess the efficiency of general public especially in non-profit organization (Kuntz, Scholtes and Vera, 2007). The used of multipliers to evaluate cross efficiency DMUs (Doyle and Green, 1994) toward problems due to inefficiencies units.

Among the various recent studies that have focused on the placement of frontier for the modified BCC were the ones conducted by Davenshar (2009) and Davenshar et al (2014). In the former, the emphasis was only on the weak part in which they suggested a new placement of weak efficient parts of the frontier in permitting hyperplane regions by employing facet analysis in real life context. As a result, efficiency values for DMUs were modified on the weak parts of frontier. While the latter study, they identified a stability region for multiple input and output in order to keep the efficiency values of efficient DMU which are located on the intersection of efficient and weak efficient frontier illustrated by a numerical example.

Based on this recent research, it was found that there has been some classification of DMUs which changes due to issues of sensitivity and stability of DEA models categories. At this point arise some proposed models with modification to the BCC model such as: Jahanshahloo, Lotfi, Shoja, Tohidi, and Razavyan (2005) whose works depend on a new attempt in finding new stability region for efficient DMUs in Production Possibility Sets (PPSs), using supporting hyper planes of PPSs before and after elimination of the DMUs under evaluation from observed DMUs set. The use of facet analysis of PPSs of modified BCC will help to develop all defining supporting hyper planes of efficient frontiers for BCC model. The modified BCC model is obtained from the classical BCC model when an upper bound is defined for its free variable u_o, using facet analysis.

Furthermore, most of the recent studies agree on the most common features of modified BCC model over BCC model:

1. The result of modified BCC is subjected to sensitivity to the selection of the inputs and outputs.
2. The numbers of efficient units on the frontier tend to increase with the numbers of inputs and outputs variable.
3. Modified BCC provide performance bench marking indicator along with the set of diagnostic for identifying the problem and the inefficiency.
4. Modified BCC enables us to prove the extreme efficiency units K to achieve an efficiency weight greater than 1 by removing the constraint.
5. Modified BCC can always be used to test for the best selection techniques.

6. Modified BCC do not need any bench mark for efficient DMUs since the common set weight is always optimal.

2.4 Super-efficiency Analysis Techniques

Super-efficiency techniques have the ability to rank both DMUs of efficient and inefficient input and output variable in ranking DMUs which was developed by (Anderson & Petersen 1993). Base on real life cases, effort has been applied to discriminate true actual performance of efficient Decision Making Units from artificial ones. However, this has yielded some irregular facets in PPS. This emerging irregularity in determining the feasibility or convexity constraint has led to the finding of this concept by Andersen & Petersen (1993) who have ranked extreme efficient units by omitting them from PPS.

In addition, previous studies for super-efficiency BCC model (Thrall and Zhu.1996) clearly defined the infeasible solution for ranking DMUs in relation to the AP model using some inputs unit as zero (Seiford and Zhu, 1999). Then it is important to identify the essential and satisfactory conditions for the infeasibility of different super-efficiency of DEA models under various assumptions of Variable Returns to Scales (VRSs). But the main differences were exposed between infeasibility and Variable Returns to Scales grouping of DMUs (Mehrabian et, al. 1999). And the main goal of establishing a super-efficiency model is to handle the problem with the infeasible, but it is advisable to change the level of ranking when the inputs of some inefficient DMUs change.

Due to random issues on super-efficiency, some studies have been put forward as a proposed new model to resolve super-efficiency problem (Tone, 2002). The newly standard super-efficiency model is approved towards measuring the efficiency of DMU that explores the slacks-based measure of efficiency. While the deficiency of this model lies in that ranking DMUs is due to the existence of zero values in any inputs, it will result in onerous computation process. Chen (2005) suggested that both input-oriented and output-oriented super-efficiency models are needed to fully characterize the super-efficiency of the evaluated DMUs. However, it does not differentiate the infeasible DMUs at all. Amirteimoori et al (2006) address an alternative super-efficiency index which is equivalent to slacks-based measure index of Tone (2002), but it may lead in infeasibility when zero values exist in data.

To sum up, it would be possible to conclude that the high quality and low costs of products are interrelated. Thus, quality and operational costs are considered as essential contributions to differentiation in products and services. The operation cost is challenging across various and competitive business-levels so it takes the form of a regression function between the supply chain and customer chain. Let's consider that the efficiency function of input will be targeted at achieving a high level of customer's satisfaction on a continuous basis, it may relatively lead to super-efficient output in products and services which is considered highly advantageous to firms. It is wise to achieve a more realistic approach by incorporating quality directly into cost of operation, which directly affect the transformation of outputs in a production technology in relation to demand of customers. All of what has been mentioned above is illustrated in the following diagram as in Figure 2 below. It shows the researcher's attempt to establish a clear picture of the operational process which in particular emphasizes on the pervasive nature of quality in relat`ion to production operation**.**

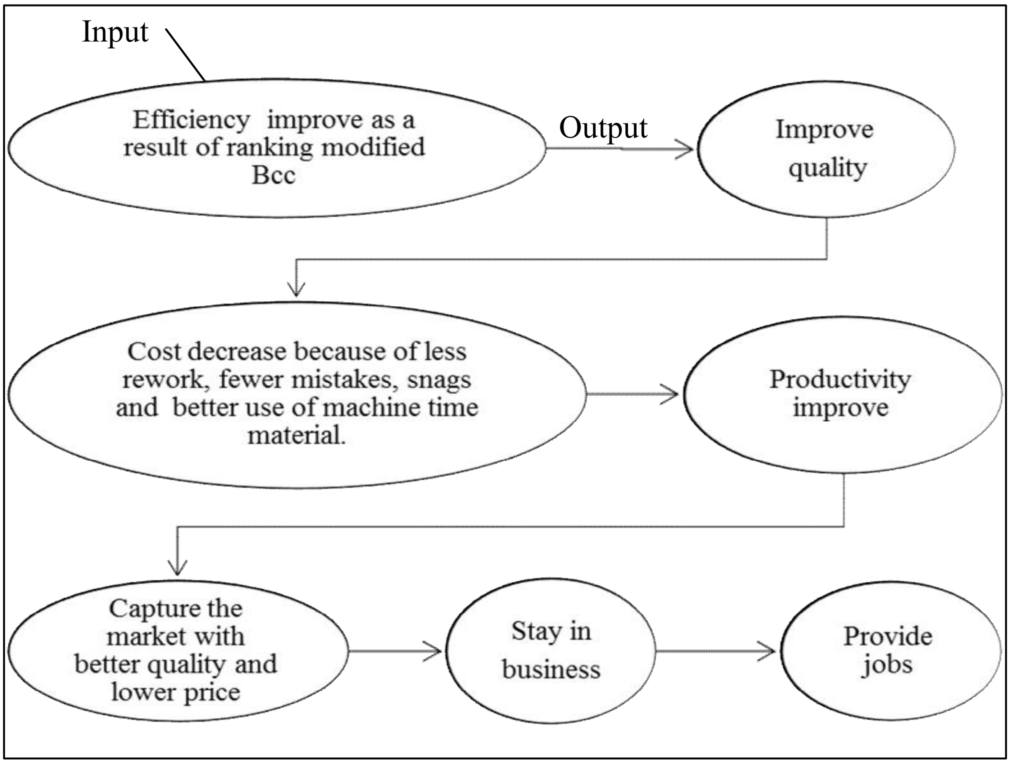

Figure 2: Pervasive Nature of Quality in Relation to Production Operation

Chapter 3

METHODOLOGY

As it is obvious, the basic notion of the current study is that if the timing of the production function and other given parameters of DMU are known, then it's is more easy to find the efficiency of the DMU. This is because if the DMUs are placed on the function, it will directly give us the feasible area of the efficiency for any DMUs, which are all located below the production curve of the efficient frontier of multiple input and output as in the diagram shown in fig. 5. But the issue on ground is that when the production function $f\left(x_1, x_2, \ldots.., x_j\right) = \left(y_1, y_2, \ldots.., y_n\right)$ of the DMU is not given in this study, so to find the efficiency of the DMUs we employed the postulate hypothesis of Production Possibility Sets, as it is clearly defined below.

In addition, we have been dealing with the pairs of positive input and output vectors (X_j, Y_j) (*j = 1, 2, n*) of *n* number of DMUs. The positive data assumption is clearly explained in such a way that all data are assumed to be nonnegative, but at least one component region of every input and output vector is positive. This phenomenon is known as semi-positive with a mathematical notation expressed by $X_j \geq 0$, $X_j \neq 0$ and $Y_j \geq 0,\ Y_j \neq 0$ for $j = 1, 2, n$. Therefore, each DMU is entitled to have a semi-positive input $X \in R^m$ and output $Y \in R^s$ as characteristics and to express the polar co-ordinate by the notation $(X\ , Y\ )$. The components of each vector (X_j, Y_j) can be regarded as a semi-positive output position in $(m+s)$ dimensional rectilinear vector space in which the superscript (m) and (s) specify the number of magnitude required to express inputs and outputs unit respectively. The sets of all possible feasible activities are known as PPSs, and are noted by *T*.

3.1 Production Possibility Sets (PPSs)

The Production Possibility Sets is well-defined as the set of all inputs and outputs element of a system in such a way that inputs elements are used to produce outputs. The PPSs of Data Envelopment Analysis model is characterized by two defined types of hyper planes (facets); strong and weak efficient facets. The definition of the strong hyper planes of the empirical Production Possibility Sets is very essential, because they can be used for determining rates of change of outputs in respect to change in inputs element in that system. Also, efficient hyper planes can also determine the nature of Returns to Scale Variable (RSV), which is the key suitable pattern for inefficient DMUs.

We can now generate hypothesis that centralize all Properties of *P* (Production Possibility Sets):

1. 1The logical feasible activities (X_j, Y_j) (j = 1, 2, n) be suitable to *P*.
2. If all the logical feasible activities $(X \ , Y \ )$ go to *P*, then the general activity $(tX \ , tY \ )$ also belongs to *P* for all positive scalar *t*. This property is known as continuous returns-to-scale (CRS) assumption.
3. For all feasible logical activity $(X \ , Y \ )$ in *P*, and any semi-nonnegative activity $(\overline{X} \ , \overline{Y} \ )$ with $X \geq \overline{X}$ and $Y \geq \overline{Y}$ are inclusive in *P*. This means that, any activity with input number $< X$ in any component position and with output number $> Y$ in any component position is feasible.

4. Any semi-positive undeviating combination of feasible activities within *P* goes to *P*.

In accordance to the data sets in matrices $X = (X_j)$ and $Y = (Y_j)$, we can define the Production Possibility Sets by satisfying (1) via (4) by

$$T_c = \{(X,Y) / \ X \geq \sum_{j=1}^{n} \lambda_j X_j,\ Y \leq \sum_{j=1}^{n} \lambda_j Y_j,\ \ \lambda_j \geq 0,\ j = 1,\ldots,n\}. \tag{3.1}$$

Where, λ = semi-positive vector in R^n .

Figure 3 (Tone, 2007), shows a typical Production Possibility Sets in two dimensional components for the single input unit and single output unit case, so that $(m\ \ ,s\ \ ) = (1,\ 1)$, respectively. In this example shown below, the possibility set is determined by *B* and the ray from the origin point *B* is the efficient frontier.

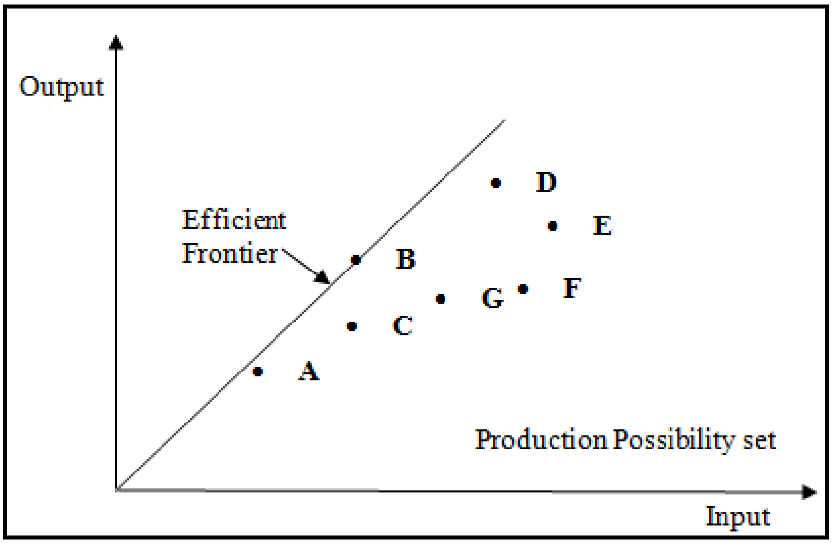

Figure 3: Production Frontier and T_C in a CCR Model

From the above graph of PPSs, the frontier of CCR model in Figure 3, let us assume that there are ' *n*' given DMUs to be determined using the index series of *j = 1,……,n*, for which each DMU is suspected to absorb *'m'* several input which is used to produce different outputs. Let $X_j = (x_{1j}, x_{2j}, ..., x_{mj})$ and $Y_j = (y_{1j}, y_{2j}, ..., y_{mj})$ be taken as inputs and outputs units vectors of a given DMU_j, all the components region of the units vectors are of positive value and each DMU has at least one observed to be non-negative input and output. If the units vectors $(X \ , Y)$ designate a production plan level, then PPSs of CCR model will be defined as:

$$T_C = \{(X, Y) / \ X \geq \sum_{j=1}^{n} \lambda_j X_j, \ Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \ \ \lambda_j \geq 0, \ j = 1, ..., n\}. \qquad (3.1)$$

After the analysis of the PPS efficiency of DMUs, the result of the DEA can now be determined using the hyper planes that define production efficiency frontier on envelope surface of DMUs, while those that do not lie on the frontier can be improved with some specified assumption, as under evaluation DMU, as DMU_o. We want to compute the maximum decrease in input values by preventing the same output in a manner that the new DMU_o remains in T_c. This means that:

$$\begin{aligned} &\min \theta \\ &\textit{subject to } (\theta X_0, Y_0) \in T_c \end{aligned} \qquad (3.2)$$

The formulation described above can be translated into a linear program, which can be resolved using linear programming model of input oriented format within the CCR model (Cooper, Charnes and banker, 1978) as shown below in model (3.3):

$$\begin{aligned} &\min \theta \\ &\textit{subject to} \\ &\textstyle\sum_{j=1}^{n} \lambda_j X_{ij} - \theta X_{i0} \geq 0 && i = 1, ..., m \\ &\textstyle\sum_{j=1}^{n} \lambda_j Y_{rj} \geq Y_{r0} && r = 1, ..., s \\ &\lambda_j \geq 0 && j = 1, ..., n \end{aligned} \qquad (3.3)$$

The dual of the above linear programming model can be written as:

$$
\begin{aligned}
& h_O = Max \quad \sum_{r=1}^{s} u_r y_{ro} \\
& subject\ to \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 && for\ j = 1,...,n \\
& \sum_{i=1}^{m} v_i x_{io} = 1 \\
& u_r \geq \varepsilon && for\ r = 1,...,s \\
& v_i \geq \varepsilon && for\ i = 1,...,m
\end{aligned}
\qquad (3.4)
$$

Where:

n is the number of DMUs.

O is the DMU being evaluated in the set of (j= *1, 2,…,* n DMUs).

h_o is the measure of efficiency of DMU_O, the DMU in the set of *(j= 1, 2,…..,* n rated relative to the others).

y_{ro} is the amount of output r produced by DMU_O.

x_{io} is the amount of resource input i used by DMU_O.

y_{rj} is the amount of service output r produced by DMU_j. When the level of r-type input is use for DMU_j.

Xij is the amount of service input i used by DMU_j. When the level of i-type input is use for DMU_j.

u_{ro} is the weight assigned to service output r, when computed in the solution of the DEA model (output-weight).

v_{io} is the weight assigned to resource input i, when computed in the solution of the DEA Model (input-weight).

M is the number of inputs used by the DMUs (when it does only assign the number of inputs data).

S is the number of outputs produced by the DMUs (when it does only assign the number of outputs).

ε is infinitesimal positive number, when the coefficients of the constraints in input and output are positive, hence removing the possibility that they will be given a zero *(0)* relative value in DEA results. (Always small positive Archimedean-infinitesimal parameter)

In addition, T_c model includes all possible feasible production plan levels, for which the CCR model helps to design its production frontier using the deviated linear mixing of the coexisting production plans level. Meanwhile, in the same view the BCC model has its production frontier spanned through by the convex hull of a given existing production plans as shown in Figure 4. The PPSs of a given BCC model is defined by;

$$T_B = \{(X,Y)/\ X \geq \sum_{j=1}^{n} X_j\lambda_j,\ Y \leq \sum_{j=1}^{n} Y_j\lambda_j, \sum_{j=1}^{n}\lambda_j = 1,\ \ \lambda_j \geq 0,\ j = 1,...,n\}. \qquad (3.5)$$

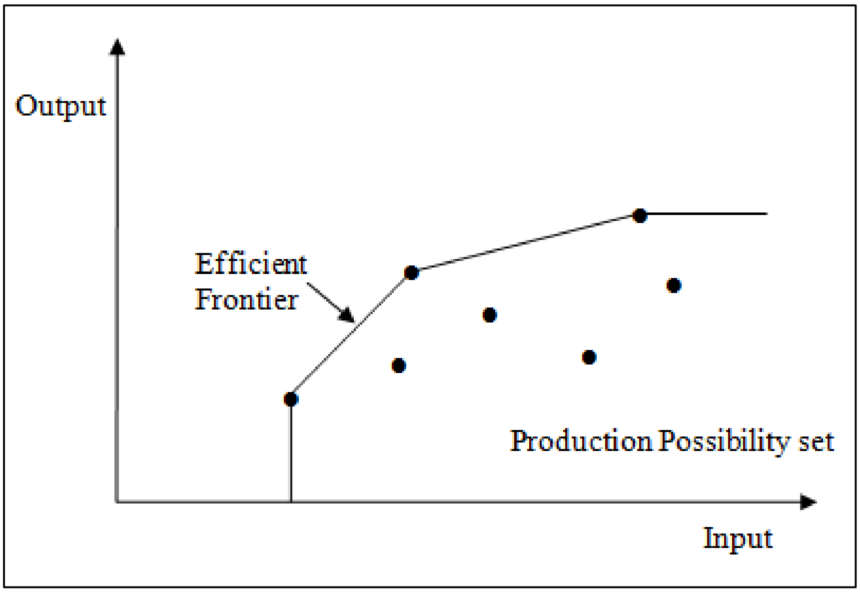

Figure 4: Production Frontier and T_B in a BCC Model

And the above BCC model is observed when a given DMUs has input and output units. If the relative efficiency of a given DMU falls within a given range of *(0- 1),* then the DMU is efficient and this only exists if there are no other DMU which uses fewer inputs to produce ≫ outputs.

In conclusion, we can say that as indicated in Figures 3 and 4 above, all the DMUs for production plans located on the production frontier are efficient.

3.2 Input Orientation of BCC Model

Input orientation in DEA means evaluating efficiency by minimizing the amount of inputs used by the production function. In most efficiency evaluating, input orientation is preferred because the companies or entities under evaluation have more control over their inputs than output. Thus, they can maximize efficiency by minimizing inputs while producing the same or equal amount of outputs (Martin and Roman, 2001). In the beverage producing industry where the managers are trying to increase profit, they can achieve efficiency by minimizing their resources and producing the same amount of output. All the models utilized in this thesis are all variable returns to scale (VRS) input orientation models. The VRS models are extension of the Constant Return to Scale (CRS) models. In particular, the BCC model is a VRS model while the CCR model is a CRS model (Charnes, Cooper, & Rhodes, 1978).

Proportional Reduction of inputs, while keeping the outputs proportions constant, is the general concept of input orientation. The CCR model efficiency score remains constant regardless of the form of orientation used. However, the BBC model of VRS takes into account the type of orientation used.

However, BCC model can cope with the present problem when assuming that the production function exhibits continuous return-to-scales. This gives rise to BCC model which is plus an additional constant variable u_0, in order to permit variable returns-to-scale as shown in the input oriented DMU related to PPSs' efficiency of set T_B linear programming model below in model (3.6):

$$
\begin{aligned}
& b_0^* = \min \ b_0 \\
& \textit{subject} \quad \textit{to} \\
& -\sum_{j=1}^{n} \lambda_j X_j + b_0 X_0 \geq 0 \\
& \sum_{j=1}^{n} \lambda_j Y_j \geq Y_0 \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0, \qquad\qquad j = 1, \ldots, n. \\
& b_0 \ \textit{free}
\end{aligned}
\tag{3.6}
$$

Where b_0^* is known as Radical Technical Efficiency (RTE) of a remark $(X_0, Y_0) \in T$.

The dual of the above linear programming model can be written as:

$$
\begin{aligned}
& h_O = Max \ \sum_{r=1}^{s} u_r y_{ro} + u_o \\
& \textit{subject to} \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_o \leq 0 \qquad \textit{for } j = 1,\ldots,n \\
& \sum_{i=1}^{m} v_i x_{io} = 1 \\
& u_r \geq 0 \qquad\qquad \textit{for } r = 1,\ldots,s \\
& v_i \geq 0 \qquad\qquad \textit{for } i = 1,\ldots,m \\
& u_o \, \textit{free}
\end{aligned}
\tag{3.7}
$$

This above model is the multiplier side of input oriented BCC linear model. It has clearly shown the efficient DMU in respect to the optimal solution of model (3.7)i.e. (U^*, V^*, u_0^*), to attain a feasible point which defines a supporting hyper plane for *T*.

The set of DMUs in the production frontier for BCC model (input or output orientation) can be subdivided into three (3) classes:

1) The strongly efficient DMUs

2) The efficient DMUs

3) The weak efficient DMUs

For which the strong efficient part consists of the DMUs which are directly located at the peaks of the frontier, while the efficient part consists of the set of efficient DMUs in which both input and output are efficient within the level of orientations and are not peaks. In contrast, the weak efficient portion consists of the set of DMUs which are efficient in the input orientation and inefficient in the output orientation or vice versa (Coelli T, 1996).

As we can see, that this paper focuses mainly on input orientation only, just as similar results can also be developed for the case of output orientation analysis.

3.2.1 Facet Analysis

Several works have been done on this area by many researchers who typically defined Return to Scale (RTS) only for single output measured condition (Banker, Charnes and Cooper, 1984; and Coelli, 1996). The extension of the notion of RTS to the several outputs situation is introduced by Banker, Charnes and Cooper (1984) and Coelli T (1996), who explicitly consider RTS only at the point that is radial technical efficient. They considered equivalent increases in input and output while keeping the input and output synthesis the same as for DMU in consideration, i.e. (X_0, Y_0). Let assume that (X_0, Y_0) is a DMU on the production frontier, which is being considered for evaluation, then we tend to direct our attention on the meeting of the Production Possibility Sets T and the hyper plane drawn from the point.

$$P = P(X_0, Y_0) = \{(X, Y) \mid X = \alpha X_0 , Y = \beta Y_0 , \alpha, \beta \geq 0\} \tag{3.8}$$

As it is clear in the following Figure 5 described by Daneshvar (2009)

$$P \cap T = \{(X,Y) \mid X = \alpha X_0 \geq \sum_{j=1}^{n} \lambda_j X_j \; Y = \beta Y_0 \leq \sum_{j=1}^{n} \lambda_j Y_j \,, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, \forall j, \alpha, \beta \geq 0\} \quad (3.9)$$

Now from the above equation let consider α and β are vertical, horizontal axes respectively. When P is considered in the new analysis α and β plane, the equivalent intersection set will be defined as:

$$\bar{T}(X_0, Y_0) = \{(\alpha X_0, \beta Y_0) \mid \alpha X_0 \geq \sum_{j=1}^{n} \lambda_j X_j, Y_0 \leq \sum_{j=1}^{n} \lambda_j Y_j \,, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, \forall j, \alpha, \beta \geq 0\} \quad (3.10)$$

Let U^*, V^* and u_0^* represent an optimal solution for the dual of BCC formulation for (X_0, Y_0). This DMU is radial technical efficient that is $b_0^* = 1$, so $U^* Y_0 + u_0^* = 1 = V^* X_0$ in input and output space.

The supporting hyper plane $U^* Y_0 + u_0^* = V^* X_0$ passes through (X_0, Y_0). The meeting of this supporting hyper plane and $\bar{T}$ is considered as the line $\beta(U^* Y_0) + u_0^* = \alpha(V^* X_0)$. If the unit of measuring magnitude of vectors $\|X_0\|$ and $\|Y_0\|$ are directed to be α and β co-ordinate axes respectively. Then the resultant line will pass through $(\alpha, \beta) = (1,1)$ for DMU under evaluation.

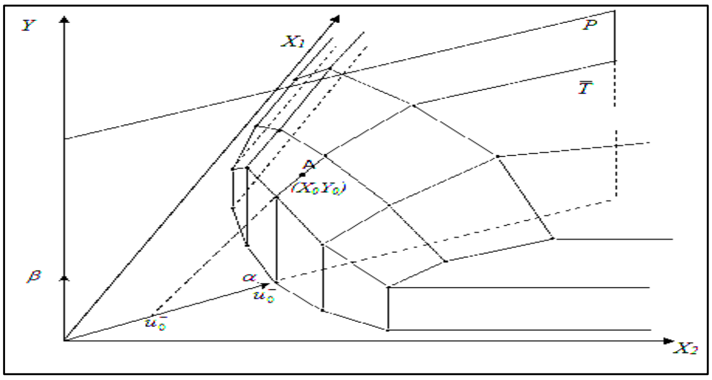

Figure 5: *P* and *T* for Two Inputs One Output

3.3 Super-efficiency Ranking BCC Model

Among various research that has been conducted on supper-efficiency DMUs, one of its great pioneers Andersen and Petersen in 1993 developed an innovative standard procedure for ranking efficient units. This approach enables an extreme efficient unit *'O'* to attain efficiency score which should be greater than one (1) by eliminating the constraint related to DMU_o in the primal formulation in model (3.7), as shown in model (3.11).

$$\begin{aligned}
& h_O = Max \quad \sum_{r=1}^{s} u_r y_{ro} + u_o \\
& subject \;\; to \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_o \leq 0 \qquad for\; j = 1,...,n,\; j \neq o \\
& \sum_{i=1}^{m} v_i x_{io} = 1 \\
& u_r \geq 0 \qquad for\; r = 1,...,s \\
& v_i \geq 0 \qquad for\; i = 1,...,m \\
& u_o \; free
\end{aligned} \tag{3.11}$$

What is wondering now in the input-oriented model is if the remaining DMUs in such a way can yield the outputs of DMU_o and it is also important to define input values that will be needed to achieve this approach. As it is mentioned earlier, we can see that any proportional increases in inputs will cause a great change in the required yield of the outputs of DMU_o, then the solution will always have $min\ h_o = h_o^* \geq 1$ with $h_0^* > 1$, which fully shows that increase in the input is always needed. The result obtained can be used for ranking of higher values of h_o* linked with DMUs that are most super-efficient.

In figure 6, there is an illustration for the input-oriented super-efficiency model where the efficient-frontier consists of a line-segments joining DMUs *B, C* and *D*. But if *DMU C* is omitted from the reference set, then this effect can be improved by constructing a new frontier consisting of the broken line-segment joining DMUs *B* and *D*. The super-efficiency of *DMU C* now becomes OC_0=OC > 100%. From the given expression, this simply implies that *DMU C* may possibly increase in inputs and still remain efficient. Figure 6 shows three (3) DMUs generating a separated single output, with a given particular strong two equal inputs $x_1 = x_2$.

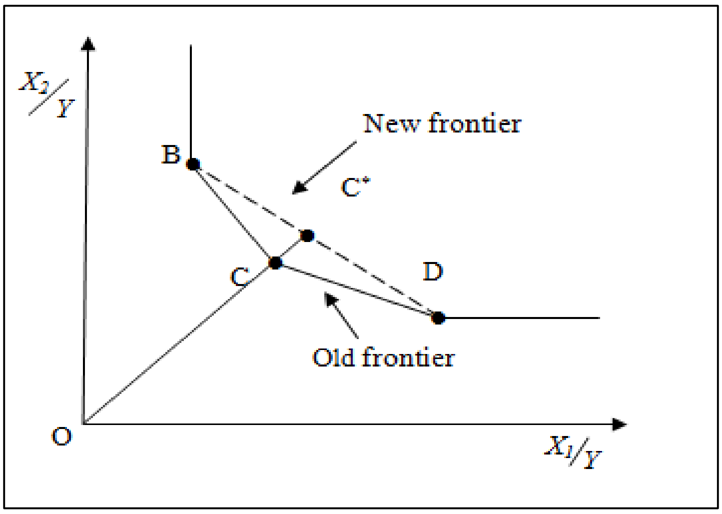

Figure 6: The Standard Super-Efficiency of *DMUC*

3.4 Modified BCC Model

Modification of models in DEA is a common scientific growth. These modifications are performed on the existing models to increase the robustness and generality of the models. As explained in the section 3.2 of the input orientation BCC model, the production frontier in DEA is divided into 3 parts, strong efficient, efficient and weak efficient. Daneshvar, Izbirak and Javadi (2014) commented that the standard BCC model proposes a biased efficiency score for DMUs located at the weak part of the production frontier. They proposed a model called the modified BCC model that takes into consideration the weak part of the efficiency frontier during efficiency evaluation. They suggest placing an upper bound on the free variable of the standard

BCC model. Accordingly, they first compute u_0^+ and u_0^- for all efficient DMUs using models (3.12) and (3.13) which are developed by Banker and Thrall**)** who described how the linear programming formulation in model (3.7) can be modified to determine these bounds. These modifications are presented below:

$$
\begin{aligned}
& u_0^+ = \max u_0 \\
& \textit{subject to} \\
& UY_0 + u_0 = 1 \\
& UY_j - VX_j + u_0 \leq 0 \qquad j = 1,\dots,n \\
& VX_0 = 1 \\
& U \geq 0 \\
& V \geq 0 \\
& u_0 \ \textit{free}
\end{aligned}
\tag{3.12}
$$

$$
\begin{aligned}
& u_0^- = \min u_0 \\
& \textit{subject to} \\
& UY_0 + u_0 = 1 \\
& UY_j - VX_j + u_0 \leq 0 \qquad j = 1,\dots,n \\
& VX_0 = 1 \\
& U \geq 0 \\
& V \geq 0 \\
& u_0 \ \textit{free}
\end{aligned}
\tag{3.13}
$$

In addition, we replaced ε as upper bound for free variable in BCC model (3.7). The definition of ε for any standard BCC model, the estimation of observed DMUs for efficient and strong efficient DMU is define as:

$$u_k^- \leq u_k^* \leq \varepsilon \leq u_k^+ \leq 1 \tag{3.14}$$

In order to do this, ε defines as follow:

$$\varepsilon = Max\{u_0^- \mid u_0^- \neq 1 \textit{ for all efficient DMUs}\} \tag{3.15}$$

After all of the above analysis regarding the modified BCC model (3.7), the following model is explored:

$$
\begin{aligned}
& z_0^* = \max UY_0 + u_0 \\
& \textit{subject to} \\
& UY_j - VX_j + u_0 \leq 0 \qquad j = 1,\dots,n \\
& VX_0 = 1 \\
& U \geq 0 \\
& V \geq 0 \\
& u_0 \leq \varepsilon
\end{aligned}
\tag{3.16}
$$

In the case where the DMU_k is weak efficient in relation to BCC model (3.6), this will cause DMUs to be located on the hyper plane of $u_0 = 1$ in PPSs. According to the recommended modified model (3.7) with respect to the constraint $u_0 \leq \varepsilon$, $(\varepsilon < 1)$, it is impossible to have any hyper plane at $u_0 = 1$, which in result DMU_k be unable to attain efficient in modified BCC model. But on the other hand, when the weak parts of frontier are modified then the efficiency of DMUs, when compared with parts of frontier, will also be modified too. The above illustration is one of the advantages of using modified BCC model.

3.5 Modified Super-efficiency Ranking Model

In ranking DMUs, a special model was designed by Andersen and Petersen (1993) (AP) who are great scholars towards developing a super-efficiency input orientation BCC model, which is basically centered on how to find the super-efficiency and ranking all DMUs as we explained in section 3.3. On other hand, when we applied this method on the data, the results of some DMUs are unbounded or have the biggest value. Because of that, with regard to the current data, it is not possible to rank the DMUs and satisfy the super-efficiency for each production line.

The value of super- efficiency in ranking the BCC model by AP method has two serious cases if the position of DMU at the end of frontier or very near to the axes. The first case can be described as if the position of DMU such as DMUA is near to the y-axis and the old frontier included DMUs *A, B, C* and *D*, the super-efficiency of *DMUA*, when the AP method is applied, would be by omitted *DMUA* from old frontier so by constructing a new frontier. After all this, a line will be drawn from the original point of axes which intersects with *DMUA* in the old frontier first, followed by the new frontier. The result from this intersection is manifested in the biggest value as shown in figure 7 below.

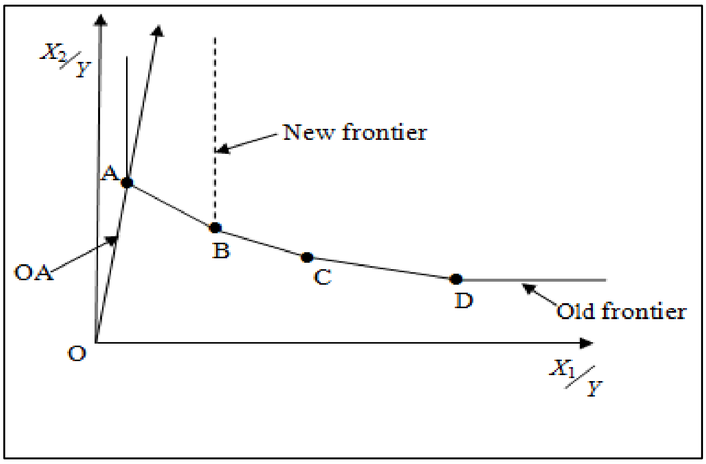

Figure 7: A Value Representation of the Biggest Super-Efficiency *DMUA*

The second case is shown in figure 8. The value of super-efficiency DMUA is unbounded in that it resulted in the DMUA being located on x-axis. Then, when the *DMUA* is omitted from old frontier, a new frontier will be constructed. As we can see in figure 8, the new frontier is parallel with the *OA* line, then the super efficiency of *DMUA* is infinite value “unbounded”.

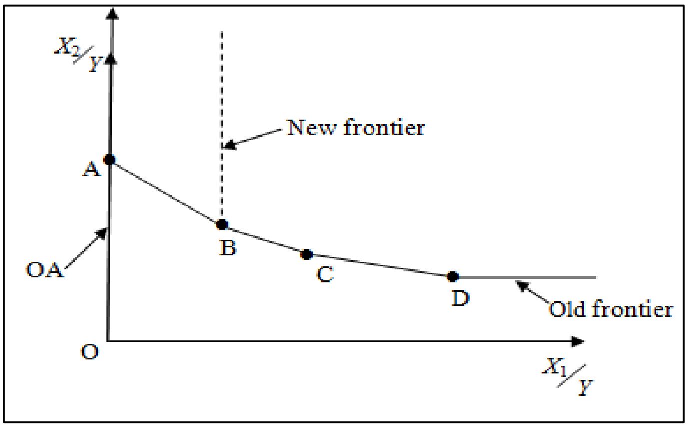

Figure 8: A Value Representation of the Unbounded Super-Efficiency *DMUA*

In the present study, a new model will be created for ranking super-efficiency. This could only done by applying the AP method on the modified BCC model which has as far as to our knowledge not been designed and applied before. This study attempted to find a feasible solution for each super efficiency DMUs which have unbounded "infeasible" or biggest value. Because in supper-efficiency, the modified BCC model could solve the problem for weak efficiency value for DMU_k by adding constraint $u_0 \leq \varepsilon, (\varepsilon < 1)$. Then, it would be possible to give a feasible supper-efficiency for each DMUs that have unbounded "infeasible" or biggest value.

The new model will be used for ranking supper-efficiency DMUs by applying the AP model on input orientation modified BCC model so as by eliminating the constraint related to DMU_o in the primal formulation in model (3.16) as it is shown below:

$$
\begin{aligned}
& z_0^* = \max \quad UY_0 + u_0 \\
& subject \quad to \\
& UY_j - VX_j + u_0 \leq 0 \qquad j = 1, \ldots, n, j \neq o \\
& VX_0 = 1 \\
& U \geq 0 \\
& V \geq 0 \\
& u_0 \leq \varepsilon
\end{aligned}
\tag{3.17}
$$

And by the new proposed model (3.17) illustrated above, we will rank supper-efficiency (SE) for all DMUs and put all DMUs in order. This makes it easier for us to find the super efficiency DMU. And Figure 9 shows how the modified super-efficiency works with the unbounded and biggest value for the output and input units. The figure has four extreme efficient DMUs, *A, B, C* and *D*. The BCC efficient frontier is *A, B, C*, and *D*, respectively. In SE-BCC model, if *DMU A* is removed from the BCC efficient frontier, we create a new frontier in order extract the value of super-efficiency DMUA. Basically, by applying the AP method with the new frontiers, the resulting values for both the standard and modified SE-BCC are significantly not the same. By such, the value of *DMU A* by intersecting of the new frontier for standard SE-BCC with *OA** line, is the biggest number or unbounded "infeasible". By contrast, the value of *DMUA* by intersecting of new frontier for modified SE-BCC with *OA*** line is feasible value.

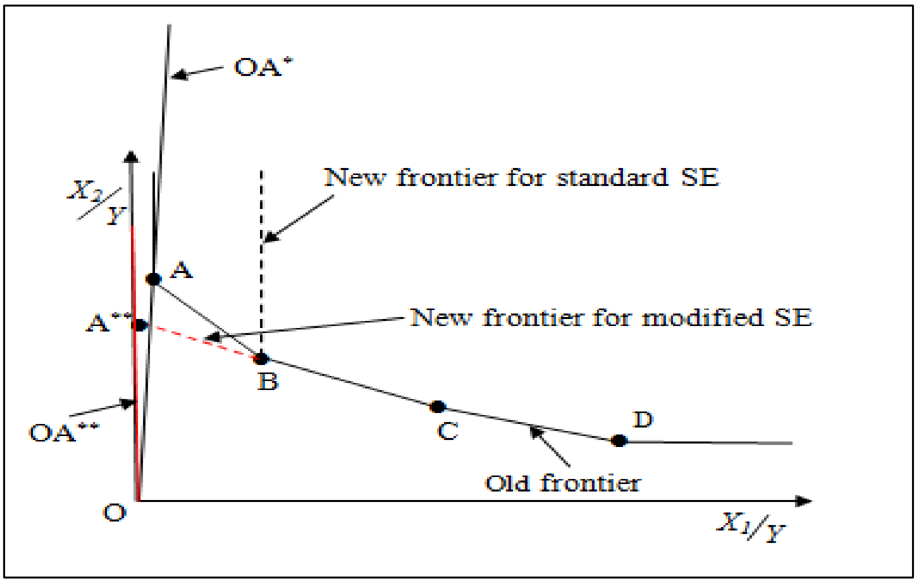

Figure 9: Modified (Newly Developed) Super-Efficiency for *DMUA*

Chapter 4

DATA COLLECTION AND EFFICIENCY ANALYSIS

Data for this research is a ready-made data which was previously collected by Saba (2016) from Ektam Kibris ltd. Ektam is a beverage producing company in Cyprus founded in the year 1982. It is one of the biggest industrial plants in the island. It produces soft drinks and holds more than 50% of the beverage markets sheer. The company produces 5 products which are considered as production lines as follows: Pet-6, Pet-2, Can, Glass Bottle and Premix lines.

This thesis focuses on ranking the efficiency of the production lines using DEA for the years 2010 up to 2015. Each production line is considered annually as a DMU and 5 DMUs on 6 years resulting in a total of 30 DMUs for efficiency evaluation.

4.1 Data Collection

This section summarizes the data in terms of definition and collection procedures for the efficiency analysis which was adapted from Ahmed (2016).

4.1.1 Input and Output Definition

The aim of efficiency evaluation and ranking is to identify the production lines that are efficient. Using historical data as a reference point for management, this can be achieved. Several factors contribute to the production process, therefore these factors all have a role to play in the efficiency of the production lines. However, all these factors cannot be considered in the efficiency evaluation.

Through a group discussion and brainstorming sessions with the management and engineers on the factory floor, the most critical and important factors are considered for the efficiency evaluation. These factors will help the management identify the ranking and performance of the production lines, thus improving the performance of the company in general through adjusting the future operations of the factory.

After the discussion and brainstorming sessions with the managers and engineers, the following factors are considered as inputs and outputs variables for the efficiency evaluation. As they believe that the following factors contribute the most to the efficiency of the production lines and ultimately the profit of the company. The inputs and outputs considered are as follows: and Table 1 gives their definitions.

Input variables

1. Amount of electricity consumed (Operational Factor).
2. Amount of fuel consumed (operational Factor).
3. Labor wages (Direct + Indirect labor) (Quality + Operation factor).
    - Direct workers who are working directly with each production line.
    - Indirect workers include the quality staffs, laboratory, management and maintenance workers.
4. Number of direct labor in each production line (Operational Factor).
5. Number of defective products for each production line (Quality Factor).

Output variables

1. Production Stock Keeping Units (SKUs), number of products approved by quality department for each production line (Quality + Operational Factor).
2. Production line contribution to income (Operational Factor).

Table 1: Inputs/ Outputs Definitions

Input Variables:	**Unit**	**Definition**
1. Amount of electricity consumed	KWh	Consumption of electricity by equipment in each production line
2. Amount of fuel consumed	Liter	Consumption of fuel by equipment in each production line
3. Labor wages (Direct + Indirect labor)	Turkish Lira	Labor wages (Direct + indirect) for each production line
4. Number of direct labor per production line	Numeral	Directly Number of labor in each production line
5. Number of defective products per production line	Numeral	Number of defected materials (Total) that are collected in each production line
Output Variables:	**Unit**	**Definition**
1. Production Stock Keeping Units (SKUs),	SKU	Total produced SKU of each production line
2. Production line contribution to income	Turkish Lira	Total contribution to income by each production line

4.1.2 Procedure for Collection of Data

In evaluating efficiency, these real-life data must be collected and calculated carefully. Some of the data cannot be directly extracted from the company's records and other data requires careful calculations to arrive at the desired value.

Inputs 1 and 2 are extracted by examining the energy (electricity and fuel) consumption report of the production lines. Human resources department provide the labor wages of the entire employees. The workers are categorized into two groups; first group includes the direct workers involved directly with the production lines, and the second group involves the indirect ones known as general utility workers. The general utility workers wages are distributed among the production lines using the Analytical Hierarchy Process (AHP). Using the weight distribution of each production line and the AHP method; data for input 3 is calculated. Input 4 is easily calculated by the head count of the direct labor involved in each production line. Data for input 5 was collected from the Quality Assurance Department annual report after categorizing them for each production line.

The data for output 1 is also from the quality assurance department report, by the summation of all confirmed quality satisfied products for each production line. The data for output 2 is from the sales department's annual sales and price change report. The data is obtained by multiplying the SKUs of each production line and the sales. Appendix A shows the complete data set used in the efficiency evaluation. To establish some sort of relationship between the inputs and outputs, and showing that these selected factors affect the efficiency of the production lines, a simple regression analysis is performed. Table 2 shows the result of the regression analysis.

Table 2: Inputs/ Outputs Correlation Matrix

	X1	**X2**	**X3**	**X4**	**X5**	**Y1**	**Y2**
X1	1	0.923	0.833	0.833	0.449	0.846	0.641
X2		1	0.934	0.813	0.532	0.948	0.8395
X3			1	0.868	0.436	0.916	0.876
X4				1	0.348	0.798	0.646
X5					1	0.679	0.620
Y1						1	0.9196
Y2							1
*(Note that **X** here refers to the inputs while **Y** refers to the outputs.)*							

4.2 Efficiency Analysis

In this section we evaluate the relative efficiency of the production lines using DEA models. The models used for the evaluation are classified into two groups, the standards models and the modified models. In the standards models, we evaluate the efficiency and ranking of the production lines using the standard BCC model, and the standard ranking model used is the supper-efficiency ranking model. For the modified evaluation, we used the modified BCC model and the modified super-efficiency model. The efficiency analysis for the production line was performed using the Lingo software for linear programming.

4.2.1 Efficiency with Standard BCC and Modified BCC Models

Using the standard BCC model (3.7) of Banker et al. (1984) and the modified BCC model (3.16) of Daneshvar et al. (2014). The efficiency of the five production lines over the six year period is evaluated. The modified BCC model identifies the DMUs whose efficiencies are exaggerated by the BCC model. This will show the DMUs that are weak efficient or compared to the weak efficient DMUs.

The BCC model efficiency is evaluated using model (3.7) and the modified BCC model efficiency is evaluated using model (3.16) and we consider the value of ε from equation (3.14).

$$\varepsilon = max\{ -83.612, 0.928\} = 0.928.$$

The efficiency scores of the production lines are shown in Table 3. The coding system using lingo for BCC and modified BCC is shown in Appendix B. In addition all of the coding lingo for u^- and u^+ and table 9 that includes the optimal value for both u^- and u^+ to determine the value of ε are given in Appendix D.

Table 3: BCC and Modified BCC Efficiency

Production Lines	DMUs	BCC Eff.	Modified BCC ($u_0 \leq 0.928$)
Pet-6 line	2010	1	1
	2011	0.96	1
	2012	1	0.959
	2013	1	0.958
	2014	1	1
	2015	1	1
Pet-2 line	2010	0.79	1
	2011	1	1
	2012	1	0.884
	2013	1	1
	2014	0.94	0.886
	2015	0.82	1
Can line	2010	1	1
	2011	1	1
	2012	1	1
	2013	1	0.989
	2014	1	1
	2015	1	1
Glass bottle line	2010	1	1
	2011	1	0.927
	2012	1	0.760
	2013	1	1
	2014	1	1
	2015	1	1
Premix line	2010	0.93	1
	2011	1	1
	2012	1	0.884
	2013	1	1
	2014	1	0.866
	2015	1	0.887

4.2.2 Efficiency with Super-efficiency and Modified Super-efficiency Models

Using the supper-efficiency model of Anderson and Peterson (1993) as shown in model (3.11), and modifying the model by placing a lower bound on the free variable as shown in model (3.17), the production lines are ranked according to this scheme.

Table 4 shows the result of the two ranking models used. The coding for super-efficiency and standard super-efficiency is shown in Appendix C.

Table 4: Super-Efficiency and Modified Super-Efficiency.

Production Lines	DMUs	Super-efficiency	Modified Super-efficiency ($u_0 \leq 0.928$)	Rank
Pet-6 line	2010	Unbounded	1.017	3
	2011	Unbounded	1.374	1
	2012	1	1	5
	2013	0.959	0.959	6
	2014	Unbounded	1.0104	4
	2015	Unbounded	1.344	2
Pet-2 line	2010	2.520	1.410	2
	2011	1.397	1.309	3
	2012	0.993	0.884	6
	2013	11.738	2.619	1
	2014	Unbounded	0.886	5
	2015	1.249	1.249	4
Can line	2010	Unbounded	1.085	3
	2011	Unbounded	1.069	4
	2012	1.0535	1.053	5
	2013	0.989	0.989	6
	2014	Unbounded	1.271	1
	2015	Unbounded	1.269	2
Glass bottle line	2010	1.117	1.0836	4
	2011	0.927	0.927	5
	2012	Unbounded	0.7605	6
	2013	Unbounded	2.488	1
	2014	Unbounded	1.399	2
	2015	Unbounded	1.229	3
Premix line	2010	Unbounded	1.109	2
	2011	Unbounded	1.059	3
	2012	0.963	0.884	5
	2013	9.045	8.857	1
	2014	Unbounded	0.866	6
	2015	Unbounded	0.887	4

Chapter 5

DISCUSSION AND CONCLUSION

In this section, results about the efficiency and ranking models are discussed. Further findings using the weights properties in DEA are used for recommendation for more improvement.

5.1 Discussion

This study focuses on evaluating the efficiency and ranking of five production lines in a beverage producing company.

5.1.1 DMUs Efficiency Results

The efficiency evaluation was performed using two models, the standard BCC model of Banker et al., (1984) model (3.7) and a modified BCC model of Daneshvra et al., (2014) model (3.16).

The modified BCC model is an extension of the BCC model that takes into consideration the weak part of the efficiency frontier. This modified model has a significant economic impact on management because it helps in differentiating the strong efficient production line from the weak efficient and highly inefficient. As explained in the modified BCC section of the available data, DMUs (production lines) are efficient and will not change their efficiency score. However, those DMUs that are weak or highly inefficient will change their efficiency score.

From Table 3 of the BCC and Modified BCC efficiency, the Pet-6 line efficiency for the years 2012 and 2013 changed from efficient to inefficient, Pet-2 line for 2012 changed from efficient to inefficient and 2014 which was inefficient became highly inefficient.

The Can line which was all efficient under BCC has one inefficient in 2013. The Glass bottle line changed from all efficient in BCC to two inefficient in 2011 and 2012. The Premix line changed from efficient in 2014 and 2015 to inefficient in both years. This shows that nine of the 30 DMUs changed their efficiency scores because they are weak efficient.

5.1.2 DMUs Super-efficiency Results

Two ranking models are used in this study to rank the production lines performance of the company. The Anderson and Peterson super-efficiency model (3.11) and a modified version of the model (3.17). The modified super-efficiency model has the same characteristics as the modified BCC model of Daneshvar et al., (2014), by taking into consideration the weak part of the frontier.

Table 4 reveals the results of the super-efficiency for the standard and the modified model. Using the standard model of super-efficiency, one cannot take an absolute decision for ranking all lines of production because the value of most super-efficiency for DMUs is unbounded. By contrast, the modified super-efficiency model shows Pet-6 line in 2011 as the highest with 1.374, followed by Pet-2 in 2013 with 2.619. According to this, line performance in Pet-6 in 2011 ranking is not unbounded as we saw in the standard super-efficiency model. In addition, in Pet-2 line is not as high as manifested in the standard super-efficiency model.

The next ranked line is the Can line in 2014 with 1.271, followed by the Glass bottle line in 2013 with 2.488. The final ranked line is Premix line in 2013 with 8.857. The significant difference between both the standard and modified super-efficiency in all lines; Can line, Glass bottle line and Premix line ranking is the fact that those production lines are not unbounded as it is clear in the standard super-efficiency model.

Accordingly, the above results would significantly help the management in having proper resources distribution for efficiency improvement and budget planning.

For example, in Pet-6 line is the highest in 2011 which means the company could adapt or manipulate the same strategy used in 2011 to achieve higher profit for the company with regards to having more production with lower costs.

5.2 DMUs Weight Calculation

Weights distribution in DEA is used to identify the variables that contribute the most to the efficiency during efficiency evaluation. Let us describe in detail the weight distribution for all efficiencies.

In Table 5, we can explain the weight distributions in relation to the BBC model which shows that in Pet-6, there is output2, inputs 4, 3, 2 that are the most efficient moving from the highest up to the lowest value. In a similar vein, Pet-2, Can, Glass bottle and Premix lines follow this exact pattern. However, output1 of all these production lines is insignificant with a weight of zero value.

Table 5: Weights for BCC Model

production lines	DMU	v1	v2	u1	u2	u3	u4	u5
Pet-6 line	2010	0	1.03	0	0	0.15	1.28	0
	2011	0	10.88	0	0.09	0.74	4.61	0
	2012	0	2.27	0	0	0	1.63	0.08
	2013	0	0	0	3.67	0	0	0
	2014	0	4.02	0.32	0	4.27	3.51	0
	2015	0	0	1.2	0	0	1.24	0
Pet-2 line	2010	0	10.88	0	0.09	0.74	4.61	0
	2011	0	2.27	0	0	0	1.63	0.08
	2012	0	0	0	3.67	0	0	0
	2013	0	4.02	0.32	0	4.27	3.51	0
	2014	0	0	1.2	0	0	1.24	0
	2015	0	17.48	0	0.15	1.2	7.41	0
Can line	2010	0	2.27	0	0	0	1.63	0.08
	2011	0	0	0	3.67	0	0	0
	2012	0	4.02	0.32	0	4.27	3.51	0
	2013	0	0	1.2	0	0	1.24	0
	2014	0	17.48	0	0.15	1.2	7.41	0
	2015	0	0.89	0.91	0	0.14	1.26	0.36
Glass bottle line	2010	0	0	0	3.67	0	0	0
	2011	0	4.02	0.32	0	4.27	3.51	0
	2012	0	0	1.2	0	0	1.24	0
	2013	0	17.48	0	0.15	1.2	7.41	0
	2014	0	0.89	0.91	0	0.14	1.26	0.36
	2015	0	1.89	0	2.06	1.28	0	0
Premix line	2010	0	4.02	0.32	0	4.27	3.51	0
	2011	0	0	1.2	0	0	1.24	0
	2012	0	17.48	0	0.15	1.2	7.41	0
	2013	0	0.89	0.91	0	0.14	1.26	0.36
	2014	0	1.89	0	2.06	1.28	0	0
	2015	0	4.49	0.27	0	5.5	2.47	0

The modified BCC model in Table 6 below explains the weight distribution as having output2, 1, with input 5 for Pet-6 line as the most significant variables. Pet-2 line has output 2, input1, 2, output1 and input 5, put in order, which include the highest value. With Can line, input1, 2, output1, input4 and output2 are having the greatest value. With regards to the Glass bottle line, there is output2, 1 and input2, 3, 5 as the most significant. The last line which is the Premix line, output1, input5, output2 and input3 are highly significant values within this line. Furthermore, the modified BCC model has identified three insignificant values which are input3 on Pet-2 line, input1, 4 on both Glass bottle and Premix lines. In all production lines, the most significant weight values are output2 and input1, 5.

By comparing the two tables, we found that the two models differ in input1, 5 in the standard BCC model have no obvious effect as opposed to the modified BCC model. The similarity between the two models is significantly revealed in ouput2 as having the highest value in them both.

Table 6: Weights for Modified BCC Model ($u0 \leq 0.928$)

Production lines	DMU	v1	v2	u1	u2	u3	u4	u5
Pet-6 line	2010	1	0	0.252	0	0.176	0.581	0
	2011	0	1.682	0	0	0	0	1.132
	2012	1.068	0	0.724	0	0	0	0
	2013	1.126	0	0	0	0.081	0.896	0
	2014	0.692	0.369	0	0.954	0	0	0
	2015	1.399	0	0	0	1	0	0
Pet-2 line	2010	0	36.913	7.539	0	0	0.617	1.214
	2011	0.412	37.274	7.656	0	0	0.612	1.379
	2012	2.243	28.484	0	15.355	0	0.121	0
	2013	5.811	86.108	37.167	1.0499	0	0	0.2161
	2014	2.498	0	1.0607	0	0	0	0.519
	2015	3.737	0	0	1.478	0	0	0.745
Can line	2010	0	1.330	2.369	0	0.378	0	0.839
	2011	0.855	0.596	0	2.149	0	0	0
	2012	0.982	0.486	0	0	0	1.833	0
	2013	0.779	0.612	0	0	0.32	1.461	0
	2014	1.519	0	0.76	0	0	0	1.085
	2015	1.898	0	0	0	0	1.833	0
Glass bottle line	2010	0	16.218	0	9.137	0	0	0.125
	2011	0.253	14.626	0	0	0	0	0.58
	2012	1.017	4.794	0	7.953	0	0	0
	2013	0	11.167	0	0	0	0	0.5
	2014	0	17.724	0	0	4.118	0	0.938
	2015	15.375	0	0	0	2.924	0	1.773
Premix line	2010	0	10.866	0	0.917	0	0	43.467
	2011	0	10.948	0	0	0	0	52.493
	2012	1.713	7.683	0	0.897	0	0	0
	2013	84.459	0	0	0	2.928	0	0
	2014	1.675	6.047	0	0	2.060	0	0
	2015	1.675	6.049	0	0	2.061	0	0

Table7 illustrates the super efficiency model for standard BCC for the weight distribution of all production lines. In Pet-6, the most significant values are input2, 3, 4 and output2, 1 while the insignificant ones are input1, 5 which have no value at all. Additionally, Pet-2 has input3, output2, input4, 2 and 5 as the highest revealed values whereas; the lowest values are within input1 which only has a direct effect on 2014.

Clearly, Can line shows the greatest values in input5, 2, 3 and output1, 2 which is different from Glass bottle line that has input5, 2 and output2, 1 as having the most evident high values. The insignificant values in Can line and Glass bottle line are input1 which shows a great effect on the year 2014. Adding to this, input4 for Glass bottle line has an insignificant value for all respective years except for 2014 but input4 has no significant effect in Can and Premix lines in all of the production years. But it is important to note here that Premix line has input5, 2, output1, 2 and input3 as the highest in weight value, compared with input1 which is insignificant in all production years except one year period 2015. In sum, let us give a clear indication of the highest weight values in all production lines which is evidently shown in input2 and output2, 1.

Table 7: Weights for Super-Efficiency BCC Model

Production lines	DMU	v1	v2	u1	u2	u3	u4	u5
Pet-6	**2010**	1	0	0	0	4.205	3.348	0
	2011	0	1.682	0	0	0	0	0
	2012	0	1.682	0	0.902	2.828	0.013	0
	2013	1.126	0	0	0	0.081	0.895	0
	2014	1.056	0	0	6.890	0.767	0	0
	2015	1.399	0	0	0	0.921	0	0
Pet-2	**2010**	0	36.91	0	0	81.121	0	16.77
	2011	11.798	0	0	0	0	0	4.595
	2012	4.340	24.23	0	18.192	0	16.98	0
	2013	0	85.47	0	0	464.57	0	0
	2014	2.498	0	13.158	0	0	47.21	0
	2015	3.738	0	0	2.738	0	0	0
Can line	**2010**	0	1.33	0	0	2.203	0	0.146
	2011	0	1.336	0	3.356	0	0	0
	2012	1.597	0	0	0	0	0	3.886
	2013	0.779	0.612	0	0	0.321	0	0
	2014	0	1	2.098	0	0	0	0
	2015	1.898	0	0	0	1.025	0	0
Glass bottle line	**2010**	0	16.218	0	29.457	0	0	0
	2011	0.253	14.627	0	0	0	0	0.58
	2012	0	6.124	0	15.008	0	0	0
	2013	0	11.167	0	0	0	0	5460.64
	2014	0	17.724	133.242	0	0	2.178	0
	2015	15.375	0	0	0	0	0	1.321
Premix line	**2010**	0	10.866	0	1111.3	0	0	4660.81
	2011	9.815	0	0	0	0	0	1.036
	2012	9.137	0	0	3.398	0	0	0
	2013	84.459	0	0	31.407	0	0	0
	2014	8.224	0	0	0	1.987	0	7.351
	2015	8.474	0	1.286	0	1.862	0	0

Table8 shows the weight distribution according to the modified super efficiency model. The production line Pet-6, there is output2, input5, output1 and input3,4 as the most significant while in Pet-2, we have output2, input2, output1 and input 3,1,5. In Can line, we have output2, input1, 2, output1, input3,4 whereas in Glass bottle line, there is input1,2,output2,1, and input3,5 as highly significant. In the last Premix line, we see output1, input5, 2, and output2 that show the greatest value. In explaining the insignificant values, we see these input1 in Pet-6 and Premix lines with zero effect revealed similar to Can and Glass bottle lines but with only one effect shown in 2014. And also, input4 has no significant impact on Glass bottle and Premix lines at all as well as Pet-2 and Can lines but these latter two lines show only one effect in one year period 2012 and 2013 respectively.

Table 8: Weights for Modified Super-Efficiency (u0 ≤ 0.928)

Production lines	DMU	v1	v2	u1	u2	u3	u4	u5
Pet-6 line	**2010**	1	0	0	0	0.1698	0.855	0
	2011	0	1.682	0	0	0	0	1.555
	2012	0	1.682	0	0	0	1.132	0
	2013	1.126	0	0	0	0.0813	0.895	0
	2014	0.698	0.363	0	1.0104	0	0	0
	2015	1.399	0	0	0	1.344	0	0
Pet-2 line	**2010**	0	37.037	5.416	12	0	0	1.226
	2011	10.069	5.702	5.927	0	0	0	3.269
	2012	2.244	28.484	0	15.354	0	0.121	0
	2013	7.044	65.058	0	0	8.978	0	0
	2014	2.498	0	1.060	0	0	0	0.519
	2015	3.738	0	0	2.738	0	0	0
Can line	**2010**	0	1.33	0	0	1.845	0	0.0448
	2011	0.47	0.928	0	2.297	0	0	0
	2012	1.597	0	0	0	0.414	0	1.036
	2013	0.779	612	0	0	0.32	1.461	0
	2014	0	1	3.319	0	0	0	0.395
	2015	1.898	0	0	0	1.832	0	0
Glass bottle line	**2010**	0	16.218	0	20.689	0	0	0.184
	2011	0.253	14.626	0	0	0	0	0.580
	2012	1.017	4.794	0	7.953	0	0	0
	2013	0	11.167	0	0	0	0	2.488
	2014	4.161	12.521	68.941	0	0	0	0
	2015	15.375	0	0	0	4.125	0	1.583
Premix line	**2010**	0	10.869	0	5.537	0	0	0
	2011	5.073	5.289	0	0	0	0	55.574
	2012	1.712	7.683	0	0.897	0	0	0
	2013	84.459	0	0	39.821	0	0	0
	2014	1.750	6.048	0	0	2.061	0	0
	2015	1.675	6.049	0	0	2.061	0	0

In sum, the most significant input is 5 and for outputs are 2 and 1 in all production lines for the modified super efficiency model. In comparison, the super efficiency for the standard BCC model gives us the finding that input2 as the most significant while, most importantly, outputs 2, 1 show the greatest estimate for both super efficiency models.

Looking at all the weight distributions suggested by the models, it can be concluded that the most contributing variables are output 2, 1 and input 1. Therefore, if the management wants to improve the efficiency of the production lines for the inefficient lines, they can start by improving the following variables: contribution to income (output 2), SKUs (output 1) and amount of electricity consumed (input 1) by increasing the output and decreasing input.

5.3 Conclusion

In this paper, we have focused on the practical applications of two specific models of DEA by shedding light on its modified nature. According to the standard BCC model, it is almost impossible to define which years are the most efficient while by using the modified super efficiency, this could be clearly yielded ranking them. Therefore, it could be concluded that for any FMCG companies, the implementation of the modified super efficiency in parallel with the modified BCC as an essential tool in examining and evaluating the production lines can significantly reveal efficiency and inefficiency with regards to operational and quality factors. This could only be achieved by having ranking for all production years. In addition, by using weight distribution values, the company's management can identify and improve the inefficient production lines by highlighting the most significant values in the input and output. In the light of all this, these significant values are distinguished as operational and quality factors. Based on all this, we could say that operational and quality factors are interrelated and have a significant impact on production lines of beverage producing companies.

5.4 Recommendation for Future Study

The complexities in efficiency evaluation create a room for further analysis, especially in the case of FMCGs where multiple factors and variables contribute to the efficiency of a production line. This research paper focuses on efficiency evaluation and ranking of the production lines using standard and modified models. The results proposed are complex for management to comprehend. A proposal for future research is that, further analysis such as Return to Scale (RS) analysis can be compared to the solution of weight distribution to see if both post efficiency analysis present parallel results for efficiency improvement of the inefficient production lines.

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APPENDICES

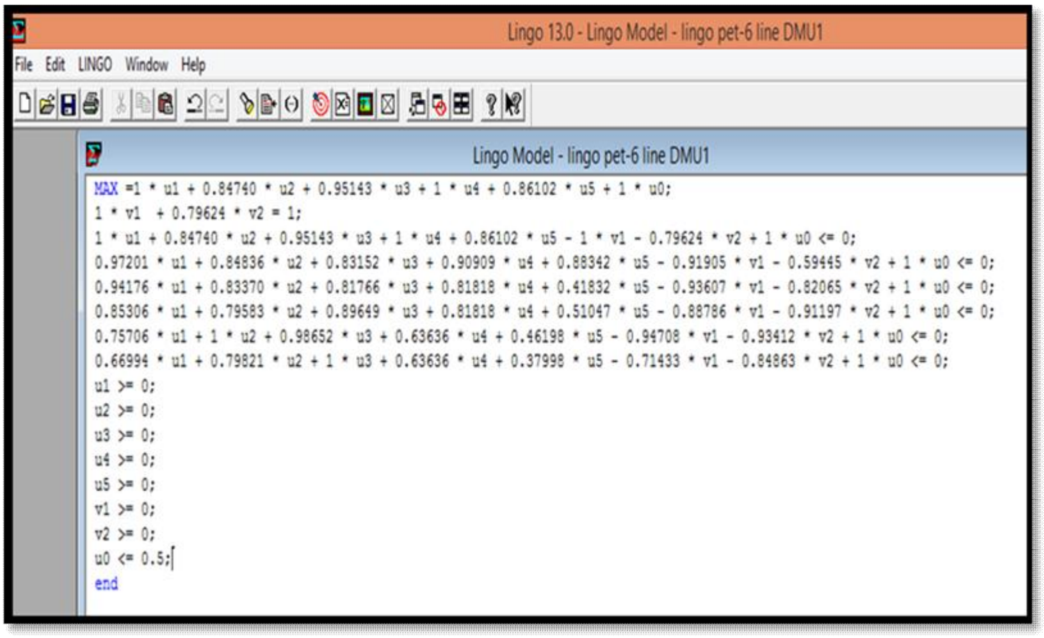
Appendix A: Data Set

Year	Production Line	DMUs	Input 1	Input 2	Input 3	Input 4	Input 5	Output 1	Output 2
2010	Pet-6 Line	DMU1	836191	48323	729,215.22	11	115235	779486	13,846,881.29
	PEt-2 Line	DMU2	33182	3665	208,132.51	5	46330	93896	471,150.00
	Can Line	DMU3	108773	25102	436,009.43	6	106928	518787	13,073,962.88
	Glass Bottle Line	DMU4	14422	2846	118,049.16	3	37039	77643	1,072,267.25
	Premix Line	DMU5	72280	11420	275,413.67	6	2513	85574	1,600,515.80
2011	Pet-6 Line	DMU6	812782	48378	637,309.70	10	118233	716385	10,337,609.31
	PEt-2 Line	DMU7	27507	1709	194,255.38	5	45598	66071	450,192.95
	Can Line	DMU8	109864	26535	398,676.46	6	105982	505381	13,013,013.82
	Glass Bottle Line	DMU9	13999	2538	106,691.63	3	37040	81866	1,156,515.12
	Premix Line	DMU10	69551	10224	249,610.83	6	2549	79419	1,588,380.00
2012	Pet-6 Line	DMU11	787492	47542	626,690.89	9	55986	729653	14,271,344.31
	PEt-2 Line	DMU12	21749	3080	204,686.23	5	14209	47865	526,322.40
	Can Line	DMU13	95105	23907	422,429.78	6	106595	487924	13,789,411.85
	Glass Bottle Line	DMU14	17266	5453	111,016.33	3	26925	166373	2,839,733.87
	Premix Line	DMU15	66760	10760	237,368.76	5	2082	85310	1,838,951.40
2013	Pet-6 Line	DMU16	713322	45382	687,102.15	9	68319	692072	15,859,469.03
	PEt-2 Line	DMU17	466	1325	223,696.97	5	16589	26421	205,064.00
	Can Line	DMU18	88014	23184	459,321.28	6	81282	478374	14,813,417.93
	Glass Bottle Line	DMU19	17529	3495	120,731.37	3	133836	113214	1,557,216.34
	Premix Line	DMU20	64905	11022	261,676.24	5	1584	88118	2,089,638.40
2014	Pet-6 Line	DMU21	633045	57025	756,110.18	7	61829	738233	16,244,660.11
	PEt-2 Line	DMU22	436378	29839	249,654.64	5	85903	311934	5,592,703.20
	Can Line	DMU23	249452	30737	522,519.00	6	95332	513083	17,390,305.56
	Glass Bottle Line	DMU24	17031	2455	163,061.18	4	17664	55085	981,231.78
	Premix Line	DMU25	63670	11770	322,098.22	6	1697	94787	2,289,852.40
2015	Pet-6 Line	DMU26	560201	45518	766,440.21	7	50855	556809	14,758,001.10
	PEt-2 Line	DMU27	329300	24694	221,205.43	4	64644	208534	3,719,996.08
	Can Line	DMU28	207499	23704	530,946.61	6	57565	410623	17,272,191.48
	Glass Bottle Line	DMU29	14585	2195	170,453.07	4	26407	50700	1,291,422.38
	Premix Line	DMU30	80034	12696	329,798.57	6	1179	91991	2,306,254.85

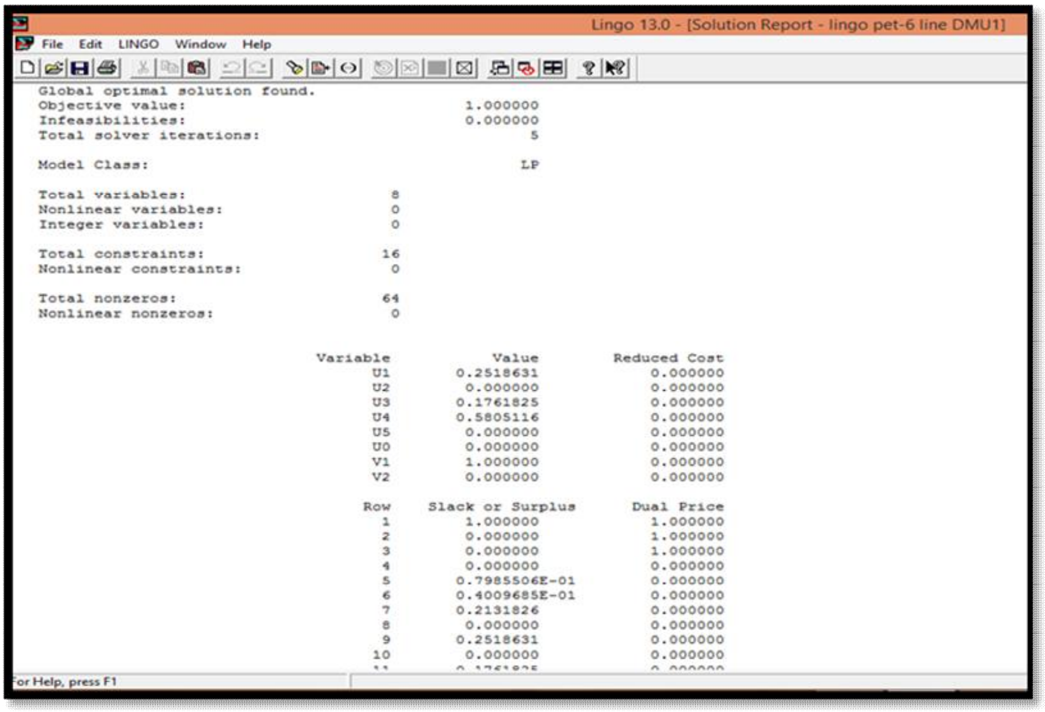
Appendix B: Lingo Coding for BCC and Modified BCC model

Pet-6 Line Coding

DMU1



u0 <= 0.928



Pet-2 Line Coding

DMU2

Lingo 13.0 - Lingo Model - Pet-2 line DMU2

Lingo Model - Pet-2 line DMU2

```
MAX = 0.0396 * u1 + 0.0642 * u2 + 0.2715 * u3 + 0.4545 * u4 + 0.3461 * u5 + u0;
0.1204 * v1 + 0.0270 * v2 = 1;
0.0396 * u1 + 0.0642 * u2 + 0.2715 * u3 + 0.4545 * u4 + 0.3461 * u5 - 0.1204 * v1 - 0.0270 * v2 + u0 <= 0;
0.0329 * u1 + 0.0299 * u2 + 0.2534 * u3 + 0.4545 * u4 + 0.3407 * u5 - 0.0847 * v1 - 0.0258 * v2 + u0 <= 0;
0.0260 * u1 + 0.0540 * u2 + 0.2670 * u3 + 0.4545 * u4 + 0.1061 * u5 - 0.0614 * v1 - 0.0302 * v2 + u0 <= 0;
0.0005 * u1 + 0.0232 * u2 + 0.2918 * u3 + 0.4545 * u4 + 0.1239 * u5 - 0.0339 * v1 - 0.0117 * v2 + u0 <= 0;
0.5218 * u1 + 0.5232 * u2 + 0.3257 * u3 + 0.4545 * u4 + 0.6418 * u5 - 0.4001 * v1 - 0.3216 * v2 + u0 <= 0;
0.3938 * u1 + 0.4330 * u2 + 0.2886 * u3 + 0.3636 * u4 + 0.4830 * u5 - 0.2675 * v1 - 0.2139 * v2 + u0 <= 0;
u1 >= 0;
u2 >= 0;
u3 >= 0;
u4 >= 0;
u5 >= 0;
v1 >= 0;
v2 >= 0;
u0 <= 0.5;
END
```

u0 <= 0.928

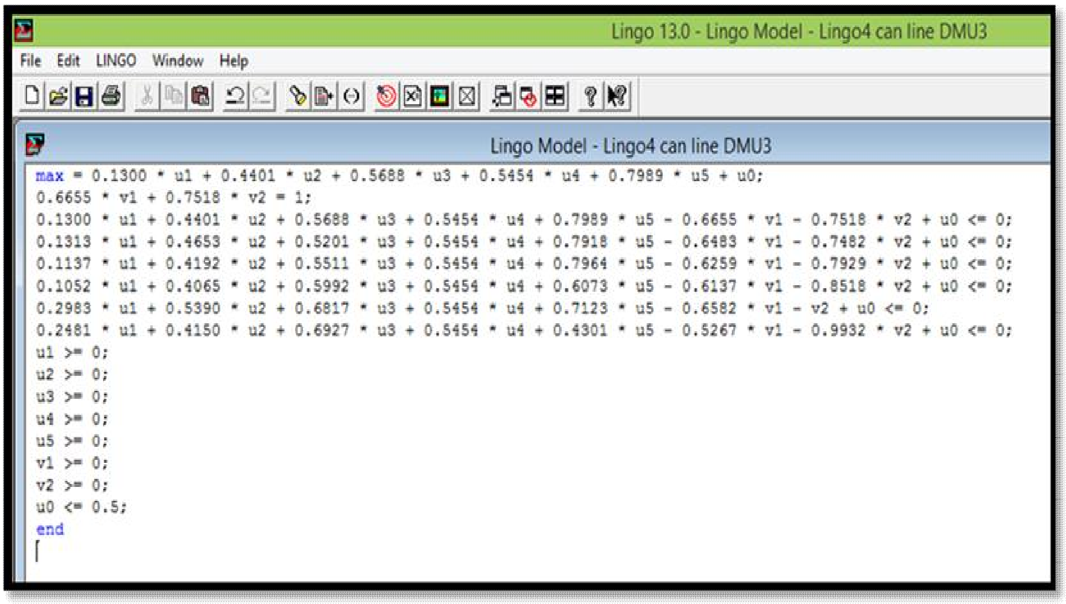
Lingo 13.0 - [Solution Report - Pet-2 line DMU2]

```
Global optimal solution found.
Objective value:                              1.000000
Infeasibilities:                              0.000000
Total solver iterations:                             6

Model Class:                                        LP

Total variables:                      8
Nonlinear variables:                  0
Integer variables:                    0

Total constraints:                   16
Nonlinear constraints:                0

Total nonzeros:                      64
Nonlinear nonzeros:                   0

                    Variable           Value        Reduced Cost
                          U1        14.49275            0.000000
                          U2        0.000000            0.000000
                          U3        0.000000            0.000000
                          U4       0.9374851            0.000000
                          U5        0.000000            0.000000
                          U0        0.000000            0.000000
                          V1        0.000000            0.000000
                          V2        37.03704            0.000000

                         Row    Slack or Surplus      Dual Price
                           1        1.000000            1.000000
                           2        0.000000            1.000000
                           3        0.000000            1.000000
                           4       0.5265700E-01        0.000000
                           5       0.3156200            0.000000
                           6        0.000000            0.000000
                           7        3.922705            0.000000
                           8        1.874106            0.000000
                           9        14.49275            0.000000
                          10        0.000000            0.000000
```

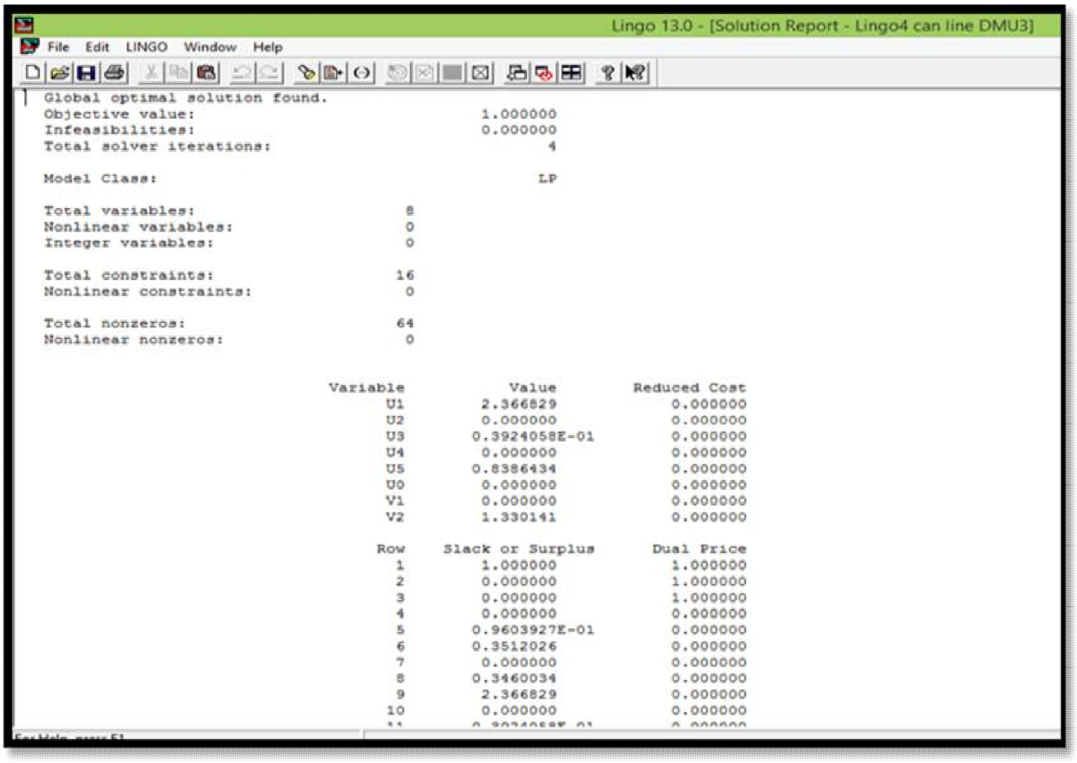
For Help, press F1

Can Line Coding

DMU3

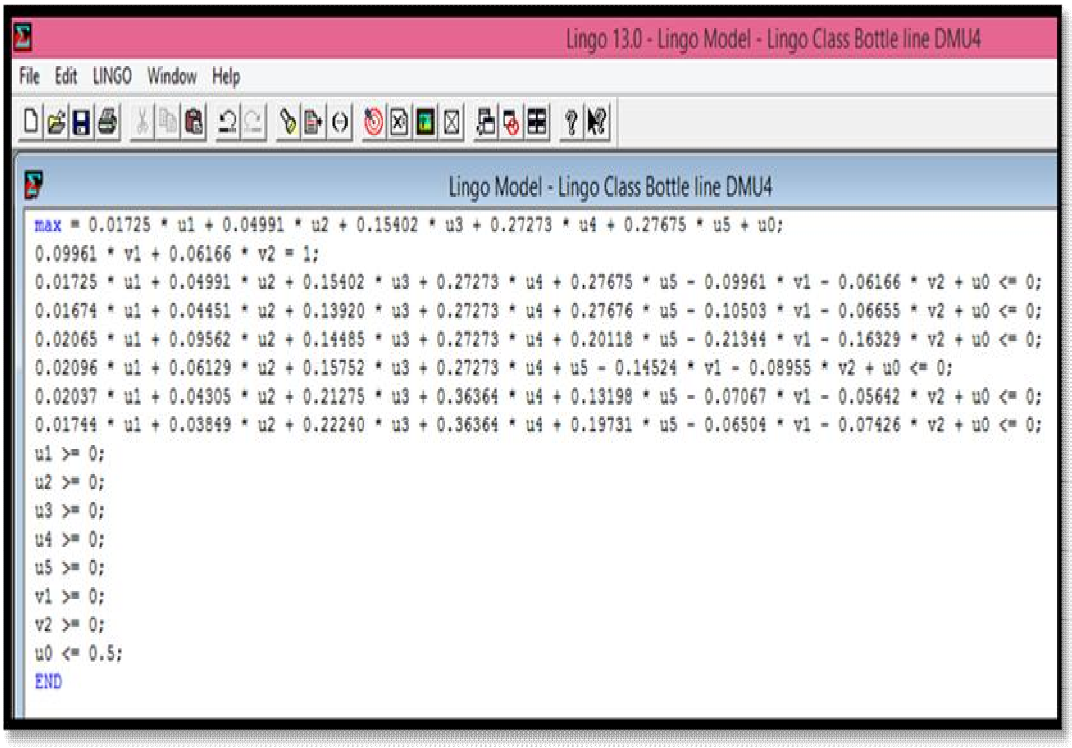


u0 <= 0.928

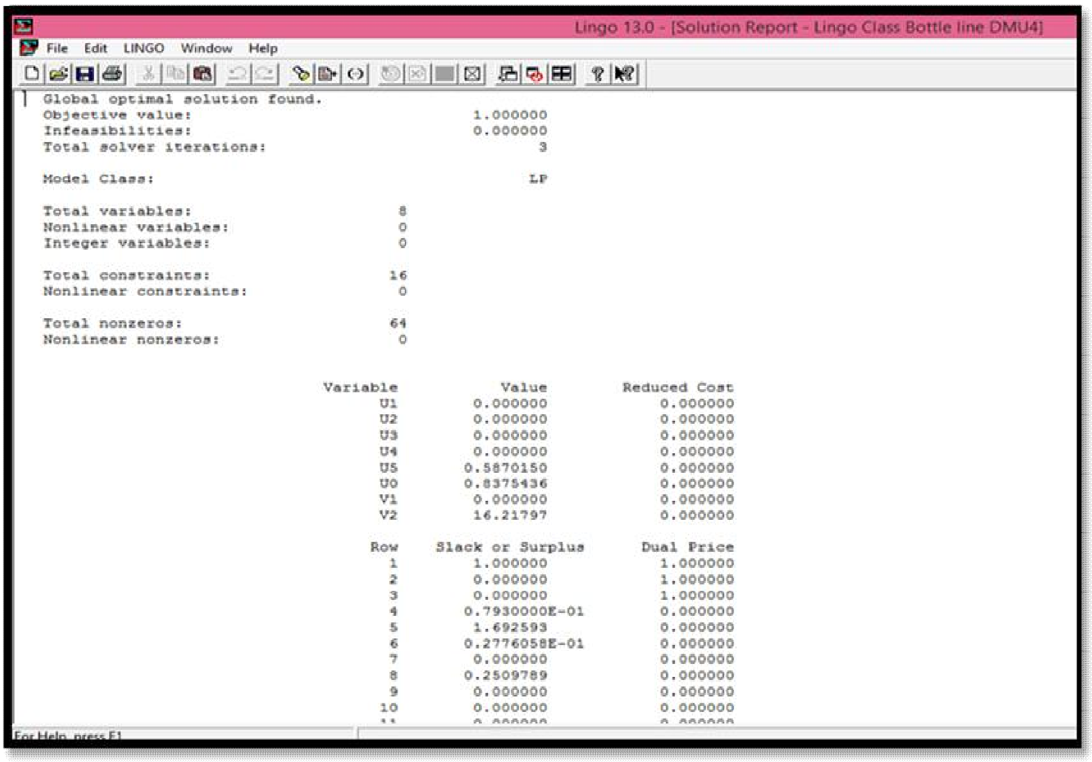


Glass Bottle Line Coding:

DMU4

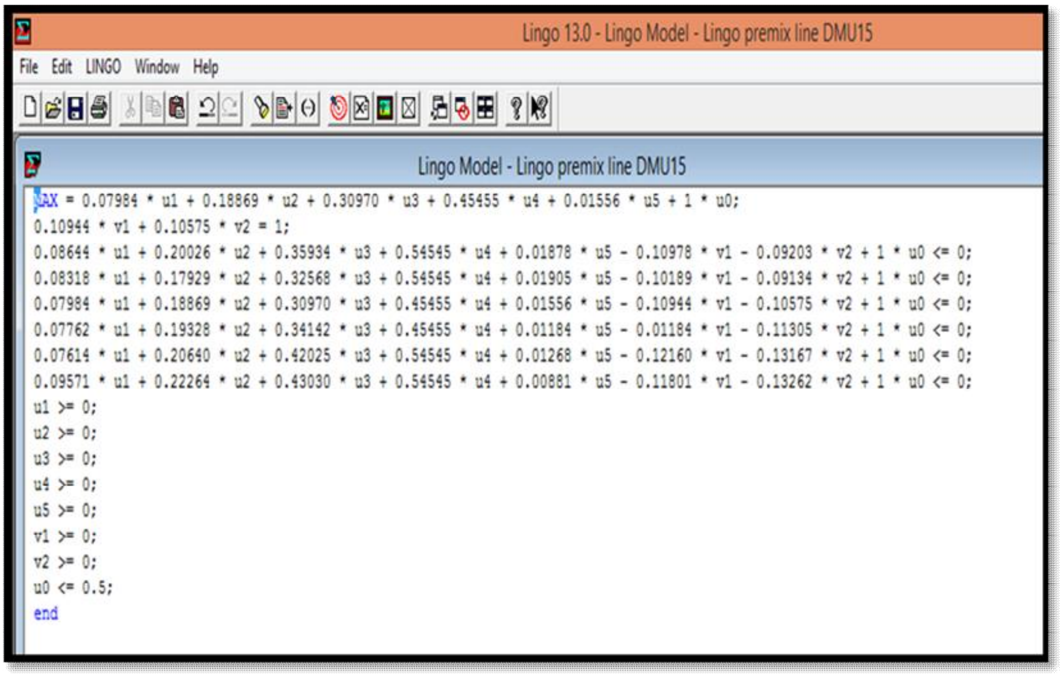


u0 <= 0.928



Premix Line Coding:

DMU15



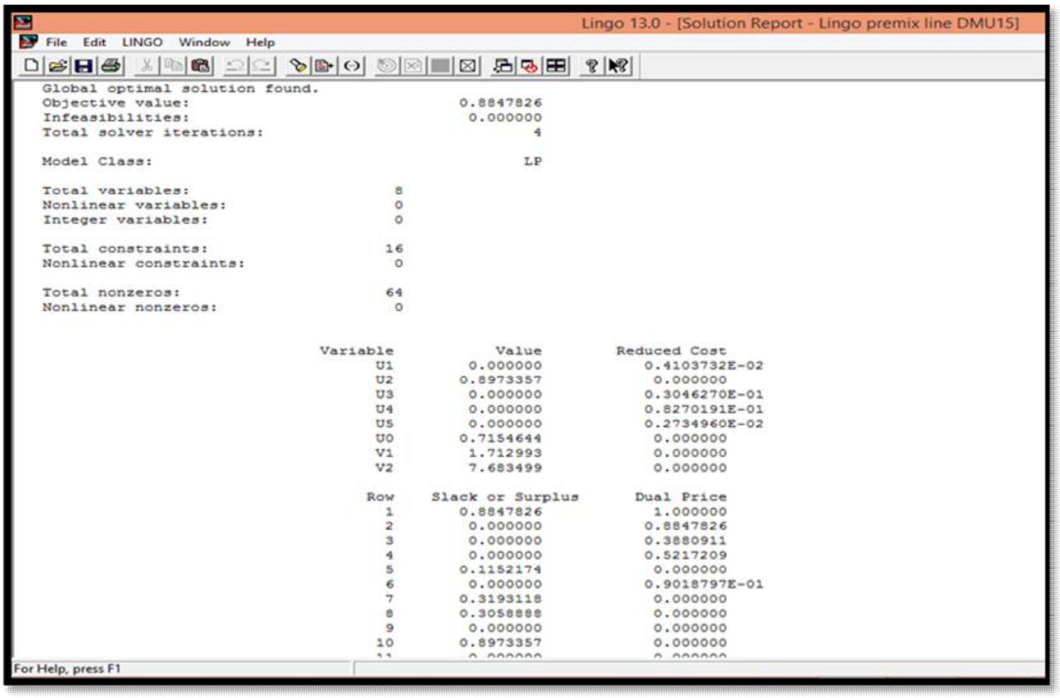
Lingo 13.0 - Lingo Model - Lingo premix line DMU15

File Edit LINGO Window Help

Lingo Model - Lingo premix line DMU15

```
MAX = 0.07984 * u1 + 0.18869 * u2 + 0.30970 * u3 + 0.45455 * u4 + 0.01556 * u5 + 1 * u0;
0.10944 * v1 + 0.10575 * v2 = 1;
0.08644 * u1 + 0.20026 * u2 + 0.35934 * u3 + 0.54545 * u4 + 0.01878 * u5 - 0.10978 * v1 - 0.09203 * v2 + 1 * u0 <= 0;
0.08318 * u1 + 0.17929 * u2 + 0.32568 * u3 + 0.54545 * u4 + 0.01905 * u5 - 0.10189 * v1 - 0.09134 * v2 + 1 * u0 <= 0;
0.07984 * u1 + 0.18869 * u2 + 0.30970 * u3 + 0.45455 * u4 + 0.01556 * u5 - 0.10944 * v1 - 0.10575 * v2 + 1 * u0 <= 0;
0.07762 * u1 + 0.19328 * u2 + 0.34142 * u3 + 0.45455 * u4 + 0.01184 * u5 - 0.01184 * v1 - 0.11305 * v2 + 1 * u0 <= 0;
0.07614 * u1 + 0.20640 * u2 + 0.42025 * u3 + 0.54545 * u4 + 0.01268 * u5 - 0.12160 * v1 - 0.13167 * v2 + 1 * u0 <= 0;
0.09571 * u1 + 0.22264 * u2 + 0.43030 * u3 + 0.54545 * u4 + 0.00881 * u5 - 0.11801 * v1 - 0.13262 * v2 + 1 * u0 <= 0;
u1 >= 0;
u2 >= 0;
u3 >= 0;
u4 >= 0;
u5 >= 0;
v1 >= 0;
v2 >= 0;
u0 <= 0.5;
end
```

u0 <= 0.928



Lingo 13.0 - [Solution Report - Lingo premix line DMU15]

File Edit LINGO Window Help

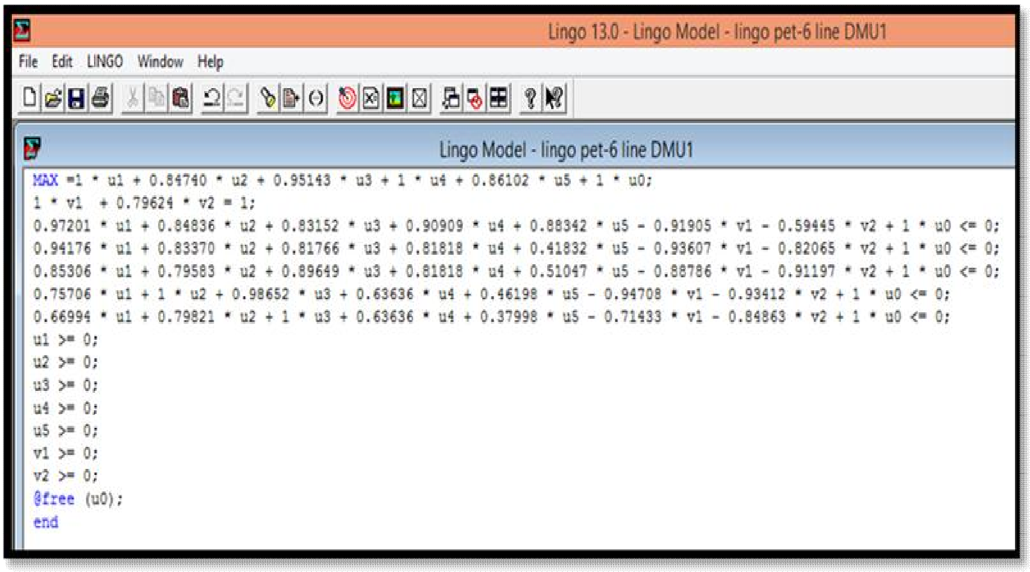
```
Global optimal solution found.
Objective value:                              0.8847826
Infeasibilities:                              0.000000
Total solver iterations:                             4

Model Class:                                        LP

Total variables:                      8
Nonlinear variables:                  0
Integer variables:                    0

Total constraints:                   16
Nonlinear constraints:                0

Total nonzeros:                      64
Nonlinear nonzeros:                   0

                      Variable           Value        Reduced Cost
                            U1        0.000000        0.4103732E-02
                            U2        0.8973357        0.000000
                            U3        0.000000        0.3046270E-01
                            U4        0.000000        0.8270191E-01
                            U5        0.000000        0.2734960E-02
                            U0        0.7154644        0.000000
                            V1        1.712993        0.000000
                            V2        7.683499        0.000000

                           Row    Slack or Surplus      Dual Price
                             1        0.8847826        1.000000
                             2        0.000000        0.8847826
                             3        0.000000        0.3880911
                             4        0.000000        0.5217209
                             5        0.1152174        0.000000
                             6        0.000000        0.9018797E-01
                             7        0.3193118        0.000000
                             8        0.3058888        0.000000
                             9        0.000000        0.000000
                            10        0.8973357        0.000000
```

For Help, press F1

Appendix C: Lingo Coding for Super-efficiency and Modified Super-efficiency Models

Pet-6 Line Coding

u0 free



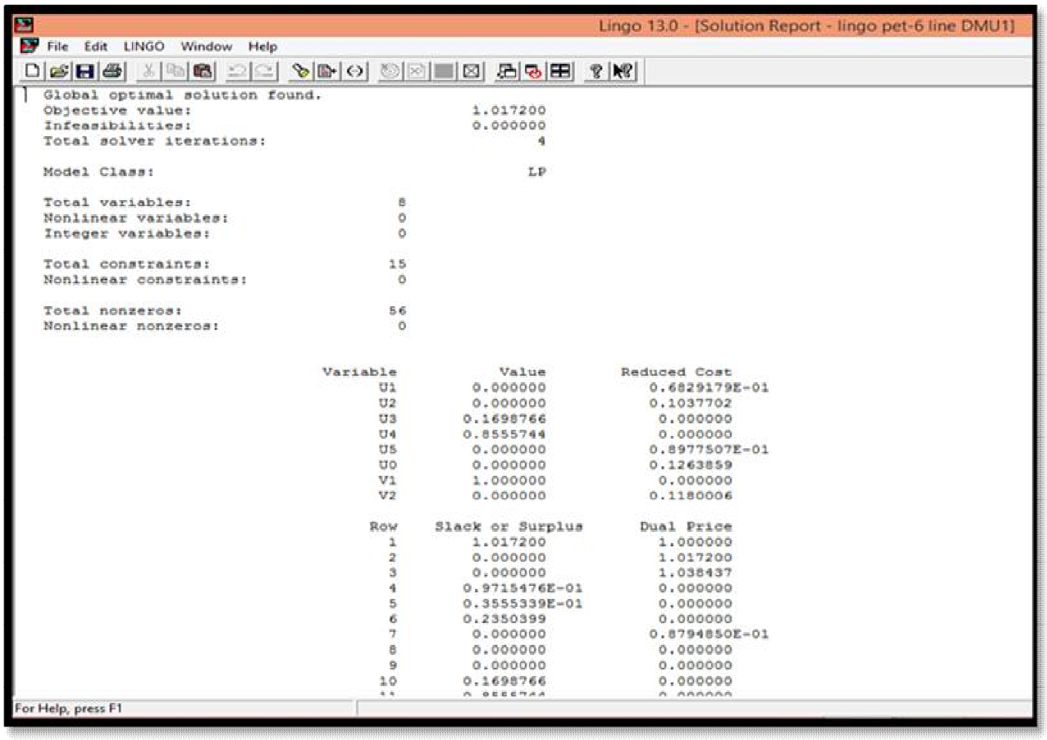
Lingo 13.0 - Lingo Model - lingo pet-6 line DMU1

File Edit LINGO Window Help

Lingo Model - lingo pet-6 line DMU1

```
MAX =1 * u1 + 0.84740 * u2 + 0.95143 * u3 + 1 * u4 + 0.86102 * u5 + 1 * u0;
1 * v1  + 0.79624 * v2 = 1;
0.97201 * u1 + 0.84836 * u2 + 0.83152 * u3 + 0.90909 * u4 + 0.88342 * u5 - 0.91905 * v1 - 0.59445 * v2 + 1 * u0 <= 0;
0.94176 * u1 + 0.83370 * u2 + 0.81766 * u3 + 0.81818 * u4 + 0.41832 * u5 - 0.93607 * v1 - 0.82065 * v2 + 1 * u0 <= 0;
0.85306 * u1 + 0.79583 * u2 + 0.89649 * u3 + 0.81818 * u4 + 0.51047 * u5 - 0.88786 * v1 - 0.91197 * v2 + 1 * u0 <= 0;
0.75706 * u1 + 1 * u2 + 0.98652 * u3 + 0.63636 * u4 + 0.46198 * u5 - 0.94708 * v1 - 0.93412 * v2 + 1 * u0 <= 0;
0.66994 * u1 + 0.79821 * u2 + 1 * u3 + 0.63636 * u4 + 0.37998 * u5 - 0.71433 * v1 - 0.84863 * v2 + 1 * u0 <= 0;
u1 >= 0;
u2 >= 0;
u3 >= 0;
u4 >= 0;
u5 >= 0;
v1 >= 0;
v2 >= 0;
@free (u0);
end
```

u0 <= 0.928



Lingo 13.0 - [Solution Report - lingo pet-6 line DMU1]

File Edit LINGO Window Help

```
Global optimal solution found.
Objective value:                              1.017200
Infeasibilities:                              0.000000
Total solver iterations:                             4

Model Class:                                        LP

Total variables:                      8
Nonlinear variables:                  0
Integer variables:                    0

Total constraints:                   15
Nonlinear constraints:                0

Total nonzeros:                      56
Nonlinear nonzeros:                   0

                   Variable           Value        Reduced Cost
                         U1        0.000000        0.6829179E-01
                         U2        0.000000        0.1037702
                         U3        0.1698766        0.000000
                         U4        0.8555744        0.000000
                         U5        0.000000        0.8977507E-01
                         U0        0.000000        0.1263859
                         V1        1.000000        0.000000
                         V2        0.000000        0.1180006

                        Row    Slack or Surplus      Dual Price
                          1        1.017200          1.000000
                          2        0.000000          1.017200
                          3        0.000000          1.038437
                          4        0.9715476E-01     0.000000
                          5        0.3555339E-01     0.000000
                          6        0.2350399         0.000000
                          7        0.000000          0.8794850E-01
                          8        0.000000          0.000000
                          9        0.000000          0.000000
                         10        0.1698766         0.000000
```

For Help, press F1

Pet-2 Line Coding

u0 free

```
Lingo 13.0 - Lingo Model - Pet-2 line DMU2
File  Edit  LINGO  Window  Help

Lingo Model - Pet-2 line DMU2
MAX = 0.0396 * u1 + 0.0642 * u2 + 0.2715 * u3 + 0.4545 * u4 + 0.3461 * u5 + u0;
0.1204 * v1 + 0.0270 * v2 = 1;
0.0329 * u1 + 0.0299 * u2 + 0.2534 * u3 + 0.4545 * u4 + 0.3407 * u5 - 0.0847 * v1 - 0.0258 * v2 + u0 <= 0;
0.0260 * u1 + 0.0540 * u2 + 0.2670 * u3 + 0.4545 * u4 + 0.1061 * u5 - 0.0614 * v1 - 0.0302 * v2 + u0 <= 0;
0.0005 * u1 + 0.0232 * u2 + 0.2918 * u3 + 0.4545 * u4 + 0.1239 * u5 - 0.0339 * v1 - 0.0117 * v2 + u0 <= 0;
0.5218 * u1 + 0.5232 * u2 + 0.3257 * u3 + 0.4545 * u4 + 0.6418 * u5 - 0.4001 * v1 - 0.3216 * v2 + u0 <= 0;
0.3938 * u1 + 0.4330 * u2 + 0.2886 * u3 + 0.3636 * u4 + 0.4830 * u5 - 0.2675 * v1 - 0.2139 * v2 + u0 <= 0;
u1 >= 0;
u2 >= 0;
u3 >= 0;
u4 >= 0;
u5 >= 0;
v1 >= 0;
v2 >= 0;
u0 <= 0.928;
END
```

u0 <= 0.928

```
Lingo 13.0 - [Solution Report - Pet-2 line DMU2]
File  Edit  LINGO  Window  Help

  Global optimal solution found.
  Objective value:                              1.410085
  Infeasibilities:                              0.000000
  Total solver iterations:                             4

  Model Class:                                        LP

  Total variables:                      8
  Nonlinear variables:                  0
  Integer variables:                    0

  Total constraints:                   15
  Nonlinear constraints:                0

  Total nonzeros:                      56
  Nonlinear nonzeros:                   0

                                Variable           Value        Reduced Cost
                                      U1        5.415691            0.000000
                                      U2        12.00030            0.000000
                                      U3        0.000000           0.1948664
                                      U4        0.000000           0.3004428
                                      U5        1.228560            0.000000
                                      U0        0.000000           0.6716361
                                      V1        0.000000       0.7262128E-01
                                      V2        37.03704            0.000000

                                     Row    Slack or Surplus      Dual Price
                                       1        1.410085            1.000000
                                       2        0.000000            1.410085
                                       3        0.000000           0.5533188
                                       4       0.1993443            0.000000
                                       5        0.000000            1.065338
                                       6        2.018158            0.000000
                                       7        0.000000       0.5297903E-01
                                       8        5.415691            0.000000
                                       9        12.00030            0.000000
                                      10        0.000000            0.000000
For Help, press F1
```

Can Line coding

u0 free

Lingo 13.0 - [Lingo Model - Lingo4 can line DMU8]

File Edit LINGO Window Help

```
max = 0.1313 * u1 + 0.4653 * u2 + 0.5201 * u3 + 0.5454 * u4 + 0.7918 * u5 + u0;
0.6483 * v1 + 0.7482 * v2 = 1;
0.1300 * u1 + 0.4401 * u2 + 0.5688 * u3 + 0.5454 * u4 + 0.7989 * u5 - 0.6655 * v1 - 0.7518 * v2 + u0 <= 0;
0.1137 * u1 + 0.4192 * u2 + 0.5511 * u3 + 0.5454 * u4 + 0.7964 * u5 - 0.6259 * v1 - 0.7929 * v2 + u0 <= 0;
0.1052 * u1 + 0.4065 * u2 + 0.5992 * u3 + 0.5454 * u4 + 0.6073 * u5 - 0.6137 * v1 - 0.8518 * v2 + u0 <= 0;
0.2983 * u1 + 0.5390 * u2 + 0.6817 * u3 + 0.5454 * u4 + 0.7123 * u5 - 0.6582 * v1 - 1 * v2 + 1 * u0 <= 0;
0.2481 * u1 + 0.4150 * u2 + 0.6927 * u3 + 0.5454 * u4 + 0.4301 * u5 - 0.5267 * v1 - 0.9932 * v2 + u0 <= 0;
u1 >= 0;
u2 >= 0;
u3 >= 0;
u4 >= 0;
u5 >= 0;
v1 >= 0;
v2 >= 0;
@free (u0);
end
```

u0 <= 0.928

Lingo 13.0 - [Solution Report - Lingo4 can line DMU8]

File Edit LINGO Window Help

```
  Global optimal solution found.
  Objective value:                              1.069339
  Infeasibilities:                              0.000000
  Total solver iterations:                             2

  Model Class:                                        LP

  Total variables:                      8
  Nonlinear variables:                  0
  Integer variables:                    0

  Total constraints:                   15
  Nonlinear constraints:                0

  Total nonzeros:                      56
  Nonlinear nonzeros:                   0


                      Variable           Value        Reduced Cost
                            U1        0.000000        0.1532488E-01
                            U2        2.298171         0.000000
                            U3        0.000000        0.8028434E-01
                            U4        0.000000        0.2313902E-01
                            U5        0.000000        0.3527747E-01
                            U0        0.000000        0.4242578E-01
                            V1       0.4696900         0.000000
                            V2       0.9295643         0.000000

                           Row    Slack or Surplus      Dual Price
                             1        1.069339          1.000000
                             2        0.000000          1.069339
                             3        0.000000         0.9764155
                             4       0.6763715E-01      0.000000
                             5       0.1458450          0.000000
                             6        0.000000        0.6601026E-01
                             7       0.2168879          0.000000
                             8        0.000000          0.000000
                             9        2.298171          0.000000
                            10        0.000000          0.000000
```

For Help, press F1

Glass bottle line coding:

u0 free

Lingo 13.0 - [Lingo Model - Lingo Class Bottle line DMU4]

File Edit LINGO Window Help

```
max = 0.01725 * u1 + 0.04991 * u2 + 0.15402 * u3 + 0.27273 * u4 + 0.27675 * u5 + u0;
0.09961 * v1 + 0.06166 * v2 = 1;
0.01674 * u1 + 0.04451 * u2 + 0.13920 * u3 + 0.27273 * u4 + 0.27676 * u5 - 0.10503 * v1 - 0.06655 * v2 + u0 <= 0;
0.02065 * u1 + 0.09562 * u2 + 0.14485 * u3 + 0.27273 * u4 + 0.20118 * u5 - 0.21344 * v1 - 0.16329 * v2 + u0 <= 0;
0.02096 * u1 + 0.06129 * u2 + 0.15752 * u3 + 0.27273 * u4 + u5 - 0.14524 * v1 - 0.08955 * v2 + u0 <= 0;
0.02037 * u1 + 0.04305 * u2 + 0.21275 * u3 + 0.36364 * u4 + 0.13198 * u5 - 0.07067 * v1 - 0.05642 * v2 + u0 <= 0;
0.01744 * u1 + 0.03849 * u2 + 0.22240 * u3 + 0.36364 * u4 + 0.19731 * u5 - 0.06504 * v1 - 0.07426 * v2 + u0 <= 0;
u1 >= 0;
u2 >= 0;
u3 >= 0;
u4 >= 0;
u5 >= 0;
v1 >= 0;
v2 >= 0;
@free (u0);
END
```

u0 <= 0.928

Lingo 13.0 - [Solution Report - Lingo Class Bottle line DMU4]

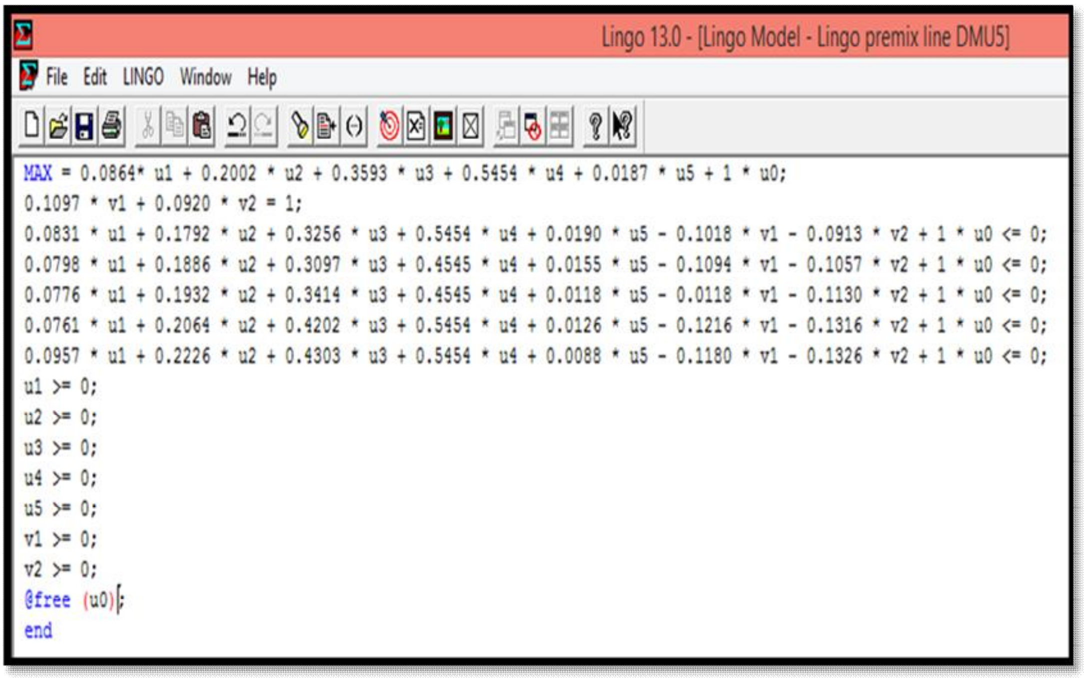
File Edit LINGO Window Help

```
Global optimal solution found.
Objective value:                              1.083622
Infeasibilities:                              0.000000
Total solver iterations:                             3

Model Class:                                        LP

Total variables:                      8
Nonlinear variables:                  0
Integer variables:                    0

Total constraints:                   15
Nonlinear constraints:                0

Total nonzeros:                      56
Nonlinear nonzeros:                   0


                            Variable           Value        Reduced Cost
                                  U1        0.000000        0.5140808E-02
                                  U2        20.68997         0.000000
                                  U3        0.000000        0.7048162E-01
                                  U4        0.000000        0.1115282
                                  U5        0.1842312        0.000000
                                  U0        0.000000        0.9479184E-01
                                  V1        0.000000        0.1920850E-01
                                  V2        16.21797         0.000000

                                 Row    Slack or Surplus      Dual Price
                                   1        1.083622         1.000000
                                   2        0.000000         1.083622
                                   3        0.1074077        0.000000
                                   4        0.6327941        0.000000
                                   5        0.000000         0.1523690
                                   6        0.000000         0.9424228
                                   7        0.3716390        0.000000
                                   8        0.000000         0.000000
                                   9        20.68997         0.000000
                                  10        0.000000         0.000000
```

For Help, press F1

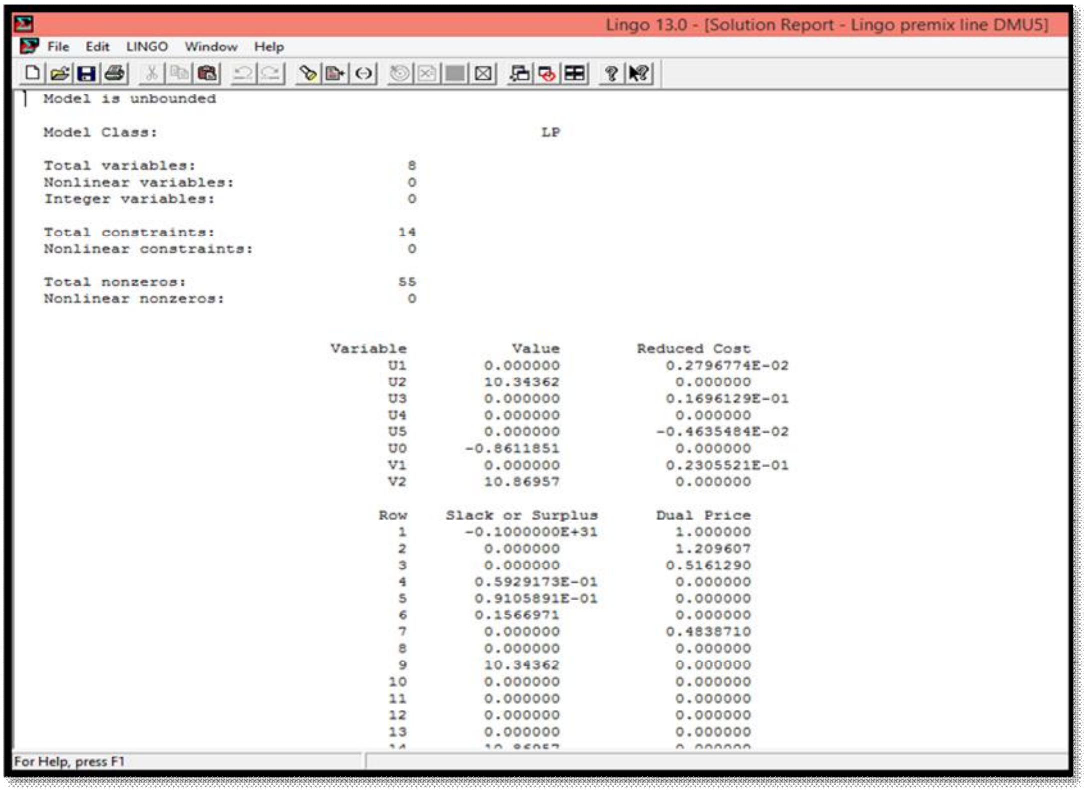
Premix line coding

u0 free




```
MAX = 0.0864* u1 + 0.2002 * u2 + 0.3593 * u3 + 0.5454 * u4 + 0.0187 * u5 + 1 * u0;
0.1097 * v1 + 0.0920 * v2 = 1;
0.0831 * u1 + 0.1792 * u2 + 0.3256 * u3 + 0.5454 * u4 + 0.0190 * u5 - 0.1018 * v1 - 0.0913 * v2 + 1 * u0 <= 0;
0.0798 * u1 + 0.1886 * u2 + 0.3097 * u3 + 0.4545 * u4 + 0.0155 * u5 - 0.1094 * v1 - 0.1057 * v2 + 1 * u0 <= 0;
0.0776 * u1 + 0.1932 * u2 + 0.3414 * u3 + 0.4545 * u4 + 0.0118 * u5 - 0.0118 * v1 - 0.1130 * v2 + 1 * u0 <= 0;
0.0761 * u1 + 0.2064 * u2 + 0.4202 * u3 + 0.5454 * u4 + 0.0126 * u5 - 0.1216 * v1 - 0.1316 * v2 + 1 * u0 <= 0;
0.0957 * u1 + 0.2226 * u2 + 0.4303 * u3 + 0.5454 * u4 + 0.0088 * u5 - 0.1180 * v1 - 0.1326 * v2 + 1 * u0 <= 0;
u1 >= 0;
u2 >= 0;
u3 >= 0;
u4 >= 0;
u5 >= 0;
v1 >= 0;
v2 >= 0;
@free (u0);
end
```

u0 <= 0.928

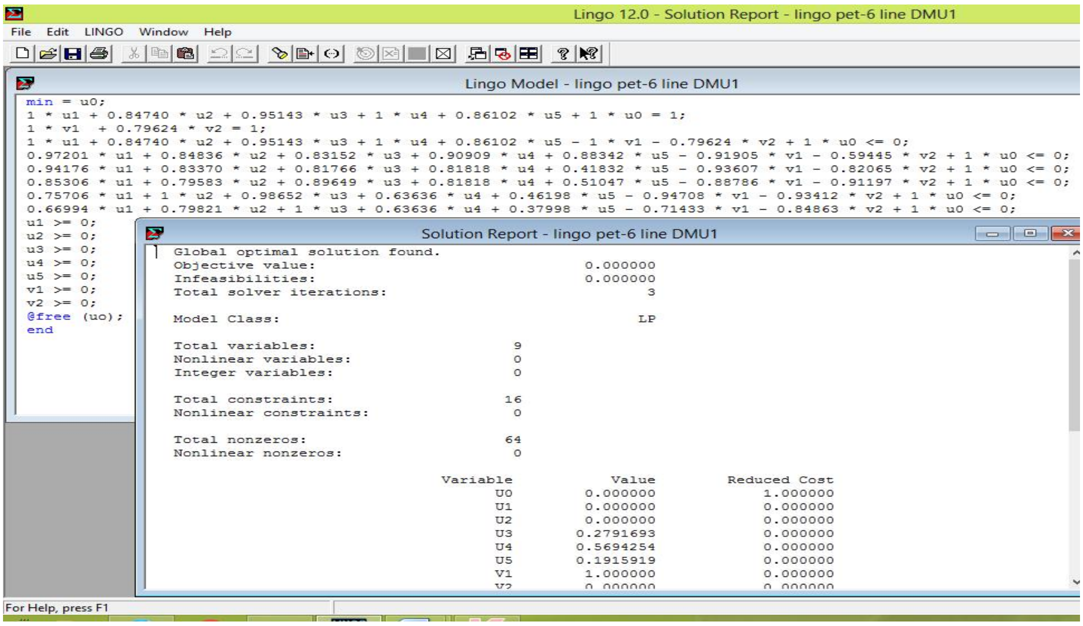



```
Model is unbounded

Model Class:                                  LP

Total variables:                    8
Nonlinear variables:                0
Integer variables:                  0

Total constraints:                 14
Nonlinear constraints:              0

Total nonzeros:                    55
Nonlinear nonzeros:                 0

                    Variable           Value        Reduced Cost
                          U1        0.000000        0.2796774E-02
                          U2        10.34362        0.000000
                          U3        0.000000        0.1696129E-01
                          U4        0.000000        0.000000
                          U5        0.000000       -0.4635484E-02
                          U0      -0.8611851        0.000000
                          V1        0.000000        0.2305521E-01
                          V2        10.86957        0.000000

                         Row    Slack or Surplus      Dual Price
                           1     -0.1000000E+31        1.000000
                           2        0.000000           1.209607
                           3        0.000000           0.5161290
                           4       0.5929173E-01       0.000000
                           5       0.9105891E-01       0.000000
                           6       0.1566971           0.000000
                           7        0.000000           0.4838710
                           8        0.000000           0.000000
                           9        10.34362           0.000000
                          10        0.000000           0.000000
                          11        0.000000           0.000000
                          12        0.000000           0.000000
                          13        0.000000           0.000000
```

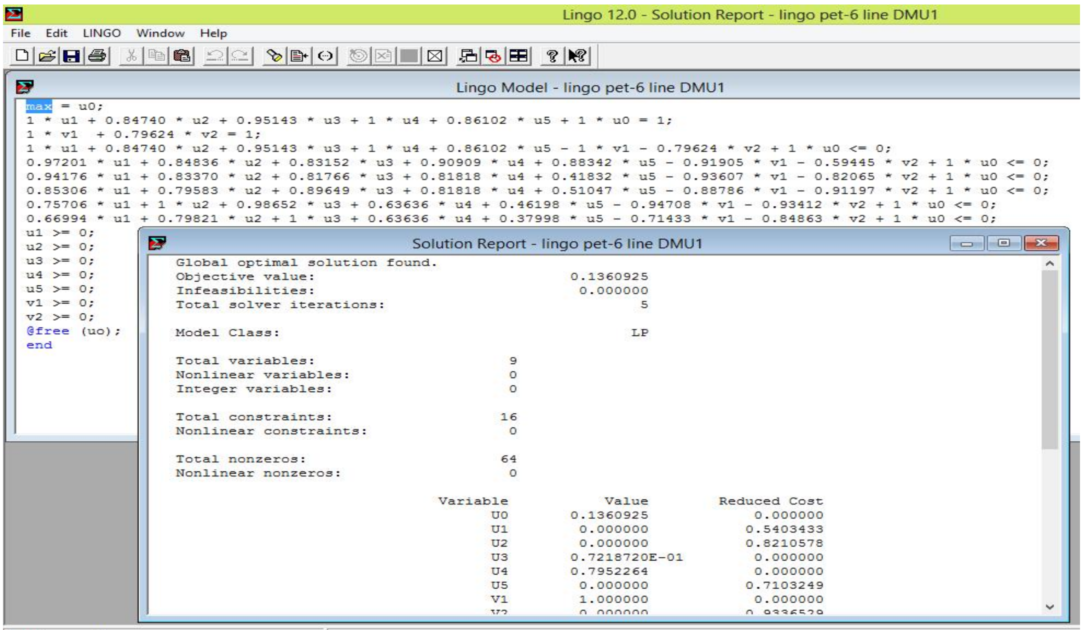
Appendix D: Lingo Coding for u^+ and u^- and including optimal value for both u^- and u^+ for determine the value of ε:

Pet-6 Line Coding:

DMU1 : u^-

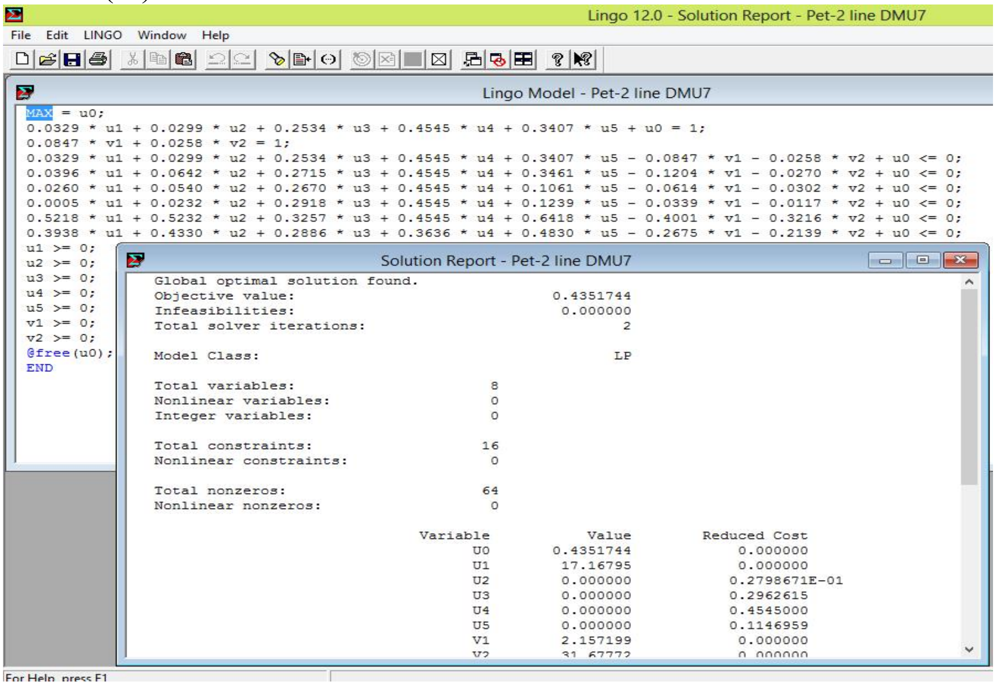


DMU1 : u^+



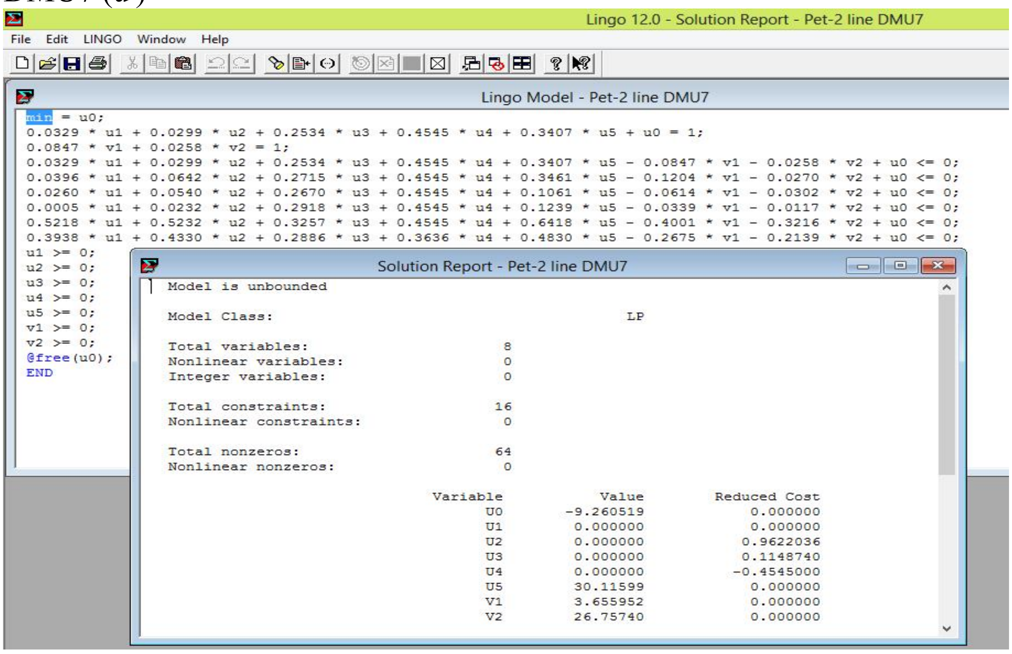
Pet-2 Line Coding:

DMU7:(u^+)



```
Lingo 12.0 - Solution Report - Pet-2 line DMU7
Lingo Model - Pet-2 line DMU7
MAX = u0;
0.0329 * u1 + 0.0299 * u2 + 0.2534 * u3 + 0.4545 * u4 + 0.3407 * u5 + u0 = 1;
0.0847 * v1 + 0.0258 * v2 = 1;
0.0329 * u1 + 0.0299 * u2 + 0.2534 * u3 + 0.4545 * u4 + 0.3407 * u5 - 0.0847 * v1 - 0.0258 * v2 + u0 <= 0;
0.0396 * u1 + 0.0642 * u2 + 0.2715 * u3 + 0.4545 * u4 + 0.3461 * u5 - 0.1204 * v1 - 0.0270 * v2 + u0 <= 0;
0.0260 * u1 + 0.0540 * u2 + 0.2670 * u3 + 0.4545 * u4 + 0.1061 * u5 - 0.0614 * v1 - 0.0302 * v2 + u0 <= 0;
0.0005 * u1 + 0.0232 * u2 + 0.2918 * u3 + 0.4545 * u4 + 0.1239 * u5 - 0.0339 * v1 - 0.0117 * v2 + u0 <= 0;
0.5218 * u1 + 0.5232 * u2 + 0.3257 * u3 + 0.4545 * u4 + 0.6418 * u5 - 0.4001 * v1 - 0.3216 * v2 + u0 <= 0;
0.3938 * u1 + 0.4330 * u2 + 0.2886 * u3 + 0.3636 * u4 + 0.4830 * u5 - 0.2675 * v1 - 0.2139 * v2 + u0 <= 0;
u1 >= 0;
u2 >= 0;
u3 >= 0;
u4 >= 0;
u5 >= 0;
v1 >= 0;
v2 >= 0;
@free(u0);
END

Solution Report - Pet-2 line DMU7
  Global optimal solution found.
  Objective value:                              0.4351744
  Infeasibilities:                              0.000000
  Total solver iterations:                             2

  Model Class:                                        LP

  Total variables:                      8
  Nonlinear variables:                  0
  Integer variables:                    0

  Total constraints:                   16
  Nonlinear constraints:                0

  Total nonzeros:                      64
  Nonlinear nonzeros:                   0

                      Variable           Value        Reduced Cost
                            U0       0.4351744            0.000000
                            U1        17.16795            0.000000
                            U2        0.000000        0.2798671E-01
                            U3        0.000000           0.2962615
                            U4        0.000000           0.4545000
                            U5        0.000000           0.1146959
                            V1        2.157199            0.000000
                            V2        31.67772            0.000000
For Help, press F1
```

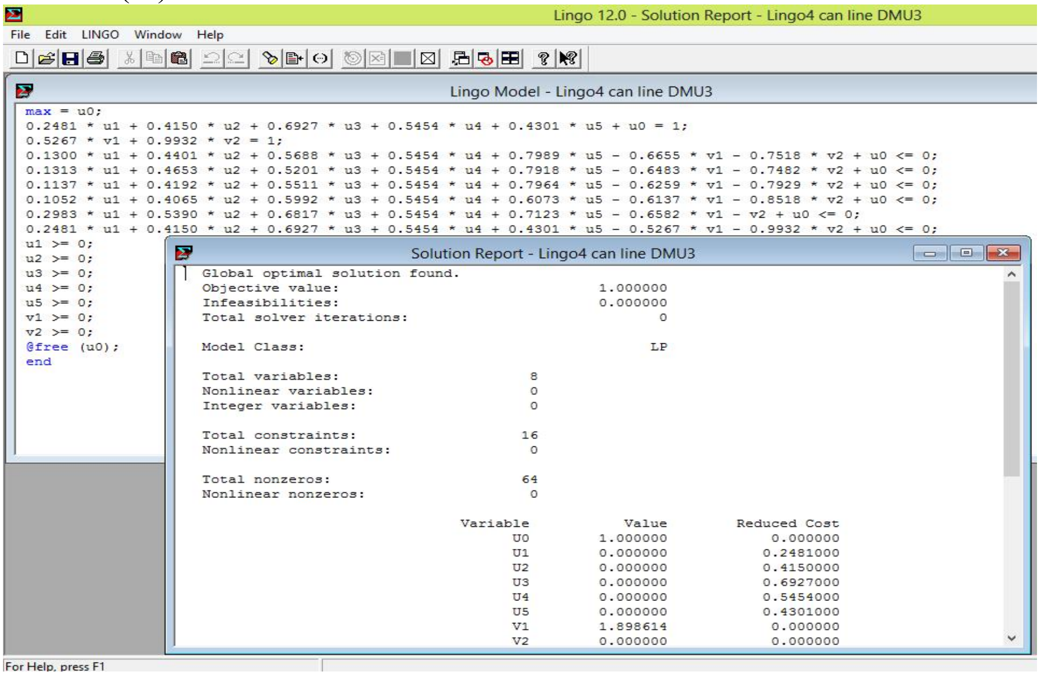
DMU7 (u^-)



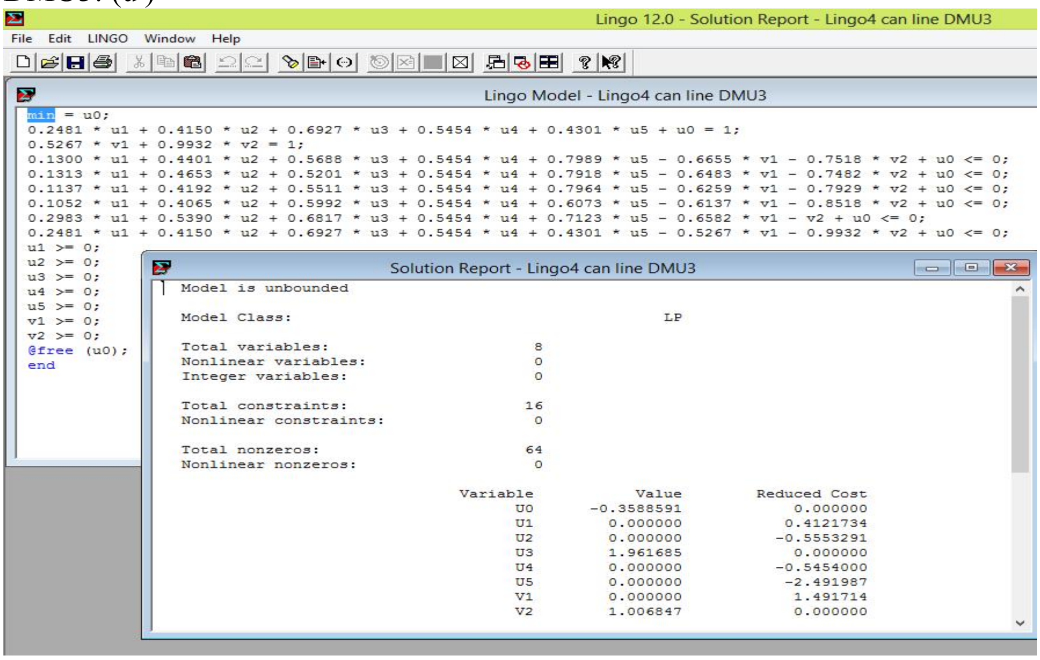
```
Lingo 12.0 - Solution Report - Pet-2 line DMU7
Lingo Model - Pet-2 line DMU7
min = u0;
0.0329 * u1 + 0.0299 * u2 + 0.2534 * u3 + 0.4545 * u4 + 0.3407 * u5 + u0 = 1;
0.0847 * v1 + 0.0258 * v2 = 1;
0.0329 * u1 + 0.0299 * u2 + 0.2534 * u3 + 0.4545 * u4 + 0.3407 * u5 - 0.0847 * v1 - 0.0258 * v2 + u0 <= 0;
0.0396 * u1 + 0.0642 * u2 + 0.2715 * u3 + 0.4545 * u4 + 0.3461 * u5 - 0.1204 * v1 - 0.0270 * v2 + u0 <= 0;
0.0260 * u1 + 0.0540 * u2 + 0.2670 * u3 + 0.4545 * u4 + 0.1061 * u5 - 0.0614 * v1 - 0.0302 * v2 + u0 <= 0;
0.0005 * u1 + 0.0232 * u2 + 0.2918 * u3 + 0.4545 * u4 + 0.1239 * u5 - 0.0339 * v1 - 0.0117 * v2 + u0 <= 0;
0.5218 * u1 + 0.5232 * u2 + 0.3257 * u3 + 0.4545 * u4 + 0.6418 * u5 - 0.4001 * v1 - 0.3216 * v2 + u0 <= 0;
0.3938 * u1 + 0.4330 * u2 + 0.2886 * u3 + 0.3636 * u4 + 0.4830 * u5 - 0.2675 * v1 - 0.2139 * v2 + u0 <= 0;
u1 >= 0;
u2 >= 0;
u3 >= 0;
u4 >= 0;
u5 >= 0;
v1 >= 0;
v2 >= 0;
@free(u0);
END

Solution Report - Pet-2 line DMU7
  Model is unbounded

  Model Class:                                        LP

  Total variables:                      8
  Nonlinear variables:                  0
  Integer variables:                    0

  Total constraints:                   16
  Nonlinear constraints:                0

  Total nonzeros:                      64
  Nonlinear nonzeros:                   0

                      Variable           Value        Reduced Cost
                            U0       -9.260519            0.000000
                            U1        0.000000            0.000000
                            U2        0.000000           0.9622036
                            U3        0.000000           0.1148740
                            U4        0.000000          -0.4545000
                            U5        30.11599            0.000000
                            V1        3.655952            0.000000
                            V2        26.75740            0.000000
```

Can Line Coding:

DMU3 : (u^+)

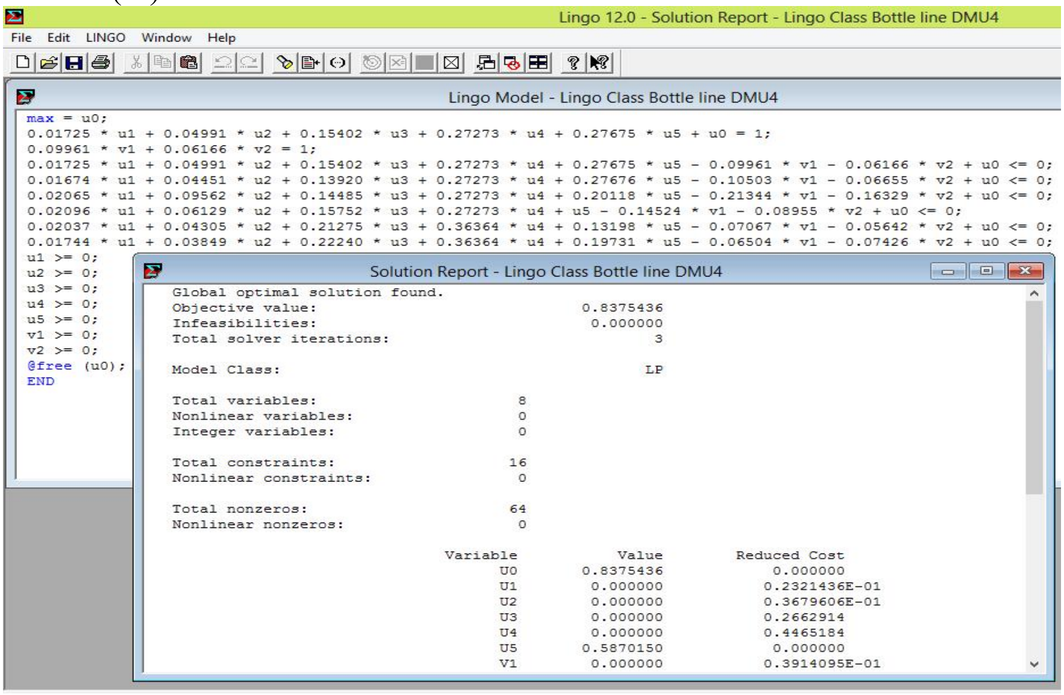


DMU3: (u^-)

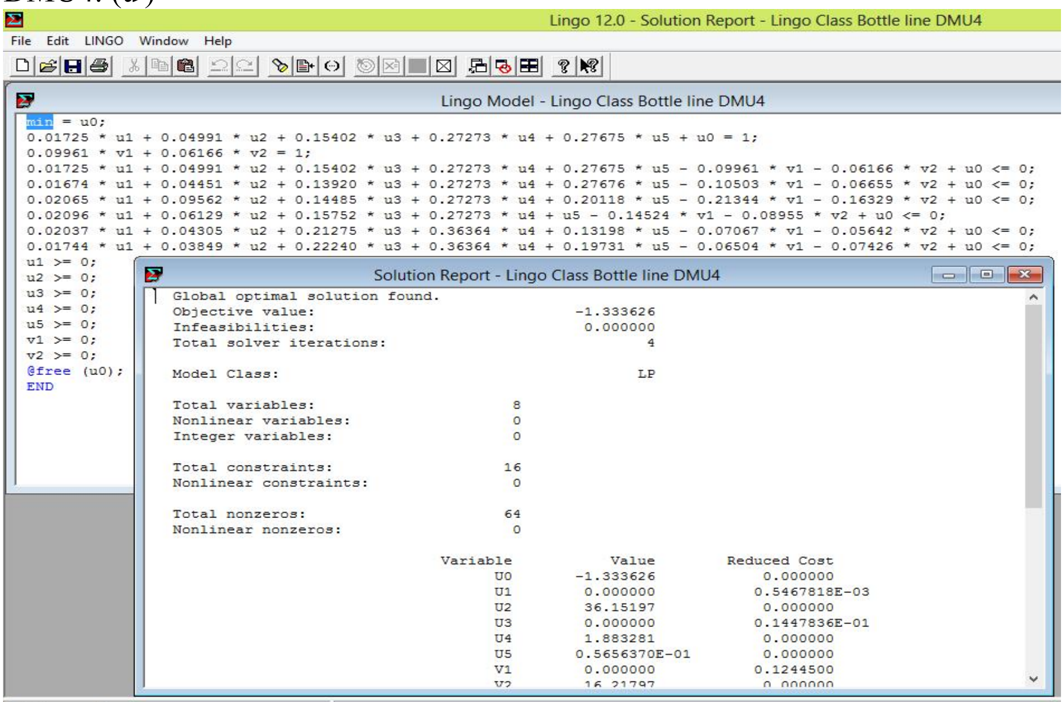


Glass bottle line coding:

DMU4: (u^+)

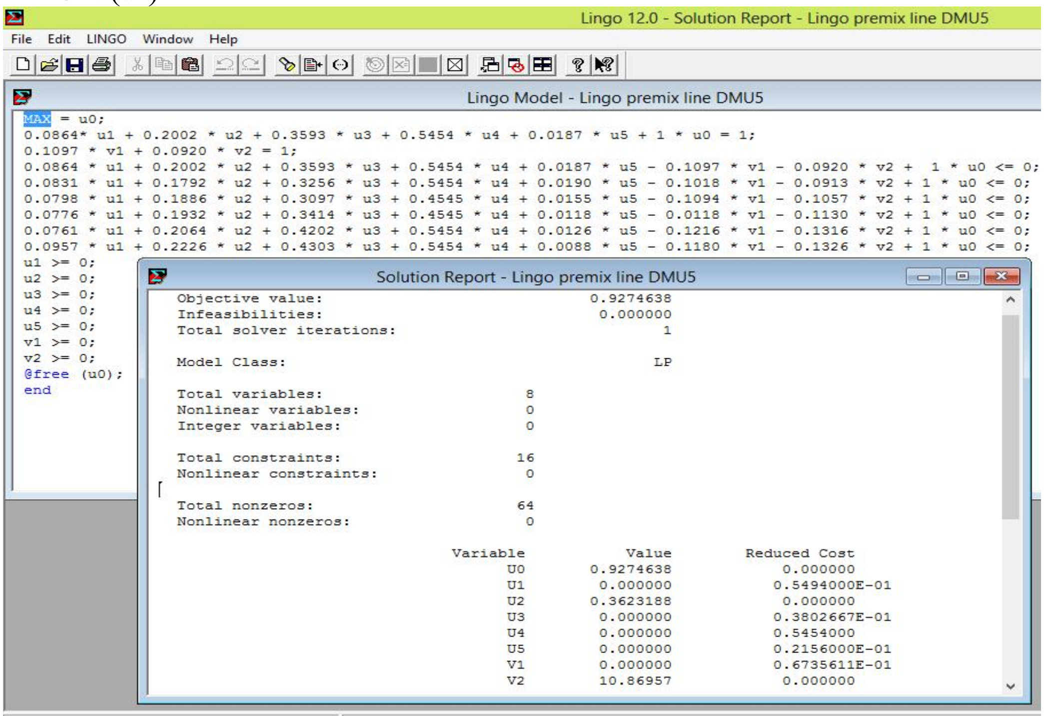


DMU4: (u^-)

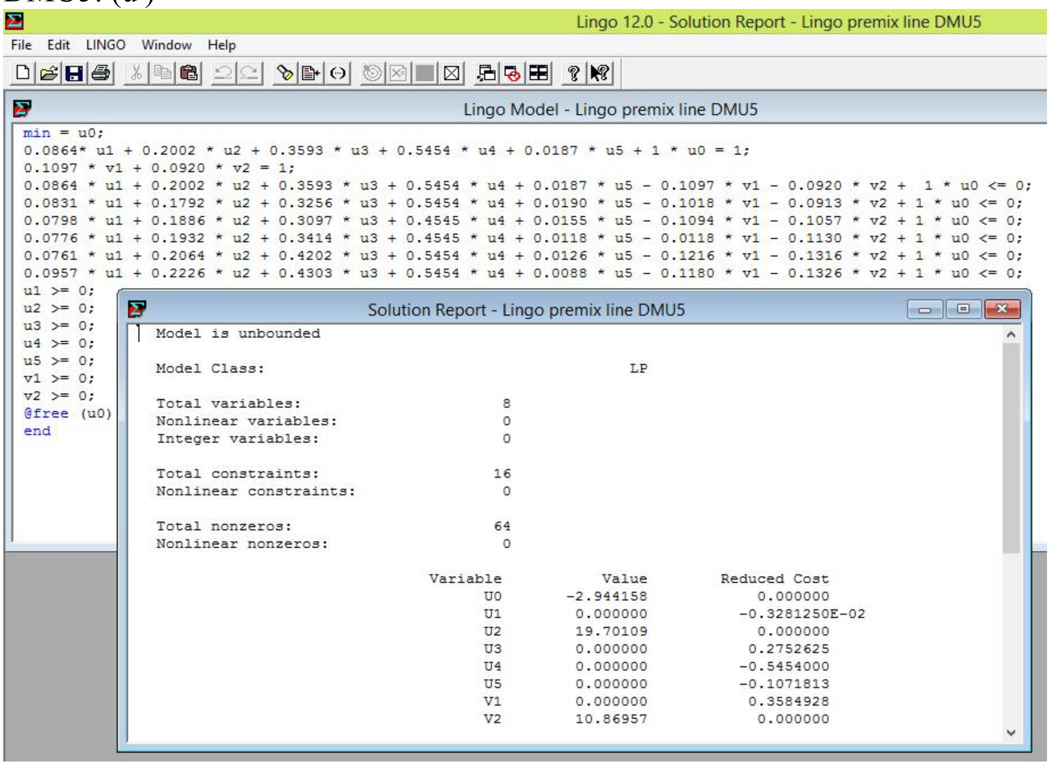


Premix line coding:

DMU5: (u^+)



DMU5: (u^-)



Optimal value for both u^- and u^+ for determine the value of ε

Production Line	**Year**	**DMUs**	**u^-**	**u^+**	**$u^- \leq u^* \leq u^+$**
Pet-6 line	2010	DMU1	0	0.136	$0 \leq u^* \leq 0.136$
	2011	DMU6	0	1	$u^* \leq 1$
	2012	DMU11	0.089	0.089	$u^* \leq 0.089$
	2013	DMU16	0.194	0.194	$u^* \leq 0.149$
	2014	DMU21	0	0.045	$u^* \leq 0.045$
	2015	DMU26	0	1	$u^* \leq 1$
Pet-2 line	2010	DMU2	$-\infty$	0.426	$u^* \leq 0.426$
	2011	DMU7	$-\infty$	0.435	$u^* \leq 0.435$
	2012	DMU12	0.066	0.066	$u^* \leq 0.066$
	2013	DMU17	$-\infty$	1	$u^* \leq 1$
	2014	DMU22	$-\infty$	1	$u^* \leq 1$
	2015	DMU27	$-\infty$	1	$u^* \leq 1$
Can line	2010	DMU3	$-\infty$	0.944	$u^* \leq 0.944$
	2011	DMU8	$-\infty$	1	$u^* \leq 1$
	2012	DMU13	$-\infty$	1	$u^* \leq 1$
	2013	DMU18	0.073	0.08	$0.073 \leq u^* \leq 0.08$
	2014	DMU23	$-\infty$	0.66	$u^* \leq 0.66$
	2015	DMU28	$-\infty$	1	$u^* \leq 1$
Glass Bottle line	2010	DMU4	-1.336	0.837	$-1.336 \leq u^* \leq 0.837$
	2011	DMU9	$-\infty$	1	$u^* \leq 1$
	2012	DMU14	$-\infty$	-0.204	$u^* \leq -0.204$
	2013	DMU19	$-\infty$	0.569	$u^* \leq 0.569$
	2014	DMU24	$-\infty$	1	$u^* \leq 1$
	2015	DMU29	$-\infty$	1	$u^* \leq 1$
Premix line	2010	DMU5	$-\infty$	0.927	$u^* \leq 0.927$
	2011	DMU10	0.928	1	$0 .928 \leq u^* \leq 1$
	2012	DMU15	0.131	0.131	$u^* \leq 0.131$
	2013	DMU20	-83.612	1	$-83.612 \leq u^* \leq 1$
	2014	DMU25	-4.559	-1.406	$-4.559 \leq u^* \leq -1.406$
	2015	DMU30	-5.814	-0.534	$-5.814 \leq u^* \leq -0.534$