Channel Estimation for Millimeter Wave Cellular Systems

Sepehr Ashtari Nakhaei

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Approval of the Institute of Graduate Studies and Research

Assoc. Prof. Dr. Ali Hakan Ulusoy
Acting Director

I certify that this thesis satisfies the requirements as a thesis for the degree of Master of Science in Electrical and Electronic Engineering.

Prof. Dr. Hasan Demirel
Chair, Department of Electrical and Electronic Engineering

We certify that we have read this thesis and that in our opinion it is fully adequate in scope and quality as a thesis for the degree of Master of Science in Electrical and Electronic Engineering.

Assoc. Prof. Dr. Ali Hakan Ulusoy
Co-Supervisor

Assoc. Prof. Dr. Ahmet Rizaner
Supervisor

Examining Committee

1. Prof. Dr. Hasan Amca
2. Assoc. Prof. Dr. Ahmet Rizaner
3. Asst. Prof. Dr. Ziya Dereboylu

ABSTRACT

The imagination of our future on wireless base networks is beyond science fiction. By emerging 2020, most of the wireless communication systems are going to suffer by the vast increase in the number of traffic in their network which leads to the lower data rate and higher latency. The current cellular network provides the stable connection for the demanding users, but in near future this technology will need a drastic improvement as it crosses its capabilities when users want more data. Therefore, 4G is going to be replaced by 5G. 5G technology, which is currently under development, is going to achieve a couple of objectives such as higher spectral efficiency, preferable battery life for handsets, higher capacity, improvement in the coverage area and lower latency. For this technology, to be realistic and operative as it is promised, some upcoming technologies such as massive MIMO, millimeter wave, beamforming and small cells are going to aid 5G system. Millimeter wave along with massive MIMO provides a high data rate within a large coverage area. Additionally, beamforming manages the massive transmission of data to all over the medium and directs the data to a specific user. In every wireless network, there are losses due to the propagation of medium, also known as channel. The channel is usually estimated by the signal processing module to assist the receiver in order to eliminate channel's severe domination. For each generation of wireless cellular networks, a model by all the factors that have an impact on the signal will be presented as a fixed modeled and then channel is estimated and compared to the modeled version to have constancy. 5G network channel should be carefully estimated due to the use of high frequency band and the massive number of antennas. In this thesis, our focus is on estimation of channel through some training procedures. We try to pre-code the incoming input signal with some code words to reach as close as possible to the actual channel. Accordingly, BER, MSE and spectral efficiency will be illustrated to provide the channel performance and stability of the mm-wave systems.

Keywords: 5G, Beamforming, Channel estimation, Precoding, Massive MIMO, Millimeter Wave and Small Cells.

ÖZ

Kablosuz ağlar üzerine kurulan hayaller bilim kurgunun ötesine geçmiş durumdadır. 2020 yılının yaklaşmasıyla beraber, birçok kablosuz haberleşme sistem ağlarında geniş çapta bir trafik artışından ötürü sıkıntı yaşanacaktır. Bundan dolayı veri hızında düşüş ve gecikmelerde artış gözlemlenecektir. Mevcut hücresel ağlar, kullanıcıların taleplerine yanıt verebilmektedir fakat yakın gelecekte bu teknolojilerin ciddi bir şekilde gözden geçirilip, kullanıcıların daha fazla veriye ihtiyaç duyacakları an için hazır hale getirilmeleri gerekmektedir. Bu gibi sebeplerden ötürü 5G sistemlerinin 4G'nin yerine geçmesi öngörülüyor. Şuan gelişme aşamasında olan 5G teknolojisinin ulaşması gereken bir takım hedefler bulunmaktadır. Bu hedeflerden öne çıkanlar yüksek spektrum verimliliği, taşınabilir cihazlar için pil ömründe artış, yüksek veri hızı, yüksek kapasite, kapsama alanında artış ve gecikmede düşüş olarak sıralanabilir. Bu teknoloji söz verildiği şekilde işlevsel olabilmesi için şuan gelişme aşamasında olan bazı teknolojilere ihtiyaç duymaktadır. 5G sistemleri, yoğun çok-girdili çok-çıktılı sistem, milimetre dalga, huzme oluşturma ve küçük hücreler gibi yeni teknolojilerin birleşmesiyle oluşturulacaktır. Milimetre dalga teknolojisinin yoğun çok-girdili çok-çıktılı sistem ile birleşmesiyle birlikte veri hızında ve kapsama alanında artış sağlanacaktır. Ayrıca, huzme oluşturma teknolojisi ise ortamdaki yoğun veri akışını düzenleyip, bu veriyi gönderilmek üzere belirlenmiş olan kullanıcıya yönlendirecektir. Tüm kablosuz haberleşme ağlarında ortamdaki yayılmadan ötürü kanal olarak adlandırılan kayıplar yaşanmaktadır. Genellikle kanalın etkisine son verip vericiye yardım etmek için bir takım sinyal işleme metotları ile kanal kestirimi yapılmaktadır. Kablosuz hücresel ağların her kuşakta sinyale etki eden bazı parametreleri vardır ve bu gibi parametreler sabit model olarak adlandırılır. Kanal kestirimi ise bu parametreler üzerinden yapılır ve tutarlı olabilmesi için modellenen kanal ile kıyaslanır. 5G sistemlerinde çok yüksek frekans bandı ve anten sayısı kullanıldığından kanal kestirimi büyük önem taşımaktadır. Bu tezde yoğunlaşılan nokta, belli bir hazırlık prosedürüne tabi tutulan kanal kestirimidir. Bu amaç doğrultusunda gelen giriş sinyaline ön-kod çalışması yapılarak var olan kanala olabildiğince yakın bir kanal kestirimi elde edilmeye çalışılmıştır. Sonrasında ise bit hata oranı, ortalama karesel hata ve spektral verimlilik gibi parametreler hesaplanarak kanal performansı ve milimetre dalga sistemlerinin kararlılığı gösterilmiştir.

Anahtar Kelimeler: 5G, huzme oluşturma, kanal kestirimi, ön kodlama, yoğun çok-girdili çok-çıktılı sistem, milimetre dalga, küçük hücreler.

DEDICATION

To my parents and my brother

For their love, supports, and encouragements

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LIST OF SYMBOLS AND ABBREVIATIONS

1G	First Generation of wireless cellular network
$\otimes$	Kronecker Product
σ	Signal Power or Variance of Signal
z^{-1}	Time Delay
F	Fourier Trasnform
F^{-1}	Inverse Fourier Transform
AMPS	Advance Mobile Phone System
AoA	Angle of Arrival
AoD	Angle of Departure
arg min	Argument of the Minimum
BER	Bit Error Rate
BS	Base Station
CDMA	Code Division Multiple Access
CFR	Channel Frequency Response
CSI	Channel State Information
DAC	Digital to Analog Convertor
$E\{.\}$	Expected Value

EHF	Extremely High Frequency
FFT	Fast Fourier Transform
GSM	Global System for Mobile
ICE	Iterative Channel Estimation
IFFT	Inverse Fast Fourier Transform
IP	Internet Protocol
IR	Impulse Response
ISI	Inter-Symbol Interference
LMMSE	Linear Minimum Mean Square Error
LoS	Line of Sight
LS	Least Square
MIMO	Multiple Input Multiple Output
MS	Mobile Station
MSE	Mean Square Error
NLoS	None Line of Sight
NMT	Nordic Mobile Telephone
PM	Parameter Model
QoS	Quality of Service
SHF	Super High Frequency

SISO	Single Input Single Output
SMS	Short Message Service
SNR	Signal to Noise Ratio
SVD	Singular Value Decomposition
TACS	Total Access Communication System
WiFi	Wireless Fidelity
WiMAX	World-Wide Interoperability for Microwave Access

Chapter 1

INTRODUCTION

1.1 Introduction

At the point when an innovation enters to industry, one endeavors to upgrade that development to its best. In terms of technology, telephone invention was a huge upheaval. First handsets' communications were through copper cables. As the time passed, wireless telephone or the First Generation (1G) network was created. The first generation of wireless telephone was born in Japan which was the first country to release the first smartphones in 1999 [1]. However, smartphones were not so popular until Apple introduced its first product (iPhone) in 2007 [2]. This product provided friendlier keypad, large touchscreen for direct finger input, and in general, it was easier to understand. This phone was operating by 1G protocol at the frequency band of 800 MHz which was low frequency compared to the new generations [3]. This low frequency has its own advantages and disadvantages. Usually, transmitting the signal through air causes a disturbance to signal, this process is called Non-Line of Sight (NLOS) which means the path between the sender and receiver is not clear and there are objects or obstacles in between antennas. That is why low frequency signal has a better chance of passing through objects and reaches to destination with less harm but this causes in disadvantage of low data rate. Line of Sight (LOS) which can be described as direct transmission of signal without crossing an object is shown in Figure 1.1.

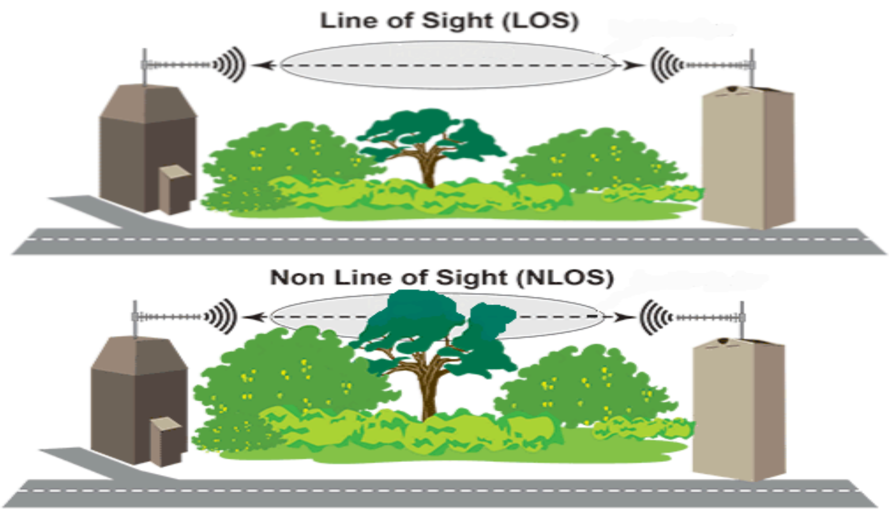

Figure 1.1: LOS and NLOS

By definition the disturbance due to the obstacles in between the sender and receiver on the transmitted signal is called attenuation that is the reduction of signal strength through the process of transmission. Additionally, as the frequency rises, absorption effect becomes more disruptive. Therefore, generally speaking, attenuation of signal caused in transmission process is due to channel. Channel was not considered important earlier and that was due to the low frequency bands. Low frequency signals are not significantly damaged by the channel and hence, receiver is able to extract the information with having less errors. Nowadays, as the technology meets the demand of high number of users in order to connect to a specific network, new frequency bands namely Super High Frequency (SHF) band, 3-30 GHz band and Extremely High Frequency (EHF) band, 30-300 GHz were deployed in order to overcome the problem of bandwidth and also data rate. Besides, the importance of channel information knowledge rockets as the frequency increases. Channel estimation has been a popular subject for every researcher as every new generation of wireless communication comes out. In this thesis, a comprehensive detail of Channel State Information (CSI) will be discussed and a mathematical form of channel estimation will be generated to meet the demand of next generation which is the Fifth Generation (5G) cellular network.

1.2 Channel State Information

CSI refers to known channel properties in the communication link. These properties show how a signal propagates from the transmitter to the receiver. The propagation could be loss due to signal power with distance or impact of phenomena such as reflection, detraction, absorption, polarization and scattering [4]. To overcome this problem, the method of channel estimation was introduced to adapt the transmission to the current channel condition, which is crucial for achieving reliable communication link with higher data rate. This procedure typically happens when a pilot signal or symbols known by both sender and receiver is transmitted through the channel. Therefore, receiver can extract the channel information and fed it back to the transmission for further improvements. Mathematically speaking, there are two different kinds of CSIs, instantaneous CSI and statistical CSI. Instantaneous CSI can be viewed as knowing the impulse response of the digital filtering. This could help to adjust the transmitter by the impulse response of channel to optimize the received signal. Statistical CSI basically shows the characteristic of the channel, for instance type of fading distribution (fast fading or slow fading), LOS component and channel gain etc. Consequently, as the channel behaves as fast fading (when channel condition varies rapidly as the time changes) the reasonable solution is to use statistical CSI and as the channel behaves as slow fading the reasonable treatment is to use instantaneous CSI. Usually the efficiency of CSI lies between these two characteristics to reach the best value of Signal to Noise Ratio (SNR) or Bit Error Rate (BER). To define CSI, firstly the communication link between the transmitter and receiver should be mathematically observed.

1.3 Basic Approach for Eliminating the Channel

As every phenomenon has its own explanation in mathematical form, wireless communication system can also be expressed in mathematical form as well. Figure 1.2 helps to discover the basic methodology of wireless communication. By assuming transmission through a channel between two base stations placing at the position of NLOS, a wireless communication system scheme by considering attenuation can be designed as

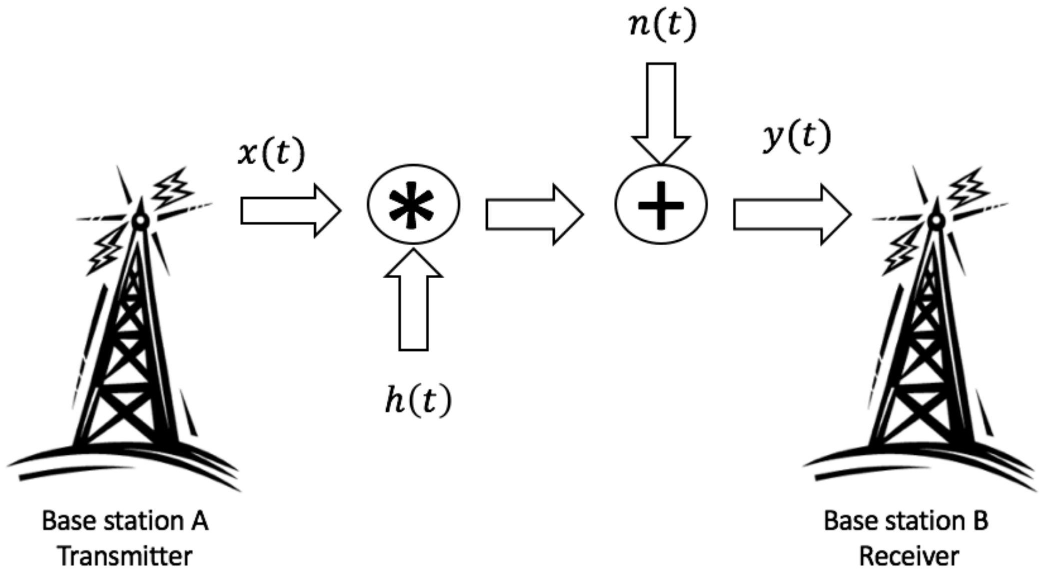

Figure 1.2: Wireless communication scheme for transmitting a package

In Figure 1.2 $y(t)$ and $x(t)$ are the received and transmitted signal vectors. $h(t)$ is the channel response and $n(t)$ is the noise [5]. We use the following notations throughout this thesis, for instance, **A** is matrix, **a** is a vector (sometimes this notation $\overline{\mathrm{A}}$ is used to show, A is a vector) and a is a scalar, whereas $\mathbf{A}^T$, $\mathbf{A}^H$, $\mathbf{A}^*$ and $\mathbf{A}^{-1}$ are its transpose, Hermitian (conjugate transpose), conjugate and inverse respectively. We use to keep the best values of the matrix in side $\mathbf{A}^\star$ in order to keep them for next stage of estimation. Notation $\mathbf{A} \circ \mathbf{B}$ is the Khatri-Rao product of **A** and **B,** $\mathbf{A} \otimes \mathbf{B}$ is the Kronecker product of **A** and **B, A ⊙ B** is the Hadamard product of **A** and **B.** Noise is usually considered as random Gaussian or Additive White Gaussian Noise (AWGN). At the position of BS-B (Figure 1.2 Base station B, receiver), to detect the transmitted information, the simplest approach is to multiply received signal by the inverse of channel response, that leads to transmitted signal plus some random noise. This simple approach for detection of signal is no longer useful for the new populated cellular networks due to the use of massive number of antennas. That is why this thesis investigates the establishment of a new way of channel estimation for reduction of channel impact on the signal which, in the case of high frequency signal modulation the effect will be much worst.

1.4 Thesis Organization

Channel estimation is always critical for wireless networks as discussed in previous sections. Additionally, by increasing the frequency, channel has more impact on the signal strength and that causes a huge disadvantage in wireless communication links. This thesis focuses to minimize the effect of channel by combining the digital and analog pre-coding matrixes for the millimeter wave radio signal.

Chapter 2 explains different generations of wireless cellular network. Explanation consists of 1G up to 5G including; history, comparison and achievements in every generation and what makes them better than the previous ones. Additionally, Chapter 2 briefly discusses the technologies aid 5G to meet the standards to act better than current cellular communication network – Fourth Generation (4G). These technologies are massive Multiple Input Multiple Output (MIMO), millimeter wave (mm-wave) and small cells. Merging all technologies finally leads to step into the future generation of wireless cellular network, 5G.

In Chapter 3, all the requirements for estimating the channel are provided. Initially, hybrid pre-coding is discussed and considered, likely to improve BER and reduce interferences. Then all the training steps which are theoretical and called dictionary learning for beamforming matrices is introduced along with array processing mechanism. Further on, 5G channel model is introduced which is a fundamental criteria for estimating the channel and establishing some simulation result to see whether this method is close enough to the actual channel. All the technical introduction mentioned above is then mathematically modeled for the simulations in Chapter 4. Finally, to define the channel capacity some mathematical approach is discussed before finding the estimated channel capacity.

Chapter 4 presents the simulation results. In this chapter, details of implementation and outcome from the simulations are shown. Firstly, channel is estimated for 5G by adding the code books and a training procedure to the design. Secondly, sending some realistic symbols and detecting them at the receiver through estimated channel and calculating BER for both actual channel and estimated channel to find the processing gain. Finally, at the end of the thesis study, a brief conclusion of all the work and all the completion that are achieved is summarized.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

5G is a promising technology and it has been predicted to show up by 2020 [3]. This thesis acknowledges the upcoming of 5G by discussing the major aid of other technologies to make this feature to revive. The mm-wave communication is one of these features for the future outdoor cellular network, and enables giga bits per second (bps) data rate, thanks to the allocation of vast bandwidth. The mm-wave technology also employs directional antennas with large array of antennas at both transmitter and receiver. In the upcoming sections, evolution of wireless cellular network and some characteristics of 5G will be demonstrated.

2.2 Evolution of Wireless Technology

Samuel F. B. Morse who was the inventor of telegraph, sent a signal by a small set of punctual and procedural signals which were known as sequences of short and long signals called “dots” and “dashes”. After a while it became popular and was called after its inventor, Morse code. This code is still in use of military and spread among most of the regular people in case of emergency like the terrorist attack. Later, this code was used by the Italian inventor, G. Marconi, who communicated letter ‘S’ by the use of Morse code. He sent this letter through a wireless channel along a distance of 3 Km in the form of three dots with the help of electromagnetic wave. This was the time that wireless communication was born which nowadays is an important part of people’s life. It all starts with 1G of wireless communication cellular network then it reaches out to current cellular network 4G, and now it can be expected that 5G will be a promising cellular network in near future.

2.2.1 First Generation

1G of wireless communication was announced in the 1980s. This communication technique was analog. It had a total data rate up to 2.4 Kbps. Some of the 1G systems were Advance Mobile Phone System (AMPS), Nordic Mobile Telephone (NMT) and Total Access Communication System (TACS). It had some disadvantages such as low data rate, low capacity for users, reckless handoff and no security. Besides, there was an unintentionally eavesdropping by the third person since voice calls were played back by the radio towers [6].

2.2.2 Second Generation

Second Generation (2G) of wireless communication was introduced in the late 1990s. This generation was the first generation that used digital communication instead of analog. For this reason and due to the low power signal, the battery life of mobile phones lasted longer. Global System for Mobile (GSM) communications was the first 2G system, mostly used for voice communication. The data rate for this system was 64 Kbps which was much more than 1G. It provided services like Short Message Services (SMS) and e-mail as well. 2G has an advance model known as 2.5G that uses 2G system frameworks, but also applying packet switching along with circuit switching [7, 8].

2.2.3 Third Generation

Third Generation (3G) was introduced in the late 2000s. The new technology brought out the data rate up to 2 Mbps. This high speed provided mobile access to services based on Internet Protocol (IP), which could help to maintain Quality of Service (QoS). This recent technology aside from the high speed, had a major disadvantage of decreasing the battery life. The architecture of 3G was also expensive to employ. Afterward, engineers have found a way to make the packet switching more efficient and made 3.5G by implementing technologies like High Speed Uplink/Downlink Packet Access (HSUPA/HSDPA). 3.5G had higher data rate (5-30 Mbps). Later on, by the aid of Worldwide Interoperability for Microwave Access (WiMAX), 3.75G came out to support broadband communication for better coverage area [7, 8].

2.2.4 Fourth Generation

4G of mobile phone communication was first released in 2010. This generation made gaming services, high-definition TV content and video conferencing possible. This generation like the previous one is based on IP and has an average data rate of 1.5 Gbps and has been standardized by the 3rd Generation Partnership Project (3GPP).

2.2.5 Fifth Generation

With an exponential increase of users in the cellular network, soon 4G will be replaced by 5G. 5G is going to make a revolution on the concept of cellular network and that is to say each user can connect to the network with a data rate up to 50 Gbps that will not drop due to growth in the number of the users. Therefore, all connected users will have the same share of data rate. Moreover, battery life will be extended by 10 times and end-to-end latency will be dropped 5 times. Consequently, new technologies such as Device-To-Device (D2D) communication or Machine-To-Machine (M2M) communication can benefit from this technology. Hence, in the near future there will be cars that are driven by themselves without any human driver or without any human interface. Figure 2.1 shows some of the possible services of 5G network.

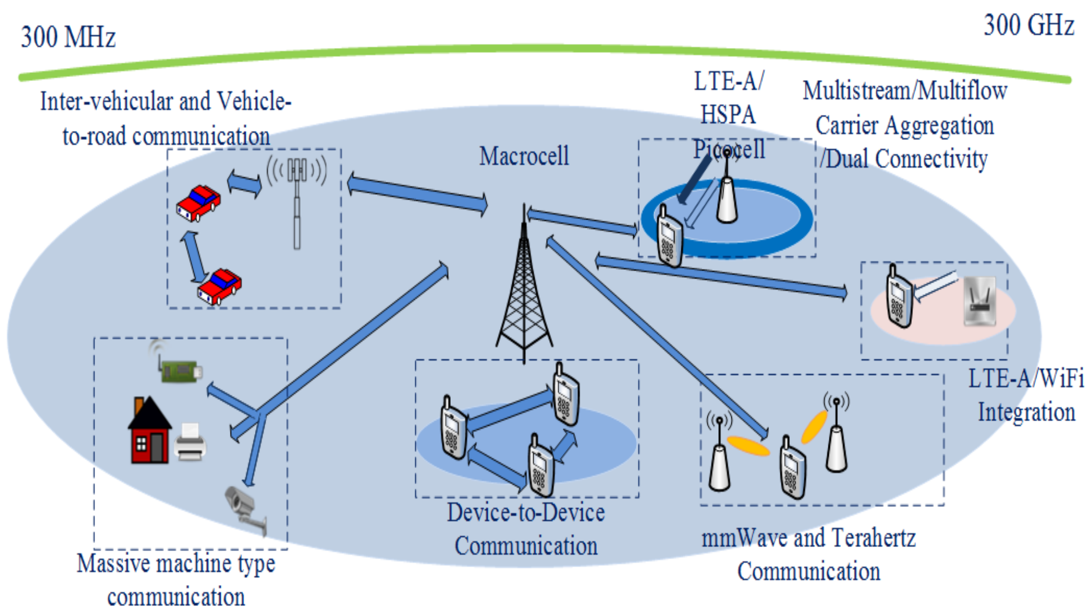

Figure 2.1: Projection of 5G network [3]

Comparison of all the generation of wireless cellular networks along with their standardization is shown in Table 2.1. It also indicates the overall aim of 5G wireless cellular network.

Table 2.1: Evolution of Wireless Technologies [3, 7, 8]

Generation	Access Technology		Data Rate	Frequency Band	Bandwidth Allocation	Forward Error Correction	Application
1G	Advance Mobile Phone Service (AMPS) (Frequency Division Multiple Access (FDMA))		2.4 Kbps	800 MHz	30 KHz	NA	Voice
2G	Global System for Mobile communications (GSM) (Time Division Multiple Access (TDMA))		10 Kbps	850/900 1800/ 1900 MHz	200 KHz	NA	Voice and date
	Code Division Multiple Access (CDMA)		10 Kbps		1.25 MHz		
2.5G	General Packet Radio Service (GPRS)		50 Kbps		200 KHz		
	Enhanced Data Rate for GSM Evolution (EDGE)		200 Kbps		200 KHz		
3G	Universal Mobile Telecommunication Systems (UMTS) / Wideband Code Division Multiple Access (WCDMA)		384 Kbps	800/850 /1800/ 1900/ 2100 MHz	5 MHz	Turbo codes	Data, voice and video calling
	CDMA		384 Kbps		1.25 MHz		
3.5G	High Speed Uplink/Downlink Packet Access (HSUPA/HSDPA)		5-30 Mbps		5 MHz		
	Evolution-Data Optimized (EVDO)		5-30 Mbps		1.25 MHz		
3.75G	Long Term Evolution (LTE) (Orthogonal / Signal Carrier Frequency Division Multiple Access (OFDMA/SC-FDMA))		100-200 Mbps	1.8 GHz, 2.6 GHz	1.4 MHz to 20 MHz	Concatenated Codes	Online gaming and high definition television
	WiMAX (Scalable Orthogonal Frequency Division Multiple Access (SOFDMA))	Fixed WiMAX	100-200 Mbps	3.5 GHz and 5.8 GHz initially	3.5 MHz and 7 MHz in 3.5 GHz band; 10 GHz in 5.8 GHz band		
4G	Long Term Evolution Advance (LTE-A) (Orthogonal / Single Carrier Frequency Division Multiple Access (OFDMA / SC-FDMA))		DL: 3 Gbps UL: 1.5 Gbps	1.8 GHz, 2.6 GHz	1.4 MHz to 20 MHz	Turbo codes	Online gaming and high definition television
	WiMAX (Scalable Orthogonal Frequency Division Multiple Access (SOFDMA))	Mobile WiMAX	100-200 Mbps	2.3 GHz, 2.5 GHz, and 3.5 GHz initially	3.5 MHz, 7 MHz, 5 MHz, 10 MHz, and 8.75 MHz initially		
5G	Beam Division Multiple Access (BDMA) and Filter Bank Multi Carrier (FBMC)		10-50 Gbps (Expected)	1.8, 2.6 GHz and Expected 30-300 GHz	60 GHz	Low Density Parity Check Code (LDPC)	Ultra-definition video and virtual reality application

2.3 Cellular Architecture

The cellular concept was first proposed by Bell Laboratories in 1947. Every cellular network has to be designed in such a way that could serve maximum users with maximum coverage area. Each region in an area of coverage is divided into hexagonal cells. These cells are the practical cells but in an ordinary area such as desert or any vacant area, the coverage shape of the antenna is circular. Figure 2.2 shows the concept of cell architecture.

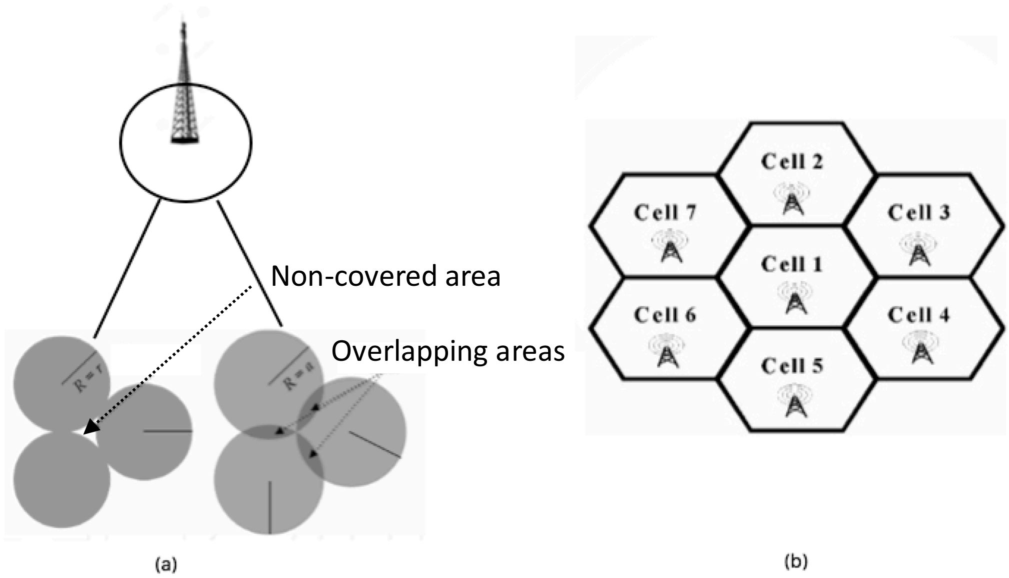

Figure 2.2: Cell Architecture, (a) wireless cell structure (b) cell model

In Figure 2.2, wireless cell structure and cell model are presented. The wireless cell structure is produced with the actual antenna pattern which in case of desert and vacant area without an obstacle is circular, as the antenna pattern are omni-directional. However, there exists some problems for the circular model. First, by placing them together, cells cannot cover the whole area and consequently, users at those specific areas have difficulty of connecting to Base Station (BS), which means no connection to the network. Secondly, in order to overcome the first problem, cells are merged together and in this case it creates overlapping and interference in the system. Then engineers came up with a smart solution and that was a hexagonal shape. In hexagonal shape which is shown in Figure 2.2 (b), by placing cells together neither any overlapping is caused nor any non-coverage area is obtained. Therefore, the initiation of cell splitting was introduced by the hexagonal shape. Each hexagonal shape has a BS in its center and users are supposed to connect through their nearest BS. 1G and other higher generation wireless cellular network until 4G have been using this phenomenon but 5G has to have different architecture due to the technologies it uses. The same hexagonal will appear in practice but infrastructure is different.

2.3.1 Fifth Generation Cellular Architecture

A general survey shows that 80% of the time wireless users stay inside a cell and only 20% of the time they move outside [9]. As it is guaranteed to use mm-wave in 5G systems for inside users to communicate with BS, signal needs to travel through the walls. As already mentioned high frequency signals have less penetration ability, and this will result in lower signal strength and high BER. That is why 5G cellular architecture has to be divided into two distinct parts as inside and outside setups [10]. In this architecture, BS will be equipped with an array of antennas to respond to the massive demand of users with high data rate. Designing cell size is another key factor. For instance, each cell must include macrocells, microcells and femtocells.

For indoor users, large antenna array will be installed at the top of every building to communicate with outdoor BS by employing of LOS components. Then, wireless Access Points (APs) will be installed inside the building and they will be connected by the cable to the outside antenna units. These devices will also be equipped with large antenna arrays for massive connectivity. These APs will have certain technologies such as WiFi, mm-wave communication and visible light communication. This will lead to energy efficiency, cell average throughput, data rate and spectral efficiency, but all of the installations will increase the infrastructure cost.

For outdoor users, one BS will be installed at every edge of cells and this BS will be connected through optical fiber cable to the main BS. The act of adding BSs at the edge of each cell will increase the installation cost. However, this will help to increase both data rate and coverage area but on the other hand, increase the hand-off due to splitting cells into smaller cells. Therefore, this feature enables the connectivity of massive users and other technologies such as D2D communication.

5G architecture incorporated D2D communication, small cell AP and the Internet of Things (IoT). This means that 5G architecture has much better scalability for handling a massive number of connected devices. However, this architecture is impossible without the help of large array of antennas. Accordingly, massive MIMO system is briefly explained in the next section.

2.4 Massive Multiple Input Multiple Output Systems

The exponential increase in the number of users for communication between each other or for connections to the internet or other networks, makes all the provider companies considerate over how they can increase the capacity without decreasing the data rate. The idea of massive MIMO is to deploy arrays of antenna at transmitter side and serve as many users by having single antenna or multi antenna at receiver. Figure 2.3 shows how MIMO system behavior is.

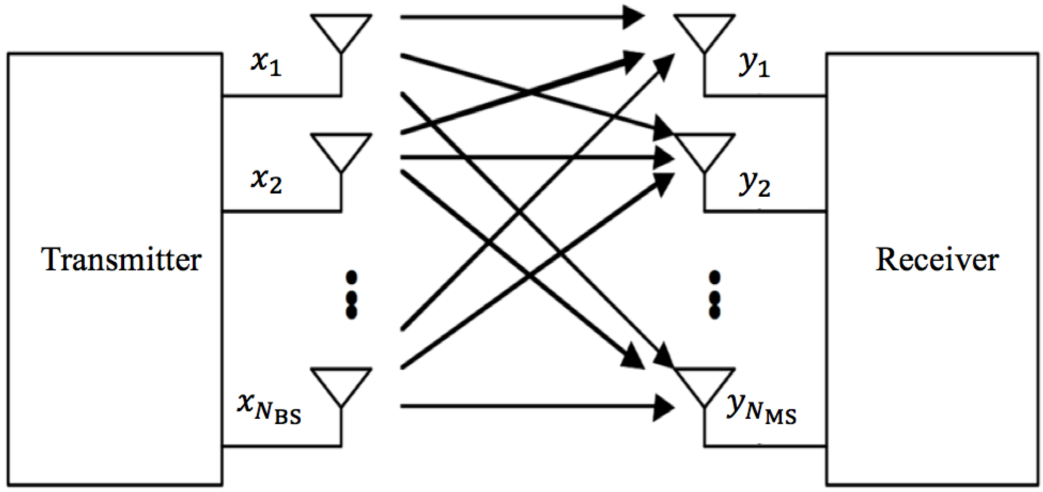

Figure 2.3: Massive MIMO system

On Section 1.3 one-way communication has been discussed which employed an antenna at receiver and another one antenna at transmitter. Here transmitter has several antennas and it uses as many antennas to send the data to receiver. Figure (1.1) illustrated that channel for lower number of antenna at BS and MS has less channel matrix components. Equation (2.1) shows the received signal model for a MIMO system which compare to the SISO model channel has more elements.

$$\begin{bmatrix} y_1 \\ \vdots \\ y_{N_{\mathrm{MS}}} \end{bmatrix} = \begin{bmatrix} h_{1,1} & \cdots & h_{1,M_{\mathrm{BS}}} \\ \vdots & \ddots & \vdots \\ h_{N_{\mathrm{MS}},1} & \cdots & h_{N_{\mathrm{MS}},M_{\mathrm{BS}}} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_{M_{\mathrm{BS}}} \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_{N_{\mathrm{MS}}} \end{bmatrix} \tag{2.1}$$

In a more compact form

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{2.2}$$

In MIMO systems, the most important and challenging issue is to accurately estimate CSI, for both uplink and down link which means for each terminal and each path made by each antenna in MIMO system. This issue calls telecommunication engineers and researchers who are working on wireless communication [12].

In massive MIMO system, BS sends streams of data at any instance of time through the medium. All the information is spread into the area reaches to each receiver antenna. Thus, every client may receive all sort of data but users want the desired data not some other random data that has been received haphazardly. On the other hand, BS does not want to send the data again every time it finds that the package has not reached the destination yet. This process is a waste of time and power. This is where beam-forming comes to action. When there are massive data in the network and only the destination user needs to be able to detect the signal, beamforming can be very helpful. Ultimately, massive MIMO can function with beamforming only. This makes the system to be more efficient, less power consuming, more spectral efficient and also increases robustness [11].

2.5 Millimeter Wave

The mm-wave signals have higher frequency band (30-300 GHz) than the previous frequency band signals used in 1G up to 4G. The mm-wave can be defined as the millimeter measurement of wavelength of high frequency band signals. Wavelengths are in millimeter scales. (2.3) shows how the mm-wave can be obtained regarding their signal frequencies.

$$\lambda = \frac{c}{f} \tag{2.3}$$

where c is the speed of light (3×10^8 m/s), λ is the wavelength and f is the signal frequency. In this equation, frequency and wavelength have a reverse relation. To be more specific, as f increases, λ decrases. That means as frequency band of 30 GHz to 300 GHz, wavelength decreases significantly and it goes to millimeter scale. For example, if we assume $f = 60$ GHz, wavelength becomes $\lambda = 5$ mm. Now it is conceived that mm-wave means wavelength is in millimeter. As mentioned before in this thesis, the reverse relation between frequency and signal strength is due to obstacles or other impacts. In mm-wave, this consequence is thoroughly high.

In order to observe the impact of signal strength decomposition as the distance between the transmitter and receiver increases, Figure 2.4 shows the increase of Free Space Path Loss (FSPL) via distance. The following equation shows how the frequency affects FSPL [38].

$$\mathrm{FSPL(dB)} = 32.44 + 20\log(f) + 20\log(d) \qquad (2.4)$$

It is observed from the path loss formula that, as *f* and distance (*d*) grows the value of path loss relatively rises. Therefore, as the path loss intensifies the negative impact on the signal strength becomes more severe.

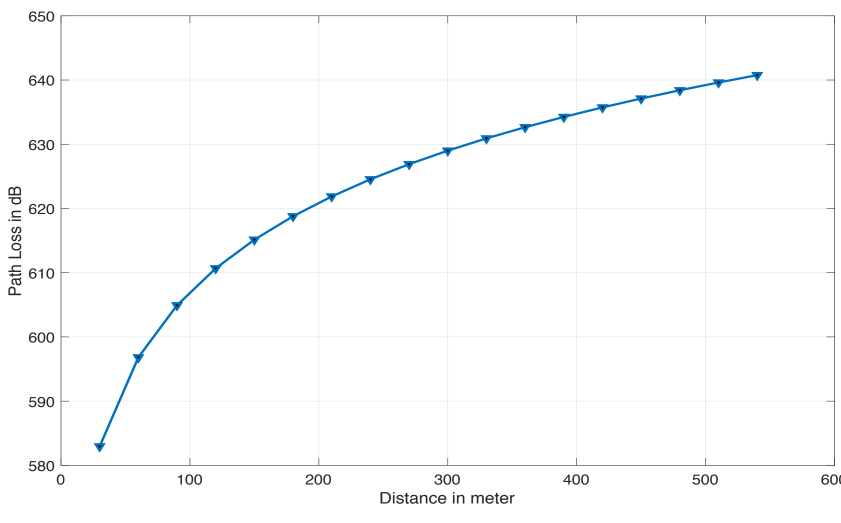

Figure 2.4: Free space path loss verses distance (*f* = 30 GHz)

As it is shown in Figure 2.4, path loss increases as logarithmic function by the increase in distance for f = 30 GHz. This is always the case for free space path loss for high frequencies.

After observing all futuristic technologies helping 5G, now is time to find the nature of the medium in between transmitter and receiver.

2.6 Channel Estimation

Each of the aspects which is examined above, were either to comprehend the complex nature or present the basic characteristics of 5G networks. Another parameter which is very crucial for the network to perform as promised is channel, the medium in between the transmitter and receiver. As massive MIMO is considered in this thesis, channel becomes more important to tackle. Usually, this parameter is calculated through some training procedures to somehow adopt the transmitter to find the next behavior of the channel and tries to eliminate that effect by equipping the transmitting symbols. To interpret the channel estimation for mm-wave, first a revision of all the traditional channel estimation techniques will be presented. Afterwards by understanding the natural phenomena, channel estimation for 5G can be proposed in the next Chapter.

2.6.1 Traditional Techniques of Channel Estimation

In general, channel estimation techniques are divided into three main categories: Channel Frequency Response (CFR), Parameter Model (PM) and Iterative Channel Estimation (ICE). CFR, which was proposed in the 1990s [31], is one of the first approaches to measure channel state parameters. It is based on channel estimation with mainly two parts: one with the statistical knowledge of CFR and the other without it. In case of statistical knowledge's absence, Least Square (LS) estimate can be used. The following equation shows how this method works [13].

$$\hat{\mathbf{H}}_{\mathrm{LS}} = \arg\min \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 \tag{2.5}$$

Considering a Gaussian noise effecting the received signal and CFR as a vector form, this technique is the same as Maximum Likelihood (ML) technique [32, 33]. Although this approach does not require any knowledge about the channel statistics it is not good enough to estimate the channel and that is because of two main reasons. Firstly, as the number of users increases in a cellular network, channel knowledge becomes essential for the system. Secondly, in urban areas, channel fluctuate is more rapid because of the existence of buildings and essence of all the attenuations made by surrounding objects. For the case that the statistical knowledge of CFR is known, estimation can be treated by a Linear Minimum Mean Square Error (LMMSE) which is for minimizing the $E\left\{\left\|\hat{\mathbf{H}} - \mathbf{H}\right\|^2\right\}$. Thus, the estimation of $\mathbf{H}$ with LMMSE can be obtained as [13]:

$$\hat{\mathbf{H}}_{\mathrm{LMMSE}} = \mathbf{R}_{\mathrm{H}} \left(\mathbf{R}_{\mathrm{H}} + \frac{1}{\gamma}\mathbf{I}\right)^{-1} \hat{\mathbf{H}}_{\mathrm{LS}} \tag{2.6}$$

where $\mathbf{R}_{\mathrm{H}} = \mathbf{H}\mathbf{H}^H$ is the correlation matrix of $\mathbf{H}$, and γ is SNR. Although, LMMSE is a better estimation method, it has a complexity due to matrix inversions.

The next approach is the PM base channel estimation method. This model is basically meant to estimate the characteristics of the channel response for instance path loss, channel gain and path delay. In order to find these parameters, signal processing techniques need to be applied and has been widely discussed in [28-30]. PM method highly improves the estimation.

The efficiency and reliability of the other method, which is called ICE, can also be achieved by PM estimation that is used for continues transmission. Hence, iterative channel estimation is more suitable for channel estimation. As every generation of cellular network gets more complicated, the channel estimation for that generation has to be developed/improved [13].

2.6.2 Channel Estimation for 4G MIMO-OFDM

OFDM uses multi sub-carrier signal to carry the data. It is called orthogonal because these sub-carriers are different from each other and each operates in different frequency bands. It is almost natural to combine OFDM with MIMO system because in MIMO system, a large number of array antennas are used and in that situation each antenna gets a different frequency range. Figure 2.5 illustrates OFDM system incorporated with MIMO.

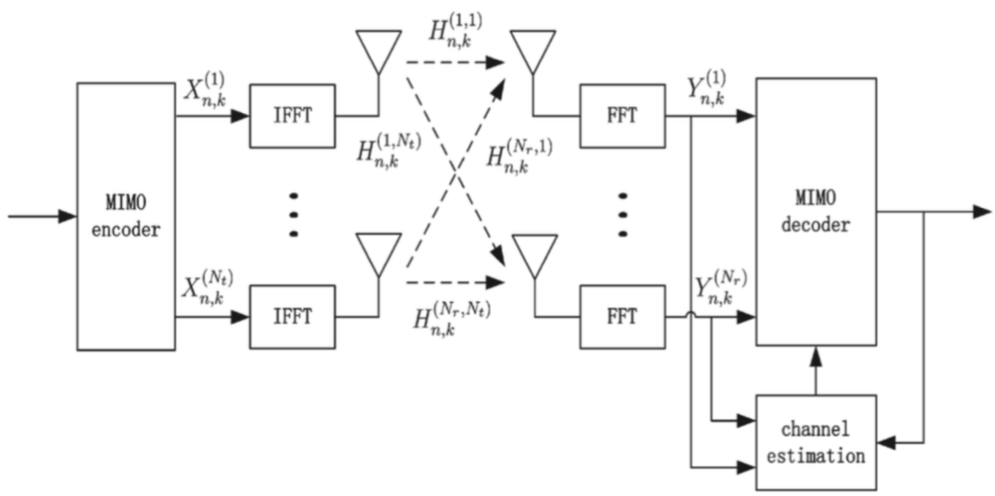

Figure 2.5: MIMO-OFDM system architecture [13]

received signal can be obtained as:

$$Y_{n,k}^{(j)} = \sum_{i=1}^{N_t} H_{n,k}^{(j,i)} X_{n,k}^{(i)} + n_{n,k}^{(j)} \quad \text{for } j = 1 \ldots N_r. \tag{2.7}$$

where N_t and N_r are the numbers of transmitter and receiver antenna respectively. $H_{n,k}^{(j,i)}$ represents the channel from the *i*-th transmitter antenna to the *j*-th receiver antenna and $X_{n,k}^{(i)}$ shows the *i*-th transmitter signal, the information signal from the *i*-th antenna. There are two approaches for estimating the channel in MIMO system. First one is the ordinary one which is sending the pilot symbols as the training symbols to see how the behavior of channel would be which also causes some problems. Such that, as there are arrays of antenna one antenna can be estimated in one instant of time while others should wait. Therefore, time consumption for estimating the channel for all the antennas along with unnecessary power usage. This process is where one-to-one transmission is happening, known as Single Input Single Output (SISO) OFDM. This method increases the training stage, thus reduces the spectral efficiency. Consequently, another method has been introduced, which will transmit pilot symbols simultaneously and leads to the following estimation.

$$\widehat{\mathbf{H}}_\mathrm{n} = \mathbf{Q}_\mathrm{n}^{-1} \mathbf{P}_\mathrm{n} \tag{2.8}$$

where

$$\widehat{\mathbf{H}}_\mathrm{n} = \left[h_{n,0}^{(i)}, h_{n,1}^{(i)}, \ldots, h_{n,L-1}^{(i)}\right]^T \tag{2.9}$$

$$\mathbf{P}_\mathrm{n} = \left[p_{n,0}^{(i)}, p_{n,1}^{(i)}, \ldots, p_{n,L-1}^{(i)}\right]^T \tag{2.10}$$

$$\mathbf{Q}_{\mathrm{n}} = \begin{bmatrix} \mathbf{Q}_{11} & \cdots & \mathbf{Q}_{1N_{\mathrm{t}}} \\ \vdots & \ddots & \vdots \\ \mathbf{Q}_{N_{\mathrm{r}}1} & \cdots & \mathbf{Q}_{N_{\mathrm{t}}N_{\mathrm{t}}} \end{bmatrix} \tag{2.11}$$

with

$$\mathbf{Q}_{(ij)} = \begin{bmatrix} q_{n0}^{(ij)} & \cdots & q_{n,-L+1}^{(ij)} \\ \vdots & \ddots & \vdots \\ q_{n,L-1}^{(ij)} & \cdots & q_{n,0}^{(ij)} \end{bmatrix} \tag{2.12}$$

with

$$q_{n,l}^{(ij)} = \sum_{k=0}^{K-1} X_{n,k}^{(j)} X_{n,k}^{(i)*} e^{j\frac{2\pi kl}{K}} \tag{2.13}$$

and finally

$$p_{n,l}^{(i)} = \sum_{k=0}^{K-1} Y_{n,k} X_{n,k}^{(i)*} e^{j\frac{2\pi kl}{K}}. \tag{2.14}$$

Channel estimation in MIMO system can be represented as a vector that corresponding value of each antenna has been obtained by the above formulas for each instance of time [14]. K is the number of paths available for the received antenna and L is the number of scattering paths, received by the receiver antenna. Finally, an observation from the complicated matrixes related to received antenna will represent the different aggregate path with unlike phases which will generate **Q**. Consequently, the estimated channel vector with having independent corresponding value for *i*-th antenna at transmitter will affect each symbol differently as they go through transmitting antennas.

2.7 Basic Criterial for 5G Channel Estimation

Apart from the traditional Channel Estimation (CE), 5G has to be equipped with the new technologies. One of the new techniques for decreasing the inter-cell interference along with D2D communication and other cellular users is precoding. Precoding simply means to give some weigh to the transmitting symbols to reduce the corruption of communication channel. As mentioned in (2.2), the transmitted signal passes through the channel and AWGN is added. In many cases, channel estimation through feedback could be time consuming and costly. Therefore, the transmitter should guess the incoming channel which is going to be done by the so-called method, precoding.

Now let us predict the channel and call it $\widehat{\mathbf{H}}$, then the information will be coded by this prediction as $\frac{\mathbf{x}}{\widehat{\mathbf{H}}}$. Resultantly, the received signal will be as follows.

$$\mathbf{Y} = \left(\frac{\mathbf{H}}{\widehat{\mathbf{H}}}\right)\mathbf{x} + \mathbf{n} \tag{2.15}$$

If the prediction is perfect, then $\mathbf{H} = \widehat{\mathbf{H}}$ and the effect of communication channel will be entirely removed and the only term which is left is the random Gaussian noise. The reason of it being called precoding is that a preprocessing technique performs transmission diversity and acts like an equalizer. In the next chapter, channel estimation for 5G cellular network will be studied in more details.

2.7.1 Antenna Beamforming

In MIMO system, transmitter sends arrays of streams of data with different direction of beamforming and estimates the channel. This procedure makes it so challenging to detect all corrupted received signals in different angles and moreover to find different behavior for different signals which leads to different values of channel. To make it more sensible, massive MIMO comes with its own challenges as much more antennas are used. As a result of more implemented antenna, more data will be sent simultaneously, and therefore, a lot of interference will be involved. The interference brings a new technology called beamforming to solve the problem. The beamforming means broadcasting the data not all over the medium but in specific directions and to specific users. This procedure is more efficient and also solves the interference problem between users. 5G can benefited from beamforming considering 5G actually uses these beamformer matrices to estimate the channel.

The working mechanism of beamforming can be explained as follows. If a handset in a cluster of cellular network tries to make a connection to the network a signal travel towards BS in order to make a connection. Through its way, signal collides with surrounding buildings and it gets different shapes and crosses with different users in the area. A massive MIMO BS receives all this data and keeps tracks of timing and directions of their arrival and then it uses signal processing algorithm to triangulate where exactly each signal is coming from, plots the best transmission direction and sends it back to the channel to each phone and the result is a coherent signal stream only transmitted to the desired user. That is why there is different measurement for different users and more complexity in estimation. Next two figures show different scenarios for channel estimation. First one performs with MIMO but without beam-forming and second one has massive MIMO with the use of beamforming.

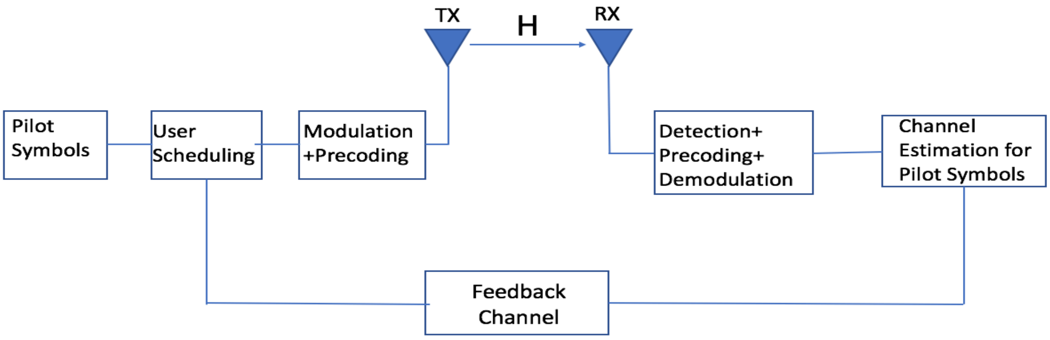

Figure 2.6: Channel estimation at one instance of time without considering beamforming for each user

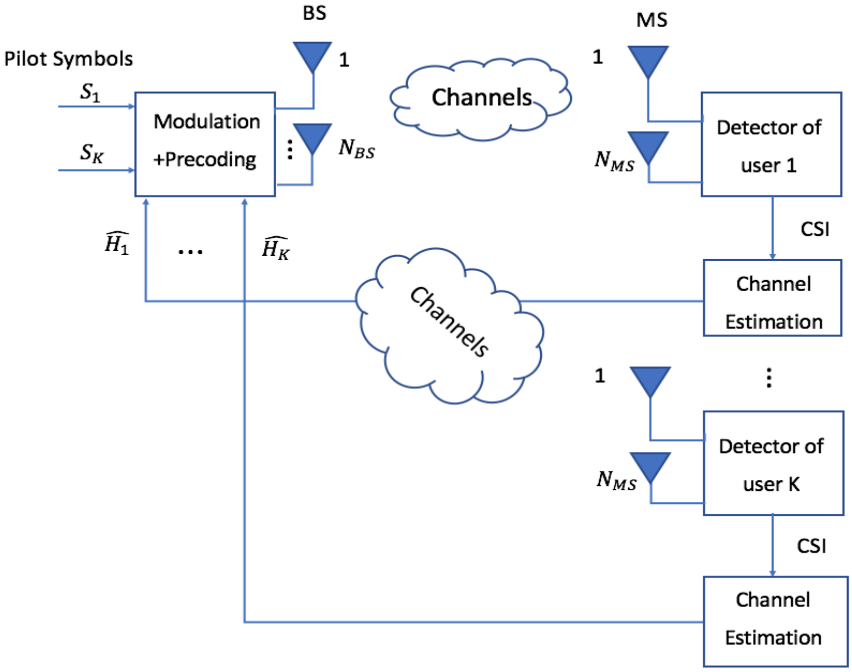

Figure 2.7: Channel estimation for multi-user with massive MIMO and beamforming

Chapter 3

CHANNEL ESTIMATION FOR MILLIMETER WAVE CELLULAR SYSTEM

3.1 Introduction

This chapter discusses the channel estimation for 5G cellular networks. Briefly speaking this chapter is all about the precoding matrices, channel model and channel estimation for 5G system. Firstly, representation of analog and digital precoding is discussed and then how the precoding matrices are going to work in 5G to reduce the channel effect and interferences are presented. Secondly, channel model will be designed by assigning different angles of departure at BS and different angles of arrival at MS along with path loss and processing gain with different number of scattering. At last, channel estimation by assuming the designed channel model for the system with mainly training the beamforming pattern will be accomplished later on.

3.2 Precoding Models

In wireless communication due to multipath propagation there could be more than one copy of the signal reaching to the receiver, resulting in Inter-Symbol Interference (ISI). One of the solutions for that is to insert guard intervals between transmitted symbols where the guard bands should be larger than the delay spread of the propagation channel. Figure 3.1, shows the effect of multipath propagation where the transmitted signal has copies of itself at receiver.

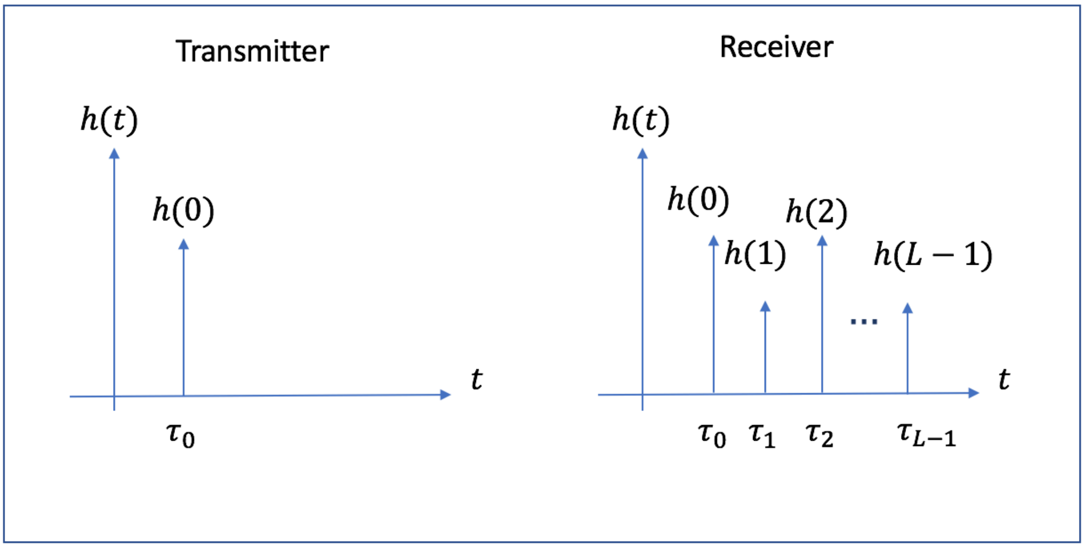

Figure 3.1: Discrete time impulse response model of multipath channel

Multipath effect in time domain results in frequency selective fading model in frequency domain. Therefore, signal occupies the whole bandwidth and results in decreasing performance in communication system. To solve this issue, signal with narrow bandwidth is transmitted where in narrow band communication system, such as IS-95 (CDMA), the signal bandwidth is much less than the channel coherent bandwidth. Hence, the narrow bandwidth experiences much lower frequency selective fading. However, narrow bandwidth implies to have much lower throughput. Therefore, in order to overcome multipath effect with higher throughput, channel equalization or precoding technique are used. The principal usage of precoding is, first the transmitter should know the CSI, then transmitted signal can be designed such that ISI in receiver is greatly mitigated.

3.2.1 Digital Precoding

Digital precoding deals with symbols in the complex field. One of the ways to solve the multipath fading is to use OFDM. If it is assumed that a signal block of $\mathbf{x}$ with a length of N_S to be transmitted through a multipath fading channel which is in order L, then the transmitted signal can be modeled as below [15, 16].

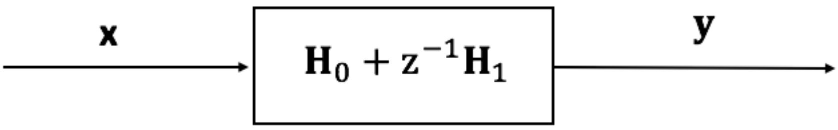

Figure 3.2: System model for multipath fading

whereas

$$\mathbf{H}_0 = \begin{bmatrix} h(0) & 0 & 0 & \dots & 0 \\ \vdots & h(0) & 0 & \dots & 0 \\ h(L-1) & \dots & \ddots & \dots & \vdots \\ \vdots & \ddots & \dots & \ddots & 0 \\ 0 & \dots & h(L-1) & \dots & h(0) \end{bmatrix} \tag{3.1}$$

$$\mathbf{H}_1 = \begin{bmatrix} 0 & \dots & h(L-1) & \dots & h(1) \\ 0 & \ddots & 0 & \ddots & \vdots \\ 0 & \dots & \ddots & \dots & h(L-1) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \dots & 0 \end{bmatrix} \tag{3.2}$$

These are called channel matrices necessary to remove the effect of the ISI. $\mathbf{H}_0$ is a lower triangular matrix and $\mathbf{H}_1$ is an upper triangular matrix with zero diagonal elements. $\mathbf{H}_1$ is applied when previous data streams are existed in the medium, due to time delay as a result of scattering. Therefore, it is concluded that those streams of data are dispensable and valueless. In that case path becomes 1, $l = 1$. In Figure 3.1, corresponding elements has been shown. Then for the i-th signal we have

$$\mathbf{y}_i = \mathbf{H}_0\mathbf{x}_i + \mathbf{H}_1\mathbf{x}_{i-1} \tag{3.3}$$

where

$$\mathbf{x}_i = [\mathrm{x}_{i,1}\ \ \mathrm{x}_{i,2} \dots \mathrm{x}_{i,p}]^T \tag{3.4}$$

$$\mathbf{y}_i = [\mathrm{y}_{i,1}\ \ \mathrm{y}_{i,2} \cdots \mathrm{y}_{i,p}]^T \tag{3.5}$$

$\mathbf{H}_1\mathbf{x}_{i-1}$ is known as Inter-Block Interference (IBI). Then if l is not equal to 1, $\mathbf{x}$ should be designed such that it leads us to no interference. That is why it is called precoding [17, 18]. In Figure 3.3 reduction of IBI and ISI is shown.

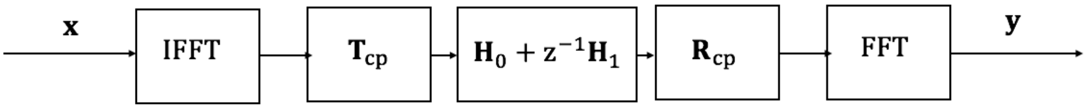

Figure 3.3: OFDM signal model and precoding design

These blocks contain Fast Fourier Transform (FFT), Inverse Fast Fourier Transform (IFFT), $\mathbf{T}_{\mathrm{cp}}$ and $\mathbf{R}_{\mathrm{cp}}$. $\mathbf{T}_{\mathrm{cp}}$ is a matrix that can paste two of last bits of signal to the top of it and $\mathbf{R}_{\mathrm{cp}}$ is the opposite action of $\mathbf{T}_{\mathrm{cp}}$. By assuming a signal with having $\mathbf{x} = [x_1\, x_2\ \ldots\ x_N]$ elements and receiving with L numbers of delays (as shown in Figure 3.1) then $\mathbf{T}_{\mathrm{cp}}$ and $\mathbf{R}_{\mathrm{cp}}$ can be written as:

$$\begin{bmatrix} x_{N_S-1} \\ x_{N_S} \\ x_1 \\ x_2 \\ \vdots \\ x_{N_S} \end{bmatrix} = \begin{bmatrix} 0_{L\times(N_S-L)} & I_L \\ & I_{N_S} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_S} \end{bmatrix} \tag{3.6}$$

where

$$\mathbf{T}_{\mathrm{cp}} = \begin{bmatrix} 0_{L\times(N_S-L)} & I_L \\ & I_{N_S} \end{bmatrix} \tag{3.7}$$

then $\mathbf{R}_{\mathrm{cp}}$ is reverse operation of $\mathbf{T}_{\mathrm{cp}}$ and that is

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_S} \end{bmatrix} = [0_{N_S \times L} \quad I_L] \begin{bmatrix} x_{N_S-1} \\ x_{N_S} \\ x_1 \\ x_2 \\ \vdots \\ x_{N_S} \end{bmatrix} \tag{3.8}$$

where,

$$\mathbf{R}_{\mathrm{cp}} = [0_{N_S \times L} \quad I_L] \tag{3.9}$$

then according to Figure 3.3 we can sum all of them as

$$\mathbf{y}_i = \mathbf{R}_{\mathrm{cp}} \mathbf{H}_0 \mathbf{T}_{\mathrm{cp}} \mathbf{x}_i + \mathbf{R}_{\mathrm{cp}} \mathbf{H}_1 \mathbf{T}_{\mathrm{cp}} \mathbf{x}_{i-1}. \tag{3.10}$$

Then we can finalize that IBI is removed by cyclic prefix as

$$\mathbf{R}_{\mathrm{cp}} \mathbf{H}_1 = 0. \tag{3.11}$$

Furthermore, to interpret the resulting diagonal matrix, the circular matrix can be calculated as

$$\mathbf{R}_{\mathrm{cp}} \mathbf{H}_0 \mathbf{T}_{\mathrm{cp}} = \mathbf{H}_0^{\mathrm{c}}. \tag{3.12}$$

Then, by calculating the circular matrix we finally can interpret the resulting diagonal matrix. Consequently, we can write down $\mathbf{H}_0^{\mathrm{c}} \mathbf{x}_i$ as a Fourier transform ($\mathcal{F}$) and invers-Fourier transform ($\mathcal{F}^{-1}$). The resulting received signal is removed from ISI and has been successfully pre-coded.

$$\mathbf{y}_i = \mathcal{F}\{\mathbf{H}_0^{\mathrm{c}}\} \mathcal{F}^{-1}\{\boldsymbol{x}_i\}. \tag{3.13}$$

Then, finally

$$\mathbf{y}_i = \mathbf{\Lambda}_{\mathrm{H}}\mathbf{x}_i \tag{3.14}$$

where $\mathbf{\Lambda}_{\mathrm{H}} = \mathrm{diag}\{\mathbf{H}_k\}_0^{Ns-1}$ and $\mathbf{H}_k = \sum_{l=0}^{L-1} h(l)\mathrm{e}^{-\mathrm{j}2\pi kl/Ns}$. Equivalently,

$$\mathrm{y}_{i,k} = \mathbf{H}_k x_{i,k} \tag{3.15}$$

where k = 1, …, N. Equation (3.15) shows that channel is chopped into k pieces and k number of signal can be transmitted simultaneously into the channel without ISI and IBI.

The procedure observed above is the presentation to simulate a digital precoding and how to detect the best symbols and bits of signal to minimize the ISI. This usually works in many wireless communication systems which are simple in interpretation just as current technologies but the new features of 5G system leave us no choice to investigate a new technology for precoding and that is called analog precoding.

3.2.2 Analog Precoding

As massive MIMO present in the system and there are arrays of antennas and RF chains. There has to be an organized way of switching and generating a beam former. Here is where analog beamformer or precoder appears. Analog precoder takes track of how the signal travels and where it travels to. As this technology has a significant advantage which is sending the signal to the desired user not all over the medium, it also has a disadvantage, which only single stream MIMO is possible. Thus, multi user MIMO cannot be supported by single use of analog precoder.

5G systems work with both digital and analog of the precoders and that is due to conquering the problem for ISI with digital precoding and to find the best beamformer with analog precoding. Thus, here is where the hybrid precoding originated from.

3.2.3 Hybrid Precoding

Hybrid precoding is the combination of digital and analog precoding. That means, there is a baseband precoding for the incoming streams of data for reduction of ISI then, an analog precoder which is after the digital stream of data transferred to the analog by Digital-to-Analog Converter (DAC). This precoder makes some switches between the phase shifters which are controlled by RF chains and finally they make the directional beam in the medium to reach to its maximum at the location of the desired user. This process of coding is applied for both transmitter and receiver. This process enables the system to transmit parallel streams of data to different users, which was previously done only for one user in case of analog beamforming. Nevertheless, with the new hybrid precoder it is possible to transmit arrays of data to different users at the same time without having interference. In Figure 3.4, hybrid precoding with multi user reliance can be contemplated.

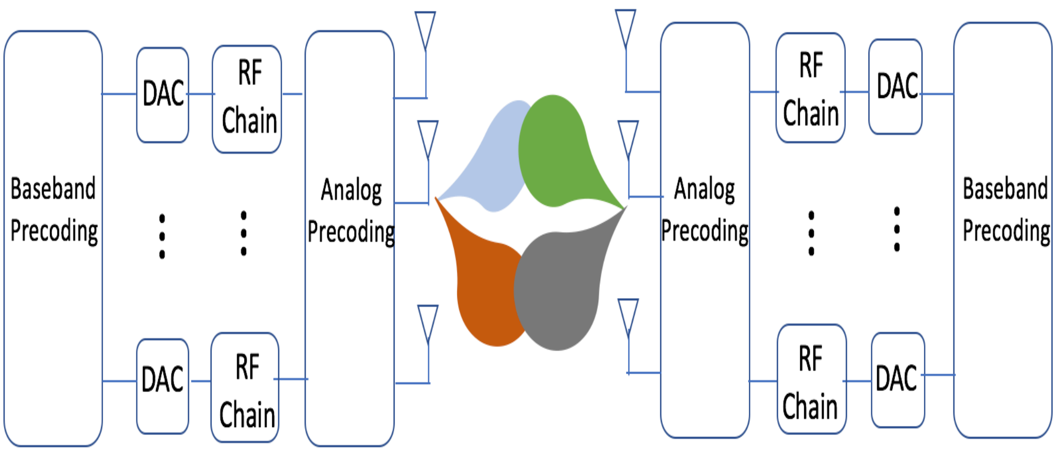

Figure 3.4: MIMO system at mm-wave with hybrid precoding

3.3 Basic Dictionary Learning Method

Dictionary Learning (DL) method is one of the techniques that helps to get a better result for channel estimation [34, 35]. DL aims to employ sparse modeling to compose efficient data representation by linearly combining few number of typical patterns of antennas which is learned from input data (pilot symbols). On the other hand, sparse matrices (or vectors) are the matrices that most of their elements are zero and it has value in the locations where information exists. DL is exploited in upcoming section where precoding matrices are going to be generated through training procedures.

3.4 Singular Value Decomposition

Singular Value Decomposition (SVD) is a factorization approach in order to decompose any $M\times N$ matrices as shown below:

$$\mathbf{H} = \mathbf{U\Sigma V}^{*} \tag{3.16}$$

where **U** is the real or complex unitary matrix which involves left eigenvectors, $\mathbf{\Sigma}$ represents a diagonal matrix with non-negative real or imaginary numbers which is known as a singular value (eigenvalue) of **H,** and **V** is unitary real or complex matrix contain right eigenvectors. SVD is one of the important tools in numerical linear algebra. In practice, SVD is used to (1) rank estimation for a matrix, (2) defining eigenvalue and eigenvector to determine the span of **H**, (3) solving unconstrained linear problems. For MIMO system, right and left eigenvectors can be characterized precoding vectors. Precoding matrices (**U** and **V**) are used for better elimination of channel effect [3, 8, 22, 27].

3.5 Array Processing

Array processing is all about the problems consist of noisy and random environment and deals with estimation theory and is considered as one of the signal processing tools [36, 37]. Estimation theory is to derive some meaningful signal's parameter from the measured data. Array structure can be defined as a set of sensors that are far away from each other spatially like antennas. Array processing is an ability to gather all the information from all the sensors (antennas) to deal with some specific estimation scenario. This technology is used in radar, anti-jamming and wireless communications. Array processing regarding for wireless communication deals with the direction of arrival or departure together with their signal wave form. For a decade, array processing is used to solve problems such as quality and performance of the wireless communication systems. This technique has lots of advantages such as speech enhancing, medical applications, smart antenna and beamforming. In 5G system, it is partially used for beamforming technique or to enhance the efficiency of the beams as the number of antennas increases at BS and MS (e.g. massive MIMO) [19-21].

3.6 5G Channel Model

5G model includes technologies such as mm-wave, massive MIMO, hybrid precoding, DL and array processing. Figure below illustrates 5G cellular network model.

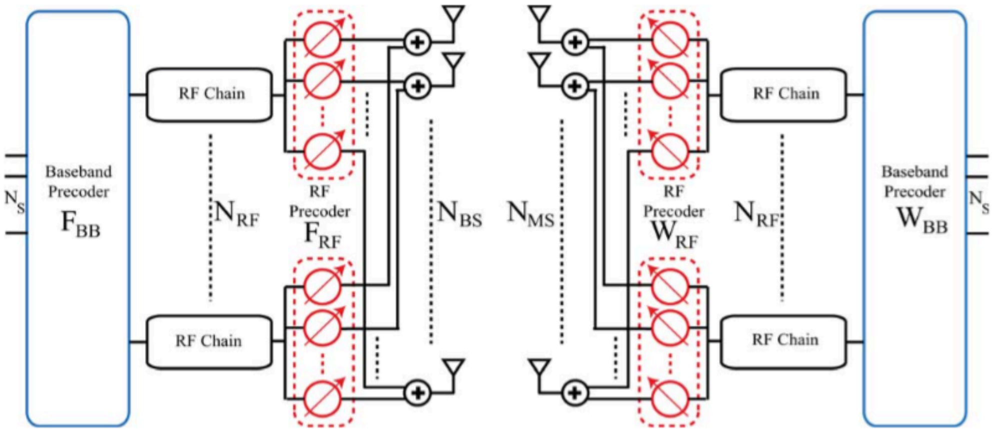

Figure 3.5: System architecture for 5G system [23]

N_s is the input data stream (voice, video or SMS) to the system, $\mathbf{F}_{BB}$ is the baseband precoder matrix which gives a gain to each symbol for defending themselves in essence of ISI. $\mathbf{F}_{RF}$ precoder chain finds the user and takes the beam former shape and radiates toward the specific user, then data reaches to the antenna array elements in order to sending the data through the medium ($\mathbf{H}$, is the channel between the sender and receiver). These processes are inversely done at the receiver to demodulate the original data and extract the best information out of the received signal. Before writing down the channel model for 5G cellular network, two important assumptions should be expressed. First, number of data stream and number of RF chains should be less than the number of antennas at both transmission and receiver such that $N_s < N_{RF} < N_{BS}$ and $N_s < N_{RF} < N_{MS}$. Commonly, number of BS antennas are always greater than the number of MS antennas $N_{MS} < N_{BS}$. Second, two mathematical expression should be kept in mind which are presented in below equations. These equations interpret the multiplication of digital and analog precoder at both transmitter and receiver, $\mathbf{F}_T$ and $\mathbf{W}_T$ respectively.

$$\mathbf{F}_T = \mathbf{F}_{RF}\mathbf{F}_{BB} \tag{3.17}$$

and

$$\mathbf{W}_{\mathrm{T}} = \mathbf{W}_{\mathrm{RF}}\mathbf{W}_{\mathrm{BB}} \tag{3.18}$$

After realizing the system components and expressions for denoting the channel, Channel model for single user communication without scattering can be written down as [23]:

$$\mathbf{H}_u = \sqrt{N_{BS}N_{MS}}\alpha_u \mathbf{a}_{\mathrm{MS}}(\theta_u)\mathbf{a}_{\mathrm{BS}}^H(\emptyset_u) \tag{3.19}$$

where α_u include the path loss and the processing gain. Then the antenna array response vectors, $\mathbf{a}_{\mathrm{MS}}(\theta_l)$ and $\mathbf{a}_{\mathrm{BS}}(\emptyset_l)$, can be written as:

$$\mathbf{a}_{\mathrm{BS}}(\emptyset_l) = \frac{1}{\sqrt{N_{\mathrm{BS}}}}[1, e^{\mathrm{j}\left(\frac{2\pi}{\lambda}\right)d\, sin(\emptyset_l)}, \dots, e^{\mathrm{j}(N_{\mathrm{BS}}-1)\left(\frac{2\pi}{\lambda}\right)d\, sin(\emptyset_l)}]^T \tag{3.20}$$

where λ is the wavelength of the signal, d is the distance between BS and MS. $\mathbf{a}_{\mathrm{MS}}(\theta_l)$ can be written in the same fashion.

Then for user with scattering, having L number of scatters, each scatter is a single propagation path between BS and MS. Then the same formula but with addition of one summation manifests the resulting model

$$\mathbf{H} = \sqrt{\frac{N_{\mathrm{BS}}N_{\mathrm{MS}}}{\rho}}\sum_{l=0}^{L} \alpha_l \mathbf{a}_{\mathrm{MS}}(\theta_l)\mathbf{a}_{\mathrm{BS}}^H(\emptyset_l) \tag{3.21}$$

where ρ denotes the path loss, α_l shows the complex gain of the l-th path where the amplitude of each path is assumed to be Rayleigh distributed. θ and $\emptyset$ are distributed between [0,2π] which are known as path's azimuth Angel of Departure (AoD) and Arrival (AoA) respectively. Thus, the channel can be written in more compact form as:

$$\mathbf{H} = \mathbf{A}_{\mathrm{MS}} \mathrm{diag}(\boldsymbol{\alpha}) \mathbf{A}_{\mathrm{BS}}^{H} \tag{3.22}$$

where

$$\boldsymbol{\alpha} = \sqrt{\frac{N_{\mathrm{BS}} N_{\mathrm{MS}}}{\rho}} [\alpha_1, \alpha_2, \dots, \alpha_L]^T \tag{3.23}$$

and

$$\mathbf{A}_{\mathrm{BS}} = [\mathbf{a}_{\mathrm{BS}}(\emptyset_1), \mathbf{a}_{\mathrm{BS}}(\emptyset_2), \dots, \mathbf{a}_{\mathrm{BS}}(\emptyset_L)] \tag{3.24}$$

$$\mathbf{A}_{\mathrm{MS}} = [\mathbf{a}_{\mathrm{MS}}(\theta_1), \mathbf{a}_{\mathrm{MS}}(\theta_2), \dots, \mathbf{a}_{\mathrm{MS}}(\theta_L)] \tag{3.25}$$

(3.24) and (3.25) carry BS and MS array response vectors. Here, it is assumed that both MS and BS have no priori knowledge of channel. Hence, a mm-wave channel estimation could be constructed according to these matrices. After the channel is modelled, a data stream is send to the medium then, the behavior of channel is observed through some training procedures. But before observing the channel, precoder matrices should be defined, basically channel paths AoA and AoD. Therefore, in the next section, precoding matrices are created.

3.7 Mathematical Approach for Hybrid Precoding

In this section, $\mathbf{F}_{\mathrm{T}}$ and $\mathbf{W}_{\mathrm{T}}$ which consist of both analog/digital precoders are constructed. In order to define these matrices, the following procedures should be traced. First, a zero matrix has to be defined that reacts to the received signal which are highly attenuated at that specific direction of arrival or departure. This matrix is called $\mathbf{G}$ matrix with a dimension of $N \times K$ where each column m containing 1's in the location where each antenna is at the best AoD range. Secondly, AoD ranges are initialized. Then after some period of time matrices for antennas at BS and MS are trained such that the best AoD or AoA is estimated then $\mathbf{F}_{\mathrm{T}}$ can be constructed. Therefore, $\mathbf{G}$ matrix is used to construct the ideal training beamforming matrices later after finding the best values of matrix $\mathbf{G}$. Precoding matrices are found by formula (3.26).

$$\mathbf{F}_{\mathrm{T}} = \mathrm{C_s}(\mathbf{A}_{\mathrm{BS,D}}\mathbf{A}_{\mathrm{BS,D}}^{H})^{-1}\mathbf{A}_{\mathrm{BS,D}}\mathbf{G} \tag{3.26}$$

where $\mathrm{C_s}$ is the normalization constant, $\mathbf{A}_{\mathrm{BS,D}}$ is the beam steering vectors, it is as same as in (3.24) but with the exception of having itself being updated for finding the best direction of angles. Further on, with using the method in [23] it can be normalized by having the number of RF chains, number of antennas and number of phase shifters, then additionally digital precoding and RF beamforming precoding can be differentiated. The same procedure is done for designing the combiners $\mathbf{W}_{\mathrm{T}}$ at receiver.

3.8 Channel Estimation for 5G Cellular Networks

Considering MIMO system, mm-wave and channel model described in previous sections, it is time to estimate the channel which means estimating different parameters of channel path such as AoA, AoD and the gain of each path. To measure these values a training procedure needs to be followed. By sending a known signal through the channel, the received signal can be written as:

$$\mathbf{r} = \mathbf{H}\mathbf{F}_{\mathrm{T}}\mathbf{s} + \mathbf{n} \tag{3.27}$$

where $\mathbf{H}$ is the channel representing mm-wave, $\mathbf{n}$ is a white Gaussian noise corrupting the transmitted signal, $\mathbf{r}$ is the received signal just before applying the combiners and $\mathbf{s}$ is the transmitted bits. Then $\mathbf{y}$ can be obtained by multiplying the received signal with the precoding matrices or combiners at the receiver.

$$\mathbf{y} = \mathbf{W}_{\mathrm{T}}^{H}\mathbf{H}\mathbf{F}_{\mathrm{T}}\mathbf{s} + \mathbf{W}_{\mathrm{T}}^{H}\mathbf{n}. \tag{3.28}$$

Now, for formulating mm-wave channel estimation, it is assumed that all the transmitted symbols are equal as $\mathbf{s} = \sqrt{\mathrm{P}}\mathbf{I}_{M_{\mathrm{BS}}}$. After applying $\mathbf{s}$ into the above formula, $\mathbf{Y}$ can be rewritten as observed in [23] such that:

$$\mathbf{Y} = \sqrt{\mathrm{P}}\mathbf{W}^{H}\mathbf{H}\mathbf{F} + \mathbf{Q} \tag{3.29}$$

where $\mathbf{Q}$ is the multiplication of the combiner with the noise.

To do the estimation process, $\mathbf{Y}$ should be investigated such that sparse representation of the channel could be defined. Sparse representation means that estimation deals mostly with beam pattern and also for simplification of matrices to have less elements in the matrices. Then vectorized received signal is illustrated as follows:

$$\mathbf{y}_{\mathrm{v}} = \sqrt{\mathrm{P}}(\mathbf{F}^{T} \otimes \mathbf{W}^{H})\,(\mathbf{A}_{\mathrm{BS}}^{*} \circ \mathbf{A}_{\mathrm{MS}})\,\alpha + \mathbf{n}_{\mathrm{Q}} \tag{3.30}$$

where α is the path gain for each different corresponding path. Then, $(\mathbf{A}_{\mathrm{BS}}^{*} \circ \mathbf{A}_{\mathrm{MS}})$, is matrix with a dimension of $N_{\mathrm{BS}}N_{\mathrm{MS}} \times L$ in which each column has a form of

$(\boldsymbol{a}_{\mathrm{BS}}^{*}(\phi_l) \otimes \boldsymbol{a}_{\mathrm{MS}}(\theta_l))$, $l = 1, 2, \ldots, L$. (3.30) implies the product of array response vectors of transmitter and receiver which are associated with AoA/AoD of the l-th path of the scattered signal caused by the medium. It is possible to write down (3.30) as a compact dictionary matrix:

$$\mathbf{A}_{\mathrm{D}} = (\mathbf{a}_{\mathrm{BS}}^{*}(\phi_l) \otimes \mathbf{a}_{\mathrm{MS}}(\theta_l)) \tag{3.31}$$

then, we have

$$\mathbf{y}_{\mathrm{v}} = \sqrt{\mathrm{P}}(\mathbf{F}^{T} \otimes \mathbf{W}^{H})\mathbf{A}_{\mathrm{D}}\alpha + \mathbf{n}_{\mathrm{Q}} \tag{3.32}$$

The above vectorized received signal $\mathbf{y}_{\mathrm{v}}$ shows the sparse representation of the channel estimation. Vectoring received signal makes the process easier to aid the processing of adaptation. Finally, by trained angles, estimation of channel is possible [23].

$$\mathbf{y}_{(1)} = \sqrt{\mathrm{P}_{(1)}}\left(\mathbf{F}_{(1)}^{T}\mathbf{A}_{\mathrm{BS,D}}^{*} \otimes \mathbf{W}_{(1)}^{H}\mathbf{A}_{\mathrm{MS,D}}\right)\alpha + \mathbf{n}_{1} \tag{3.33}$$

$$\mathbf{y}_{(2)} = \sqrt{\mathrm{P}_{(2)}}\left(\mathbf{F}_{(2)}^{T}\mathbf{A}_{\mathbf{BS,D}}^{*} \otimes \mathbf{W}_{(2)}^{H}\mathbf{A}_{\mathrm{MS,D}}\right)\alpha + \mathbf{n}_{2} \tag{3.34}$$

$$\vdots$$

$$\mathbf{y}_{(\mathrm{S})} = \sqrt{\mathrm{P}_{(\mathrm{S})}}\left(\mathbf{F}_{(\mathrm{S})}^{T}\mathbf{A}_{\mathrm{BS,D}}^{*} \otimes \mathbf{W}_{(\mathrm{S})}^{H}\mathbf{A}_{\mathrm{MS,D}}\right)\alpha + \mathbf{n}_{\mathrm{S}} \tag{3.35}$$

This adaptive algorithm helps to make a better AoA and AoD which at the end adjusts the direction of the next stage. Firstly, by initializing some angles for AoA and AoD then finding the **G** matrix which is a sparse vector with non-zero elements in the locations that corresponds to the angles of arrival and departure. Then angles are updated and the best **G** matrix is found. Eventually, from the **G** matrix the best precoding matrices ($\mathbf{F}_{\mathrm{T}}$) are constructed by (3.26). Afterward, real data stream is sent through the channel with the channel model in (3.21). From the received signal, it is possible to find the best AoAs/AoDs, which leads to estimation of the channel as in [23]:

$$\widehat{\mathbf{H}} = \widehat{\mathbf{A}}_{\mathrm{MS}} \, \mathbf{diag}(\widehat{\boldsymbol{\alpha}}) \widehat{\mathbf{A}}_{\mathrm{BS}}^{H} \tag{3.36}$$

After estimation of the channel, optimal precoding matrices are calculated by SVD of estimated channel matrix ($\widehat{\mathbf{H}}$). To eliminate the channel effect, SVD is resulting to an optimal way of calculating $\mathbf{F}_{\mathrm{T}}$ and $\mathbf{W}_{\mathrm{T}}$. This process causes BER to have accurate received symbols which are less corrupted by the channel. Then, it is possible to measure and find out whether the estimation method is close enough to the actual channel. To do this lets first summarize the estimation method through the algorithm 1.

In algorithm 1, N and K are the number of resolution and number of beamforming vectors per stage. L is the number of actual channel paths and L_e is the estimated number of channel paths. At the initial stage both the BS and MS use to generate KL_e beamforming vectors. Then AoA and AoD are sliced in to sub-ranges each. Afterwards, according to the maximum power detected at the sliced section L_e can be defined. To fine maximum power, received signal is calculated through initiation of $\mathbf{F}_{\mathrm{T}}$ and $\mathbf{W}_{\mathrm{T}}$. Further, this process repeated to find the required resolution. First only one path is estimated, second this path is stored into the $\mathbf{U}^{\mathrm{BS}}$ and $\mathbf{U}^{\mathrm{MS}}$ to be used in later iterations, $\mathbf{U}$ matrix is generated to store the best values of AoA, AoD and path gain. This will continue till the iterative number of steps finishes. Then, this procedure is done for the next generated antenna lobe. Finally, for each stored path we can find the maximum power and consequently, the range of the quantized AoA/AoD and the path gain are defined. Finally, channel can be estimated through (3.36).

Algorithm 1 adaptive Estimation Algorithm for Multi-Path mm-wave Channel [23]

Input: BS and MS knows *N*, *K* and L_e

Initialization: $\mathbf{U}^{\mathrm{BS}}_{(1,1)} = [1, \ldots, 1]$, $\mathbf{U}^{\mathrm{MS}}_{(1,1)} = [1, \ldots, 1]$, $E = \log_K({}^{N}/_{L_e})$

for $k \leq L_e$ **do**

for $l \leq E$ **do**

for $m_{\mathrm{BS}} \leq KL_e$ **do**

BS precoder $\mathbf{F}_{(l,\mathbf{U}^{\mathrm{BS}}_{(k,l)})}$

for $m_{\mathrm{MS}} \leq KL_e$ **do**

MS combiner $\mathbf{W}_{(l,\mathbf{U}^{\mathrm{MS}}_{(k,l)})}$

Then

$y_{m_{\mathrm{BS}}} = \sqrt{P_l}\, \mathbf{W}_{(l,\mathbf{U}^{\mathrm{MS}}_{(k,l)})} \mathbf{H}\, \mathbf{F}_{(l,\mathbf{U}^{\mathrm{BS}}_{(k,l)})} + \mathbf{n}_{m_{\mathrm{BS}}}$

$y_{(l)} = [y_1^T, y_2^T, \ldots, y_K^T]^T$

for $p = 1 \leq k - 1$ **do**

let's define the g matrix for previous path contributions

$\mathbf{g} = \mathbf{F}^T_{(l,\mathbf{U}^{\mathrm{BS}}_{(p,l)})} [\mathbf{A}_{\mathrm{BS,D}}]^*_{:,\mathbf{U}^{\mathrm{BS}}_{(p,l)}(1)} \otimes \mathbf{W}^H_{(l,\mathbf{U}^{\mathrm{BS}}_{(p,l)})} [\mathbf{A}_{\mathrm{MS,D}}]_{:,\mathbf{U}^{\mathrm{MS}}_{(p,l)}(1)}$

$\mathbf{y}_{(l)} = \mathbf{y}_{(l)} - \mathbf{y}^H_{(l)} \mathbf{g}(\mathbf{g}^H \mathbf{g})\mathbf{g}$

$(m^*_{\mathrm{BS}}, m^*_{\mathrm{MS}}) = \arg\max[\mathbf{Y} \odot \mathbf{Y}^*]_{m_{\mathrm{MS}}, m_{\mathrm{MS}}}$

$\mathbf{U}^{\mathrm{BS}}_{(k,l+1)}(1) = K(m^\star_{\mathrm{BS}} - 1) + 1$

$\mathbf{U}^{\mathrm{MS}}_{(k,l+1)}(1) = K(m^\star_{\mathrm{MS}} - 1) + 1$

for $p = 1 \leq k - 1$ **do**

$\mathbf{U}^{\mathrm{BS}}_{(k,l+1)}(p) = \mathbf{U}^{\mathrm{BS}}_{(k,l+1)}(1)$

$\mathbf{U}^{\mathrm{MS}}_{(k,l+1)}(p) = \mathbf{U}^{\mathrm{MS}}_{(k,l+1)}(1)$

Using G matrix, we have

$\hat{\phi}_k = \bar{\phi}_{\mathbf{U}^{\mathrm{BS}}_{(k,E+1)}}(1)$, $\hat{\theta}_k = \bar{\theta}_{\mathbf{U}^{\mathrm{BS}}_{(k,E+1)}}(1)$, $\hat{\alpha}_k = \sqrt{{}^{\rho}/_{P_{(E)} G_{(E)}}}\, \mathbf{y}^H_{(\mathrm{E})}\, \mathbf{g}/\mathbf{g}^H \mathbf{g}$

3.9 Principle of Channel Capacity

Channel capacity emphasize the maximum limit which data stream can be transmitted through the medium. Mathematical approaches for calculating the channel capacity is introduced. To begin with, entropy is explained, as entropy is the basic knowledge and mathematical form of calculating the channel capacity. Later, in Chapter 4 all the obtained equations will be simulated which result to actual channel capacity in comparison with estimated channel capacity.

3.9.1 Entropy

Entropy is referred to uncertainty of the state which is a measurement of distributed random variables in a set or the average information content. For example, in a summer season whether broad cast announce the temperature as 20 degrees for a certain day. In this case the entropy is low as everyone expected to have that temperature but if the next day they announce the temperature as -2 degrees, this information has a high entropy value as no one is expecting that temperature. Entropy is one of the important measurement in communication system where there are random realizations and by guessing some variables behavior of the system, entropy can be concluded. Thus, it is possible to recover the information from the corrupted signal as it passes through the channel. Entropy is often shown as

$$E(X) = -\sum_{x \in X} p(x) \log_2 \, p(x) \tag{3.37}$$

where $E(X)$ is an entropy of a discrete random variable X with probability distribution $p(x)$.

Another measurement is the joint entropy of pair of two discrete random variables (X, Y),

$$E(X,Y) = -\sum_{x\in X}\sum_{y\in Y} p(x,y)\log_2 p(x,y) \tag{3.38}$$

where $p(x, y)$ is the joint probability distribution.

Now by knowing the joint entropy, it is conceivable to write down the conditional entropy

$$E(Y|X) = -\sum_{x\in X}\sum_{y\in Y} p(x,y)\log_2 p(y|x) \tag{3.39}$$

where

$$p(y|x) = \frac{p(x,y)}{p(x)} \tag{3.40}$$

3.9.2 Mutual Information

As entropy denotes the uncertainty about the channel input before observing the channel output, and the conditional entropy presents the uncertainty about the channel output after observing the output, mutual information is the uncertainty about the channel input that is resolved by the channel output. Mutual information between two random variables is a measure of mutual dependence between the two sets. Therefore, mutual information can be defined as:

$$I(X;Y) = \sum_{x\in X}\sum_{y\in Y} p(x,y)\log_2\frac{p(x,y)}{p(x)p(y)} \tag{3.41}$$

or can be expressed as subtraction between entropy and conditional entropy.

$$I(X;Y) = E(X) - E(X|Y) \tag{3.42}$$

3.9.3 Channel Capacity

After formulizing the mutual information, channel capacity of a communication system can be constructed. For the case when there is a single antenna at transmitter and receiver capacity can be defined as

$$C = \max\ \{ I(X;Y)\} \tag{3.43}$$

After solving (3.43) when there is a transmitted symbol and at the receiver, received symbols. Channel capacity is

$$C = \frac{1}{2}\log_2\left(1 + \frac{P}{N}\right) \tag{3.44}$$

where *P* is the transmitted signal power and *N* is the power of the Gaussian noise or noise variance. Equation (3.44) is more known as the Shannon Capacity and can be summarized as:

$$C = B\ \log_2(1 + \mathrm{SNR}) \tag{3.45}$$

where *B* is the bandwidth of the channel. For multi antenna at transmitter and receiver, the capacity can be obtained as:

$$C = \max \textstyle\sum_i \frac{1}{2}\log_2\left(1 + \frac{P_i}{N_i}\right) \tag{3.46}$$

3.9.4 Channel Capacity for MIMO System with CSIT

When CSI (channel matrix) is known at transmitter (CSIT), the channel capacity for MIMO system can be obtained as:

$$C = \max \log_2 \left|\mathbf{I}_N + \frac{1}{\sigma^2}\mathbf{H}\mathbf{H}^H\right| \tag{3.47}$$

where **I** is the identity matrix having *N* diagonal elements. *N* is the equal to the rank of $\mathbf{H}$ and $|\quad|$ is the determinant operation. Therefore, channel capacity for MIMO system is defined from (3.47) [24-26]. Where by using (3.47) channel capacity for both modeled channel and estimated channel can be obtained. Therefore, this calculation illustrates the performance of algorithm used for estimation. In the next chapter, simulation results to strengthen the performance algorithm will be presented.

Chapter 4

EXPERIMENTAL SETUP AND SIMULATION RESULTS

4.1 Introduction

This chapter is about the implementation details and the design of each mathematical approach that is illustrated in Chapter 3. Matlab 2017b platform is used to obtain the simulation results.

4.2 Details of Implementation

In order to simulate the system model presented in previous chapter with formulas there should be an organized way of doing it. Thus, a flow chart or a mind map is presented to demonstrate the implementation approaches in Figure 4.1. This flow chart consists of blocks where each block represents an action or an initialization. There is a basic path to start simulating a system. For instance, in wireless communication to model a channel, three main steps can construct the simulation:

- Presenting the initial values
- Main simulation block
- Representation of the resulting, calculated from the main block

In the Figure 4.1 the main steps to reach to the objective of the thesis can be viewed. Here, first, channel is modeled by setting some initial values such as number of antennas allocated at BS and MS, number of RF chains and number of phase shifters.

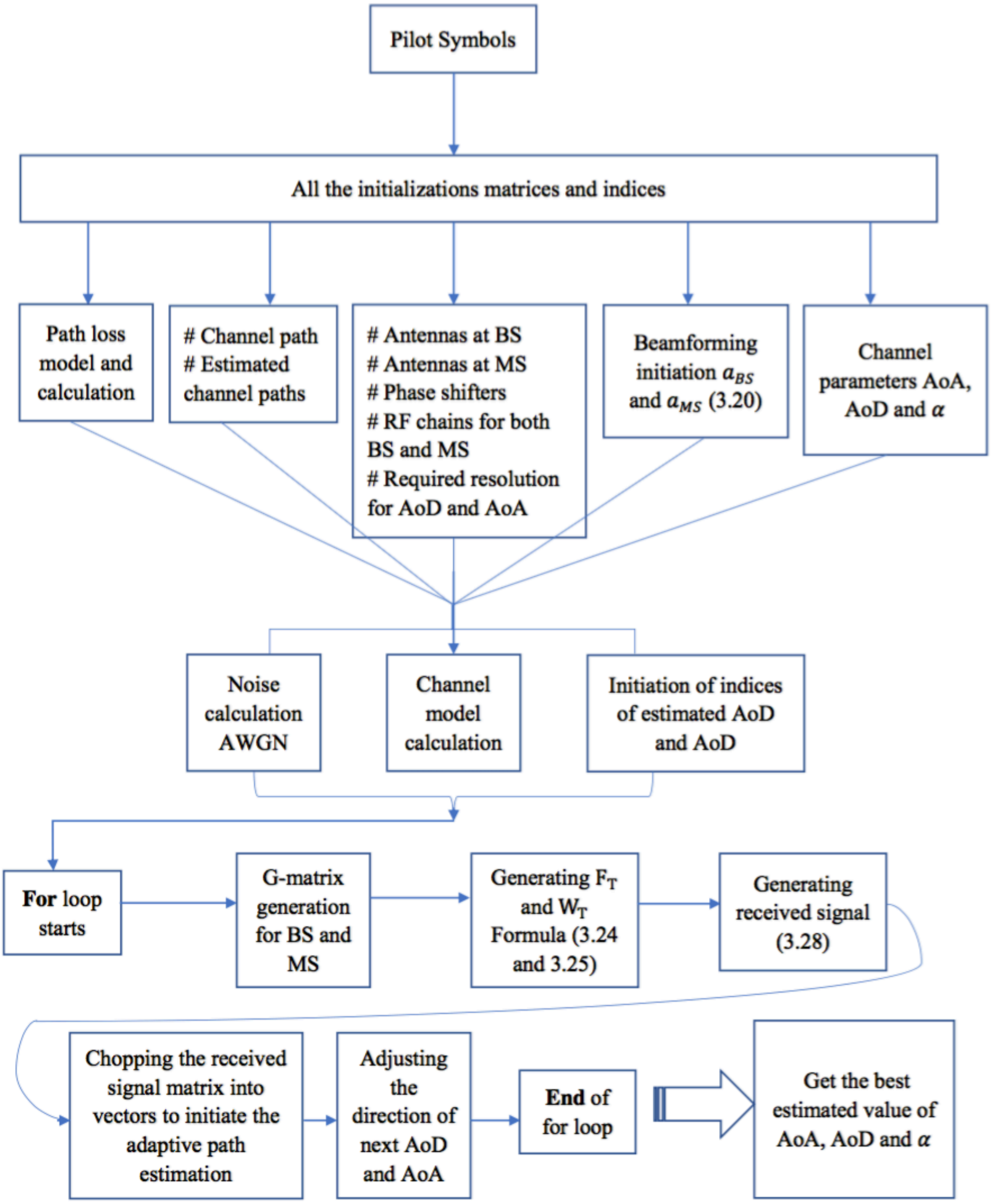

Figure 4.1: Flowchart for computing best angels of arrival and departure

As there are scattering in the medium channel paths are initialized for the number received signals. Later, as channel is modeled the main simulation block is initiated.

In the main simulation block channel estimation is measured. This simulation is through training procedures which appears in the mind map.

After estimation mechanism is done, pilot signals are sent through the system to observe the received signal. Then an iterative loop is presented to update AoA and AoD to accomplish the best estimation through some training steps.

Later, by accomplishing the best channel estimation further calculation can be detected. As discussed SVD of both estimated and actual channels are obtained to get the optimum precoding matrices. These precoding matrices are used later on to measure BER. BER is calculated with real case scenario such that real symbols have been sent to the medium with different modulations techniques to see how many bits get to the receiver with no errors. Later on, simulation and presentation of BER graphs will be stated. Below chart Figure 4.2 shows the final steps of simulation.

The goal in this thesis is to accomplish and observe the channel estimation and moreover to see how this method works in real life situation to observe BER and Mean Square Error (MSE). In the next section, observations and results of simulations are discussed.

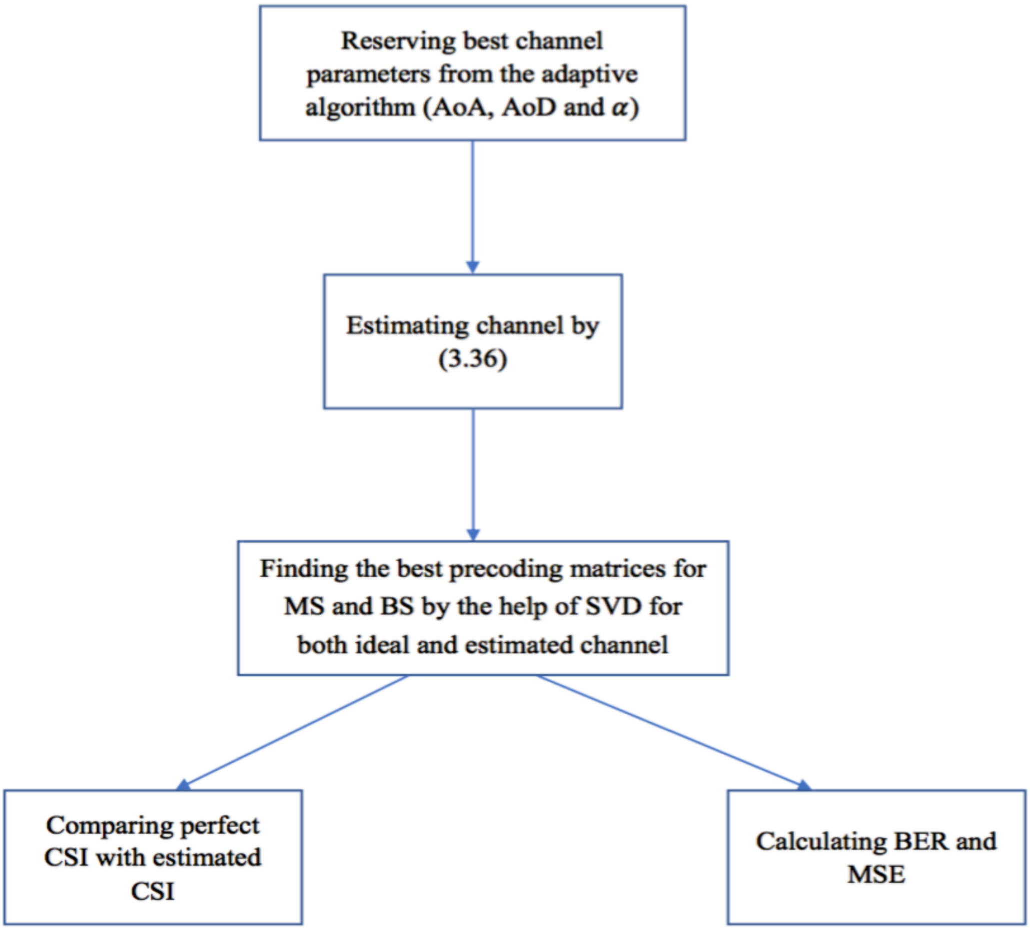

Figure 4.2: Flowchart for computing last steps of simulation, BER, MSE and comparison of CSI

4.3 Simulation Results

In this part, experimental results are illustrated. Firstly, it is assumed that the number of antennas at MS is half of the number of antennas at BS and that is because not many number of antenna can be installed in MS as there is size limitation. Secondly, a carrier frequency of 30 GHz is considered for mm-wave system. By calculating the power at transmitter and path loss, the antenna patterns at both BS and MS are observed as in Figures 4.3 and 4.4 according to (3.20).

It can be observed from the figures that antenna pattern consists of main lobe and back lobe. It is observed that there are two main beams one of them is observed as main and the other one is back lobe. The same pattern is valid for MS but with less power. These patterns are called AoA and AoD. These patterns are constructed to be the initial values for creating the channel as $\mathbf{A}_{\mathbf{MS}}$ and $\mathbf{A}_{\mathbf{BS}}$ as in (3.24) and (3.25).

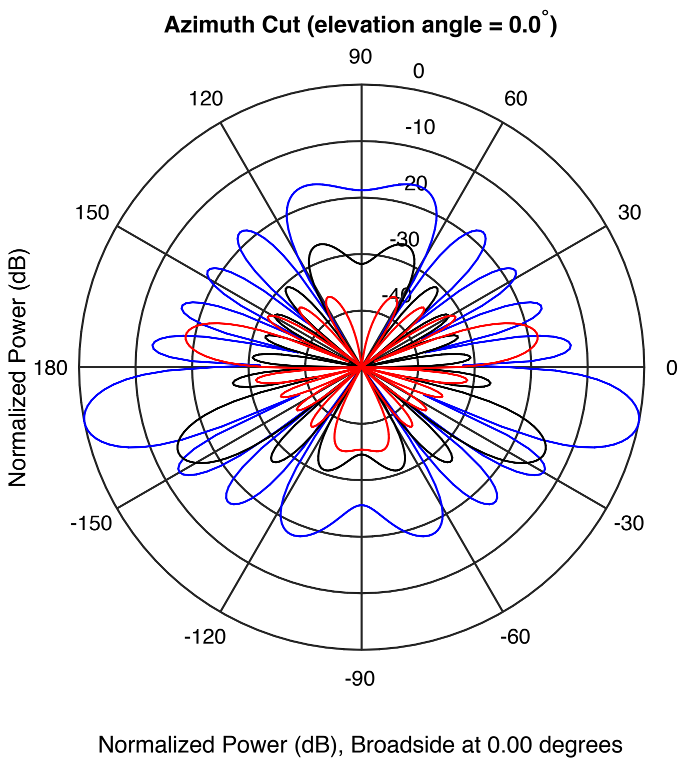

Figure 4.3: BS antenna pattern for 1 realization (f = 30 GHz and L = 3)

As it is realized there are two main differences between the radiated pattern of antenna for BS and MS. First, BS has narrower beams which is due to the frequency. As frequency increases the main lobe beam becomes narrower. Therefore, due to the fact that MS has a limited power and cannot radiate with higher frequency, MS has a wider lobe. Second, when directional antenna is used, side lobes and back lobe are gone. The reason behind seeing the back lobe and main lobe in Figures 4.3 and 4.4 is because omni-directional antenna with the horizontal elevation angle is used.

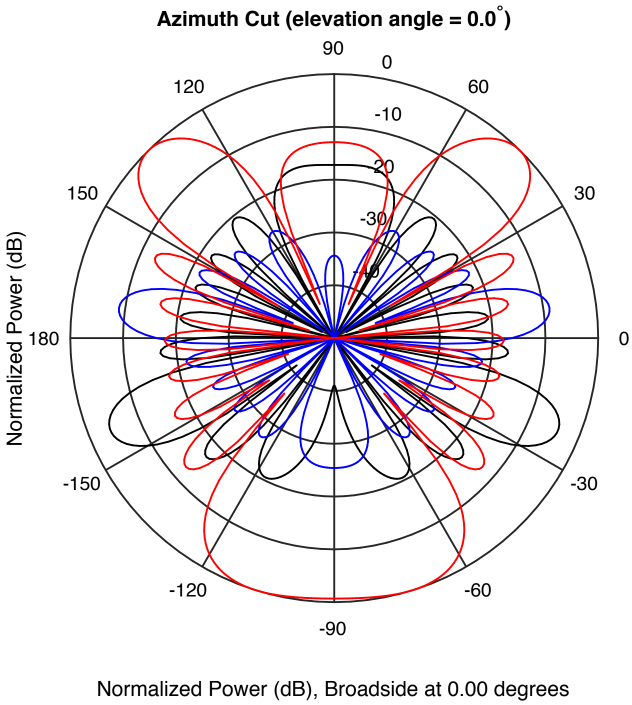

Figure 4.4: MS antenna pattern for 1 realization (f = 30 GHz and L = 3)

The aim to define antenna patters is to initialize AoA and AoD that later by training procedures are used for estimating these patterns for the best angels where they radiate with highest power to finally estimate the channel.

Figures 4.3 and 4.4 show the patterns for only one resolution, as the number of resolutions increases the probability of having better learning and training measurement increases, as stated in [23]. This paper has proven that 192 resolution samples are enough in terms of having less number of training but making the best

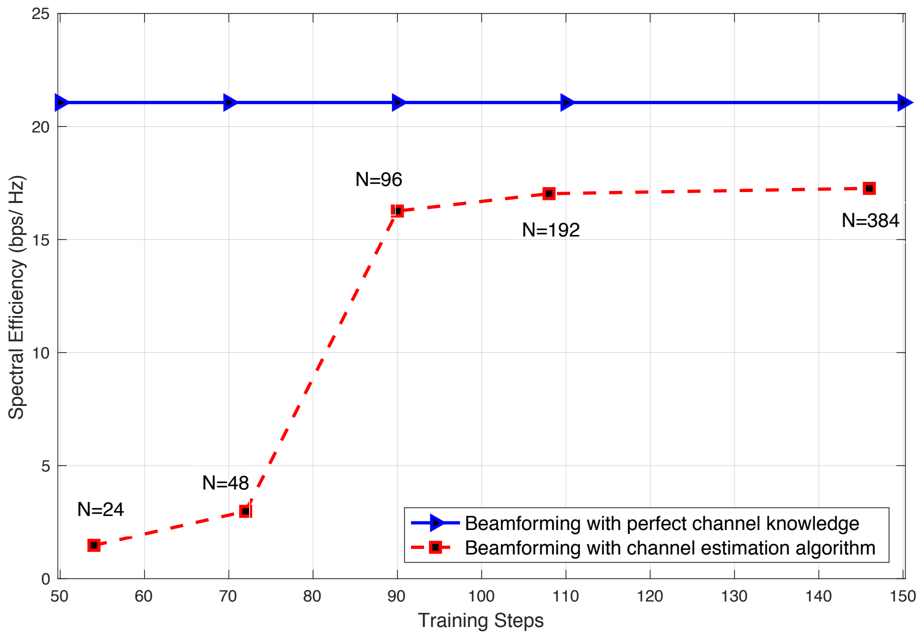

Figure 4.5: Improvement of the spectral efficiency with different resolution steps (SNR=0 dB, f = 30 GHz and L = 3)

beamforming matrices. Likewise, it is proved by simulations that 192 resolution samples are efficient, as presented in Figure 4.5.

More beamforming patterns results to realistic patterns. Thus, for both BS and MS different beam pattern with higher realization to generate the AoA and AoD is obtained but as the simulation for both is almost the same, only for BS is shown in Figure 4.6. Despite the fact that differentiating between the patterns as realization increases is impossible.

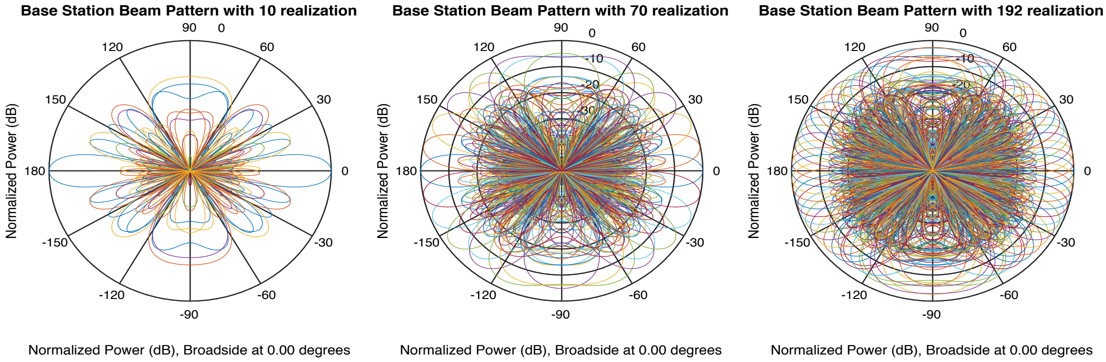

Figure 4.6: BS antenna pattern with different realization (*f* = 30 GHz)

It is concluded that 192 resolution parameter is enough for beamforming matrices to adapt themselves in the measurement of channel estimation.

Channel Impulse Responses (IR) is now introduced since channel is modeled and estimated through antenna pattern observed above. As the channel matrices are very large in dimension one realization of the channel is presented in Figure 4.7, where x-axis is time and y-axis is magnitude. As the channel is immense and there are high number of channel parameters, illustrating the whole channel matrix, results to have many components merging to each other's. Therefore, only one realizations are shown to make it more visible.

Received signal IR is measured and illustrated through Figure 4.8. The intention to get IR of the received signal is to later improve the estimation algorithm by vectorizing the received signal and train the transmitted symbols according to the received symbols corrupted by the channel.

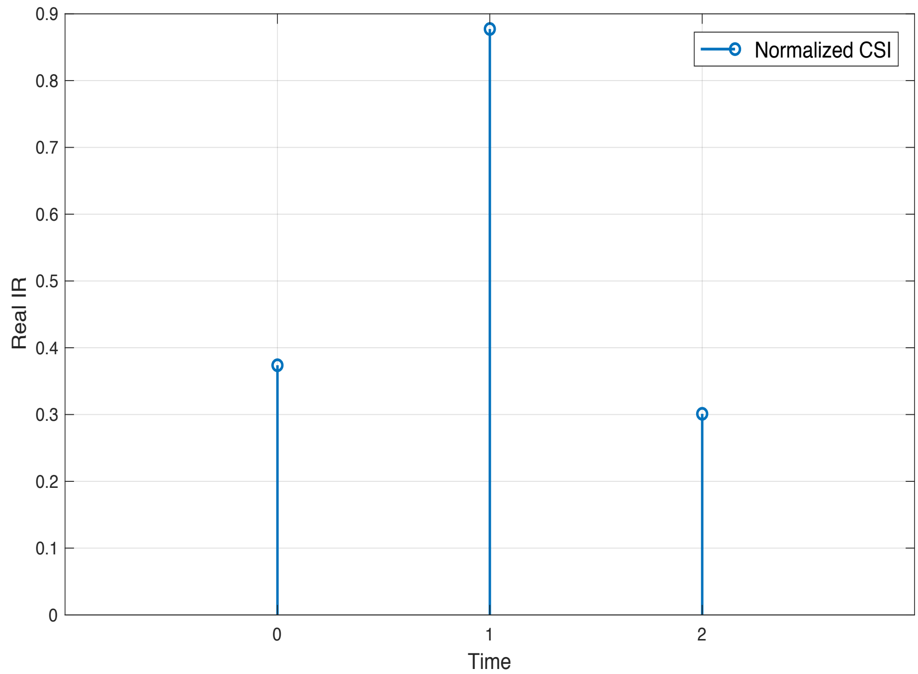

Figure 4.7: Magnitude of the normalized channel IR

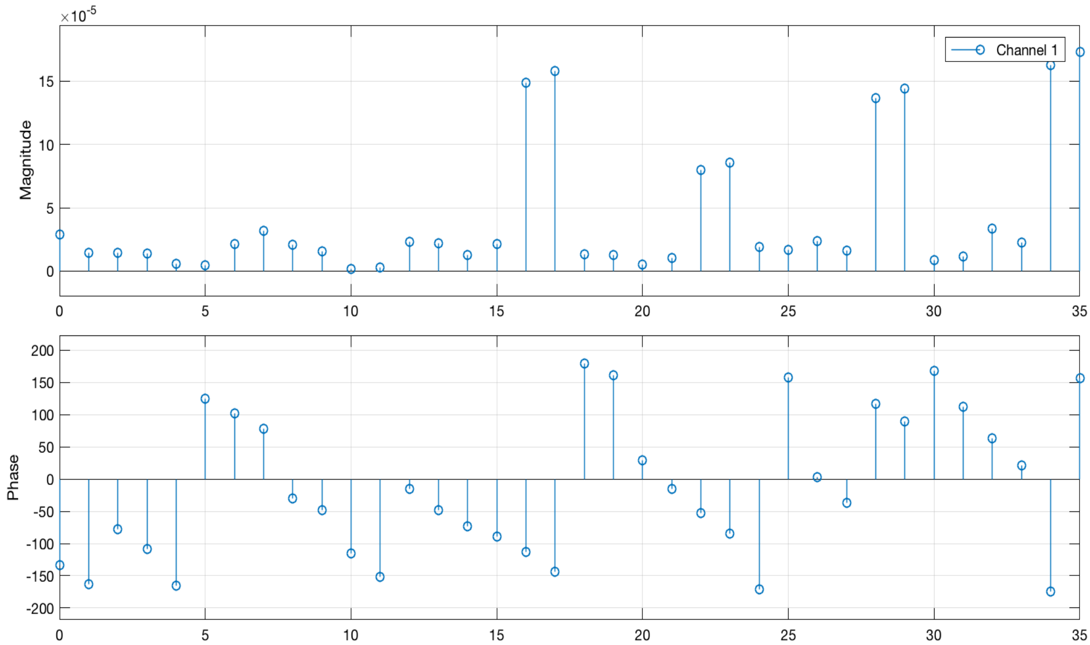

Figure 4.8: Impulse response of received signal

In Figure 4.8, magnitude and phase of the received signal in terms of direction and angel of each antenna pattern at receiver are shown.

Subsequently, channel capacity can be observed after estimation algorithm is perfectly performed. Channel capacity for 5G system can be observed through (3.47). Hybrid precoding for perfect channel knowledge is obtained by taking SVD of modeled channel and normalizing it, then processing it through hybrid precoding function to get the best precoders and combiners. Further on, hybrid precoding for estimated CSI is obtained in exactly same manner but with the difference of using channel estimation. Therefore, channel capacity is obtained and calculated. Figure 4.9 is the illustration of spectrum efficiency versus SNR through (4.1). In this graph SNR is lower than expectation and that is because of the bandwidth is high in mm-wave and this is without the array gain.

$$R = log_2 \left| I_{N_s} + \frac{P}{N_s} \mathbf{W}_{\mathrm{T}}^{H} \widehat{\mathbf{H}}\, \mathbf{F}_{\mathrm{T}} \left(\mathbf{W}_{\mathrm{T}}^{H} \widehat{\mathbf{H}}\, \mathbf{F}_{\mathrm{T}} \right)^{H} \right| \tag{4.1}$$

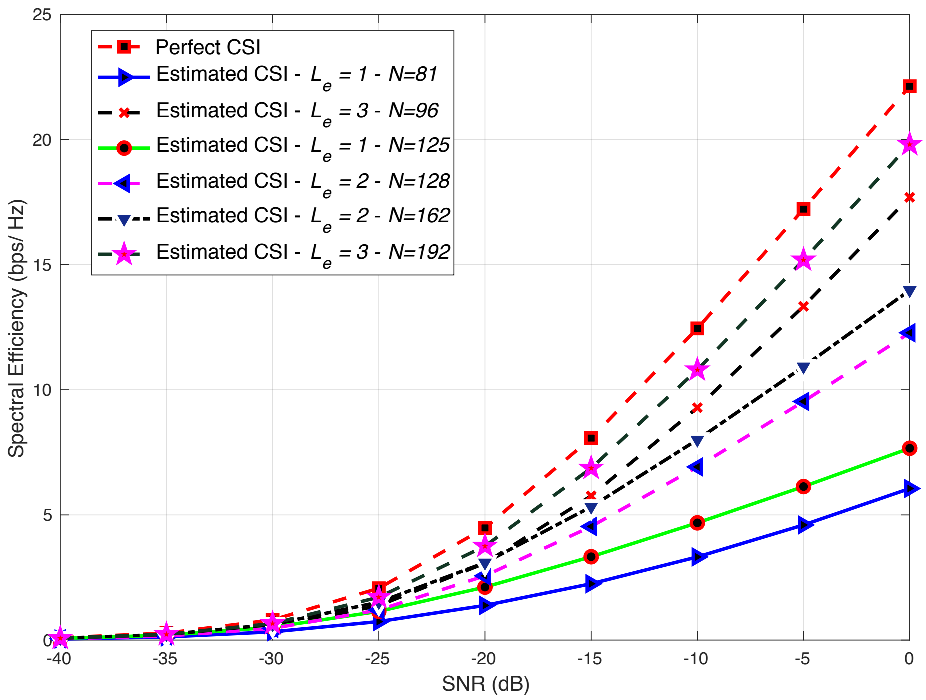

Figure 4.9: Channel capacity for perfect CSI together with estimated CSIs (f = 30 GHz and L = 3)

The aim in Figure 4.9 is to obtain the differences between the resolution parameters and state that Figure 4.5 is in fact correct (same simulation result can also be viewed in [23]). It is observed from Figure 4.9 that, the results of $N = 96$ and $N = 192$ are very close to each other. As the number of antenna increases even at the low SNR, spectral efficiency is significant. Low SNR is due to the low power of transmission and effect of high power of noise which leads to lower values of SNR.

Spectrum efficiency can be varied as the number of iteration and performing hybrid coding changes. Thus, treatment of different precoding is performed to accomplish the best precoding matrix that gives the best channel capacity. First, channel capacity for ideal hybrid precoding matrices is obtained in Figure 4.10. Secondly, same precoding matric but with normalization is performed in Figure 4.11.

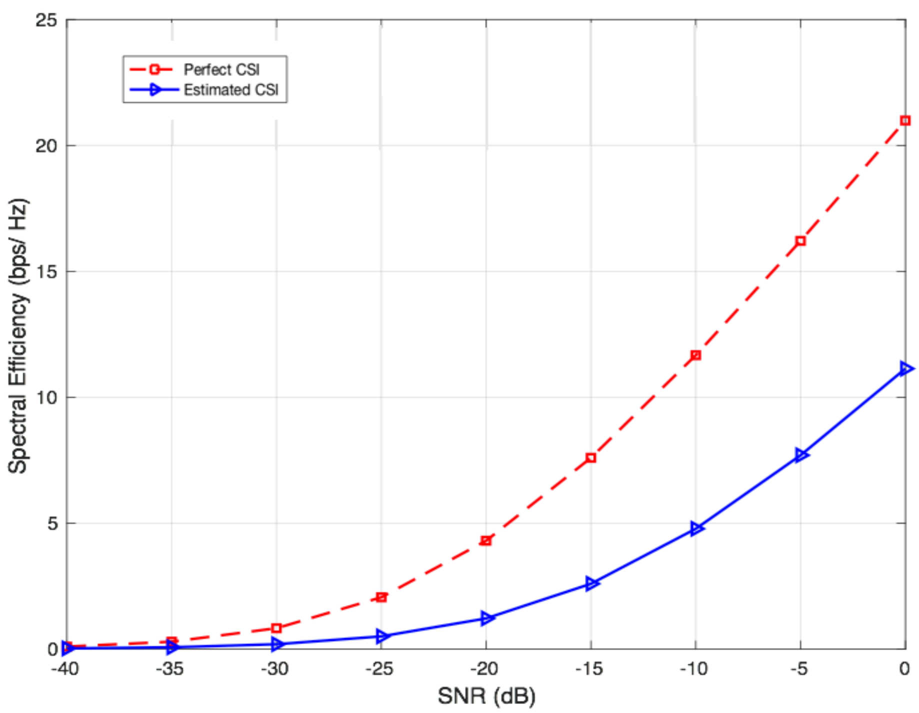

Figure 4.10: Channel capacity for estimated CSI with ideal precoding matrices (f = 30 GHz and L = 3)

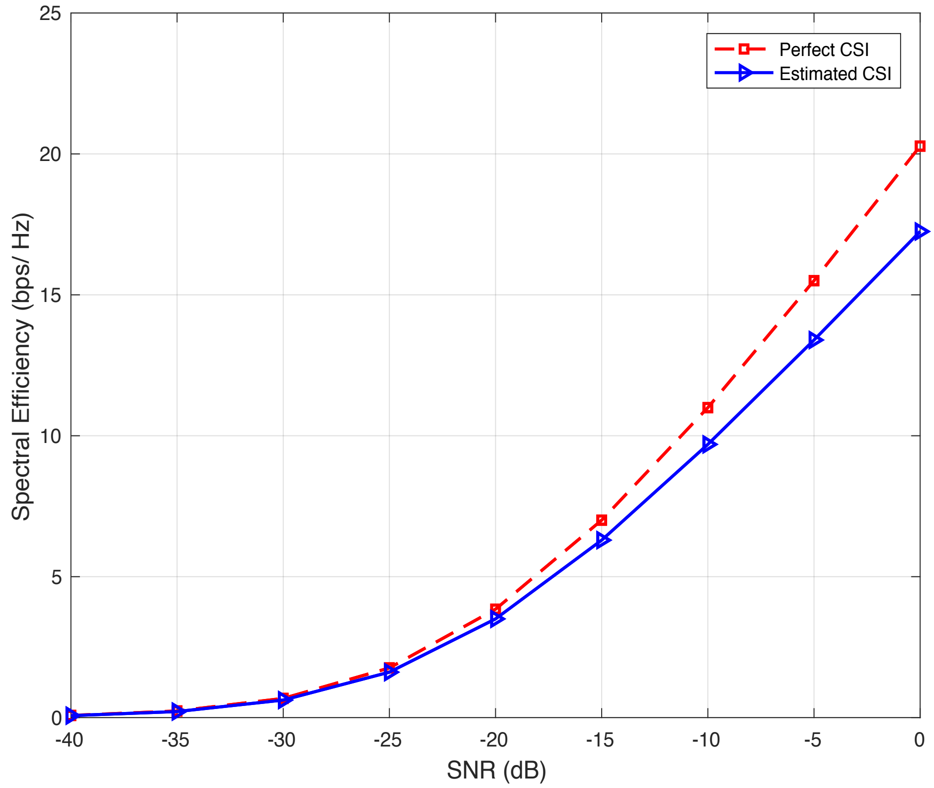

Figure 4.11: Channel capacity for estimated CSI with ideal normalized precoding matrices (f = 30 GHz and L = 3)

Results obtained above state that the precoding matrices with normalized measurements are much closer to the perfect CSI. Therefore, using these matrices are more efficient. Performance of system with higher order of training steps can be time consuming which in 5G this cannot happen due to the promising E2E latency.

After channel performance is obtained, other parameters can be manipulated in order to establish new results. Figure 4.12 illustrate that if the number of channel paths (L) considered in the modeled channel is 9, then with different number of estimated paths L_e equals to 3, 6 and 9 for estimation algorithm, will result to have different spectral efficiency versus SNR. Figure 4.12 demonstrates the comparison of high number of scattering while Figure 4.13 illustrates the realistic

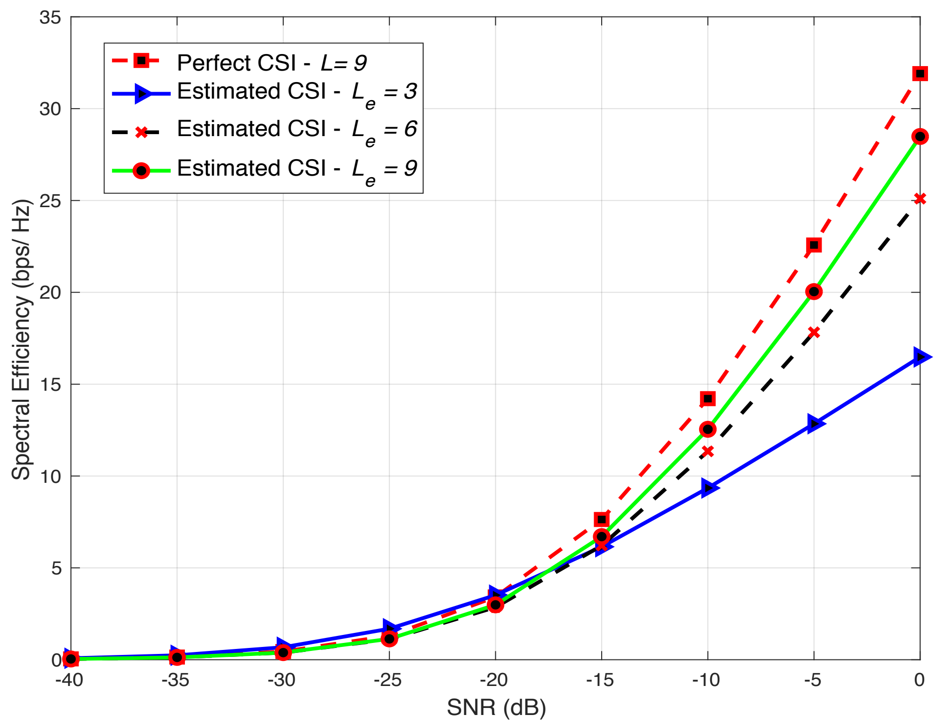

Figure 4.12: Different channel capacity with different channel paths assumed for estimation comparison with perfect CSI ($L = 9$ and $f = 30$ GHz)

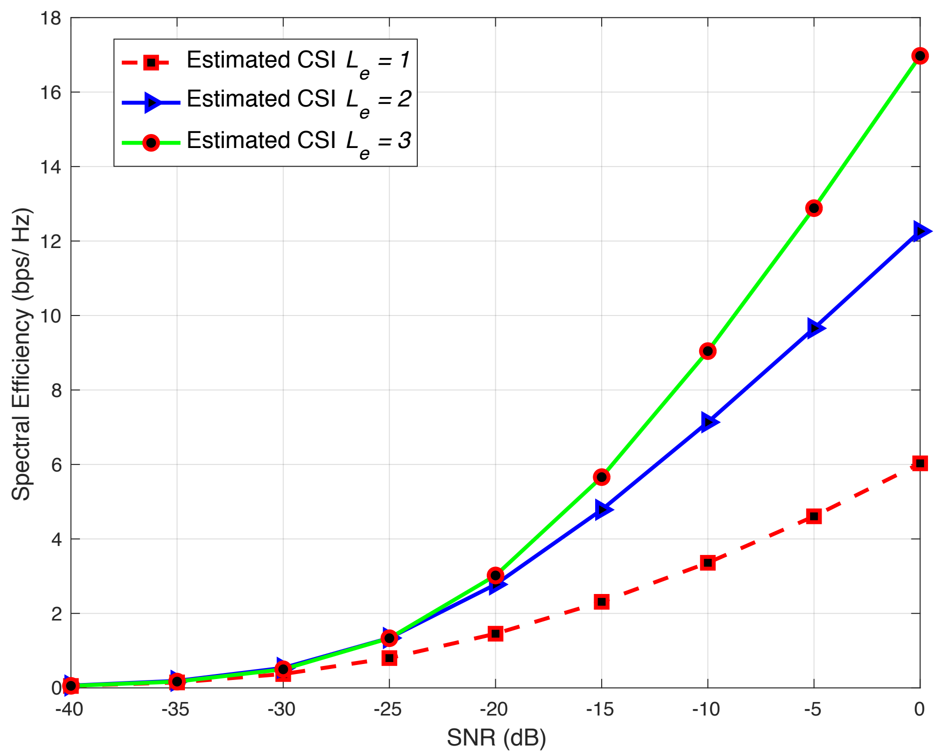

Figure 4.13: Different channel capacity with different channel paths assumed for estimation ($f = 30$ GHz and $L = 3$)

number of scattering that can actually be considered in a system. Usually, receiver considers the limited number of scattering due to the complexity of the system. As the number of scattering increases the complexity for detecting the signal highly increases. Therefore, real case scenario where actual scattering is considered needs to be observed.

As it is observed, considering $L = 3$ is the optimum value for getting the best throughput and is the maximum standardization for accepting the number of scattering.

4.3.1 BER Calculation

In order to make the communication link reliable, engineers always calculate how many transmitted bits is received without error. In order to make more bits reaches without an error, some realizations with different CSI with some realistic bits with different modulations scheme is performed. Thus, the aim is to detect the error per each realization and try to improve them. Two modulation techniques are used here, one is Quadrature Amplitude Modulation (QAM) and the other one is Binary Phase Shift Keying (BPSK). BER for BPSK, 4-QAM, 16-QAM and 64-QAM could be observed in Figure 4.14.

As expected the least value of error is for BPSK. For QAM error decreases respectively as the number of constellation point increases. As the point in constellation diagram increases points get closer to each other and it makes the decision more difficult for demodulation to decide which binary sequence is sent. Further on, BER for known channel parameters and estimated channel parameters are shown in Figure 4.15 to find the accuracy of the estimation algorithm.

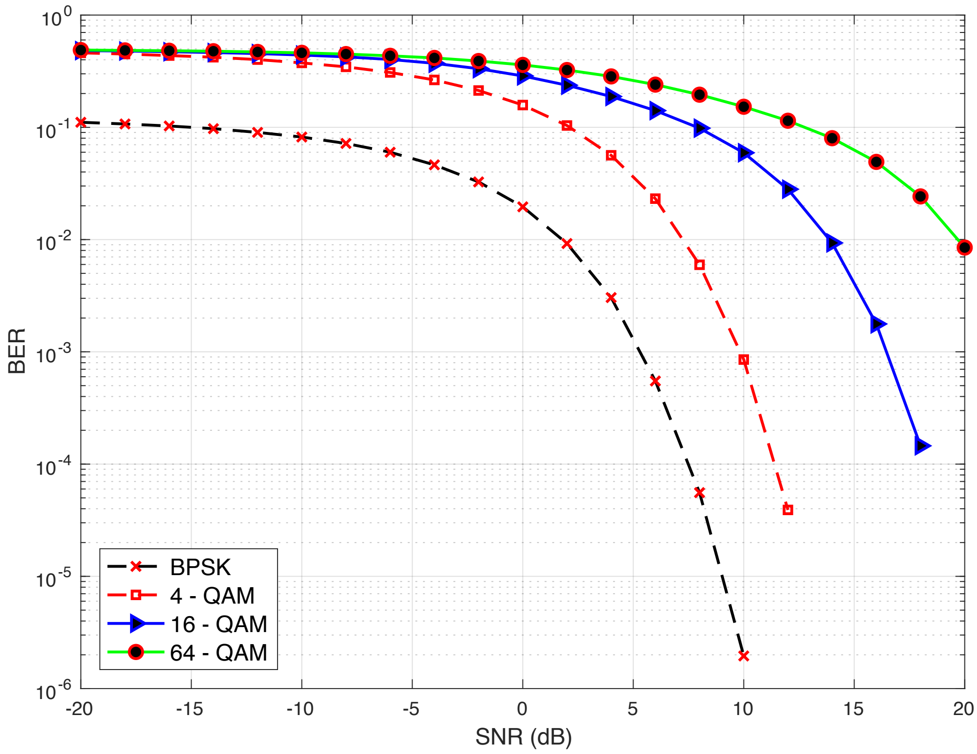

Figure 4.14: Bit error rate versus SNR (f = 30 GHz and L = 3)

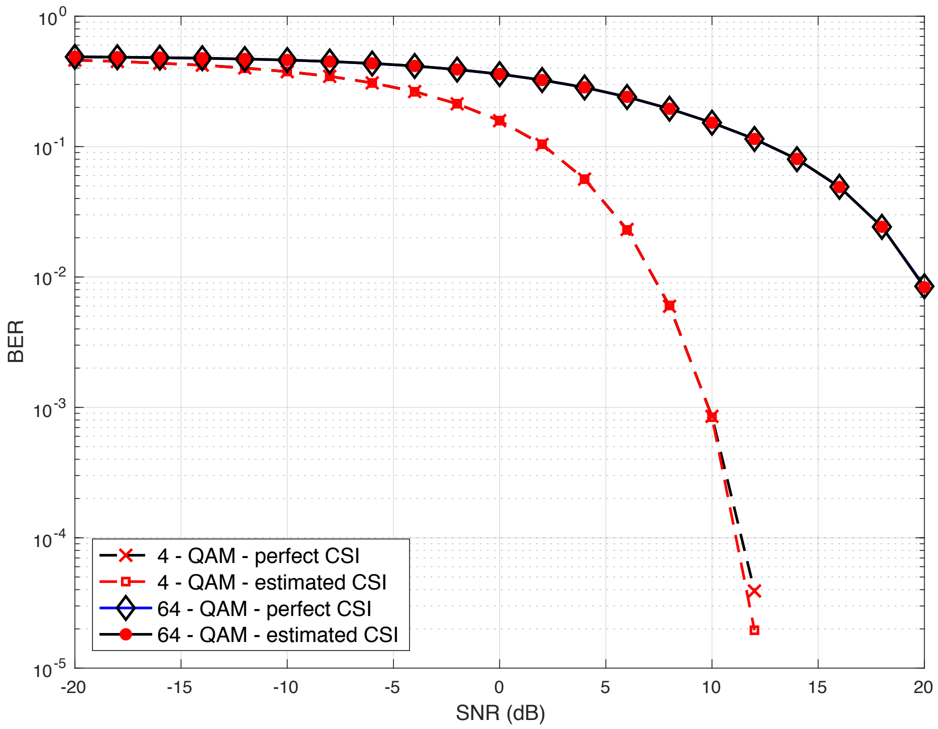

Figure 4.15: BER of known channel and estimated channel (f = 30 GHz and L = 3)

4.3.2 MSE Calculation

In this section, MSE of channel estimation is observed. This measurement is calculated for the equal power allocation in order to observe the slope of each graph to comprehend the fast changes of error. Further on, different frequency band can be simulated for this methodology in order to observe the effect of different frequency values on average probability error curve. First, we calculate MSE for estimated channel with considering of equal power distribution in Figure 4.16 through (4.2).

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\mathbf{H}_i - \widehat{\mathbf{H}}_i)^{\mathbf{2}} \tag{4.2}$$

By recognizing the behavior of the estimated CSI, it is possible to manipulate the channel with different frequency band. Additionally, observation of different path loss exponent is considered, as there are different channel situations in different environments facing users to connect to the network. Figure 4.17 and Figure 4.18 illustrate MSE for different frequencies and different path loss exponents respectively.

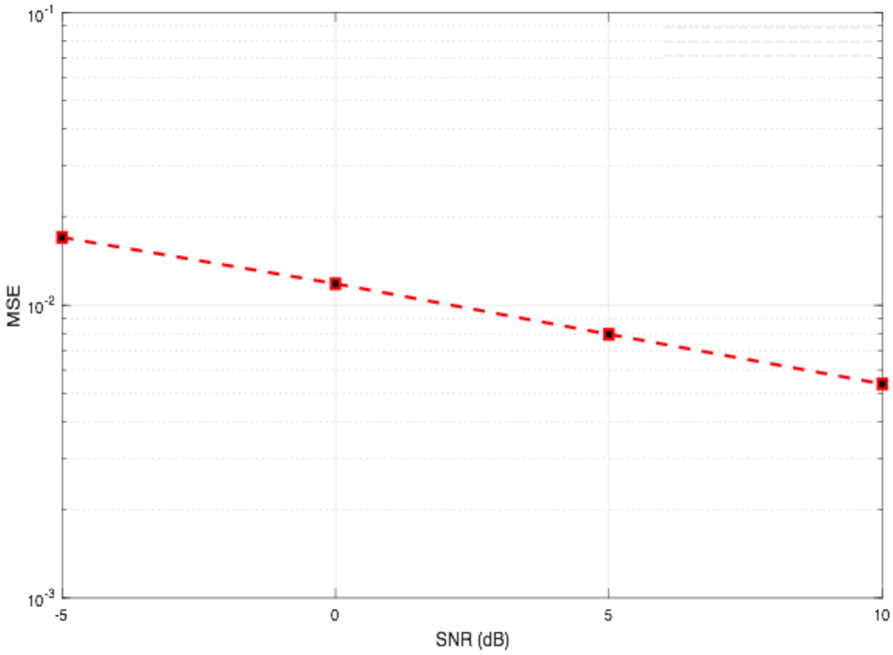

Figure 4.16: MSE for estimation CSI with equal power allocation (f = 30 GHz and L = 3)

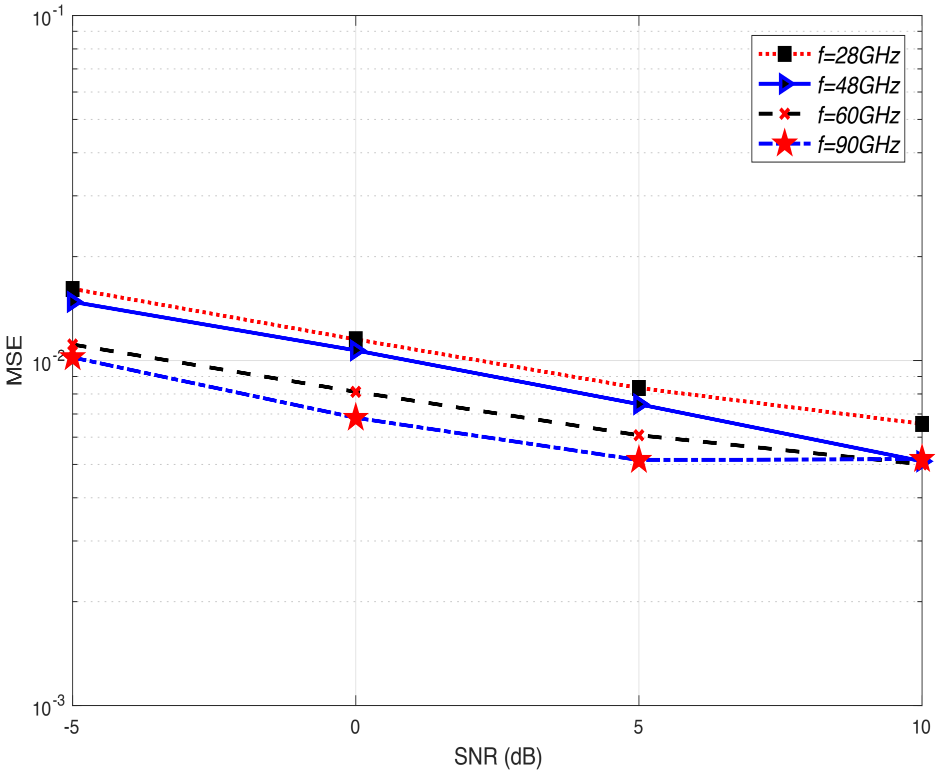

Figure 4.17: MSE for different frequencies (L = 3)

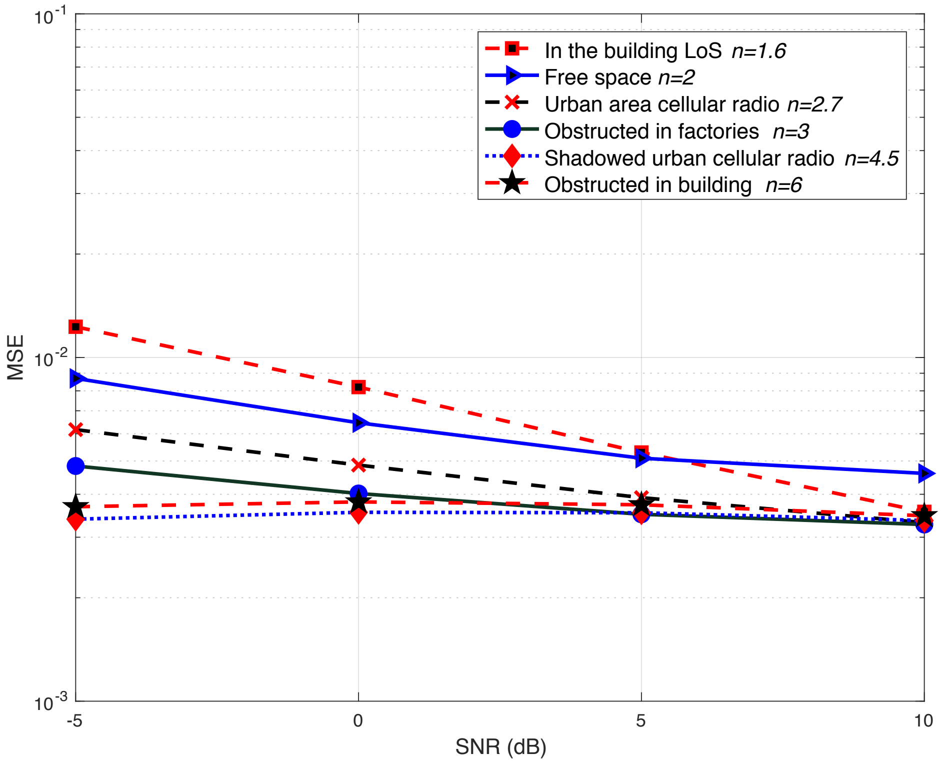

Figure 4.18: MSE for different path loss exponent (f = 30 GHz and L = 3)

Chapter 5

CONCLUSION AND FUTURE WORK

In this thesis, a comprehensive discussion of all the cellular wireless networks together with the emerging new generation (5G) and all the beneficial advantages were explained. How new technologies are going to be helpful to fulfill the requirements of 5G were explained. For instance, using the frequency band of 30 GHz up to 300 GHz enables to transmit more data with higher data rate with an instance of time. Other technologies such as massive MIMO that includes arrays of antenna at both transmitter and receiver and antenna beamforming which helps to recognize the receiver position to aid the transmitted signal with having least bumping, to decrease the interference level. Therefore, all together can fulfill to increase the capacity of the system, decrease the latency and extend the battery life of mobile handsets. However, there is always a channel effect on the transmitted signal which should be estimated before sending the data to manipulate the transmitted signal to receive least impact while traveling through the channel. Therefore, the channel is estimated and compared to the actual one. After estimating the channel, the effect of different modulation schemes on the BER is presented. Through the thesis AoA and AoD where the most important parameters for mm-wave due to the fact that channel estimation and channel model is done by these two matrices. Those matrices were trained to estimate the best channel through some iteration. After, channel and channel estimation matrices were produced, channel capacity for MIMO system having mm-wave presented. Additionally, number of iteration to obtain the best value of channel matrix calculated, then used in order to find the different channel capacities with different scattering paths. Moreover, BER for different modulation scheme were presented for strengthen the literature. Later, MSE showed the behavior of the error curve versus different values of frequency and path loss exponent. Then resulting simulation leads us to have significant improvement and the improvements can finally be the drawing line for initiating 5G of cellular networks.

For future work, new power allocation algorithm can be deployed and simulated such that it increases the power of each antenna in MIMO system, in order to increasing the data rate and channel capacity. Additionally, RF chains and analog precoders can be studied further to have better performance on channel capacity. Finally, channel can be modeled and estimated with considering random blockages between the BS and MS.

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