Iterative Decoding of Turbo Product Codes (TPCs) Using the Chase-Pyndiah Turbo Decoder

Muath Ghazi Abdel Qader Ghnimat

Submitted to the
Institute of Graduate Studies and Research
in partial fulfilment of the requirements for the degree of

Master of Science
in
Electrical and Electronic Engineering

Eastern Mediterranean University
February 2017
Gazimağusa, North Cyprus

Approval of the Institute of Graduate Studies and Research

Prof. Dr. Mustafa Tümer
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science in Electrical and Electronic Engineering.

Prof. Dr. Hasan Demirel
Chair, Department of Electrical and Electronic Engineering

We certify that we have read this thesis and that in our opinion it is fully adequate in scope and quality as a thesis for the degree of Master of Science in Electrical and Electronic Engineering.

Prof. Dr. Erhan A. İnce
Supervisor

Examining Committee

1. Prof. Dr. Hasan Amca ______________________________

2. Prof. Dr. Hasan Demirel ______________________________

3. Prof. Dr. Erhan A. İnce ______________________________

ABSTRACT

The ground breaking error correction codes that could achieve low bit error rates (near Shannon's limit) were Turbo Codes (TCs) introduced by Berrou, Glavieux in 1993. The encoders for these outstanding codes were created by parallel concatenation of two recursive systematic convolutional codes separated by an interleaver. For decoding TCs Log-MAP algorithm could be used due to its gain in computational speed and improvement in precision. The only problem with turbo coding and decoding is that, the choice of interleaver for the encoder may cause an error floor due to their inherent poor distance properties.

Turbo product codes (TPCs) are powerful linear block codes formed by combining more than one simple linear block codes. They classify under serially concatenated codes however unlike TCs they do not rely on an interleaver and hence they have no error floor problem. TPCs which date as early as 1954, are multi-dimensional codes that are constructed by two or more linear block codes also known as component codes. The product code is obtained by placing ($k_1 \times k_2$) information bits in an array with k_1 rows and k_2 columns. Important parameters of a product code are the length of the codeword (*n*), the length of the information (*k)* and the minimum distance (*d*). Turbo product decoding is possible using either hard or soft decoding. In hard decision decoding, the input must be a binary sequence, whereas in soft decision decoding the values are immediately processed by the decoder to gauge a code sequence. A SISO decoder can be utilized to generate the soft decision decoder outputs. Some of the soft decoding algorithms include: the maximum a posteriori probability (MAP) algorithm, Chase-Pyndiah iterative decoder, symbol based MAP algorithm, the Soft Output Viterbi Algorithm, sliding-window based MAP decoder, and the forward recursion based MAP decoder. The work presented in this thesis presents the bit-error-rate (BER) versus signal-to-noise-ratio (SNRdB) for soft decision (SD) decoding. The modulation type is BPSK and encoded symbols are transmitted over AWGN and Rayleigh fading channels. For SD the Chase-Pyndiah decoding algorithm was simulated to lower the bit error rate from iteration to iteration. We assumed an information array of (4×4) and the coded array had dimensions (7×7) for the first code, and an information array of (11×11) and the coded array had dimensions (15×15) for the second one. BER results were obtained as ensemble average of many runs (repetitions).

Simulation results indicate that the SD with Chase-Pyndiah algorithm provides nearly 1.1dB lower BER in comparison to the uncoded BPSK over AWGN channel for $(7,4)^2$ and 2.6dB when using the $(15,11)^2$ at target BER of 10^{-3}. For flat fading Rayleigh channel, the simulation achieved at around 18dB SNR value for the $(7,4)^2$ TPC and at 15dB for the $(15,11)^2$ at target BER of 10^{-3}, whereas the uncoded BPSK could achieve the same target BER beyond 20dB.

Keywords: Turbo product codes, Iterative decoding, Chase-Pyndiah algorithm.

ÖZ

Shannon sınırının yakınında düşük bit hata oranlarına ulaşabilen hata düzeltme kodları, 1993 yılında Berrou ve Glavieux tarafından önerilen Turbo Kodlar (TC) idi. Bu seçkin kodlar için kodlayıcılar, bir serpiştirici ile ayrılmış iki ardışık sistematik evrişimsel kodun paralel birleştirilmesi ile oluşturulmaktaydı. TC'lerin şifresini çözmek için hesaplama hızındaki kazanç ve hassaslığı artırması nedeniyle Log-MAP algoritması kullanılabilir. Turbo kodlama ve kod çözme ile ilgili tek problem, kodlayıcıdaki serpiştirici seçimine bağlı olarak, serpiştiricinin içsel zayıf mesafe özelliklerinden dolayı bir hata zeminine neden olabilmesidir.

Turbo çarpım kodları (TPC'ler) birden fazla basit, doğrusal blok kodunu birleştirerek oluşturulan güçlü doğrusal blok kodlarıdır. TPCler sıralı olarak birleştirilmiş kodlar altında sınıflandırırlar, ancak TC'lerin aksine bir kodlayıcıya dayanmazlar ve dolayısıyla hata zemini problemleri yoktur. 1954 yılına kadar uzanan TPC'ler, bileşen kodları olarak da bilinen iki veya daha fazla doğrusal blok kodla oluşturulmuş çok boyutlu kodlardır. Çarpım kodu, (k_1×k_2) bilgi bitlerini k_1 satır ve k_2 sütunları içeren bir diziye yerleştirerek elde edilir. Bir çarpım kodunun önemli parametreleri, bilginin uzunluğu (n), kod sözcüğünün uzunluğu (k) ve en ufak mesafe (d) dir. Turbo çarpım kodlarının çözümü, sert veya yumuşak kod çözme yöntemleri kullanarak gerçekleştirilebilir. Sert karar tabanlı çözme işleminde girdi ikili bir dizi olmalı, yumuşak kararlı çözücülerde ise değerler bir kod sırasını ölçmek için kod çözücü tarafından hemen işlenmelidir. Yumuşak kararlı kod çözücünün çıktılarını üretmek için bir SISO kod çözücü kullanılabilir. Yumuşak kod çözme algoritmalarından bazıları şunlardır: en büyük sonsal olasılık (MAP) algoritması, Chase-Pyndiah iteratif kod çözücüsü, sembol tabanlı MAP algoritması, yumuşak çıkışlı Viterbi algoritması, kayan-pencere tabanlı MAP dekoder ve ileri yineleme tabanlı MAP dekoder. Bu çalışmada, yumuşak kararlı (SD) bir kod çözücü için farklı sinyal / gürültü oranlarında (SNRs) bit hata oranları (BERs) hesaplanmıştır. Modülasyon tipi olarak BPSK kullanılmış ve kodlanmış sembollerin AWGN ve düz sönümlemeli Rayleigh kanalları üzerinden iletimi yapılmıştır. Benzetimler esnasında farklı iterasyonlardaki bit-hata-oranını düşürebilmek için Chase-Pyndiah şifre çözme algoritması kullanılmıştır. Bilgi dizisinin (4×4) olduğu ilk kod için kodlanmış dizinin boyutları (7×7) ve bilgi dizisinin (11×11) olduğu ikinci kod için kodlanmış dizinin boyutu (15×15) idi. Bit-hata-oranı sonuçları, çok sayıdaki benzetimin toplam ortalaması olarak elde edilmiştir.

Simülasyon sonuçları göstermiştir ki iteratif Chase-Pyndiah şifre çözme algoritması kullanılıp, AWGN kanalı üzerinde $(7,4)^2$ TPC kodlu veri iletimi gerçekleştirildiğinde 10^{-3} hedef BER şifrelenmemiş BPSKye göre 1.1dB daha önce sağlanmıştır. $(15,11)^2$ kodlu veri iletimi için ise bu değer 2.6dB daha önce sağlanabilmektedir. Düz sönümlemeli Rayleigh kanalı üzerinde 10^{-3} hedef BER $(7,4)^2$ ve $(15,11)^2$ TPC kodlu veri iletimleri için sırası ile 18dB ve 15dB de elde edilirken kodlanmamış BPSK aynı hedef bit-hata-oranını ancak 20dBden sonar sağlayabilmektedir.

Anahtar Kelimeler: Turbo çarpım kodları, iteratif kod çözücüsü, Chase-Pyndiah algoritması.

DEDICATION

To My Family

ACKNOWLEDGMENT

I would like to thank my supervisor Prof. Dr. Erhan A. İnce for his inspiration and continuous feedback. Without his careful guidance this study would not have been successively completed.

A special thanks to Dr. Mahmoud Nazzal for having discussions with me from time to time and showing me how to use MATLAB efficiently. Great thanks to all of my friends for their presence, as it enhanced my motivation by making me feel at home.

Finally, I especially wish to thank my lovely mother, father, brothers, sisters and uncles who have provided me with immeasurable support and encouragement during all these years.

TABLE OF CONTENTS

ABSTRACT ……………………………………………………………………...iii

ÖZ…………………………………………………………………………………...v

DEDICATION ……………………….……………………………………………..vii

ACKNOWLEDGMENT ……………………………………………………..viii

LIST OF FIGURES ………………………………………………………….. xi

LIST OF ABBREVIATIONS ……………….……………….…………………... xiii

1 INTRODUCTION 1

1.1 Literature Review 2

1.2 Thesis Outline... 4

2 TURBO PRODUCT ENCODER.. 5

2.1 Concatenated codes 6

2.2 Turbo Product Encoder... 8

3 TURBO PRODUCT DECODER: STRUCTURE AND ALGORITHMS 11

3.1 Hard Input Hard Output Decoding 11

3.2 Soft Input Soft Output Decoding.. 12

3.3 Structure of a Soft Input Soft Output Decoder 12

3.4 The Maximum a Posteriori Probability (MAP) Algorithm 14

4 CHASE DECODERS AND CHASE-PYNDIAH ALGORITHM 17

4.1 Introduction 17

4.2 Type-I Algorithm.. 18

4.3 Type-II Algorithm 19

4.4 Type-III Algorithm 19

4.5 Chase-Pyndiah Algorithm 19

5 SIMULATION BASED BER PERFORMANCE 25

5.1 Simulation results over AWGN Channel 26

5.2 Simulation results over Rayleigh Fading Channel 28

6 CONCLUSION AND FUTURE WORKS 32

6.1 Conclusion 32

6.2 Future work 33

REFERENCES…………………………..…………………………………………….35

APPENDIX 39

Appendix A: MATLAB implementation of Chase-Pyndiah Decoder 40

LIST OF FIGURES

Figure 2.1: Concatenated Codes. 7

Figure 2.2: Concatenated Encoder and Decoder with Interleaver. 7

Figure 2.3: The construction structure of TPCs ($P = C^1 \times C^2$). 8

Figure 2.4: Placing Information Bits in a ($k_1 \times k_2$) Array. 9

Figure 2.5: Encoding the k_2 Rows to produce n_1 Columns using C^1.. 9

Figure 2.6: Encoding the n_1 Columns to produce n_2 Rows using C^2. 10

Figure 2.7: Construction of $(7,4)^2$ TPC…………………………………………….10

Figure 3.1: Turbo Product Decoder Component Decoder 13

Figure 3.2: Architecture of Turbo Product Decoder. 14

Figure 4.1: Geometric Sketch for decoding algorithm. 18

Figure 4.2: Two-Dimensional TPC Iterative Decoding Process............................... 21

Figure 5.1: Transmission over the AWGN Channel. 26

Figure 5.2: Bit error rate performance for Chase-Pyndiah decoder for information transmitted over the AWGN channel after $(7,4)^2$ encoding [BPSK modulation, Repetition = 7000, number of least reliable bits (p)=4]. 27

Figure 5.3: Bit error rate performance for Chase-Pyndiah decoding of information transmitted over the AWGN channel after $(15,11)^2$ encoding [Code Rate ≈ 0.54, BPSK modulation , Repetition=1000, number of least reliable bits (p) =10]............ 28

Figure 5.4: Simulation Model for transmission over the Rayleigh fading channel. .. 29

Figure 5.5: Frequency response for a flat fading Rayleigh channel. 30

Figure 5.6: Bit error rate performance for Chase-Pyndiah decoding of information transmitted over the Rayleigh fading channel using $(7,4)^2$ encoding [Code Rate ≈ 0.33, BPSK modulation , Repetition=1000, number of least reliable bits (p) =4]..... 31

Figure 5.7: Bit error rate performance for Chase-Pyndiah decoding of information transmitted over the Rayleigh fading channel using $(15,11)^2$ encoding [Code Rate ≈ 0.54, BPSK modulation , Repetition=1000, number of least reliable bits (*p*) =10]... 31

Figure 6.1: An case of 4 least reliable value……….……..………...………..……..33

LIST OF ABBREVIATIONS

TCs	Turbo Codes
TPCs	Turbo product codes
SISO	Soft Input Soft Output
MAP	Maximum A Posteriori Probability
BER	Bit Error Rate
SNR	Signal to Noise Ratio
SD	Soft Decision
ECC	Error Correcting Coding
AWGN	Additive White Gaussian Noise
LDPC	Low Density Parity Check Codes
LAN	Local Area Network
HD	Hard Decision
FEC	Forward Error Correction
HIHO	Hard Input Hard Output
SDD	Soft Decision Decoding

LLR	Log Likelihood Ratio
BPSK	Binary Phase Shift keying
CTC	Convolutional Turbo Code
LOS	Line of Sigh
GA	Genetic Algorithm
PSO	Particle Swarm Optimization

Chapter 1

INTRODUCTION

In the nearby past years, there was a quick development in wireless mobile communication systems because the demand for applications that uses wireless mobile communication systems had been increasing very quickly. The major goal in communication systems are minimize the error probability by design an efficient systems of power and bandwidth resources, while reducing the complexity in order to save the time and reduce the cost .

Communication engineering attempt to transmit the information bits from the source to the destination via a channel with high reliability. Indeed, this is a difficult task since there are many factors to cause faults. Detecting and correcting errors is important while sending information bits. If we cannot detect the occurred errors and so we cannot correct them, the received information bits at the destination site will be differ from the original source information bits. These defect in the data sequence what the communication engineers try to reduce.

The information bits include any coding scheme matched to the nature of the data. The error correcting coding (ECC) encoder takes data bits as input from the source and adds redundant bits as additive white Gaussian noise (AWGN), then moves to the process of modulating a signal, sending it over a communication channel, demodulating it, and finally decoding the data bits to get the original information bits.

1.1 Literature Review

Reliability measured by the ability of errors correction and received information bits at receiver without errors or with a little percentage of errors, so it considered as one of the major aims when sending information bits over any medium. As long as the channel capacity is greater than the transmission rate, the reliability is possible even over channels with noise (Cover & Thomas, 1991). Claude Shannon shows this outcome for an AWGN channel in 1948, is known as the Shannon's Theorem or noisy channel coding theorem (Shannon, 1948).

Channel coding is used to protect the information bits from interference and noise and minimize the number of bit errors. The main idea is the transmitter encodes the information bit by adding redundant bits by using an ECC. The scientist Richard Hamming pioneered this field in the 1940s and created the first ECC in 1950 and called it the Hamming (7, 4) code.

This redundant bit allows detecting and correcting the bit errors at the received information bits. The cost of using channel coding to protect the information is an expansion in bandwidth or a reduction in data rate.

There are two principle sorts of channel codes, in particular block codes (also called algebraic codes) and convolutional codes (Lin & Costello, 1983) (Wicker, 1995). There are numerous contrasts between these two distinct sorts of coding systems. This difference was fundamentally taking into account three perceptions. Firstly, block codes are proper for protecting blocks of information bits independent one from the other whereas convolutional codes are suitable for protecting continuous streams of information bits. Secondly, the code rates of block codes are close to unity, while the code rates of convolutional codes are lower. Finally, block code decoding is sort of the hard input decoding and using soft input decoding for convolutional codes.

Presently, these differences are setting out toward blur. Convolutional codes can undoubtedly be fitting to encode blocks and soft inputs decoders have been accepted in block codes. Block code can be minimizing values of code rates to become equal with convolutional codes.

The requirement of modern coding is concatenated structures. The concatenated structures are using a few basic encoders and whose decoding is performed by iterated entries in the related decoders. The sort of iterative processing was opened by Turbo codes (1993) and from that point numerous concatenated structures depended on iterative decoding have been rediscovered (or envisioned). Some of these include turbo codes (TCs) and, low-density parity-check codes (LDPC). Each of these codes have been received in universal standards, and comprehension their coding forms and their decoding algorithms is a premise sufficiently wide to handle whatever other norms of disseminated coding and of related iterative decoding.

In this thesis, one of the linear block code types, called Block turbo codes also known as a Turbo product codes (TPCs) is presented. TPCs which are obtained by serial concatenation of simpler block codes were first proposed by Elias in 1954 (Elias, 1954).

1.2 Thesis Outline

Chapter 2 provides a brief introduction to the idea behind Turbo Product Codes (TPCs) and provides details on how the encoder for TPCs operates. It also explains how to calculate the rate of the TPC given the two constituent codes.

Chapter 3 presents the general structure for a soft input soft output (SISO) iterative decoder that can be used to decode TPCs, and also explains in detail the Maximum a Posteriori Probability (MAP) algorithm.

Chapter 4 points out the reason for choosing the Chase-Pyndiah iterative decoder for the decoding of TPCs and after explaining the three versions of the Chase algorithms provides full details on how to implement the Chase-Pyndiah decoder.

Chapter 5 studies the performance of TPCs through computer simulation using the MATLAB platform. Additive White Gaussian and Rayleigh fading channels are realized and information encoded using various TPCs are transmitted over the channels and retrieved at the receiver side using a Chase-Pyndiah decoder.

Chapter 6 concludes the thesis and gives some directions for future work.

Finally, the Appendix-A includes the 9 MATLAB functions developed while implementing the Chase-Pyndiah decoder.

Chapter 2

TURBO PRODUCT ENCODER

Turbo coding was first presented during 1993 by Claude Berrou. Until that time, scientists believed that it would be impossible to attain BER performances near Shannon's bound without vast complexity.

Turbo codes are obtained by two (or more) constituent codes which are also known as serial or parallel concatenation. There are two known forms of constituent codes: (i) block codes and (ii) convolutional codes. Turbo Product Codes (TPCs) which are also known as Block Turbo Codes (BTCs) is a variation on the conventional Turbo Coding but TPC encoding/decoding does not have an error floor problem as with TCs.

Turbo product codes are mainly used in mobile communications, wireless Local Area Networks (LAN), wireless internet access and satellite communications (M, Gao, & Vilaipornawai, 2002).

The ideas below constitute the base of turbo product codes:

1. Block codes are employed as an alternative of generally used systematic or non-systematic convolutional codes.

2. SISO decoding (SDDs) is employed as an alternative to HDDs.

3. Construct a long block code with moderate decoding complexity by combining shorter codes. Uses iterative decoding.

2.1 Concatenated codes

In 1954, Elias presented concatenated codes which assist in improving the authority of FEC (Forward Error Correction) codes. The theory supplies the production of outer encoder that contributes to a new encoder. The inner encoder is recognized as the concluding encoder prior to the channel. The consequential composite code is evidently more intricate than any of the character codes. On the other hand it can willingly be decoded: we basically pertain each of the factor decoders in turn, from the internal to the outer (Burr, 2001).

This straight forward system suffers a down side which is called error propagation. If a decoding error occurs in a codeword, it frequently consequences in a number of data error. When these are approved on to the subsequent decoder they may over power the capability of that code to correct the errors. The presentation of the outer decoder may be enhanced if these errors were circulated between a numbers of detached codeword. This is achieved via an interleaver/de-interleaver.

This interleaver contains of a two-dimensional array. Then, the data is interpreted all alongside its rows. After the array is filled, the information is interpreted out by columns, thus changing the arrangement of the information.

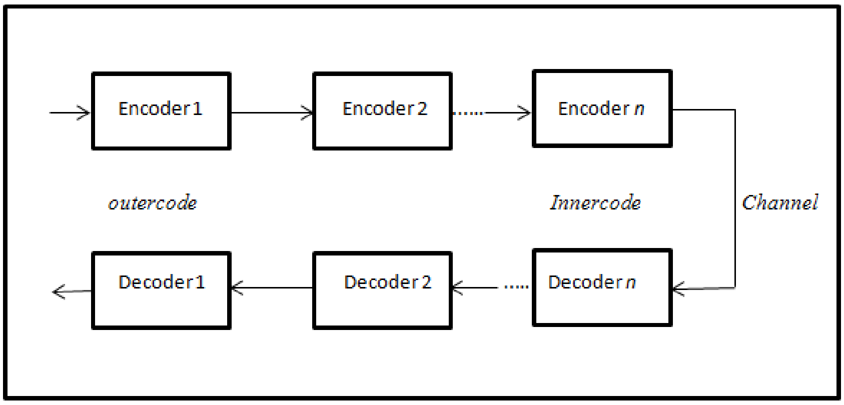


Figure 2.1: Concatenated Codes (Burr, 2001).

Interleaver could be positioned among the inner and outer encoders of a concatenated code that practices two element codes, the de-interleaver linking the outer and inner decoders in the recipient, as you can see in Figure 2.2. Afterwards, only if the rows of the interleaver are at the slightest provided that the outer codeword's, plus the columns are at the slightest on condition that the inner data blocks, every information bit of an inner codeword fall into a singular outer codeword. Therefore, so long as the outer code is capable of correcting no less than one error, it can at all times handle with particular decoding errors in the inner code.

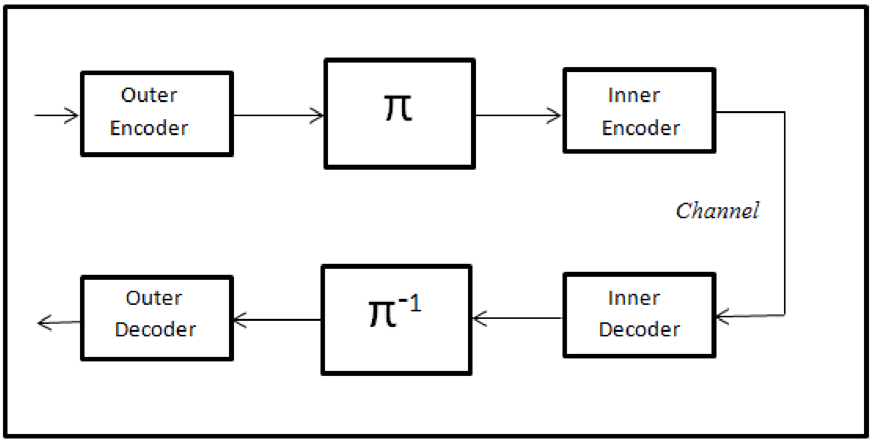


Figure 2.2: Concatenated Encoder and Decoder with Interleaver (Burr, 2001)

2.2 Turbo Product Encoder

TPCs are obtained by serial concatenation of linear block codes (MacWilliams & Sloane, 1978). The idea is to construct simple yet powerful codes with large minimum Hamming distance.

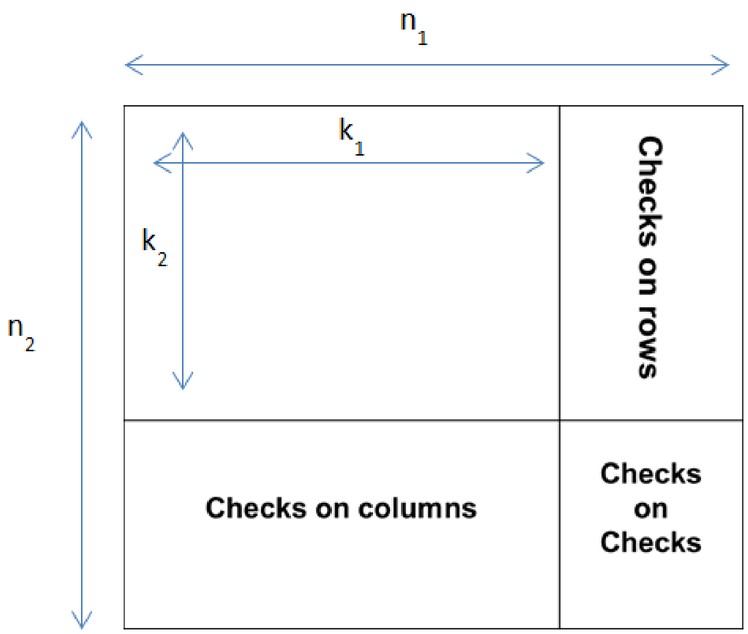


Figure 2.3: The construction structure of TPCs ($P = C^1 \times C^2$) (Al Muaini, AlDweik, & AlQutayri, 2011).

Consider we have two systematic linear block codes C^1 with parameters (n_1, k_1, $d^{(1)}{}_{\text{min}}$) and C^2 with parameters (n_2 , k_2 , $d^{(2)}{}_{\text{min}}$), where n_i , k_i and d_i , (i=1,2) indicates to codeword length, number of information bits, and minimum Hamming distance, respectively. The block codes C^1 and C^2 are named component or constituent codes and the resulting product code P can be represented as ($n_1 \cdot n_2$, $k_1 \cdot k_{2,}$ $d^{(1)}{}_{\text{min}} \cdot d^{(2)}{}_{\text{min}}$). As shown in Figure 2.3, the turbo product code is obtained as follows (Al Muaini, AlDweik, & AlQutayri, 2011) :

4. ($k_1 \times k_2$) data is placed in an array of k_2 rows and k_1 columns.

5. encode the k_2 rows using C^1.

6. encode the n_1 columns using C^2 to produce n_2 rows.

The code rate R for the constructed product code P would be $R_P = R_1 \times R_2$, where R_i is the code rate of code C^i and the correcting capabilities of turbo product code is denoted by $t = [(d_{\min} -1) / 2]$. Thus, we can shape much extended block codes with huge MHD by assembly short codes that have slight MHD.

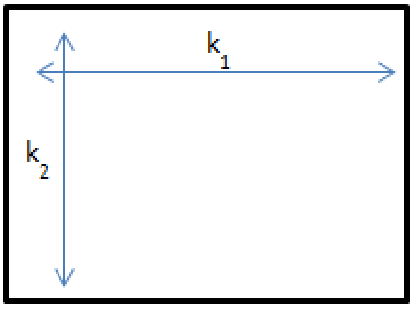


Figure 2.4: Placing Information Bits in a ($k_1 \times k_2$) Array.

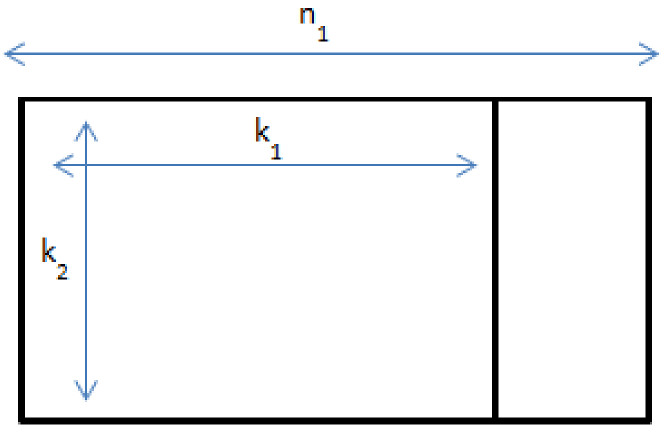


Figure 2.5: Encoding the k_2 Rows to produce n_1 Columns using C^1.

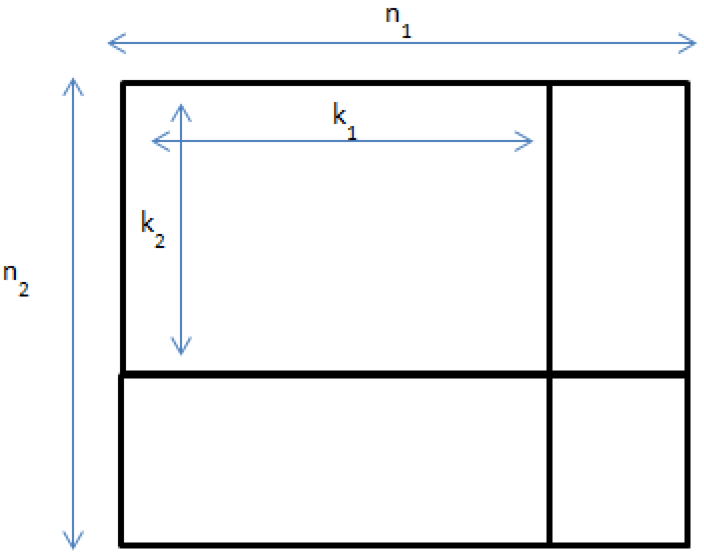


Figure 2.6: Encoding the n_1 Columns to produce n_2 Rows using C^2.

Note that the (n_2 - k_2) final rows are codewords of C^1 and (n_1 – k_1) final columns of the array are codewords of C^2 (Prasad & Nee, 2004). Figure 2.7 shows the construction of (7,4)^2 TPC and is an extract from (He & Ching, 2007).

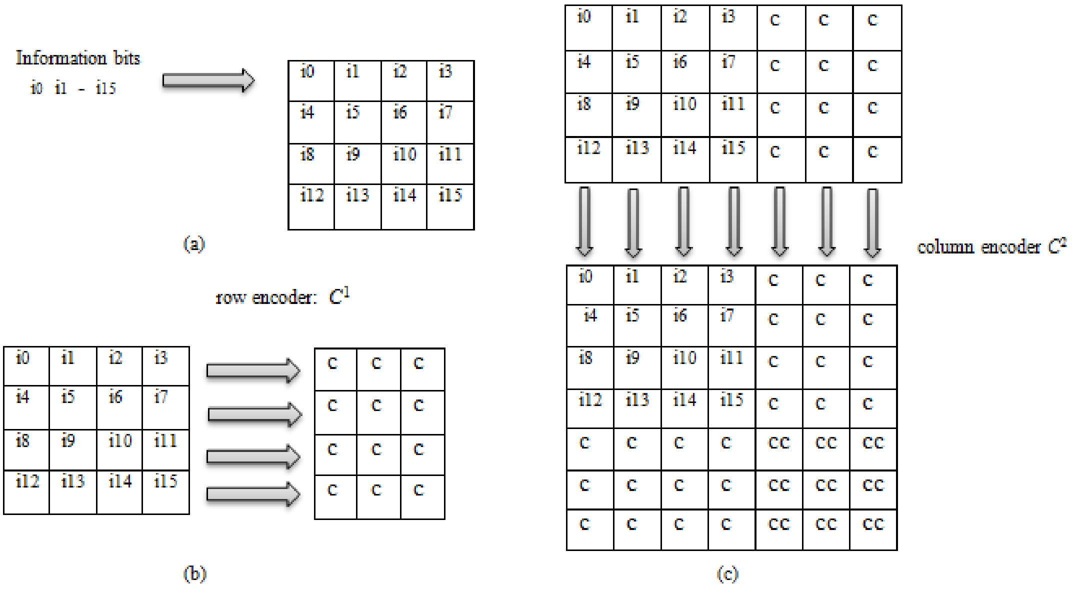


Figure 2.7: Construction of (7,4)^2 TPC (He & Ching, 2007).

Transmission encoded using TPCs are decoded using sequential “columns/rows” decoding to lower the decoding complexity. To attain better performance than the HDDs, we have to use SD of the constituent codes.

Chapter 3

TURBO PRODUCT DECODER: STRUCTURE AND ALGORITHMS

In this chapter we present algorithms for decoding Turbo Product Codes (TPCs) when the inputs are either hard or soft data. We first provide the general structure for a soft input soft output (SISO) iterative decoder and later explain in detail the Maximum a Posteriori Probability (MAP) algorithm.

3.1 Hard Input Hard Output Decoding

For a hard input hard output (HIHO) system, the output of the demodulator needs to be converted to binary. The output $\boldsymbol{H} = \left(h_{1,1}, \cdots, h_{n1,n2}\right)$ where $h_{i,j} \in \{0,1\}$ can be calculated using:

$$\boldsymbol{H} = 0.5[\operatorname{sign}(\boldsymbol{R}) + 1] \qquad (3.1)$$

Where ***R*** denotes the corresponding received sequence.

Once ***R*** has been computed, the maximum likelihood decoding (MLD) can be achieved searching for the codeword ***D*** that gives the minimum Hamming distance to ***H***:

$$\boldsymbol{D} = C^i \text{ if } \left|\boldsymbol{H} - C^i\right|^2 \leq |\boldsymbol{H} - C^l|^2 \qquad \forall\, l \in [1, 2^k], l \neq i \qquad (3.2)$$

Unfortunately, the complexity of the search for ***D*** will become high for large *n* values and the solution is to partition ***H*** into two separate vectors that can be decoded independently. For independent decoding of the individual parts row/column decoding can be used. In the first half iteration n_1 rows will be decoded and in the following half iteration n_2 columns will be decoded. For example, the Berlekamp-Massey algorithm described in (Berrou, Codes and Turbo Codes, 2010) can be used for HIHO decoding.

3.2 Soft Input Soft Output Decoding

For the soft input soft output (SISO) decoding maximum likelihood decoding is achieved by searching for the codeword $\boldsymbol{D}$ that minimizes the distance between $\boldsymbol{D}$ and $\boldsymbol{R}$. Similar to the HIHO case explained in section 3.1, $\boldsymbol{R}$ is partitioned into smaller row/column vectors. Afterwards, each vector (row/column) is decoded using soft decision decoding (SDD). One efficient SDD algorithm suggested for decoding of TPCs was first proposed in 1994 by R. Pyndiah (Pyndiah, Alain, Picart, & Jacq, 1994). This iterative decoder which is based on the Chase II algorithm (Chase, 1972) is known as Chase-Pyndiah iterative decoder. Each iteration of a Chase-Pyndiah decoder is composed of two half iterations. In the first half iteration the rows of the product code are decoded first. Similarly in the second half-iteration the columns are decoded. To reduce the bit error probability the algorithm will take multiple iterations. According to Kim there is a fine balance between performance and complexity of decoder. Therefore, the implementation of SDD is a very important issue.

3.3 Structure of a Soft Input Soft Output Decoder

The component decoder for a turbo product code is as depicted in Figure 3.1. Given the information received from the channel ($\boldsymbol{R}_\mathrm{i}$) the SISO decoder computes the soft output (LLR_i) during the first half-iteration. Since for the remaining half-iterations the demodulator does not provide actual soft information the algorithm has to compute extrinsic information (***E***i) from ***R***i and LLRi by multiplying LLRi by w_i^a and then subtracting ***R***i from the answer. Note that w_i^a is a weight factor.

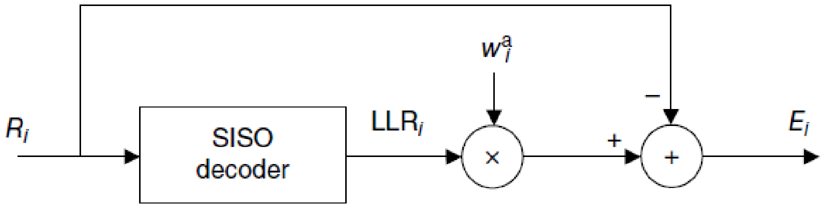


Figure 3.1: Turbo Product Decoder Component Decoder (Kim, 2015).

The algorithms have high complexity are Log-MAP algorithm or optimal MAP algorithm. Suboptimal variants like Max-log-MAP algorithm and Log-MAP algorithm are utilized in practice. The SOVA has low complexity so it considered as desirable algorithm. However, the Max-log-MAP algorithm has better performance than SOVA. The Log-MAP decoding scheme is the modified version of the MAP decoding scheme and is computationally less complex than the original MAP decoding algorithm (Kim, 2015).

Trellis of block codes can be represented, which allows us to implement ML or MAP decoding, but the complexity increments whenever the codeword length increments. So is appropriate for column or row codes or in general, for any short length codes.

The turbo product decoder in the receiver utilizes soft decision data bits sequence. Figure 3.2 represents the architecture of turbo product decoder.

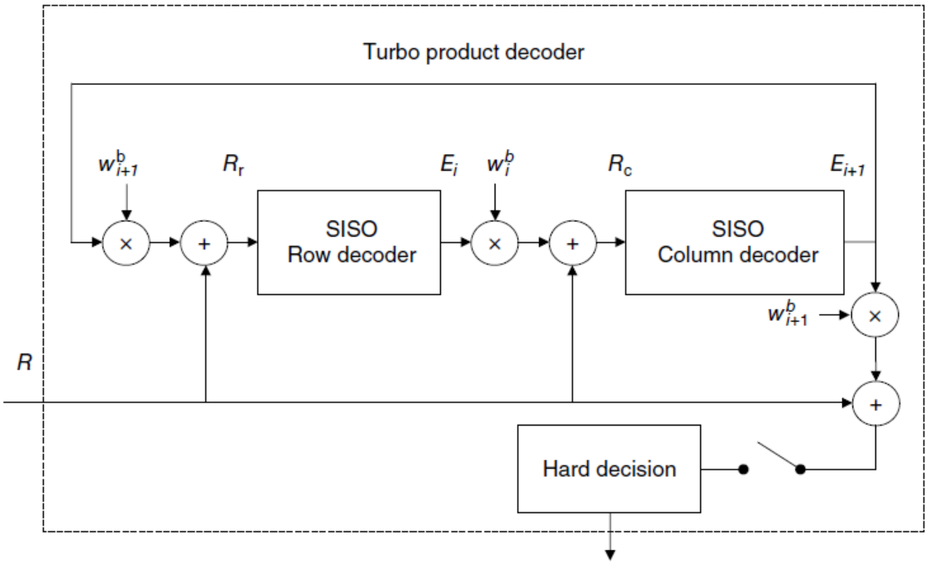


Figure 3.2: Architecture of Turbo Product Decoder (Kim, 2015).

3.4 The Maximum a Posteriori Probability (MAP) Algorithm

The reduction of the bit error rate in the decoding process is considered the main objective of MAP algorithm. Hence, in the wake of accepting the data out of the channel, the decoder set the most probable input bits, depends on the received bits. Since the input bits are binary, it is customary to shape a log-likelihood ratio (LLR) (Vucetic & Yuan, 2000) and base the bit gauges on correlations taking into account size of the probability proportion to a threshold. The ratio of log-likelihood for the input bits indexed at time t is known as

$$\Lambda(x_t) = ln\frac{P(x_t = 1|\mathrm{r})}{P(x_t = 0|\mathrm{r})} \qquad (3.3)$$

The goal of this method is to examine the received sequence and to calculate the a posteriori probabilities of the input information bits.

The expression of $P(x_t = i|\mathrm{r})$ indicates to the posteriori likelihood of the data bit, $(x_t = i)$ where $i \in \{0,1\}$, according to the received information r. Based on the values of the log-likelihood proportion, the decoder creates estimates of the data bits. The soft output which passed in the wake of processing to the next decoder as an a priori data is known the magnitude of LLR. The sign of log-likelihood decides the hard estimate of the original data sequence. The estimator complies with the accompanying following rule:

$$x_t = \{ 1 \text{ if } \wedge(x_t) \geq 0 \ , \ 0 \ otherwise \} \tag{3.4}$$

The LLR should be calculated exactly to perform the decoding and that can be defined as follows:

$$\wedge(x_t) = \frac{\sum_{l`=0}^{M_s-1} \alpha_{t-1}\,(l`)\beta_t(l)\gamma_t^1(l`,l)}{\sum_{l`=0}^{M_s-1} \alpha_{t-1}\,(l`)\beta_t(l)\gamma_t^0(l`,l)} \tag{3.5}$$

Where, α, β, and γ, are characterized as:

$$\alpha_t(l) = \sum_{l`}^{M_s-1} \sum_{i\in(0,1)} \alpha_{t-1}\,(l`)\gamma_t^i(l`,l) \tag{3.6}$$

$$\beta_t(l) = \sum_{l`=0}^{M_s-1} \sum_{i\in(0,1)} \beta_{t+1}\,(l`)\gamma_{t+1}^i(l,l`) \tag{3.7}$$

$$\gamma_t^i(l`,l) = p_t(i)exp \sum_{i\in(0,1)} r_t c_t \tag{3.8}$$

The value of $\alpha_t(l)$ indicates to a probability state at time (t) in light of a forward recursion over the trellis. It is values calculated iteratively and its quantity is a function of the received bits with time indices minimal than or equivalent to t. Moreover, according to its recursive form, $\alpha_0(l)$ value is initialized depends on the process of encoding. Since the encoder starts in the zero case for each frame $\alpha_0(0) = 0$ and $\alpha_0(l) = 1\, for\, l\ \neq 0$ (Vucetic & Yuan, 2000) (Hagenauer & Robertson, 1994).

The value of $\beta_t(l)$ also indicates to a probability state. Be that as it may, it depends on a backward recursion through the trellis. it is starting at time index t = τ. the quantity of $\beta_\tau(l)$ at the backward recursion likewise should be introduced like $\alpha_t(l)$. In the event that the last condition of the encoder is known, i.e. Sτ = j, then $\beta_\tau(j)$=1 and $c_\tau(l)$=0 $for\ l \neq j.$ However, if the last state is obscure, implying that was not ended to the all zero state at the end of the encoding process, then set $\beta_\tau(l)$=1 β for every one of the state.

The transitional probability $\gamma_t^i(l`, l)$ is the last part of the MAP algorithm. This term using the a priori information and the path metric to calculate the likelihood of the state when moving from state l` to l at time t where i ∈ {0,1}.

To avoid numerical instabilities, the terms α_i(m) and β_i(m) should to be scaled at every decoding stage, such that $\sum_m \alpha_i(m) = \sum_m \beta_i(m) = 1$ (Morelos-Zaragoza, 2006).

Chapter 4

CHASE DECODERS AND CHASE-PYNDIAH ALGORITHM

4.1 Introduction

The family of Chase algorithms (Chase I, Chase II and Chase III) are a set of sub-optimal decoding procedures that are designed to work with binary decoders that can correct up to $\lfloor (d-1)/2 \rfloor$ errors. A binary decoder in essence determines the codeword $\mathbf{X}_m$ which differ from the received sequence $\mathbf{Y}$ in as few locations as possible given that the difference is less than or equal to $\lfloor (d-1)/2 \rfloor$. If we define an "error sequence" as a sequence that contains 1s in places where $\mathbf{X}_m$ and $\mathbf{Y}$ differ and denote the binary weight of such a sequence as $\mathrm{W}(\mathbf{Z}_m)$ then the objective of a binary decoder is to find the error sequences (codeword) that has $\mathrm{W}(\mathbf{Z}_m) \leq \lfloor (d-1)/2 \rfloor$.

The family of Chase algorithms were proposed since the trellis-based Maximum a Posteriori Probability (MAP) used for decoding of convolutional turbo codes is computationally complex. Instead of using the MAP algorithm, the Chase algorithm is repeatedly applied among rows and columns of the Turbo Product Codes to obtain extrinsic information for each bit position. Chase III algorithm which is also known as Chase-Pyndiah algorithm generates test patterns using p least reliable bits which are computed using a reliability sequence obtained by the soft decoding of the received signal.

According to the error pattern generation the Chase algorithms can be classified into Type-I, Type-II and Type-III. The sections below give details about the three variants of the Chase decoder.

4.2 Type-I Algorithm

A very large collection of error patterns is considered for this algorithm. The algorithm uses the entire collection of error patterns inside a sphere of radius (d – 1) about **Y** which is known as the received sequence (refer to Fig. 4.1). Hence, all error patterns of binary weight fewer than or equivalent to (d – 1) are considered. While selecting the error patterns since the analog weight instead of the binary weight is used it is possible to select an error pattern with $\lfloor (d-1)/2 \rfloor$members which extends the error-correcting capability of the code (Chase, 1972).

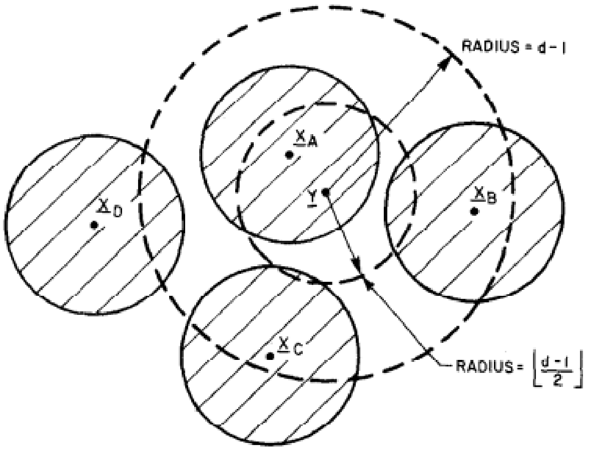


Figure 4.1: Geometric Sketch for decoding algorithm (Chase, 1972).

.

4.3 Type-II Algorithm

As opposed to the Type-I algorithm the Type-II Chase algorithm (Chase, 1972) considers a smaller collection of potential error patterns. It only considers the error patterns with less than $\lfloor (d-1)/2 \rfloor$errors located outside the set that contains $\lfloor d/2 \rfloor$ lowest channel measurements. The error patterns examined now contain less than (d - 1) errors and we no longer need to test all possible error patterns.

A set of test patterns, $\{\mathbf{T}\}$, that can generate all the required error patterns is used by the Type-II algorithm. All combinations of binary sequences at the *p* least reliable bit positions is considered and there are at most $2^{\lfloor d/2 \rfloor}$ test patterns (including all zero pattern). The Type-II algorithm has a significantly reduced complexity in return for a slightly inferior performance.

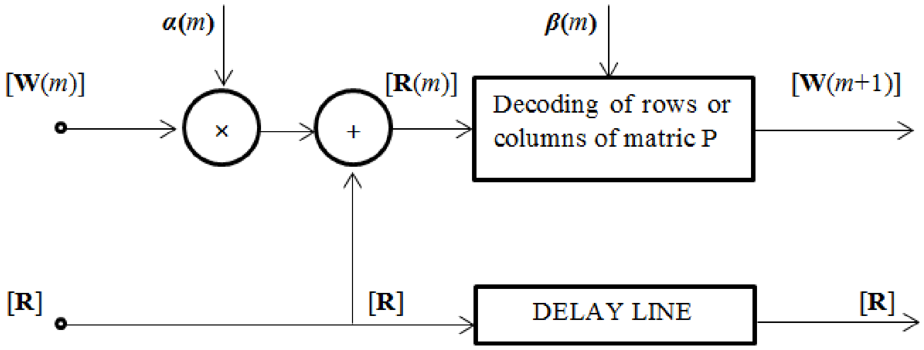
4.4 Type-III Algorithm

This algorithm is similar to the Type-II Algorithm, however rather than using $2^{[d/2]}$ test patterns it uses [(d/2) + 1] instead. Each test pattern has i 1’s situated in the i-th positions of minimal confidence values (Chase, 1972).

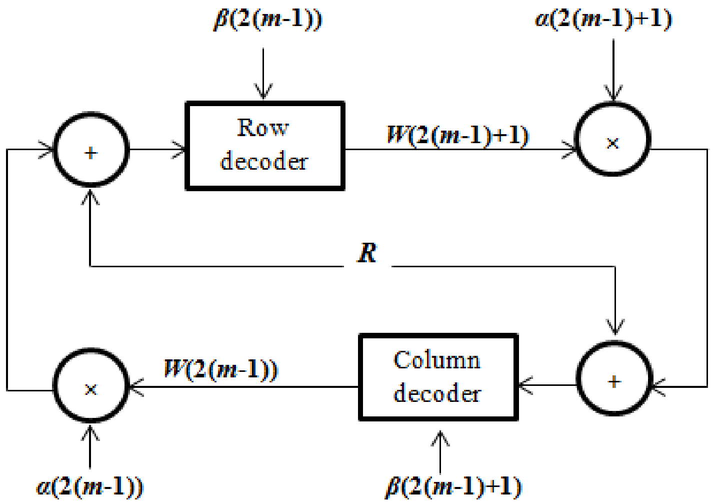
4.5 Chase-Pyndiah Algorithm

In 1972, Chase (Adde, Gomez Toro, & Jego , 2012) had proposed algorithms that approximate the optimum Maximum-Likelihood (ML) decoding of block codes with low computing complexity and small performance degradations. Later on in 1994, R. Pyndiah et al. (Wang, tang, & Yang, 2010) presented a new iterative decoding algorithm for decoding product codes, based on the iterative SISO decoding of concatenated block codes. What follows gives a summary of the well-known Chase-Pyndiah algorithm.

For linear block codes, the Chase-Pyndiah algorithm uses the values received from the transmission channel to approximate the performance of the maximum a posteriori (MAP) decoder. Pyndiah who was inspired by the idea of convolutional TC decoding (Berrou, Glavieux, & Thitimajshima, Near Shannon limit error-correcting coding and decoding: Turbo Codes, 1993) tried to improve the decoding algorithm by introducing the concept of a soft value that is computed at the output for each decoded bit. With iterative decoding, it became probable for the row and column decoders of a product code to exchange extrinsic information about the bits. This new algorithm which can decode the product codes in a turbo manned was named Chase-Pyndiah decoder. The block diagram for the Chase-Pyndiah decoder that works in a turbo fashion is as depicted in Figure 4.2 (a) at the *k*-th half iteration. For a two dimensional turbo product code the decoder must have two elementary decoders which are known as row and column decoders. Each elementary decoder will use the observation ***R*** (soft input from the channel) and the extrinsic information ***W*** (output of the second elementary decoder) at the prior half-iteration. In Figure 4.2 (b), *m* denotes the half-iteration number. One iteration is two half iterations and each elementary decoder only work during one half iteration and is idle during the second. Figure 4.2 (b) depicts how the two elementary decoders share extrinsic information with each other for iteratively improving the error rate. α is called the weighting factor and utilized to replace the unreliability of the extrinsic information specially at the initial iterations. It is zero at the primary half-iteration and increments up to one as the half-iteration follow. The reliability factor β is utilized to transform the output of the hard decision to a soft decision worth. It increments also up to 1 in the same way as α. β additionally gives ideal circumstances, where it is set as an element of the BER.



(a)Turbo decoder at *k*-th half-iteration (Pyndiah R. M., 1998).



(b)Extrinsic information sharing during decoding (Cho & Sung, 2011).

Figure 4.2: Two-Dimensional TPC Iterative Decoding Process.

In this thesis, the values of α and β used were the same as in (Pyndiah R. M., 1998) and are as depicted below:

$$\alpha(m) = [0.0, 0.2, 0.3, 0.5, 0.7, 0.9, 1.0, 1.0, 1.0, 1.0].$$

$$\beta(m) = [0.2, 0.4, 0.6, 0.8, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0].$$

What follows below give details on how to compute the *p* (least reliable positions), create the test sequences and the perturbed sequences, compute the syndromes and the metrics for each codeword, find the most likely codeword, compute the reliability for each bit and calculate the extrinsic information for each bit.

Assuming that $\boldsymbol{r} = (r_1, r_2, \cdots, r_n)$ is the received word coming from the transmission channel then the input of the SISO decoder's for rows at half iteration *m* is given by $\mathrm{R_{row}} = \mathrm{R_{row}} + \mathrm{W}\big(2(m-1)\big) * \alpha(2(m-1))$ and for columns $\mathrm{R_{col}} = \mathrm{R_{col}} + \mathrm{W}(2(m-1)+1) * \alpha(2(m-1)+1)$. The steps of the Chase-Pyndiah algorithm with *p* least reliable positions are given below:

Step 1: Create the reliability $\boldsymbol{R_{abs}} = (|r_1|, |r_2|, \cdots, |r_n|)$ and generate the sign sequence $\boldsymbol{Y} = (y_1, y_2, \cdots, y_n)$ from the observation $\boldsymbol{R}$, where

$$y_i = \begin{cases} 1, & if\ \mathrm{r}_i > 0 \\ 0, & \text{else} \end{cases}$$

Step 2: Set *p* from $\boldsymbol{R_{abs}}$ acquired at previous Step.

Step 3: Generate $\boldsymbol{q}(\boldsymbol{2^p})$ test patterns $\boldsymbol{T_q}$ which includes each element of zero and one on the selected positions, and all 0s on the rest of the positions.

Step 4: Create the perturbed sequence set by:

$$\boldsymbol{Z}_q = \boldsymbol{y} \oplus \boldsymbol{T}_q \tag{4.1}$$

Where $\oplus$ denotes to modulo-2 addition.

Step 5: Compute the set of syndrome values $\boldsymbol{S_i} = (s_1^i, s_2^i, \cdots, s_q^i)$ by using:

$$\boldsymbol{S}^{\mathrm{i}} = \boldsymbol{Z}^i \, . \, H^i \qquad (4.2)$$

For each sequence $\boldsymbol{Z}^i$ in the set of the perturbed sequence $\boldsymbol{Z}^q$ and the transpose of parity check matrix $\boldsymbol{H}$. If the computed syndrome is 0 that implies that there no errors. Else that, we take transpose syndrome for each row and comparing it with the columns of parity check matrix. The number of similar column that indicate to the number of bit that has an error. So we have to change it from 0 to 1 or vice versa and after that shape the concurrent codeword set $\boldsymbol{C}^q$ under the supposition that there is one error (Berrou, Codes and Turbo Codes, 2010).

Step 6: Compute the metrics for each $\boldsymbol{C}^i$ in the set of the concurrent codeword $\boldsymbol{C}^q$ by using the following equation:

$$\boldsymbol{M}_i = \sum_{1 \le j \le n} \boldsymbol{r}_j (1\text{-} \, 2\boldsymbol{c}_{ij} \,) \qquad (4.3)$$

Step 7: Determine the index of the minimum metrics and thus Codeword $\boldsymbol{c}_{pp}$ which is the most apparent one

$$\boldsymbol{M}_{pp} = \min\{\boldsymbol{M}_i\}$$

Step 8: Compute the reliability for each bit *j* such that:

$$\boldsymbol{F}_j = {}^{1}\!/_{4}\ (\min\{\boldsymbol{M}_i\ , c_{ij} \neq\ c_{pp,j}\} - \ \boldsymbol{M}_{pp}\ ) \tag{4.4}$$

The reliability $\boldsymbol{F}_{\mathrm{j}}$ will be a fixed value β when there isn't any concurrent words for which the j-th bit is not the same as $\mathrm{c}_{\mathrm{pp,j}}$

Step 9: Calculate extrinsic information or in other words, the output of the SISO decoder, for each bit j by using the following equation:

$$\boldsymbol{W}_j = (\ 2\ \times\ \boldsymbol{c}_{pp,j} - 1\ ) \times\ \boldsymbol{F}_j - \boldsymbol{r}_j \tag{4.5}$$

After that, extrinsic information values are shared between column and row decoders.

Chapter 5

SIMULATION BASED BER PERFORMANCE

It is a known fact that the power of FEC codes increases with length *k* and approaches the Shannon bound only at very large *k*. However, for large *k* values the decoding complexity will also rise. This suggests that it would be desirable to build a long, complex code out of much shorter component codes, which can be decoded more easily. Both in the literature and in this thesis it was shown that turbo product codes (TPCs) with long *k* can be achieved by concatenating similar or different rate Hamming Codes. Since decoding of TPCs can be costly using the maximum a posteriori decoder in this chapter the iterative decoder proposed by Chase-Pyndiah (described in Chapter 4) is used to generate the soft outputs.

This chapter presents bit-error-rate (BER) versus signal-to-noise-ration (SNRdB) plots for iterative decoding of information encoded using turbo product codes such as $(7,4)^2$ and $(15,11)^2$. Binary phase shift keying (BPSK) modulation is assumed and system BER performance over AWGN and Rayleigh fading channels is obtained using the MATLAB platform. The decoder is a Chase-Pyndiah decoder using the Chase-II algorithm for reduced computational complexity. The code rates for $(7,4)^2$ and $(15,11)^2$ TPCs were respectively 0.327 and 0.538.

5.1 Simulation results over AWGN Channel

The system block diagram depicted in Figure 5.1 shows how the information bits are encoded using TPCs and then modulated. Since the channel is an additive channel to each modulation symbol a respective white Gaussian noise is added. At the receiver a BPSK demodulator followed by iterative decoder for TPCs is used to decode the received sequence. Afterwards, decoded bits and the transmitted bits are compared to compute the bit error rate (BER) of the system.

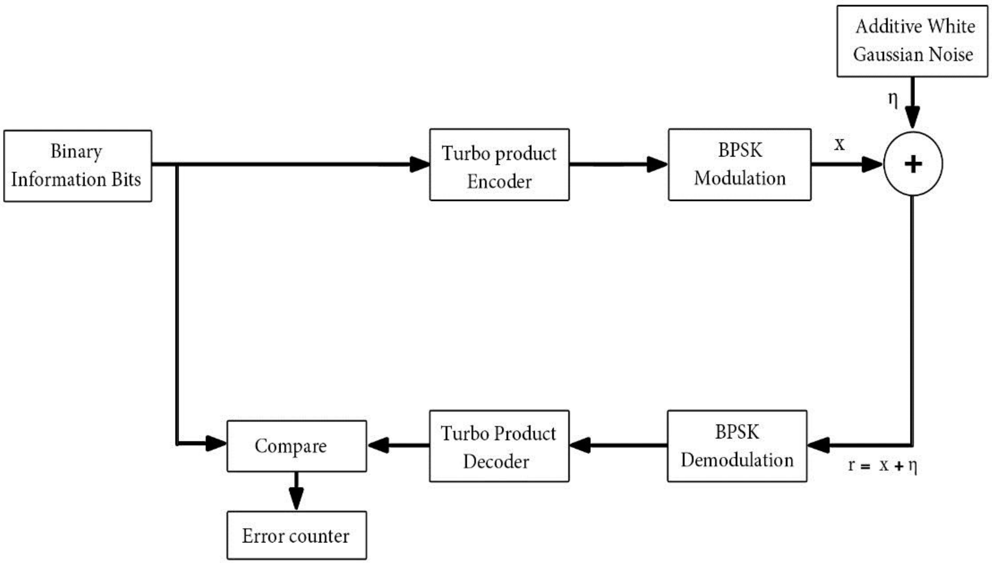


Figure 5.1: Transmission over the AWGN Channel (Ali, 2001).

During simulations the $(7,4)^2$ and $(15,11)^2$ TPCs were considered. The BER performance of the iterative decoder for the $(7,4)^2$ TPC is as provided in Figure 5.2. The plot depicts BER curves for the first five iterations of the iterative decoder and the uncoded BPSK performance. Please note that for the first couple of iterations the BER performance is worse than that of the uncoded BPSK. However, with increasing iterations the BER for the $(7,4)^2$ coded information will improve beyond the uncoded BPSK results. For instance at a BER of 10^{-3} the iterative decoder will be approximately 1.1dB better than the uncoded BPSK results. The results depicted in Fig. 5.2 have been obtained by taking an ensemble average of 7000 repetitions when p=4 (the number of least reliable bits).

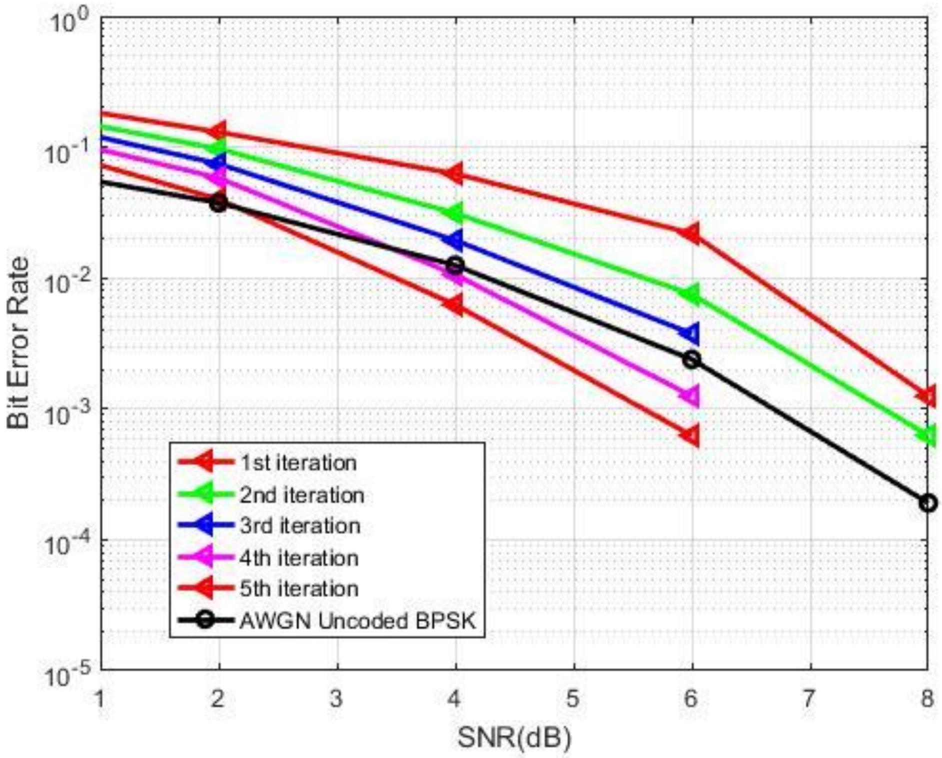


Figure 5.2: Bit error rate performance for Chase-Pyndiah decoder for information transmitted over the AWGN channel after $(7,4)^2$ encoding [BPSK modulation, Repetition = 7000, number of least reliable bits (*p*)=4].

The soft decoding results using the Chase-Pyndiah iterative decoder on $(15,11)^2$ encoded transmission over AWGN channel is provided in Figure 5.3. Here the information size is 121 bits. After the linear encoding the block size becomes 225 bits with code rate 0.54. The simulation curves show that as before at each iteration the turbo like decoding helps improve the achievable BER. However, this does not mean that BER will continue to further improve as iteration size is increased. As can be seen from Fig. 5.3 at iteration 5 the BER values are approximately the same as those at iteration 4. This indicates that with only four iterations a BER of 10^{-5} can be achieved at 6dB. For comparing performance of $(15,11)^2$ encoding against the $(7,4)^2$

encoding we again check to see what is the gain over the uncoded BPSK for a BER of 10^{-3} and this time the gain is approximately 2.6dB. This indicates that using a $(15,11)^2$ TPC will help attain 1.5 dB more gain over the system using the $(7,4)^2$ TPC.

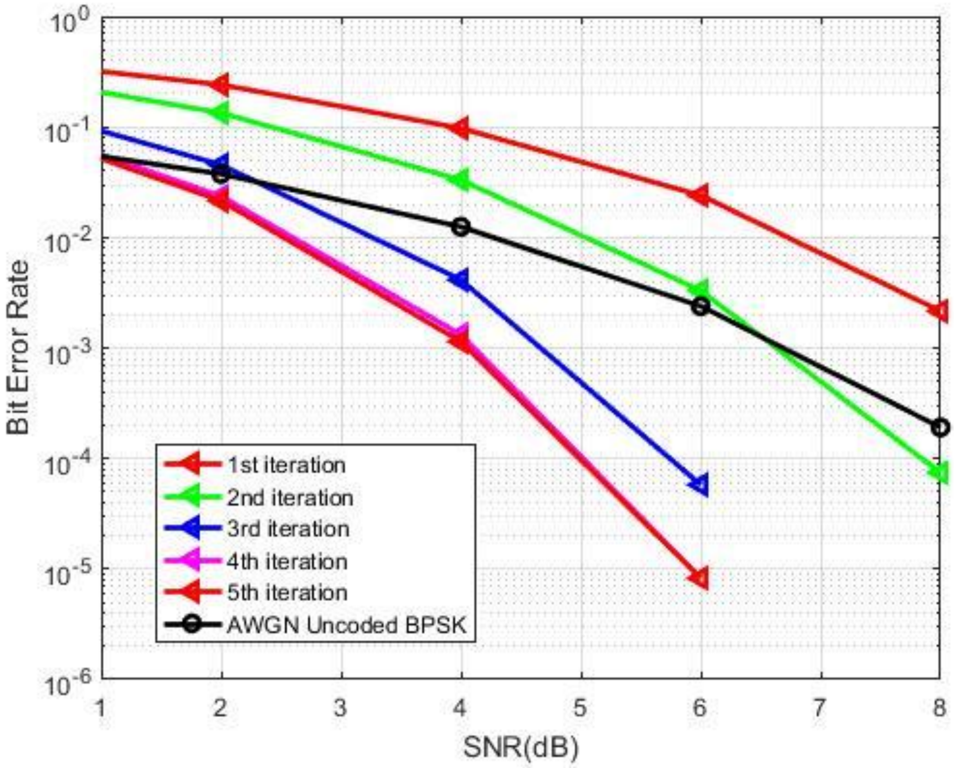


Figure 5.3: Bit error rate performance for Chase-Pyndiah decoding of information transmitted over the AWGN channel after $(15,11)^2$ encoding [Code Rate ≈ 0.54, BPSK modulation , Repetition=1000, number of least reliable bits (*p*) =10].

5.2 Simulation results over Rayleigh Fading Channel

In this section the results attained for transmission over the Rayleigh Fading channel are provided. The TPCs used are same as before [$(7,4)^2$ and $(15,11)^2$]. Figure 5.4 depicts the system model for transmission over the Rayleigh fading channel. The decoder used is the iterative Chase-Pyndiah decoder.

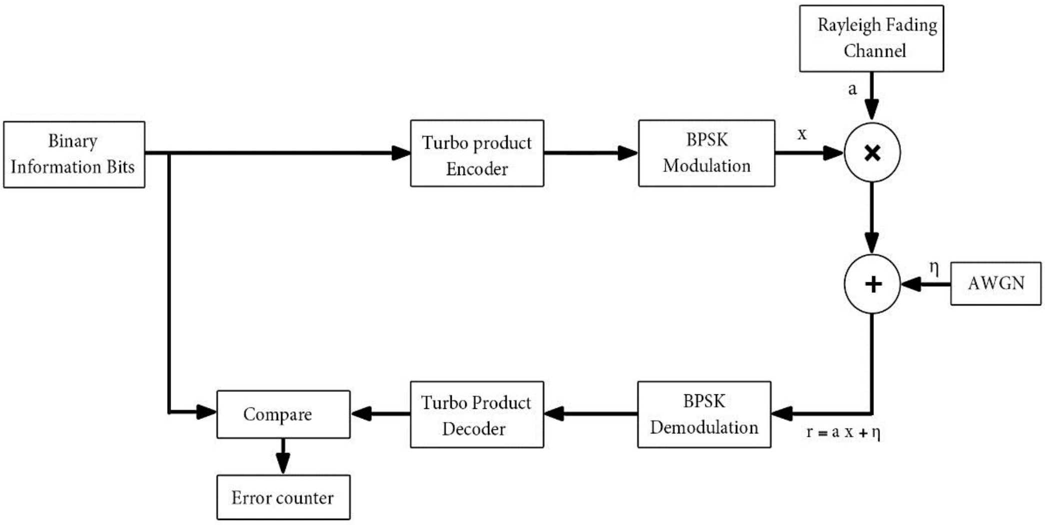


Figure 5.4: Simulation Model for transmission over the Rayleigh fading channel (Ali, 2001).

The Rayleigh fading channel usually referred to as the worst fading-channel is a statistical model for the effect of propagarion environment on a radio channel. A Rayleigh fading channel corrupts the transmitted signal with multiplicative fading and additive white Gaussian noise at the same time. When a signal passes through a Rayleigh channel its amplitude will fade according to a Rayleigh distribution. The received signal for transmission over the Rayleigh channel can be modelled as $r = a \cdot x + \eta$ (refer to Fig. 5.4) where a denotes the normalized Rayleigh fading factor and η the additive white noise. Such modelling is also known as the single-tap flat fading channel. Performance over a Rayleigh fading channel is expected to degrade due to the nulls in its frequency response as shown in Figure 5.5.

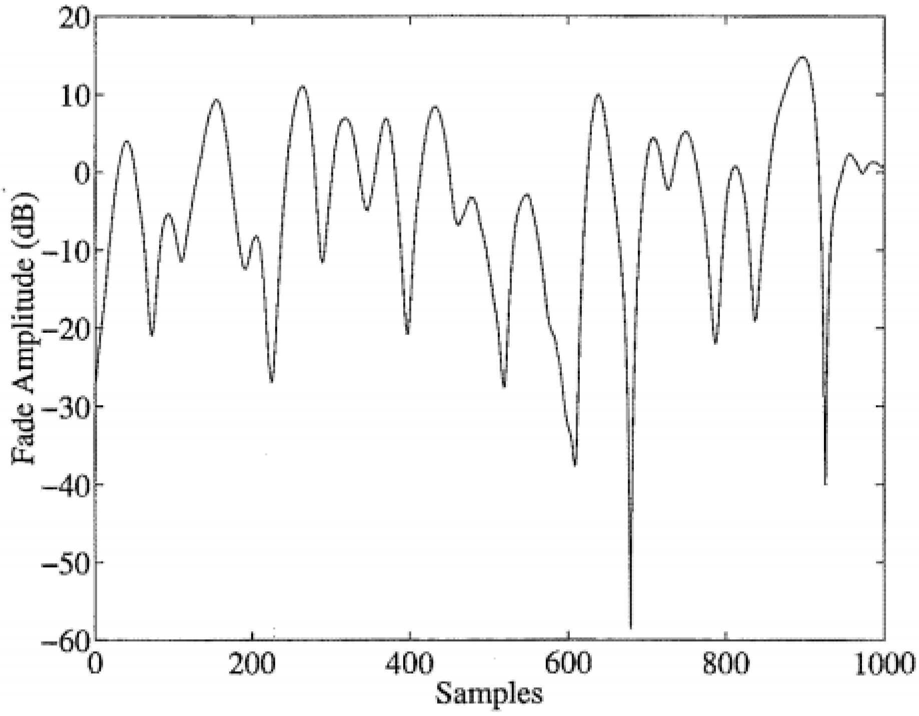


Figure 5.5: Frequency response for a flat fading Rayleigh channel (Rappaport, 2002).

Figures 5.6 and 5.7 below provide the BER performance of the iterative decoder for transmission over the Rayleigh fading channel using the $(7,4)^2$ and $(15,11)^2$ TPCs as encoder.

Note that the BER performance over a Rayleigh fading channel is lower in comparison to the performance over the AWGN channel. This is due to the deep nulls that are experienced in the frequency response of a Rayleigh fading channel and due to the fact that there is no line-of-sight (LOS) component available. Note that, a target BER of 10^{-3} is achieved approximately at SNR of 18dB for the $(7,4)^2$ TPC and at 15dB for the $(15,11)^2$ TPC. The uncoded BPSK will achieve target BER of 10^{-3} beyond 20dB.

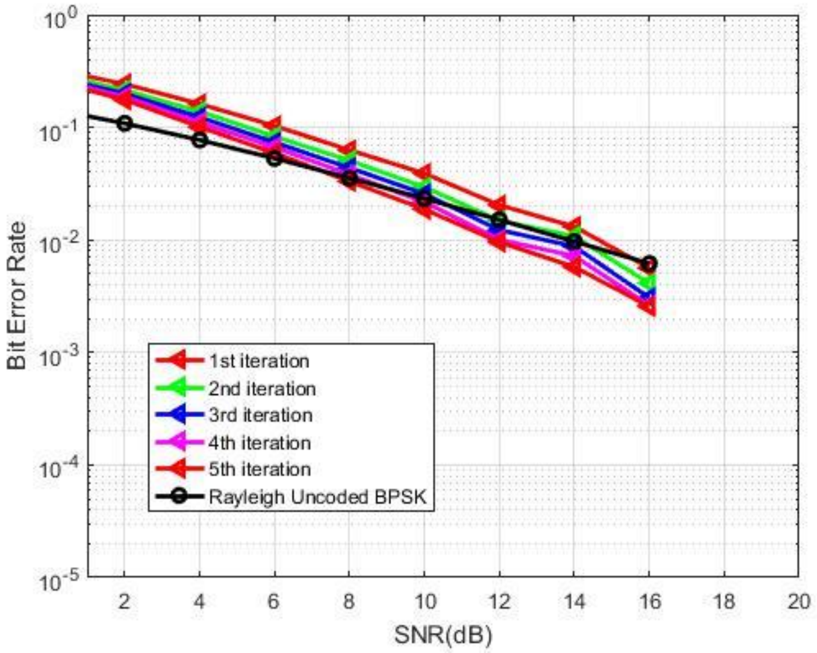


Figure 5.6: Bit error rate performance for Chase-Pyndiah decoding of information transmitted over the Rayleigh fading channel using $(7,4)^2$ encoding [Code Rate ≈ 0.33, BPSK modulation , Repetition=1000, number of least reliable bits (p) =4].

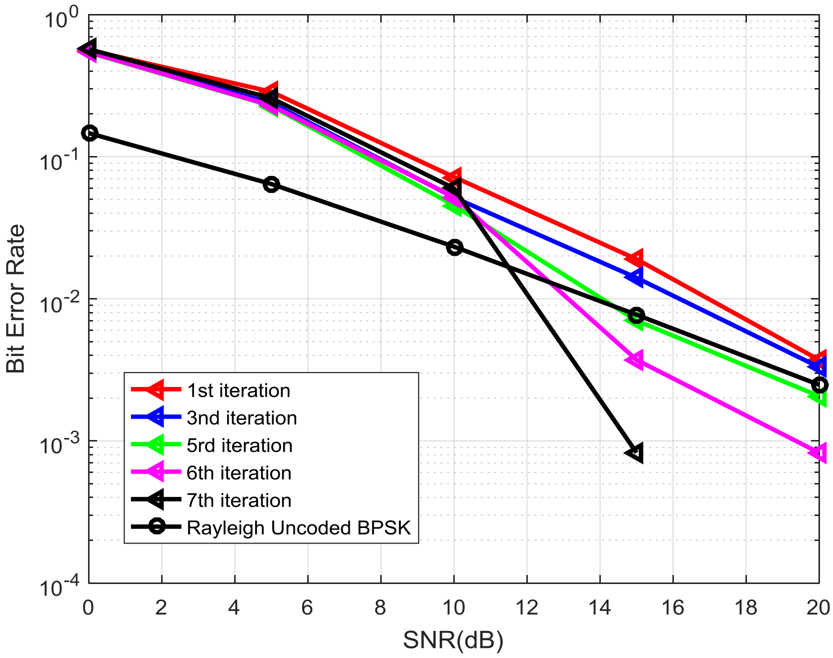


Figure 5.7: Bit error rate performance for Chase-Pyndiah decoding of information transmitted over the Rayleigh fading channel using $(15,11)^2$ encoding [Code Rate ≈ 0.54, BPSK modulation , Repetition=1000, number of least reliable bits (p) =10].

Chapter 6

CONCLUSION AND FUTURE WORKS

6.1 Conclusion

In the thesis we presented the BER vs SNR(dB) results for transmission of information bits encoded using $(7,4)^2$ and $(15,11)^2$ Turbo Product Codes. Channels assumed were the AWGN and the flat fading Rayleigh channel. For generation of the soft outputs the Chase-Pyndiah decoding algorithm was used. Simulation results have showed that for transmission over both the AWGN and Rayleigh channels the $(15,11)^2$ TPC will help achieve a better performance gains at a target BER of 10^{-3}. Under the AWGN channel the gain obtained over uncoded BPSK at target BER of 10^{-3} was 1.1dB for the $(7,4)^2$ encoded transmission and 2.6dB when using the $(15,11)^2$ encoded transmission. Over the AWGN channel, the iterative Chase-Pyndiah decoder will converge for both the $(7,4)^2$ or $(15,11)^2$ TPCs after 5 or so iterations.

We note that the BER performance over the Rayleigh fading channel was worse when using either of the two TPCs in comparison to performance attained over the AWGN channel. Over the flat fading Rayleigh channel a target BER of 10^{-3} was achieved at around 18dB SNR value for the $(7,4)^2$ TPC and at 15dB for the $(15,11)^2$ TPC. The uncoded BPSK could achieve the same target BER beyond 20dB.

6.2 Future work

Recently (Cho & Sung, 2011) has proposed the use of a threshold based three step algorithms, to decrease the computational complexity of iterative Chase-Pyndiah decoder. The objective is to reduce the value of p (minimize the error positions considered). The Algorithm can be summarized as follows:

Step1: Arrange the reliability of the selected least reliable positions obtained in ascending order.

Step2: Compute the absolute difference between each neighboring positions, then start from the pair that has the smallest value.

Step3: Set the value of threshold and except the location of higher reliability and all the locations exceeding the higher reliability value if the difference of the pair overtakes that value, then stop the process. Else do nothing and moves to the next pair.

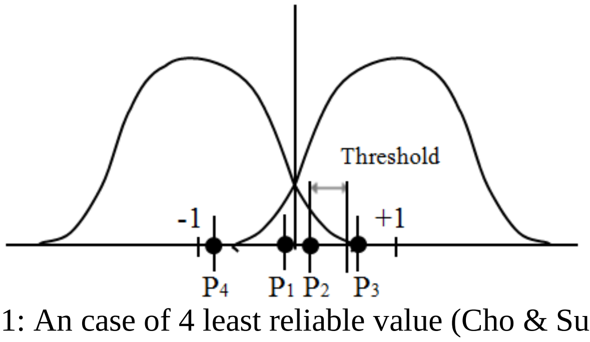


Figure 6.1: An case of 4 least reliable value (Cho & Sung, 2011).

For an example, if we suppose the least reliable value is 4, then we have four positions p_1, p_2, p_3, and p_4 with observation values -0.1, 0.2, 0.7, and -0.9, respectively as shown in Fig. 6.1, and the threshold is set to 0.4. Firstly, we sorted the value in ascending order based on their magnitudes. Note that, the position p2 is not excluded because the difference of reliability for p_1 and p_2 is 0.1 and that is smaller than the threshold, 0.4. Continually, the difference of reliability between p_2 and p_3 is 0.5 and this value is greater than the threshold. So, p_3 and p_4 are excepted also and the process stops. Least reliable changed from four to two positions, as explained in our example.

Future work should implement this threshold based algorithm and further reduce the computational complexity of the decoder and investigate to see how much performance loss will be experienced.

Recently Yamuna E.B. et al has proposed using the particle swarm optimization (PSO) for decoding of block turbo codes. He has showed that genetic algorithm (GA) based decoders have large latency and has proposed to overcome this problem using the PSO approach. As future work the 12-step PSO algorithm described in (Sudharsan , Vijay Karthik , Vaishnavi , & Yamuna, 2016) can be coded and its performance versus latency can be evaluated.

REFERENCES

Adde, P., Gomez Toro, D., & Jego , C. (2012). Design of an efficient Maximum Likelihood soft decoder for systematic short block codes. *IEEE TRANSACTIONS ON SIGNAL PROCESSING,*, pp.3914-3919.

Al Muaini, S., AlDweik, A., & AlQutayri, M. (2011). BER PERFORMANCE OF NONSEQUENTIAL TURBO PRODUCT CODES OVERWIRELESS CHANNELS. *IEEE GCC conf.* Dubai: Khalifa University.

Ali, S. (2001). *Performance Analysis of Turbo Codes.*

Argon , C., & McLaughlin, S. (2001). Turbo Product Codes for Performance Improvement of Optical CDMA Systems. *IEEE*, PP.1505-1509.

Berrou, C. (2010). *Codes and Turbo Codes.* Paris: Springer.

Berrou, C., Glavieux, A., & Thitimajshima, P. (1993). Near Shannon limit error-correcting coding and decoding: Turbo Codes. *ICC*, pp.1064-1070.

Burr, A. (2001). Turbo Codes: the ultimate error control codes. *ELECTRONICS Rr COMMUNICATION ENGINEERING JOURNAL*, pp.155-165.

Chase, D. (1972). A Class of Algorithms for Decoding Block Codes with Channel Measurement Information. *IEEE Transactions on Information Theory*, pp. 170–182.

Cho, J., & Sung, W. (2011). Reduced complexity Chase-Pyndiah decoding algorithm for turbo product codes. *IEEE,* pp.210-215.

Cover, T. M., & Thomas, J. A. (1991). *Elements of Information Theory.* New York: Wiley Interscience.

Elias, P. (1954). *Error-Free Coding," IRE Transactions on Information Theory, vol. PGIT-4, pp. 29–37,.* CAMBRIDGE, MASSACHUSETTS: MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

Goalic, A., Karine Cavalec-Amis, K., & Ker, V. (2002). Real-Time Turbo Decoding of Block Turbo Codes using the Hartmann-Nazarov algorithm on the DSP TEXAS TMS320C6201". *IEEE,* pp.1716-1720.

Hagenauer, J., & Robertson, P. (1994). Iterative Turbo Decoding of Systematic Convolutional Codes with MAP and SOVA algorithms. *ITG Conference on Source and Channel Coding.* Munich.

He, Y., & Ching, P. (2007). Performance Evaluation of Adaptive Two-Dimensional. *International Conference on Integration Technology,* (pp. PP. 103-107). Shenzhen, China.

Kim, H. (2015). *Wireless Communications Systems Design.* United Kingdom: John Wiley & Sons, Ltd.

Lin, S., & Costello, D. J. (1983). *Error Control Coding: Fundamentals and Applications.* New Jersey: Englewood Cliffs: Prentice-Hall.

M, S. R., Gao, Y., & Vilaipornawai, U. (2002). *The Theory of Error Correcting Codes.* United States of America: KLUWER ACADEMIC.

MacWilliams, F., & Sloane, N. (1978). *The Theory of Error Correcting Codes.* United States of America: North-Holland.

Morelos-Zaragoza, R. H. (2006). *The Art of Error Correcting Coding.* England: John Wiley & Sons Ltd,.

Pradhan, H. (2009). *Block Turbo Code and its application to OFDM for wireless local area network.* Rourkela: National Institute of Technology.

Prasad, R., & Nee, R. V. (2004). *OFDM for Wireless Communications systems.* London: Artech House.

Pyndiah, R. M. (1998). Near-Optimum Decoding of Product. *IEEE Transactions on Communications*, pp.1003-1010.

Pyndiah, R., Alain, G., Picart, A., & Jacq, S. (1994). Near optimum decoding of products codes. *IEEE GLOBECOM'94 Conf* (pp. pp.339–343). Bert, France: Telecome Bretagne.

Rappaport, T. S. (2002). *Wireless Communications Principles and Practice.* Prentice Hall.

Reddy, S., & Robinson, J. (1972). Random error and burst corrections by iterated codes. *IEEE Transactions on Information Theory*, 182–185.

Shannon, C. E. (1948). A Mathematical Theory of Communication. *Bell System Technical Journal*, 379-423 (Part One), 623-656 (Part Two).

Sudharsan , A., Vijay Karthik , B., Vaishnavi , C., & Yamuna, E. (2016). Performance enhanced iterative soft-input soft-output decoding algorithms for block turbo codes. *Journal of Telecommunications, Electronic and Computer Engineering*, pp.105-110.

Vucetic, B., & Yuan, J. (2000). *Turbo Codes Principles and Applications.* New York: Kluwer Academic.

Wang, F.-G., tang, Y., & Yang, F. (2010). The Iterative Decoding Algorithm Research of Turbo Product Codes. *IEEE*, pp.97-100.

Wicker, S. B. (1995). *Error Control Systems for Digital Communications and Storage.* New Jersey: Englewood Cliffs: Prentice-Hall.

APPENDIX

Appendix A: MATLAB implementation of Chase-Pyndiah Decoder

Programs required for Chase-Pyndiah decoding of information transmitted over AWGN channel after encoding with $(15,11)^2$ TPC with BPSK modulation.

1. *calc_ExtrinsicInfo_W.m*

2. *calc_metrics.m*

3. *calc_SyndromesandCodewords.m*

4. *calc_test_patterns.m*

5. *find_rel.m*

6. *gen_perturbed_sequence.m*

7. *half_iteration.m*

8. *received_signal.m*

9. *main_simulation_program.m*

```
function W=calc_ExtrinsicInfo_W(D,F,R_row)

num_cols=size(D,2);

    for j=1:num_cols

    W(j)=((2*D(j)-1)*F(j))- R_row(j);

    end

end
```

```
function [M,Mpp,D,loc]=calc_metrics(R_row,C)

num_rows=size(C,1);
num_cols=size(C,2);

for i=1:num_rows
    Ci = C(i,:);
    sum=0;
    for j=1:num_cols
        sum=sum+R_row(j)*(1-2*Ci(j));
    end
    M(i)=sum;

    Mpp = min(M);
    Loc = find(M==Mpp);
    Loc = loc(1);

    D = C(loc,:);
end
```

```
function [C,Si]=calc_SyndromesandCodewords(Z)
[parmat00,genmat,n,k] = hammgen(4);

num_col=size(Z,2);
C=Z;
Si = [];

for i=1:size(Z,1)
    hi=Z(i,:);
    syndrome = mod(hi * parmat00' ,2);
    Si=[Si;syndrome];
    col=syndrome';
    ci=hi;
    for k=1:num_col
        parmat00_col=parmat00(:,k);
        cond=numel(find(col==parmat00_col))==4;
        if cond
            ci(k)=~hi(k);
        end
    end

    C(i,:)=ci;
end
```

```
function Tq=calc_test_patterns(R,p,P)
num_rows=2^p;
num_cols=size(R,2);
Tq=zeros(num_rows,num_cols);
perm_mat=double(dec2bin(0:2^p-1))-48;

for i=1:num_rows
    Tq(i,P)=perm_mat(i,:);
end
```

```
function F=find_rel(M,C,D,Mpp,beta)
num_rows=size(C,1);
num_cols=size(C,2);

        for j=1:num_cols
            store_M= 1000*ones(size(M));
            j;
            Cj=C(:,j);
            for ii=1:num_rows
                ii;
                if Cj(ii)~= D(j)
                    store_M(ii)=M(ii);
                end
            end
            min_CM=min(store_M); % minimum changed M
           if min_CM == 1000
                   %F(j)=beta*D(j);
                   F(j)=beta;
           else
                 F(j)=(min_CM - Mpp)/4;
               end
        end
end
```

```
function Zq=gen_perturbed_sequence(Y,Tq)
num_rows=size(Tq,1);

for i=1:num_rows
    Zq(i,:)=xor(Y,Tq(i,:));
end
```

```
function [W,D]=half_iteration(R_row,beta)

[parmat00,genmat,n,k] = hammgen(4);

p=10;
% step1
R_abs= abs(R_row);
Y=(R_row>0);

% step2
[AA,index] = sort(R_abs);
Pos=index(1:p);

% step3
Tq=calc_test_patterns(R_row,p,Pos);

% step4
Zq=gen_perturbed_sequence(Y,Tq);

% step5
[C,Si]=calc_SyndromesandCodewords(Zq);

% step6
[M,Mpp,D,loc]=calc_metrics(R_row,C);

% step7
F=find_rel(M,C,D,Mpp,beta);

% step8
 W=calc_ExtrinsicInfo_W(D,F,R_row);
```

```
% This is the main simulation program that uses the functions.

clear all
close all
clc

alpha=[0.0  0.2  0.3  0.5  0.7   0.9  1.0  1.0  1.0 1.0 ];
beta =[0.2  0.4  0.6  0.8  1.0   1.0  1.0  1.0  1.0 1.0 ];

 %alpha=[0.0 0.1 0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.55 0.6 0.65 0.7 0.9 1.0 1.0];
 %beta =[0.2 0.3 0.4 0.45 0.5 0.55 0.6 0.65 0.7 0.75 0.8 0.9  1.0 1.0 1.0 1.0];

n=15; k=11;                              %Parameters of Linear block
code
info_word_length=k*k;

SNRdB=0:2:8;                            %SNR in dB
SNR=10.^(SNRdB./10);                    %SNR in linear scale

ber1=zeros(length(SNR),1);               %Simulated BER fod SOFT Decoding
ber2=zeros(length(SNR),1);
ber3=zeros(length(SNR),1);
ber4=zeros(length(SNR),1);
ber5=zeros(length(SNR),1);
%
for i=1:length(SNR)
    SNR(1,i)
    xj1=0;
    xj2=0;
    xj3=0;
    xj4=0;
    xj5=0;
    repetition=1000;

    for jj=1:repetition

% ==========================================================
% Start iterative decoding using Chase-Pyndiah algorithm
%
[R,msg,v]=received_signal(SNR(i));


    % initializing W:
        W=zeros(15,15);
```

```
            m=1; % iteration-1

                R1=R+W*alpha(2*(m-1)+1);              % Input of row
decoding for 1st iteration
                for k=1:15
                R_row=R1(k,:);
                [W1,D]=half_iteration(R_row,beta(2*(m-1)+1));
                W(k,:)=W1;
                end



                R2=R+W*alpha(2*(m-1)+2);  % Input of column decoding for
1st iteration
                for k=1:15
                R_col=R2(:,k);
                [W1,D]=half_iteration(R_col',beta(2*(m-1)+2));
                W(:,k)=W1;
                end

           out_it1 = W>0;
           z1=length(find((msg ~= out_it1(5:15,5:15))));
           xj1=xj1+z1 ;




                m=2;


                R1=R+W*alpha(2*(m-1)+1);  % Input of row decoding for 2nd
iteration
                for k=1:15
                    R_row=R1(k,:);
                    [W1,D]=half_iteration(R_row,beta(2*(m-1)+1));
                    W(k,:)=W1;
                end

                 R2=R+W*alpha(2*(m-1)+2);    % Input of column decoding
for 2nd iteration
                for k=1:15
                    R_col=R2(:,k);
                    [W1,D]=half_iteration(R_col',beta(2*(m-1)+2));
                    W(:,k)=W1;
                end

               out_it2 = W > 0;
               z2=length(find((msg ~= out_it2(5:15,5:15))));
               xj2=xj2+z2;



               m=3;

                R1=R+W*alpha(2*(m-1)+1);  % Input of row decoding for 2nd
iteration
                for k=1:15
                    R_row=R1(k,:);
                    [W1,D]=half_iteration(R_row,beta(2*(m-1)+1));
                    W(k,:)=W1;
                end

                 R2=R+W*alpha(2*(m-1)+2);    % Input of column decoding
for 2nd iteration
                for k=1:15
                    R_col=R2(:,k);
                    [W1,D]=half_iteration(R_col',beta(2*(m-1)+2));
                    W(:,k)=W1;
                end

               out_it3 = W > 0;
               z3=length(find((msg ~= out_it3(5:15,5:15))));
               xj3=xj3+z3;



                m=4;

                R1=R+W*alpha(2*(m-1)+1);  % Input of row decoding for 2nd
iteration
                for k=1:15
                    R_row=R1(k,:);
                    [W1,D]=half_iteration(R_row,beta(2*(m-1)+1));
                    W(k,:)=W1;
                end

                 R2=R+W*alpha(2*(m-1)+2);    % Input of column decoding
for 2nd iteration
                for k=1:15
                    R_col=R2(:,k);
                    [W1,D]=half_iteration(R_col',beta(2*(m-1)+2));
                    W(:,k)=W1;
                end

               out_it4 = W > 0;
               z4=length(find((msg ~= out_it4(5:15,5:15))));
               xj4=xj4+z4;



                m=5;

                R1=R+W*alpha(2*(m-1)+1);  % Input of row decoding for 2nd
iteration
                for k=1:15
                    R_row=R1(k,:);
                    [W1,D]=half_iteration(R_row,beta(2*(m-1)+1));
                    W(k,:)=W1;
                end

                 R2=R+W*alpha(2*(m-1)+2);    % Input of column decoding
for 2nd iteration
                for k=1:15
                    R_col=R2(:,k);
                    [W1,D]=half_iteration(R_col',beta(2*(m-1)+2));
                    W(:,k)=W1;
                end

               out_it5 = W > 0;
               z5=length(find((msg ~= out_it5(5:15,5:15))));
               xj5=xj5+z5;

end
[xj1 xj2 xj3 xj4 xj5 ]

ber1(i,1)=xj1/(repetition*info_word_length);
ber2(i,1)=xj2/(repetition*info_word_length);
ber3(i,1)=xj3/(repetition*info_word_length);
ber4(i,1)=xj4/(repetition*info_word_length);
ber5(i,1)=xj5/(repetition*info_word_length);

end

[SNRdB' ber1 ber2 ber3 ber4 ber5]

figure
semilogy(SNRdB,ber1(:,1),'r-<','linewidth',2.0)
hold on;
```

```
semilogy(SNRdB,ber2(:,1),'g-<','linewidth',2.0)
semilogy(SNRdB,ber3(:,1),'b-<','linewidth',2.0)
semilogy(SNRdB,ber4(:,1),'m-<','linewidth',2.0)
semilogy(SNRdB,ber5(:,1),'r-<','linewidth',2.0)

theoretical_awgn=0.5*erfc(sqrt(SNR));  %Theoretical uncoded BPSK AWGN
semilogy(SNRdB,theoretical_awgn,'ko-','LineWidth',2); grid on
legend('1st iteration','2nd iteration','3rd iteration','4th
iteration','5th iteration','AWGN Uncoded BPSK');
xlabel('SNR(dB)');
ylabel('Bit Error Rate');
axis([1 8 10^-6 1])
```