Stock Market Credibility upon Trading Volume: Bohmian Quantum Potential Approach

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ABSTRACT

Price return is an interesting factor for many investors; however, it is expected that the price return credibility to be affected by the trading volume of any given market as a complex system. In this study, the Bohmian quantum mechanics is used due to the time correlation of return and volume of the stock markets under consideration to investigate the relationship between these variables. The obtained results show that the quantum potential functions in the same manner for trading volume as the price return, and confines the variations of the volume to a specific interval. The joint quantum potential as a function of return and volume, defined by the probability distribution function (pdf) of a given market, serves as a suitable instrument to check the credibility of the market at higher volumes. As a result of the behavior of the pdf and the corresponding quantum potential, the variations of the price return at higher volumes decrease as the trading volume increases, making the market more credible which is more pronounced in developed markets.

Further, it is shown that, the distance between the quantum potential walls of price returns can be a proxy for the risk of the relative stock index. In other words, the investigation of different return frequencies shows that the market risk increases as the distance between the potential walls increases. The magnitude of the risk is different for different indices allowing the traders to decide on their portfolio selection and their investment horizon. Our results are consistent with the behavior of the developed and emerging markets.

Keywords: Price return, Trading volume, Joint quantum potential, Risk

ÖZ

Fiyat getirisi birçok yatırımcı için ilgi çekici bir faktördür; bununla birlikte, fiyat getirisinin güvenilirliğinin, belirli bir pazarın ticaret hacminin karmaşık bir sistem olarak etkilenmesinden kaynaklanması beklenmektedir. Bu çalışmada Bohmian kuantum mekaniği, söz konusu borsaların fiyat getirisi ve hacminin zaman korelasyonundan dolayı kullanılmıştır. Elde edilen sonuçlar, kuantum potansiyelinin, fiyat getiri olarak alım satım hacmi için aynı şekilde işlev gördüğünü ve hacmin varyasyonlarını belirli bir aralıkla sınırlandığını göstermektedir. Belirli bir pazarın olasılık dağılımı fonksiyonu (odf) ile tanımlanan geri dönüş ve hacim fonksiyonu olarak ortak kuantum potansiyeli, pazarın daha yüksek hacimlerde güvenilirliğini kontrol etmek için uygun bir araç olarak hizmet edebileceği gösterilmiştir.

Olasılık dağılımı fonksiyonunun davranışı ve buna karşılık gelen kuantum potansiyelinin bir sonucu olarak, işlem hacminin artmasıyla, daha yüksek hacimlerdeki fiyat getirisinin değişmesi azalarak, gelişmiş piyasalarda daha belirgin olan piyasayı daha güvenilir kılmaktadır. Ayrıca, fiyat iadelerinin kuantum potansiyel duvarları arasındaki mesafenin, ilgili hisse senedi endeksi riski için bir gösterge olabileceği gösterilmiştir. Başka bir deyişle, farklı dönüş frekanslarının araştırılması, potansiyel riskler arasındaki mesafe arttıkça piyasa riskinin arttığını göstermektedir. Risklerin büyüklüğü, yatırımcıların portföy seçimine ve yatırım ufkuna karar vermelerine izin veren farklı endeksler için farklıdır. Sonuçlarımız gelişmiş ve gelişmekte olan piyasaların davranışları ile tutarlıdır.

Anahtar Kelimeler: Fiyat getirisi, İşlem hacmi, Ortak kuantum potansiyeli, Risk

To My Loving and Supportive Wife

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LIST OF ABBREVIATIONS

ACT	Algorithmic Complexity Theory
AMH	Adaptive Markets Hypothesis
BFT	Behavioral Finance Theory
DAX	Deutsche Boerse AG German Stock Index
DJIA	Dow Jones Industrial Average
EMH	Efficient Market Hypothesis
PDF	Probability Density Function
SSEC	Shanghai Stock Exchange Index
S&P500	Standard and Poor's 500
TOPIX	Tokyo Stock Price Index

Chapter 1

INTRODUCTION

In this introductory chapter the background for the subject together with the importance and purpose of the study will be presented. Further, the basic research questions, objectives, terminology, definitions and limitations will be explained. Finally, the structure and organization of the chapters are outlined to give an overview to the reader.

1.1 Background of the Study

The field of finance dates back to 1960s. The fundamental theories of finance such as the efficient market hypothesis, the Capital Asset Pricing Model (CAPM), the option pricing method, and the modern portfolio theory were developed since then. One of the main objectives of this field is to study and forecast the evolution of stock markets. Stock markets, as one of the most complex interacting systems, cannot be investigated by a deterministic method accounting for each stock. The sources of complexities are due to the global economic conditions, systematic and non-systematic risks, multiple agent interactions and numerous other factors. Consequently, a probabilistic and statistical model based on the stochastic calculus must be employed to describe such systems. For example, powerful approaches from statistical physics have been used for modelling the markets. In his pioneer work, Bachelier (1900), for instance, focused on pricing of options, using the random walk formalization of Brownian motion to obtain an appropriate differential equation for the probability distribution function of price changing. This innovative statistical approach became the starting point for some alternative models. For example, Mandelbrot (1963) proposed a model which shows that the Levy or other stable distribution functions fit better with the real data distributions than the Gaussian ones. Also, Black and Scholes (1973) obtained a formula for the pricing of European call options and stocks on the basis of random walk probability distribution.

Another important step dating back to eighties was the exponential increase of the high frequency electronic data availability due to the developing of new technologies. This, in turn, attracted more attention of several researchers that have conducted many studies in stock markets over years, where each of them focuses on a specific aspect of capital market behavior (Mantegna and Stanley, 2000). Beginning with several hypotheses the building blocks of the Econophysics as an interdisciplinary approach were initiated by applying the physical methods into the economics and complex financial markets. In this respect, both classical and quantum mechanical models were applied and during the past 20 years a lot of studies have been done and a collection of papers have been amassed in the literature that we will refer to some of them later. Using Econophysical models to solve the financial problems, does not imply that the investors must give up using classical analysis of financial economists in their activities and start to use Econophysics as an alternative approach. In fact, the methodology considered by financial economists is a top-down method (e.g., starting from a priori first principles) while Econophysicists rather are interested to work with empirical data or follow a bottom-up approach.

Due to the strong correlations and subsequent entanglement of the markets, quantum mechanics can be used as a suitable toolkit for studying the evolution of these entangled systems. Khrennikov (1999), who is a pioneer in this area, applied Quantum Mechanics into modeling some financial systems. In a series of papers, Choustova (2002, 2004, and 2009) introduced a mathematical modeling based on Classical and Quantum Mechanics to investigate the dynamics of the financial systems. They argued that the real financial conditions are comprised of hard as well as soft components. The former component (e.g., industrial manufactures, natural resources, goods and services and etc.) may be governed by the classical Hamiltonian mechanics, while the latter (e.g., phycological behavior of traders, financial information and etc.) is described by Bohmian quantum mechanics. The important feature of Bohmian quantum mechanics is due to the notion of quantum potential which plays a significant role in applying this method to financial problems. It helps to describe the collective behavior of the complex financial system without exploring the detail interactions between its individual constituents. The models that are used in Econophysics are made on the basis of the efficient market hypothesis, where the traders are mostly rational and the information affects the prices randomly and follow the random walk process. However, the empirical investigations show that the situation is not compatible with the complete random walk statistics and deviates from Gaussian white nose into levy or any other fat-tailed distribution.

Using empirical data, Tahmasebi et al. (2015) employed the quantum potential method to describe the mechanism of the fluctuations of price returns. They found that the existence of vertical potential walls could be responsible for this issue through the time entanglement of the price return. In addition, their findings showed that the probability distribution function of the price return of the markets obeys a power law behavior indicating a scale invariance of the price return, which, in turn, enables one to get information about the behavior of the emerging and mature markets. Very recently, Shen and Haven (2017) estimated the classical as well as the quantum potential function, using the empirical data for the commodity markets. They could confirm the existence of the potential walls and the scaling behavior of the return variations. Emphasizing different information contents of the classical and quantum potentials, which reflect the hard and soft market conditions respectively, they pointed out the correlation between these two potentials.

In the present study, the collective behavior of some targeted emerging and developed markets is investigated and using the empirical data of the market indices it is shown that the quantum potential walls confine the variations of the price return into a definite interval where the distance between the walls can be a proxy for the risk of the respective stock index. Furthermore, by following the same logic adopted by Tahmasebi et al. (2015) and Shen and Haven (2017), which will be discussed later, the relation between trading volume and price return is explored by introducing the joint distribution function and corresponding joint quantum potential for these variables. The advantages of this study are a) proposing a quantitative indicator for risk measurement on the basis of distance between the quantum potential walls, b) introducing, for the first time, the joint quantum potential which allows the exploring the mutual interaction of price return and trading volume as well as their confinement mechanism for the stock markets.

1.2 Purpose and Motivation of Study

During the last few decades, the strong correlation between stock markets has increased, which inevitably motivated interested researchers to use the Bohmian quantum mechanics to investigate these markets. They have shown that there exists a quantum potential which confines the price return variations into a definite interval. However, it is expected that the price return credibility to be affected by the trading volume for a given market. To study the relationship between the price return and the trading volume, we have extended the single variable quantum potential to the stock markets described by multi-variable quantum potential functions. This method is applied to some stock market indices and a bidirectional causality relation is obtained between their price return and trading volume.

1.3 Research Questions

This study aims to answer the following questions:

1. How a quantitative risk indicator could be defined for the stock markets?
2. Why short-term high price return and/or trading volume variations are not experienced?
3. What is the joint pdf of these variables?
4. Are these variables inherently independent?
5. What is the impact of trading volume on the stock market credibility?

1.4 Research Objectives

The study seeks to model the dynamics of the stock markets as the complex and highly correlated systems. In this respect, the Bohmian quantum mechanics and inherent quantum potential function is employed to investigate the behavior of the price return and trading volume as two most significant variables. Further, the model is capable of proposing a quantitative indicator for different stock markets risks, as well.

The methodology introduced in this research includes the multi-variable quantum potential function of stock indices using the empirical data to construct the joint probability distribution function. As a special case, when one deals with trading volume and price return as two variables, one may study the mutual impacts of these variables for any given market.

1.5 Limitations

The study is using a novel mathematical framework based on quantum potential dynamics (Bohmian mechanics) to describe the dynamics of the stock market and its link with the trading volume. It builds the contribution on previous findings and a joint quantum potential is created to observe the joint pdf of price returns and trading volume. Although, the study attempted to present in a clear manner and construct the visualization of concrete quantum potential that could potentially explain the joint dynamics of the price returns and trading volume, there are some unavoidable shortcomings and limitations.

First, due to data availability, the data used throughout the work is limited to relatively short period of time. Thus, the lack of data obviously reduces the reliability and accuracy of the results. Second, the study is confined to the quantum effects by employing the quantum potential of Bohmian quantum mechanics, however, a better look to the financial markets may be followed by considering the combination of classical and quantum potential effects.

1.6 Key Terms and Definitions

Price return: Percentage of logarithmic change of price for any given market during one day calculated by $q(t) = \ln p(t + dt) - \ln p(t)$, where $q(t)$, $p(t)$ and dt are the price return, price and time interval of the price change, respectively.

Trading volume: Number of transactions or the number of traded shares for any given market during one day. Variations of the trade volume may cause the considerable of price changes. Increasing the trading volume for a stock market could be considered as an indicator of increasing of its liquidity.

Credible market: The market in which the prices are more likely real as well as stable. This is an important property of any developed market.

Risk: Amount of reduction of expected return to the actual one in an investment. Here, the difference between the upper bound and lower bound of quantum potential walls.

Quantum potential: The term in Bohmian representation of quantum mechanics (which is obtained by inserting the polar form of the wave function in the Schrödinger equation) that guides the motion of quantum particles in a similar fashion as the classical particle moves on the deterministic trajectories under the Newtonian classical dynamins. In this study the quantum potential will governs the evolution of the price return or trading volume considered as the dynamical variables of the stock markets.

Correlation: Quantitative statistical dependence referring to how close the random variables (i.e., today and prior day prices in the case of stock markets) are. In the case of strong correlation, we will deal with the entanglement concept as the signature of the quantum behavior of the stock markets.

Entanglement: The quantum state of each particle in an entangled system of two particles, even separated by a relatively large distance, must be considered as a whole and cannot be described independently. In other words, the existence of two particles expressed by a single wave function depends on each other.

Probability distribution function (pdf): The probability function of each of the values of a random variable (in the case of a discrete variable) or the probability function of a variable being located in a specified interval (in the case of a continuous random variable).

Joint probability distribution function: The simultaneous probability distribution of two random variables in a specific domain. In this study, the probability distribution of price return and trading volume as two important variables of the stock market are jointly pointed out.

White noise (Gaussian) distribution: A white (Gaussian) noise is a statistical process with pdf equal to that of the normal distribution. Its mean value is zero and the variables are completely random and uncorrelated at different times. It denotes the behavior of efficient markets.

1.7 Disposition

In chapter 2 the literature regarding the theoretical and empirical studies on stock markets modeling will be reviewed. In chapter 3 the research data and methodology applied to this research work will be proposed. In chapter 4 results and discussions will be presented. Chapter 5 will be devoted to conclusions, remarks and possible future studies.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

In this study, the literature of theoretical framework together with empirical foundations regarding the stock markets is reviewed by considering finance and Econophysics articles and books. Most important theories are discussed with relative empirical studies. Furthermore, several studies regarding the relationship between price return and trading volume are also referred.

2.2 Theoretical Background

In recent years, the new approaches in the field of finance have focused on the complexities of financial markets. Given the weaknesses of traditional portfolio theories and the efficient market hypothesis, and the increasing decline in their acceptability, these new approaches have been discussed among researchers in this field. Reducing the acceptability of the aforementioned theories is due to the complexity of the real world and the impact of various economic factors, individual and social psychology and etc., on financial markets. Also, these traditional theories of portfolios and the efficient market hypothesis are not able to answer the questions of researchers about the possibilities of arbitrage pricing of financial assets, the impact of information on the stock prices, and so on. The results of number of researches (Rozeff and Kinney, 1976; Harris and Gurel, 1986; Hirshleifer and Shumway, 2003) conducted in developed markets show that traditional theories are not appropriate approaches for illustrating the complexities of financial markets in the third millennium, and investigating financial phenomena needs more advanced approaches that embody behavioral science and psychology in this field. The results of these researches indicate that the interpretation of the capital market in the real world is different and complex with what Fama and his associates are proposing in the efficient market hypothesis. In other words, operating in real markets are much more complex than forming of such a simple theories and models. Despite all the efforts made to model the financial markets, reaching a unified theory to investigate the dynamics of the financial markets is still an open debate. In the following sections, some important theories and methods for governing the dynamical evolution of financial markets are discussed.

2.2.1 Efficient Market Hypothesis

Efficient Market Hypothesis (EMH) was initially formulated by Fama (1965, 1970) and was later formalized by Samuelson (1965), and it may be the most well-known hypothesis in finance theory under debate. The simplicity of this hypothesis (for example, rational investors, normal distribution of returns) led to new mathematical models attempting to clarify the mechanism of price formation, by the analysis of the extensive literatures that followed. Most of the models assume completely rational investors with an unbiased behavior capable of incorporating all new information entering the market, completely and immediately. As a result, available information on the market determines the price levels, and the new information entering the market leads to changes. Due to the random nature of the new information arrival, price changes will be random as well. Therefore, stochastic processes can model price changes. Bachelier (1990) proposed such an approach for the first time that was followed by many other authors. Shiryaev (1999) and Mantegna and Stanley (2000) present a detailed discussion on the history of asset price stochastic models. This paradigm studied the utilization of deterministic models in assessing the financial series of price dynamics, as the financial price series dynamics (volume, price, number of transactions) is like a random process characterized by the unpredictable nature of future outcomes. Critics have constantly scrutinized EMH since its introduction, both empirically (Malkiel, 2003) and theoretically (LeRoy, 1976). However, according to Fama (1970), EMH critics can be summarized within the underlying price formation model critics. Thus, in case of no consensus on a model describing the mechanism of price formation, the theory cannot be tested competently. However, a limitative theoretical framework and irregular financial time series have led to heavy criticism toward the EMH validity. It was mainly argued that a martingale model is not capable of explaining different irregularities including high volatility, irregular returns or the financial bubbles development. Some experimental research (Cutler et al., 1989; Biondo et al., 2013) demonstrated that financial price series dynamics cannot always be explained by random procedures and also, the changes in prices are not invariably stochastic.

Many theoretical and empirical debates are inferred from the financial time series statistical analysis that do not let the martingale model to be a proper solution to describe the mechanism of price formation. It is almost evident that the price returns distributions for asset cannot be Gaussian for high intervals (over a week); however, this might not be the case for lower intervals. Mandelbrot (1963) warned about fat tails persistence in return series and suggested that high price variations (>5%) can often be explained through a distribution with Gaussian nature. He demonstrated price variations are unexplainable through processes with normally having stable nature, except by Pareto-stable procedures or α-stable Levy procedures and argued that processes governing the returns distribution may have a local Gaussian nature.

Many other authors, including Fama (1965) demonstrated empirically that fat tails exist in financial return series distribution. Subsequently, many models emerged that described the financial returns dynamics principally on the basis of the assumption that power law is dominant on the distributed price returns. Jondeau et al. (2007) presented a comprehensive view of the references on non-Gaussian modeling of price returns. Mandelbrot (1967) proposed fractals and fractal Brownian motion for the first time to model the financial returns long-term dependency— is frequently lead to stable processes, thereby, called procedures with memory. He demonstrated that financial time series with an irregular nature are scale invariable, following a behavior with fractal nature. These variations are similar for different scales, and their statistical features are similar without considering the scale of the series. The alternative to the EMH, the Fractal Markets Hypothesis was proposed by Peters (1994) showing the importance of investment horizon and information importance in investor's performance. He argued that liquidity is the major impetus of equilibrium in market, acting like a surrogate to efficiency.

From an economic perspective, Grossman and Stiglitz (1980) argued that EMH is not sustainable, because efficiency signals no information costs that would finally cause indifference toward collecting financial information. Instead, they suggested that while comparing two contrasting markets (for example, futures vs. spot), efficiency should be considered in relative terms. LeRoy and Porter (1981) and Shiller (1981) showed high volatilities in returns unjustifiable through the dividends changes and inferred that models with rational behavior of investor cannot explain the volatility. Lo and Mackinlay (2001) suggested that through a competitive advantage (for example, financial innovation, superior technology, higher quality of information), the market can be beaten over limited time periods. If such incentives are taken into consideration for financial innovation, more flexible predictive models are created. Various financial markets irregularities were studied by Malkiel (2003) Rubinstein (2001), and Ball (2009) (for instance asymmetry of information, market momentum, seasonality, mean reverting, effects of day of the week, volatility clustering) in an EMH compatible manner. They argue to describe rationality less restrictively, and even argue that several types of rationality exist that if one considers non-uniform distribution of information across market participants, it is an acceptable assumption.

Lo and MacKinley (2001) considered EMH as the simplified version of infinit possible models. The answer may be shifting from EMH framework to models with more flexible and less confining nature supporting efficiency as well as predictability to some extent. Campbell et al. (1997) demonstrated that even within the perfectly rational agents in modern economic theory, certain degrees of predictability can be existed in dynamics of financial asset prices. Elements affected by the varied conditions of market and business atmosphere such as structure of market, costs of trading, and investors' demands can lead to certain degrees of predictability. They emphasized that in order to offset the risk, investors are eager to adopt a degree of predictability is necessary.

2.2.2 Behavioral Finance Theory

The market participants' biased behavior is mostly claimed to be the cause of financial markets inefficiency. In the literature, many sets of such deviations from hypothesized rationality have been studied; outside of the EMH framework, there exists a model for almost any financial irregularity putting that on the bias of a particular investor. This led to the development of Behavioral Finance Theory (BFT) introduced by Shiller (1990). In order to review the investors' decision-making procedure -using the scientific research standards to the investors' behavioral and socio-cognitive biases-, and its effect on conditions of market, behavioral models combine human psychology elements with neoclassical economics elements. According to the BFT, risk aversion, strategy development, and allocation of resources are influenced by cognition and preferences.

Barberis and Thaler (2002) argued that the two major reasons for market inefficiency are: 1) the limited power of arbitrage leading to conditions that the market price is not reflective of its basic features, although profit generating opportunities do not exist and 2) the biased investors' behavior leading to seemingly unreasonable decisions. De Long et al. (1990) used a model of overlapping generations in which irrational noise investors (trading on the basis of short-term shocks in information) exist to explain the financial anomalies like mean reverting, volatility clustering, and sub-evaluation of closed mutual funds. The authors attribute the irregularities to poor financially educated investors. The difference of prices from their basic values can be resulted and maintained by their apparently irrational behavior. Because of the short-term horizon and risk limitations the arbitrageurs may adopt as their own resources, such phenomena may not be offset by them, because their resources are borrowed in most of the times and the owners are looking for limited risks and short-term returns. The mistakes of the irrational investors cannot be offset by rational investors at all times.

De Bondt and Thaler (1985) investigated the investor's inclination to overreact to new information and demonstrated that financial assets with the highest return in the previous periods are usually of lower return in subsequent time span and vice versa. Barberis et al. (1997) deducted that investors emphasize more on the latest price values and give negligible importance to causation determining their dynamics. Haugen (1996) suggested that interim overreactions caused by market momentum result in mean reverting episodes in distant future, because the markets find the disequilibrium and correct the prices. Chan et al. (1997) demonstrated that investors absorb and process the newly-entering formation to the market gradually, so that the under/over evaluations periods become longer for prices. A partial explanation for the slow integration might be the persistent investors' priorities and the retarded variations in prospects. As Daniel and colleagues (1998) emphasized, the nature of information is the other important factor. They deducted that investors show higher reactions toward private information but underreact to public data and information. Smith et al. (1988) made a simulation of the environment of a financial market in controllable conditions of laboratory and determined financial bubbles leading to the market crash in about two thirds of the cases. They emphasize that the deviation between the expected price and its prime value is continued even in cases with skillful investors, but the gap and investors' experience are negatively correlated, showing a learning-by-doing procedure. Hubermann and Regev (2001) demonstrated that pessimism and optimism could be very contagious, causing a rapid increase of price variations over short time interval. Shanthikumar (2004) demonstrated that the small investments by the household are more prone to adopting a deviated treatment in comparison with the skilled investors of a strongly and soundly financial education.

However, Haigh and List (2005) argued that the latter individuals show more noticeable distortions on certain behaviors. However, such models are constrained to be used to describe the phenomenon they are developed for- that are very specific market architecture cases and investors' conduct- and they cannot present a more comprehensive perception of the financial systems functioning. Whilst BFT presents models explaining particular irregularities in financial markets, a major restriction of such an approach is the fact that it does not present a comprehensive model capturing the entire financial ecosystem facets.

2.2.3 Theory of Algorithmic Complexity in Stock Markets

Algorithmic Complexity Theory (ACT) was presented by Kolmogorov (1965) and Chaitin (1966) postulating that the time series can be considered unpredictable, if the information volume is not compressible in a more compressed format. That is, the most competent algorithm reproducing the series is of the same length of the series, itself. Considering that one major implication of EMH is the fact that no one can predict the future price values on the basis of historic series, the efficiency can be interpreted from a complexity hypothesis viewpoint. Interlink of market efficacy and unpredictability of return is that a time series with aggregated non-excessive information on economy (similar to EMH theory) is of the same features as a series generated randomly. The significant volume of information embedded in financial prices leads to challenging identification of a subgroup belonging to an algorithm detectable in series that might be utilized for prediction of prospective implications. Therefore, the price volatility prediction challenge is due to the plentiful data and information and not due to the absence of information. The market is not fully efficient if new information causes non-random changes in prices, but this adjustment exactly grants us the chance to reach the price series entered by the new market information. Arbitrageurs exploit this incompetence as far as all new information is integrated by the market and it regains its efficiency. Patterns of trading are resulted by different size heterogeneous investor groups, accessibility of information and expertise. In contrast to EMH schema, in which market assimilates information freely and instantly, the worthy information is more costly in reality, and different time durations are required by investors to fully disseminate the data. Therefore, the assimilation of information is a step-by-step data inflow illustrating only partial market information instead of being an instantaneous process.

Ivkovic and Weisbenner (2005) argued that trading conduct can be considered as the implication of the investors' financial education and expertise, that is not resulted by any behavioral or psychological bias. Coval and Moskowitz (2001) and Malloy (2005) demonstrated that locally trading investors reach higher returns compared to the investors trading across divers' geographic locations. Also, Kacperczyk et al. (2005) demonstrated that mutual funds managing portfolios were focused in sections in which they are expert (an informational benefit) with greater returns compared to the ones with diversified portfolios in multiple sections. These experimental outcomes demonstrate the behavioral approach utilization on the basis of information theory. In classic economics theory of information on the basis of investors' rationality (presented by Grossman and Stiglitz, 1980), the investors are assumed to correctly evaluate the information, and thereby, ready for paying a fixed amount to receive its access, but the main limitation of this hypothesis is the fact that the information process by the investors is not explained by the theory.

The Shannon's entropy theory (1948) describing the transmission procedure of information for systems of communication presented a more robust framework from this perspective. He developed a criterion to quantify the information received from a source by a system. A similar procedure is applicable for living organisms, such as financial markets investors. Chen (2005) presented the information interpretation as reduced entropy both mathematically (the case of Shannon entropy) and physically.

The physical costs of information seem to be closely related with the economic costs. This approach may define some of the major features of information analysis in the stock markets, e.g. in fact the cost of useful information is of higher. Additionally, the data amount assimilated by investors is constrained by the asymmetry level of information and volume of efficient information processing. A certain level of expertise formed over long time periods is necessary to understand the information, and the number of investors understanding the value of information has negatively correlated with it. An investor buying a company shares before its success, will receive higher returns than the one buying the shares of the company after the company is in high demands. Honge and Stein (2003) demonstrated that heterogeneity of the investors is obtained through level of financial wisdom and the cost paid by the investors to retrieve information. The authors emphasized that in the process of price formation, heterogeneity can play a decisive role. Although consumerized trading and financial service companies' development led to significant decrease of cost of financial information, due to the more public access to information, its value also declined.

According to empirical studies, in financial markets, price patterns are closely correlated with information processing patterns, emphasizing the link between behavioral finance and information theory. Chen (2003) demonstrated that most of psychological patterns show biological or physical constraints (for example, the needs for water and housing) or an evolutionary adaptation to informative analysis (for example, educating, and changes in strategy). This recent practice is within Lo's (2004) proposed theoretical framework, i.e. that financial market might be considered from an evolutionary viewpoint in which certain investor species receive more efficiency in processing of information and thus, while other species get extinct, they can survive in the financial ecosystem.

2.2.4 Adaptive Markets Hypothesis

While perfect rationality versus bounded rationality namely the essential premises of EMH and BFT, apparently makes them mutually incompatible, they sustain providing perceptions on the functioning of financial market at various time horizontals and scales. Scholars like Samuelson (1965), Malkiel (2003) and Khandani and Lo (2008) suggest that in the long run, all the uncertainties associated with the efficiency of market is eliminated. Because in the long run, the disequilibrium in prices is adjusted when irregularities tend to offset each other. In the long run, market consistently reaches to a phase composed of rational investors and efficiently processed information. Whilst in the long run, the description of financial markets is presented by EMH, BFT presents local short-term interpretations for functioning of markets.

The Adaptive Markets Hypothesis (AMH) formalized by Lo (2004), presents a new solution for studying financial phenomenon through application of the foundations of evolution such as natural selection and competition for financial interaction. According to AMH, resource optimization conception in neoclassical theory is replaced by satisfaction concept which is a suboptimal resolution. The latest one is a flexible procedure resembling natural selection, which has been founded on consecutive trial and error stages until reaching the local equilibrium. The changes in conditions of market may finally cause changes in equilibrium circumstances. Lo (2014) made a comparison between markets and ecosystems consisting of different stakeholders with various structures and sizes where the returns resemble the nutrition resources. In this environment, food availability determines the market dynamics, which consequently specifies changes in the structure and number of current species. By analyzing the events leading to and resulted by the global financial crisis from 2008, Verheyden et al. (2013) identify the ecosystem elements that once making far from equilibrium, triggers a sequential impact that leads to appearance of certain types until reaching a new equilibrium. The efficiency degree changes are consistent with the financial market evolutionary interpretation over time. In the long run, information is processed efficiently by the markets until an exogenous disturbing agent disrupts the equilibrium. The investors should adapt to the new circumstances of the market until a renewed equilibrium is reached, and during the same process, when judged by past circumstances of the market, they may show an apparently irrational behavior. Once equilibrium is reached again, the investors become rational toward the renew circumstances of the market and the information is processed in an efficient manner.

In AMH framework, both BFT and EMH are just very special states of boundless spectra of market prospects. Thus, financial markets may not be absolutely efficient of inefficient; instead due to the adjustment of the functional institutional, or structural features of market, their efficiency degree changes over time. The hypothesis is not completed yet and shows some inconsistencies in its concept. As Verheyden et al. (2013) pointed out, considering the fact that the investor species with certain biases in their behavior can survive the shock, natural selection and evolutionary theory cannot explicate the entire procedure of financial markets becoming efficient again after their being taken out of equilibrium. Although, AMH may not describe the entire financial systems functioning, it provides a more flexible context that allows application of more heterodox practices toward modeling the dynamics of financial time series. As Segal and Segal (1998) emphasized, quantum effects can explain some of the anomalies in financial asset prices variations resulted by investors' apparent unreasonable behavior. Haven (2003 and 2005) also use quantum theory to explain random procedures of financial market.

2.2.5 Econophysics and Quantum Mechanical Method

A complex system is constructed by many constituents interacting with each other. Financial markets could be looked as highly complicated systems. Broadly speaking, economics may be described as the science studying the procedure in which economic factors make efficient use of resources for production and distribution of services and goods. Mantegna and Stanley (2000) defined financial market as a complicated open system in which non-linear interactions of investors determine a behavior change (through integration of feedbacks). Like the mechanical systems, the financial markets' functioning is governed by rules that remain unchanged over longer time periods.

Scholars of other research fields attracted to the study of the statistical features of financial time series have focused on financial markets complexity, their dynamics uncertainty, and large volumes of financial information. While Econophysics is still deemed as an emerging interdisciplinary context combining mathematics, economy and physics, it uses elements of probability theory, chaos theory and statistical analysis to investigate the nature of economic phenomenon and markets functioning. Financial market is constantly evaluated, fed by the generalized electronic trading implementation, causes the collection of very massive financial information datasets. This phenomenon led to establishment and experimental verification of deterministic models in financial price dynamics. Many studies were carried out on the similarities between economic and physical systems. As alternative approach to representative agent models including the studies on macroeconomic issues, Econophysicists proposed the methods on the basis of statistical concepts such as disordered systems and chaos theories in studying financial systems. It is approved that even in economic systems, techniques like extraction of mean property from the dynamical behavior of a system's constituent parts are helpful. Mechanical physics models let the Econophysicists to investigate the total behavior of financial system without the need for detailed study of the behavior of their component parts in advance. Using concepts such as correlation effects, scaling theory, stochastic dynamics, self-similarity and self-organizing systems, this can be done without the need for a definite understanding of the system functioning and structure at micro levels.

In comparison with the physics principles and laws, stylized facts can be determined for economic and financial phenomena. Chakraborti er al. (2011) presented the volatility clustering and availability of fat tails which confirmed experimentally by several investigations. The other scholars including Pagan (1996), Guillaume et al. (1997) and Cont (2001) presented additional stylized facts (heavy tails and peakedness, slow decay of the autocorrelation and etc.). Gopikrishnan et al. (1999) examined the return distribution of Hang-Seng, S&P500, and NIKKEI indicators for various time intervals concluding that for time intervals less than four days, a power law with exponent equal to 3 governs the return distribution that shows the fat tails presence. The distributions follow a Gaussian convergence for frequencies over 4 days. Pagan (1996), and Cont et al. (1997) concluded that even for lags of one minute, the autocorrelation function converges to zero rapidly, showing the lack of correlation between returns. The first author to emphasize the financial time series heteroscedasticity was Mandelbrot (1963)- big changes lead to big price changes, while small changes lead to small price changes-, but Sornette (2003) made a comparison between this phenomenon and a phase transition launched by perpetual competition and mimetic behavior between investors.

Thermodynamics and seismology ideas are utilized to interpret and define risk diversification, bubble formation, and asset price dynamics. Black and Scholes (1973) and Merton (1973) facilitated the conditions of pricing method which is a powerful technique in financial studies. In certain conditions, they demonstrated that the time evolution of the price of a financial option is defined using the heat diffusion equation. Sornette (2009) investigated the speculative bubble formation mechanics and the consequent market manias and deducted that these severe events are of a higher frequency than the one delivered by power law. Additionally, the causes leading to burst of financial bubble are endogenous to system and show systemic instability accumulation specified through mimetic behavior and, unrealistic and overoptimistic expectations of investors. Battiston et al. (2009) who used dynamic models to define individual risk development as coupled random procedures were partially in contradiction with the generic notion that diversification of portfolio leads to risk minimization. The authors inferred that when the risk sharing network complexity between people exceeds a certain threshold, the diversification affirmative effect is ignored. Instability of market is also increased by the increased complexity of financial instruments used. Due to adaptive nature of financial markets, the indices for well-timed detection of these events can be constructed. Sornette (2003) recorded the major market crashes episodes occurring in emerging and developed countries and concluded that these events can be predicted. Zumbach et al. (2000), Mailet and Michel (2003), and Negrea (2014) presented indicators for estimating market crises. Weber et al. (2007) demonstrated that instantly after a major decline in market index, an Omori process (power-law decay of seismicity after an earthquake) is the best to describe the volatility. Additionally, Omori processes also describe the volatilities of the aftershocks, indicating the presence of a memory for the return volatility on various time horizons and its tendency to reproduce itself, which are the phenomena explicated in scaling theory.

Investigation of financial phenomena using concept of quantum mechanics is a special research field within Econophysics. Financial markets are complicated systems in such framework, in which every investor interacts in the same way as the particles interacting in physical systems. As a result of uncertainties associated with financial market, instruments and advanced theories, statistical physics have been utilized for modeling the financial systems dynamics (For example, path-integral technique, perturbation theory, random matrix theory, differential manifolds). Meyer (1999), and Eisert et al. (1999) presented the application of overlaid financial functions and quantum cryptography to establish trading strategies, leading to formation of quantum game theory.

In the Newtonian classical mechanics, a deterministic hypothesis is assumed in which the position of a particle at any given time *t* is determined exactly. The only use of such framework is for describing the financial asset price dynamics without volatility. Nevertheless, the position of a particle cannot be specified precisely in quantum mechanics. Alternatively, a probability space can be estimated for all possible positions of the particle at any given moment in time. This framework determines the financial markets framework more accurately, where uncertainty characterizes asset prices dynamics and only a probability can be assigned to a certain event by the investors. Now, the comparison between the position of a particle and the price of financial asset (with non-null volatility) seems to be acceptable. Another major comparison can be the use of the same equations to define the financial and physical systems dynamics. Haven (2002) demonstrated that the equation of Black-Scholes-Merton for prices dynamics behaves as a special form of the Schrodinger's quantum mechanics. Additionally, in a financial systems framework, the Heisenberg's uncertainty rule holds as well, since price volatility (momentum), and price level (position) cannot be measured accurately, and simultaneously.

Bohm and Hiley (1993), and Hiley and Pylkkanen (1997) investigated Bohmian mechanics and its results within the cognitive field of study. Segal and Segal (1998) suggested that quantum effects can explain the irregularities of changes in financial asset prices. Haven (2003 and 2005) applied the quantum principles explaining random procedures of financial market. Piotrowski (2003) and Piotrowski and Sladkowski (2004) have suggested alternative game theory models on basis of quantum mechanics to study financial system. Biological studies show that at a neuronal level, quantum mechanics may play a decisive role in the decision-making procedure. Khrennikov (2007) emphasized that social sciences can receive the formalism of quantum mechanics with possible profound results in the functioning of financial markets. Choustova (2007) applied Bohmian mechanics to develop a model describing the dynamics of a financial system, using the quantum potential concept which capturing the impact of investor's behaviors and interactions. Dima et al. (2015) have presented a formulation to investigate the dynamical system volatility as well as a pricing procedure representative on the basis of heterogeneous groups of investors.

Quantum physics basics are applicable if financial markets are deemed as complicated systems where investors interact each other similar to that of quantum particles interactions. The preference of a quantum-like model in contrast with a behavioral model in which investors' cognitive and psychological profiles are required is the fact that all the micro level specifications of the system are represented at a macro level.

2.3 Price Return and Trading Volume Relationship

Investigating the relationship between trading volume and stock return has attracted the attention of many finance and economics researchers. The available evidence shows that some stock market studies have focused on stock price and its behavior over the time. However, due to the variability and non-stationary behavior of stock price, most researchers mainly focus on stock return, defined as the logarithm of the relative change of price (Campbell et al., 1993; Todorova and Soucek, 2014) rather than stock price or raw price return. When the trading volume of a stock market is concerned, different definitions are proposed by the relevant literature, including the number of shares traded (Ying, 1966; Hiemstra, 1994; Gervais et al., 2001), the number of transactions (Conrad, 1994; Ansary, 2012), and the turnover ratio (Lo and Wang, 2000). In the present work, the number of traded shares is used as a proxy for trading volume.

In Karpoff's (1987) view, there are four reasons for the importance of discussing the relationship between volume and stock return as follows. First, the existing models in the financial markets predict the volume-return relationships of the stocks according to the volume of the inputs into the market, the dissemination of information, the size of the market and the conditions of the transactions. Thus, exploring the relationship between trading volume and stock return may help distinguish and decide between different hypotheses proposed about the market structure. Second, for those studies that use a combination of volume and stock return data, it is important to know how these two are interrelated. Third, volume-return relation is critical to the debate over the empirical distribution of speculative pricing. Finally, the mechanism and modality of the relationship between trading volume and return have important implications for future market studies; where the price changes have a considerable impact on the volume of futures contracts.

The correlation between trading volume and price return is widely studied and discussed by many authors. For example, Granger and Morgenstern (1963) conducting an empirical study and using the data from the New York Stock Exchange found that daily price changes has no relation with trading volume, both in absolute terms. Habib (2011) using OLS and GARCH models concluded that there exists no casual relation between volume and return for the Egyptian market. On the other hand, Campbell et al. (1993) and Wang (1994) have shown that the relation between volume and return is not a simple linear relationship; rather there exists a complex nonlinear relation between them. Podobnik et al. (2009) studied the behavior of volume changes and their relationship to price changes using the data recorded for the (S&P500) index and obtained a power law cross-correlation between them. Ausloos and Ivanova (2002) by generalizing the classical technical analysis and considering the trading volume introduced a mechanistic approach to predict the evolution of the stock markets. Ahmad and Sarr (2016), investigated the relationship between price return and trading volume using the monthly data of Muscat security market from 2009 to 2013. They concluded the existence of a significant interaction between trading volume and returns for this market. Using sequential arrival information hypothesis, Copeland (1976) illustrates the existence of at least a unidirectional causality between these variables, since the information dissemination does not evolve contemporaneously among market participants. Unidirectional relationship from volume to return has been acknowledged by Saatcioglu and Starks (1998) as well. The existence of bidirectional causality between return and volume is supported by Chen et al. (2001) and Chuang et al. (2009).

Among the different approaches employed to study the behavior of the financial markets, two have been attractive for some other disciplines as well: approaches from Physics and Statistics. For instance, the starting point of investigating the price-volume relations dates back to Osborne (1959) who modeled the stock price trend using a diffusion process and showed theoretically that the volume could affect the price variance. Mantegna and Stanley (2000) obtained new ideas about financial markets' behavior by implementing statistical and physical methods. Chakraborti et al. [2011) and Chen (2015) applied the chaos theory to study the dynamics of a financial system. Moreover, Baaquie (2013) and Baaquie et al. (2014) investigated the basic concepts of economics based on the statistical mechanics, using classical potential and Hamiltonian dynamics. During the last decades, the correlation between the stock markets and their corresponding variables has increased to be inevitably entangled. This property and the collective behavior of the financial assets have persuaded researchers to use the quantum potential model taken from Bohmian quantum mechanics. Since the pioneering work of Khrennikov (1999), many researchers have been engaged in this area. Haven (2004 and 2005), for instance, utilized the principles of quantum mechanics to describe the stochastic processes inherent in the financial markets. Similarly, Choustova (2008) used Bohmian quantum mechanics to construct a theoretical model for describing the evolution of the stock markets. Tahmasebi et al. (2015) showed that the entanglement between today's and yesterday's prices of stock markets implies the existence of quantum potential which confines the price return changes into a specific domain. Shen and Haven (2017) followed the same method by considering both classical and quantum potentials and concluded that, in addition to stock markets, there exist potential walls for commodity markets as well. The details of the Bohmian quantum mechanics, as the methodology of this research, will be presented in the following chapter.

Chapter 3

DATA DESCRIPTION AND METHODOLOGY

3.1 Introduction

This chapter devotes to the date used in the study and describes a mathematical framework based on quantum potential dynamics, namely the Bohmian mechanics. The Kernel density function will be discussed together with the joint probability distribution. Then, the methodology will be introduced and explained in details.

3.2 Data Description

The data used in the present study were extracted from Thomson Router database for the Dow Jones Industrial Average (DJIA), the Standard and Poor's 500 (S&P 500), the Deutsche Boerse AG German Stock Index (DAX), Tokyo Stock Price Index (TOPIX) as the developed markets, and Shanghai Index (SSEC) as the emerging market indices, from January 2010 to December 2017, to investigate the collective behavior of their price returns and its possible link with the trading volume.

3.3 Kernel Density Estimator

Kernel density estimator is used as a data smoothing technique. Different functions and packages are introduced in this respect. Some free parameters such as, variance and mean value in Gaussian Kernel, are used to adjust the distribution optimally with the given data. These parameters have significant influence on the smoothing procedure. Here we use the standard Gaussian Kernel and adjust its variance as the single parameter named as bandwidth parameter in such a way that the corresponding pdf to be optimally smoothed and close to the true probability distribution function. By this smoothing procedure we consider the impact of the neighboring discredited cells on the bin under consideration.

3.4 Methodology

Application of the Quantum mechanics to different fields such as: psychology (Zimmerman, 1979; Schwartz et al., 2005; Bruza et al., 2015), cognitive sciences (Busemeyer and Townsend, 1993; Bruza et al., 2009; Busemeyer et al., 2014), microbiology (Bridson and Gould, 2000; Arndt et al., 2009; Trevors and Masson, 2011) and genetics (Jorgensen, 2011; Asano et al., 2013) are explored by different studies. Its application to finance and economics known as Econophysics is another harvest of this method. A sub-field of Econophysics is known as the Quantum Finance in which the financial subjects are investigated using the quantum-like models. A frequently used approach in this area to model the stock market dynamics is the Bohmian quantum theory. In this section, we describe how Bohmian quantum mechanics helps us to understand the dynamics of stock markets.

David Bohm (1952), presented a theory that is known as Bohmian quantum mechanics. The dynamical equations governing the evolution of the system in Bohmian quantum mechanics is obtained by inserting the wave function $\psi(q,t) = R(q,t)\exp(i\frac{s(q,t)}{h})$ in the Schrodinger equation $ih\frac{\partial\psi}{\partial \mathrm{t}} = -\frac{h^2}{2m}\frac{\partial^2\psi}{\partial q^2} + V\psi$ and obtaining two coupled equations for $R(q,t)$ and $S(q,t)$ as follows (Holland, 2000):

$$\frac{\partial R^2}{\partial t} + \frac{1}{m}\frac{\partial}{\partial q}(R^2\frac{\partial S}{\partial q}) = 0, \tag{1}$$

$$\frac{\partial S}{\partial t} + \frac{1}{2m}(\frac{\partial S}{\partial q})^2 + (V - \frac{h^2}{2mR}\frac{\partial^2 R}{\partial q^2}) = 0, \tag{2}$$

where, $R(q,t)$ and $S(q,t)$ represent the amplitude and the phase of the wave function, respectively. h, q and m are the Plank constant, position and mass of the particle, respectively. In equation 2, in addition to classical potential, V, there is another potential:

$$U(q,t) = \frac{h^2}{2mR}\frac{\partial^2 R}{\partial q^2}, \tag{3}$$

which is called the quantum potential.

In applying the above method to a given stock market, $q(t)$ denotes the price return, m and h indicate the market value and the uncertainty in price and price change, respectively, and $S(q,t)$ represents the phase of the market quantities. When a single market is concerned, m and $S(q,t)/h$ are assumed to be constant, however, when the interconnection of at least two different markets is concerned, the values of m and S must be specified.

Let $q(t) = \ln p(t+dt) - \ln p(t)$, where $p(t)$ and dt are the price and time interval of the price change, respectively. Let $R(q,t)$ denote the probability distribution function (pdf) for the price return, which could be extracted using the data described in section 3.2 for all indices.

It seems that modeling a real stock market as a complex system, may not be performed by considering only a single variable of price return. In addition, the existing evidence shows that different factors (such as volume) have their impacts on the behavior of the probability density function (Andersen, 1996; Llorente et al., 2002; Ahmad and Sarr, 2016). Given this, one may like to generalize the method adopted by Tahmasebi et al. (2015) and Shen and Haven (2017) to a system of more than one variable.

Two central equations of Bohmian quantum mechanics describing the dynamics of the n-dimensional systems may be generalized as follows:

$$\frac{\partial R^2}{\partial t} + \frac{1}{m}\sum_{i=1}^{n} \frac{\partial}{\partial q_i}\left(R^2 \frac{\partial S}{\partial q_i}\right) = 0 \tag{4}$$

$$\frac{\partial S}{\partial t} + \frac{1}{2m}\sum_{i=1}^{n} \left(\frac{\partial S}{\partial q_i}\right)^2 + \left(V - \frac{h^2}{2mR}\sum_{i=1}^{n} \frac{\partial^2 R}{\partial {q_i}^2}\right) = 0 \tag{5}$$

which are obtained by inserting the time dependent wave function of n-independent variables i.e., $\psi(q_1, q_2, \ldots, q_n, t) = R(q_1, q_2, \ldots, q_n, t)\exp(i\frac{S(q_1,q_2,\ldots,q_n,t)}{h})$, in the Schrodinger equation:

$$ih\frac{\partial \psi(q_1,q_2,\ldots,q_n,t)}{\partial t} = -\frac{h^2}{2m}\sum_{i=1}^{n} \frac{\partial^2 \psi(q_1,q_2,\ldots,q_n,t)}{\partial {q_i}^2} + V(q_1, q_2, \ldots, q_n)\psi(q_1, q_2, \ldots, q_n, t), \tag{6}$$

Where $R(q_1, q_2, \ldots, q_n, t)$ and $S(q_1, q_2, \ldots, q_n, t)$ are the amplitude and the phase of the wave function, and h, q_i and m are the Plank constant, the i-th component of the position, and the mass of the particle, respectively (Holland, 2000). So, the quantum potential will be like below:

$$U(q_1, q_2, \dots q_n, t) = \frac{\hbar^2}{2mR} \sum_{i=1}^{n} \frac{\partial^2 R(q_1, q_2, \dots q_n, t)}{\partial {q_i}^2} = \sum_i^n U_i\,(q_1, q_2, \dots q_n, t), \qquad (7)$$

which is known as the quantum potential for an n-dimensional system. Note that, if R in equation 7 is a separate function of n independent variables, the corresponding quantum potential reduces to the sum of n one-dimensional quantum potentials. In this special case, as is shown in figure 6, the domains of the variables are fixed and confined by the corresponding separable quantum potentials. However, as will be shown later, at least in our data this is not the case and the evidence does not always allow for the separation of the variable technique to solve the problem. This means that R, in general, is not a separable function of n independent variables $(q_1, q_2, \dots, q_n)$. Nevertheless, one may still express the total quantum potential as the summation of n quantum potentials $U_i, i = 1, 2, \dots, n$ as a function of $(q_1, q_2, \dots, q_n)$ family, governing q_i coupled with the remaining dependent group of variables of the family. Even, one may consider various cases intermediating the above extreme limits, where the group of n variables could be divided into independent subgroups of dependent variables. Corresponding to each independent variable in a given subgroup, one may define a quantum potential of partially coupled dependent variables.

Chapter 4

RESULTS AND DISCUSSIONS

4.1 Introduction

In this chapter we present the results obtained by applying the method of Bohmian Quantum Mechanics, as described in previous chapter, to investigate the evolution of markets as dynamical complex systems. The time variation of price returns, the probability distribution functions, the single variable and multi-variable quantum potentials for various indices are shown and explained. Quantitative risk analysis for the same indices are also done upon the difference of upper and lower bound of the quantum potential domain.

4.2 Results for Single-variable Case

We first consider the case of markets with a pdf as a function of a single variable. Thus, according to the previous chapter we have $n = 1$, and for the price return we denote, $q_1 \rightarrow r$ and the corresponding pdf represented by $R(r)$ is obtained through integrating on all variables and the time except r, and for the trading volume $q_1 \rightarrow v$, and corresponding pdf represented by $R(v)$ is obtained through integrating on all variables and the time except v.

The results for the price, price return and single-variable probability distribution function $R(r)$ of the Dow Jones index are plotted in figures 1(a), 1(b) and 1(c), respectively.

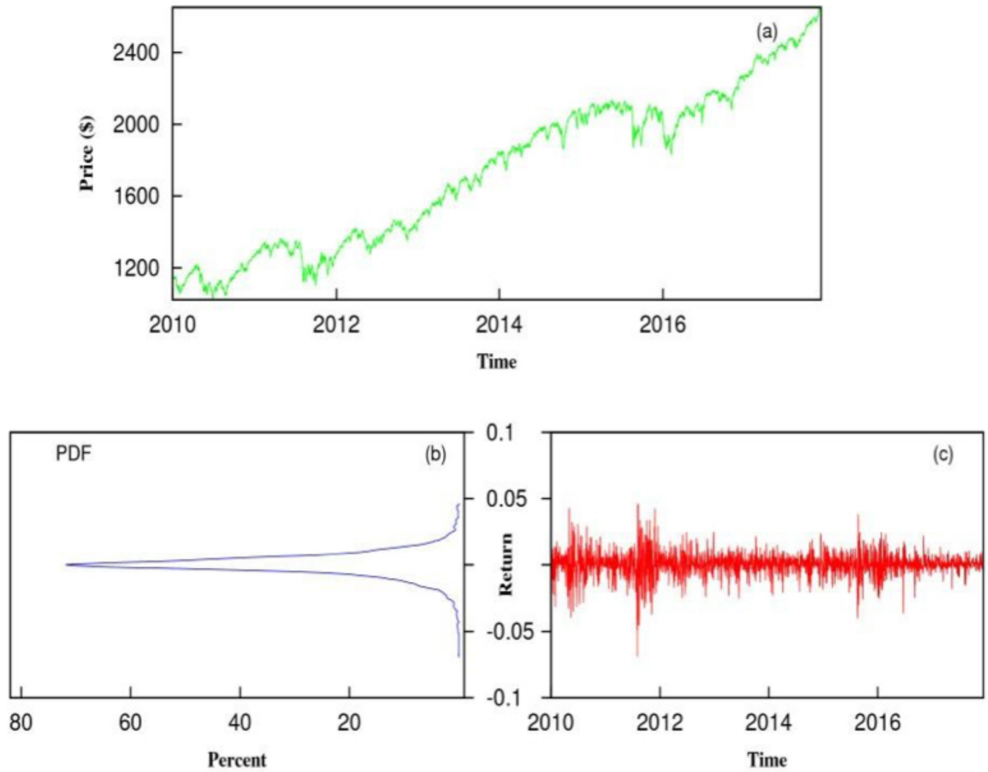

Figure 1: a) The time plot of DJIA index; b) the plot of the price return for the DJIA index; c) the pdf of the DJIA index

Note that there is a trend stationarity in price, as expected, and increases in the course of time, however, the corresponding pdf for the price return is almost stationary and behaves as a Gaussian-like function around zero return value. Following Tahmasebi et al. (2015) and Shen and Haven (2017), we calculated the single-variable quantum potential for each index by applying the corresponding pdf in equation 3. The results for the quantum potentials for all indices were calculated on daily, weekly, seasonal and yearly basis. The quantum potential for Dow Jones index as a developed market is shown in figure 2.

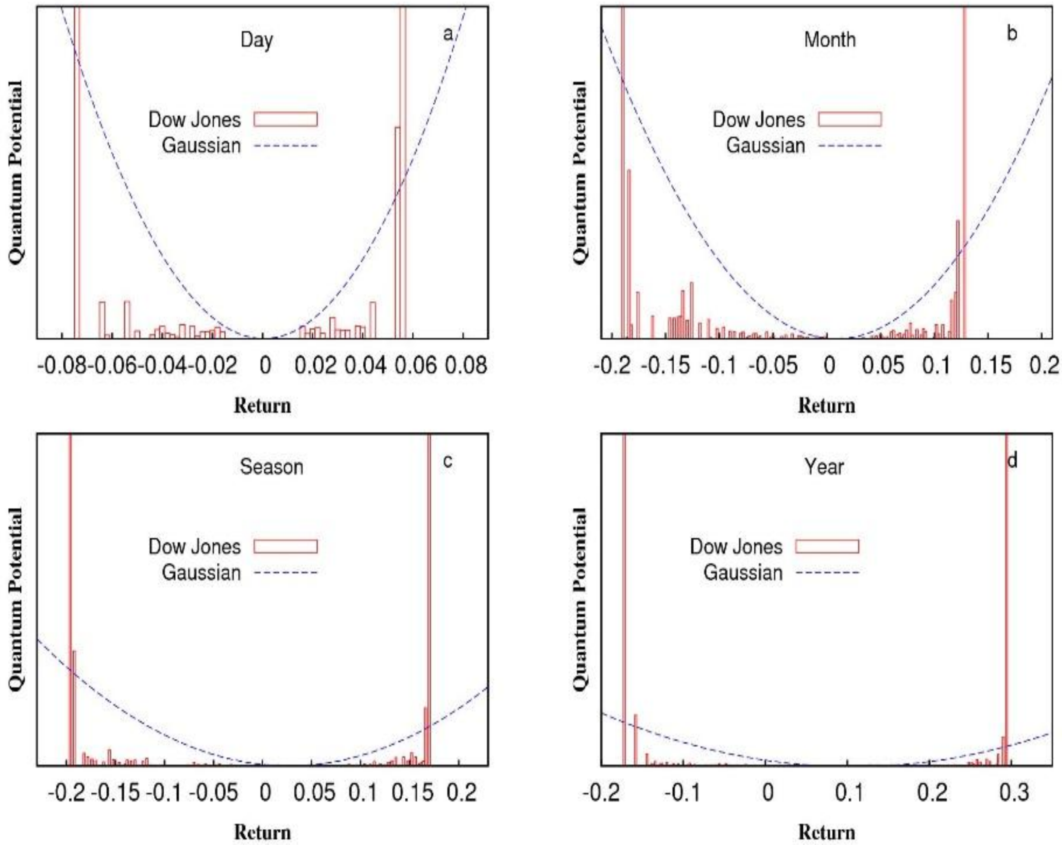

Figure 2: The plot of a) daily, b) monthly, c) seasonal and d) yearly quantum potential versus the price returns; the dashed line corresponds to the white noise quantum potential.

To check the model, the quantum potential of the Gaussian white noise, which is unbiased in prices at different times correlation, was compared with the quantum potential of Dow Jones index with the same variance. In order to derive the quantum potential for the Gaussian white noise of the respective index, one may insert the corresponding function, R, represented by $exp(-(q-q_0)^2)/2\sigma^2$, into equation 3, and after some mathematical calculations obtain,

$$U(q,t) = \frac{4(q-q_0)^2}{4\sigma^4} - \frac{2q}{2\sigma^2}, \quad (8)$$

where σ^2 represents the variance and q_0 is the average variation of price returns for the market under examination.

In figure 2, the quantum potential of the real data (solid line) is compared with the quantum potential of the Gaussian white noise (dashed curve). It can be seen from the above four panels that there exist quantum potential walls which confine the returns into a specified domain. The variations of the returns increase from 6 percent, on a daily basis, to 30 percent on an annual timescale, where the Bohmian quantum potential walls tend to be Gaussian like. This is expected to occur in real situations, where the stock market prices do not change considerably during a day in contrast to the yearly variations.

In figure 3, the same plot as in figure 2 shows Shanghai index as emerging one. Note that in figure 3 for SSEC as an emerging market, the quantum potential function, in contrast to Dow Jones index, is not a well predictable function and the potential walls are rather fragile ones and its return domain has wider variations. The different behavior of developed and emerging markets may due to some reasons such as transparency, information asymmetry, maturity period, governmental interventions and etc.

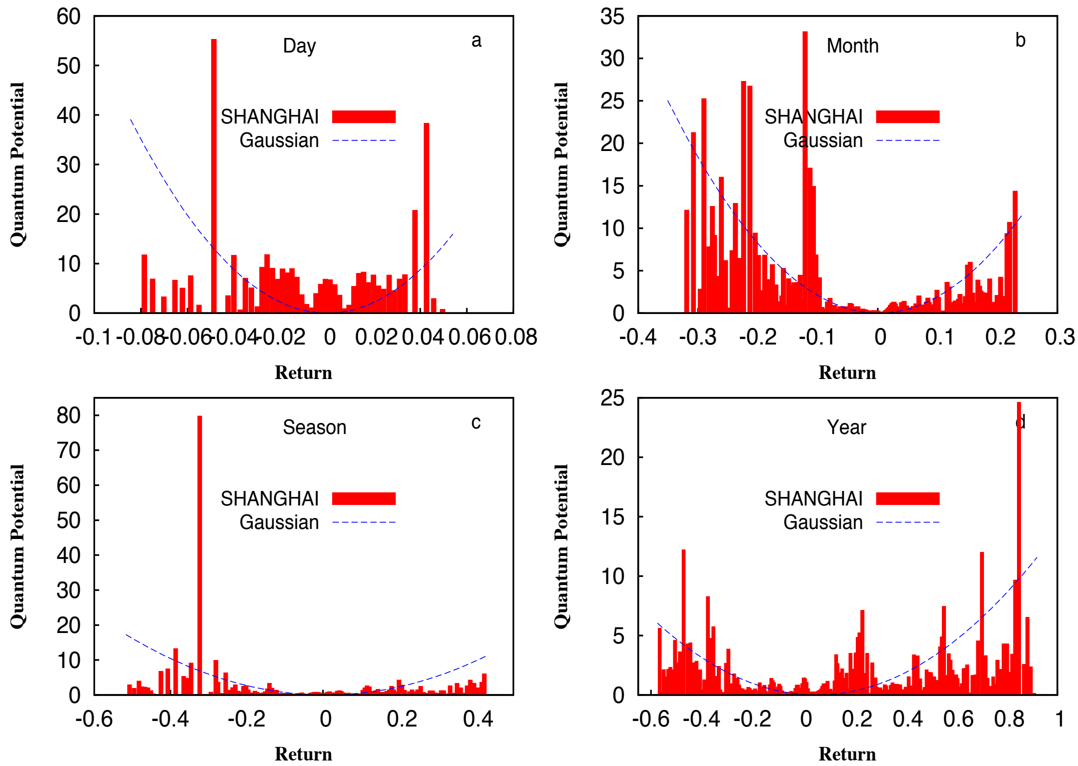

Figure 3: The same as figure 2 for SSEC index

Investment, as a financial decision, has always entailed two components, ‘risk’ and ‘return’, and their trade-off offers different investment portfolios. On the one hand, investors seek to maximize their profits from an investment and, on the other hand, they are faced with the uncertainty surrounding the financial markets, which, creates an uncertainty in the access to the investment returns. In other words, all investment decisions are based on the relationship between the risk and return. According to a clear consensus observed for the existence of positive relations between the risk and return, the quantum potential can be used as a useful indicator for comparing the risks of different indices against each other. As shown in figure 4, where the daily and yearly quantum potentials for different indices are plotted, a wider range of the potential walls means a higher return, and consequently, a higher risk.

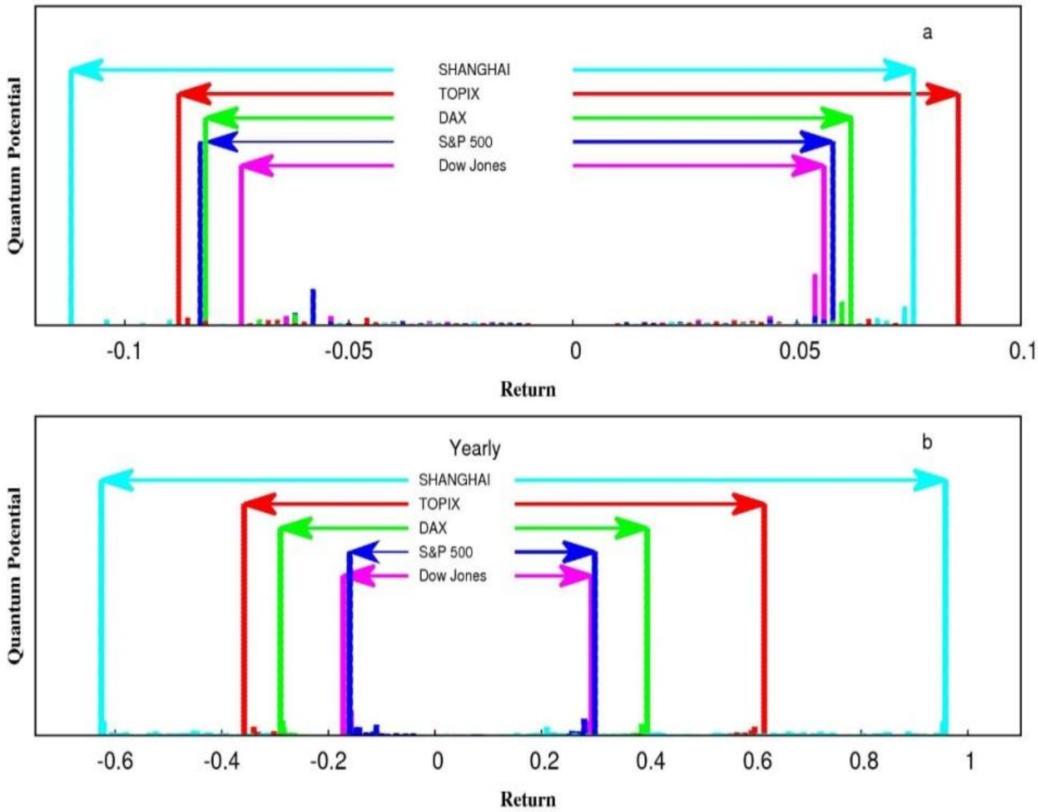

Figure 4: a) The daily and b) yearly quantum potentials for the selected indices

Shanghai index as an emerging market has a relatively wide range of returns. Therefore, the corresponding wide ranges of the quantum potential walls indicate a higher risk both in daily and yearly time scales. In contrast, the relatively short ranges of the quantum potential walls indicate a lower risk for the Dow Jones and S&P 500, as the developed markets indices. These results are consistent with those of Derrabi and Leseure (2005) who showed that the emerging markets have higher returns in comparison to the developed markets, while being riskier.

Tables 1 and 2 show the results of the risk and the average return based on the quantum potential permissions, respectively. As shown in table 1, it is found that the amount of the risk is highest for Shanghai as compared to other indices.

According to table 2, the daily average returns of all indices, from 2010 to 2017, are negative as also shown in figure 4 above. This is due to the fact that the quantum potentials for all indices are left-oriented with respect to the mean value. The situation is reversed for the yearly time scale, where the average returns become positive for all indices and the corresponding quantum potential walls behave as right-oriented. This is due to the collective behavior of the stock markets that reveals the lower price return for the short run than for the long run.

Table 1: Risk of the selected indices based on the quantum potential permissions (between walls) in the period of January 2010 to December 2017.

	Daily	Monthly	Seasonally	Yearly
S & P 500	0.1421	0.3167	0.3924	0.4600
DOW JONES	0.129	0.3183	0.3662	0.465
SHANGHAI	0.1876	0.6114	1.0048	1.5855
TOPIX	0.175	0.43744	0.6236	0.9766
DAX	0.1461	0.4905	0.6596	0.6880

Table 2: Quantum potential restrictions on the price return is not symmetric for positive and negative values. This table shows the average returns of the selected indices based on these restrictions. (January 2010- December 2017).

	Daily	Monthly	Seasonally	Yearly
S & P 500	-0.00167	0.0156	0.0453	0.1664
DOW JONES	-0.00185	0.01618	0.04772	0.1578
SHANGHAI	-0.00268	-0.00072	0.001515	0.01059
TOPIX	-0.00164	0.00969	0.02527	0.07397
DAX	-0.00184	0.0108	0.03088	0.11903

In figure 5(a), $R(r)$ for all possible values of volumes during a time interval of Jan. 2010 to Dec. 2017 is plotted for the DJIA and SSEC indices. It is seen that $R(r)$ of the DJIA index is more localized and peaked around the zero return values than that of the SSEC index. Could it indicate that the former is a more credible market compared to the later?

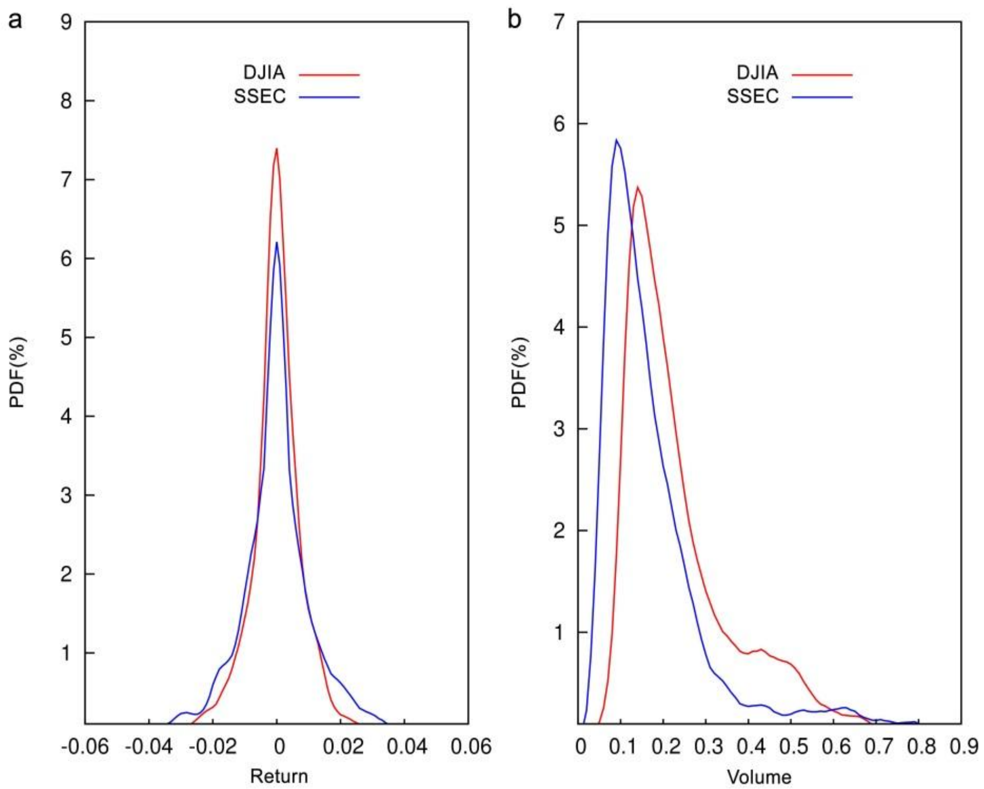

Figure 5: a) The plot of the pdf for price returns from Jan. of 2010 to Dec. of 2017 for the DJIA (red line) and SSEC (blue line) indices. b) The same plot for volume (divided by its maximum value).

In figure 5(b), the same plot of $R(v)$ for all possible values of price returns during the time interval of Jan. 2010 to Dec. 2017, shows that the higher (lower) trading volumes in the DJIA index are more (less) probable than that of the SSEC index. This may be due to the willingness of the traders to trade more in the stable market conditions which, in turn, enhances the credibility. According to figure 5(b), there exists a *threshold* value of trading volume for each market below which the respective market cannot perform and will not be valid. In other words, the market will crash below the threshold volume. We will come back to this when we consider the joint pdfs and the corresponding joint quantum potentials.

Following the same procedure, where the quantum potential for price return is obtained, one may plot the corresponding quantum potential for trading volume. These are shown in figures 5(a) and (b).

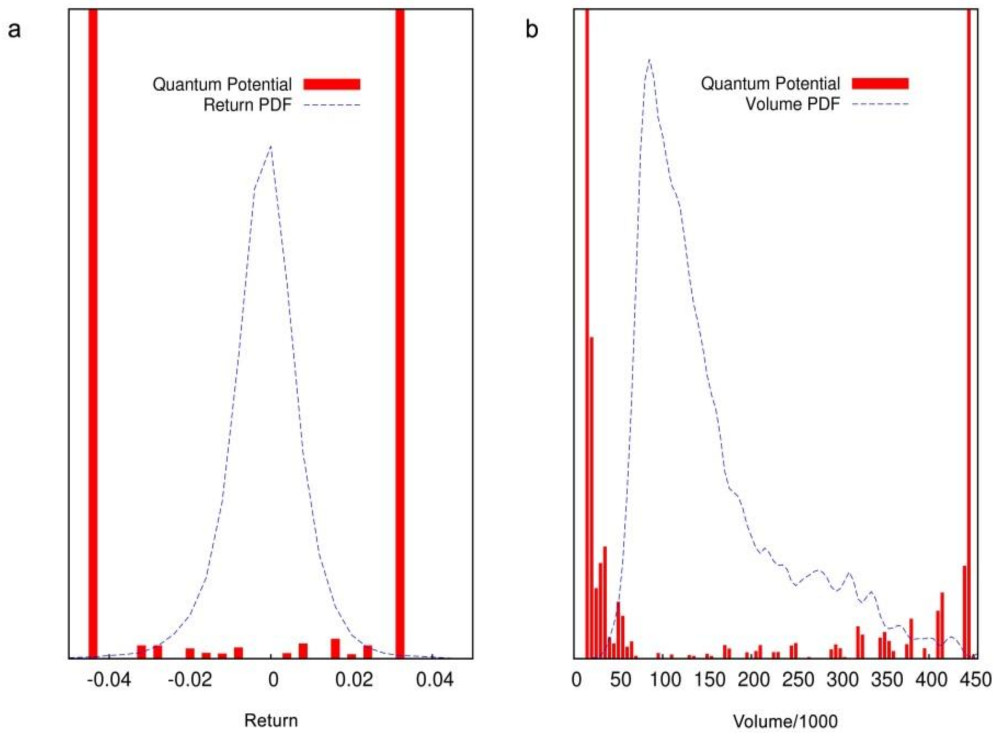

Figure 6: The plot of the quantum potentials for a) price return from Jan. of 2010 to Dec. of 2017 for the DJIA index and b) trading volume for the same period and index.

In a similar vein, one may argue that like the return quantum potential shown in figure 6(a) which confines the daily return variations to a specific interval, there also exists a volume quantum potential which confines the daily variations of the trading volume to a specific interval (as shown in figure 6(b)). Note that the existence of the threshold value for the volume might be due to the foot point location of the left side wall of the quantum potential.

4.3 Results for Double-variable Case

As outlined before, the functional behavior of real markets, known as the complicated dynamical systems, is expected to be affected by several variables. In other words, examining the evolution and outcomes of a market by a single variable model, may be a nonchalant approach to the problem and far from the reality. To answer the questions 'why short-term high return and/or trading volume variations are not experienced', 'What is the joint pdf of these variables' and 'whether these variables are inherently independent', one needs to have information about the functional behavior of the pdf in its general form. Going a step further, one may consider the simultaneous functions of price return and trading volume and search for a joint probability distribution function of these two variables. In this respect, according to previous chapter we have $n = 2$, $q_1 \to r$, $q_2 \to r$ and the corresponding joint probability distribution function, $R(r, v)$ is obtained by integrating all variables and the time except r and v, is constructed by choosing suitable bins in (r, v) plane and plotted for the DJIA and SSEC indices in figures 7(a) and 7(b), respectively.

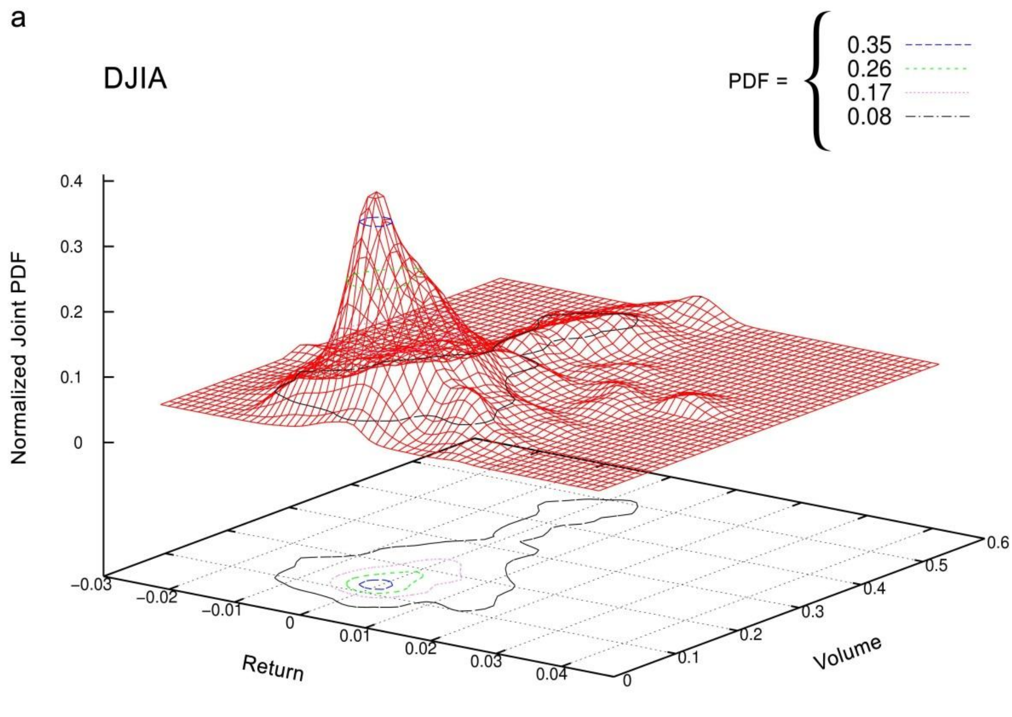

Figure 7: a) The joint pdf from Jan. of 2010 to Dec. of 2017 for the DJIA index. The trace of intersections of a plane perpendicular to the vertical axis for different values of the pdf is shown in (r, v) plane to obtain isoprobability contours.

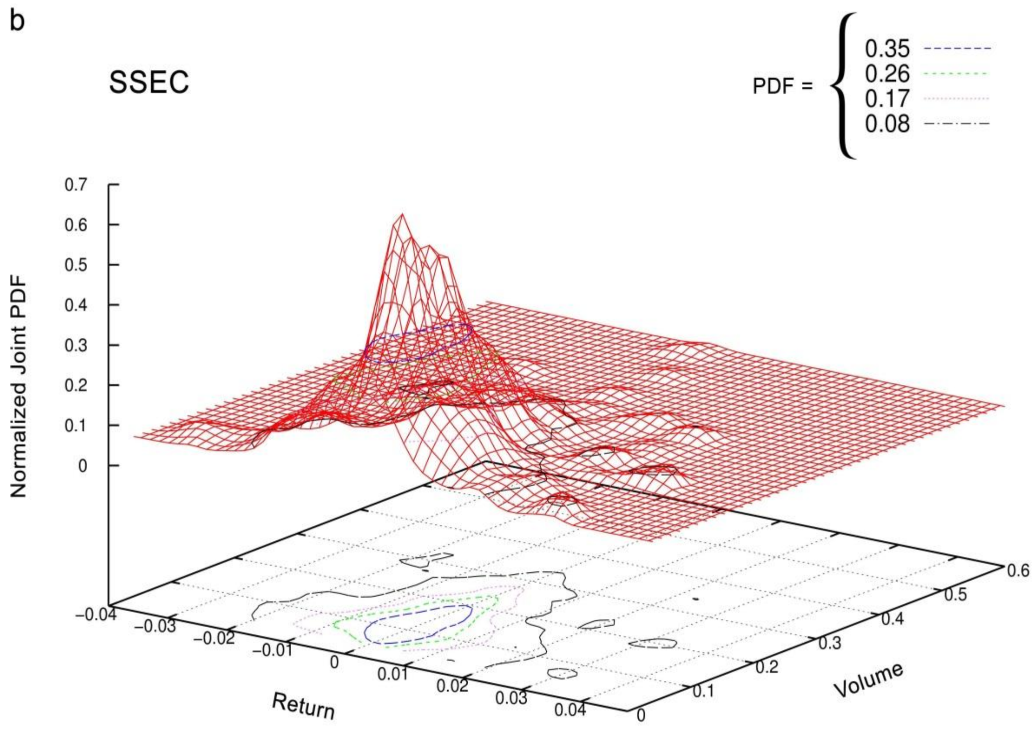

Figure 7: b) The same plot as in figure 7(a) for the SSEC index

According to figures 7(a) and 7(b) it is understood that the absolute value of price return increases until its maximum value as the trading volume increases, and then starts to decrease while the volume increases to make the market credible. For each index, the turning point occur where the corresponding price return reaches the maximum value. The results are not the same for different volume intervals, i.e. the return increases by increasing volume before the turning point and decreases after that up to a certain value (see equation 10) and becomes more or less stable thereafter.

Note that the probability decreases for the volumes and returns far from 0.1 and zero, respectively. However, the local maxima of probability distribution function occur more or less around the zero return for all possible volumes, indicating the credibility. Of note, the procedure of being credible, as seen in the figures 7(a) and (b), is not the same for the indices under consideration. The DJIA index is, relatively, more credible than the SSEC index, consistent with the developed and emerging nature of these indices, respectively.

In addition, the probability for higher returns decreases for any fixed trading volume. This means that it is not possible to have any arbitrary return by increasing the volume. For example, in figure 7(a), it is not possible to have a return value of 0.02, with the probability of 0.08, or a return value of 0.01 with the probability of 0.17. This guides the investors to assess their chance for gaining a prerequisite return. Note that the probability for DJIA index as developed market decreases faster than SSEC index as emerging market consistent with our former result of risk analysis.

An interesting finding in figures 7(a) and 7(b) is that r and v do not behave as two independent variables. If they were, the traces shown in these figures would have rectangular shapes rather than irregular ones as it is. In other words, the two-dimensional pdf surface would become the surface of a pyramid with rectangular cross section. The dependence of r and v means that the trading volume has its impact on price return and vice versa. Note that due to different credibility levels which occur at higher volumes, the mutual impacts of r and v are not the same for DJIA and SSEC indices. This result agrees with those of Chuang et al. (2009) and Lin (2013) that show trading volume has positive and negative impacts on price return using quintile regressions method.

To make an instructive sense of the interdependence of r and v we suggest a parametric analytical function for the joint probability distribution, $R(r, v)$ and try to fit it to the data for appropriate values of parameters. Referring to figures 5(a) and (b) for the plot of $R(r)$ and $R(v)$ as the pdfs of DJIA and SSEC indices, respectively and looking at the schematical behaviors of $R(r)$ (as a Gaussian-like function) and $R(v)$ (as a Maxwell-Boltzmann-like function) one simple expression may be:

$$R(r, v) \propto e^{\frac{-r^2}{\sigma_r^2(v)}} v^2 e^{-\frac{v^2}{\alpha^2}}, \tag{9}$$

where α is a constant to be determined by fitting the normalized form of $R(r, v)$ with data and $\sigma_r^2(v)$ denotes the *volume dependent* variance for the price return Gaussian-like distribution function. For example, as it is seen by figures 7(a) and **Error! Reference source not found.**, the symmetric form of pdf with respect to the zero-return axis is almost preserved going in direction of increasing volume.

However, the width (or variance) of the pdf increases at the beginning, then decreases and finally become constant for different volume intervals. Almost, the same behavior is seen for the isoprobability contours in figures 7(a) and (b) in the (r, v) plane. Thus, as a simple model, we assume the following linear expressions for $\sigma_r^2(v)$:

$$\sigma_r{}^2(v) = \left\{ \begin{array}{ll} \beta_1 v + \gamma_1, & 0.05 < v < 0.1, \\ \beta_2 v + \gamma_2, & 0.1 < v < 0.3, \\ \beta_3 v + \gamma_3, & 0.3 < v < 0.5 \end{array} \right\}, \quad (10)$$

and try to find the utilized parameters by fitting the corresponding pdf with the given data. As an example, we plotted equation 9 as shown in figure 8 and by using equation 10 and fitting the corresponding $R(r, v)$ with the data given for DJIA index, we obtained $\alpha = 0.1$, $\beta_1 = 0.07$, $\beta_2 = -0.05$, $\beta_3 = 0.00$, $\gamma_1 = 0.01$, $\gamma_2 = 0.02$ and $\gamma_3 = 0.01$.

Note that the joint probability distribution function $R(r, v)$ defined by equation 9, is not a separable function of two variables r and v. In other words, it is not the product of two independent functions of $R(r)$ and $R(v)$, but they are coupled together through $\sigma_r{}^2(v)$. We will come back to this point when we argue about the joint quantum potential.

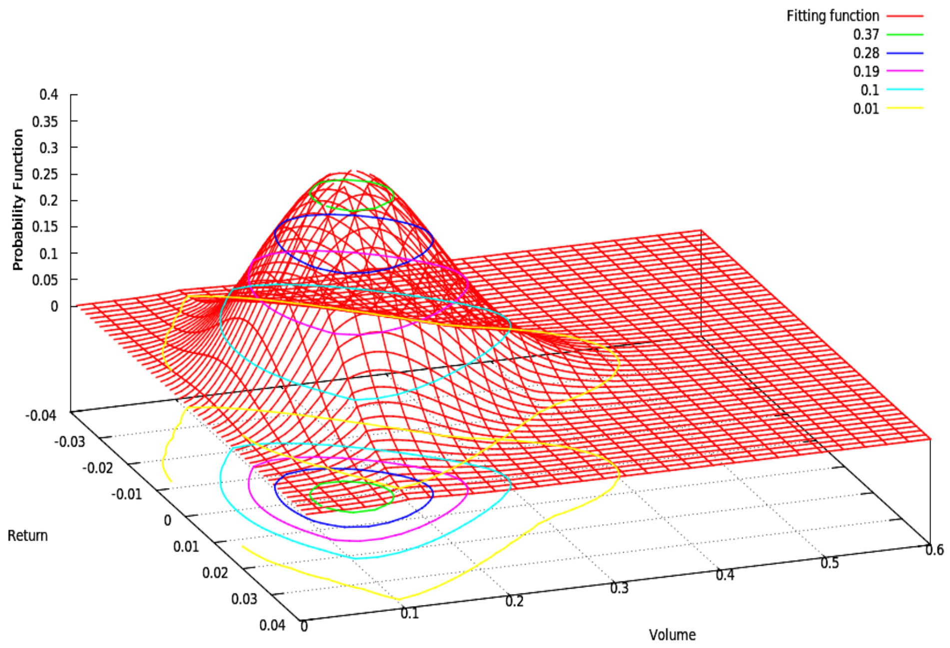

Figure 8: The plot of joint probability function, $R(r,v)$ given by equations 9 and 10 with the parameters chosen to fit with the DJIA index data

Note that the slices chosen in figure 8 which gives the isoprobability contours in (r,v) plane, are almost the same as the pdf values of figure 7. Other suitable forms for ${\sigma_r}^2(v)$ and employing possibly more parameters one may get better fitting to the data.

What is the reason for the pdf being more or less a localized function of r and v confined to a specific region of (r,v) plane in figures 7(a) and (b)? The answer is originated in one-dimensional quantum potentials $U(r)$ and $U(v)$ which confine the return and volume into specific domains of r and v variables, respectively (as seen in figures 6(a) and (b)). Similarly, a two-dimensional joint quantum potential $U(r,v)$ confines the joint pdf into a specific region in (r,v) plane, which in turn, restricts return and volume values.

In figures 9(a) and (b), the joint quantum potential $U(r, v)$ is plotted in (r, v) plane. It is seen that the variations of the return and volume are confined between the potential walls. There are limiting values controlled by the quantum potential, marked by yellow to blue colors, where the trading volume and price return, cannot exceed them. The trace of the quantum potential walls is similar to the isoprobability contour plots of the pdfs in figures 7(a) and 7(b). Thus, similar interpretations of the results can be made here, too. That is, as we argued in the previous section, here also the quantum potential rules threshold values for trading volume, where the market starts to be valid. In fact, one may argue that the joint quantum potential is responsible for the behavior of the joint pdf, such as the impact of trading volume on price return, credibility of the market, and local maxima of the joint pdf around the zero return.

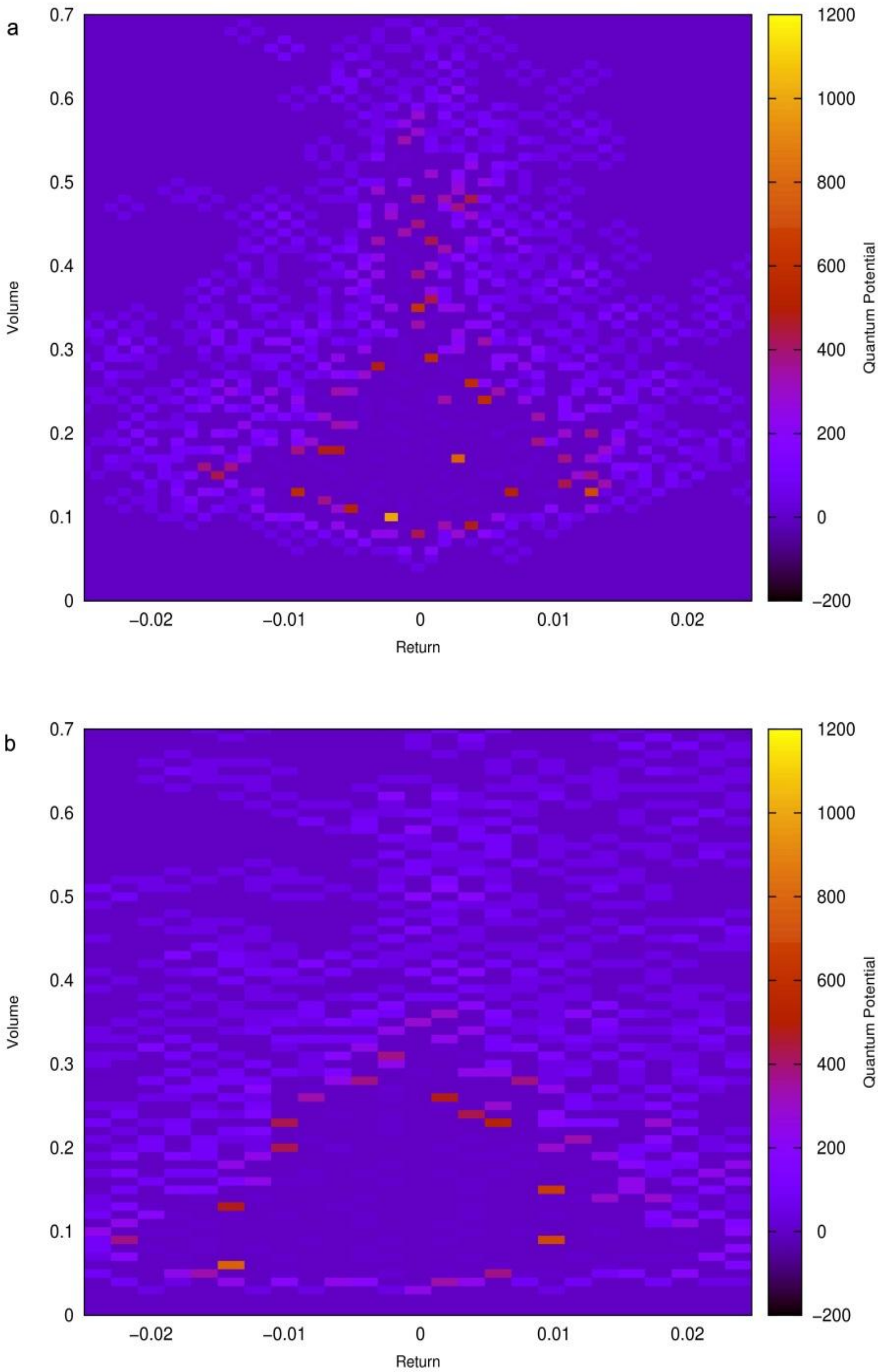

Figure 9: a) The plot of the quantum potential in (r, v) plane for DJIA index. b) The same plot for SSEC index. Different colors show the strength of the quantum potential as calibrated at the right.

It should be noted that in addition to different characteristics that we identified for the indices under consideration, the region defined by the foot points of the quantum potential walls for the DJIA index (developed market) are located more regularly and driven out to the boundaries than the SSEC index (emerging market). This might be due to the fact that the efficient stock markets as mature markets are more regulated and legislated. As it is seen in figures 5(b), 7(a), 7(b), 9(a) and 9(b), the threshold value for the DJIA index is relatively higher than that of the SSEC index. This property might be due to the nature of the markets that are performed in two different economies influenced by different kinds of governmental interventions.

Chapter 5

SUMMARY AND CONCLUSIONS

5.1 Conclusions

In this study, we have employed the Bohmian quantum mechanics for investigating five selected stock indices of the emerging and developed markets. Calculating their quantum potentials, we compared them with the corresponding quantum potentials of the Gaussian white noise having the same variances. We have found that in the short-run, where the entanglement in prices is higher, the quantum potential walls are robust and narrower than those of the long-run pattern, where the entanglement is lower.

Presumably, the trading strategy in an individual investor scale is a personal decision. However, it seems that in the collective scales there is a global pattern that governs the behavior of the stock markets. This global strategy pattern which is reflected as an average outcome in the real data, though not known in detail, is embedded in the quantum potential. In other words, the variance of the individual scale decisions is not much far from that of the other investors and follows a global pattern, which confines the events into a domain wall.

It is instructive to note that the exact modeling for evolution of a market, being considered as an extremely complicated dynamical system, could not be solely determined by a restricted number of variables. Too much soft (behavioral and psychological) and hard (economical) factors are present and have their own impact on the markets returns. This is the reason for generalizing the single-variable methodology as to a multi-variable system. However, all that should not disappoint the researchers from trying to model the issue by a simplistic as well as scientific point of view. In spite of the fact that such modeling, with presumably maximum number of simplifications, does not consider all relevant ingredients, but can still detect the essential identity of the real problem without worrying about its full complexity. Although, as we referred before, some authors have investigated the effect of the price return together with the trading volume, most of the researchers have assumed the price return as a single variable describing the evolution of the markets which have been amassed in the Econophysics literature.

The joint probability distribution of return and volume introduced here shows that these two variables are not independent from each other but have their own mutual impacts. Due to the localized nature of the probability distribution function around the lower returns, further evidence can be provided by the isoprobability contours. That is, starting from a threshold value for trading volume, any increase in volume subsequently leads to an increase in the absolute value of price return but reversely to a decrease in return, thus making the market more credible.

Another important finding reveals that behind the observed behavior of the joint probability distribution function and the corresponding isoprobability contours, there exists a joint quantum potential due to the correlation between a price and a volume and their prior-day price and volume, respectively. Therefore, one answer to the question of ‘why it is not possible to have higher absolute returns in higher volumes’ can be the constraints embedded in such a joint quantum potential function. In fact, the credibility could be interpreted and understood better in terms of the joint quantum potential which governs the variations of price return and trading volume together. In other words, when one deals with price return or volume quantum potentials, separately, as shown in figures 6(a) and (b), one has robust return or volume intervals confined by the corresponding fixed potential walls. Conversely, the joint quantum potential, as shown in figure 9, yields flexible return and volume intervals redounding the credibility of the markets having lower return at higher volumes.

As explained before, with the use of the quantum potential method in studying stock markets, one may distinguish between four distinct cases. The first case deals with representing the markets by means of the quantum potential as a function of single independent variable, ignoring the impact of all other possible variables. In this case, the interval of variations of the variable is fixed and confined by the corresponding quantum potential. This method has been adopted and discussed for instance by Tahmasebi et al. (2015) and Shen and Haven (2017) arguing that the quantum potential is a function of price return as a single variable with a fixed variation interval. In the second case, that is considered here, the markets are represented by the quantum potential as a function of two joint variables leading to a bidirectional causality relation between return and volume. Thirdly, the quantum potential of the markets is taken as a function of more than two variables without any subgroup structures and is argued to govern the fluctuations of each variable through the impact of all remaining groups of variables. Finally, the quantum potential is again taken as a function of more than two variables; however, in contrast to the former case, here the variables could be categorized into different subgroup structures. In this case, the quantum potential, when obtained, could shed light on deeper layers of the corresponding markets.

Furthermore, in order to maximize the expected returns of the selected portfolio, one may need to diversify the risk through selecting different stocks from different markets. The relation between the risk and average return shows that the long-run investments in the developed stock markets could be safer and more profitable than short-run investments.

5.2 Future Studies

In this study we considered an individual market as an isolated dynamical system without considering the impacts of other existing markets. However, considering the globalization of stock markets, such a simplified model could not explain the resulting complicated situation. Thus, it is noteworthy for future studies to investigate the interconnections of different correlated stock markets by introducing an appropriate quantum potential based on the markets quantum interference.

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