

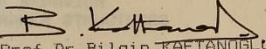


**SOME ASPECTS OF BEHAVIOR IN
FRAMED TUBE SKYSCRAPER
STRUCTURES**

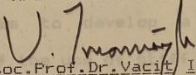
**A MASTER'S THESIS
in
Building Science
(Department of Architecture)
Middle East Technical University**

**By
Yonca HÜROL
May, 1987**

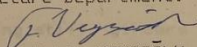
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I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science in Building Science Program of the Architecture Department.


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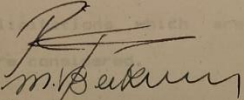
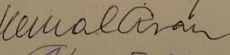
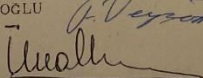
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ABSTRACT

SOME ASPECTS OF BEHAVIOR
IN FRAMED TUBE SKYSCRAPER STRUCTURES

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M.S. in Building Science, Architecture
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May 1987, 110 pages

The aim of this study is to develop a clear picture regarding the effect of the use of stiff girders on the efficiency of framed tubes, and to show the advantages of using tubular configuration in skyscrapers. The first part of the study discusses the limitations which affect the heights and the slenderness ratios of skyscrapers as their efficiency is measured by their height. The behavior of framed tube skyscraper structural systems and the role of using stiff girders on their behavior is then examined. Lastly the study deals with what is gained by using tubular geometry, and proportions of framed tube systems are compared with those of planar frames when the limitations which are discussed in the beginning part are considered.

Key words - Skyscraper, Framed tube, Planar frame,
Stiffness

ÖZET

TÜBÜLER ÇERÇEVE SİSTEMLERİN BAZI DAVRANIŞ ÖZELLİKLERİ

HÜROL, Yonca
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Bu çalışmanın amacı tübüler çerçeve sistemlerde rijit yatay elemanlar kullanılmasının sebeplerini açıklığa kavuşturmak ve tübüler çerçeve sistemlerin erişebildikleri yükseklikleri diğer çerçeve sistemlerle mukayese etmektir. Gökdelen strüktürlerinin erişebilecekleri yükseklik onların strüktürel malzemeyi verimli kullanışlarının göstergesi olduğundan, çalışmanın ilk kısmında gökdelenlerin biçimlenmelerine kısıtlar olarak etki eden faktörler tartışılmakta, daha sonra tübüler çerçeve sistemlerin davranışı ve rijit yatay elemanların bu davranışa etkisi açıklığa kavuşturulmaktadır. Son olarak ise tübüler çerçeve sistemleri kullanmanın avantajlarını saptamak için tübüler çerçeve sistemlerin erişebileceği maksimum yükseklik bilinen çerçeve sistemlerin erişebilecekleri yüksekliklerle ilk kısımda belirlenen kısıtlar açısından karşılaştırılmaktadır.

Anahtar Kelimeler - Gökdelen, Tübüler çerçeve, Çerçeve sistemler, Rijitlik

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- F_H Horizontal force.
- F_M Magnification factor.
- F_{OS} Factor of safety for overturning.
- F_u Bolt (lateral) load acting on surface of the structure.
- F_v Vertical force.
- F_w Distributed lateral load.
- F_y Yield stress for the structural material.

LIST OF SYMBOLS

P_1, P_2	Lateral reactions of the earth against structure.
r	Radius of gyration.
h	Height.
h_s	Storey height.
A	Area.
A_f	Floor area.
b	Width.
c	Constant.
D	Width of the building.
d	Depth of a flexural member.
d_1	Distance of the area from the neutral axis.
E	Modulus of elasticity. (Young's modulus)
F	Lateral load.
f	Total compressive stress acting on the structure.
f_{bmax}	Maximum bending stress in a flexural member.
f_c	Compressive stress due to axial loading.
f_{callow}	Allowable compressive stress for the structural material.
f_{callow_s}	Allowable compressive stress for soil.
F_h	Horizontal force.
FM	Magnification factor.
F_{So}	Factor of safety for overturning.
F_u	Unit lateral load acting on surface of the structure.
F_v	Vertical force.
F_w	Distributed lateral load.
f_y	Yield stress for the structural material.

F1, F2	Lateral reactions of the earth against overturning.
G	Shearing modulus.
h	Height of the building.
hs	Storey height.
I	Moment of inertia.
i	Radius of gyration.
Ih	Moment of inertia of a horizontal structural member.
IT	Moment of inertia of the framed structure.
Iv	Moment of inertia of a vertical member.
k	Factor related to shear stress.
l	Length of the structural member.
lc	Length of a column.
lg	Length of a girder.
lh	Length of a horizontal member.
lv	Length of a vertical member.
M	Bending moment.
Mo	Overturning moment.
MR	Resisting moment against overturning moment.
MR1, MR2	Components of MR.
n	Number of storeys, slices etc..
Nc	Axial force acting on a column.
NCR	Resultant compressive stresses.
Q	Shear.
Qh	Shear acting on a horizontal member.
Qv	Shear acting on a vertical member.
P	Vertical load acting on the skyscraper.
Pf	Vertical reaction of the earth.

P_u	Unit vertical load acting on each floor.
r	Slenderness ratio of a flexural structure.
r_b	Slenderness ratio for buckling.
S	First moment of an area.
x	Lever arm between resultant vertical load acting on the structure, and resultant reaction of the earth to this load.
y_{beams}	Deflection of beams.
y_c	Cantilever deflection.
$y_{columns}$	Deflection of columns.
y_m	Deflection due to shear acting on the structural members. (y_{mv}, y_{mh})
y_{mh}	Deflection due to deflection of horizontal members in one storey.
y_{mv}	Deflection due to deflection of vertical members in one storey.
y_s	Shearing displacement.
y_T	Total deflection of a structural member.
y_{Ts}	Total deflection of a skyscraper.
	Angle.

1.1 STRUCTURAL SYSTEMS USED IN SKYSCRAPERS

The history of high-rise structures starts with masonry structures. The Monadnock Building shown in Figure 1.1, was accepted as a high-rise structure when it was constructed in 1891. It is 16 storeys high, but it is not a slender structure.

(1.1) A.S.C.E., I.A.S.C.E., 'Proceedings of the International Conference on Planning and Design of Tall Buildings', Vol. 1, A.S.C.E., Pennsylvania 1972, p. 287-290.

1. INTRODUCTION

The term 'skyscraper' is a popular term, used to describe high and slender buildings. Skyscrapers can be classified under the heading of high-rise buildings. The difference between the two is the slenderness of skyscrapers. The height and slenderness of skyscrapers cause unusual structural problems, and because of this, different structural systems are used for different height ranges, depending especially on their stiffness.

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- (1.1) A.S.C.E., I.A.B.S.E., 'Proceedings of the International Conference on Planning, and Design of Tall Buildings', Vol.1.a, A.S.C.E., Pennsylvania 1972, p.567-590.



Figure 1.1. Monadnock Building.

Source: ASCE, IABSE, 'Planning and Design of Tall Buildings, Vol 1.a, p.573.

The skyscraper idea was born in the 1880's with the steel frame of the 10 storey high Home Insurance Building ^(1.2) shown in Figure 1.2. It became necessary to classify comparatively high and slender buildings in a different category, after the construction of this building.

(1.2) Eberhard, J. P., 'A Systematic Approach to Tall Building Requirements', A.S.C.E., I.A.B.S.E., 'Planning, and Design of Tall Buildings', Vol.1.a, p.133.

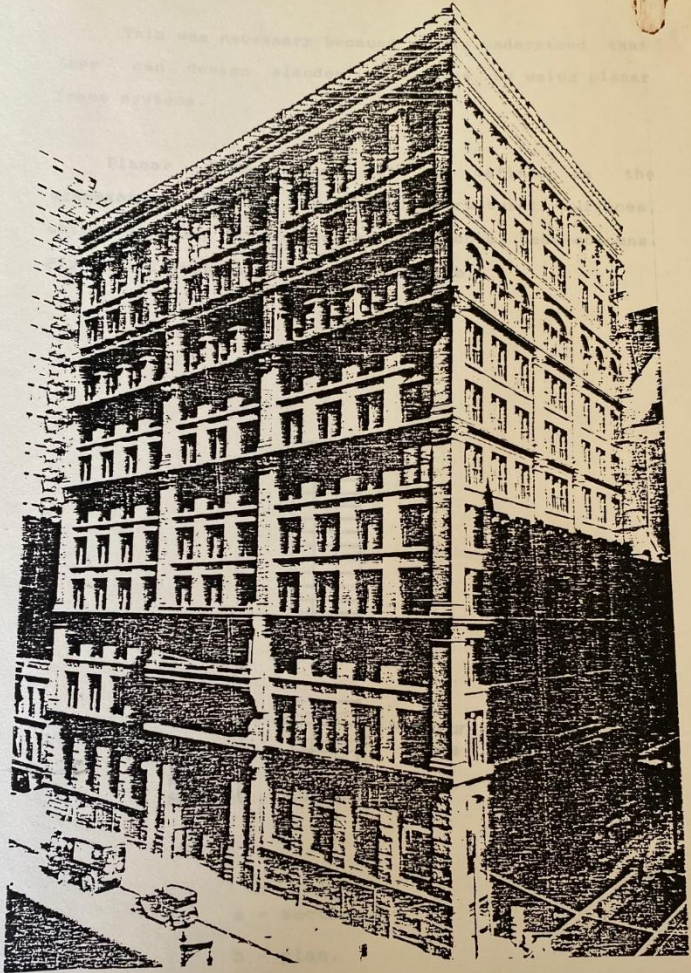
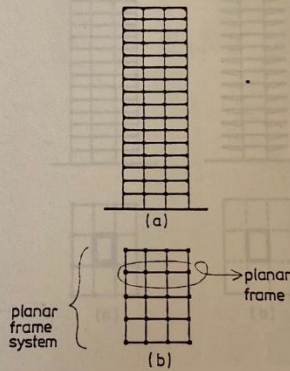


Figure 1.2. Home Insurance Building.

Source : Andrews, W., 'Architecture in Chicago', p.36.

This was necessary because people understood that they can design slender structures by using planar frame systems.

Planar frame systems can be defined as the aggregation of two dimensional structural surfaces, which consists of vertical and horizontal beam-columns. Figure 1.3 shows a planar frame system.



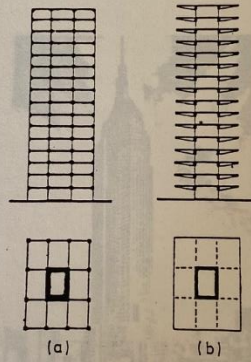
a - section.

b - plan.

Figure 1.3. Example of planar frame system.

Source; Schueller, W., 'High-rise Building', p.53.

Another attempt to increase efficiency is to use shear wall systems. This resulted in the planar frame with shear walls, or simple shear wall systems, some examples of which are shown in Figure 1.4.



- a. Shear wall integrated with planar frame.
- b. Shear wall system.

Figure 1.4. Alternatives of using shear wall systems.

Source: Schueller, W., 'High-rise Building', p.53.

Up to the 1930's the heights and slenderesses of buildings were increased, as a result of the use of planar frame systems and planar frame systems with shear walls. Between the 1910's and the 1930's very high buildings were constructed by using these structural systems. One of these buildings is the Empire State Building which is shown in Figure 1.5. Its height is 381 meters, and it remained as the tallest building in the world for 40 years. (1.3)

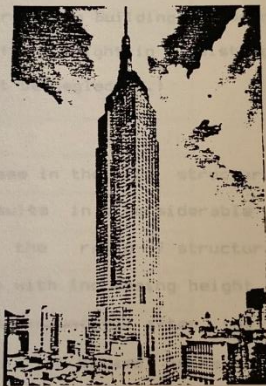


Figure 1.5. Empire State Building.

Source: ASCE, IABSE, 'Planning and Design of Tall Buildings, Vol.C, p.125.

(1.3) Klein, H.H., 'Great Structures of the World', The World Publishing Company, N.Y. 1968.

A list of important skyscrapers is given in Table 1.1 together with some of their characteristics. In this table, one can see a sudden decrease in the heights of buildings constructed between the 1930's and the 1960's. After the construction of planar frame and shear wall systems like those of the Empire State and the Chrysler buildings, it was understood that the selection of a more appropriate structural system is very important for skyscrapers, because these were very heavy structures. (In Table 1.1 weight of brick exterior walls of Empire State Building has been neglected. But the effect of the weight in resisting lateral loads however can not be neglected.)

An increase in the unit structural weight of a skyscraper results in a considerable increase in its cost, because the ratio of structural cost to total cost increases with increasing height and slenderness ratio. (1.4) Consequently, after the construction of these buildings, it became necessary to search for another way of reaching similar heights, by using less structural material.

(1.4) Conlin, W.F., 'Economics of High-rise Buildings', A.S.C.E., I.A.B.S.E., 'Planning and Design of Tall Buildings', Vol.1.a, p.126.

Table 1.1. Information about important high-rise buildings.

Building Name	Place	Year	Structural system	Height (m)	Slenderness ratio	Height (storey)	Structural material	Unit weight(kg/m ³)	Importance	Ref.
Monadcock B.	-	1891	Bearing wall	-	-	16	Masonry	-	-	(1.5)
Ingalls B.	Cincinnati	1901	Planer frame system	64	-	16	Reinforced concrete	-	First R.C. skyscraper	(1.6)
Marina City Towers	Chicago	1962	Planar frame shear core plus	180	-	60	"	-	Tallest at the time of const.	(1.7)
Royal Bank of Canada	Montreal	1962	"	200	-	40	"	-	-	(1.6)
Chesnut De Witt B. (SOM)	Chicago	1965	Framed tube	-	-	43	"	-	First tubular structure	(1.6)
Australia Square B.	Sydney	1967	Planar frame plus shear core	183	-	45	Light weight concrete	-	-	(1.6)
One Shell Plaza B. (SOM)	Texas	1971	Tube in tube	218	5,5	52	"	-	Tallest light weight conc. skv	(1.6)
Standart Bank Center	-	-	-	170	-	31	Prestressed concrete	-	-	(1.5) (1.6)
One Shell Square B. (SOM)	New-Orleans	1972	Framed tube	-	-	51	Composite	-	-	(1.6)
Woolworth T.	New York	1913	Planar frame system	242	-	60	Steel	-	-	(1.7)
Chrysler B.	"	1929	"	319	-	77	"	-	-	(1.7)
Empire State	"	1930	planar frame plus shear wall	381	9,3	102	"	206 for skeleton	Tallest at the time of const.	(1.6) (1.10)
Brunswick B. (SOM)	Chicago	1962	Planer frame plus shear wall	-	-	38	"	-	-	(1.7)
Chase Manhattan Bank (som)	New York	1960	Planar frame system	243	7,3	60	"	270	-	(1.8)
John Hancock B. (SOM)	Chicago	1968	Trussed tube	344	7,9	100	"	145	-	(1.9)
Standard Oil of Indiana B.	"	1973	Tube	346	-	80	"	-	-	(1.6)
World Trade Center	New York	1973	Framed tube	410	6,9	110	"	180	Tallest at the time of const.	(1.7)
Sears and Roebuck B. (SOM)	Chicago	1974	Bundled tube	442	6,4	110	"	161	Tallest in the world at the pre.	(1.9)

(1.5) ASCE; IABSE; "Proceeding of the International Conference on Planning and Design of Tall Buildings"; vol. 1.a,1972.

(1.6) ASCE; IABSE; "Proceeding of the International Conference on Planning and Design of Tall Buildings"; vol. c,1972.

(1.7) Mainstone, R.; "Developments in Structural Form", M.I.T Press, Massachusetts, 1975.

(1.8) Howard, S. H.; "Structure: An Architects Approach", Mc Graw Hill Book Company, New York, 1975.

(1.9) Schmertz, M. F.; "Office Building Design".

(1.10) Klein, H. A.; Klein, M. C.; "Great Structure of the World".

Thus, the tube concept was first developed in the 1960's by Fazlur Khan and the firm Skidmore, Owings, and Merrill. (1.11) The first tubular skyscrapers of the world were the reinforced concrete Chestnut De Witt Apartments which were constructed in 1965 (1.12) (Figure 1.6). They were framed tubes.

In 1973 the twin towers of World Trade Center were constructed and they became the tallest in the world. Their structural systems were also framed tubes. In the case of framed tubes the structural material which resists the effect of lateral loads was placed at the perimeter of the building in the form of surrounding framed surfaces. The spacing between the columns of the perimeter structure is very small, as seen in Figure 1.7. The maximum spacing used between columns of framed tubes is 4,5 meters. (1.13)

-
- (1.11) White, R.N., et. al., 'Structural Engineering', Vol. 3, John Wiley, and Sons, N.Y., 1974, p.540.
- (1.12) Schmertz, M.F., 'Office Building', Mc.Graw Hill, N.Y., 1975, p.182.
- (1.13) Khan, F., Navinchandra, R. A., 'Analysis, and Design of Framed Tube Structures for Tall Concrete Buildings', A.C.I., 'Response of Multi-storey Concrete Structures to Lateral Loads', 2. edition, A.C.I., Detroit 1973, p.41.

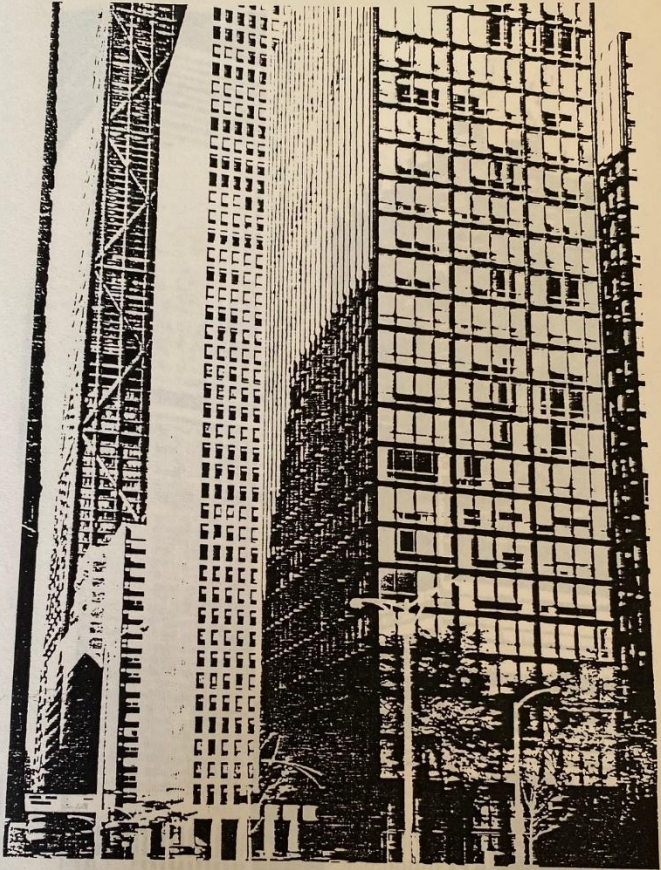


Figure 1.6. Chestnut De Witt Apartments. (White building at the middle)

Source: Mainstone, R., 'Structural Form', p.274.

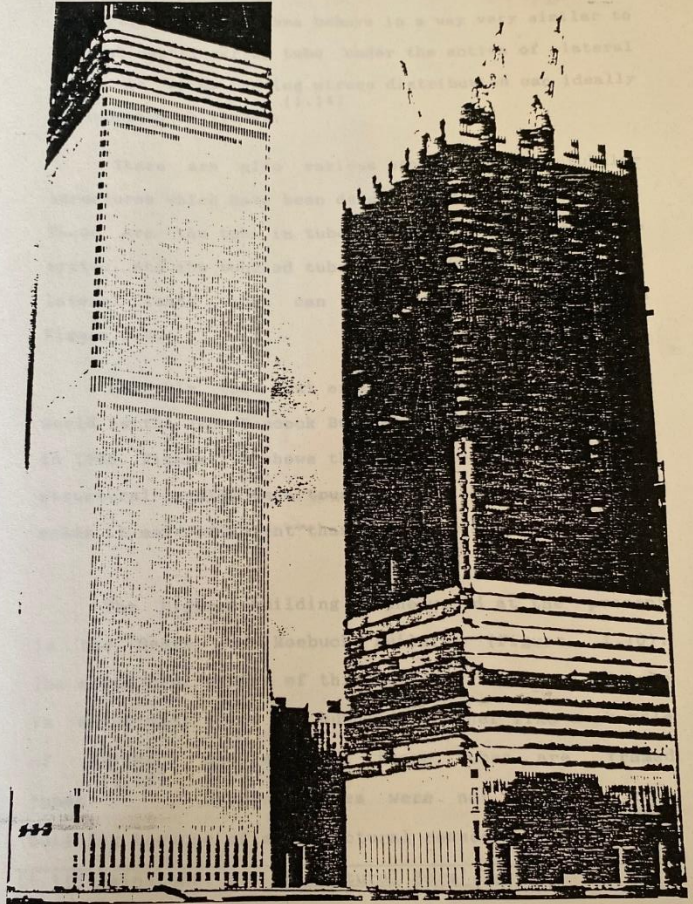


Figure 1.7. World Trade Center.

Source : Mainstone, R., 'Structural Form', p.276.

It has been stated that because of these properties, framed tubes behave in a way very similar to a real cantilevering tube under the action of lateral loads, in which bending stress distribution can ideally be accepted as linear. (1.14)

There are also various other types of tubular structures which have been developed after framed tubes. These are the tube in tube system, the trussed tube system, and the bundled tube system. Under the action of lateral loads each can be modeled as shown in Figure 1.8.

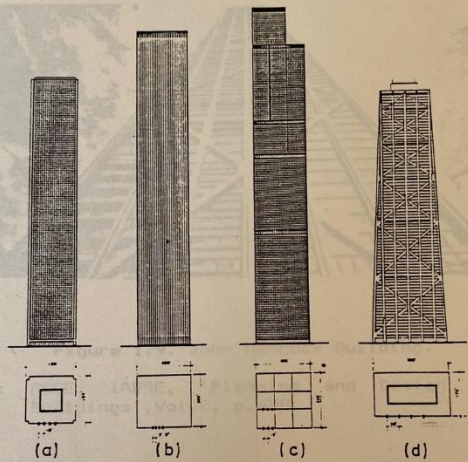
Probably the most efficient skyscraper in the world is the John Hancock Building which was constructed in 1968. Figure 1.9 shows the John Hancock Building. Its structural system is a trussed tube, and this system makes it more efficient than other skyscrapers. (1.15)

The highest building in the world at the present is the Sears and Roebuck Building (Figure 1.10). The structural system of this building is a bundled tube in which many tubes are bundled together like a bundle of sticks, and each of these tubes are framed tubes. (1.16) Trussed tubes were not used in this building because of architectural reasons.

(1.14) Mainstone, R., 'Structural Form', p.88.

(1.15) Schmertz, M.F., 'Office Building', p.185.

(1.16) Schmertz, M. F., 'Office building'.



One can not get an idea about the appropriate heights of skyscraper structural systems by studying Table 1.1 because it is possible to use any structural system if stability is not considered. Consequently, appropriate heights of skyscraper structural systems have been determined only as a function of wind load as shown in Table 1.2.

a - Tube in tube.
 b - Framed tube.
 c - Bundled tube.
 d - Trussed tube.

Figure 1.8. Models of tubular structures under the action of lateral loads. (Existing examples.)

Source: Schueller, W., 'High-rise Building', p.102.

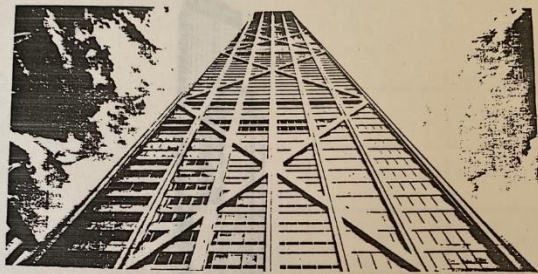


Figure 1.9. John Hancock Building.

Source: ASCE, IABSE, 'Planning and Design of Tall Buildings', Vol.C, p.498.

One can not get an idea about the appropriate heights of skyscraper structural systems by studying Table 1.1 because it is possible to use any structural system if efficiency is not considered. Consequently, appropriate height of skyscraper structural systems have been suggested by Schueller (1.17) as shown in Table 1.2.

(1.17) Schueller, W., 'High-Rise Building', John Wiley and Sons, Canada, 1977, p.114.

Table 1.3. Approximate height limits of structural systems which are extensively used for high-rise structures.

STRUCTURAL MATERIAL	MAXIMUM HEIGHT (STORIES)	MAXIMUM HEIGHT (FEET)
REINFORCED CONCRETE	20	70
PRECAST CONCRETE	25	100
STEEL	50	175
STEEL-TUBE	55	190
STEEL-TUBE WITH BRACED CORE	65	230
STEEL-TUBE WITH BRACED CORE AND PERIPHERAL BRACING		250
STEEL-TUBE WITH BRACED CORE AND PERIPHERAL BRACING AND DIAPHRAGMS		105
STEEL-TUBE WITH BRACED CORE AND PERIPHERAL BRACING AND DIAPHRAGMS AND BRACED CORE		140
STEEL-TUBE WITH BRACED CORE AND PERIPHERAL BRACING AND DIAPHRAGMS AND BRACED CORE AND PERIPHERAL BRACING		200
STEEL-TUBE WITH BRACED CORE AND PERIPHERAL BRACING AND DIAPHRAGMS AND BRACED CORE AND PERIPHERAL BRACING AND DIAPHRAGMS		300
STEEL-TUBE WITH BRACED CORE AND PERIPHERAL BRACING AND DIAPHRAGMS AND BRACED CORE AND PERIPHERAL BRACING AND DIAPHRAGMS AND BRACED CORE		305

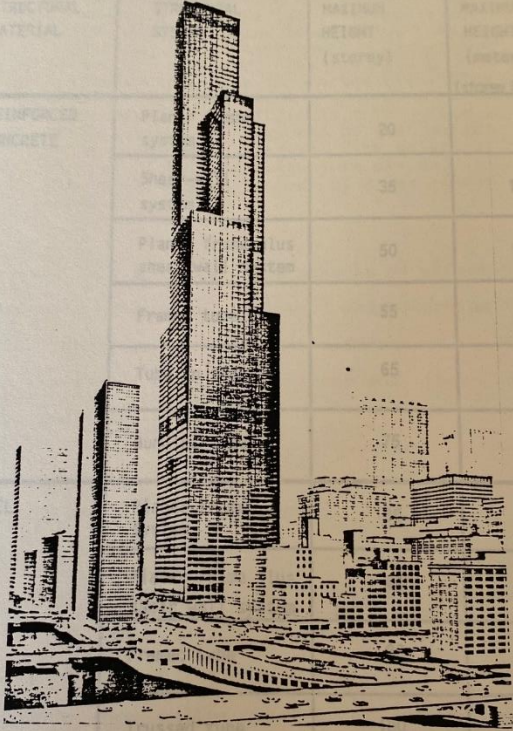


Figure 1.10. Sears and Roebuck Building.

Source: ASCE, IABSE, 'Planning and Design of Tall Buildings', Vol. C, p. 500.

Table 1.2. Appropriate height limits of structural systems which are extensively used for high-rise structures.

STRUCTURAL MATERIAL	STRUCTURAL SYSTEM	MAXIMUM HEIGHT (storey)	MAXIMUM HEIGHT (meters) (storey height=2.5)
REINFORCED CONCRETE	Planar frame system	20	70
	Shear-wall system	35	120
	Planar frame plus shear wall system	50	175
	Framed tube	55	190
	Tube in tube	65	230
	Bundled tube	75	260
STEEL	Planar frame system	30	105
	Planar frame plus shear wall system	40	140
	Framed tube	80	280
	Trussed tube	100	350
	Bundled tube	110	385

Schueller states that this table was prepared by
(1.18)
Fazlur Khan, and he adds:

"....The structural systems given for certain heights should not be considered an absolute rule. In fact, the 102 storey Empire State Building is characterized by a rigid frame-shear wall interaction system, indicated as applying to buildings less than 40 stories high. The chart is organized according to structural efficiency (i.e., optimization) as measured by the weight per square foot; that is, the weight of the total building structure divided by the total square footage of gross floor area..."

Table 1.2 also gives a measure of effectiveness of skyscraper structural systems, and according to this table there are important differences between the efficiency of tubular structures and those of other structural systems.

1.2. AIM AND SCOPE OF THE THESIS.

Since framed tube structures are the forerunners of tubes, the behavior of tubular structures can be understood by studying the behavior of framed tubes. The difference between the behavior of earlier structural systems and framed tubes is due to two important differences in configuration. These are:

(1.18) Ibid., p.114.

1 - Placing the structural material which resists lateral loads at the perimeter of the building instead of placing it close to the neutral axis.

2 - Using extremely short girders.

The aim of this thesis is to develop a clear picture regarding the effect of the use of stiff girders on the behavior of framed tubes under the action of lateral loads, and to show advantages of tubular configuration. The discussion is limited to framed tubes that act independently of any core or other inner structure.

Heights that can be reached by skyscrapers are determined by the structural requirements of strength, stability, stiffness and structural efficiency. Thus, heights that can be reached by skyscrapers are discussed generally in chapter two, where the structural requirements listed above are examined.

The configuration and behavior of the perimeter structures of framed tubes are studied in the third chapter, which examines how lateral and vertical loads are transferred to the ground in the case of ideal framed tubes and the effect of using stiff girders on the behavior of perimeter structures of the framed tubes.

In chapter four heights which can be reached by framed tubes is compared with heights which can be reached by planar frames. This is done in order to understand what is lost without using tubular geometry. Comparisons are made by using the concepts examined in the second chapter.

Throughout the study similarities between cantilevering beams and skyscrapers are used. They are considered as cantilevering beam-columns which have different moment of inertias and different bending stress distributions. The structures are assumed to be square shaped in plan, and all kinds of loadings are assumed as uniform and static for purposes of simplification.

1.1. BEHAVIOR AND PROPORTIONS OF SKYSCRAPERS.

Generally it is said that to be classified as a skyscraper, a high-rise structure must behave like a cantilevering beam under the action of lateral loads, if such loadings are assumed as static.

(2.1) Crister, L. E., 'Theory of Modern Steel Structures', Vol. 1, 2nd edition, The Macmillan Company, N.Y., 1949, p.320.

2. STRUCTURAL REQUIREMENTS FOR SKYSCRAPERS

In architecture and structural engineering the term 'skyscraper' is used to describe the buildings which are both high and slender. This definition can give a rough idea about the difference between the structural behavior of skyscrapers and that of ordinary structures. But it is necessary to know their proportions in order to discuss their structural requirements.

2.1. BEHAVIOR AND PROPORTIONS OF SKYSCRAPERS.

Generally it is said that to be classified as a skyscraper, a high-rise structure must behave like a cantilevering beam under the action of lateral loads, if such loadings are assumed as static.

(2.1) Grinter, L. E., 'Theory of Modern Steel Structures', Vol. 1, 2. edition, The Macmillan Company, N.Y., 1949, p.320.

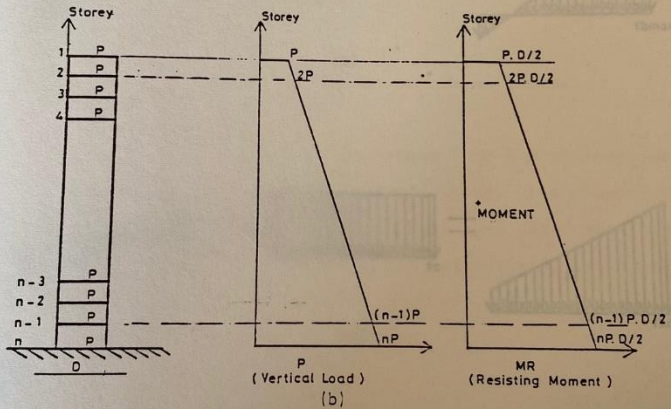
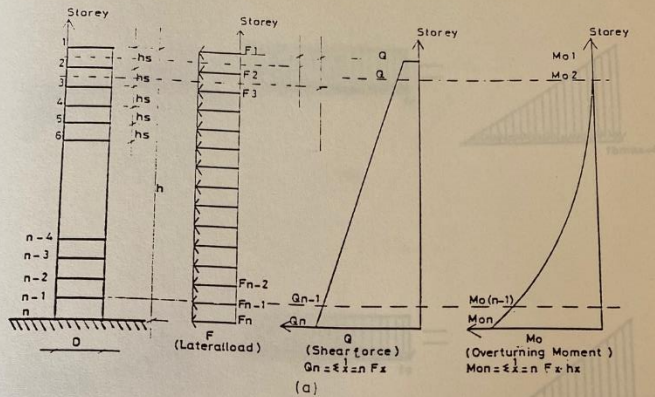
Lateral loads create bending moment in the structure, similar to the case of cantilevering beams. This moment is called the 'overturning moment', because it tends to overturn the structure.

In reality, skyscrapers behave as cantilevering beam-columns, because of the vertical loads acting on them. The vertical loads tend to balance the overturning moment by creating a moment in the opposite direction. As shown in Figure 2.1, ^(2.2) the rate of increase of overturning moment from the top of the structure towards the ground is greater than the rate of increase of resisting moment which is due to vertical loads.

Depending on the magnitudes of lateral and vertical loads, and the proportions of the structure three different types of stress distributions may occur at the ground level. These are shown in Figure 2.2.

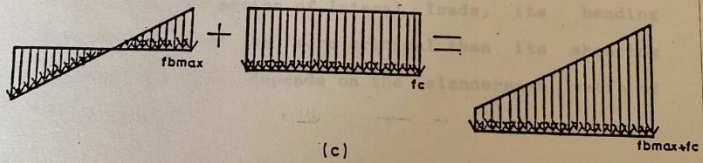
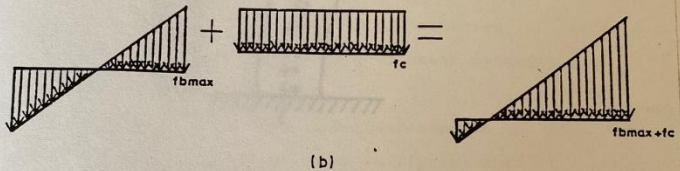
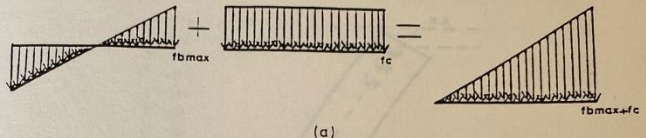
Whatever the stress distribution is, the structure bends as seen in Figure 2.3, because shortening of one side of the structural member will always be more than the shortening of the other side under the action of lateral loads.

(2.2) Schueller, W., 'High-rise Building Structures', p.128.



- a. Overturning moment due to lateral load.
 (lateral load has been accepted as uniform.)
- b. Resisting moment. (vertical load per area has
 been accepted as equal for all storeys.)

Figure 2.1. Overturning and resisting moments.



f_{bmax} = Maximum bending stress.
 f_c = Compressive stress.

- a. If $f_{bmax} = f_c$
- b. If $f_{bmax} > f_c$
- c. If $f_{bmax} < f_c$

Figure 2.2. Stress distribution alternatives at the ground level.

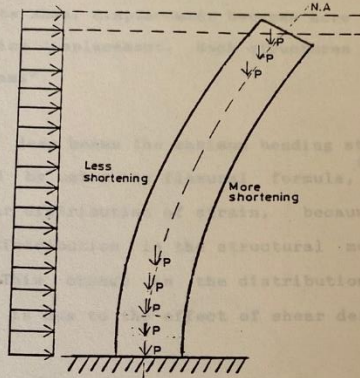


Figure 2.3. Deflection of a beam-column.

In order for a beam to behave like a cantilevering beam under the action of lateral loads, its bending deformation must be more critical than its shearing deformation. This depends on the slenderness ratio of the structure :

$$r = \frac{h}{D} \quad (\text{Eq. 2.1})$$

where h = Height of the building.
 D = Smallest width of the building.

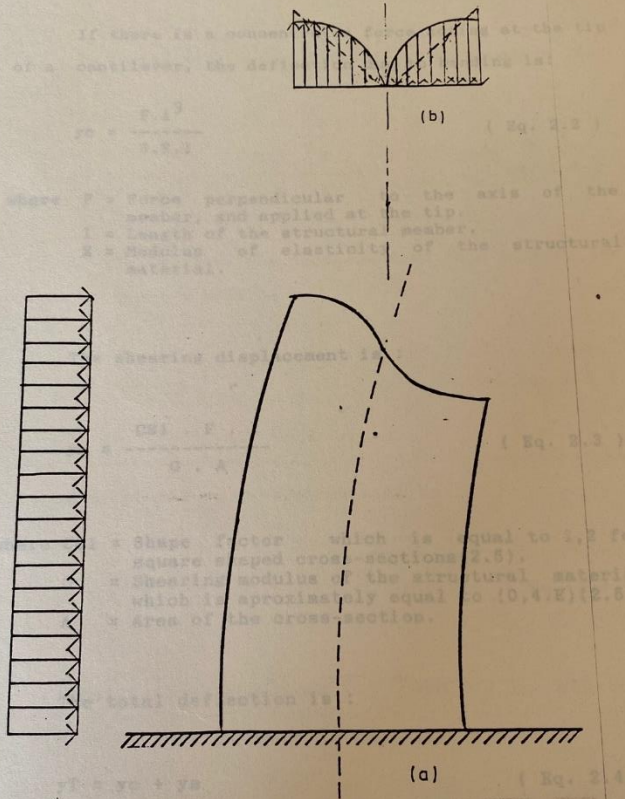
If the slenderness ratio is less than a certain value, the shear displacement becomes more critical than the bending displacement. Such structures are named as 'deep beams'.

In deep beams the maximum bending stress can not be found by using the flexural formula, ^(2.3) based on the linear distribution of strain, because the bending stress distribution in the structural member is not linear. This change in the distribution of bending stresses, is due to the effect of shear deformations.

In deep beams, the shear stresses in the structure cause deformations as shown in Figure 2.4. In other words, sections which are plane before bending, do not remain plane after bending. ^(2.4)

To examine the lateral deformation of skyscrapers under lateral loads, let us first examine the case of solid cantilevering beams. The total deflection of a solid cantilevering beam is equal to the deflection due to bending moment plus the shearing displacement.

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- (2.3) White, R.N., et. al., 'Structural Engineering'; Vol.3; p.62.
- (2.4) Oden, J.T., Ripperger, E.A., 'Mechanics of Elastic Structures', 2. ed., Mc Graw Hill, N.Y., 1981, p.126.



- a. Deflection of a deep beam.
- b. Bending stress distribution.

Figure 2.4. Bending stress distribution in deep beams.

If there is a concentrated force acting at the tip of a cantilever, the deflection due to bending is:

$$y_c = \frac{F \cdot l^3}{3 \cdot E \cdot I} \quad (\text{Eq. 2.2})$$

where F = Force perpendicular to the axis of the member, and applied at the tip.
 l = Length of the structural member.
 E = Modulus of elasticity of the structural material.

The shearing displacement is :

$$y_s = \frac{CS1 \cdot F \cdot l}{G \cdot A} \quad (\text{Eq. 2.3})$$

where $CS1$ = Shape factor which is equal to 1,2 for square shaped cross-sections(2.5).
 G = Shearing modulus of the structural material which is approximately equal to $(0,4 \cdot E)$ (2.5).
 A = Area of the cross-section.

The total deflection is :

$$y_T = y_c + y_s \quad (\text{Eq. 2.4})$$

(2.5) White, R.N., et. al., 'Structural Engineering', Vol. 2, p.96.

Substituting equation 2.2 and 2.3 in equation 2.4 yields;

$$y_T = \frac{F \cdot l^3}{3 \cdot E \cdot I} + \frac{CS \cdot l \cdot F \cdot l}{G \cdot A} \quad (\text{Eq. 2.5})$$

If the shape of the cross-section is a square, the moment of inertia will be :

$$I = \frac{d^4}{12} \quad (\text{Eq. 2.6})$$

Area of the cross-section will be :

$$A = d^2 \quad (\text{Eq. 2.7})$$

Substituting equations 2.6 and 2.7 in equation 2.5 yields ;

$$y_T = \frac{12 \cdot F \cdot l^3}{3 \cdot E \cdot d^4} + \frac{3 \cdot F \cdot l}{E \cdot d^2} \quad (\text{Eq. 2.8})$$

$$y_T = \frac{3 \cdot F \cdot l}{E \cdot d^2} \cdot \left(\frac{1,33 \cdot l^2}{d^2} + 1 \right) \quad (\text{Eq.2.9})$$

Equation 2.9 shows that the deflection due to bending moment is $(1,33 \cdot (l/d)^2)$ times the shearing displacement, ^(2.6) for cantilevers having solid square shaped cross-sections under the stated conditions.

It is generally accepted that, bending deformation starts to gain importance if the length to depth ratio of a cantilever is more than 2, and the importance of bending deformation dominates if the ^(2.7) length to depth ratio is more than 5.

It can be accepted that the slenderness ratios of skyscrapers change between 5 and 7. ^(2.8) The maximum slenderness ratio of a skyscraper should be around 7, because of the stiffness requirements to satisfy user ^(2.9) comfort.

(2.6) Ibid., p.57.

(2.7) Lin, T.Y., Stotesbury, S.D., 'Structural Concepts, and Systems for Architects, and Engineers', John Wiley, and Sons, Manhattan, 1981, p.218.

(2.8) Schueller, W., 'High-rise Building Structures', p.57.

(2.9) Chang, F.K., 'Human Responce Factors', A.S.C.E., I.A.B.S.E., 'Planning, and Design of Tall Buildings', V.C, p.137.

Skyscrapers are not solid structural members, and shear in the structure causes large shear deformations on all of its structural members. Because of this, deflection of these structures is equal to deflection due to overturning moment plus deflection due to shear forces acting on the structure.

The shearing mode of deflection is greater than the bending deflection in the case of all framed structures. In order not to increase all modes of deflection and stresses in the structural members, maximum slenderness ratio of the skyscrapers is accepted approximately as 7.

2.2. STRENGTH REQUIREMENTS OF SKYSCRAPERS.

2.2.1. Strength expectations from skyscrapers.

The effect of loadings acting on skyscrapers change from time to time, especially because of the dynamic character of lateral loads acting on the structure.

Skyscraper structures must be designed to resist both the usual and the maximum expected lateral loads. Because the expected life time of a skyscraper is much longer than that of an ordinary building and the maximum lateral load which may occur in the life time of the structure should not cause collapse of it. Because of this, the ductility and yield capacity of the structure becomes important. Figure 2.5 shows the expected load and deflection capacity of skyscrapers.

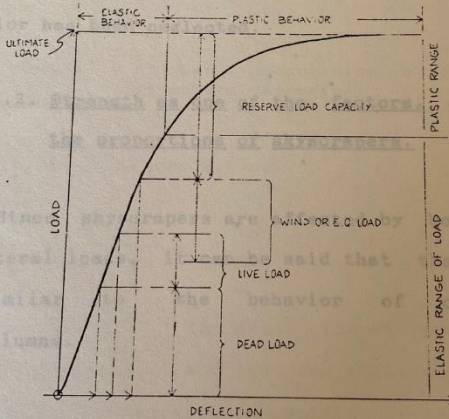


Figure 2.5. Expected load versus deflection capacity of skyscrapers.

Source: Lin, T., Stotesbury, S., 'Structural Concepts and Systems', p.150.

The reserve load capacity of the structure must satisfy safety requirements under the action of the maximum expected lateral loads. (2.10)

The stiffness of the structure becomes unimportant in the case of maximum expected loadings, but when usual conditions are considered it is the stiffness requirements that determine the dimensions of the cross-sections.

As this thesis deals only with usual conditions, the elastic behavior and the stiffness of the structure have been given greater importance and the plastic behavior has been neglected.

2.2.2. Strength as one of the factors, determining the proportions of skyscrapers.

Since skyscrapers are affected by both vertical and lateral loads, it can be said that their behavior is similar to the behavior of cantilevering beam-columns.

The maximum compressive stress in beam-columns can be found as :

$$f = f_c + f_{bmax} \quad (\text{Eq. 2.10})$$

(2.10) Schueller, W., 'High-rise Building', p. 48.

$$f = \frac{P}{A} + \frac{M \cdot d}{2 \cdot I} \quad (\text{Eq. 2.11})$$

If it is assumed that,

$$\frac{P}{A} = \frac{M \cdot d}{2 \cdot I} \quad (\text{Eq. 2.12})$$

the stress distribution in the cross-section will be as shown in Figure 2.2.a, and it can be said that,

$$\frac{P}{A} = \frac{M \cdot d}{2 \cdot I} = \frac{f_{callm}}{2} \quad (\text{Eq. 2.13})$$

where f_{callm} = Allowable compressive stress for the structural material.

If, as in Figure 2.2.c,

$$\frac{P}{A} > \frac{M \cdot d}{2 \cdot I} \quad (\text{Eq. 2.14})$$

one can say that the axial forces acting on the structure dominate, and that the axial stresses in the member must be decreased especially by increasing the area of the cross-section. If, on the other hand, as in Figure 2.2.b,

$$\frac{P}{A} < \frac{M \cdot d}{2 \cdot I} \quad (\text{Eq. 2.15})$$

one can say that, the bending forces acting on the structure dominate, and that moment of inertia of the cross section must be increased in order to decrease the bending stresses in the structure.

Tensile stresses will occur if maximum bending stress is greater than axial stresses, and this will bring many complications to the foundation design, especially when maximum expected lateral loads are concerned. Because of this, the limiting case for skyscrapers under the action of usual loads can be accepted as :

$$\frac{P}{A} \gg \frac{M_o \cdot D}{2 \cdot I} \quad (\text{Eq. 2.16})$$

This factor becomes more important, if the structural material is reinforced concrete, because of the weakness of concrete in resisting tensile stresses.

As understood from the explanation above, factors controlling stress distribution in the cross-section of the structure are :

- 1 - Proportions of the structure.
- 2 - Magnitude of lateral and vertical loads acting on the structure.
- 3 - Area and moment of inertia of the cross section.

(2.11) Schueller, W., 'High-rise Building', p.130.

2.3. STABILITY REQUIREMENT OF SKYSCRAPERS.

2.3.1. Overturning instability.

Since it will be better to reach condition in equation 2.16 under the action of usual lateral loads, resultant stress distribution in the structure will be as shown in Figure 2.2.a.

According to the above explanations, the equilibrium of the superstructure will be satisfied as shown in Figure 2.6.

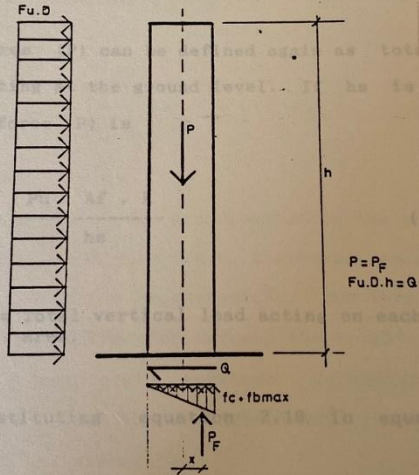


Figure 2.6. Equilibrium of superstructure under the action of usual lateral loads.

At the ground level overturning moment due to lateral loads is :

$$M_o = \frac{F_u \cdot D \cdot h^2}{2} \quad (\text{Eq. 2.17})$$

where F_u = Lateral load acting on each unit area of the surface.

This overturning moment must be balanced by resisting moment due to vertical loads which is :

$$MR = P \cdot x \quad (\text{Eq. 2.18})$$

Force (P) can be defined again as total vertical load acting at the ground level. If h_s is the storey height, force (P) is :

$$P = \frac{P_u \cdot A_f \cdot h}{h_s} \quad (\text{Eq. 2.19})$$

where P_u = Total vertical load acting on each unit floor area.

Substituting equation 2.19 in equation 2.18 yields;

$$MR = \frac{P_u \cdot A_f \cdot h}{h_s} \cdot x \quad (\text{Eq. 2.20})$$

Since,

$$M_o = MR \quad (\text{Eq. 2.21})$$

$$\frac{F_u \cdot D \cdot h^2}{2} = \frac{P_u \cdot A_f \cdot h}{hs} \cdot x \quad (\text{Eq. 2.22})$$

It can be said that, the height which can be reached by skyscrapers is :

$$h = 2 \cdot \frac{P_u}{F_u} \cdot \frac{D \cdot x}{hs} \quad (\text{Eq. 2.23})$$

Since the stress distribution on the cross-section is as shown in Figure 2.2.a, the magnitude of x depends on the shape of the cross-section, and this value may control the difference between the height ranges of various structural systems depending on the distribution of the structural material in the cross section of the structure.

2.3.2. Buckling instability.

Buckling of a structural member can be defined as a sudden movement away from the line of action of the compressive force. The tendency to buckle increases with decreasing bending stiffness.

Depending on their slenderness ratios, columns can be classified as long columns, intermediate columns, and short columns.

Short columns do not have such buckling instability problems, because under the action of compressive forces they may fail due to crushing, and this is not a stability problem.

In the case of long columns buckling is a stability problem. They buckle suddenly before using the whole capacity of the cross-section. But just after buckling, they lose all their reserve load capacity.

(2.12) White, R.N., et. al., 'Structural Engineering', Vol.3, p.67.

It is stated in many references that, skyscrapers behave like long columns under the action of vertical loads, (2.13) but in reality there will always be a lateral load acting on the structure. The combination of vertical and lateral loads is also seen in the case of beam-columns, and by studying the behavior of beam columns one can have an idea about buckling tendency of skyscrapers.

A beam-column does not buckle suddenly. Because as seen in Figure 2.7, it is already deflected under the action of lateral loads, and the application of a vertical load results in increasing deflection, (2.14) because of the additional moment which causes deflection of it.

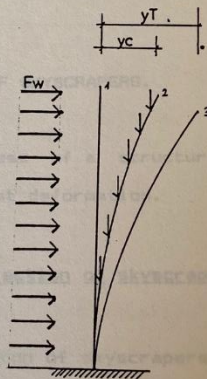


Figure 2.7. Deformation of a beam-column.

- (2.13) Salvadori, M., 'Structure in Architecture', 2. ed., Prentice Hall Inc., New Jersey 1975, p.198.
- (2.14) White, R. et al., 'Structural Engineering', V.3, p.97.

The explanation above shows that, the failure of a beam-column can be due to excessive deflection.

After the application of compressive force (P),
(2.15)
new deflection will be :

$$y_T = y_c \cdot FM \quad (\text{Eq. 2.24})$$

where FM= Magnification factor due to distributed axial force acting on a cantilever.
y_c= Deflection due to bending moment.

2.4. STIFFNESS OF SKYSCRAPERS.

The stiffness of a structure is a measure of its tendency to resist deformation.

2.4.1. Deflection of skyscrapers.

The deflection of skyscrapers under the action of lateral loads can be studied under three different headings. These are :

(2.15) Ibid., p.99.

- 1 - Deflection due to bending moment.
- 2 - Deflection due to shear in the structure.
- 3 - Increase of deflection, due to the effect of vertical loads.

2.4.1.1. Deflection due to bending moment: The deflection of a structure under the action of bending moment is very similar to the deflection of a cantilevering beam.

As the deflection of a cantilevering beam under the action of distributed lateral load is: (2.16)

$$y_c = \frac{1}{8} \cdot \frac{F_w \cdot l^4}{E \cdot I} \quad (\text{Eq. 2.25})$$

for skyscrapers, this amounts to :

$$y_c = \frac{1}{8} \cdot \frac{F_u \cdot D \cdot h^4}{E \cdot I} \quad (\text{Eq. 2.26})$$

As is apparent from equation 2.26, the deflection due to bending moment changes depending on the moment of inertia of the structural system.

(2.16) Ibid., p.19.

2.4.1.2. Deflection due to shear : The shear displacement of a solid flexural member is very small and it is usually neglected especially if the member is slender. But in the case of framed structures, the deflection due to shear in structural members becomes much more important than deflection due to bending as a whole. (2.17)

In the case of even the stiffest skyscrapers, the deflection due to shear in structural members is at least 3 times bigger than the deflection due to bending, and thus neither mode of deflection can be neglected.

Consider one bay of a planar frame under the action of lateral loads as shown in Figure 2.8. The deflection due to shear in structural members is :

$$y_m = y_{mv} + y_{mh} \quad (\text{Eq. 2.27})$$

where y_{mv} = Deflection due to the deformation of columns in one storey.

y_{mh} = Deflection due to the deformation of the girders in one storey.

(2.17) Ghali, A., Neville, A., 'Structural Analysis', 2.ed., Chapman and Hall Ltd., London 1978, p.356

(2.18) Schmertz, M.F., 'Office Building', p.182.

Figure 2.8 shows effect of the deformation of girders and columns on the deflection of the structure. (2.19) The deflection of the structure shown in Figure 2.8 is :

$$y_m = (y_{\text{column}}) + (\theta \cdot l_c) \quad (\text{Eq. 2.28})$$

$$y_m = (y_{\text{column}}) + \left(\frac{y_{\text{girder}}}{l_g} \cdot l_c \right) \quad (\text{Eq. 2.29})$$

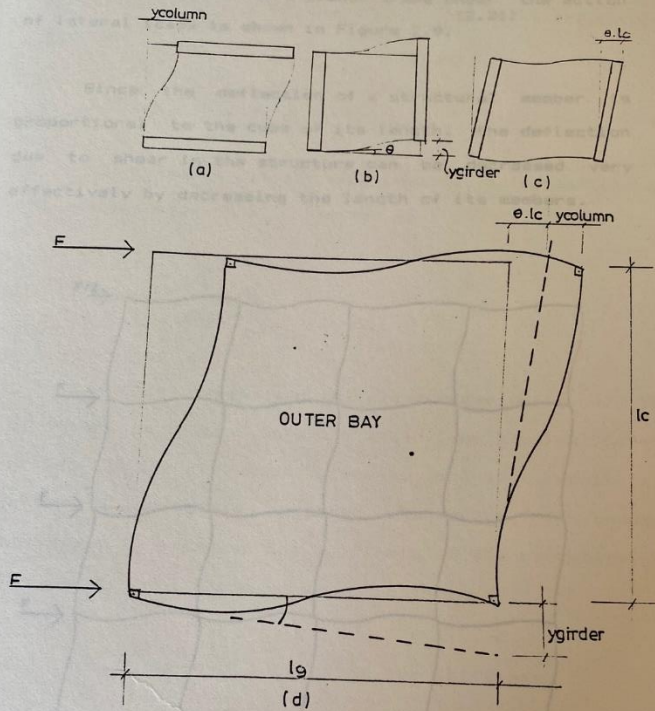
Equations 2.28 and 2.29 define the deflection of one bay. The total lateral deflection of a framed structure due to the bending moment acting on its members is equal to the total deflection of all storeys due to moment acting on the members.

If one considers the lower storeys and the interior structural members, by assuming that change of deflection for each bay is linear, the deflection due to shear in the structural members is approximately: (2.20)

$$y_m = \frac{y_{mv} + y_{mh}}{2} \cdot n \quad (\text{Eq. 2.30})$$

where n = Number of storeys.
 y_{mv} = Deflection due to deflection of vertical members in one storey.
 y_{mh} = Deflection due to deflection of horizontal members in one storey.

-
- (2.19) Grinter, L., 'Steel Structures', Vol.1, p.323.
 (2.20) Lin, T. Stotesbury, S., 'Structural Concepts and Systems', p. 234.



- a- Deflection of columns if girders are infinitely stiff.
- b- Deflection of girders if columns are infinitely stiff and if right lower joint can move up or down.
- c- Deflection of girders if columns are infinitely stiff and if joints can not move upwards.
- d- Deflection of a bay.

Figure 2.8. Deflection of girders, and columns of a framed structure.

The deflection of a planar frame under the action of lateral loads is shown in Figure 2.9. (2.21)

Since the deflection of a structural member is proportional to the cube of its length, the deflection due to shear in the structure can be decreased very effectively by decreasing the length of its members.

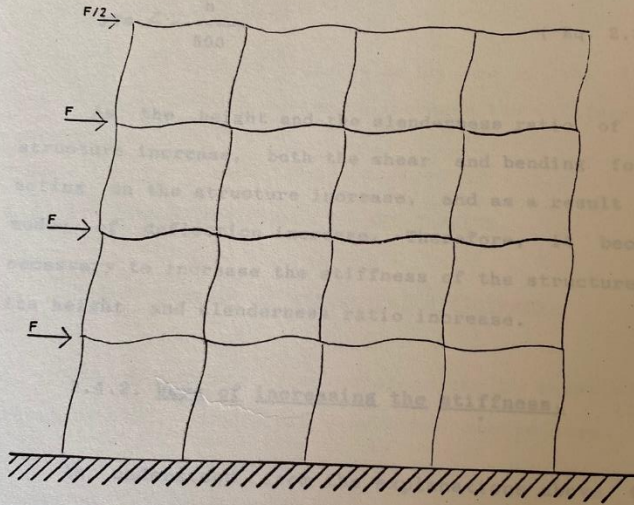


Figure 2.9. Deflection of a planar frame, under the action of lateral loads

(2.21) Faber, J., Mead, F., 'Reinforced Concrete', E. and F.N. Spon Ltd., 1961, p.210.

2.4.1.3. Increase of deflection due to the effect of vertical loads : When the effect of axial loads on a deflected structure is considered, the total deflection becomes ;

$$y_{Ts} = (y_c + y_m) \cdot FM \quad (Eq. 2.31)$$

In order to satisfy user comfort, total deflection of a skyscraper must be limited as : (2.22)

$$y_{Ts} \leq \frac{h}{500} \quad (Eq. 2.32)$$

As the height and the slenderness ratio of the structure increase, both the shear and bending forces acting on the structure increase, and as a result all modes of deflection increase. Therefore, it becomes necessary to increase the stiffness of the structure, if its height and slenderness ratio increase.

2.4.2. Ways of increasing the stiffness.

As understood from the explanations above there can be several basic ways of increasing the stiffness of a structure. These are :

-
- (2.22) Finzi, L., 'Summary Report', A.S.C.E., I.A.B.S.E., 'Planning and Design of Tall Buildings', Vol. 2, p.248.

- 1 - Decreasing height of the skyscraper.
- 2 - Increasing width of the skyscraper.
- 3 - Increasing the moment of inertia of the structure, without changing its slenderness ratio and height. (As in the case of tubes.)
- 4 - Increasing the stiffness of the structural members.
- 5 - Increasing the amount of the structural material used.

It is usually preferred to use one of the first four alternatives in order to increase the stiffness of structures.

2.5. STRUCTURAL EFFICIENCY OF SKYSCRAPERS.

2.5.1. Concept of structural efficiency.

The efficiency of skyscrapers can be measured by measuring their structural weight per usable area. (2.23)

Structural weight per usable area tends to increase with increasing slenderness ratio and with increasing height. (2.24)

-
- (2.23) Sokolow, A., 'Weight Analysis', ASCE, IABSE, 'Planning and Design of Tall Buildings', V.1a, p.603
- (2.24) Lin, T.Y., Stotesbury, S.D., 'Structural Concepts and Systems', p.131.

In the case of skyscrapers, it becomes very important to select an efficient structural system and to optimize it. If the structural system is not efficient, additional structural material is necessary to decrease its deflection. Increasing the stiffness by using additional structural material results in a parabolical increase in the structural weight per usable area versus height. (See Figure 2.10) The efficiency problem of skyscrapers is mostly due to the effect of lateral loads. As shown in Figure 2.10, the vertical loads affect the increase of structural weight (2.25) per usable area linearly.

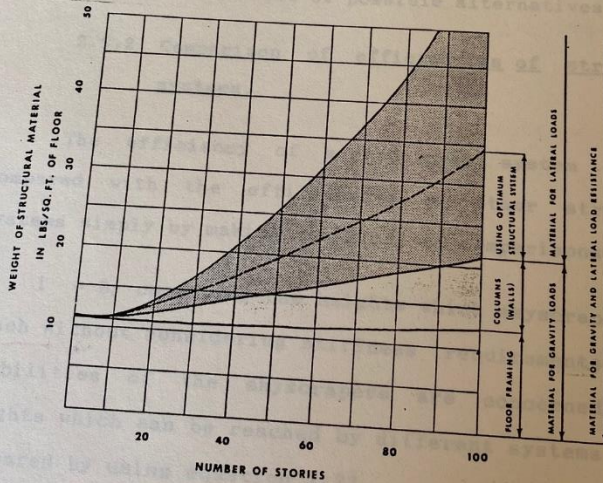


Figure 2.10. Variation of structural weight per usable area with increasing height.

Source: Schueller, W., 'High-rise Building', p.115.

(2.25) Schueller, W., 'High-rise Building', p.115.

The same figure shows that, if the selected structural system is efficient, the lateral loads also affect the increase of structural weight per usable area linearly. (2.26)

If proportions of a building are usual, the selection of an appropriate structural system and sizing of members can be done by the designer with the help of experience. But if the proportions are not usual, as in the case of skyscrapers, alternative structural systems and their alternative proportions must be studied with the help of appropriate analysis and design methods in order to select the best of possible alternatives.

2.5.2. Comparison of efficiencies of structural systems.

The efficiency of a structural system can be compared with the efficiencies of other structural systems simply by making two kinds of comparisons:

1 - By comparing the heights which skyscrapers can reach without considering stiffness requirements. When stabilities of the skyscrapers are concerned, the heights which can be reached by different systems can be compared by using equation 2.23.

2 - By comparing the heights which skyscrapers can reach, taking their stiffness into account.

(2.26) Ibid., p.115.

1- CONFIGURATION AND BEHAVIOR OF FRAMED TUBE SKYSCRAPERS

Since the lateral deflection due to the bending moment in the structure is inversely proportional to the moment of inertia, it can be said that :

$$\frac{yc1}{yc2} = \frac{I2}{I1} \quad \text{(Eq. 2.33)}$$

According to equation 2.30 the shear mode of deflections of two structures can be compared as:

$$\frac{ym1}{ym2} = \frac{ymh1 + ymv1}{ymh2 + ymv2} \quad \text{(Eq. 2.34)}$$

The distance between the opposite perimeter walls of framed tubes can reach 60 meters. At the first look, it may be seen as advantageous to use only the perimeter structure in resisting vertical loads in order to increase the magnitude of resisting moment, by increasing the magnitude of resisting moment, by increasing ratio of unit vertical load to unit lateral load. But in reality spanning distances like 60 meters is not economic for skyscrapers, because of two important reasons:-

(3.2)

(3.1) Schuller, M., 'High-Rise Buildings', p.103.
 (3.2) Lin, T.Y., Brodesbury, S.D., 'Structural Concepts and Methods', p.168.

3- CONFIGURATION AND BEHAVIOR OF FRAMED TUBE SKYSCRAPER STRUCTURES.

3.1. CONFIGURATION

Framed tubes, which are also called 'hollow tubes',^(3.1) are framed structures, but the structural members which resist lateral loads are placed at the perimeter of the structure in the form of surrounding framed surfaces and distance between the perimeter columns of the structure is at a maximum 4,5 meters. (See Figure 3.1)

The distance between the opposite perimeter walls of framed tubes can reach 60 meters. At the first look, it may be seen as advantageous to use only the perimeter structure in resisting vertical loads in order to increase the magnitude of resisting moment, by increasing ratio of unit vertical load to unit lateral load. But in reality spanning distances like 60 meters is not economic for skyscrapers, because of two important reasons:^(3.2)

-
- (3.1) Schuller, W., 'High-Rise Building', p.103.
(3.2) Lin, T.Y., Stotesbury, S.D., 'Structural Concepts, and Systems', p.168.

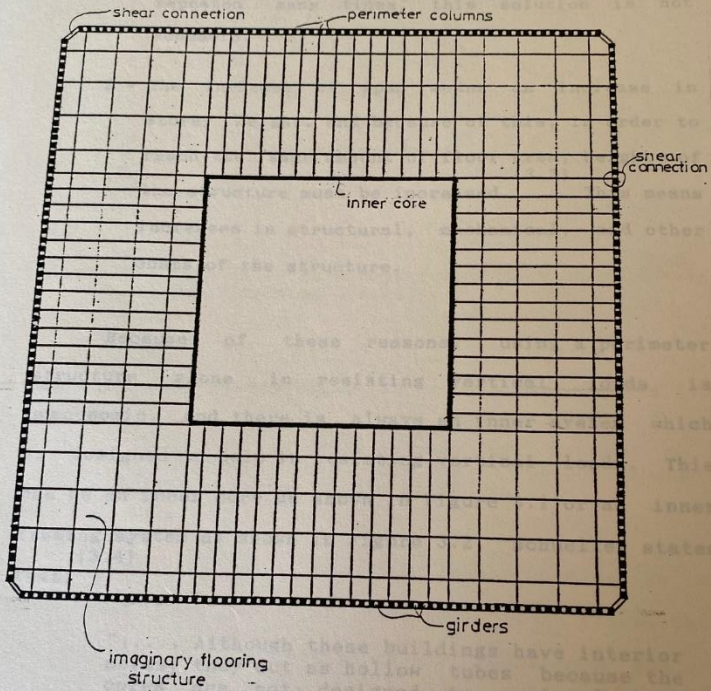


Figure 3.1 Plan of an imaginary framed tube which has an inner core.

1 - The structural material which is necessary for the flooring structure increases, and if one considers that the same flooring structure is repeated many times, this solution is not economic.

2 - The increase of span means an increase in storey height, and because of this, in order to reach the same amount of floor area, height of the structure must be increased. (3.3) This means increases in structural, mechanical, and other costs of the structure.

Because of these reasons, using a perimeter structure alone in resisting vertical loads is uneconomic, and there is always an inner system which is designed to help in resisting vertical loads. This can be an inner core as shown in Figure 3.1 or an inner framing system as shown in Figure 3.2. Schueller states (3.4) that:

"..... Although these buildings have interior cores, they act as hollow tubes because the cores are not designed to resist lateral loads...."

(3.3) Schueller, W., 'High-rise Building', p.188.

(3.4) Ibid., p.103.

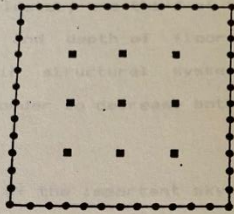


Figure 3.2 Plan of an imaginary framed tube which has an inner framing system.

(3.4)

Schueller also states that:

"..... The interior columns are assumed to carry gravity loads only and do not contribute to the exterior tube's stiffness...."

These statements should be interpreted so as to mean that the perimeter structure is so stiff that the effect of inner system on the resistance against lateral loads is very small, and this effect can be neglected. (In the case of tube in tube systems the inner core is especially designed or stiffened in order to resist lateral loads also.)

The core inside the perimeter structure can decrease the span of the flooring structure to a span which is approximately one third of the building width. Since weight and depth of flooring structures are important, their structural system must be selected carefully in order to decrease both their weight and depth.

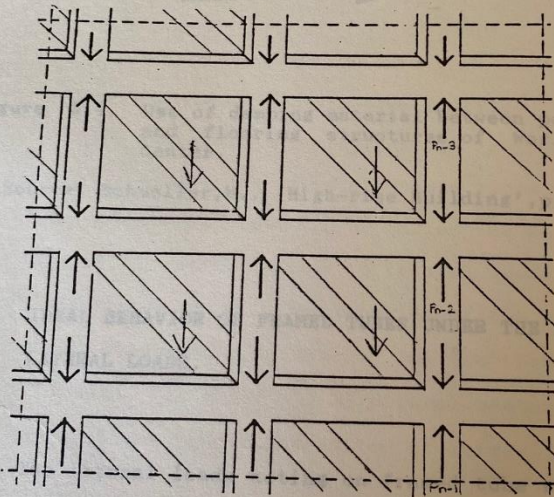
In most of the important skyscrapers, like the World Trade Center, composite floor systems have been used. (3.5) By using such a composite floor system, the strength and stiffness of the floor increases between 15 and 30 percent. (3.6) For example in the World Trade Center a composite flooring system, approximately 80 centimeters in depth, is used to span 18 meters.

Gravity loads transferred by the flooring structure act directly on the perimeter columns. Figure 3.3 indicates that the axial loads acting on the columns can not cause shear in the perimeter girders. Also, the use of damping material between flooring system and perimeter structure only enables the transfer of vertical loads to the columns. Figure 3.4 shows use of damping material between the flooring structure and the perimeter walls of the World Trade Center.

(3.5) Ibid., p.202.

(3.6) Ibid., p.198.

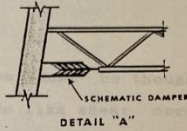
Damping materials are viscous (i.e. tend to flow under pressure like a liquid) and elastic (i.e. return to their original position, after the removal of the load). Viscoelastic materials resist shear forces, but they do not store energy like a spring. They convert energy into heat. After the forces are released, the material does not snap back like a spring, but slowly returns to its unstressed position. It works like a mechanism which is used in slowing down the closing of doors. (3.7) And as understood from the explanation above they mostly help in resisting dynamic lateral loads.



$$P_n > P_{n-1} > P_{n-2} > P_{n-3}$$

Figure 3.3. Transfer of vertical loads to the perimeter structure.

(3.7) Ibid., p.120



DETAIL "A"

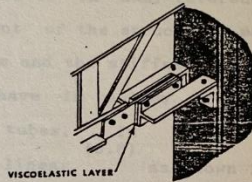


Figure 3.4. Use of damping material between perimeter, and flooring structures of World Trade Center.

Source: Schueller, W., 'High-rise Building', p.122.

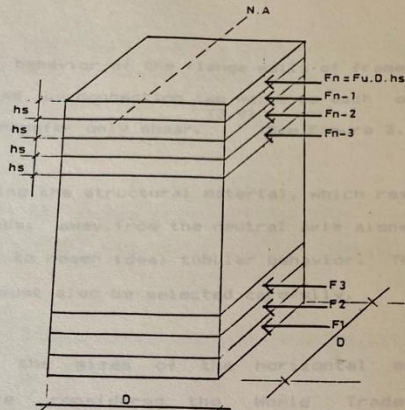
3.2. IDEAL BEHAVIOR OF FRAMED TUBES UNDER THE ACTION OF LATERAL LOADS.

The lateral loads acting on framed tube structures can be modeled as shown in Figure 3.5.

These structures must be thought of as three dimensional structures like shear cores, 'I' beams, etc. It is stated in many references that, because of the placement of the structural material away from the neutral axis and the stiffness of the girders, these structures behave in a way very similar to real cantilevering tubes, in which bending stress distribution is linear, (3.8) as shown in Figure 3.6.

Since the stress distribution in framed tubes is similar to the stress distribution in solid tubular cantilevering beams, the walls of framed tubes which are parallel to lateral loads can be called the web walls, and the walls perpendicular to lateral loads can be called the flange walls. The behavior of the web walls of framed tubes must be similar to the behavior of planar frames, in which all structural members behave like beam-columns under the action of lateral loads.

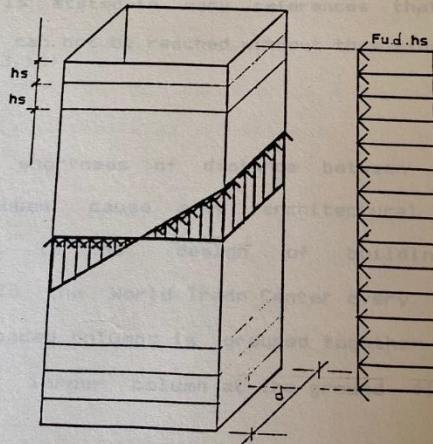
(3.8) Mainstone, R., 'Structural Form', p.88.



F_u = Unit pressure acting on the surface structure.

It is assumed that $F_n = F_{n-1} = F_{n-2} = \dots = F_1$

Figure 3.5. Lateral loads acting on a framed tube.



($F_u . d . hs$): Lateral load acting per hs.

Figure 3.6. Bending stress distribution in solid cantilevering beams.

The behavior of the flange walls of framed tubes, is realized by connecting two walls to each other in order to transfer only shear. (3.9) (See Figure 3.1)

Placing the structural material, which resists the lateral loads, away from the neutral axis alone is not sufficient to reach ideal tubular behavior. The length of girders must also be selected carefully.

When the sizes of the horizontal structural members are considered the World Trade Center constitutes a good example. The depth of the 1 meter long girders at the World Trade Center towers is around 130 centimeters. They are very stiff flexural members, and it is stated in many references that, tubular behavior can not be reached without this kind of stiff girders. (3.10)

The shortness of distance between columns of framed tubes cause many architectural problems, especially in the design of buildings lower storeys. In the World Trade Center every triplet of closely spaced columns is grouped together to form a single but larger column at the ground floor level.

(3.9) Khan, F., Navinchandra, R.A., 'Analysis and Design of Framed Tubes', p.55.

(3.10) Ibid., p.41.

In many framed tubes this problem is solved by increasing the distance between columns in different ways. Some examples of the solution are given in reference 3.11.

3.3 EFFECT OF GIRDER STIFFNESS ON THE BEHAVIOR OF FRAMED TUBES UNDER THE ACTION OF LATERAL LOADS.

3.3.1. Effect of girder stiffness on the behavior of web walls.

At the first look it seems like there are similarities between the behavior of the web walls of framed tubes and planar frames. Thus, as a beginning it can be said that, by studying the effect of girder stiffness on the behavior of planar frames one may have an idea about how the behavior of a web wall can be affected by stiffness of its girders.

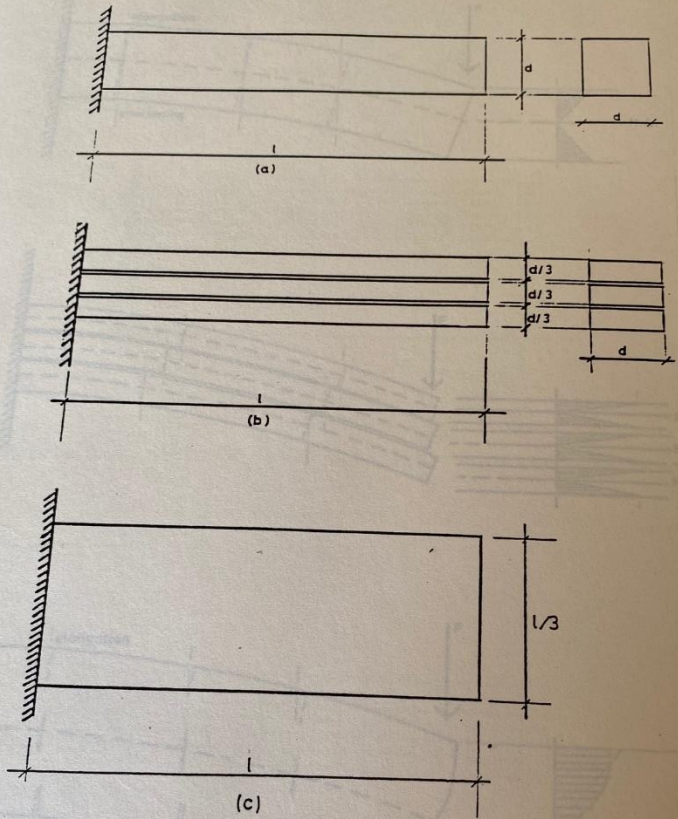
A convenient approach to understand the internal forces, which act on different kinds of planar frames under the action of lateral loads, is to study the stresses in differently configured solid cantilevering beams.

(3.11) Rich, R.C., 'Parking Design, and Requirements for High-rise Buildings', A.S.C.E., I.A.B.S.E., 'Planning, and Design of Tall Buildings', p.227.

3.3.1.1. Shear strain as a controlling factor of bending stress distribution in cantilevering beams: In order to understand the factors which control the magnitude and distribution of bending stresses in cantilevering beams, one can study the stresses in the three cantilevering beams shown in Figure 3.7. The deflected shapes and corresponding stress distributions of these beams are shown in Figure 3.8.

The monolithic cantilevering beam can also be thought of as a beam which consists of many rigidly connected slices, (Figure 3.9.) and the cantilevering beam, which consists of a number of slices, can be modeled as in Figure 3.10.

The bending stiffness of vertical connectors and joints approach infinity in the first one, and can be accepted as zero in the second one. The deflected shapes of these two alternatives are shown in Figure 3.11.



a- Monolithic cantilevering beam.

b- Cantilevering beam, consisting of a number of unconnected slices.

c- Deep beam.

Figure 3.7. Three different alternatives of cantilevering beams.

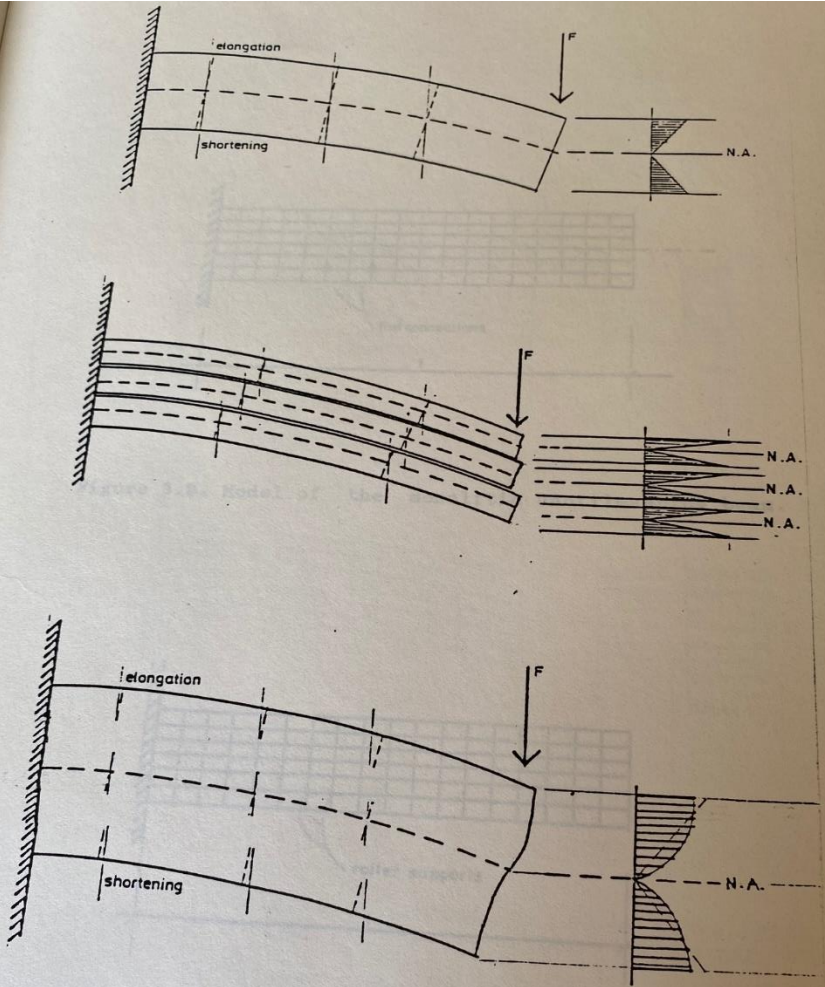


Figure 3.8. Deflected shapes and bending stress distributions of the cantilevering beams shown in Figure 3.7.

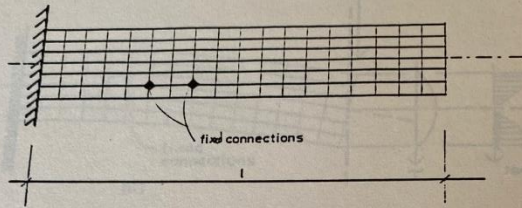


Figure 3.9. Model of the monolithic cantilevering beam.

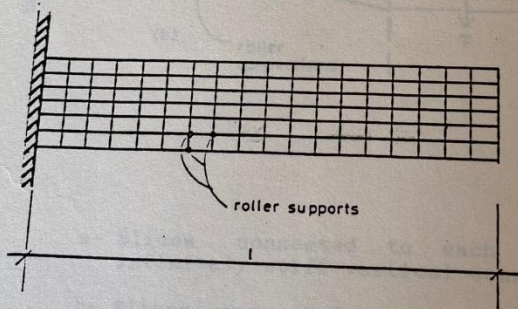
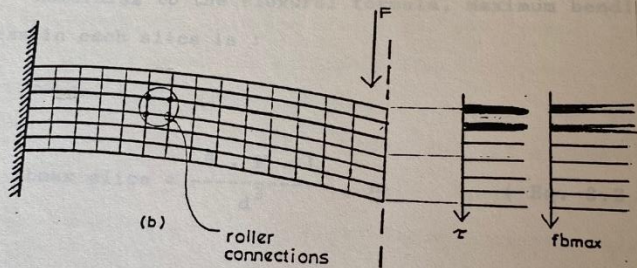
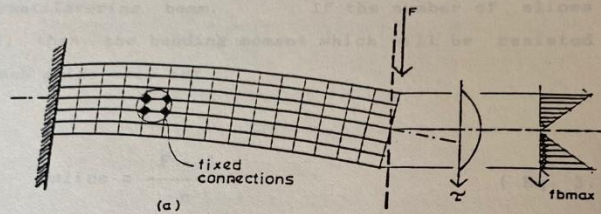


Figure 3.10. Model of the multi-piece cantilevering beam.



a- Slices connected to each other with infinitely stiff vertical connectors.

b- Slices connected to each other in order only to transfer vertical loads.

Figure 3.11. Deflected shapes of cantilevering beams which consist of many slices and bending and shear stress distribution in them.

If the slices are connected to each other so as to transfer only vertical loads, then the shear in the connecting members all through the cross section is zero, and each slice behaves as an individual cantilevering beam. (3.12) If the number of slices is n , then the bending moment which will be resisted by each slice will be:

$$M_{\text{slice}} = \frac{F \cdot l}{n} \quad (\text{Eq. 3.1})$$

According to the flexural formula, maximum bending stress in each slice is :

$$f_{b\text{max slice}} = \frac{6 \cdot F \cdot l}{d^3} \cdot n \quad (\text{Eq. 3.2})$$

Since the deflection of a solid cantilevering beam is:

$$y_c = \frac{M \cdot l^2}{3 \cdot E \cdot I} = \frac{P \cdot l^3}{3 \cdot E \cdot I} \quad (\text{Eq. 3.3})$$

(3.12) Salvadori, M., Heller, R., 'Structure in Architecture', p.149.

then the deflection of this cantilevering beam which consists of n unconnected pieces will be :

$$y_{\text{piece}} = y_{\text{solid}} \cdot n^2 \quad (\text{Eq. 3.4})$$

In reality if a structural member is not solid, the stiffness of joints which connect slices and vertical connectors can neither reach infinity and nor be zero. They can only be comparatively more or less stiff. In order to understand the effect of stiffness of the joints on the deflected shape, one should examine Figure 3.11.

If connectors between slices are not rigid they tend to deform under the action of shear forces. This affects their elongation and shortening as in the case of deep beams. (Figure 2.4.) Therefore, if connectors are not rigid, the stress in the interior slices increases when compared with solid slender cantilevers.

3.3.1.2. Effect of deflection of girders on the behavior of planar frames: Consider a planar frame consisting of horizontal girders and vertical columns. The girders of the planar frame can be thought of as shear connectors between vertical columns when the frame is deforming under the action of lateral loads. If the girders are stiff enough then the axial forces

acting on columns will be approximately as shown in
(3.13) Figure 3.12, and shear force acting on the girders
will be distributed as in the case of solid
cantilevering beams.

In order to satisfy the condition in Figure 3.12,
the girders must be stiff and the slenderness ratio of
(3.14) the structure must be more than 5.

The stiffness of a flexural structural member is
inversely proportional to the cube of its length. This
means that, if length of the girders increases as;

$$l_2 = l_1 \cdot c \quad (\text{Eq. 3.5})$$

deflection of the member becomes;

$$y_2 = y_1 \cdot c^3 \quad (\text{Eq. 3.6})$$

Thus, as the stiffness of girders decreases, the
effect of shear deformations increases especially at
places where shear is a maximum. Figures 3.13 and 3.14
show change of the distribution of axial forces in
columns, when the girders are not stiff. (3.15)

(3.13) Grinter, L.E., 'Steel Structures', Vol.2, p.320.

(3.14) Ibid., p.321.

(3.15) Grinter, L.E., 'Steel Structures', Vol.1, p.319.

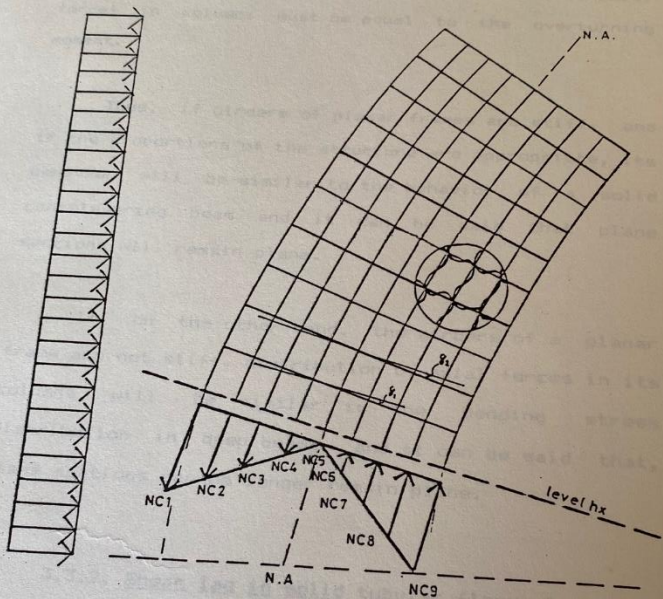


Figure 3.12. Deflection of girders of a planar frame and axial forces in its columns if girders are stiff.

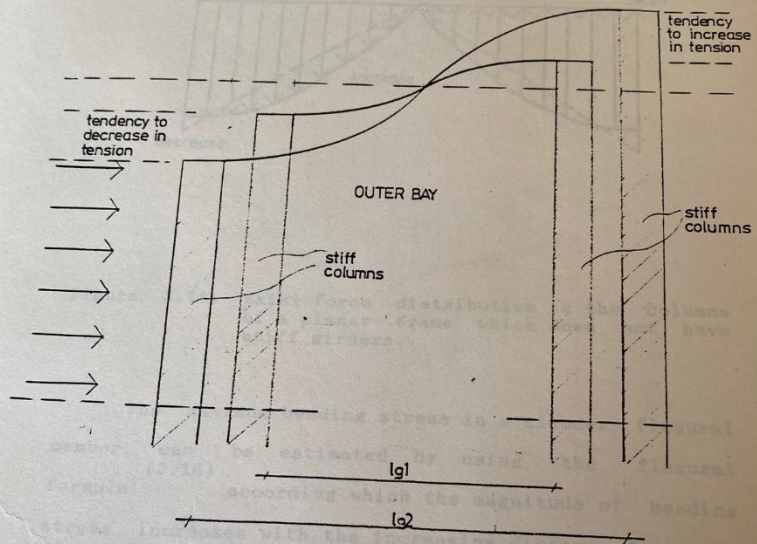
Since the axial forces which act on the inner columns tend to increase because of the deformation of the girders, the axial forces acting on outer columns must decrease, because the total moment created by axial forces in columns must be equal to the overturning moment.

Thus, if girders of planar frames are stiff, and if the proportions of the structure are appropriate, its behavior will be similar to the behavior of a solid cantilevering beam and it can be said that plane sections will remain plane.

If, on the other hand, the girders of a planar frame are not stiff, distribution of axial forces in its columns will be similar to the bending stress distribution in deep beams, and it can be said that, plane sections can no longer remain plane.

3.3.2. Shear lag in solid tubular flexural members

There are similarities between the behavior of framed tubes and tube shaped flexural members. In order to understand how similar they are, the factors which control tubular behavior must be clarified.



$lg2 = lg1 \cdot 1,5$
 $ygirder2 = ygirder1 \cdot (1,5)^3$
 Figure 3.13. Effect of girder deflection on axial forces in columns.

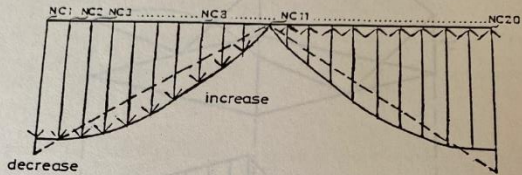


Figure 3.14. Axial force distribution in the columns of a planar frame which does not have stiff girders.

The maximum bending stress in a slender flexural member can be estimated by using the flexural formula, (3.16) according which the magnitude of bending stress increases with the increasing distance from the neutral axis. The bending stress distribution and the shear stress distribution in a solid tubular cross-section is as shown in Figure 3.15.

-
- (3.16) Lin, T.Y., Stotesbury, S.D., 'Structural Concepts and Systems', p.67.
- (3.17) Ersoy, U., Wasti, T., 'Introductory Mechanics of Deformable Bodies', M.E.T.U., Ankara, 1984, p.200

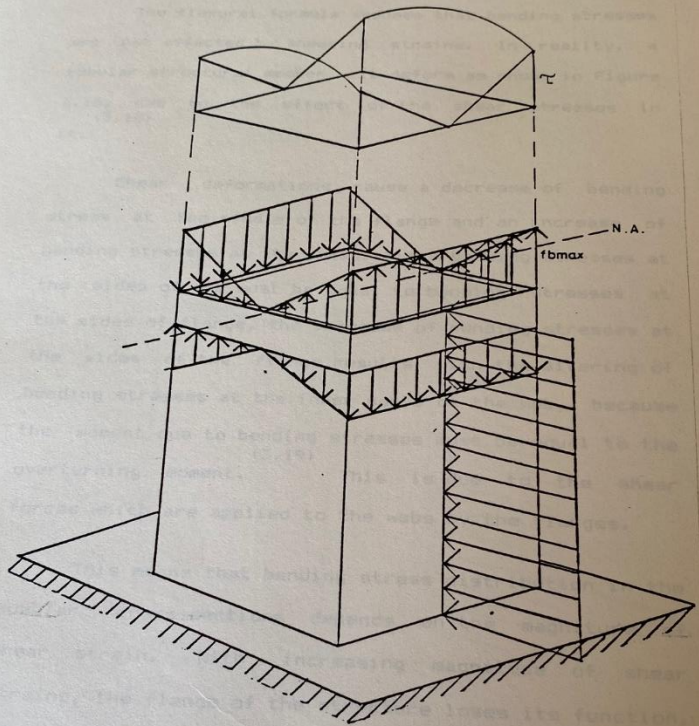


Figure 3.15. Shear stress distributions in tubular cross sections.

As seen in Figure 3.15, the shear forces are resisted largely by the web of the cross-section, and the bending forces are resisted mostly by the flange.

The flexural formula assumes that bending stresses are not affected by shearing strains. In reality, a tubular structural member will deform as shown in Figure 3.16, due to the effect of the shear stresses in it. (3.18)

Shear deformations cause a decrease of bending stress at the middle of the flange and an increase of bending stresses at the sides. Since bending stresses at the sides of web must be equal to bending stresses at the sides of flange, the increase of bending stresses at the sides of the flange results with the altering of bending stresses at the inner parts of the web, because the moment due to bending stresses must be equal to the overturning moment. (3.19) This is due to the shear forces which are applied to the webs by the flanges.

This means that bending stress distribution in the tubular cross-sections depends on the magnitude of shear strain. With increasing magnitude of shear strains, the flange of the structure loses its function. This kind of a structural member should not be accepted as a real tube.

(3.18) Oden, J., Ripperger, E., 'Mechanics', p.129.

(3.19) White, R.N., et. al., 'Structural Engineering', Vol.3, p.543.

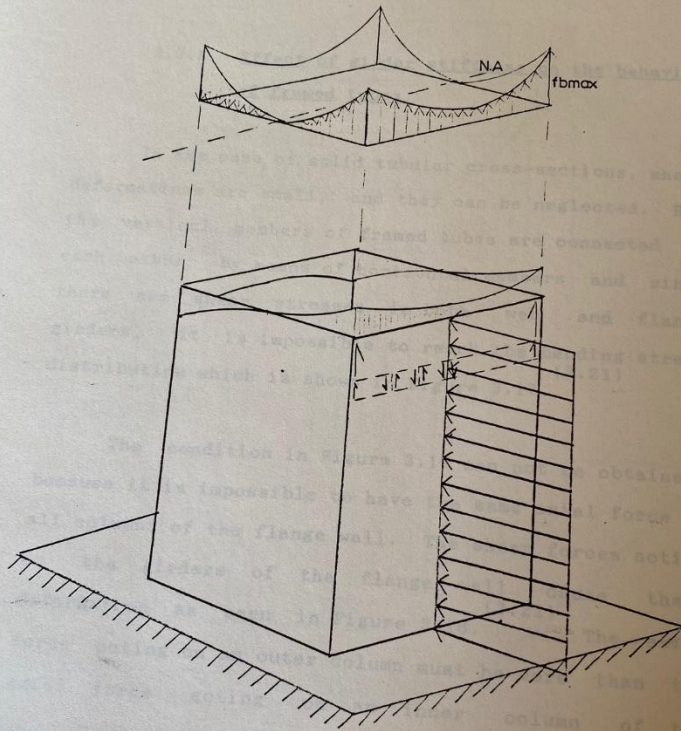


Figure 3.16. Effect of shear strains on the bending stress distribution in a tubular cross section.

This effect of shear strains on bending stress distribution is known as 'shear lag', and it may cause significant variations in the bending stress distribution in wide flange tubular and I beams. (3.20)

3.3.3. Effect of girder stiffness on the behavior of framed tubes.

In the case of solid tubular cross-sections, shear deformations are small, and they can be neglected. But the vertical members of framed tubes are connected to each other by means of horizontal members and since there are shear stresses in both web and flange girders, it is impossible to reach the bending stress distribution which is shown in Figure 3.17. (3.21)

The condition in Figure 3.17 can not be obtained, because it is impossible to have the same axial force in all columns of the flange wall. The shear forces acting on the girders of the flange wall cause their deformation as seen in Figure 3.18. (3.22) The axial force acting on an outer column must be more than the axial force acting on an inner column of the

(3.20) Oden, J.T., Ripperger, E.A., 'Mechanics', p.128.

(3.21) White, R.N., et. al., 'Structural Engineering', Vol.3, p.543.

(3.22) Ibid., p.543.

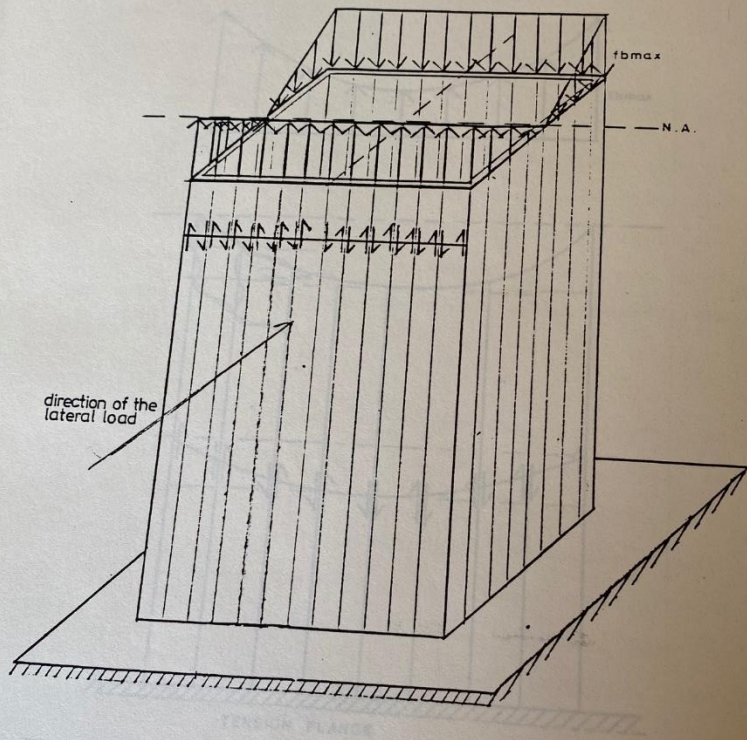


Figure 3.17. Ideal bending stress distribution which can not be reached by framed tubes.

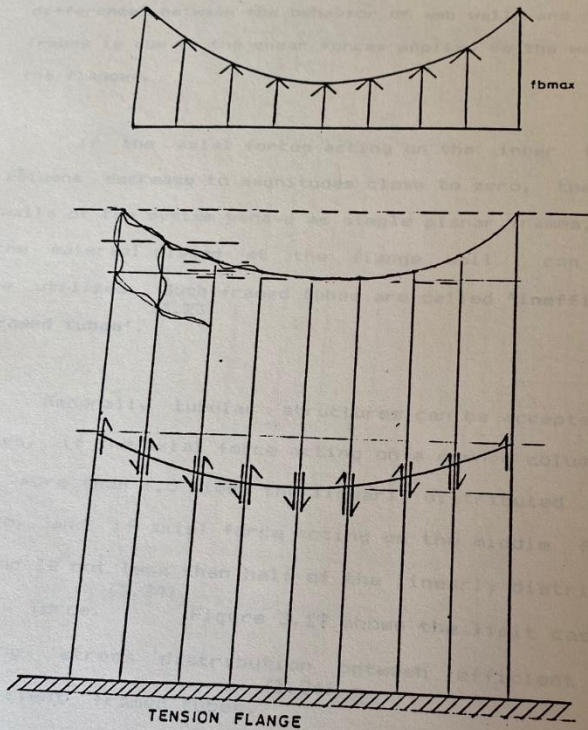


Figure 3.18. Deflection of girders of tension flange.

flange wall. Consequently, the axial forces acting on the inner columns of the web walls must also alter, because the total moment due to the axial loads acting on columns can not exceed the overturning moment. The difference between the behavior of web walls and planar frames is due to the shear forces applied to the webs by the flanges.

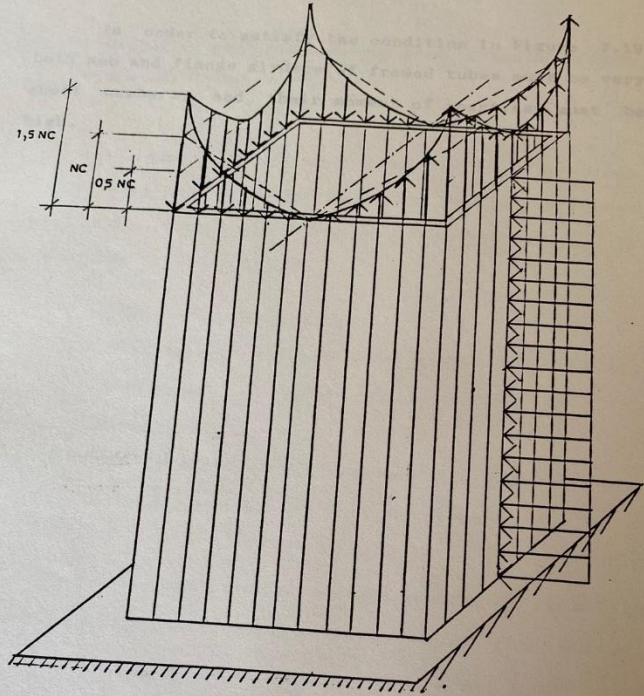
If the axial forces acting on the inner flange columns decrease to magnitudes close to zero, the web walls of the system behave as simple planar frames, and the material placed at the flange wall can not be utilized. Such framed tubes are called 'inefficient framed tubes'.
(3.23)

Generally tubular structures can be accepted as tubes, if the axial force acting on a corner column is not more than 1,5 times the linearly distributed axial force, and if axial force acting on the middle flange column is not less than half of the linearly distributed axial force.
(3.24)

Figure 3.19 shows the limit case of bending stress distribution between efficient and inefficient framed tubes.
(3.24)

(3.23) Smith, B.S., Coull, A., 'Elastic Analysis of Tall Concrete Buildings', A.S.C.E., I.A.B.S.E., 'Planning, and Design of Tall Buildings', p.157.

(3.24) Schmertz, M., 'Office Building', p.183.



----- When there is no shear lag.
 ———— When there is effect of shear lag.

Figure 3.19. Limit case of distribution of bending stresses, between efficient and inefficient framed tubes.

In order to satisfy the condition in Figure 3.19 both web and flange girders of framed tubes must be very short members, and their moment of inertias must be high.

The results which may be obtained are the maximum heights that can be reached by rigidly framed and framed tubes and how to obtain maximum heights by considering:

- 1 - The overturning stability.
- 2 - The lateral deflection regarding the overall stiffness of the cross-section of the structure.
- 3 - The lateral deflection regarding the stiffness of the vertical and horizontal members.

The study with respect to the moment of inertia of the tubes and the lateral deflection of the section will give an idea about the overall stiffness of lateral members. The study with respect to the individual members will give an indication about the lateral stiffness of the overall structure.

4. QUANTITATIVE COMPARISON OF THE PROPORTIONS OF PLANAR FRAMES AND FRAMED TUBES.

The reasons which may lie behind the different heights that can be reached by planar frames and framed tubes can best be studied quantitatively, by considering:

- 1 - The overturning stability.
- 2 - The lateral deflection regarding the overall stiffness of the cross-section of the structure.
- 3 - The lateral deflection regarding the stiffness of the vertical and horizontal members.

The study with respect to the moment of inertias of the section will give an idea about the overall bending mode of lateral deformation. The study with respect to the stiffness of the individual members will give an indication about the lateral displacement due to the overall shearing mode of deformation.

4.1. COMPARISON BASED ON OVERTURNING STABILITY.

It was stated in chapter two that when overturning stability is considered, the height which can be reached by a skyscraper is :

$$h = 2 \cdot \frac{P_u}{L_u} \cdot \frac{D \cdot x}{h_s} \quad (\text{Eq. 4.1})$$

where x = Lever arm between resultant vertical forces.

The magnitude of x depends on the type of distribution of the axial forces in the columns due to the vertical and lateral loads, and this value controls the difference between height ranges of different structural systems.

For slender planar frames, which have stiff girders, the axial force distribution will be as shown in Figure 4.1.

Planar frames which do not have stiff girders, will have an axial force distribution as shown in Figure 4.2.

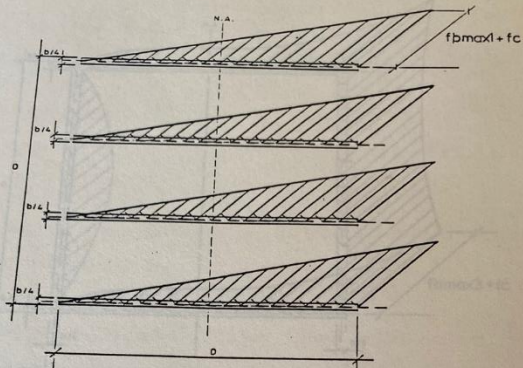


Figure 4.1. Axial force distribution in slender planar frames which have stiff girders.

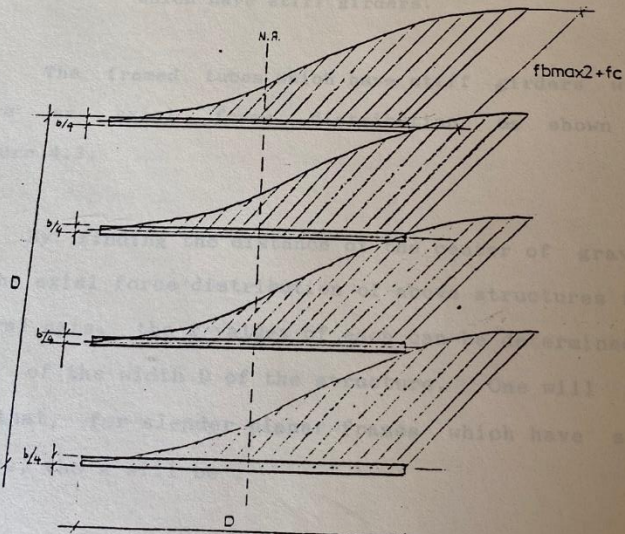


Figure 4.2. Axial force distribution in planar frames which do not have stiff girders.

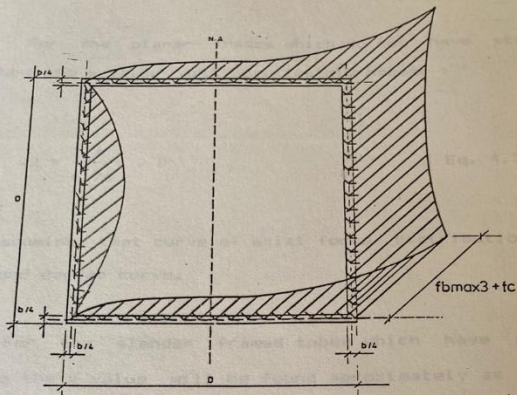


Figure 4.3. Axial force distribution in framed tubes which have stiff girders.

The framed tubes which have stiff girders will have an axial force distribution as shown in Figure 4.3.

By finding the distance of the center of gravity of the axial force distribution of above structures from neutral axis, the x values of each can be determined in terms of the width D of the structure. One will find out that, for slender planar frames which have stiff girders, the x will be :

$$x_1 = \frac{1}{6} \cdot D \quad (\text{Eq. 4.2})$$

For the planar frames which do not have stiff girders the x value will approximately become :

$$x_2 = \frac{1}{4.76} \cdot D \quad (\text{Eq. 4.3})$$

by assuming that curve of axial force distribution is a second degree curve.

For the slender framed tubes which have stiff girders the x value will be found approximately as :

$$x_3 = \frac{1}{4.95} \cdot D \quad (\text{Eq. 4.4})$$

by assuming that curve of axial force distribution is a second degree curve.

The calculation of the x values of these structures is given in Appendix 1. By using these values the maximum heights which can be reached by different structural systems can be compared as :

$$\frac{h_1}{h_2} = \frac{P_{u1} \cdot F_{u2} \cdot x_1}{P_{u2} \cdot F_{u1} \cdot x_2} \quad (\text{Eq. 4.5})$$

and the following results will be reached by using equation 4.5.

$$\frac{h(\text{framed tube})}{h(\text{stiff p. frame})} = 1,21 \quad (\text{Eq. 4.6})$$

$$\frac{h(\text{framed tube})}{h(\text{planar frame})} = 0,96 \quad (\text{Eq. 4.7})$$

$$\frac{h(\text{stiff p. frame})}{h(\text{planar frame})} = 0,78 \quad (\text{Eq. 4.8})$$

Since the planar frames which have stiff girders are preferred in the case of skyscrapers, because of the stiffness requirements it can be said that height which can be reached by framed tubes is 20 % greater than the height which can be reached by planar frames when stability of the structure is considered.

4.2. COMPARISON BASED ON THE LATERAL DEFLECTION OF STRUCTURAL SYSTEMS.

4.2.1. Comparison of deflection of the structural systems in the bending mode.

If the moment of inertia of the structure is constant all through the height of the structure, the deflection of a cantilever under the action of distributed lateral loads will be described as in equation 2.26. According to this, the deflection of skyscrapers due to overturning moment is inversely proportional to their moment of inertias.

The moment of inertia of a cross-section consisting of column sections can be computed as :

$$I = \sum (A \cdot d_1^2) \quad (\text{Eq. 4.9})$$

where A = Area of column.

d₁ = Distance of the area from neutral axis.

The cross-sections of planar frames are likely to vary between two alternatives which are shown in Figure 4.4. The moment of inertia of the cross-section, which is shown in Figure 4.4.a is approximately:

$$I(\text{plan frame}) = \frac{A}{4} \cdot \left(\frac{D}{2}\right)^2 \cdot 4 = \frac{A \cdot D^2}{4} \quad (\text{Eq. 4.10})$$

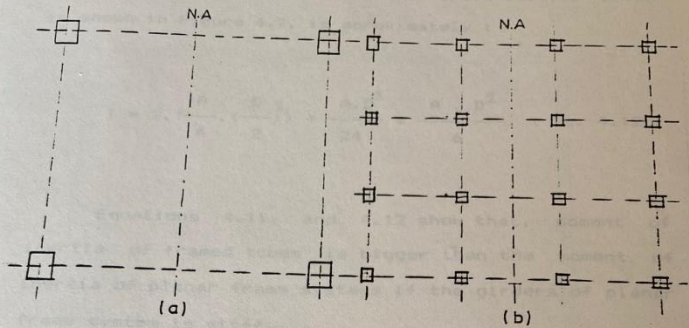
The moment of inertia of the cross-section of the planar frame which is shown in Figure 4.4.b must be very close to the moment of inertia of the cross-section which is shown in Figure 4.5.

The moment of inertia of the cross-section, which is shown in Figure 4.5 is approximately :

$$I(\text{stiff pla.fra.}) = 4 \cdot \left(\frac{(b/4) \cdot D^3}{12}\right) = \frac{A \cdot D^2}{12} \quad (\text{Eq. 4.11})$$

In the case of framed tubes, the structural material which resist both axial and bending forces is distributed in the cross-section as shown in Figure 4.6.

The moment of inertia of a framed tubes cross section must be very close to the moment of inertia of the tubular cross-section which is shown in Figure 4.7.



- a. Sparcely placed columns.
- b. Closely placed columns.

Figure 4.4. Distribution of the structural material in the cross-section of planar frames.

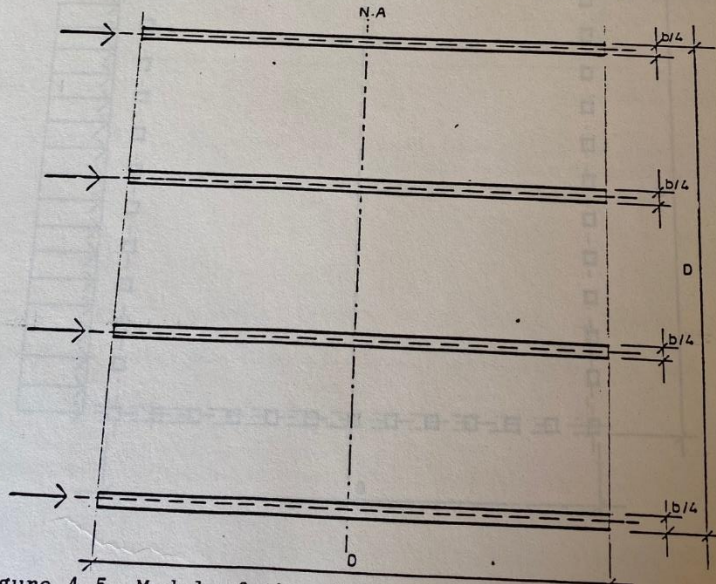


Figure 4.5. Model of the planar frame which is shown in Figure 4.4b, when the moment of inertia is considered.

The moment of inertia of the cross-section which is shown in Figure 4.7, is approximately :

$$I = 2 \cdot \left(\frac{A}{4} \cdot \left(\frac{D}{2} \right)^2 \right) + \frac{A \cdot D^2}{24} = \frac{A \cdot D^2}{6} \quad (\text{Eq. 4.12})$$

Equations 4.11, and 4.12 show that, moment of inertia of framed tubes is bigger than the moment of inertia of planar frame systems if the girders of planar frame system is stiff.

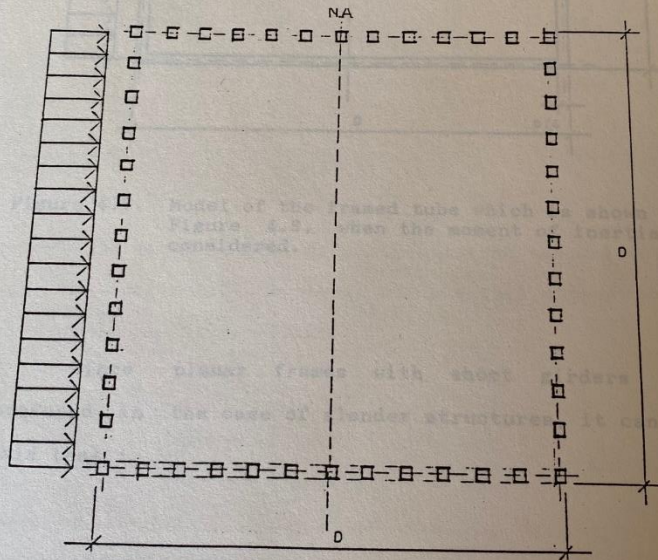


Figure 4.6. Distribution of the structural material in the cross-section of the framed tubes.

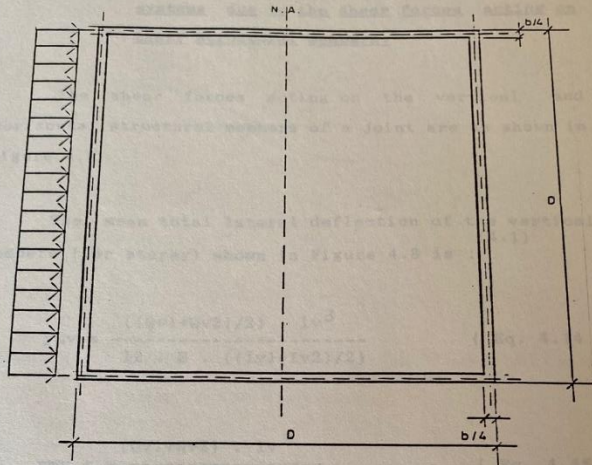


Figure 4.7. Model of the framed tube which is shown in Figure 4.6, when the moment of inertia is considered.

Since planar frames with short girders are preferred in the case of slender structures, it can be said that :

$$\frac{y_c(\text{plan. frame})}{y_c(\text{framed tube})} = 2 \quad (\text{Eq. 4.13})$$

4.2.2. Comparison of the deflections of structural systems due to the shear forces acting on their structural members.

The shear forces acting on the vertical and horizontal structural members of a joint are as shown in Figure 4.8.

The mean total lateral deflection of the vertical members (per storey) shown in Figure 4.8 is : ^(4.1)

$$y_{mv} = \frac{((Q_v1+Q_v2)/2) \cdot l_v^3}{12 \cdot E \cdot ((I_v1+I_v2)/2)} \quad (\text{Eq. 4.14})$$

$$y_{mv} = \frac{(Q_v1+Q_v2) \cdot l_v^3}{12 \cdot E \cdot (I_v1+I_v2)} \quad (\text{Eq. 4.15})$$

The total lateral deflection due to the deflection of the horizontal structural members (per storey) which are shown in Figure 4.8 is : ^(4.2)

$$y_{mh} = \frac{(Q_h1+Q_h2) \cdot l_h^3}{12 \cdot E \cdot (I_h1+I_h2)} \cdot \frac{l_v}{l_h} \quad (\text{Eq. 4.16})$$

(4.1) Schueller, W., 'High-rise Building', p.172

(4.2) Lin, T., Stotesbury, S., 'Structural Concepts and Systems'.

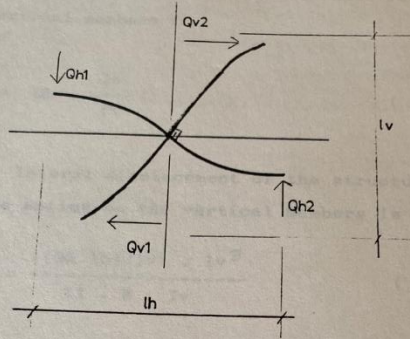


Figure 4.8. Shear forces acting on vertical and horizontal structural members of a joint in a framed structure.

According to equations 4.15 and 4.16 deflection of the structure due to shear in its members is proportional to magnitude of shear force acting on its structural members, and cube of the length of its structural members.

The shear force acting on the columns of systems can be determined from the equilibrium of the joint which is shown in Figure 4.8, by writing:

$$Q_h \cdot l_h = Q_v \cdot l_v \quad (\text{Eq. 4.17})$$

According to equation 4.17, the shear force acting on the vertical members is :

$$Q_v = Q_h \cdot \frac{lh}{lv} \quad (\text{Eq. 4.18})$$

The lateral displacement of the structure due to shear force acting on the vertical members is therefore:

$$y_{mv} = \frac{((Q_h \cdot lh)/lv) \cdot lv^3}{12 \cdot E \cdot I_v} \quad (\text{Eq. 4.19})$$

$$y_{mv} = \frac{Q_h \cdot lh \cdot lv^2}{12 \cdot E \cdot I_v} \quad (\text{Eq. 4.20})$$

According to equation 2.34 the deflection of structural systems due to the shear forces acting on them can be compared as :

$$\frac{y_{m1}}{y_{m2}} = \frac{y_{mv1} + y_{mh1}}{y_{mv2} + y_{mh2}} \quad (\text{Eq. 4.21})$$

It can be said that there will be differences between the girder lengths of planar frames and framed tubes as :

$$lh_2 = c \cdot lh_1 \quad (\text{Eq. 4.22})$$

This is due to the fact that it is easier to increase stiffness of planar frame systems because they are not affected from shear lag. Thus, it can be said that the girder length of a stiff planar frame can be more than the girder length of a framed tube.

By substituting equations 4.16 and 4.20 in equation 4.21 it can be said that :

$$\frac{y_{m1}}{y_{m2}} = \frac{Q_{h1}}{Q_{h2}} \cdot \left(\frac{1}{c} \right) \cdot \frac{I_{h2}}{I_{h1}} \cdot \frac{1}{l_{v2}} \cdot \frac{1 + \left(\frac{l_{v1}}{l_{h1}} \cdot \frac{I_{h1}}{I_{v1}} \right)}{1 + \left(\frac{l_{v2}}{l_{h1} \cdot c} \cdot \frac{I_{h2}}{I_{v2}} \right)}$$

The derivation of equation 4.23 is shown in appendix 2.

Using equation 4.23 with the pertinent values of the structural systems to be compared, one can obtain a comparison of the possible proportions of these systems.

According to Schmertz the shearing mode of deflection of framed tubes is 3 times greater than their bending mode of deflection. (4.3) Drecho states that shearing mode of deflection of planar frames is 7 or 10 times greater than their bending mode of deflection. (4.4) Depending on the explanations above it can be said that shearing mode of deflection of planar frames is more than shearing mode of deflection of framed tubes.

4.2.3. Comparison of total deflections of the framed structural systems.

In the case of skyscrapers total deflection due to the overturning moment and shear forces must be multiplied with the magnification factor because of the effect of vertical loads acting on them. (see eq. 2.31)

If one neglects this factor, the total deflection of different structural systems can be compared as :

$$\frac{y_{Ts1}}{y_{Ts2}} = \frac{y_{c1} + y_{m1}}{y_{c2} + y_{m2}} \quad (\text{Eq. 4.24})$$

(4.3) Schmertz, M.F., "Office Building", p.182.

(4.4) Drecho, A. T., "Frames and frame shear wall systems", ACI, "Multistory concrete structures", p.23.

According to the explanations above it can be stated that the height which can be reached by framed tubes is more than the height which can be reached by planar frames. The same result can be obtained by examining Table 1.2.

All structural systems must satisfy the requirements of:

- 1 - Strength,
- 2 - Stability,
- 3 - Stiffness,
- 4 - Structural efficiency.

By experience, it is known that during the structural design of ordinary structures, the requirement of strength dominates over the other requirements. But as the overall slenderness ratio of the structure increases, three important factors start to determine the relative importance of these structural requirements. These factors can be described as follows:

1 - With increasing overall slenderness ratio, the tendency to overturn increases.

2 - With increasing overall slenderness ratio, the tendency to deflect laterally under lateral loads increases.

5. CONCLUSION

All structural systems must satisfy the requirements of;

- 1 - Strength.
- 2 - Stability.
- 3 - Stiffness.
- 4 - Structural efficiency.

By experience, it is known that during the structural design of ordinary structures, the requirement of strength dominates over the other requirements. But as the overall slenderness ratio of the structure increases, three important factors start to determine the relative importance of these structural requirements. These factors can be described as follows:

- 1 - With increasing overall slenderness ratio, the tendency to overturn increases.
- 2 - With increasing overall slenderness ratio, the tendency to deflect laterally under lateral loads increases.

3 - The deflection of structures must be limited for user comfort. Thus, in order to satisfy the stiffness requirement, the amount of structural material used tends to increase with increasing slenderness ratio.

The factors above show that, with increasing overall slenderness ratio, the requirements of stability and stiffness start to be more important than the strength requirement, especially under usual conditions.

In order to understand factors determining the height which can be reached by the skyscraper structures, the factors of stability and stiffness have been considered in the second chapter of this thesis.

In the third chapter of the thesis configuration of framed tubes is explained and it is shown that tubular behavior can not be reached without the use of stiff girders.

In the fourth chapter the heights which can be reached by framed tubes have been compared with the heights which can be reached by planar frames when their stabilities and stiffnesses are considered.

It is shown that the height which can be reached by framed tubes is 20 % greater than the height which can be reached by planar frames which have stiff girders, when their stability requirements are considered.

In order to compare stiffnesses of two structures both their bending mode and shearing mode of deflections have been compared. It is shown that bending mode of deflection of planar frames which have stiff girders is two times the bending mode of deflection of framed tubes.

The shearing mode deflection of two systems can be compared by considering individual structures. The equations necessary for this comparison have been presented, and it is explained that shearing mode of deflection of planar frames is more than shearing mode of deflection of framed tubes.

In conclusion, it can be said that by replacing columns to the perimeter of the structure and by using stiff girders, the height which can be reached by the structure increases, and this is mainly due to the change in the stiffness of the structural members.

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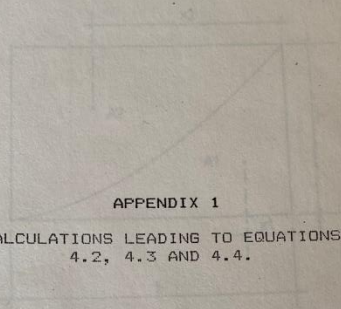
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APPENDICES

APPENDIX 1. Calculations leading to equations 4.2, 4.3 and 4.4.

4.1. Area and centre of mass of parts of a rectangle

Curve to the left down



APPENDIX 1

CALCULATIONS LEADING TO EQUATIONS 4.2, 4.3 AND 4.4.

Figure 4.1. Area of curves to the left down.

$$A_1 = \int_0^x y \, dx \quad \text{(Eq. 4.1)}$$

$$A_2 = \int_x^a y \, dx \quad \text{(Eq. 4.2)}$$

$$A = \int_0^a y \, dx \quad \text{(Eq. 4.3)}$$

$$A_1 = \int_0^x y \, dx \quad \text{(Eq. 4.4)}$$

APPENDIX 1. Calculations leading to equations 4.2, 4.3, and 4.4.

A.1.1. Areas and centroids of parts of a rectangle.

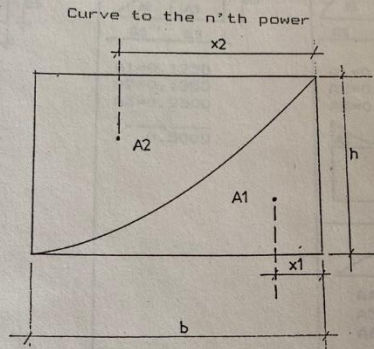


Figure A1.1. Areas of curves to the n^{th} power.

$$A1 = \frac{b \cdot h}{n + 1} \quad (\text{Eq. A1.1})$$

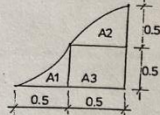
$$x1 = \frac{b}{n + 2} \quad (\text{Eq. A1.2})$$

$$A2 = \frac{n \cdot b \cdot h}{n + 1} \quad (\text{Eq. A1.3})$$

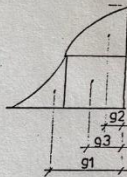
$$x2 = \left(\frac{n + 3}{2} \right) \cdot x1 \quad (\text{Eq. A1.4})$$

Table A1.1. Calculations leading to equations 4.2, 4.3, and 4.4.

PLANAR FRAME
(not stiff)



$$\begin{aligned}
 A1 &= 0.083D \\
 A2 &= 0.167D \\
 A3 &= 0.250D \\
 + \text{-----} \\
 &0.500D
 \end{aligned}$$



$$\begin{aligned}
 g1 &= 0.625D \\
 g2 &= 0.188D \\
 g3 &= 0.250D
 \end{aligned}$$

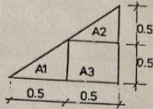
$$\begin{aligned}
 g1 \cdot A1 &= 0.052D^2 \\
 g2 \cdot A2 &= 0.031D^2 \\
 g3 \cdot A3 &= 0.062D^2
 \end{aligned}$$

$$\begin{aligned}
 + \text{-----} \\
 &0.145D^2
 \end{aligned}$$

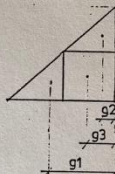
$$\begin{aligned}
 g_{tot} &= 0.290D \\
 0.5D - 0.290D &= 0.21D
 \end{aligned}$$

$$\frac{1}{4.76} \cdot D$$

PLANAR FRAME
(stiff)



$$\begin{aligned}
 A1 &= 0.125D \\
 A2 &= 0.125D \\
 A3 &= 0.250D \\
 + \text{-----} \\
 &0.500D
 \end{aligned}$$



$$\begin{aligned}
 g1 &= 0.167D \\
 g2 &= 0.666D \\
 g3 &= 0.250D
 \end{aligned}$$

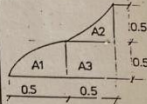
$$\begin{aligned}
 g1 \cdot A1 &= 0.021D^2 \\
 g2 \cdot A2 &= 0.083D^2 \\
 g3 \cdot A3 &= 0.063D^2
 \end{aligned}$$

$$\begin{aligned}
 + \text{-----} \\
 &0.167D^2
 \end{aligned}$$

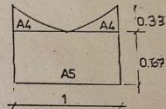
$$\begin{aligned}
 g_{tot} &= 0.330D \\
 0.5D - 0.330D &= 0.17D
 \end{aligned}$$

$$\frac{1}{6} \cdot D$$

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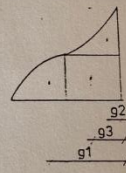


$$\begin{aligned}
 A1 &= 0.167D \\
 A2 &= 0.083D \\
 A3 &= 0.250D
 \end{aligned}$$



$$\begin{aligned}
 A4 &= 0.111D \\
 A5 &= 0.667D \\
 A6 &= 0.222D
 \end{aligned}$$

$$A_{tot} = 2.000D$$



$$\begin{aligned}
 g1 &= 0.688D \\
 g2 &= 0.125D \\
 g3 &= 0.250D
 \end{aligned}$$

$$\begin{aligned}
 g1 \cdot 2A1 &= 0.229D^2 \\
 g2 \cdot 2A2 &= 0.021D^2 \\
 g3 \cdot 2A3 &= 0.125D^2 \\
 g4 \cdot A4 &= 0.000D^2 \\
 g5 \cdot A5 &= 0.000D^2 \\
 g6 \cdot A6 &= 0.222D^2
 \end{aligned}$$

$$\begin{aligned}
 + \text{-----} \\
 &0.597D^2
 \end{aligned}$$

$$\begin{aligned}
 g_{tot} &= 0.298D \\
 0.5D - 0.298D &= 0.202D
 \end{aligned}$$

$$\frac{1}{4.95} \cdot D$$

APPENDIX 2
DERIVATION OF EQUATION 4.23

APPENDIX 2. Derivation of equation 4.23.

According to equation 4.21 :

$$\frac{y_{m1}}{y_{m2}} = \frac{y_{mh1} + y_{mv1}}{y_{mh2} + y_{mv2}} \quad (\text{Eq. A2.1})$$

$$y_{mh} = \frac{Q_h \cdot I_h^2 \cdot I_v}{12 \cdot E \cdot I_h} \quad (\text{Eq. A2.2})$$

$$y_{mv} = \frac{Q_h \cdot I_h \cdot I_v^2}{12 \cdot E \cdot I_v} \quad (\text{Eq. A2.3})$$

$$\frac{y_{m1}}{y_{m2}} = \frac{\frac{Q_{h1} \cdot I_{h1}^2 \cdot I_{v1}}{12 \cdot E \cdot I_{h1}} + \frac{Q_{h1} \cdot I_{h1} \cdot I_{v1}^2}{12 \cdot E \cdot I_{v1}}}{\frac{Q_{h2} \cdot I_{h2}^2 \cdot I_{v2}}{12 \cdot E \cdot I_{h2}} + \frac{Q_{h2} \cdot I_{h2} \cdot I_{v2}^2}{12 \cdot E \cdot I_{v2}}} \quad (\text{Eq. A2.4})$$

$$\frac{y_{m1}}{y_{m2}} = \frac{Q_{h1}}{Q_{h2}} \cdot \frac{(I_{h1}^2 \cdot \frac{I_{v1}}{I_{h1}}) + (I_{h1} \cdot \frac{I_{v1}^2}{I_{v1}})}{(I_{h2}^2 \cdot \frac{I_{v2}}{I_{h2}}) + (I_{h2} \cdot \frac{I_{v2}^2}{I_{v2}})} \quad (\text{Eq. A2.5})$$

$$\frac{y_{m1}}{y_{m2}} = \frac{Q_{h1}}{Q_{h2}} \cdot \frac{I_{h1}}{I_{h2}} \cdot \frac{\frac{I_{v1} \cdot I_{h1}}{I_{h1}} + \frac{I_{v1}^2}{I_{v1}}}{\frac{I_{v2} \cdot I_{h2}}{I_{h2}} + \frac{I_{v2}^2}{I_{v2}}} \quad (\text{Eq. A2.6})$$

$$\frac{y_{m1}}{y_{m2}} = \frac{Q_{h1}}{Q_{h2}} \cdot \frac{l_{h1}}{l_{h2}} \cdot \frac{I_{h2} \cdot I_{v2}}{I_{h1} \cdot I_{v1}} \cdot \frac{(I_{v1} \cdot l_{v1} \cdot l_{h1}) + (I_{h1} \cdot l_{v1}^2)}{(I_{v2} \cdot l_{v2} \cdot l_{h2}) + (I_{h2} \cdot l_{v2}^2)}$$

(Eq. A2.7)

$$\frac{y_{m1}}{y_{m2}} = \frac{Q_{h1}}{Q_{h2}} \cdot \frac{l_{h1}}{l_{h2}} \cdot \frac{I_{h2} \cdot I_{v2}}{I_{h1} \cdot I_{v1}} \cdot \frac{l_{v1}}{l_{v2}} \cdot \frac{(I_{v1} \cdot l_{h1}) + (I_{h1} \cdot l_{v1})}{(I_{v2} \cdot l_{h2}) + (I_{h2} \cdot l_{v2})}$$

(Eq. A2.8)

$$\frac{y_{m1}}{y_{m2}} = \frac{Q_{h1}}{Q_{h2}} \cdot \frac{l_{h1}}{l_{h2}} \cdot \frac{I_{h2}}{I_{h1}} \cdot \frac{l_{v1}}{l_{v2}} \cdot \frac{l_{h1} + (l_{v1} \cdot \frac{I_{h1}}{I_{v1}})}{l_{h2} + (l_{v2} \cdot \frac{I_{h2}}{I_{v2}})}$$

(Eq. A2.9)

$$\frac{y_{m2}}{y_{m1}} = \frac{Q_{h1}}{Q_{h2}} \cdot \frac{1}{c} \cdot \frac{I_{h2}}{I_{h1}} \cdot \frac{l_{v1}}{l_{v2}} \cdot \frac{1 + (\frac{l_{v1}}{l_{h1}} \cdot \frac{I_{h1}}{I_{v1}})}{1 + (\frac{l_{v2}}{l_{h1} \cdot c} \cdot \frac{I_{h2}}{I_{v2}})}$$

(Eq. A2.10)