Simultaneous Scheduling of Preventive Maintenance and Production for Single and Parallel Machines

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ABSTRACT

In the last decades, the simultaneous scheduling of production and preventive maintenance has been receiving a considerable attention. Initially, in most researches, maintenance activities were treated as tasks with a fixed period. However, this assumption leads to create a hole in the time horizon. Recently, the variations in maintenance times were addressed, but the starting time is still fixed and known in advance in most of the works. There are few researches that consider the maintenance starting times as decision variables, especially in the non-preemptive case. In this study, the expected total completion time is minimized in the case of a single machine and random failures. The probability of machine failure is an increasing function of the age and the length of the time interval, and preventive maintenance reduces the machine age to zero. The problem is represented by a nonlinear integer programming model which is reduced later to an unconstrained 0-1 optimization problem. Subsequently, a method for solving the unconstrained model by identifying the preventive maintenance decisions is proposed.

Moreover, the problem for minimizing the expected makespan on the single machine for the same above mentioned maintenance conditions is addressed and two heuristics methods were proposed to solve the problem.

Additionally, the problem of parallel machines which are under the same reliability conditions, but they may have different values of maintenance parameters is discussed. An approximation method based on the bin packing's first fit algorithm as well as an exact branch and bound method were introduced to solve the problem.

Finally, numerical examples were provided to illustrate each solution procedure of the proposed methods and some analysis was performed. The results show the benefits of integrating both decisions of production and maintenance, because some savings in the values of the discussed performance measures were obtained.

Keywords: Production, Preventive Maintenance, Single machine, Multi-machine, Integrating Schedule.

ÖZ

Son yıllarda, üretim ve koruyucu bakımın aynı anda çizelgelendirilmesi büyük oranda dikkat çekmeye başladı. Önceden, çoğu araştırmada, bakım etkinlikleri belli dönemlerde yapılan işler olarak değerlendiriliyordu. Ancak bu varsayım zaman ufkunda bir boşluk oluşmasına neden oluyordu. Son zamanlarda yapılan çalışmalarda, bakım zamanlarındaki değişiklikler de ele alınmış, ancak bakım başlangıç zamanı sabit ve çoğunda da bu zaman önceden biliniyor. Bakım başlama zamanlarını karar değişkeni olarak kullanan, özellikle önleyici olmayan durumlarda, az sayıda araştırma bulunmaktadır. Bu çalışmada, tek makine ve rassal arıza durumunda toplam tamamlama süresi enküçüklenmiştir. Makine arızası olasılığı, makine yaşının ve zaman aralığı uzunluğunun artan bir fonksiyonudur ve koruyucu bakım, makine yaşını sıfıra indirir. Bu problem, doğrusal olmayan tamsayılı programlama modeli olarak gösterilmiş ve daha sonra da kısıtsız bir 0-1 optimizasyon problemine indirgenmiştir. Devamında da, koruyucu bakım kararlarını tanımlayarak kısıtsız modeli çözecek bir yöntem önerilmiştir.

Ayrıca tek makine ve yukarıda bahsedilen bakım koşullarında tüm işlerin tamamlanma süresini enküçükleyecek iki sezgisel yöntem önerilmiştir.

Bunlara ek olarak, aynı güvenilirlik koşullarında ancak farklı bakım parametre değerlerine sahip paralel makineler de tartışılmış ve problemi çözmek için pin paketleme ilk fit algoritması ve yanısıra kesin dal-sınır yöntemine dayanan bir yaklaşıklama yöntemi de sunulmuştur.

Son olarak, önerilen her yöntemin çözüm yordamlarını gösteren sayısal örnekler verilmiş ve bazı çözümlemeler yapılmıştır. Sonuçlar, performans göstergelerindeki iyileşmelerden dolayı, üretim ve bakım kararlarının bütünleştirilmesinin yararlarını göstermektedir.

Anahtar Kelimeler: Üretim, Koruyucu Bakım, Tek Makine, Çoklu Makine, Bütünleştiren Çizelge.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS AND ABBREVIATIONS

a_0	Age of the machine prior to making sequencing and PM decisions.
a_j	Age of the machine after performing a job in position j and before the j+1 PM decision.
a_{ik}	Age of machine i before starting the job in k^{th} position.
C_{avg}	Average completion time.
C_i	Completion time for job i.
C_{max}	Maximum completion time.
d_i	Job due date
E_i	Earliness of job i.
EMS_{k_i}	Expected makespan in preemptive case with k_i PM times.
EMS_{mi,L,k^i}	Expected makespan for all the jobs on machine i with PM times k^i.
$E\{N(P_i)\}$	Expected number of failures for the machine during the period P_j when its age at the beginning is zero.
$E\{N(P_i;t_s)\}$	Expected number of failures for the machine during the period P_j when its age at the beginning is t_s.
F_{avg}	Average flow time.
F_i	Flow time.
$F(t)$	Probability density function of Weibull distribution.
k_i	Preventive maintenance times in preemptive case (i = 1, 2).
L_{avg}	Average lateness.
L_{max}	Maximum lateness.
LS	List containing the given jobs and the machine age listed in LPT.
p_j	Processing time of job j.

m	Number of machines.
$N(t)$	Number of failures during a period of time (t).
S	List of subsets $(s_1, s_2, \ldots, s_k)$ and each subset represent a segment between two preventive maintenance decisions.
s_{st}	The length of the shortest segment produced.
t	Time period.
t_m	Preventive maintenance time.
t_{m_i}	Preventive maintenance time for machine *i*.
t_r	Repair time.
t_{r_i}	Repair time for machine *i*.
x_{ij}	equal to 1 if job *i* is in position *j* and 0 if not.
x_{ijk}	The job *j* on the machine *i* at the *k* position (x_{ijk} = 0 or 1).
y_j	PM decision on machine before the job at the j^{th} position, (0 or 1).
y_{ik}	PM decision before the job in position *k* on machine *i*.
$\bar{y}_j$	The complementary value of y_j.
$Z(t)$	Hazard function of the Weibull distribution.
$\lambda(t)$	Failure rate function.
η	scale parameter
η_i	Scale parameter for machine *i*.
β	shape parameter.
β_i	Shape parameter for machine *i*.
π_j	The total age of the machine after processing the next job P_j.
$\dot{\tau}$	Best time to perform the preventive maintenance for a machine when its failure rate represented by Weibull distribution, the failure repair is minimal and preventive maintenance repair is perfects.

Ψ_j	The value of the machine age such that the expected value of the completion time of the job without PM equals its expected value with PM.
DT	Machine down time.
ECT	Expected completion time.
EDD	Early due date.
EMS	Expected makespan.
ETCT	Expected total completion time.
FMS	Flexible manufacturing system.
F_{tot}	Total flow time.
JIT	Just in time.
LFJ	Least flexible job first.
LPT	Longest processing time.
L_{tot}	Total lateness.
MDT	Mean down time.
MTBR	Mean time between renewals.
MTTF	Mean time to failure.
NHPP	Nonhomogeneous Poisson process.
PM	Preventive maintenance.
RL	Remaining load (unscheduled jobs)
RP	Renewal process.
SPT	Shortest processing time.
UT	Machine up time (working time).

Chapter 1

INTRODUCTION

1.1 Interrelationship between Production and Maintenance

Nowadays, the profit margins are limited and the competition is increased. Thus, the conditions of the production or service systems are the major determinants to introduce products or services have the ability to compete (Sloan, 2008, pp. 116-117).

Maintenance operations, whether to repair faults or preventable in some of production systems are highly sensitive. For example, delays in aircrafts repair may result in significant damage or precious asset out of service, even on a temporary basis. Also, in the pharmaceutical equipment; the delay in maintenance may cause its contamination which is leading to contamination of products. For the same context, in the shop machinery the deterioration in the cutting tool and delaying its replacement may affect the quality of products as well as inability to meet the demand (Wang, 2002, pp. 469 – 479 and Sloan, 2008, pp. 116).

Maintaining the system efficiency by relying on an excess inventory covering the shortage in products and services due to malfunctions (not to give enough attention to the maintenance process) is expensive and impractical. It is not possible to keep expensive planes as a spare and to replace potentially defective aircraft in the fleet. In the same logic, the establishment of a stockpile for medicines which have short validity with high inventory costs is not the appropriate economic policy to compete. Also, this policy did not consider the rapid technological changes and the customized products. Moreover, some production policies did not permit for a large stock such as production with JIT policy. On the other hand, the achievability of the reliable system which can work with full capacity without breakdowns or defective products is not possible in the real life. Thus, performing the proper maintenance to the system can improve its performance by minimizing the breakdowns and the defective products (Waeyenbergh, Pintelon and Gelders, 2000, pp. 439 – 470). However, the applied maintenance strategy plays an important role in maintaining the effectiveness of the system.

In some maintenance strategies, the decision to perform the maintenance activities depends on the failure occurrence (corrective maintenance strategy). Therefore, only production decisions needs to be planned. In such cases, the production plan likely to be inaccurate because of the failures interruptions which are not considered in the production plan. Other maintenance strategies make maintenance decisions depending on monitoring of some measurable factors (preventive strategy) such as some reliability measures which depend on the machine age to determining the best time to perform the preventive maintenance. Decisions of these strategies plan for production interruptions to perform the preventive maintenance in addition to the possible failures interruptions. Also, they did not take the production schedule and its conditions such as the resumability (some production models are nonresumable and if the job processing is interrupted then, the job will be reprocessed from the beginning) status into account. So, made both of maintenance and production decisions independently may did not lead to ensure an effective performance of the system and then the inability to meet the required production capacity in a timely fashion. Thus, it seems to be the working towards for harmonizing of these decisions is necessary to ensure the required efficiency.

In fact, considered the production schedule and preventive maintenance plan decisions simultaneously is not new. For more than two decades ago a lot of researches started to work in this area. Their results can be roughly classified into three types according to how they dealing with maintenance:

1. Some researchers considered the maintenance activities during a certain periods of time. These periods start at known time as well as their durations are known in advance. This type often referred as “production schedule with machine unavailability constraints”.
2. Some researchers considered the existence of unavailability periods in the planning horizon of production schedule; the starting time is a decision variable and the lengths of these periods is a linear increasing function in the starting time or the work load.
3. Other researchers considered the existence of unavailability periods in the planning horizon of production schedule due to preventive maintenance which is assumed to have a fixed value. Moreover, these models estimated the expected failures and their expected repair times. The time to perform the preventive maintenance is a decision variable and it is affecting the failure function.

The second and the third types often referred in the literatures as “integrated production scheduling and preventive maintenance plan” or “scheduling of production and maintenance simultaneously”. Also, some researchers referred the first and second types as a deterministic type of integrated schedules and the third one as probabilistic due to the way of considering of the failures.

In this research, models for integrating the production schedule and preventive maintenance planning in case of single machine and multi-machines in parallel are discussed. Both models are probabilistic models and the failures estimated according to the most recommended distribution in literature to represent the mechanical machines failures. Two types of repair are proposed; minimal repair for the sudden failures which restores the machine to the functional status "as old as bad" and perfect repair for the preventive maintenance which restore the machine to a good status "as good as new". Both models are assumed to be resumable models where the jobs interrupted by failures continued after repair without any additional penalty and the preventive maintenance will be performed only before and/or after the job processing. The models constituted a constrained nonlinear binary integer programming problems.

1.2 Dissertation Outline

Chapter 2, introduces some preliminaries about the scheduling in the shop floor and their complexity especially for the considered models. Additionally, some important fundamentals regarding the considered systems reliability measures, failures modeling and maintenance strategies. Chapter 3 is a survey on some literatures in this area. It summarizes the systems with their conditions and assumptions, and the proposed technique to solve the problem if it exists. The content of chapter 4 is a single machine model which is minimizing the total expected completion time with some related proven lemmas which support their solution procedure. In chapter 5, a model to minimize the expected makespan on a single machine is discussed with some proven facts which lead to the proposed solution procedures. Moreover, a model for multi-machines to minimizing the maximum expected makespan and their proposed solution procedures are reported also in this chapter.

The two above mentioned models in chapters 4 and 5 are non-preemptive models but the preemptive case solution is determined in chapter 5 and then used to define a solution for the non-preemptive problem. Finally, the conclusion of the study and recommended extensions for the expected future work are given in chapter 6.

1.3 Summary of Contributions

- For the proposed model to minimize the expected total completion time some lemmas has been introduced and proved.
- Based on the proven statements, the single machine model to minimize the expected total completion time is simplified from constrained nonlinear binary integer programming model to unconstrained nonlinear binary integer programming model.
- Based on the proven statements, an algorithm to determining the optimal integrating solution for minimizing the total completion time on a single machine is provided.
- Some lemmas are proved and used to minimize the expected makespan on a single machine under the assumptions and conditions of the model.
- Two heuristics for generating the optimal or near optimal solution is proposed.
- A heuristic method based on the first fit algorithm of the bin packing problem and on the properties of optimal solution for the preemptive case is proposed to minimize the maximum expected makespan on the parallel machines.

- A branch and bound algorithm to minimize the maximum makespan on parallel multi-machines was suggested.

Chapter 2

FUNDAMENTALS IN SHOP SCHEDULING AND MAINTENANCE

2.1 Introductory Remarks

The scheduling problem in the shop floor, its classification and the common used performance measures, the categories of production scheduling models, solution methods and their complexity are briefly presented. Some more details for the considered models in this work which are single and multi-identical machines are given. Maintenance, maintenance methods and the types of repairs are outlined with some basic concepts in the reliability theory. The counting process and its four types are discussed and the required details for Nonhomogeneous Poisson process (NHPP) and Renewal process (RP) which will modeling the failure behavior and preventive maintenance policy, respectively, for the current study are provided. Moreover, the Weibull distribution and their failure function that used as the failure function in NHPP are addressed. Finally the interrelationships between production and maintenance and their integration are introduced.

2.2 Scheduling

A lot of researches concerning to the optimization problems have been made in the previous decades; scheduling is the most addressed topic among of those problems and still has considerable attention so far. Scheduling in each area has many forms according to their constraints, measured criteria and its environment (Xhafa and Abraham, 2008, pp. VII).

In the shop floor area, production schedule is the sequence of the jobs through machine(s). The schedule is determined such that it is optimal for certain performance measure. There are some parameters in this type of scheduling (sequencing) problems to define its form which are:

a) Job characteristics (non-preemptive or preemptive, precedency, arrival date, etc.)
b) System environment (single machine, parallel machines, flow shop, etc.)
c) Performance measure (Total completion time, number of tardy jobs, makespan, flowtime, etc.)
d) Static or dynamic schedule (number of considered jobs and their availability time).

In spite of the large amount of researches in the scheduling, it is still receiving a considerable attention of researchers. This fact can be justified in two main causes according to Garey and Johnson (1979):

1. Most of scheduling problems are hard computationally, so there is always need to search for simpler and efficient techniques.
2. The continuous improvements in the production environment which increases the restrictions.

2.2.1 Regular and Irregular Performance Measures

Regular performance measures are those measures which are nondecreasing in the job completion (Pindo, 2010, pp.19). Let, N be the number of jobs in the scheduling problem and m be the number of machines (processors or system units); some commonly used performance measures are as follows:

- Total completion time $= \sum_j C_j$
- Average completion time $= \left(\frac{1}{N}\right) \sum_j C_j$

- Maximum completion time (Makespan)= $\max_i C_i$, where i is the machine index
- Average flow time $= \left(\frac{1}{N}\right)\sum_j F_j$
- Maximum lateness $(L_{max}) = \max(L_1, \dots, L_n)$, where $L_j = C_j - d_j$ and d_j is the due – date of job j.
- Total weighted tardiness $\sum w_j T_j$, where $T_j = \max\left[L_j,\ 0\right]$ and w_j is the weight of job j.
- Maximum tardiness $= \max T_j$
- Total tardiness $= \sum_j T_j$
- Average tardiness $= \left(\frac{1}{N}\right)\sum_j T_j$

A performance measure is irregular if it is not nondecreasing in the completion time. The most important such measures are based on the earliness (earliness penalty) which is defined as follows:

$$\text{Earliness } E_j = max\{d_j - C_j, 0\}$$

A notation used to specify the shop scheduling problems is **A/B/C**, where **A** describes the machine(s) with flow pattern and it should has just one entry, **B** describing the operations constraints and it can have more than one entry or it can be empty, **C** represents the objective function (performance measure) and most probably has one entry (Pindo, 2010, pp.14).

2.2.2 Scheduling Models

Field **A** above can have, but not limited to, one of the following scheduling models (Sule, 2008, pp. 10-12):

1. ***Single machine* (1)**: There are one server and the available jobs requiring to be processed on this machine (server) one by one.
2. ***Flow shop* (*Fm*)**: The jobs must be processed on more than one machine and in the same machine order. However, the required time to processing each job on each machine may differ from job to another.
3. ***Parallel shop* (*Pm*)**: *m* identical machines and each of them can process any of the jobs. In some cases, may there is dependency between the jobs.
4. ***Job shop* (*Jm*)**: There are m different machines and the job may need some or all of these machines. Each job moves to the required machines in specific sequence.
5. ***Open shop* (*Om*)**: There are m different machines and the job may need some or all of these machines. Each job moves to the required machines in any sequence. So, this model is similar to the job shop model except that in the model there is no specific route for the job operations on the required machines.

Field **B** can describing one or more restrictions such as *prmu* in case of the permutation is allowed, *rcrc* to show the recirculation (the job may visit the machine more than once) and *prec* for precedence constraint (some jobs must be processed before some others).

Field **C** can have one of the performance measures introduced in section 2.2.1.

2.2.3 Classification of the Scheduling Problems in Shop Floor

Production schedule models in the shop floor can be classified according to Xhafa and Abraham (2008) to:

1) Depends on the jobs arrival

The scheduling problem is ***static*** if all the jobs available at the beginning of the horizon time and ***dynamic*** if they have different arrival times. In the latter case a rescheduling process may needed.

2) Depends on the job processing time and machine availability

   The problem can be classified as ***deterministic*** problem if the job processing time and machine's unavailability times are known in advance. If the jobs processing times or machine's unavailability times are unknown prior then, the problem is ***probabilistic***.

3) based on the number of system stages

   if the job has only one process which requires one machine then, the system is ***single stage*** and if it has multi processes that may need multi machines then, the system is ***multi-stages***

4) based on the number of machines and the jobs path through the system

   the scheduling problem can be for ***single machine***, ***multi machine in parallel***, ***two machine flow shops***, ***multi-machines flow shop***,...etc.

5) Depend on the production and its inventory plan

   The scheduling problem called ***open*** if the produced products are made based on customer order and ***closed*** if the produced products are made to be kept and waited the estimated orders.

In the past decades, researchers concentrated on the static and deterministic scheduling models. Since two decades ago, the probabilistic models has attracted researches but not explored enough yet.

2.2.4 Single Machine Scheduling Problem

It was the first discussed scheduling problem for the shop floor environment where the jobs visit one machine only once. The research findings have been applied on more complicated problems especially in the serial systems that have a bottleneck machine. One of the most complicated models in the single machine scheduling is sequence with dependent setup times, the problem is NP-hard and only small size problems can be solved efficiently.

2.2.5 Identical Parallel Machines Scheduling Problem

In multi-machine scheduling problem, the machines can be in parallel or series or mixed (some of them in series and the others are identical and any one of them can be used). In this work, the interest is for the identical parallel case only. This problem can represent many cases in the real life such as docks and ships, teachers and students, technical assistance staff in hospitals and patients, in computer science for processors and operations, etc.

In the parallel case the jobs can be processed on any of the available m machines with the same processing time and the makespan is the objective function. The most cases of the identical parallel machines problems are NP-hard. The problem can be solved optimally in an easy way for some performance measures and fast algorithm can approximate the optimal solution only for some others (Robert and Vivien, 2010, pp.86) and (Xhafa and Abraham, 2008, pp.5). The scheduling decision consists of two parts, assigning the jobs on the available machines (all available machines must be used) and determining the sequence of the assigned jobs on each machine. For example, it can be solved optimally for minimizing the total flow time problem (SPT list dispatching rule), and approximately for minimizing the maximum makespan (large size problems).

For minimizing the maximum makespan on the classical (deterministic) identical machines ($P_m \parallel C_{max}$) problem, the jobs assigned on the machine can be in any sequence and the problem is to balance the load (jobs) on the *m* machines. Thus, the lower bound of the problem is:

$$C_{max} \geq max\left\{\max_j\{p_j\}, \frac{1}{m}\sum_j p_j\right\}$$

The most famous algorithm for this problem is introduced by Graham (1969). The algorithm is creating a List Scheduling for the given jobs in nonincreasing order then, the jobs assigned according to the longest processing time (LPT) rule. The algorithm has a tight bound of $\frac{4}{3} - \frac{1}{3m}$ for LPT list and $2 - \frac{1}{m}$ in general as shown in Graham (1966) for an arbitrary list (List Scheduling).

2.2.6 Complexity of Shop Scheduling Problems

Some special cases of the shop scheduling problems can be solved by algorithms in polynomial time; all of them are belong to NP class. Generally, they become more complex if the number of system units (machines) more than three. Thus, it can be solved only by deterministic algorithm in exponential behavior. In other words, when the size of the problem increased (number of jobs) the required time to solve the problem increased exponentially (Xhafa and Abraham, 2008, pp.8).

2.2.7 Solution for Shop Scheduling Problems

The interested researches to optimizing the problems of shop scheduling discussed two ways to define the solution of those problems which are:

1. Exact algorithms

The exact algorithms provide the optimal solution in a bounded time. However, because of most of shop scheduling problems are NP-hard; finding an algorithm to solve such problem in polynomial time does not exist. Practical problems with large size requiring an exponential computation time to solve by the exact algorithms. The addressed exact algorithms are mixed integer programming, Branch and Bound, and decomposition methods.

2. Approximate algorithms

Because of difficulties of using the exact algorithms in the mentioned problems; the need to find approximate methods became inevitable. The approximation methods providing a near solution to the optimal and in some cases it may lead the optimal solution. Heuristics and Meta-Heuristics are the two types of the approximation methods.

a) Heuristics Algorithms

Blum and Roli (2003) classifying the heuristics algorithms in the shop scheduling problem to:

(i) Constructive: The solution root is empty and the heuristics start to build it from scratch. In each step a part of the solution is added and a partial solution is generated. Constructive algorithms are fast algorithms so they are suitable for the problems with large inputs. The dispatching rule is an example for the constructive heuristics.

(ii) Local Search: Start with solution root or an initial solution which generated by constructive method or randomly and then looking for a better solution in the set of neighborhood solutions. The method generating the neighborhood set by changing parts of the initial solution; if a better solution found, then it called a local optimal solution.

b) Meta-Heuristic Algorithms

Meta-Heuristics method discussed first by Glover (1986) and it has a good attention in the nowadays researches. They are merging the heuristics methods in efficient framework. The aim of the new methodology is to efficiently and effectively explore the search space driven by logical moves and knowledge of the effect of a move facilitating the escape from locally optimum solutions. Meta-heuristic methods advantage is in terms the robustness for the provided solutions. However, the implementing of the Meta-heuristic methods is not easy as they required special information about the problem to be solved.

2.3 Maintenance

Maintenance includes preventive and corrective actions performed to keep or restore the system to a satisfactory functional condition. A providing the best possible system reliability and safety in a minimum cost are depending on the maintenance policies that carried out for the system (Sherif and Smith,1981, pp.47). Determining the system reliability and availability is the first required step to design the maintenance policy of the complex system. However, this is not a simple task especially when the subsystem's failure rate functions did not have the same distribution.

2.3.1 Maintenance Types

Blischke and Murthy (2003) divided the maintenance activities to two types as follows:

I. Preventive Maintenance

Preventive maintenance is preplanned activity aiming to improve the system reliability and increasing its lifetime. The system will not be available during the preventive maintenance and its time interval depends on the planned actions. The time to perform the preventive maintenance depends on the maintenance policy which is defined by the decision maker. These policies can be categorized to:

a) **Time based maintenance:** In the time based maintenance policy the preventive maintenance performed according to predefined time table.

b) **Age based maintenance:** The preventive maintenance performed depending on the component age.

c) **Condition based maintenance:** Condition based maintenance: the maintenance actions are performed depending on the value of some measured variables, which characterize the system wear out status, and/or reliability measures. However, often measuring the required variables is difficult, so other measurable variables maybe used to estimate the required variables.

d) **Usage based maintenance:** Depend on continuous monitoring of the item during its usage period. This policy is suitable for some products such as tires.

e) **Opportunity based maintenance:** Performing preventive or corrective maintenance in the system may be given an opportunity to maintain another items and avoiding the system shutdown again to maintain them. Its applicable policy for the system has large number of items.

Generally, whatever the preventive maintenance policy implemented, it should improve the system's performance and its availability. Jardine and Buzacott (1985) studying the effect of maintenance policy on the some of the reliability measures.

II. Corrective Maintenance

Corrective maintenance or as called by some researchers "repair" is the maintenance actions that performed due to failures and to restore the repairable system to a functional status. Repairs operations were classified according to the status of the system after repair as follows:

a) **Perfect repair:** restore the item to good condition after repair and, in the literature, is often referred as ‘as good as new’.

b) **Minimal repair:** restore the item to a functional status, and its age after repair remains the same as before repair. In the literature, this is often referred to as ‘as bad as old’. Failure events occur according to a nonhomogeneous Poisson process (NHPP).

c) **Imperfect repair:** restore the item to a condition between the two streams in (i) and (ii), and in some literature, this is described as ‘general repair’.

d) **Worse repair:** the maintenance process undeliberately causes the system to be worse after maintenance than it was before failure, but it will not fail.

e) **Worst repair:** the maintenance process undeliberately leads the system to breakdown.

The ***maintenance strategy*** is a combination of corrective and preventive maintenance policies.

2.3.2 Counting Process

When the considered system is a repairable system; event recurrence (failures) is expected which may make the system not available. Immediately, after each event the system will be restored to a functional status by the repairing process and it will be available again until the next event. The time period between these two consecutive events called the time between failures or interarrival time. Modeling these sequential events is the purpose of the counting process. In short, the four types of counting process are (Rausand and Hoyland, 2004, pp. 232-295):

I. Homogeneous Poisson process

In this type of counting process the mean time between failures is exponentially distributed and independent as well as the failure rate is constant and just one failure can be happen in the same time. The expected number of failures during t period is λt; where λ is the failure rate. Homogeneous Poisson process is a special case in both nonhomogeneous Poisson process and renewal process.

II. Renewal process

When the time between failures in the counting process is independent and identically distributed; it is called a renewal process. This means that the failed item will be replaced or restored to a good functional status “as good as new”. In the repair terminology, is said to be perfect repair.

Consider a system starts its operations at age zero and upon failures it will be repaired and restored to a good condition. Additionally, assume that the interoccurance times and repair times are independent and identically distributed with MTTF (mean up time) and MDT (mean down time) respectively. Thus, as shown in Figure 2.1, the renewal periods T_i are:

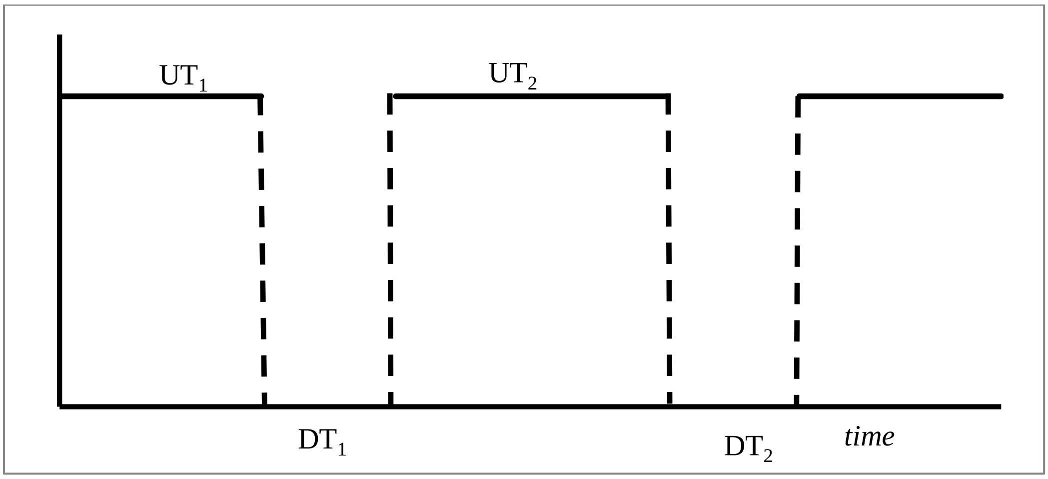

Figure 2.1: Alternating Renewal Process

$$T_i = UT_i + DT_i, \ \forall \ i = 1, 2, 3, \ldots \tag{1}$$

and;

$$MTBR = MTTF + MDT \tag{2}$$

where,

MTBR: the mean time between renewals

The previous process is said to be alternating renewal process. Depending on the perfect repair policy the system availability is:

$$A = \frac{Mean\ up\ time}{Mean\ up\ time + Mean\ down\ time} = \frac{MTTF}{MTTF + MDT} \tag{3}$$

III. Nonhomogeneous Poisson process

If the Poisson process is nonstationary process and the failure rate function is varied with time (it is function of *t*) then, the counting process is said to be nonhomogeneous Poisson process. Because of the nonstationary increment property of the nonhomogeneous Poisson process it has different probability for the failure occurrence at each epoch of time. In other words, the probability of failure occurrence may have probability to occur at certain time more than at others. Thus, the mean time between failures are not identical distributed and not independent.

As mentioned earlier NHPP has independent increments then, the number of failures $N(t)$ during the time interval $(t_1, t_2]$ is independent from the number of failures before t_1 and then from the interoccurance times. Therefore, the conditional rate of occurrence of failures $\lambda_C\left(t \middle| \mathcal{H}_{t_1}\right)$ for the next interval is $\lambda(t)$ and it is independent on the history $\mathcal{H}_{t_1}$ up to t_1 (see Rausand and Hoyland, 2004, pp. 278). It implies that the conditional rate of failures ($\lambda_C(t)$) after repair directly is the same just before the failure. The repair process under these assumptions is called minimum repair. Minimal repair introduced and formulated first by Barlow and Hunter (1961). The replacement or a good repair for the failed component in a system consisting of many parts will not add a significant improvement to the overall system reliability. Therefore, assuming the reliability of the system just before repair will be the same after repair immediately is a factual approximation. The repairable system models which use the non-homogeneous Poisson process, are considered as a "***Black Box***" where there is no attention about how is the inside structure of the system (Rausand and Hoyland, 2004, pp. 278). Assume the failures for the system will happen at T_1,

T_2, T_3 and T_4 as shown in Figure 2.2 and let the process be nonhomogeneous Poisson process with failure rate $\lambda(t)$.

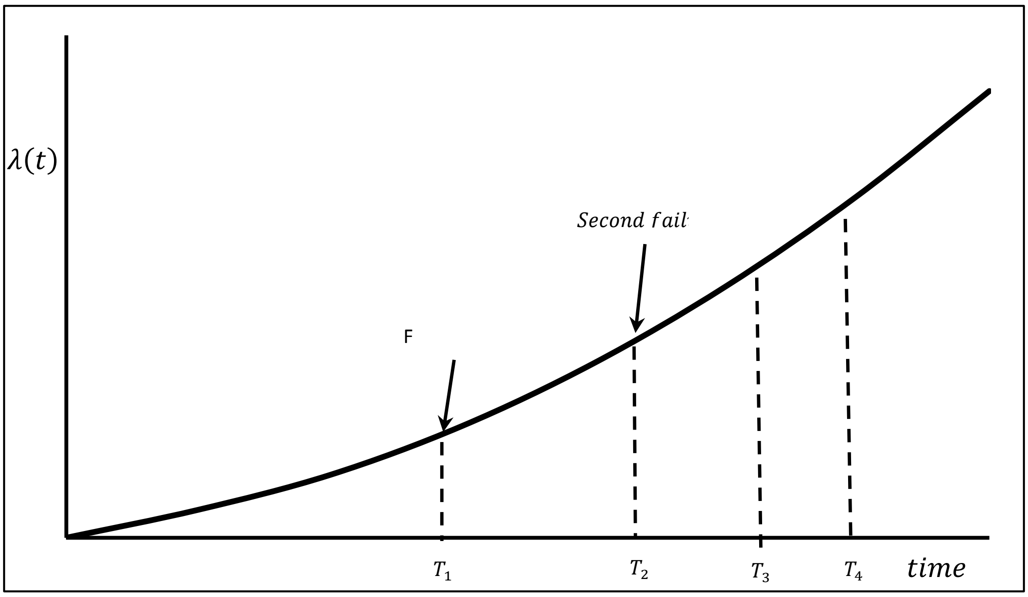

Figure 2.2: The failures occurrence rate

Accordingly, the expected number of failures during the time span (0, t) is: (Ross, 1996, pp. 78-80).

$$Pr(N(t) = n) = \frac{[\lambda(t)]^n}{n!} e^{-\lambda(t)}, \ \forall n = 0, 1, 2, \dots \quad (4)$$

IV. Imperfect repair

Renewal process and Nonhomogeneous Poisson process are the two main processes which are offered models to describe the failures behavior of the repairable system. Both processes are using two extreme repair methods; the renewal process assumes the repair as perfect process where the system reliability condition after the repair is good (good as new). On the other side, the nonhomogeneous Poisson process assumes that the repair is minimal where the system reliability condition after the repair is bad (bad as old). Other models have been proposed for the repair between these models (perfect and minimal) and known as normal repair or imperfect repair.

The proposed imperfect repair models can be categorized upon their effect on the system after repair as follows:

(i) Decreases the rate of the failure function after repair.

(ii) Decreases the age of the system after repair.

Because of the imperfect repair models are out of the scope of this work; the reader can have access to more details about them from some researches such as Pham and Wang (1996) and Hokstad (1997).

In this work, the failures modeled according to the black box approach but before that the distribution which will represent the failure rate in the process should be defined.

2.3.3 Weibull Distribution

The Weibull distribution which was developed by the Swedish scientist Waloddi Weibull (1887-1979) is extensively used to describe the life behavior in many studies interested in the analysis of reliability. One of the best advantages of the Weibull distribution is its capability to represent the rates of the event occurrence in different ways by describing their parameters appropriately. Moreover, it can represent the rate of the failures occurrence when it is constant, increasing or decreasing. Blischke and Murthy (2003) as well as Rausand and Hoyland (2004) mentioned that the Weibull distribution is the best distribution to describe the failure behavior for the mechanical equipment, bearings, semiconductors, and so on.

The Weibull distribution function, probability density function and hazard function respectively are:

$$F(t) = Pr(T \leq t) = \begin{cases} 1 - exp^{-\left(\frac{t}{\eta}\right)^{\beta}} & for\ t > 0 \\ \\ 0 & othewise \end{cases} \quad (5)$$

where,

η : scale parameter

β : shape parameter

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} exp^{-\left(\frac{t}{\eta}\right)^{\beta}}, \quad for\ t > 0 \quad (6)$$

$$Z(t) = \frac{\beta}{\eta^{\beta}} t^{\beta-1} \quad (7)$$

The failure rate function (hazard function, $Z(t)$) for the Weibull distribution is decreasing function if the shape parameter values less than one ($\beta < 1$) , constant for the shape parameter value equal to one ($\beta = 1$) and increasing function for the shape parameter values greater than one ($\beta > 1$) as shown in Figure 2.3.

To modeling the component's failures two phases should be considered. First, modeling the first failure which is relies on the component reliability and; second, the next failures which are rely on both the reliability and the used rehabilitation action (Blischke and Murthy, 2003, pp. 523).

2.3.4 The First Failure

The time up to the first failure event can be estimated using the failure distribution as the follows:

$$F(t) = \Pr(T \leq t) \quad (8)$$

Different formulas can be determined by equation (8) depending on the considered failure distribution. In this work, the Weibull distribution represents the failure function as it is the most recommended one for the mechanical equipment. Thus, formulas in (5), (6) and (7) will represent the failure distribution, failure function and failure rate respectively.

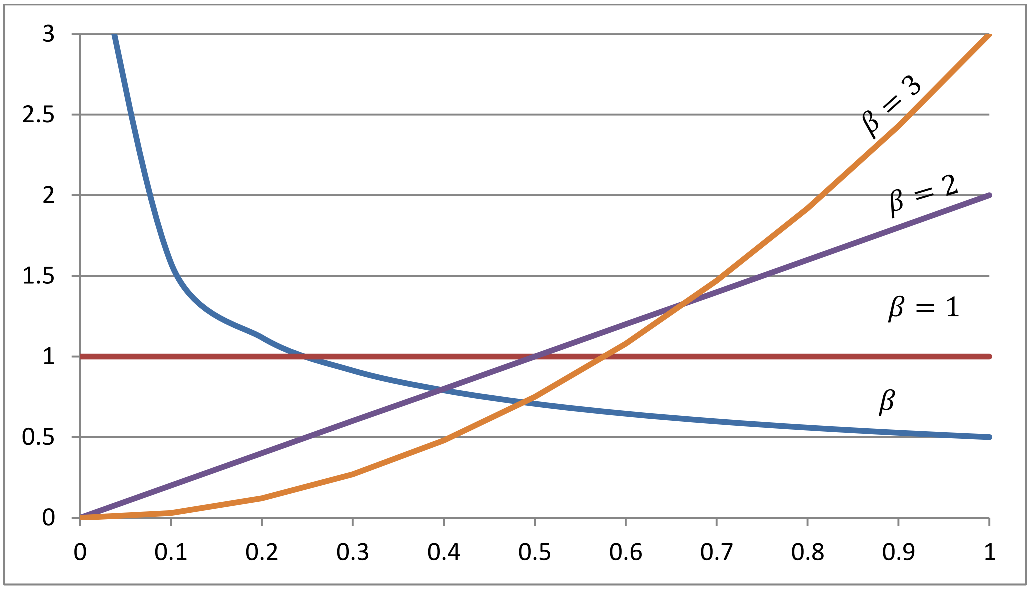

Figure 2.3: Failure rate function with $\eta = 1$

2.3.5 The Subsequent Failure

As mentioned earlier, the subsequent failures rely on the reliability of the component and the restoration action. Minimal repair is one of restoration types that discussed earlier in the nonhomogeneous Poisson process.

Based on this policy; the expected number of failures E{N(t)} for a machine with some or many parts can be modeled as a nonhomogeneous Poisson process (nonstationary process) with an intensity function given by the failure rate function (Murthy, 1991, 245-246). The probability of n failures occurring during a given time t can be expressed as follows:

$$P\{N(t) = n\} = \frac{e^{-\lambda(t)}\,[\lambda(t)]^n}{n!}$$

where $\lambda(t)$ is given by

$$\lambda(t) = \int_0^t Z(x)\,dx \tag{9}$$

The expected number of failures over this period is:

$$E\{N(t)\} = \lambda(t) = \int_0^t Z(x)\;dx \tag{10}$$

When the failure function is described by the Weibull distribution, the expected number of failures during the period $t \in\ [t_1, t_2]$ is

$$E\{N(t = t_2 -\ t_1)\} = \int_{t_1}^{t_2} \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1}\,dx \tag{11}$$

$$E\{N(t)\} = \left(\frac{t_2}{\eta}\right)^{\beta} - \left(\frac{t_1}{\eta}\right)^{\beta} \tag{12}$$

When the processing time is P_i and the previous age is t_s, the expected number of failures can then be described generally as follows:

$$E\{N(P_i; t_s)\} = E\{N(P_i + t_s; 0)\} - E\{N(t_s; 0)\}$$

or

$$E\{N(P_i; t_s)\} = E\{N(P_i + t_s)\} - E\{N(t_s)\}$$

If the stating time is zero ($t_s = 0$) then,

$$E\{N(P_i; 0)\} = E\{N(P_i)\} = \left(\frac{P_i}{\eta}\right)^{\beta} \tag{13}$$

2.3.6 Machine Availability with NHPP and Weibull Distribution

The machine availability in the production environment such as Flexible Manufacturing Systems (FMS) and Just-in-Time (JIT) is a key issue due to its direct effect on the production volume and quality. Machine's breakdown consuming the production time and affect the product quality (Al-Najjar, 2007, pp. 262). Therefore, maximizing the machines availability increases the manufacturer ability for the challenges in nowadays industries.

In addition to considering the Nonhomogeneous Poisson process (***black box policy***) which has the Weibull failure rate function as the failure behavior in this work, the preventive maintenance will be performed according to the alternating renewal process where the repair after preventive maintenance is perfect. Accordingly, the machine status next to preventive maintenance action is a good as new and the renewal points are the start of the operation and the end of the preventive maintenance.

Assume that the system average cycle time includes τ of working time, t_m of preventive maintenance time and $E\{N(\tau)\}t_r$ of expected repair time during the τ period. The expected repair time during the τ period of working time is:

$$E\{N(\tau)\}t_r = \left(\frac{\tau}{\eta}\right)^{\beta} t_r \qquad (14)$$

Referring to equation (3) the machine availability is (Cassady and Kutanoglu, 2003, pp. 505)

$$A(\tau) = \frac{MTTF}{MTTF + MDT} = \frac{\tau}{\tau + t_m + \left(\frac{\tau}{\eta}\right)^{\beta} t_r}$$

The best time to perform the preventive maintenance is $\dot{\tau}$ which satisfies the equations:

$$\frac{dA(\tau)}{d\tau} = 0$$

Then,

$$\frac{\left(\tau + t_m + \left(\frac{\tau}{\eta}\right)^{\beta} t_r\right) - \tau\left(1 + \beta\left(\frac{\tau}{\eta}\right)^{\beta-1} \frac{1}{\eta} t_r\right)}{\left(\tau + t_m + \left(\frac{\tau}{\eta}\right)^{\beta} t_r\right)^2} = 0$$

$$t_m + \left(\frac{\tau}{\eta}\right)^{\beta} t_r - \tau\beta\left(\frac{\tau}{\eta}\right)^{\beta-1} \frac{1}{\eta} t_r = 0$$

$$t_m + \left(\frac{\tau}{\eta}\right)^{\beta-1} \left(\left(\frac{\tau}{\eta}\right) t_r - \frac{\tau\beta t_r}{\eta}\right) = 0$$

$$t_m + \left(\frac{\tau}{\eta}\right)^{\beta-1} \left(\frac{\tau}{\eta}\right) t_r(1-\beta) = 0$$

$$\left(\frac{\tau}{\eta}\right)^{\beta} = \frac{-t_m}{t_r(1-\beta)} = \frac{t_m}{t_r(\beta-1)}$$

$$\dot{\tau} = \eta\left[\frac{t_m}{t_r(\beta-1)}\right]^{1/\beta} \quad (15)$$

2.4 Interrelationship between Production and Maintenance

The task of maintaining production system has some requirements such as qualified human resources, special tools, spare parts, etc. The interruptions of the production processes due to preventive maintenance are additional constraints on the production schedule.

According to Coudert, Grabot and Archimède (2002), there are three hierarchical levels in industrial companies for the production and maintenance:

- Preventive maintenance can be carried out when the machine is in unloaded periods. Therefore, production has higher level. This case is more applicable for the machines that can be maintained easily.

- Preventive maintenance periods are determined first and then positioned on the machine calendar. Second, scheduling the production operations. Thus, the maintenance has the higher level.
- Both of production and maintenance on a par having the same position hierarchically which needs coordination or cooperation.

Maintenance and production coordination have some purposes which are:

- Ensuring that the required time for maintenance activities are taken into account in the production schedule and carried out at a timely manner.
- To provide a quick response for maintaining the machine faults.

Optimizing the schedules of both production and maintenance individually most probably will require some adjustments to be compatible with each other in the same process. Therefore, the modified schedules may lose their optimality conditions (Löfsten, 1999, pp. 718). Thus, meeting the scheduled production necessitated on a real schedule for the production which is considering the known interventions such as preventive maintenance activities. Moreover, increasing the level of coordination and cooperation can help to ensure a better optimality as well as meeting its conditions. Generally, these concepts are the base for what is called "integrated schedule of production and maintenance".

2.5 The Current Study

In this study, two production models will be considered. First, a single machine problem which is aiming to optimize the production and preventive maintenance decisions simultaneously for minimizing the expected total completion time. Second, the multi parallel machines problem will be addressed for minimizing the maximum expected makespan. In the latter case, minimizing the expected makespan on a single machine should be investigated also. In both models the preventive maintenance is a decision variable and its repair is perfect. Additionally, the probability of machine failures is considered and modeled according to nonhomogeneous Poisson process where the failure's repair is minimal (***black box***). Weibull failure rate function is the failure rate function in nonhomogeneous Poisson process.

In the next chapter, a review for some literature related to the shop scheduling with availability constraints problems will be introduced.

Chapter 3

RELATED LITERATURE

3.1 Introduction

In the last fifty years the shop floor scheduling problem received a considerable attention. Most of this effort has been concentrated on the deterministic schedule (Pinedo, 2010, pp. 13). The majority of the research in this area supposes that the machines consumes all the horizon time in the production process. However, machines during the manufacturing processes stop for preventive maintenance or to repair the faults. During the maintenance time the machine is not available for production. Therefore, the scheduling model which is considering these unavailability periods is more realistic.

Once the production process is interrupted due to the preventive maintenance or failures; three situations arise regarding the situation of the job when the processing starts again. These are called resumable, semiresumable and nonresumable situations. The model is called resumable if the job can be continued after the down period without any penalty, semiresumable if a job cannot be finished before the next down period of a machine and the job has to partially restarted after the machine has become available again, and nonresumable where the job needs to restart totally.

In the following sections, a summary of related literature considering the integration of production and maintenance decisions is discussed. The considered cases are one machine, two machines flow shop, multi machines and process industry.

3.2 Single Machine Problem

Simultaneously scheduling of production and maintenance started to receive a considerable attention of researchers for more than two decades ago (Xu, Wan, Liu and Yang, 2015, pp.1). However, although of the amount of research in this field it is still not yet explored and it still has a great attention so far.

Graves and Lee (1999) assumes the unavailability period (T) for a single machine is fixed and the maintenance must be performed within it. The time to perform the preventive maintenance is a decision variable and it can be achieved just once during the time horizon. The models were semiresumable and aiming to optimize two performance measures, minimizing the total weighted job completion times and minimizing the maximum lateness. Two scenarios regarding the length of T were analysed. First, when T is short in relation to the horizon time, the problem was NP – complete for both measures and pseudopolynomial dynamic programming is suggested for both as well. Second, if T is long in relation to the horizon time, the minimizing of total weighted completion time was NP – complete, while the Earliest Due Date rule (EDD) is optimal for minimizing the maximum lateness.

Minimizing the weighted tardiness was the objective of the resumable mathematical model built by Cassady and Kutanoglu (2003). The model assumes that the preventive maintenance activities cannot interrupted the job processing and they should be performed before or after finishing the job processing. Moreover, the model takes into account the probability of the machine failures according to the Weibull distribution and suppose just one failure can be happen during the job processing and then the expected repair time estimated according to the possible failure scenarios for the given jobs. The preventive maintenance restores the machine to a good (perfect repair) condition and the repair due to failure is just keeping the machine in functional (minimal repair) condition. The model solved using the enumerative method and the $n! \times 2^n$ (n is the number of jobs) possible solutions must be investigated.

Later, Sortrakul, Nachtmann and Cassady (2005) proposed heuristics based on genetic algorithm to solve the model introduced by Cassady and Kutanoglu (2003) which is integrated the production and preventive maintenance plan. The authors emphasize on its effectiveness in solving the small, medium and large size problems. Pan, Liao and Xi (2010) proposed a mathematical model aiming to minimize the maximum weighted tardiness for a single machine by integrating the preventive maintenance plan and production schedule. In their model, maintenance time is a variable related to machine degradation, and the preventive maintenance did not interrupt the job process, whereas the Weibull function represented the failure function for the model. An enumerative method was used to find the best sequence and preventive maintenance plan for the minimum weighted tardiness. Fitouhi and Nourelfath (2012) had more objectives by considering the preventive maintenance and production planning for a single machine. Minimizing preventive and corrective maintenance cost, backordering cost, setup cost and production cost were considered simultaneously. The model must satisfy the demand during the given horizon period as well as the minimal repair policy used for the machine repair. The problem was solved by comparing the results of multi-product capacitated lot sizing problems. The model gained some value by integrating the decisions and the noncyclical preventive maintenance when the demand varied from one period to another. A single machine subject to multi-maintenance activities and minimizing makespan, total completion time and total weighted completion time was introduced by Kim and Ozturkoglu (2013). The preventive maintenance restored the deteriorated processing time to the original processing time; the problem was formulated as an integer programming model and was solved using some heuristics and a genetic algorithm. Nie, Xu and Tu (2014) discussed a model with several objective functions: maximizing the average timeliness level and minimizing the total weighted completion time. The model incorporated both maintenance planning and production scheduling for a single machine under a fuzzy environment. The computational results for a numerical example were used to illustrate the algorithm's value and demonstrate its efficiency. For a single machine subject to sudden failures according to the Weibull function, Lu, Cui and Han (2015) introduced a model that makes the decisions of the PM times, job sequence and the jobs completion times proactively and simultaneously. Additionally, to optimize both the system stability and robustness, a genetic algorithm based on the properties of the optimal schedule was proposed to solve the problem, and the experimental results showed the effectiveness and efficiency of the algorithm in solving the desired problem. The model provided by Xu, Wan, Liu and Yang (2015) for minimizing the total completion time on a single machine assumes that the maintenance activities start at a fixed and known time and that the maintenance durations are functions of the machine load (nonnegative and increasing functions). The case is non–preemptive, and the maintenance activities could not interrupt the jobs processing. They concluded that, in case that the derivation of the maintenance time function is greater than or equal to one, the case can be solved optimally using a polynomial time algorithm. If the derivation of the maintenance time function is less than one, the case can be solved by the proposed polynomial time approximation scheme. Another work by Lou, Chang and Ji (2015) assumed that the maintenance time is a positive and increasing function in terms of the maintenance start time (workload) for a non–preemptive case. However, the maintenance time here has a deadline that should not be exceeded when beginning the maintenance duration. The job sequence and maintenance starting time are decision variables used to minimize the total completion time, the number of tardy jobs, the maximum lateness and the makespan. Finally, they showed that the problems for all of the above-mentioned measures could be solved in polynomial time by the proposed algorithms. Minimizing the maximum delivery time for a set of jobs which have release dates and tails is the objective of the work performed by Hfaiedh, Sadfi, Kacem and Alouane (2015) on a single machine. The machine has an unavailability period during a known interval (t_1, t_2), and the case is not preemptive. The job sequence is a decision variable, and the release date (r_j) cannot be in the maintenance period. A branch and bound method was proposed to solve the problem, and the numerical results demonstrated its ability to solve large problems. Minimizing the cost of M preventive maintenance tasks that should be scheduled on M machines where each machine must be maintained exactly once is the aim of the work introduced by Rebai, Kacem and Adjallah (2012). They assume that the preventive maintenance tasks are continued during the time horizon due to the limitations in the maintenance resources and its expensive costs. In this study, the preventive maintenance tasks have optimistic and pessimistic dates to start and starting the tasks of preventive maintenance before or after these two dates will increase its costs. The preemtion while performing the tasks is not allowed but the idle time is permitted. The problem formulated for minimizing the total earliness and tardiness for M jobs (preventive maintenance tasks) on single machine. The problem solved using linear programming with branch and bound as an exact method, the local search method approach and a genetic algorithm used as a meta heuristic method.

3.3 Flow Shop Problem

Lee (1997) discussed a problem for minimizing the makespan on two machine flow shop with availability constraints and the unavailability period known in advance. The case was resumable and there is at least one machine available all the time. The unavailability period assumed on the machine one first and then studied when it on the second machine. The problem is NP-hard (as approved) and pseudo-polynomial dynamic programming algorithm was introduced to solve the problem. Additionally, depending on the unavailability period on which machine; two heuristics was proposed to solve the problem with worst case error bound 1/2 on machine one and 1/3 on machine two. Subsequently, the same author (Lee, 1999) introduces a semiresumable model for two-machine flow shop with the same assumption that the preventive maintenance period is known in advance. The model doesn't consider the machine failures and the jobs can be interrupted for preventive maintenance. Moreover, two special cases are studied, resumable and nonresumable. They conclude that the problem is NP-hard except when the unavailability periods for both machines is in the same time and the case is resumable. Also, a dynamic programming algorithm is provided to solve the problem.

In order to improve the work provided by (Lee, 1997); Breit (2004) addressed the resumable case when the unavailability period on the second machine only and an approximation algorithm with relative bound error of 5/4 is proposed. Kubzin and Strusevich (2002) discussing the problem of no - wait process in addition to the unavailability constraints for two machine flow shop. The job must start its operation on the second machine immediately when its operation on the first machine finished. Three scenarios are examined, resumable, semiresumable and nonresumable. The problem with single hole in time horizon and traveling salesman problem was discussed and some approximation methods were provided. Allaoui and Artiba (2006) describing a hybrid system of two stages, the first stage with one machine only and all jobs require it, the second with *m* identical machines and any jobs can be processed on any one of them. The job(s) waiting between the two stages is allowed but the preemption is not. Each machine in the described system is subject to at most one availability period which is known in advance. In order to minimizing the makespan a branch and bound algorithm is suggested. The problem decisions determined in two stages, first, sequencing the jobs on single machine and, second, balancing the jobs on *m* machines. Under various scenarios of the unavailability constraints Kubzin, Potts and Strusevich. (2009) studied of minimizing the makespan on two machines flow shop when the case is resumable and semiresumable. First, for the multi holes in the time horizon of the first machine with resumable case they provide fast (3/2) approximation algorithm. Second, for the problem with single hole on the time horizon for one of the two machines with semiresumble case a polynomial-time approximation scheme was suggested. Later, Hadda (2009) discussing the case of several unavailability periods on the second machines with resumable assumption and provided a (4/3) – approximation algorithm. Immediately thereafter, Hadda (2010) introduced improved algorithm with (4/3) error bound when the several holes on the first machine in the resumable case. Zhao and Tang (2011) proposing a model for two machines no-wait flow shop with unavailability constraints. They assumed that the job processing time is a simple linear nondecreasing function of its starting time and the unavailability period is on one machine only. They found that the problem is NP-hard in ordinary sense for only one hole with resumable case and NP-hard in strong sense for more than one hole. A branch and bound algorithm reported by Chihaoui et al. (2011) for two machine no-wait flow shop with assumptions that each machine has unavailability period, the unavailability period for each machine is fixed and known in advances as well as they overlap and the case is nonresumable. The aim is to minimizing the makespan when the jobs have release dates and several lower and upper bounds is proposed. Reverse of the hybrid system described by Allaoui and Artiba (2006) is the two - stage assembly system discussed by Hadda, Dridi and Gabouj (2014). The first stage has m machines and the second stage has only one machine, the assembly operation for each job performed on the machine at stage two after finishing all its operations in stage one. The model is semiresumable and the target is to minimizing the makspan (maximum completion time of the jobs) when one of the $m + 1$ machines has unavailability period during the study time. A heuristics with a tight ratio bound of 2 is provided and they approved that when the second stage has a hole there is no polynomial heuristic with finite bound.

3.4 Multi Machine

For multi-machines Sehmidt in 1984 was the first in considering the unavailability periods while scheduling *n* jobs with preemptive case (Lee and Wu, 2008, pp.363).

An expert system was the tool for making the plan and management decisions which is used by Brandolese, Franci and Pozzetti (1996). Scheduling of the production and maintenance simultaneously is in the core of the expert system. The system has one stage consisting of multi-machines working in parallel to produce multi-products.

The target is minimizing the production cost which is affected by the job to be processed and the used machine, set-up cost that expended depending on the machine and the jobs sequence and maintenance costs. Moreover, the system aims to meet the due date and minimizing the total utilization of the plant (total of processing, set-up, maintenance and idle times).

Minimizing the total weighted completion time for *n* jobs on *m* independent machines with unavailability constraints was the model discussed by Lee and Chen (2000). The machines working in parallel and each of them has once unavailability period for maintenance during the horizon time. Scheduling the jobs and maintenance activities simultaneously in case of sufficient resources to maintain any number of machines at the same time was the first considered scenario, the second was if the available resources are sufficient to maintain just one machine and machines need maintenance services should wait until finishing the machine under maintenance. Their conclusion was, the problem is NP-hard in both cases even the jobs have equal weights. The branch and bound is based on column generation and solves the medium size problems in reasonable time.

Kenne, Boukas and Gharbi (2003) discussed the production scheduling and preventive maintenance planning for a flexible manufacturing system having several identical machines. The machines can produce different types of parts and they are subject to the sudden failures. Preventive maintenance increases the system productivity by increasing the machine availability. The decision variables are the inventory levels which are affected by the production and repair rates. The work aims to minimize the surplus and repair costs and a computational algorithm is employed for that. Minimizing the makespan of parallel multi-machines with availability constraints of the jobs and machines was a studied by Gharbi and Haouari (2005). They consider the release dates and delivery times of the jobs and the unavailability period of each machine. The schedule is said to be feasible when the job is performed on one machine without interruption and a branch and bound algorithm has been used to solve the problem. The computational result shows its ability to solve the problem up to 700 jobs and 20 machines within moderate CPU time. Liao, Shyur & Lin (2005) partition the problem of minimizing the makespan of two machines into four sub-problems. One of the two machines has unavailability period during specific time and length. The two cases resumable and nonresumable are considered, and each one of the sub-problem are solved optimally by an algorithm where each of them has an exponential time complexity, but they were efficient in solving the large problems. Considering the unavailability and eligibility (each job should be treated on specific machine) constraints for M machines in parallel and *n* unrelated jobs; Liao and Sheen (2008) introduced a binary search algorithm for non-preemptive scheduling problem. The algorithm is verifying the infeasibility of the problem or find the optimal maximum makespan if it exists by solving a series of maximum flow problems. Lee and Wu (2008) assumed the job processing time is increasing linearly in its starting time and the unavailability periods are known in advance for scheduling the production on multi identical machine problem. The objective was minimizing the makespan in both cases resumable and non resumable and a lower bound heuristic algorithm is proposed for each case. Sheen, Liao and Lin (2008) determine the optimal solution that minimizes the maximum lateness on parallel machines with availability and eligibility constraint using branch and bound method. The case is non-preemptive and the unavailability period on each machine is known in advance. A comparison has been made between the branch and bound method with some priority dispatching rules (LFJ, LPT and EDD) and the results showed the accuracy of the solution by the dispatching rules is not good. Three exact methods which are mixed integer linear programming, a dynamic programming and branch and bound were proposed by Mellouli, Sadfi, Chu and Kacem (2009) for minimizing the total completion times on identical parallel machines where each of them has an unavailability period. Additionally, some heuristics are presented and the experimental results showed the ability of the introduced heuristics to suggest satisfactory solutions. Xu, Cheng, Yin and Li (2009) discussed the minimizing of makespan on two parallel machines where one machine is not available within a periodical period. They show that the worst case ratio of LPT rule is 3/2 for offline problem and 2 for online schedule. Sun and Li (2010) suggested two models to schedule a set of nonpreemptive jobs on two identical parallel machines which are subject to multiple maintenance activities. First, the model objective is to minimize the makespan and the maintenance activities are performed periodically. Second, the model minimizes the total completion time by integrating the decisions of production and maintenance. Finally, two algorithms are proposed to solve the above mentioned models. Preventive maintenance and prior assignments are the two unavailability causes that were discussed for m parallel machines by Fu, Huo and Zhao (2011) to minimize the total weighted completion time. Their study showed that there was no approximation algorithm when the unavailability period (due to PM) was planned for all of the m machines. To simplify this problem, some assumptions have been made: one machine is always available, the processing time is equal to the weight for each job, and there are a fixed number of unavailability periods. Consequently, a polynomial–time approximation scheme was developed. Additionally, the prior assignment at a certain time of n jobs having different weights on specific machines showed that there is no constant approximation with two constant jobs (two prior assigned jobs) for one machine or with one constant job for each machine in the case of two machines. Tan, Chen and Zhang (2011) considering the scheduling problem of production and maintenance on m machines where k of m machines ($1 \leq k \leq m$) are unavailable during specific periods of time and $m - k$ are always available. Their target is to minimizing the total completion time and they show that the worst case ratio of SPT algorithm is at most $[1 + (m-1)/(m-k)]$ when $k < m$. Also, it can be at most $[2 + (k-1)/(m-1)]$ if the unavailability periods do not overlap and exactly one unavailable period on each of the first k machines. A mathematical programming model built by Xu and Yang (2013) for minimizing the makespan on two parallel machines where one of them is unavailable periodically. They transforming the setting of the two machines into one machine setting and the computational experiments show the LPT rule beat the SPT rule in the average case analysis for 96% of the problem parameters combinations. Wang, Zhou, Ji, and Wang (2014) assume that the jobs processing times are increased linearly of their starting time. The m machines are subject to unavailability period and may not all of them is ready at time zero. All the jobs are available at the beginning of the time horizon and the case is not preemptive. Heuristic algorithms are proposed to minimizing the logarithm of makespan. A resumable model with multiple availability constraints for minimizing the makespan on parallel machines was the work introduced by Hashemian, Diallo and Vizvári (2014). The small size problems formulated as a mixed integer linear programming and solved optimally by CPLEX. Additionally, an implicit enumeration algorithm employing the lexicographic order is presented for large size problems. The numerical analysis show the validity and performance improvement of the enumeration method.

3.5 Process Industry

The simultaneous schedule of the production and preventive maintenance is not limited for shop scheduling problems. It is also applicable in the process industry. Ashayeri, Teelen & Selen (1996) introduced a model minimize the maintenance costs (preventive and corrective), inventory and backorders costs and production costs. The reliability of the system affected by the preventive maintenance decisions which are considered at the same time with the production schedule. The problem formulated as a mixed integer linear programming problem in flexible way to be applicable for other production conditions (discrete manufacturing) in JIT environments. The model solved using special branch and bound method (restricted) and the authors suggested to producing a heuristic for large scale problems. Chang, Zhou and Li (2016) analyzed a model that integrated the production and maintenance decisions for a machining system with a constant demand and a single product. Both the cutting tool life and the machine reliability were affected by the production rate, so the machine's preventive maintenance and cutting tool replacement were considered. The preventive maintenance achieved after N cutting tool replacements, and whenever a failure occurs, the corrective maintenance took place. The demand shortage can be reduced by controlling the stock. To minimize the unit production cost, decisions about the production rate and N (the number of cutting tool replacements) should be made simultaneously. The model was solved numerically to determine the optimal joint policy.

3.6 The Current Study

This study can be divided to two major lines:

1. The first main result is a model integrating preventive maintenance and production schedule for a single machine in order to minimize the total completion time. The model assumes that more than one failures can be happening during the job processing time on the contrary of Cassady and Kutanoglu's (2003) assumption and the expected number of failures estimated according to the non-homogenous Poisson process which has Weibul hazard function as a failure rate function. The repair after failures is minimal to keep the machine in functional status and preventive maintenance cannot interrupt the job processing, so the model is a resumable model. Preventive maintenance repair is perfect and restore the machine to a good condition (good as new) as well as it can be performed just before or after finishing the job processing. The job sequence and the time to achieve preventive maintenances are decision variables.
2. The second main result is a model integrating the preventive maintenance and production schedule for minimizing the maximum expected makespan on multi-identical machine. The possible number of failures and estimated repair time for each machine estimated according to non-homogenous Poisson process and Weibull distribution with same or different parameters. The schedule of preventive maintenance is decision variable and it cannot interrupt the job processing. The balancing of the load on the available machines is the target of this model as well as minimizing the difference between maximum and minimum makespan.

However, to analyze the makespan on the several machines the one should know its behavior on the single machine. Because of this a sufficient analysis has been made to exploring the measure behavior. During the entire study, it has been assumed that the maintenance resources are enough to maintain any number of machines at any time.

In the next chapter, Chapter 4, the single machine model which minimizes the expected total completion time is introduced with its solution method.

Chapter 4

SIMULTANEOUS SCHEDULING OF PRODUCTION AND PREVENTIVE MAINTENANCE ON A SINGLE MACHINE

4.1 Introductory Remarks

For approximately 50 years, production scheduling on a single machine has received increased attention (Haddad, 2014, pp. 6543). Machine being available to work at any time is the common assumption that has been made before two decades in most of the proposed scheduling models. Actually, the machine could be unavailable due to many practical reasons such as maintenance, breakdowns and prior assignments. Considering such factors in the scheduling problems will increase the ability to carry out the proposed schedules in the real life.

One of the most important factors that affect the performance of a production system is the implemented maintenance strategy. In this chapter, a model for integrating maintenance and production schedules to minimize the expected total completion time (ETCT) for a single machine will be discussed. Minimizing the total expected completion time according to Oyetunji and Oluleye (2008) will lead to minimizing some other important measures, such as total flow time (F_{tot}), total lateness (L_{tot}), average completion time (C_{avg}), average flow time (F_{avg}) and average lateness (L_{avg}). Therefore, the model can be used to optimize other functions. The Weibull distribution will represent the failure function $(\beta > 1)$ of the nonhomogeneous Poisson process which estimates the model failures as shown in chapter 2, equations from (8) to (12), and if any preventive maintenance is performed before starting the job processing, preventive maintenance will restore the machine age to zero (perfect repair) according to the renewal process in section 2.3.2 – 2 and 2.3.6. The repair due to failure is minimal, and the model is resumable after repair.

Based on some lemmas, the model of the problem is reduced from a nonlinear 0-1 constrained problem to a nonlinear 0-1 unconstrained problem in section 4.4. A method for solving the unconstrained model and to identify the PM decisions will be introduced and illustrated with numerical example. Finally, a numerical analysis is performed to investigate the effect of changes in the model parameters on the solution. The results show its sensitivity to changing the preventive maintenance plan and the expected value of the total completion time depending on the changed parameters and the value of the change of each parameter.

4.2 The Integrating Model

The proposed model simultaneously considers the production schedule and preventive maintenance plan for minimizing the expected total completion time. It is assumed that PM transforms the machine age to zero. The initial age of the machine, which is denoted by a_0, can be nonzero.

4.2.1 Notations

i: index of job; 1, 2, …., n.

j: index of the job sequence on the machine, 1, 2, ., n.

a_0 : age of the machine prior to making sequencing and PM decisions.

a_j: age of the machine after performing a job in position j and before the j+1 PM decision.

C_j: completion time for a job in position j on the machine.

F(t): cumulative distribution function of Weibll distribution.

p_i : processing time for the i^{th} job on the machine.

N_j (p_j; a_{j-1}): number of failures of the machine due to processing a job in position j with respect to the age of the machine a_{j-1}, and if $a_{j-1} = 0$, $N_j = (p_j)$.

t_r: repair time for the machine in case of failure; it is constant.

t_m: preventive maintenance time for the machine.

x_{ij}: equal to 1 if job i is in position j and 0 if not.

y_j: PM decision on machine before the job at the j^{th} position, (0 or 1).

4.2.2 The Model

The completion time of the first job if job i is in the first position (j = 1) is

$$C_1 = p_i + t_m y_1 + (E\{N_1(p_i, (1 - y_1)a_0)\})t_r$$

The machine age after processing the first job is

$$a_1 = (1 - y_1)a_0 + p_i$$

The completion time of the second job if job k is in the second position (j = 2) is

$$C_2 = p_k + t_m y_2 + (E\{N_2(p_k, (1 - y_2)a_1)\})t_r + C_1; \;\; k \neq i$$

The completion time of the job in the j^{th} position, i.e., C_j, can be determined in a similar way. The natural requirement that each job must be assigned to a position and only one job can be assigned to a position can be described by the usual assignment problem constraints; see equations (19) and (20) below. A general model that minimizes the total expected completion time can be described as follows:

$$Min \sum_{j=1}^{n}(n - j + 1)\left[\sum_{i=1}^{n} p_i x_{ij} + t_m y_j + \left(E\left\{N_j\left(\sum_{i=1}^{n} p_i\, x_{ij}\,, (1 - y_j)a_{j-1}\right)\right\}\right)t_r\right] \qquad (16)$$

subject to:

$$a_j = (1 - y_j)a_{j-1} + \sum_{i=1}^{n} p_i \ x_{ij} \tag{17}$$

$$E\left\{N_j\left(\sum_{i=1}^{n} p_i\, x_{ij}\,, (1 - y_j)a_{j-1}\right)\right\} = \left(\frac{(1 - y_j)a_{j-1} + \ \sum_{i=1}^{n} p_i\, x_{ij}}{\eta}\right)^{\beta} - \left(\frac{(1 - y_j)a_{j-1}}{\eta}\right)^{\beta} \tag{18}$$

$j = 1, 2, \dots, n$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \forall\, j = 1, 2, \dots, n \tag{19}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \forall\, i = 1, 2, \dots, n \tag{20}$$

$$\forall\, i, j:\ x_{ij} = 0 \text{ or } 1$$

$$\forall\, j:\ y_j = 0 \text{ or } 1$$

It is clear that the foregoing model represents a nonlinear 0-1 constrained problem, and determining its optimal solution requires implicite or explicite enumeration all of the possible ($n! \times 2^n$) solutions. In the next section, some properties of the problem will be used to simplify the solution procedure.

4.3 The Incremental Failure Function and the SPT Rule

The employed failure function is the Weibull function with $\beta > 1$ and it is an increasing function in the job processing time and machine age; thus, the SPT rule will be optimal for minimizing the total completion time, as shown below.

4.3.1 Lemma 1

For integrating the PM plan and production schedule on a single machine, if

(i) the failure function is increasing function of both the machine age and length of the time interval, and

(ii) the repair time and the preventive maintenance time (t_r, t_m) are fixed, then the expected value of the ***total completion time*** on the machine is minimal only if the jobs are in SPT order.

Proof:

Assume that in an order of jobs, there are two consecutive jobs with processing times $P_1 \leq P_2$. The expected number of failures on the machine while processing any of these jobs is:

$$E\{N(P_k; (1-y_j)t_s)\}$$

where,

P_k: processing time of the job (k is job index), $k = 1, 2$

y_j: preventive maintenance decision prior to job k in position j regardless of whether it is job 1 or 2 and $j = 1, 2$.

t_s: the age of the machine before processing any of the jobs and before carrying out any preventive maintenance action.

The basic idea of the proof is to determine the case when the interchange of the two jobs improves/does not improve the objective function. The latter case holds if the following inequality is true. The left-hand side of the inequality describes the case before the interchange, and the right-hand side describes the case after the interchange.

$P_1 + E\{N(P_1; (1- y_1)t_s)\}t_r + y_1 t_m + P_1 + E\{N(P_1; (1- y_1)t_s)\}t_r + y_1 t_m +$

$P_2 + E\left\{N\left(P_2; (1-y_2)\big((1- y_1)t_s + P_1\big)\right)\right\}t_r + y_2 t_m \leq P_2 + E\{N(P_2; (1-$

$y_1)t_s)\}t_r + y_1t_m + P_2 + E\{N(P_2;\ (1 - y_1)t_s)\}t_r + y_1t_m + P_1 +$

$$E\left\{N\left(P_1; (1 - y_2)\big((1 - y_1)t_s + P_2\big)\right)\right\}t_r + y_2t_m \tag{21}$$

This inequality can be written as

$2P_1 + 2E\{N(P_1;\ (1 - y_1)t_s)\}t_r + 2y_1t_m + P_2 + E\left\{N\left(P_2; (1 - y_2)\big((1 -\right.\right.$

$\left.\left.y_1)t_s + P_1\big)\right)\right\}t_r + y_2t_m \le 2P_2 + 2E\{N(P_2;\ (1 - y_1)t_s)\}t_r + 2y_1t_m + P_1 +$

$$E\left\{N\left(P_1; (1 - y_2)\big((1 - y_1)t_s + P_1\big)\right)\right\}t_r + y_2t_m \tag{22}$$

Then,

$P_1 + 2E\{N(P_1; (1 - y_1)t_s)\}t_r + E\left\{N\left(P_2; (1 - y_2)\big((1 - y_1)t_s + P_1\big)\right)\right\}t_r \le P_2 +$

$$2E\{N(P_2;\ (1 - y_1)t_s)\}t_r + E\left\{N\left(P_1; (1 - y_2)\big((1 - y_1)t_s + P_2\big)\right)\right\}t_r \tag{23}$$

Four cases regarding the values of y_1 and y_2 can be presented:

a) If $y_1 = y_2 = 1$,

In this case, the inequality in (23) will be

$$P_1 + 2E\{N(P_1;\ 0)\}t_r + E\{N(P_2; 0)\}t_r \le P_2 + 2E\{N(P_2;\ 0)\}t_r + E\{N(P_1; 0)\}t_r$$

$$P_1 + E\{N(P_1;\ 0)\}t_r \le P_2 + E\{N(P_2;\ 0)\}t_r \qquad (23.\boldsymbol{a})$$

As $P_1 \le P_2$ and the expected number of failures during a period equal to P_1 is less than the expected number of failures during P_2, the forgoing inequality holds in this case.

b) If $y_1 = y_2 = 0$,

Return to the inequality in (23):

$$P_1 + 2E\{N(P_1;\ t_s)\}t_r + \ E\big\{N\big(P_2;(t_s +\ P_1)\big)\big\}t_r \ \le\ P_2 + 2E\{N(P_2;\ t_s)\}t_r + E\big\{N\big(P_1;(t_s +\ P_2)\big)\big\}t_r$$

The previous inequality can be written as

$$P_1 +\ 2E\{N(P_1 + t_s)\}t_r - 2E\{N(t_s)\}t_r + E\{N(P_1 + P_2 + t_s)\}t_r - E\{N(P_1 + t_s)\}t_r \ \le\ P_2 +\ 2E\{N(P_2 + t_s)\}t_r - 2E\{N(t_s)\}t_r + E\{N(P_1 + P_2 + t_s)\}t_r - E\{N(P_2 + t_s)\}t_r$$

$$P_1 + E\{N(P_1 + t_s)\}t_r \le\ P_2 + E\{N(P_2 + t_s)\}t_r \qquad (23.\boldsymbol{b})$$

It is clear that the left-hand side is always less than the right-hand side, and the inequality holds.

c) If $y_1 = 1$ and $y_2 = 0$,

$$P_1 + 2E\{N(P_1;\ 0)\}t_r +\ E\{N(P_2;P_1)\}t_r \le\ P_2 + 2E\{N(P_2;\ 0)\}t_r +\ E\{N(P_1;P_2)\}t_r$$

$$P_1 +\ 2E\{N(P_1)\}t_r + E\{N(P_1 + P_2)\}t_r - E\{N(P_1)\}t_r \ \le\ P_2 +\ 2E\{N(P_2)\}t_r + E\{N(P_1 + P_2)\}t_r - E\{N(P_2)\}t_r$$

$$P_1 +\ E\{N(P_1)\}t_r \ \le\ P_2 +\ E\{N(P_2)\}t_r \qquad (23.\boldsymbol{c})$$

The previous result supports the claim for the inequality in (21).

d) If $y_1 = 0$ and $y_2 = 1$,

$$P_1 + 2E\{N(P_1;\ t_s)\}t_r +\ E\{N(P_2;0)\}t_r \ \le\ P_2 + 2E\{N(P_2;\ t_s)\}t_r +\ E\{N(P_1;0)\}t_r$$

$$P_1 +\ 2E\{N(P_1 + t_s)\}t_r - 2E\{N(t_s)\}t_r + E\{N(P_2)\}t_r \ \le\ P_2 +\ 2E\{N(P_2 + t_s)\}t_r - 2E\{N(t_s)\}t_r + E\{N(P_1)\}t_r$$

$$P_1 +\ 2E\{N(P_1 + t_s)\}t_r - E\{N(P_1)\}t_r \ \le\ P_2 +\ 2E\{N(P_2 + t_s)\}t_r - E\{N(P_2)\}t_r \qquad (23.\boldsymbol{d})$$

$$P_1 + (E\{N(P_2)\}t_r - E\{N(P_1)\}t_r) \ \le\ P_2 +\ 2(E\{N(P_2 + t_s)\}t_r - E\{N(P_1 + t_s)t_r\})$$

As $P_1 \leq P_2$, the failure function is an increasing function in age and/or operation time, and P_1 and P_2 are increased by the same level (t_s) on the right-hand side; thus, the following relationship will always be correct.

$$(E\{N(P_2)\} - E\{N(P_1)\}) \leq 2(E\{N(P_2 + t_s)\} - E\{N(P_1 + t_s)\})$$

Then, the inequality in (21) will hold for all possible cases, and the proof is complete.■

Now, it can be concluded that regardless of the preventive maintenance plan, the optimal sequence will be the sequence that respects the SPT rule. Actually, this will not lead directly to the optimal solution of the model because the optimal preventive maintenance plan must still be explored.

4.3.2 Lemma 2

The total completion time for two jobs can be minimized by switching the preventive maintenance plan if one of the following two inequalities holds.

- If $y_1 = 1$ and $y_2 = 0$,

$$2E\{N(P_1;\ 0)\}t_r + t_m + E\{N(P_2; P_1)\}t_r \geq 2E\{N(P_1;\ t_s)\}t_r + E\{N(P_2; 0)\}t_r \tag{24}$$

- If $y_1 = 0$ and $y_2 = 1$,

$$2E\{N(P_1;\ t_s)\}t_r + E\{N(P_2; 0)\}t_r \geq 2E\{N(P_1;\ 0)\}t_r + t_m + E\{N(P_2; P_1)\}t_r \tag{25}$$

Proof:

Switching the preventive maintenance plan and keeping the jobs in the shortest processing time are advantages if the following inequality holds.

$$2P_1 + 2E\{N(P_1;\ (1-\ y_1)t_s)\}t_r +\ 2y_1t_m +\ \ P_2\ +\ E\left\{N\left(P_2;(1-y_2)\left((1-y_1)t_s +\ P_1\right)\right)\right\}t_r +\ y_2t_m\ \geq\ 2P_1 + 2E\{N(P_1;\ (1-\ y_2)t_s)\}t_r +\ 2y_2t_m +\ \ P_2\ + E\left\{N\left(P_2;(1-y_1)\left((1-\ y_2)t_s +\ P_1\right)\right)\right\}t_r +\ y_1t_m$$

$$2E\{N(P_1;\ (1-\ y_1)t_s)\}t_r +\ y_1t_m +\ E\left\{N\left(P_2;(1-y_2)\left((1-\ y_1)t_s +\ P_1\right)\right)\right\}t_r\ \geq\ 2E\{N(P_1;\ (1-\ y_2)t_s)\}t_r +\ y_2t_m +\ E\left\{N\left(P_2;(1-y_1)\left((1-\ y_2)t_s +\ P_1\right)\right)\right\}t_r \tag{26}$$

Two cases should be discussed here:

a) If $y_1 = 1$ and $y_2 = 0$, by substituting in (26):

$$2E\{N(P_1;\ 0)\}t_r +\ t_m +\ E\{N(P_2;P_1)\}t_r\ \geq\ 2E\{N(P_1;\ t_s)\}t_r +\ E\{N(P_2;0)\}t_r$$

b) If $y_1 = 0$ and $y_2 = 1$, by substituting in (26):

$$2E\{N(P_1;\ t_s)\}t_r +\ E\{N(P_2;0)\}t_r\ \geq\ 2E\{N(P_1;\ 0)\}t_r +\ t_m +\ E\{N(P_2;P_1)\}t_r$$

The two conditions hold, and the proof has been finished.■

4.3.3 Lemma 3

The expected completion time of a job *j* can be decreased by switching the preventive maintenance decision selected before that job from Yes ($y_j = 1$) to No ($y_j = 0$) if and only if the following inequality holds.

$$E\{N(P_j;0)\}t_r +\ t_m\ \geq\ \ E\{N(P_j;t_s)\}t_r \tag{27}$$

Proof:

$$P_j + E\{N(P_j;\ (1-\ y_j)t_s)\}t_r +\ y_jt_m \geq\ P_j + E\{N(P_j;\ (1-\ \bar{y}_j)t_s)\}t_r +\ \bar{y}_jt_m$$

where $\bar{y}_j\ = 1-\ y_j$; then,

$$E\{N(P_j;(1-y_j)t_s)\}t_r + y_j t_m \geq E\{N(P_j;(1-\bar{y}_j)t_s)\}t_r + \bar{y}_j t_m$$

Let $y_j = 1$, then $\bar{y}_j = 0$ and

$$E\{N(P_j;0)\}t_r + t_m \geq E\{N(P_j;t_s)\}t_r \blacksquare$$

The inequality in (27) states that y_j can be switched from 1 to 0 if and only if the expected repair time due to operating the machine for a period equal to the job processing time, considering its previous age to be zero, plus the preventive maintenance time is greater than the total expected repair time due to operating the machine for a period equal to the job processing time considering the previous age ($t_s > 0$).

In the same context, the following can be shown.

4.3.4 Lemma 4

The expected completion time of a job can be minimized by switching the preventive maintenance decision selected before the job from No ($y_j = 0$) to Yes ($y_j = 1$) if and only if the following inequality holds.

$$E\{N(P_j;0)\}t_r + t_m \leq E\{N(P_j;t_s)\}t_r$$

Note: Lemmas 3 and 4 show that the total completion time of a job in its position can be minimized by switching the PM plan decision prior to the job, but this cannot guarantee that the expected total completion time of all jobs in their sequence will also be minimized. However, in the next section, the previous facts will be used to rebuild the model in (16).

4.4 Unconstrained Model

By referring to the problem of integrating the production schedule and the preventive maintenance plan and the model introduced in (16), suppose that

$$P_1 \leq P_2 \leq P_3 \leq \cdots \leq P_n$$

Depending on lemma 1, the two constraints in (19) and (20) are assigned in advance using the SPT rule, and the mentioned model can be rewritten as:

$$Min\ ETCT = \sum_{i=1}^{n}(n-i+1)P_i + \left(\sum_{i=1}^{n}(n-i+1)E\ (N[P_i; a_{i-1}])\right)t_r + \left(\sum_{i=1}^{n}(n-i+1)\ y_i\right)t_m \quad (28)$$

where y_i and a_{i-1} have the same definition as y_j and a_{j-1}.

Because the model assumes that the Weibull distribution function is a failure function,

$$Min\ ETCT = nP_1 + n\left[\left(\frac{(1-y_1)a_0+P_1}{\eta}\right)^{\beta} - \left(\frac{(1-y_1)a_0}{\eta}\right)^{\beta}\right]t_r + ny_1t_m + (n-1)P_2 + (n-1)\left[\left(\frac{(1-y_2)\left((1-y_1)a_0+P_1\right)+P_2}{\eta}\right)^{\beta} - \left(\frac{(1-y_2)\left((1-y_1)a_0+P_1\right)}{\eta}\right)^{\beta}\right]t_r + (n-1)y_1t_p + \cdots + P_n + E[N(P_n; a_{n-1})]t_r + y_nt_m \quad (29)$$

Let $\bar{y}_i = 1 - y_i$. If $\bar{y}_i = 0$, then PM will be performed, and if $\bar{y}_i = 1$, then no PM will be performed.

$$Min\ ETCT = nP_1 + n\left[\left(\frac{a_0\bar{y}_1+P_1}{\eta}\right)^{\beta} - \left(\frac{a_0\bar{y}_1}{\eta}\right)^{\beta}\right]t_r + nt_m - n\bar{y}_1t_m + (n-1)P_2 + (n-1)\left[\left(\frac{a_0\bar{y}_1\bar{y}_2+P_1\bar{y}_2+P_2}{\eta}\right)^{\beta} - \left(\frac{a_0\bar{y}_1\bar{y}_2+P_1\bar{y}_2}{\eta}\right)^{\beta}\right]t_r + (n-1)t_m - (n-1)\bar{y}_2t_m + \cdots + P_n +$$

$$\left[\left(\frac{a_0\bar{y}_1\bar{y}_2\bar{y}_3\ldots\bar{y}_n+P_1\bar{y}_2\bar{y}_3\ldots\bar{y}_n+P_2\bar{y}_3\ldots\bar{y}_n+\cdots+P_{n-1}\bar{y}_n+P_n}{\eta}\right)^{\beta}-\right.$$

$$\left.\left(\frac{a_0\bar{y}_1\bar{y}_2\bar{y}_3\ldots\bar{y}_n+P_1\bar{y}_2\bar{y}_3\ldots\bar{y}_n+P_2\bar{y}_3\ldots\bar{y}_n+\cdots+P_{n-1}\bar{y}_n}{\eta}\right)^{\beta}\right]t_r+t_m-\bar{y}_n t_m \qquad (30)$$

In general,

$$Min\ ETCT=\sum_{i=1}^{n}(n-i+1)P_i+\frac{n(n+1)}{2}t_m-\sum_{i=1}^{n}(n-i+1)\,t_m\bar{y}_i$$
$$+\left(\sum_{k=1}^{n}(n-k+1)\left[\left(\frac{a_0}{\eta}\prod_{i=1}^{k}\bar{y}_i+\sum_{i=2}^{k}\frac{P_{i-1}}{\eta}\prod_{i}^{k}\bar{y}_i+\frac{P_k}{\eta}\right)^{\beta}\right.\right.$$
$$\left.\left.-\left(\frac{a_0}{\eta}\prod_{i=1}^{k}\bar{y}_i+\sum_{i=2}^{k}\frac{P_{i-1}}{\eta}\prod_{i}^{k}\bar{y}_i\right)^{\beta}\right]\right)t_r \qquad (31)$$

The model becomes an unconstrained nonlinear 0–1 function that can be generated and solved more easily than the first function.

In the next section, lemmas 2, 3, 4 and the yet to be introduced lemmas 5 and 6 with the equation in (15) will be used to find the optimal solution for the previous model.

4.5 Solution Procedure

To solve the nonlinear unconstrained model in (31), which aims to minimize the expected total completion time, some important values should be defined.

4.5.1 The Best Time to Achieve the Preventive Maintenance Action

To maximize the machine availability, the preventive maintenance action should be achieved at $\dot{\tau}$ as shown in chapter 2, section 2.3.6 by equation (15):

$$\dot{\tau}=\eta\left[\frac{t_m}{t_r(\beta-1)}\right]^{\frac{1}{\beta}}$$

where $\dot{\tau}$ is the best time to perform preventive maintenance if $\beta > 1$. $\dot{\tau}$ is true for an infinite time horizon and for preemptive technology.

When the age of the machine before beginning to process the next job is greater than or equal to $\dot{\tau}$, the preventive maintenance should be executed to open another cycle ($\dot{\tau}$), which will maximize the machine availability. Conversely, because the problem has a bounded time horizon and non-preemptive technology, the full utilization of the opened cycle by the remaining load (RL) is uncertain all of the time. Therefore, opening a new cycle without sufficient utilization may cause an increase in the value of the performance measure. A reference point that can help in making a decision at each $a_{j-1} \geq \dot{\tau}$ may be defined as follows.

The expected down time period during each $\dot{\tau}$ period is:

$$t_m + \left(\frac{\dot{\tau}}{\eta}\right)^{\beta} t_r \qquad (32)$$

Assume that $RL < \dot{\tau}$; then, ρ_k is the period of time to the breakeven point between the total expected downtime during the period $\dot{\tau}$ (PM + repair time) and is the expected total downtime during the period ρ_k considering the machine's previous age (repair time during ρ_k considering a_{j-1}).

Let,

$$t_m + \left(\frac{\dot{\tau}}{\eta}\right)^{\beta} t_r = \left[\left(\frac{a_{j-1} + \rho_k}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right] t_r \qquad (33)$$

where *k* is the cycle index.

Before using the ρ_k value, two facts should be known:

1. The number of failures function *N(t)* is a strictly convex function for all values of *t* (β >1).

$$N(t) = \left[\left(\frac{a_{j-1} + t}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]$$

$$\frac{d}{dt}\left(\frac{dN(t)}{dt}\right) = \frac{\beta(\beta - 1)}{\eta^2}\left(\frac{a_{j-1} + t}{\eta}\right)^{\beta-2} > 0$$

2. The second fact can be introduced by the following lemma:

4.5.2 Lemma 5

For all initial ages a_{j-1}, there is at most one RL ($\bar{\rho}$) such that the expected values of the downtime with PM and without PM are equal.

Proof:

If $a_{j-1} = 0$, then the expected downtime without PM is strictly less than with PM as $t_m > 0$. Assume that $a_{j-1} > 0$ and;

Let ρ be the RL. The function of the expected number of failures with PM is

$$N(\rho) = t_m + \left(\frac{\rho}{\eta}\right)^{\beta} t_r$$

and;

$$\frac{dN(\rho)}{d\rho} = \frac{\beta}{\eta}\left(\frac{\rho}{\eta}\right)^{\beta-1} t_r$$

where $\rho \leq \dot{\tau}$.

The function of the expected number of failures without PM at the beginning of a cycle of length $\dot{\tau}$ is

$$N(\rho) = \left[\left(\frac{a_{j-1} + \rho}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right] t_r$$

and;

$$\frac{dN(\rho)}{d\rho} = \frac{\beta}{\eta}\left(\frac{a_{j-1} + \rho}{\eta}\right)^{\beta-1} t_r$$

Note that

$$\frac{\beta}{\eta}\left(\frac{\rho}{\eta}\right)^{\beta-1} t_r \leq \frac{\beta}{\eta}\left(\frac{a_{j-1} + \rho}{\eta}\right)^{\beta-1} t_r$$

Thus, because the first derivative of the function of the expected number of failures with PM is always less than or equal to the first derivative of the function of the expected number of failures without PM, it can be concluded that there is at most only one breakeven point between these two functions in the period $\dot{\tau}$.■

Now, by using equation (33), the value ρ_k can be calculated as:

$$\rho_k = \eta \left[\frac{t_m}{t_r} + \left(\frac{\dot{\tau}}{\eta}\right)^{\beta} + \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]^{\frac{1}{\beta}} - a_{j-1} \qquad (34)$$

Let μ_k be the age of the machine at the breakeven point. Then,

$$\mu_k = \rho_k + a_{j-1} = \eta \left[\frac{t_m}{t_r} + \left(\frac{\dot{\tau}}{\eta}\right)^{\beta} + \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]^{\frac{1}{\beta}} \qquad (35)$$

Therefore, the conclusion is that when the age of the machine before beginning to process the next job is greater than or equal to $\dot{\tau}$ ($a_{j-1} \geq \dot{\tau}$) and the remaining work load is less than $\dot{\tau}$, the preventive maintenance decision cannot be made according to $\dot{\tau}$ only, and the decision should also be investigated from the performance measure side.

The other problem that causes the main difficulty in solving model (31) is that the PM can be performed only before beginning to process a job. Therefore, when $a_{j-1} < \dot{\tau}$ and $a_j = a_{j-1} + p_i > \dot{\tau}$, an appropriate criterion to decide when the PM should be performed (to do it early or to delay it) is required. Otherwise, all of the possible scenarios for PM should be investigated. Before defining this criterion, other important values will be introduced.

4.5.3 The Time When the Expected Repair Time Equals the Preventive Maintenance Time

Let a_{j-1} be the age of the machine. The expected repair time is equal to the preventive maintenance time at load $\emptyset_j$. If

$$\left[\left(\frac{a_{j-1} + \emptyset_j}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right] t_r = t_m \tag{36}$$

then,

$$\left(\frac{a_{j-1} + \emptyset_j}{\eta}\right)^{\beta} = \frac{t_m}{t_r} + \left(\frac{a_{j-1}}{\eta}\right)^{\beta}$$

$$\emptyset_j = \eta\left[\frac{t_m}{t_r} + \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]^{\frac{1}{\beta}} - a_{j-1} \tag{37}$$

where $\emptyset_j$ is the additional time added to the machine age to make the expected repair time equal to the PM time. If

$$\gamma_j = \emptyset_j + a_{j-1} = \eta\left[\frac{t_m}{t_r} + \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]^{\frac{1}{\beta}} \tag{38}$$

then γ_j is the total machine age required to make the expected repair time equal to the PM time. Let π_j is the total age of the machine after processing the next job P_j ($\pi_j = a_{j-1} + P_j$). If $\pi_j = \gamma_j$, then

$$\left[\left(\frac{\gamma_j}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right] t_r = t_m \tag{39}$$

If $\pi_j > \gamma_j$, then

$$\left[\left(\frac{\pi_j}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right] t_r > t_m \tag{40}$$

and;

If $\pi_j < \gamma_j$, then

$$\left[\left(\frac{\pi_j}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right] t_r < t_m \tag{41}$$

4.5.4 The Breakeven Point for (age value) the Expected Completion Time with Preventive Maintenance and without Preventive Maintenance

Referring to Lemma 3, it is known that the job completion time decreases by changing the PM decision from yes to no (y_j= 1 to y_j = 0 or $\bar{y}_j = 0 \; to \; \bar{y}_j = 1$) if and only if

$$E\{N(P_j; 0)\}t_r + t_m \geq E\{N(P_j; t_s)\}t_r$$

The next question to be answered is the following: when will the job expected completion time (ECT) with preventive maintenance equal the job expected completion time without preventive maintenance?

$$ECT_{y_j=1} = P_j + t_m + \left(\frac{P_j}{\eta}\right)^{\beta} t_r$$

and;

$$ECT_{y_j=0} = P_j + \left[\left(\frac{a_{j-1} + P_j}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right] t_r$$

If $\pi_j = \gamma_j$, then, depending on (39) and, $ECT_{y_j=1} > ECT_{y_j=0}$ then an additional value, say Δ_j, should be added to γ_j to balance the difference due to the value of $\left(\frac{P_j}{\eta}\right)^{\beta} t_r$, and then

$$\left[\left(\frac{\gamma_j + \Delta_j}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right] t_r = t_m + \left(\frac{P_j}{\eta}\right)^{\beta} t_r \tag{42}$$

From equation in (39),

$$\left[\left(\frac{\gamma_j}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right] t_r = t_m$$

Then,

$$\left[\left(\frac{\gamma_j + \Delta_j}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right] t_r = \left[\left(\frac{\gamma_j}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right] t_r + \left(\frac{P_j}{\eta}\right)^{\beta} t_r$$

$$\Delta_j = \eta \left[\left(\frac{\gamma_j}{\eta}\right)^{\beta} + \left(\frac{P_j}{\eta}\right)^{\beta}\right]^{\frac{1}{\beta}} - \gamma_j \tag{43}$$

Also from equation in (42), it can be shown that

$$\Delta_j = \eta \left[\frac{t_m}{t_r} + \left(\frac{P_j}{\eta}\right)^{\beta} + \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]^{\frac{1}{\beta}} - \gamma_j \tag{44}$$

Let,

$$\Psi_j = \gamma_j + \Delta_j = \eta \left[\frac{t_m}{t_r} + \left(\frac{P_j}{\eta}\right)^{\beta} + \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]^{\frac{1}{\beta}} \tag{45}$$

where Ψ_j is the value of the machine age such that the expected value of the completion time of the job without PM equals its expected value with PM.

If $\pi_j = \Psi_j$, then $ECT_{y_j=1} = ECT_{y_j=0}$;

if $\pi_j > \Psi_j$, then $ECT_{y_j=1} < ECT_{y_j=0}$; and

if $\pi_j < \Psi_j$, then $ECT_{y_j=1} > ECT_{y_j=0}$.

It has become appropriate to provide the 6th and 7th lemmas that will be used later to solve the model referred to in (31) according to the basis that when the machine availability is increased, the expected total completion time of a job or a set of jobs will be decreased.

4.5.5 Lemma 6

Assume that:

(i) the index order of the jobs is an SPT order;

(ii) the failure function has the Weibull distribution with $\beta > 1$;

(iii) both the PM and the repair times are constants and are t_m and t_r, respectively and;

(iv) the PM can be performed only before the start of the process of a job.

(v) the first job such that a PM decision must be made before the start of the job, is job j.

If $j < n$ and $a_{j-1} < \dot{\tau} < \pi_j < \Psi_j$ then, $\bar{y}_j = 1$ ($y_j = 0$ for the model in (16)) if and only if one of the following two inequalities holds.

(i) $(q-j+1)\left[\left(\frac{a_{j-1}+P_j}{\eta}\right)^\beta - \left(\frac{a_{j-1}}{\eta}\right)^\beta\right] t_r + (q-j)\left(\frac{P_{j+1}}{\eta}\right)^\beta t_r \le \left(\frac{P_j}{\eta}\right)^\beta t_r +$

$$(q-j)\left(\frac{P_j+P_{j+1}}{\eta}\right)^\beta t_r + t_m \quad (46)$$

(ii) $(q-j+1)\left[\left(\frac{a_{j-1}+P_j}{\eta}\right)^\beta - \left(\frac{a_{j-1}}{\eta}\right)^\beta\right] t_r + (q-j)\left[\left(\frac{a_{j-1}+P_j+P_{j+1}}{\eta}\right)^\beta - \left(\frac{a_{j-1}+P_j}{\eta}\right)^\beta\right] t_r \leq (q-j+1)\left(\frac{P_j}{\eta}\right)^\beta t_r + (q-j+1)t_m + (q-j)\left[\left(\frac{P_j+P_{j+1}}{\eta}\right)^\beta - \left(\frac{P_j}{\eta}\right)^\beta\right] t_r$ (47)

Otherwise, $\bar{y}_j = 0$, where $q = j + 1$

Proof:

If $j < n$, there are at least two decisions for PM. One is before j, and the other is before j+1. Therefore, the possible PM plans expressed by the complementary variables ($\bar{y}_j$, $\bar{y}_{j+1}$) are (1, 1), (1, 0), (0, 1) and (0, 0). Recall that Ψ_j is the breakeven point where the down times of a period of length P_j for machine age a_{j-1} with PM and without PM are equal. It follows from (42) that the down time during P_j is greater with PM than without PM. If the statement is true, then (1, 0) and/or (1, 1) are better than (0, 0) and (0, 1).

Because $\pi_j < \Psi_j$, then (1, 0) is always better than (0, 0), which can be shown as follows:

Let P_j, P_{j+1} be two consecutives jobs, where (1, 0) is at least as good as (0, 0) if and only if

$$(q-j+1)P_j + (q-j+1)\left[\left(\frac{a_{j-1}+P_j}{\eta}\right)^\beta - \left(\frac{a_{j-1}}{\eta}\right)^\beta\right] t_r + (q-j)P_{j+1} + (n-j)\left(\frac{P_{j+1}}{\eta}\right)^\beta t_r + (q-j)t_m \leq (q-j+1)P_j + (q-j+1)\left(\frac{P_j}{\eta}\right)^\beta t_r + (q-j+1)t_m + (q-j)P_{j+1} + (q-j)\left(\frac{P_{j+1}}{\eta}\right)^\beta t_r + (q-j)t_m$$

$$(q-j+1)\left[\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}-\left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]t_r \leq (q-j+1)\left(\frac{P_j}{\eta}\right)^{\beta}t_r+(q-j+1)t_m$$

According to equation in (42) and because $\pi_j < \Psi_j$, the previous inequality holds. Therefore, for $\bar{y}_j = 1$ to be true, one of the two plans (1, 0) or (1, 1) or both must be better than (0, 1).

(1, 0) is at least as good as (0, 1) if and only if:

$$(q-j+1)P_j+(q-j+1)\left[\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}-\left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]t_r+(q-j)P_{j+1}+$$

$$(q-j)\left(\frac{P_{j+1}}{\eta}\right)^{\beta}t_r+(q-j)t_m \leq (q-j+1)P_j+(q-j+1)\left(\frac{P_j}{\eta}\right)^{\beta}t_r+$$

$$(q-j+1)t_m+(q-j)P_{j+1}+(q-j)\left[\left(\frac{P_j+P_{j+1}}{\eta}\right)^{\beta}-\left(\frac{P_j}{\eta}\right)^{\beta}\right]t_r$$

$$(q-j+1)\left[\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}-\left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]t_r+(q-j)\left(\frac{P_{j+1}}{\eta}\right)^{\beta}t_r \leq \left(\frac{P_j}{\eta}\right)^{\beta}t_r+$$

$$(q-j)\left(\frac{P_j+P_{j+1}}{\eta}\right)^{\beta}t_r+t_m$$

This inequality is the first inequality of the statement.

(1, 1) is at least as good as (0, 1) if and only if:

$$(q-j+1)P_j+(q-j+1)\left[\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}-\left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]t_r+(q-j)P_{j+1}+$$

$$(q-j)\left[\left(\frac{a_{j-1}+P_j+P_{j+1}}{\eta}\right)^{\beta}-\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}\right]t_r \leq (q-j+1)P_j+(q-j+1)t_m+$$

$$(q-j+1)\left(\frac{P_j}{\eta}\right)^{\beta}t_r+(q-j)P_{j+1}+(q-j)\left[\left(\frac{P_j+P_{j+1}}{\eta}\right)^{\beta}-\left(\frac{P_j}{\eta}\right)^{\beta}\right]t_r$$

Hence,

$(q-j+1)\left[\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}-\left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]t_r +$

$(q-j)\ \left[\left(\frac{a_{j-1}+P_j+P_{j+1}}{\eta}\right)^{\beta}-\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}\right]t_r\ \leq\ (q-j+1)t_m+(q-j+$

$1)\left(\frac{P_j}{\eta}\right)^{\beta}t_r+(q-j)\left[\left(\frac{P_j+P_{j+1}}{\eta}\right)^{\beta}-\left(\frac{P_j}{\eta}\right)^{\beta}\right]t_r$

The inequality is the second inequality of the statement.

If (1, 0) and/or (1, 1) cannot be better than (0, 1), then (0, 1) is better than (1, 0) and (1, 1), and thus, $\bar{y}_j=0$. ■

4.5.6 Lemma 7

Assume that:

(i) the index order of the jobs gives the SPT order;

(ii) the failure function has the Weibull distribution with β >1;

(iii) both the PM and repair times are constants and are t_m and t_r, respectively and;

(iv) the PM can be done only before starting the processing of a job, and the first job such that no PM decision has been made before the jobs is job *j*.

If $j<n$, $a_{j-1}<\ \dot{\tau}\ <\ \pi_j$ and $\Psi_j\ \leq\ \pi_j$, then $\bar{y}_j=0$.

***Proof*:**

There are at least two decisions for PM again, which is similar to the proof of lemma 6. It will be shown below that (0, 0) is better than (1, 0) and (0, 1) is better than (1, 1). Therefore, $\bar{y}_j=0$ is the optimal solution.

a) for (0, 0) and (1, 0)

$$(q-j+1)P_j+(q-j+1)\left(\frac{P_j}{\eta}\right)^{\beta}t_r+(q-j+1)t_m+(q-j)P_{j+1}+$$

$$(q-j)\left(\frac{P_{j+1}}{\eta}\right)^{\beta}t_r+(q-j)t_m\ \leq (q-j+1)P_j+(q-j+1)\left[\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}-\right.$$

$$\left.\left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]t_r+(q-j)P_{j+1}+(q-j)\left(\frac{P_{j+1}}{\eta}\right)^{\beta}t_r+(q-j)t_m$$

$$(q-j+1)\left(\frac{P_j}{\eta}\right)^{\beta}t_r+(q-j+1)t_m\ \leq (q-j+1)\left[\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}-\left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]t_r$$

According to (42) and because $\pi_j\ \geq \Psi_j$, the previous inequality holds.

b) for (0, 1) and (1, 1)

$$(q-j+1)P_j+(q-j+1)\left(\frac{P_j}{\eta}\right)^{\beta}t_r+(q-j+1)t_m+(q-j)P_{j+1}+$$

$$(q-j)\left[\left(\frac{P_j+P_{j+1}}{\eta}\right)^{\beta}-\left(\frac{P_j}{\eta}\right)^{\beta}\right]t_r\ \leq (q-j+1)P_j+(q-j+1)\left[\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}-\right.$$

$$\left.\left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]t_r+(q-j)P_{j+1}+(q-j)\left[\left(\frac{a_{j-1}+P_j+P_{j+1}}{\eta}\right)^{\beta}-\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}\right]t_r$$

$$(q-j+1)\left(\frac{P_j}{\eta}\right)^{\beta}t_r+(q-j+1)t_m+(q-j)\left[\left(\frac{P_j+P_{j+1}}{\eta}\right)^{\beta}-\left(\frac{P_j}{\eta}\right)^{\beta}\right]t_r\ \leq$$

$$(q-j+1)\left[\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}-\left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]t_r+(q-j)\left[\left(\frac{a_{j-1}+P_j+P_{j+1}}{\eta}\right)^{\beta}-\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}\right]t_r$$

According to (42) and because $\pi_j\ \geq \Psi_j$,

$$(q-j+1)\left(\frac{P_j}{\eta}\right)^{\beta}t_r+(q-j+1)t_m\ \leq (q-j+1)\left[\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}-\left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]t_r$$

It follows from the convexity of the failure function that:

$$(q-j)\left[\left(\frac{P_j+P_{j+1}}{\eta}\right)^{\beta}-\left(\frac{P_j}{\eta}\right)^{\beta}\right]t_r \le (q-j)\left[\left(\frac{a_{j-1}+P_j+P_{j+1}}{\eta}\right)^{\beta}-\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}\right]t_r \blacksquare$$

4.6 PM Decision Procedure

Referring to the model in (31), for any set of jobs $P_1, P_2, \ldots, P_{j-1}, P_j, P_{j+1}, \ldots, P_n$, where $P_{j-1} \le P_j \le P_{j+1}$, the PM decisions $\bar{y}_j$ can be made according to the following steps:

From j = 1 to n

1. Calculate $\pi_j = a_{j-1} + P_j$, $\dot{\tau}$ and Ψ_j; go to 2.
2. If $a_{j-1} < \dot{\tau}$ and $\pi_j \le \dot{\tau}$, then $\bar{y}_j = 1$ and $a_j = a_{j-1} + P_j$; go to 7. Otherwise, go to 3.
3. If $a_{j-1} \ge \dot{\tau}$, then go to 3.1. Otherwise, go to 4.

3.1 If $RL \ge \dot{\tau}$, then $\bar{y}_j = 0$ and $a_j = P_j$; go to 7. Otherwise, go to 3.2.

3.2 If $RL < \dot{\tau}$ and $\pi_j < \Psi_j$, then go to 5. Otherwise, go to 3.3.

3.3 $a_{j-1} \ge \dot{\tau}$ and $\pi_j > \Psi_j$, so $\bar{y}_j = 0$ and $a_j = P_j$; go to 7.

4. If $a_{j-1} < \dot{\tau}$ and $\dot{\tau} < \pi_j < \Psi_j$, then go to 5. Otherwise, go to 6.
5. If $j < n$, then go to 5.1. Otherwise, go to 5.3.

5.1 If and only if at least one of the following two inequalities holds,

(i) $2\left[\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}-\left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]t_r+\left(\frac{P_{j+1}}{\eta}\right)^{\beta}t_r \le \left(\frac{P_j}{\eta}\right)^{\beta}t_r+\left(\frac{P_j+P_{j+1}}{\eta}\right)^{\beta}t_r+t_m$

(ii) $2\left[\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}-\left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right]t_r+\left[\left(\frac{a_{j-1}+P_j+P_{j+1}}{\eta}\right)^{\beta}-\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}\right]t_r \le 2\left(\frac{P_j}{\eta}\right)^{\beta}t_r+2t_m+\left[\left(\frac{P_j+P_{j+1}}{\eta}\right)^{\beta}-\left(\frac{P_j}{\eta}\right)^{\beta}\right]t_r$, then $\bar{y}_j = 1$ and $a_j = a_{j-1} + P_j$; go to 7. Otherwise, go to 5.2.

5.2 $\bar{y}_j = 0$ and $a_j = P_j$; go to 7.

5.3 $j = n$, If $\left[\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right] t_r < \left(\frac{P_j}{\eta}\right)^{\beta} t_r + t_m$, then $\bar{y}_j = 1$; go to 7.

Otherwise, $\bar{y}_j = 0$ and $a_j = P_j$; go to 7.

6. If $a_{j-1} < \dot{\tau}$ and $\dot{\tau} < \pi_j \geq \Psi_j$, then $\bar{y}_j = 0$ and $a_j = P_j$; go to 7.

7. If $j < n$, then $j = j + 1$; go to 1. Otherwise, go to 8.

8. Stop.

Figure 4.1 shows a diagram for the different possible paths of making the PM decisions.

In the next section, a numerical example illustrating the procedure to make preventive maintenance decisions will be discussed, and a numerical analysis to demonstrate how the different parameters in the model affect the PM decisions will be presented.

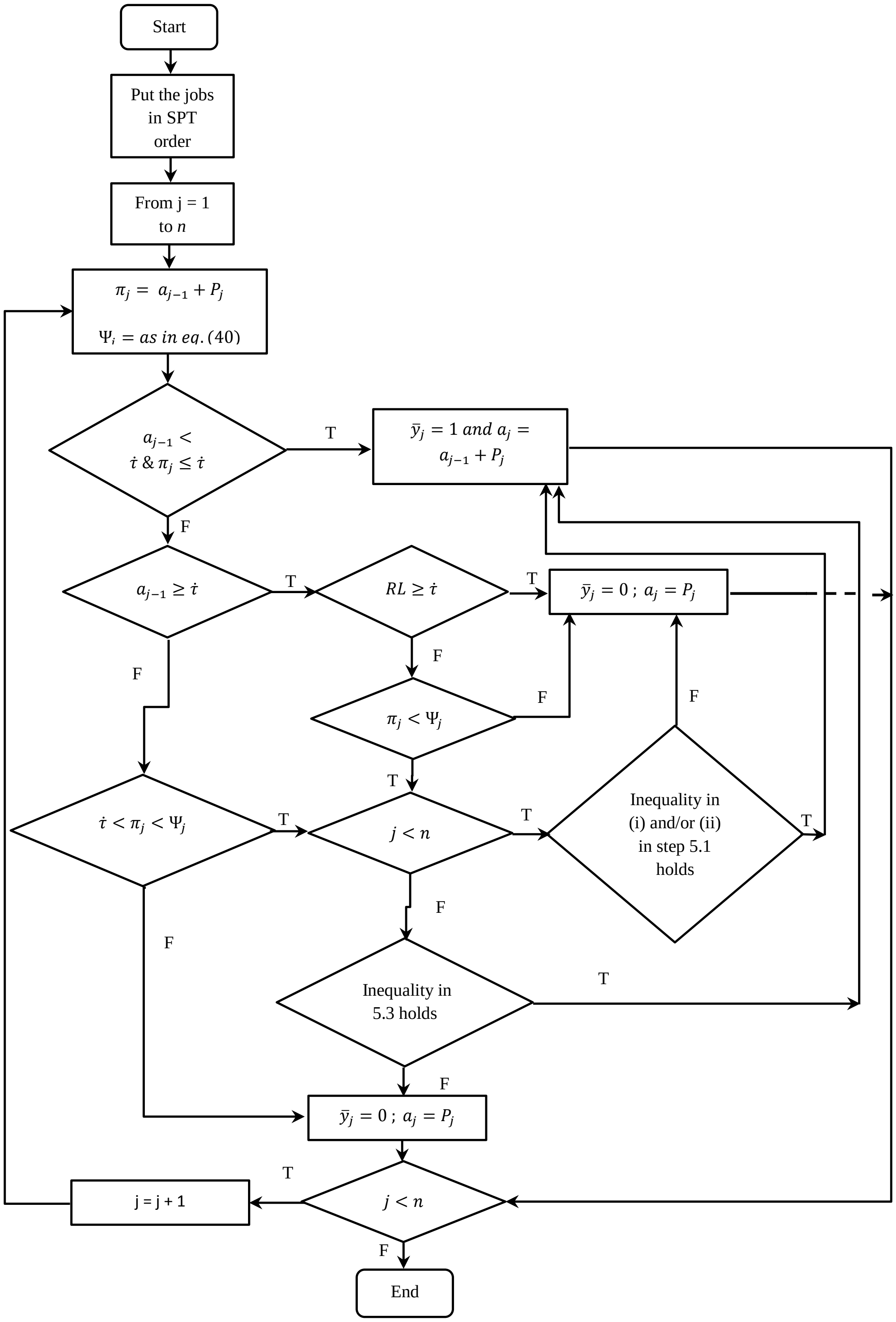

Figure 4.1: The different possible paths of making the PM decisions

4.7 Numerical Example and Analysis

4.7.1 Numerical example (*n* = 4)

i	1	2	3	4
P_i	41	27	25	33
β = 2, η = 70, t_r = 15, t_m = 5, a_o = 33				

Solution:

First, the jobs in SPT order are the following: $P_1 = 25,\ P_2 = 27, P_3 = 33, P_4 = 41.$

By referring to the model in equation (31), the problem in this example can be written as:

$$Min\ ETCT = \sum_{i=1}^{4}(4-i+1)P_i + \frac{4(4+1)}{2}(5) - \sum_{i=1}^{4} 5(4-i+1)\,\bar{y}_i$$
$$+\left(\sum_{k=1}^{4}(4-k+1)\left[\left(\frac{33}{70}\prod_{i=1}^{k}\bar{y}_i + \sum_{i=2}^{k}\frac{P_{i-1}}{70}\prod_{i}^{k}\bar{y}_i + \frac{P_k}{70}\right)^2 - \left(\frac{33}{70}\prod_{i=1}^{k}\bar{y}_i + \sum_{i=2}^{k}\frac{P_{i-1}}{70}\prod_{i}^{k}\bar{y}_i\right)^2\right]\right)15$$

Then, the final form is:

$$Min\ TECT = 4(25) + 3(27) + 2(33) + (41) + \tfrac{4(4+1)}{2}(5) - (4)5\bar{y}_1 - (3)5\bar{y}_2 - (2)5\bar{y}_3 - 5\bar{y}_4 - (5)\bar{y}_5 + \left(4\left[\left(\tfrac{33}{70}\bar{y}_1 + \tfrac{25}{70}\right)^2 - \left(\tfrac{33}{70}\bar{y}_1\right)^2\right] + 3\left[\left(\tfrac{33}{70}\bar{y}_1\bar{y}_2 + \tfrac{25}{70}\bar{y}_2 + \tfrac{27}{70}\right)^2 - \left(\tfrac{33}{70}\bar{y}_1\bar{y}_2 + \frac{25}{70}\bar{y}_2\right)^2\right] + 2\left[\left(\tfrac{33}{70}\bar{y}_1\bar{y}_2\bar{y}_3 + \tfrac{25}{70}\bar{y}_2\bar{y}_3 + \tfrac{27}{70}\bar{y}_3 + \tfrac{33}{70}\right)^2 - \left(\tfrac{33}{70}\bar{y}_1\bar{y}_2\bar{y}_3 + \tfrac{25}{70}\bar{y}_2\bar{y}_3 + \tfrac{27}{70}\bar{y}_3\right)^2\right] + \left[\left(\tfrac{33}{70}\bar{y}_1\bar{y}_2\bar{y}_3\bar{y}_4 + \tfrac{25}{70}\bar{y}_2\bar{y}_3\bar{y}_4 + \tfrac{27}{70}\bar{y}_3\bar{y}_4 + \tfrac{33}{70}\bar{y}_4 + \tfrac{41}{70}\right)^2 - \left(\tfrac{33}{70}\bar{y}_1\bar{y}_2\bar{y}_3\bar{y}_4 + \tfrac{25}{70}\bar{y}_2\bar{y}_3\bar{y}_4 + \tfrac{27}{70}\bar{y}_3\bar{y}_4 + \tfrac{33}{70}\bar{y}_4\right)^2\right]\right)15$$

Second, using the algorithm in section 4.6 (PM solution procedure), the value of variables $\bar{y}_1, \bar{y}_2,\ \bar{y}_3$ and $\bar{y}_4$ will be defined.

Iteration 1:

$$j = 1, a_0 = 33, \qquad P_1 = 25, \qquad \dot{\tau} = 40.4 \text{ (by equation in (15))}$$

$$\pi_1 = a_0 + P_1 = 58$$

By equation in (45);

$$\Psi_1 = 70\left[\frac{5}{15} + \left(\frac{25}{70}\right)^2 + \left(\frac{33}{70}\right)^2\right]^{\frac{1}{2}} = 57.86$$

$$a_0 < \dot{\tau}\,, \dot{\tau} < \pi_1 \text{ and } \pi_1 > \Psi_1$$

then, $\quad \bar{y}_1 = 0$, according to step 6 in section 7.4

Iteration 2:

$j = 2,\ \ a_1 = 25,\ P_2 = 27,\ \ \pi_2 = a_1 + P_2 = 52,\ \Psi_2 = 54.66$

$a_1 < \dot{\tau}\,,\ \dot{\tau} < \pi_2$ and $\pi_2 < \Psi_2$, then the problem is now at step 4 of section 6.4. Additionally, because $j < n$, the decision is at step 5.1.

Whereas condition (i) in step 5.1 in section 6.4 holds (16.06 < 18.25), $\bar{y}_2 = 1$ and $a_2 = a_1 + P_2 = 25 + 27 = 52$.

Iteration 3:

$$j = 3,\ a_2 = 52,\ P_3 = 33,\ \pi_3 = 52 + 33 = 85$$

$$RL = P_3 + P_4 = 33 + 41 = 74$$

$$a_2 > \dot{\tau} \text{ and } RL > \dot{\tau}$$

Then, $\bar{y}_3 = 0$, according to step 3.1 in section 6.4. $a_3 = P_3 = 33$

Iteration 4:

$j = 4,\ a_3 = 33 < \dot{\tau},\ P_4 = 41,\ \pi_4 = 33 + 41 = 74 > \dot{\tau},\ \Psi_4 = 66.36 < \pi_4$

$$\dot{\tau} < \pi_4 \text{ and } \pi_4 > \Psi_4$$

Then, the problem is now at step 6 of section 6.4 and $\bar{y}_4 = 0$.

Therefore, the solution is [0-1-0-0] and the ETCT value is 361.56. The preventive maintenance will be performed before the first, third and fourth jobs; there is no preventive maintenance before the second job. If the PM decisions are made independently of the production, the jobs will be ordered in shortest processing time and then according to the value of $\dot{\tau}$, the PM will be achieved each $\dot{\tau}$ period. If at that time the machine was busy, the PM will be delayed until the machine finishes the current job. In this case, the $\bar{y}_j$ are [1-0-1-0] and the ETCT = 365.3. The difference in the ETCT value depends on the problem parameters and the problem size. In the next section, the differences in the ETCT in both cases (simultaneously and independently) will be noted when the model parameters have been changed.

4.7.2 Computational Analysis

It is not possible to give a closed formula for the effect of the parameters as the underlying nonlinear integer programming problem is NP-complete. However, the model seeks the optimal solution using an optimization process between the total PM time and the total CM time for a set of jobs sorted in SPT order. Therefore, the effect of those parameters on the PM decisions depends on the behavior formed by the given combination of their values, which is reflected by the failure function and the best time to perform the PM action ($\dot{\tau}$). To demonstrate these ideas, a computational analysis was carried out by designing an experiment for four parameters (β, η, t_r and t_m) with two levels of each parameter. There are 2^4 trials conducted on a problem of 16 jobs generated randomly (15, 16, 17, 24, 25, 29, 30, 30, 37, 37, 48, 48, 50, 51, 54 and 60). The values of the parameter levels used in this experiment were used by *Cassady* and *Kutanoglu* (2003) to investigate a similar but not identical problem.

Table 4.1 shows the 16 experimental trial results (16 trials in the case of the integrated decisions using the introduced model "ETCT(Integ.)" and 16 trials in the case of the two decisions made separately "ETCT (Sep.)"). From these results, the following can be summarized:

1) Increasing the PM time by any value while the other factors remain fixed will lead to an increase in the ETCT. The value of $\dot{\tau}$ will increase, and then the number of PM times may decrease or remain fixed because of the non-preemptive case. If the number of PM times decreases, the number of failures will increase, and then the repair time will increase as well. If the number of PM times remains fixed, the ETCT will increase due to the increase in the PM time. This can be noticed clearly by monitoring each of the two consecutive trials in Table 4.1.
2) If the repair time increases, $\dot{\tau}$ will decrease, and this will lead to an increase in the number of PM times required to minimize the failures and will then reduce the increase in the total repair time, as shown in the trials pairs (1, 3), (2, 4),… etc. In some other cases and because the amount of the increase in the repair time does not motivate enough of a change in the number of PM times even if $\dot{\tau}$ decreases, the model changes the PM plan only (changes in the PM positions). Additionally, the PM plans may remain fixed due to the number of PM times achieving the maximum value (n or n-1 if $a_0 = 0$). However, in all of the above-mentioned cases, the ETCT is increased.
3) The increase in the scale parameter value will decrease the expected number of failures, will increase the $\dot{\tau}$ value and, thus, will decrease the required number of PM times. All of this will lead to a decrease in ETCT and vice versa. This can be noted by comparing the trials pairs (1, 9), (2, 10), (3, 11)…, etc.
4) The shape parameter has a different effect that cannot be evaluated in isolation from the rest of the parameters in the failure function ($\eta, t = a_0 + P_j$) and the best time for the PM action $\dot{\tau}$ (t_r, t_m). If the value of t is greater than η, any

increase in the value of β will increase the number of failures, and if t is less than η, any increase in β will decrease the number of failures. Conversely, t has no effect on $\dot{\tau}$, but the increase in β will decrease the $\dot{\tau}$ value up to a specific value and it will then begin to increase. Therefore, here, the length of the job processing time plays an important role in the PM decision. For example, in trial pair (1, 5), the expected failures is decreased because of an increase in β, and $\dot{\tau}$ also decreases, but the number of PM times remains fixed. The trial pair in (9, 13) has a similar case, but η = 100, which is large for the processing time values compared with η = 60 in trial pair (1, 5). For the two pairs in (3, 7) and (11, 15), the expected number of failures decreased, but the value of $\dot{\tau}$ increased, and the number of PM times remained fixed. All of these lead to the conclusion that the effect of the shape parameter is influenced by the other parameters.

Finally, the model responds to the changes in its parameters by changing the value of the ETCT and the proposed PM plan. The number of PM times may increase or decrease or may remain fixed but change positions (change the plan). This change depends on the changed parameter(s), the value of the change and the status of the remaining parameters. The behavior that is formed in the model and controls its decisions is constituted by all of its parameters. In all experiment trials, the value of the ETCT obtained by the integrated model is always better than that one obtained by scheduling of the production and PM plan separately, and the machine with parameter values in trial 13 is the best at scheduling these jobs to achieve a minimum ETCT.

Table 4.1: Experimental results

Trail No.	η	β	t_r	t_m	$\dot{\tau}$	Integrated Solution					ETCT (Sep.)
						No. of PM Times	Total PM time	No. of Failures	Total Expected Failure Time	ETCT (Integ.)	
1	60	2	15	5	34.6	13	65	6.9	103.6	4881.9	4950.7
2	60	2	15	10	49.0	10	100	8.4	126.1	5255.7	5398.3
3	60	2	25	5	26.8	14	70	6.7	167.0	5272.6	5337.0
4	60	2	25	10	37.9	12	120	7.3	182.7	5740.0	5892.2
5	60	3	15	5	33.0	13	65	5.3	79.6	4680.6	4858.4
6	60	3	15	10	41.6	12	120	5.8	87.5	5112.6	5292.1
7	60	3	25	5	27.8	14	70	5.1	126.9	4935.1	5040.2
8	60	3	25	10	35.1	13	130	5.3	132.7	5405.6	5650.4
9	100	2	15	5	57.7	9	45	3.6	53.5	4421.7	4451.2
10	100	2	15	10	81.6	5	50	5.5	82.8	4634.4	4646.5
11	100	2	25	5	44.7	11	55	2.8	70.1	4617.2	4644.9
12	100	2	25	10	63.2	9	90	3.3	83.5	4905.9	4931.7
13	100	3	15	5	55.0	10	50	1.6	24.6	4285.8	4365.1
14	100	3	15	10	69.3	8	80	2.6	39.3	4541.7	4583.3
15	100	3	25	5	46.4	11	55	1.4	35.4	4386.3	4428.9
16	100	3	25	10	58.5	9	90	2.0	50.1	4689.2	4788.1

4.8 Summary

Preventive maintenance activities consume some production time, but delaying these activities due to production demands will increase the probability of machine failures and increase production costs.

In this chapter, a model for integrating preventive maintenance planning and production scheduling to minimize the expected total completion time is introduced. The work in this chapter can be divided into two parts. First, a mathematical model was built depending on some assumptions; this has been achieved by representing the problem using a nonlinear constrained 0-1 model. Second, the model is solved. Using some proven facts, such as the job sequence, is optimal if the jobs are ordered with respect to the SPT rule, the problem has been simplified, and the model becomes a nonlinear unconstrained function. Moreover, depending on the best time to perform the preventive maintenance and the breakeven point between the expected completion time for a job with preventive maintenance and that without it, a method has been proposed to define the PM decisions for the unconstrained model. A numerical example to illustrate the proposed solution procedure was introduced, and a computational analysis was carried out to investigate the effect of changes in the model parameters on its PM decisions. The analysis shows that the model responds to the changes in its parameters in different ways. Additionally, different experiment trials confirm the benefits of integrating the maintenance and production decisions; and the beneficial value depends on the values of the parameters and the size of the problem, where large problems accumulate more benefits.

In the next chapter, minimizing the maximum expected makespan on parallel machines will be discussed and a heuristic method as well as branch and bound method will be introduced to solve the model. To study the makespan problem on multi-machines it is necessary to know about the expected makespan problem on a single machine. Therefore, the problem is addressed and analyzed as well.

Chapter 5

INTEGRATED PREVENTIVE MAINTENANCE PLANNING AND PRODUCTION SCHEDULING FOR PARALLEL MACHINES

5.1 Introductory Remarks

Doing production scheduling and preventive maintenance planning simultaneously is not an easy task especially if the probability of the machine failures is taking into the account. Moreover, the performance measure for the problem it can make the problem more difficult. In the next sections of this chapter, two problems will be investigated. First, the behavior of the performance measure (makespan) for a subset of jobs on a single machine will be examined to know the best way in sequencing those jobs on the assigned machine. Second, the assignment problem for a set of jobs on m machines. Both problems will be solved in order to minimize the given performance measure (makespan) by assigning the appropriate load for each machine. An exact and approximation method will be introduced to integrate the preventive maintenance planning with production schedule for minimizing the mentioned measure.

5.2 Minimizing Makespan on a Single Machine

In deterministic case the makespan is just the sum of the processing times of jobs assigned on the given machine, and it is the same whatever the order of those jobs. If the probability of the machine failures and its repair time as well as the preventive maintenance and its time are taken into consideration, then the expected repair time for the machine will be affected by the length of the non-preemptive jobs and the decision when the preventive maintenances take place. Minimizing the expected value of makespan on a single machine is scheduling the jobs and the preventive maintenance. In the following, the effect of the sequence of the jobs on the makespan is explored.

5.2.1 Lemma 8

Assume that:

i) The number of machine failures is drawn from Weibull distribution with $\beta > 1$ and η.

ii) The preventive maintenance time and expected repair time are deterministic and have values t_m and t_r respectively.

Then, there is an optimal solution of the minimization of the expected makespan on a single machine such that the last two jobs are in SPT order.

Proof:

Let P_j and P_{j+1} two consecutive jobs on a machine and $P_j \leq P_{j+1}$. The SPT order gives a smallest completion time for the second job if and only if

$$P_j + y_j t_m + \left[\left(\frac{(1-y_j)a_{j-1}+P_j}{\eta}\right)^{\beta} - \left(\frac{(1-y_j)a_{j-1}}{\eta}\right)^{\beta}\right] t_r + P_{j+1} + y_{j+1} t_m +$$

$$\left[\left(\frac{(1-y_{j+1})\left((1-y_j)a_{j-1}+P_j\right)+P_{j+1}}{\eta}\right)^{\beta} - \left(\frac{(1-y_{j+1})\left((1-y_j)a_{j-1}+P_j\right)}{\eta}\right)^{\beta}\right] t_r \leq P_{j+1} + y_j t_m +$$

$$\left[\left(\frac{(1-y_j)a_{j-1}+P_{j+1}}{\eta}\right)^{\beta} - \left(\frac{(1-y_j)a_{j-1}}{\eta}\right)^{\beta}\right] t_r + P_j + y_{j+1} t_m +$$

$$\left[\left(\frac{(1-y_{j+1})\left((1-y_j)a_{j-1}+P_{j+1}\right)+P_j}{\eta}\right)^{\beta} - \left(\frac{(1-y_{j+1})\left((1-y_j)a_{j-1}+P_{j+1}\right)}{\eta}\right)^{\beta}\right] t_r$$

Where, the jobs in the left side are in SPT order and in LPT order at the right side.

The possible PM plans for y_j and y_{j+1} are (0 0), (0 1), (1 0) and (1 1):

(i) (0, 0)

By substituting the $y_j = 0$ and $y_{j+1} = 0$ in the left hand side (LHS) of the previous inequality, the follows is holds:

$$P_j + \left[\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right] t_r + P_{j+1} + \left[\left(\frac{a_{j-1}+P_j+P_{j+1}}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta}\right] t_r$$

Also, by substituting the $y_j = 0$ and $y_{j+1} = 0$ in the right hand side (RHS) of the previous inequality, the follows is holds:

$$= P_{j+1} + \left[\left(\frac{a_{j-1}+P_{j+1}}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right] t_r + P_j + \left[\left(\frac{a_{j-1}+P_{j+1}+P_j}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}+P_{j+1}}{\eta}\right)^{\beta}\right] t_r$$

The two sides (LHS and RHS) are equals and they can be reduced as follows:

$$\left[\left(\frac{a_{j-1}+P_j+P_{j+1}}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right] t_r = \left[\left(\frac{a_{j-1}+P_{j+1}+P_j}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right] t_r$$

Even in the case, if the age a_{j-1} is zero they will be equals.

(ii) (0, 1)

If $y_j = 0$ and $y_{j+1} = 1$, by substituting in the LHS, the follows is holds:

$$= P_j + \left[\left(\frac{a_{j-1} + P_j}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right] t_r + P_{j+1} + t_m + \left(\frac{P_{j+1}}{\eta}\right)^{\beta} t_r$$

Also, the RHS is:

$$= P_{j+1} + \left[\left(\frac{a_{j-1}+P_{j+1}}{\eta}\right)^{\beta} - \left(\frac{a_{j-1}}{\eta}\right)^{\beta}\right] t_r + P_j + t_m + \left(\frac{P_j}{\eta}\right)^{\beta} t_r$$

For both sides and based on the facts that $P_{j+1} \geq P_j$ and the function $\left(\frac{x}{\eta}\right)^{\beta}$ is convex if $x > 0$, and $\beta > 1$ then, the LHS is always less or equals the RHS. Thus, the follows inequality is hold:

$$\left(\frac{a_{j-1}+P_j}{\eta}\right)^{\beta} t_r + \left(\frac{P_{j+1}}{\eta}\right)^{\beta} t_r \leq \left(\frac{a_{j-1}+P_{j+1}}{\eta}\right)^{\beta} t_r + \left(\frac{P_j}{\eta}\right)^{\beta} t_r$$

(iii) (1, 0)

By substituting the $y_j = 1$ and $y_{j+1} = 0$ in the LHS of the inequality, the follows is holds:

$$= P_j + t_m + \left(\frac{P_j}{\eta}\right)^{\beta} t_r + P_{j+1} + \left[\left(\frac{P_j+P_{j+1}}{\eta}\right)^{\beta} - \left(\frac{P_j}{\eta}\right)^{\beta}\right] t_r$$

The RHS:

$$P_{j+1} + t_m + \left(\frac{P_{j+1}}{\eta}\right)^{\beta} t_r + P_j + \left[\left(\frac{P_{j+1} + P_j}{\eta}\right)^{\beta} - \left(\frac{P_{j+1}}{\eta}\right)^{\beta}\right] t_r$$

The two sides are equals and they can reduce to:

$$\left(\frac{P_j+P_{j+1}}{\eta}\right)^{\beta} t_r = \left(\frac{P_{j+1}+P_j}{\eta}\right)^{\beta} t_r$$

Here it is clear that, when the effect of the machine age is terminated by PM then there is no effect of the job sequence.

(iv) (1, 1)

If $y_j = 1$ and $y_{j+1} = 1$, by substituting in the left hand side (LHS), the follows is holds:

$$= P_j + t_m + \left(\frac{P_j}{\eta}\right)^{\beta} t_r + P_{j+1} + t_m + \left(\frac{P_{j+1}}{\eta}\right)^{\beta} t_r$$

The RHS is:

$$= P_{j+1} + t_m + \left(\frac{P_{j+1}}{\eta}\right)^{\beta} t_r + P_j + t_m + \left(\frac{P_j}{\eta}\right)^{\beta} t_r$$

The two sides are equals and the job sequence in this case has no effect on the jobs makespan.■

The conclusion here is that, whatever the criteria which was used to generate the optimal job sequence, the last two jobs must be in SPT order to grantee it is the best.

Note: making the last two jobs in the sequence is in SPT order does not mean the sequence is optimal, but if they are not, the sequence may not optimal even the previous jobs is in optimal order.

5.2.2 Lemma 9

Assume that in a single machine problem:

i) The number of the machine failures have Weibull distribution with parameters $\beta > 1$ and η.

ii) The preventive maintenance and expected repair time are deterministic and have values t_m and t_r respectively.

iii) The age of the machine is zero at the beginning of the horizon time ($a_0 = 0$).

Then, for every preventive maintenance plan and any scheduling of the jobs there is a preventive maintenance plan with the reversed (opposite) scheduling of the jobs such that the expected values of the two makespans are equals.

Proof:

Let $\boldsymbol{S}$ be a set of $\boldsymbol{n}$ jobs ordered in arbitrary sequence and the assigned preventive maintenance plan for it (whatever if it optimal or not) dividing those jobs to $\boldsymbol{q}$ subset ($\boldsymbol{S}_1$, $\boldsymbol{S}_2$,…, $\boldsymbol{Sq}$) and each set has T_h length (h = 1, 2, …, q) as shown in Figure 5.1, where $i_1, i_2, \ldots, i_N$ is the number of jobs in subset 1, 2,…., q respectively and $i_1 + i_2 + \cdots + i_N = n$.

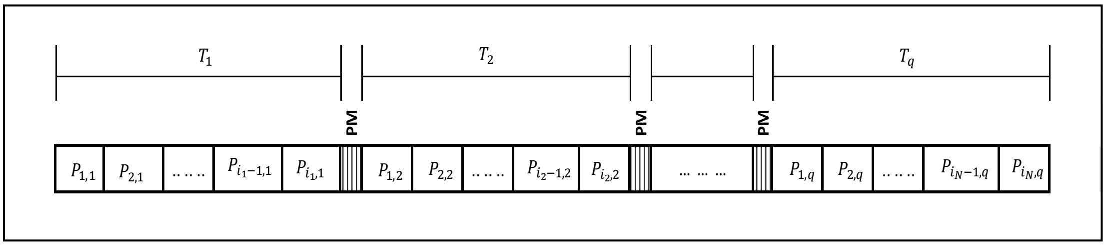

Figure 5.1: An arbitrary sequence for a set of jobs with assigned PM plan

Such that,

$$S_1 \cup S_2 \cup \ldots \cup S_q = S$$

$$S_r \cap S_g = \emptyset, \ \ \forall \ r = 1, 2, .. q \ , \ g = 1, 2, .. q \text{ and } r \neq g$$

Assume that PM takes place between every two subsets. Furthermore, assume the subsets are arranged in opposite order and the jobs in each subset are arranged in opposite order as well. Also, let the preventive maintenance plan arising with the new sequence respecting the previous spans length T_1, T_2,…, T_q. Thus, the jobs are ordered in the opposite sequence as shown in Figure 5.2.

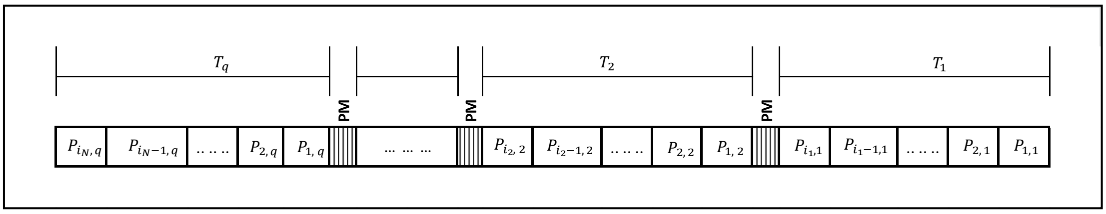

Figure 5.2: The opposite sequence with the accompaniment PM plan

The expected makespan for the first sequence is:

$$EMS_1 = \left(P_{1,1} + P_{2,1} + \cdots + P_{i_1,1}\right) + \left(\frac{P_{1,1}+P_{2,1}+\cdots+P_{i_1,1}}{\eta}\right)^{\beta} + t_m + \left(P_{1,2} + P_{2,2} + \cdots + P_{i_2,2}\right) + \left(\frac{P_{1,2}+P_{2,2}+\cdots+P_{i_2,2}}{\eta}\right)^{\beta} + t_m + \cdots + t_m + \left(P_{1,q} + P_{2,q} + \cdots + P_{i_N,q}\right) + \left(\frac{P_{1,q}+P_{2,q}+\cdots+P_{i_N,q}}{\eta}\right)^{\beta}$$

Thus,

$$EMS_1 = T_1 + \left(\frac{T_1}{\eta}\right)^{\beta} + t_m + T_2 + \left(\frac{T_2}{\eta}\right)^{\beta} + t_m + \cdots + t_m + T_q + \left(\frac{T_q}{\eta}\right)^{\beta}$$

The expected makespan for the second sequence (opposite to the first) is:

$$EMS_{2(opp.Seq.)} = \left(P_{i_N,q} + P_{i_N-1,q} + \cdots + P_{1,q}\right) + \left(\frac{P_{i_N,q}+P_{i_N-1,q}+\cdots+P_{1,q}}{\eta}\right)^{\beta} + t_m + \cdots + \left(P_{i_2,2} + P_{i_2-1,2} + \cdots + P_{1,2}\right) + \left(\frac{P_{i_2,2}+P_{i_2-1,2}+\cdots+P_{1,2}}{\eta}\right)^{\beta} + t_m + \left(P_{i_1,1} + P_{i_1-1,1} + \cdots + P_{1,1}\right) + \left(\frac{P_{i_1,1}+P_{i_1-1,1}+\cdots+P_{1,1}}{\eta}\right)^{\beta}$$

$$EMS_{2(opp.Seq.)} = T_q + \left(\frac{T_q}{\eta}\right)^{\beta} + t_m + \cdots + T_2 + \left(\frac{T_2}{\eta}\right)^{\beta} + t_m + T_1 + \left(\frac{T_1}{\eta}\right)^{\beta}$$

Thus,

$$EMS_1 = EMS_{2(opp.Seq.)}$$

The expected makespans are equals but the two sequences are in the opposite order and the accompaniment preventive plans will be different unless the jobs are equal ■

Table 5.1 summarizing that plans for three jobs (plans start with 1 was discarded because $a_0 = 0$. Additionally, it can be conclude that if the age of the machine at the beginning of the time horizon is zero, then there is no difference between sorting the jobs in SPT or LPT. But the best sequence may be a different one.

Table 5.1: Equivalent PM plans

Jobs in SPT order	Jobs in LPT order
(0 0 0)	(0 0 0)
(0 1 0)	(0 0 1)
(0 0 1)	(0 1 0)
(0 1 1)	(0 1 1)

5.2.3 Lemma 10

Assume that T_0, T_1 and T_2 are three consecutive times when the age of the machine is zero. Let t_1 be the total processing time of jobs between T_0 and T_1, and t_2 be the total processing time of jobs between T_1 and T_2. Assume that the same jobs are arranged into two other parts, such that the total processing times of the two parts are $\bar{t}_1$ and $\bar{t}_2$ (See Figure – 5.3). Assume that:

$$|t_1 - t_2| > |\bar{t}_1 - \bar{t}_2|$$

Then the schedule obtained by substituting the jobs of the $[T_0,\ T_1]$ interval by the jobs of the part having total processing time $\bar{t}_1$ and the jobs of the $[T_1,\ T_2]$ interval by the jobs of the part having total processing time $\bar{t}_2$, and everything else is unchanged, has shorter makespan.

Proof:

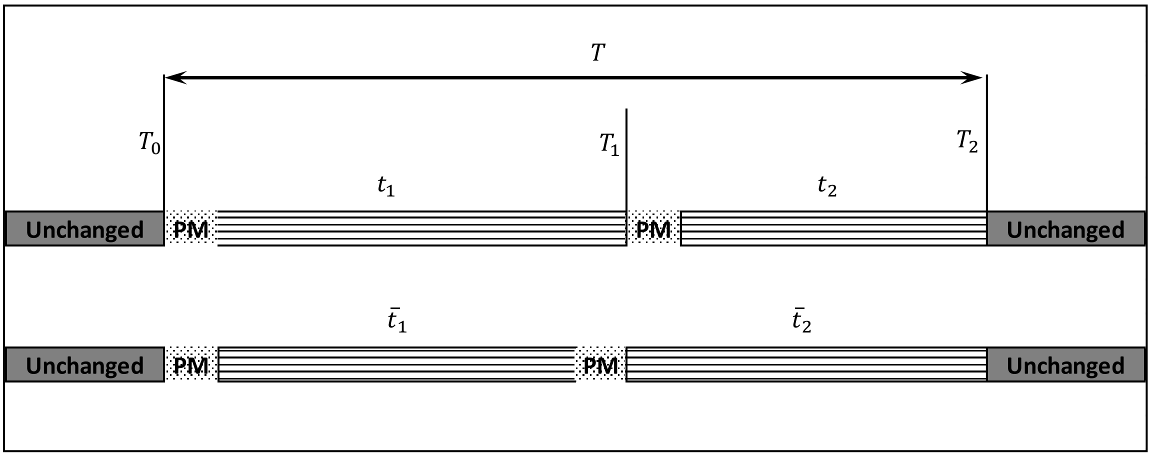

Figure 5.3: Descriptive of case in lemma 10

The expected value of the lengths of the two solutions between T_0 and T_2 are as follows:

$$t_m + t_1 + \left(\frac{t_1}{\eta}\right)^{\beta} t_r + t_m + t_2 + \left(\frac{t_2}{\eta}\right)^{\beta} t_r$$

and;

$$t_m + \bar{t}_1 + \left(\frac{\bar{t}_1}{\eta}\right)^{\beta} t_r + t_m + \bar{t}_2 + \left(\frac{\bar{t}_2}{\eta}\right)^{\beta} t_r$$

where,

$$(t_1 + t_2) = (\bar{t}_1 + \bar{t}_2)$$

The comparison of the two quantities is equivalent to:

$$\left(\frac{t_1}{\eta}\right)^{\beta} t_r + \left(\frac{t_2}{\eta}\right)^{\beta} t_r \sim \left(\frac{\bar{t}_1}{\eta}\right)^{\beta} t_r + \left(\frac{\bar{t}_2}{\eta}\right)^{\beta} t_r$$

It can be reduced farther to:

$$t_1^{\beta} + t_1^{\beta} \sim \bar{t}_1^{\beta} + \bar{t}_1^{\beta}$$

Let,

$$f(t) = t^{\beta} + (T - t)^{\beta}$$

where, t is the length of the first group of jobs and (T – t) representing the length of second group of jobs.

$$f'(t) = \beta t^{\beta-1} - \beta(T-t)^{\beta-1} = \begin{cases} > 0 \; if \; t > T/2 \\ = 0 \; if \; t = T/2 \\ < 0 \; if \; t < T/2 \end{cases}$$

Thus,

$$f(t) \; is \begin{cases} increasing \; if \; t > T/2 \\ minimum \; if \; t = T/2 \\ increasing \; if \; t < \; T/2 \end{cases}$$

So, when the difference between these two groups of jobs is minimal then, the expected makespan of these jobs will be shorter. ■

Additionally, from the previous lemma it can be concluding that if at any stage of the time horizon the age (a_{j-1}) is greater than zero then the job(s) with the complementary value to $\dot{\tau}$ or at least the job(s) with less penalty is favored for the expected makespan. Table 5.2 shows a numerical illustration for the previous conclusions of three lemmas. In all cases (A and B) the $\eta = 60.63, \; t_m = 5$ and $t_r = 15, \beta = 2, \dot{\tau} = 35$. And $a_0 = 10$ for case A and $a_0 = 0$ for case B. Each sequence of the six possible sequences presented with its best PM plan of eight possible PM scenarios.

Even it is true that the difference between the optimal expected makespan and the value gained by the two previous rules (LPT and SPT) is slight, but it is valuable to know about it. This fact will be used later for multi-identical machines.

Table 5.2: Numerical Illustration

Case	No.	Job Seq.			PM Plan			EMS	Remarks
A	1	4	25	38	0	0	1	83.691	1. The optimal value is in sequence 1& 3 and both of them with the age a_0 giving the best dividing of the T value (10+4+25~38). 2. In 1&3 the last two jobs in SPT and if they are switched will create 2&4 which is not optimal.
	2	4	38	25	0	0	1	85.176	
	3	25	4	38	0	0	1	83.691	
	4	25	38	4	0	1	0	83.789	
	5	38	4	25	0	1	0	84.425	
	6	38	25	4	0	1	0	84.425	
B	1	4	25	38	0	0	1	81.324	When $a_0 = 0$, the expected makespan for SPT and LPT is equals.
	2	38	25	4	0	1	0	81.324	

It is known that the difference in the expected makespan between any two sequences is due to the difference in the repair time and preventive maintenance time, and this difference arising due to the penalty of doing the preventive maintenance early (early decision for PM lead to loss $\left[(\dot{\tau}/\eta)^{\beta} - \left(P_j/\eta\right)^{\beta}\right] t_r$, where $P_j < \dot{\tau}$) or late (late decision for PM lead to add $\left[\left(\left(P_j + P_k\right)/\eta\right)^{\beta} - (\dot{\tau}/\eta)^{\beta}\right] t_r$, where $\left(P_j + P_k > \dot{\tau}\right)$) of its time ($\dot{\tau}$). This variation in the time of $\dot{\tau}$ to perform the preventive maintenance is because of the job processing cannot be interrupted for the preventive maintenance. So, any sequence of jobs can minimizing this penalty it will be the optimal or very near and then the difference will be very small.

5.2.4 Minimizing the Expected Makespan by Minimizing the Penalty of Doing Preventive Maintenance Early or Later: A Heuristic Method

Assume there are n jobs to be scheduled on a machine with initial age a_0. Let LS and S be two lists.

LS: is a list containing the given jobs and the initial age of the machine listed in non-increasing order from $j = 1$ to $j = n + 1$, where $n + 1$ is the number of jobs plus one item representing the initial age of the machine.

S: is a list of subsets $(s_1, s_2, \dots, s_k)$ and each subset represent a segment between two preventive maintenance decisions. Each subset containing item(s) representing job(s) and one of them including the machine age at time zero when $a_0 > 0$.

Assume that the subsets $s_1, \dots, s_{k-1}$ have been already defined and subset s_k is under construction.

The set s_k is created by optimization problems as follows:

$$G = max \sum_{j \in LS \setminus \cup_1^k s_l} P_j x_j + a_{s_k}$$

Sub.to:

$$\sum_{j \in LS \setminus \cup_1^k s_l} P_j x_j \leq \dot{\tau} - a_{s_k}$$

$$\forall j: x_j = 0 \text{ } or \text{ } 1$$

and

$$H = min \sum_{j \in LS \setminus \cup_1^k s_l} P_j x_j + a_{s_k}$$

Sub.to:

$$\sum_{j \in LS \setminus \cup_1^k s_l} P_j x_j \geq \dot{\tau} - a_{s_k}$$

$$\forall j: x_j = 0 \text{ } or \text{ } 1$$

Where, a_{s_k} is the length of the segment k after each added item to it. It is zero at the beginning.

Let RL is the remaining load at the end of each iteration of the scheduling process. If the RL is less than $\dot{\tau}$ then, an investigation should be made to see if it is better to add this load to the shortest created segment (s_{st}) or doing the preventive maintenance and creating a new segment for minimizing the performance measure. Therefore, if EMS is the value of the expected makespan for the created segment up to the last one (without RL and RL < $\dot{\tau}$) then, a new segment will be created for the remaining load if the following inequality holds.

$$EMS + t_m + \left(\frac{RL}{\eta}\right)^{\beta} t_r < EMS - \left(\frac{s_{st}}{\eta}\right)^{\beta} t_r + \left(\frac{s_{st} + RL}{\eta}\right)^{\beta} t_r$$

$$\left(\frac{t_m}{t_r}\right) + \left(\frac{RL}{\eta}\right)^{\beta} < \left(\frac{s_{st} + RL}{\eta}\right)^{\beta} - \left(\frac{s_{st}}{\eta}\right)^{\beta}$$

Where, s_{st} is the length of the shortest segment produced.

The following is a heuristic to minimizing the expected makespan for a set of jobs on a machine.

Note: P_j representing an item in LS whatever it job processing time or machine age.

Heuristic (H_1):

1. Create the LS (with items $n + 1$ if $a_0 > 0$ and n if $a_0 = 0$) in non-increasing order. Let $k = 1$ and $a_{s_1} = 0$.
2. For $j = 1$ to the last remaining item in LS.
3. If $P_j \geq \dot{\tau}$ then, put P_j in s_k as one segment in S and go to 7. Otherwise, go to 4.
4. $P_j < \dot{\tau}$, put P_j in s_k, $a_{s_k} = a_{s_k} + P_j$, resort LS and go to 5.

5. Find G and H. Go to 6.

6. If $\left[(\dot{\tau}/\eta)^{\beta} - (G/\eta)^{\beta}\right]t_r < \left[(H/\eta)^{\beta} - (\dot{\tau}/\eta)^{\beta}\right]t_r$, then put the item(s) in G in s_k with P_j, close segment k as one segment in S and go to 7. Otherwise, put item(s) in H in s_k with P_j, close segment k as one segment in S and go to 7.

7. $k = k + 1$, $a_{s_k} = 0$, resort LS and go to 8.

8. If $RL > \dot{\tau}$ then, go to 2. Otherwise, go to 9.

9. If $\left(\frac{t_m}{t_r}\right) + \left(\frac{RL}{\eta}\right)^{\beta} > \left(\frac{s_{st}+RL}{\eta}\right)^{\beta} - \left(\frac{s_{st}}{\eta}\right)^{\beta}$ then, put the remaining item(s) in s_{st} and go to 10. Otherwise, put the remaining item(s) in s_k as one segment in S and go to 10.

10. Stop.

With this sequencing process, a PM plan is constructed where after each decision taken depend on the steps in 3 and 6 a PM is performed after the created segment with except is of the last one. A numerical example introduced in section 5.4.1 to illustrate the heuristic solution procedure.

5.2.5 A Preemptive Method to Minimizing the Makespan for Non-preemptive Jobs: A Heuristic Method

When the problem is preemptive, the job processing can be interrupted at any time for the repair and then it can be resumed without any additional penalty. As it is shown earlier, the best time to perform the preventive maintenance for a machine is determined by the non-homogenous Poisson process with Weibull failure rate function and is $\dot{\tau}$. As the problem now is assumed to be preemptive, then it can be performed on the time. The total processing time may not be an integer multiplier of the $\dot{\tau}$, so two questions should be answered. First, how many is the optimal number of the preventive maintenances. Second, given the optimal number of preventive maintenances when should they be performed?

Let;

- the age of the machine be a_0 at the beginning of the time horizon.
- P: be the total processing time $\left(\sum_{j=1}^{n} p_j\right)$.

Assume that $\dot{\tau} << \mathrm{P} + a_0$. If $\mathrm{P} + a_0$ is the multiple of $\dot{\tau}$ then the total number of preventive maintenance times is $k = ((P + a_0)/\dot{\tau}) - 1$, provided that k is integer. However, if it is not then, the number of preventive maintenance times is $k_1$or k_2 where:

$$k_1 = \left\lfloor \frac{P+a_0}{\dot{\tau}} \right\rfloor - 1,\ k_2 = \left\lceil \frac{P+a_0}{\dot{\tau}} \right\rceil - 1$$

The expected makespan as a function in k is a convex function and at k_1 or k_2 will have the minimum expected value. Figure – 5.4 shows the relation between the expected makespan (EMS) and the number of preventive maintenance times (k) for the example in section 5.4.2.

Given the number of preventive maintenance times is k_1 or k_2, say k:

For k = 0, 1, 2, ….

If k = 0, then

The expected makespan (EMS) = $P + \left[\left(\frac{P+a_0}{\eta}\right)^{\beta} - \left(\frac{a_0}{\eta}\right)\right] t_r$

Furthermore, it is easy to see that if the number of preventive maintenances is k then the optimal length of the $k + 1$ intervals are equal to $(P + a_0)/(k + 1)$.

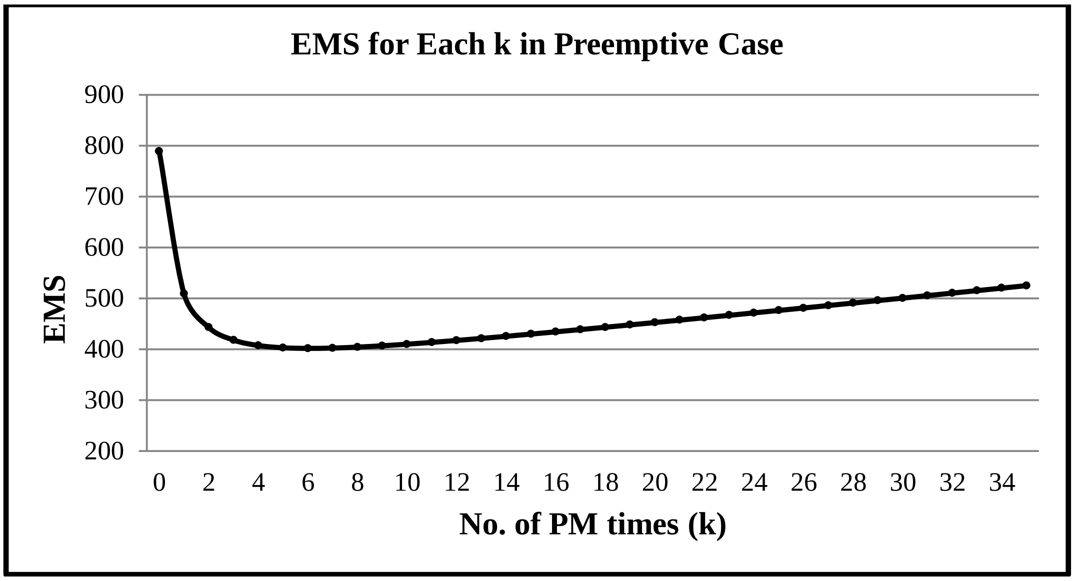

Figure 5.4: EMS as a function in k

Then, the optimal value of the expected makespan for a set of jobs on a single machine when the case is preemptive is:

$$EMS_k = min\{EMS_{k_1}, EMS_{k_2}\}$$

where,

$$EMS_{k_i} = P + k_i t_m + k_i \left(\frac{P + a_0}{(k_i + 1)\eta}\right)^{\beta} t_r + \left[\left(\frac{P + a_0}{(k_i + 1)\eta}\right)^{\beta} + \left(\frac{a_0}{\eta}\right)^{\beta}\right] t_r$$

and $i = 1, 2$

In the next section, the problem will be considered as non-preemptive problem gradually by making subsets of the jobs which should be processed without interruptions. The subset s_r containing one or more jobs and the total processing time of that job(s) called a segment.

A dummy job is added to the given n jobs and its value equals the machine age at the beginning of the time horizon a_0 $(a_0 < \dot{\tau})$. Moreover, a list for $n + 1$ items representing the n given jobs plus the dummy job listed in non-increasing order will be created.

Note: a_0 must be less than $\dot{\tau}$. Otherwise, the machine must be maintained and this action will not be included in the time horizon for scheduling the n jobs.

Let LS and S be the lists already introduced in section 5.2.4. Furthermore, let

s_r: subset of jobs such that they are processed continuously without interruptions, and;

A_s: the expected makespan of the subset(s) in S where the jobs in each subset will be processed continuously.

Before going to introduce the heuristic, the calculation of the expected makespan for the ***first iteration*** will be illustrated to make a clear vision about the calculation method. In next iterations the calculations should consider the status of the dummy job (a_0) if it is already assigned or not yet.

Assume p_1 is the first job in LS and it is not a preemptive job whilst the remaining real load (*RL*) are preemptive. Therefore, the expected makespan is:

If $p_1 \geq \dot{\tau}$ (p_1 cannot be the dummy job (a_0) because it is greater than $\dot{\tau}$) then,

$$Remaining\ Load\ (RL) = \sum_{j=1}^{n+1} p_j - p_1 - a_0 = \sum_{j=2}^{n+1} p_j - a_0$$

$$k_1 = \left\lfloor \frac{RL + a_0}{\dot{\tau}} \right\rfloor - 1,\ k_2 = \left\lceil \frac{RL + a_0}{\dot{\tau}} \right\rceil - 1 \text{ and assume that } k_2 = k_1 + 1$$

$$EMS_{k_i} = A_s + RL + k_i t_m + k_i \left(\frac{\sum_{j=2}^{n+1} p_j}{(k_i + 1)\eta} \right)^{\beta} t_r + \left[\left(\frac{\sum_{j=2}^{n+1} p_j}{(k_i + 1)\eta} \right)^{\beta} - \left(\frac{a_0}{\eta} \right)^{\beta} \right] t_r$$

where $i = 1, 2$ and,

$$A_s = p_1 + \left(\frac{p_1}{\eta}\right) t_r + t_m$$

$$EMS_k = min\{EMS_{k_1}, EMS_{k_2}\}$$

If $p_1 < \dot{\tau}$ then,

$$G = max \sum_{j \in LS} p_j \, x_j$$

Sub.to:

$$\sum_{j \in LS} p_j \, x_j \leq \dot{\tau} - p_1, \qquad j \neq 1$$

and,

$$H = min \sum_{j \in LS} p_j \, x_j$$

Sub.to:

$$\sum_{j \in LS} p_j \, x_j \geq \dot{\tau} - p_1, \qquad j \neq 1$$

Note: p_1 is not an element of the sets G and H by definition.

The expected makespan calculations in the two previous cases depend on the state of the dummy job as the follows:

For G:

- If $p_1 = a_0$ (p_1 is the dummy job)

$$Remaining\ Load\ (RL) = \sum_{j=1}^{n+1} p_j - \ p_1 - G = \sum_{j=2}^{n+1} p_j - G$$

$$k_1 = \left\lfloor \frac{RL}{\dot{\tau}} \right\rfloor - 1, \ \ k_2 = \left\lceil \frac{RL}{\dot{\tau}} \right\rceil - 1 \text{ and } \ k_2 = k_1 + 1$$

$$EMS_{k_i}^{G} = A_s + RL + k_i t_m + (k_i + 1)\left(\frac{RL}{(k_i + 1)\eta}\right)^{\beta} t_r$$

where $i = 1, 2$ and,

$$A_s = G + \left[\left(\frac{p_1 + G}{\eta}\right)^{\beta} - \left(\frac{p_1}{\eta}\right)^{\beta}\right] t_r + t_m$$

where, $p_1 = a_0$.

$$EMS_k^G = min\{EMS_{k_1}^G, EMS_{k_2}^G\}$$

- If p_1 is a real job and the dummy job (a_0) is in G

$$Remaining\ Load\ (RL) = \sum_{j=1}^{n+1} p_j - p_1 - G = \sum_{j=2}^{n+1} p_j - G$$

$$k_1 = \left\lfloor \frac{RL}{\dot{\tau}} \right\rfloor - 1,\ \ k_2 = \left\lceil \frac{RL}{\dot{\tau}} \right\rceil - 1 \text{ and } \ k_2 = k_1 + 1$$

$$EMS_{k_i}^{G} = A_s + RL + k_i t_m + (k_i + 1)\left(\frac{RL}{(k_i + 1)\eta}\right)^{\beta} t_r$$

where $i = 1, 2$ and,

$$A_s = p_j + G - a_0 + \left[\left(\frac{p_1 + G}{\eta}\right)^{\beta} - \left(\frac{a_0}{\eta}\right)^{\beta}\right] t_r + t_m$$

Remember a_0 is one of the item(s) in G.

$$EMS_k^G = min\{EMS_{k_1}^G, EMS_{k_2}^G\}$$

- If a_0 is not p_1 nor in G

$$Remaining\ Load\ (RL) = \sum_{j=1}^{n+1} p_j - p_1 - G - a_0 = \sum_{j=2}^{n+1} p_j - G - a_0$$

$$k_1 = \left\lfloor \frac{RL + a_0}{\dot{\tau}} \right\rfloor - 1,\ \ k_2 = \left\lceil \frac{RL + a_0}{\dot{\tau}} \right\rceil - 1 \text{ and } \ k_2 = k_1 + 1$$

$$EMS_{k_i}^{G} = A_s + RL + k_i t_m + k_i\left(\frac{RL + a_0}{(k_i + 1)\eta}\right)^{\beta} t_r + \left[\left(\frac{RL + a_0}{(k_i + 1)\eta}\right)^{\beta} - \left(\frac{a_0}{\eta}\right)^{\beta}\right] t_r$$

where $i = 1, 2$ and,

$$A_s = p_j + G + \left(\frac{p_1 + G}{\eta}\right)^{\beta} t_r + t_m$$

Remember a_0 is not p_1 nor in G.

$$EMS_k^G = min\{EMS_{k_1}^G, EMS_{k_2}^G\}$$

For H:

In the same manner and by replacing G by H the expected makespan for statement H can be calculated.

Finally, the decision for the jobs that will be in the subset s_r will be made according to:

$$EMS_k = min\{EMS_{k_1}^G, EMS_{k_2}^G, EMS_{k_1}^H, EMS_{k_2}^H\}$$

If the selected one was including the dummy job (a_0) then the corresponding subset is the first (s_1) in S. Otherwise, it can take another position in the sequence. The preventive maintenance decision will be made according to the next heuristic. An illustrative example in section 5.4.2 to illustrate the solution procedure in H_2 will be introduced.

Heuristic (H_2)

1. Create LS, $A_s = 0$, S = {∅}.
2. Let $j = 1$ and go to 3.
3. If $p_j \geq \dot{\tau}$ then, put job p_j as one subset in S, $A_s = A_s + p_j + \left(\frac{p_j}{\eta}\right) t_r + t_m$, resort LS, do PM if it is followed by another subset and go to 5. Otherwise go to 4.
4. $p_j < \dot{\tau}$ then, find G and H and go to 4.1.

4.1 If p_j is a_0 then go to 4.1.1. Otherwise, go to 4.2.

4.1.1 If $\min_i\{EMS_{k_i}^G\} < \min_i\{EMS_{k_i}^H\}$ then, put job(s) in G at the beginning of S as one subset, $A_s = A_s + G + \left[\left(\frac{p_j+G}{\eta}\right)^\beta - \left(\frac{p_j}{\eta}\right)^\beta\right] t_r + t_m$, do PM if it is followed by another set, resort LS and go to 5; otherwise, go to 4.1.2.

4.1.2 Put job(s) in H at the beginning of S as one subset, $A_s = A_s + H + \left[\left(\frac{p_j+H}{\eta}\right)^\beta - \left(\frac{p_j}{\eta}\right)^\beta\right] t_r + t_m$, do PM if it followed by another set, resort LS and go to 5.

4.2 If one of the jobs in G is a_0 then go to 4.2.1. Otherwise, go to 4.3.

4.2.1 If $\min_i\{EMS_{k_i}^G\} < \min_i\{EMS_{k_i}^H\}$ then, put p_j and job(s) in G without a_0 at the beginning of S as one subset, $A_s = A_s + p_j + (G - a_0) + \left[\left(\frac{p_j+G}{\eta}\right)^\beta - \left(\frac{a_0}{\eta}\right)^\beta\right] t_r + t_m$, do PM if it is followed by another set, resort LS and go to 5. Otherwise, go to 4.2.2.

4.2.2 If one of the jobs in H is a_0 then, go to 4.3.2. Otherwise, put p_j and job(s) in H in S as one subset, $A_s = A_s + p_j + H + \left(\frac{p_j+H}{\eta}\right)^\beta t_r + t_m$, do PM if it is followed by another set, resort LS and go to 5.

4.3 If one of the jobs in H is a_0 then go to 4.3.1. Otherwise, go to 4.4.

4.3.1 If $\min_i\{EMS_{k_i}^G\} < \min_i\{EMS_{k_i}^H\}$ then, put p_j and job(s) in G in S as one subset, $A_s = A_s + p_j + G + \left(\frac{P_j+G}{\eta}\right)^\beta t_r + t_m$, do PM if it is followed by another set, resort LS and go to 5. Otherwise, go to 4.3.2.

4.3.2 Put p_j and job(s) in H without a_0 at the beginning of S as one subset, $A_s = A_s + p_j + (H - a_0) + \left[\left(\frac{P_j+H}{\eta}\right)^\beta - \left(\frac{a_0}{\eta}\right)^\beta\right] t_r + t_m$, do PM if it is followed by another set, resort LS and go to 5.

4.4 If $\min_i\{EMS_{k_i}^G\} < \min_i\{EMS_{k_i}^H\}$ then, put p_j and job(s) in G in S as one subset, $A_s = A_s + p_j + G + \left(\frac{P_j+G}{\eta}\right)^\beta t_r + t_m$, do PM if it is followed by another set, resort LS and go to 5. Otherwise go to 4.5.

4.5 Put p_j and job(s) in H in S as one subset, $A_s = A_s + p_j + H + \left(\frac{P_j+H}{\eta}\right)^\beta t_r + t_m$, do PM if it is followed by another set, resort LS and go to 5.

5. If LS is empty then, go to 6. Otherwise, go to 2.

6. Stop.

Figure – 5.5 introduces a flow chart illustrating the heuristic (H_2) method.

It is clear that the problem of minimizing the makespan on a single machine is a problem of distributing the jobs on $(k + 1)$ periods in optimal way. Optimal way means the load is balanced in the possible best way.

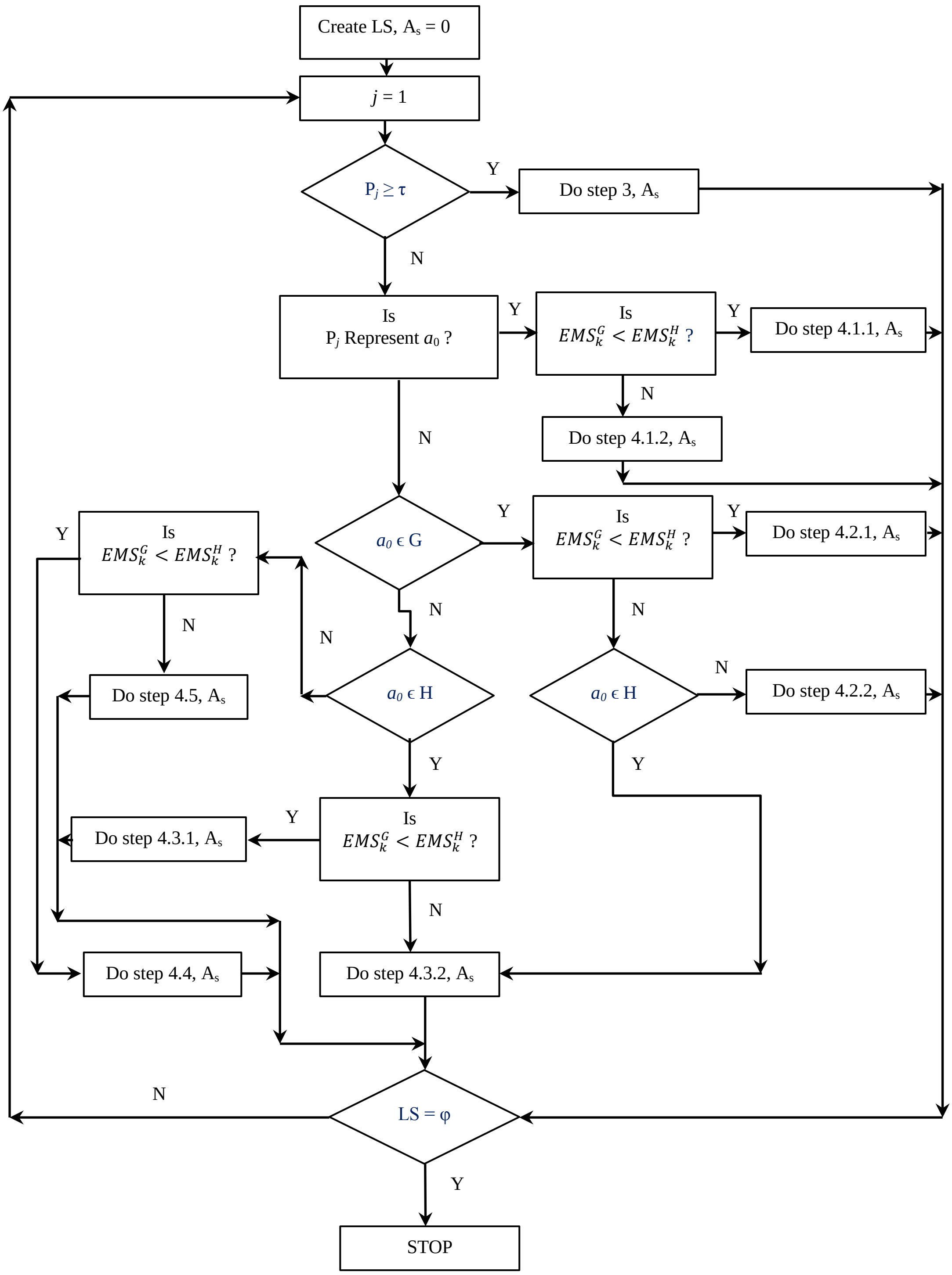

Figure 5.5: H_2 flow chart

5.3 Parallel Machine

In what follows, the simultaneous schedule of production and preventive maintenance is discussed in the production cell. It is assumed that each machine can perform every job and the processing time of a job does not depend on the machine. However, the machines have different reliability measures.

Minimizing the expected makespan for a set of jobs on parallel machines containing two tasks, the jobs should be assigned on the available machines that can achieve the same duty but can be in different times span due to the difference in their reliability measures. Also, the subset of the jobs assigned on each machine should be ordered in a way that keeping the expected makespan minimal. In the following sections, the problem of minimizing the expected makespan on the multi-machines working in parallel that may have same or different maintenance parameters will be introduced, and then an exact and approximation methods will be proposed to solve it.

5.3.1 Modeling of the Problem

The general model of the problem uses the notations as the follows:

x_{ijk} : the job *j* on the machine *i* at the *k* position (x_{ijk} = 0 or 1).

P_j : Processing time of job *j*.

y_{ik} : PM decision before the job in position *k* on machine *i*.

a_{ik} : age of machine *i* before starting the job in k^{th} position.

t_{m_i}: Preventive maintenance time for machine *i*.

t_{r_i} : Repair time for machine *i*.

η_i : Scale parameter for machine *i*.

β_i : Shape parameter for machine *i*.

$$Min\ C_{max} = \min \max_i\{C_i\}$$

$$Min\ \{C_{max} - C_{min}\} = min(\max_i\{C_i\} - \min_i\{C_i\})$$

$$\sum_{k=1}^{n}\sum_{j=1}^{n}\left[P_j x_{ijk} + t_{m_i} y_{ik} + t_{r_i}\left(\left(\frac{a_{ik-1}(1-y_{ik}) + P_j}{\eta_i}\right)^{\beta_i} - \left(\frac{a_{ik-1}(1-y_{ik})}{\eta_i}\right)^{\beta_i}\right) x_{ijk}\right] \leq C_{max}$$

i= 1, 2, 3, ….,m

$$a_{ik} = a_{ik-1}(1-y_{ik}) + \sum_{j=1}^{n} p_j x_{ijk} \quad i = 1,2,3,\dots,m \quad \text{and} \quad k = 1,2,3,\dots,n$$

To ensure that there is only one job for each position on the machine.

$$\sum_{j=1}^{n} x_{ijk} \leq 1 \quad i = 1,2,3,\dots,m \quad \text{and} \quad k = 1,2,3,\dots,n$$

However, because of the assumption that each machine must be used, so the possible positions on each machine are $n - m + 1$ and to ensure that each job must be processed one time on one machine.

$$\sum_{i=1}^{m}\sum_{k=1}^{n-m+1} x_{ijk} = 1 \quad j = 1,2,3,\dots,n$$

Each machine must be used.

$$\sum_{j=1}^{n}\sum_{k=1}^{n} x_{ijk} \geq 1 \quad i = 1,2,3,\dots,m$$

It is necessary to avoid any idle period on the machine. In other words, if any position is empty then, the next positions must be empty also.

$$\sum_{j=1}^{n} x_{ijk} \geq \sum_{j=1}^{n} x_{ijk+1} \quad i = 1,2,3,\ldots.,m \quad \text{and } k = 1,2,3,\ldots.,n$$

$$x_{ijk} = 0 \; or \; 1 \,,\; y_{ik} = 0 \; or \; 1 \,,\; a_{ik} \geq 0$$

5.3.2 Model Solving

The problem is to distribute the jobs on the machines in a manner that the maximum expected makespan is minimal, and as the second objective, the difference between the maximum and minimum makespan is minimal.

In the deterministic case, the good balance of the jobs leads to minimum of maximum makspan as well as minimizing the difference between maximum and minimum makespan. On the other hand, if the expected repair and preventive maintenance are considered then, the case becomes more difficult, especially if the machines reliability measures are different. Moreover, the balance can occur on different levels of the makespan due to the different expected response for each job in each machine. This will call the dummy balance. Dummy balance is if the difference of the maximum and minimum makespans of machines is small, however the makespan of the cell is not minimal. Example in section 5.4.3 shows this case. Also, it shows that even the less malfunction machine has the maximum load and minimum makespan, doesn’t guarantee that the expected makespan is minimal.

The non-preemptive problem is relaxed to a preemptive problem. The optimal solution of the relaxed problem gives a lower bound of the non-preemptive problem. A branch and bound method is applied to obtain the optimal solution of the non-relaxed problem. A first – fit heuristic generates a good feasible solution and accelerates the branch and bound method.

5.3.3 The Preemptive Problem and its Solution

Assume that there are m machines, and n jobs (n > 2m) to be distributed on that machines. The case is preemptive and the jobs can be interrupted for maintenance and resumed without additional penalty (resumable case). In order to simplifying the problem for readers assuming there are two machines, m_1 and m_2, and the total load ($L = \sum_{j=1}^{n} P_j$) should be divided on both of them in a way that the expected makespans of both are equals.

5.3.4 Lemma 11

Assume that all jobs are preemptive. The age of the machines at the beginning are a_i ($a_i \geq 0, i = 1, 2, \dots, m$). In any optimal solution the expected values of the makespan on the machines have the same value.

Proof:

Assume that there are at least two machines, say machines A and B, having different expected values, say $T_A < T_B$. Then it is possible to reassign a small part of the load of machine B to A such that the new expected values of the makespans satisfy the inequality $T_A < T_A^{new} = T_B^{new} < T_B$. The overall makespan of the new schedule is not greater than that of the previous schedule. If not all makespans are equal then the method can be repeated■

In what follows, a near optimal solution is generated first and then, this solution is the starting point that will be used in Newton's method to determine the optimal solution.

Let,

l_1: the amount of load assigned to machine 1.

l_2: the amount of load assigned to machine 2.

Let $L = l_1 + l_2$

EMS_{mi,L,k^i}: minimum expected makespan for all the jobs on machine i (i = 1, 2) in the same sequence with preventive maintenance times k^i.

Where,

if $a_0 = 0$ then,

$$EMS_{mi,L,k_d} = \sum_{j=1}^{n} P_j + k_d t_{mi} + (k_d + 1)\left(\frac{T_d}{\eta_i}\right)^{\beta} t_{ri}, \quad \forall\, d = 1,2$$

and,

$$k_1 = \left\lfloor \frac{\sum_{j=1}^{n} P_j}{\dot{\tau}} \right\rfloor - 1 \text{ or } k_2 = \left\lceil \frac{\sum_{j=1}^{n} P_j}{\dot{\tau}} \right\rceil - 1$$

$$T_d = \frac{\sum_{j=1}^{n} P_j}{(k_d + 1)}$$

Thus,

$EMS_{m_i,L,k^i} = min\{EMS_{m_i,L,k_d} \mid \forall d = 1,2\}$ and $\forall i = 1, 2, \dots, m$

Otherwise, if $a_0 > 0$ then,

$$EMS_{mi,L,k_d} = \sum_{j=1}^{n} P_j + k_d t_{mi} + \left[\left(\frac{T_d}{\eta}\right)^{\beta} - \left(\frac{a_0}{\eta}\right)^{\beta}\right] t_{ri} + k_d \left(\frac{T_d}{\eta_i}\right)^{\beta} t_{ri}$$

and,

$$k_1 = \left\lfloor \frac{a_0 + \sum_{j=1}^{n} P_j}{\dot{\tau}} \right\rfloor - 1 \text{ or } k_2 = \left\lceil \frac{a_0 + \sum_{j=1}^{n} P_j}{\dot{\tau}} \right\rceil - 1$$

$$T_d = \frac{a_0 + \sum_{j=1}^{n} P_j}{(k_d + 1)}$$

Thus,

$$EMS_{m_i,L,k^i} = min\{EMS_{m_i,L,k_d} \mid \forall d = 1,2 \} \text{ and } \forall i = 1, 2, \dots, m$$

Ordering the machines on their makespan values where:

$$EMS_{m_1,L,k^1} \leq EMS_{m_2,L,k^2}$$

Then,

$$\lambda_i = {EMS_{m_1,L,k^1}} / {EMS_{m_i,L,k^i}} \quad \text{ and } \quad \lambda_1 \geq \lambda_2$$

Let;

$$\lambda_t = \sum_i \lambda_i$$

∴ approximately $l_i = \left(L / \lambda_t \right) \lambda_i$, where l_i is the load on machine i and $i = 1, 2$

For the assigned load of each machine and depending on the best number of preventive maintenance times calculated according to this load as shown earlier $\left(k^i = k_1 \text{ or } k_2\right)$, the expected makespan on each machine is:

$$EMS_{m_1,l_1,k^1} = l_1 + k^1 t_{m1} + (k^1 + 1) \left(\frac{l_1}{(k^1 + 1)\eta_1} \right)^{\beta_1} t_{r1}$$

and;

$$EMS_{m_2,l_2,k^2} = l_2 + k^2 t_{m2} + (k^2+1)\left(\frac{l_2}{(k^2+1)\eta_2}\right)^{\beta_2} t_{r2}$$

The expected makespan on both machines should be equals, and to ensure that the load on the machines should be adjusted and this step can be achieved using Newton’s method.

Since $EMS_{m_1,l_1,k^1} = EMS_{m_2,l_2,k^2}$ then,

$$l_1 + k^1 t_{m1} + (k^1+1)\left(\frac{l_1}{(k^1+1)\eta_1}\right)^{\beta_1} t_{r1} = l_2 + k^2 t_{m2} + (k^2+1)\left(\frac{l_2}{(k^2+1)\eta_2}\right)^{\beta_2} t_{r2}$$

$$l_1 + k^1 t_{m1} + (k^1+1)\left(\frac{l_1}{(k^1+1)\eta_1}\right)^{\beta_1} t_{r1} = (L - l_1) + k^2 t_{m2} + (k^2+1)\left(\frac{L-l_1}{(k^2+1)\eta_2}\right)^{\beta_2} t_{r2}$$

For Newton’s method let,

$$f(l_1) = 2l_1 - L + k^1 t_{m1} - k^2 t_{m2} + (k^1+1)\left(\frac{l_1}{(k^1+1)\eta_1}\right)^{\beta_1} t_{r1}$$

$$-(k^2+1)\left(\frac{L-l_1}{(k^2+1)\eta_2}\right)^{\beta_2} t_{r2} = 0$$

$$f'(l_1) = 2 + \frac{\beta_1 t_{r1}}{\eta_1}\left(\frac{l_1}{(k^1+1)\eta_1}\right)^{\beta_1 - 1} + \frac{\beta_2 t_{r2}}{\eta_2}\left(\frac{L-l_1}{(k^2+1)\eta_2}\right)^{\beta_2 - 1}$$

Then, the adjusted load is:

$$l_1(new) = l_1 - \frac{f(l_1)}{\grave{f}(l_1)}$$

Because there is no too much difference in the makespan values before the adjustment, so the Newton’s method adjusts the difference in a few steps.

The previous can be generalized for *m* - machines as the following:

$$L = \sum_{i=m}^{m} l_i \qquad \forall\, i = 1,2, \dots, m$$

$$\lambda_t = \sum_{i=1}^{m} \lambda_i \qquad \forall\, i = 1,2, \dots, m$$

where

$$l_i = \frac{L}{\lambda_t} \lambda_i \qquad \forall\, i = 1,2, \dots, m$$

and;

$$l_i = L - \sum_{r=1}^{m} l_r \qquad \forall\, r = 1,2, \dots, m \text{ and } r \neq i$$

$$\lambda_i = {}^{EMS_{m_1,L,k^1}}\!\big/_{EMS_{m_i,L,k^i}} \qquad \forall\, i = 1,2, \dots, m$$

$$EMS_{m_i,l_i,k^i} = l_i + k^i t_{mi} + \left(k^i + 1\right)\left(\frac{l_i}{(k^i+1)\eta_i}\right)^{\beta_i} t_{ri} \qquad \forall\, i = 1,2, \dots, m$$

The machines loads can be adjusted by Newton's method according to the follows:

$f : R^n \rightarrow R^n$

l_0 : is an initial point.

The linear approximation of the function is:

$$f(l) = \begin{pmatrix} f_1(l) \\ f_2(l) \\ . \\ . \\ f_m(l) \end{pmatrix}$$

The Jacobian's matrix of $f(l)$ is:

$$A = \begin{pmatrix} \frac{df_1(l_0)}{dl_1} & \cdots & \frac{df_1(l_0)}{dl_m} \\ \vdots & \ddots & \vdots \\ \frac{df_m(l_0)}{dl_1} & \cdots & \frac{df_m(l_0)}{dl_m} \end{pmatrix}$$

Assume that the inverse of Jacobian's matrix is existing, then:

$f(l)$ around l_0 is approximately:

$$f(l) = f(l_0) + A(l - l_0)$$

The approximation equation:

$$f(l_0) + A(l - l_0) = 0$$

$$A(l - l_0) = -f(l_0)$$

$$l - l_0 = -A^{-1}f(l_0)$$

$$l_{new} = l_0 - A^{-1}f(l_0)$$

Therefore, the optimal integrated solution of the preemptive problem is reached and in the next step this solution will representing the initial infeasible solution of the non-preemptive problem. It gives a lower bound for the optimal objective function value.

5.3.5 Non-preemptive problem

Depending on the solution of preemptive problem introduced in the previous section, two different methods will be proposed. First, the branch and bound method will employed for the exact solution and, second, an approximation method for a good solution.

1. An Exact Method (Branch and bound)

Branch and bound (B&B) is a method designed to solving discrete and combinatorial optimization problems. The mechanism of the B & B is as follows: in the total enumeration tree, at any node, if there is no promising solution in any of its descendants, then they should be discarded. Hence, it can "prune" the tree at that node. Prune enough branches of the tree will reduce the computations in a manageable size. The decision to ignore the solutions in the pruned branches has been made after ensuring that the optimal solution cannot be at any one of these nodes. Thus, the B&B approach is an exact optimization method and it is not a heuristic, or approximating procedure. Often, it is not difficult to find a feasible solution for the problem. So, it is possible to use some heuristics to obtain that solution. This incumbent solution gives the upper bound (UB) in the case of a minimization problem. Additionally, at any node, a "bound" for the best solution expected of the descendants of this node must be computed, this bound denoted by the lower bound (LB). If the incumbent solution (upper bound), is better than any expectations for any solution (lower bound) resulting of that node then, there is no promising branch of this node and it should be prune. However, the branch can be prune because of other cases and this depends on the kind of the problem and some advance information such as the upper bound.

The solution of the preemptive problem will represent the node zero (infeasible) for the branch and bound method and the expected makespan at this node will representing the lower bound at this node. Solutions except at node zero or any node with a feasible solution are partially non-preemptive and the expected makespan will calculate depending on the assigned non-preemptive job(s) and the remaining assigned load for the machine.

In the following, a description for the branch and bound method to minimizing the maximum makespan for a set of jobs on multi machines working in parallel will be presented.

Branching**:** each job can be on each machine. Therefore, all the possible scenarios of the assignment problem $(m^n - m)$ can be generated if it is necessary. Branches that will lead to a load greater than the load on the UB for the machine has a maximum expected makespan will be discarded. Moreover, because of the condition that all machines must be used, so any branch will propose all jobs on one machine or leaving any machine empty is discarded.

The Lower Bound**:** to calculate the lower bound of each node the non-preemptive problem will be relaxed to be preemptive. At node zero the problem will be preemptive completely (infeasible for non-preemptive case) and the load on each machine will assigned as shown earlier in section 6.3.2.1. In the subsequent nodes, the subproblem at each node will be partially non-preemptive and the lower bound at each of them will be defined as follows:

a) Calculate the expected makespan on each machine (EMS_{m_i,l_i,k^i}) depend on its assigned load (l_i) and the assigned job(s) as follows:

Let, a_A be the age at the end of non-preemptive part and the beginning of preemptive part.

If $a_A = 0$ then,

$$EMS_{mi,l_i,k_d} = A_s + Rl_i + k_d t_{mi} + (k_d + 1)\left(\frac{T_d}{\eta_i}\right)^{\beta} t_{ri} \quad \forall\, d = 1,2$$

where,

A_s is the expected makespan of the assigned jobs

Rl_i is the remaining load on machine i ($Rl_i = l_i - \sum P_j \mid j \in jobs\ in\ A_s$)

and,

$$k_1 = \left\lfloor \frac{Rl_i}{\dot{\tau}} \right\rfloor - 1 \text{ or } k_2 = \left\lceil \frac{Rl_i}{\dot{\tau}} \right\rceil - 1$$

$$T_d = \frac{Rl_i}{(k_d + 1)}$$

Thus,

$$EMS_{m_i,l_i,k^i} = min\{EMS_{m_i,L,k_d} \mid \forall d = 1,2\} \text{ and } \forall i = 1, 2, \dots, m.$$

Otherwise, if $a_A > 0$ then,

$$EMS_{mi,L,k_d} = A_s + Rl_i + k_d t_{mi} + \left[\left(\frac{T_d}{\eta}\right)^{\beta} - \left(\frac{a_A}{\eta}\right)^{\beta}\right] t_{ri} + k_d \left(\frac{T_d}{\eta_i}\right)^{\beta} t_{ri}$$

and,

$$k_1 = \left\lfloor \frac{a_A + Rl_i}{\dot{\tau}} \right\rfloor - 1 \text{ or } k_2 = \left\lceil \frac{a_A + Rl_i}{\dot{\tau}} \right\rceil - 1$$

$$T_d = \frac{a_A + Rl_i}{(k_d + 1)}$$

Thus,

$$EMS_{m_i,L,k^i} = min\{EMS_{m_i,L,k_d} \mid \forall d = 1,2\} \text{ and } \forall i = 1, 2, \dots, m$$

b) Using Newton's method the remaining loads (preemptive part) on the machines will be adjusted to make the expected makespans on all machines equals.

Upper bound: the upper bound can be determined using the approximation method described in the next section.

Fathoming procedure: a node is fathomed in one of the following causes:

a) If a feasible solution has been reached as the optimal solution of the relaxed (preemptive) problem.

b) If the LB of the node is greater than or equals to the UB. This leads to conclude that there is no better feasible solution in the node than the one which gives the upper bound.

c) At any node, if the machine which is produced the UB (maximum makespan for the solution obtained by the approximation method at the beginning) has a part of its total load as a non-preemptive load (the jobs that assigned to this machine) and that part plus the smallest remaining job has a value greater than or equal to its value in the above mentioned feasible solution then, this node will be fathomed if the remaining non-assigned jobs including the smallest one cannot be on the other machines without exceeding the UB. This procedure is like to the test procedure in implicit enumeration. Mathematically this can be described as follows:

Let l_i be the current non-preemptive load of the machine at the node.

l_{UB} : the non-preemptive load at the upper bound

F : set of free jobs (non-assigned jobs)

If

$$l_i + \min_{j \in F} P_j > l_{UB}$$

and

$$\min C_{max} > UB$$

Whilst;

$$EMS\left(l_k + \sum_{j \in F} x_{kj} P_j\right) \leq C_{max} \; ; \quad \forall\, k \neq i,$$

$$\sum_{k \neq i} x_{kj} = 1 \, , \quad j \in F$$

$$\forall\, j \in F,\ \ x_{kj} = 0 \text{ or } 1$$

A numerical example illustrating the solution procedure of the problem is introduced in section 5.4.4.

2. The Approximation Method

In this method, the jobs will be assigned to the machines according to the assigned load which is calculated with the assumption that the problem is preemptive problem, so most probably it is difficult to meet the assigned load exactly. A first fit algorithm will be used to assign the load for each machine and then the remaining unfitted jobs are added according to the specified criteria in the algorithm below:

Fit Algorithm

1. Put the jobs in LPT order.
2. Start with machine one (the least likely to malfunction) and fit the jobs in scheduling list (SL) in their order.
3. If the job does not fit the current machine (the bound of each machine is l_i which is defined by the optimal solution of the preemptive problem) then, fit it on the next machine (machine with the next index).
4. If all jobs have been fitted then stop, otherwise go to 5.
5. The remaining unfitted jobs will be added according to LPT rule but for the makespan not the load (less expected makespan takes longest remaining job) and ties always broken for the machine with smallest index.

Up to here a good feasible solution has been generated, and it may need some improvement in order to get an optimal or better solution. The simple and best way to improve the solution is switching some loads between the machines have the maximum makespan and minimum makespan (depending on load assigned, shortage of the load should be assigned and extra load should be moved) to decrease the first and increase the second. By repeating this procedure it may lead to optimal or better solution. Also, by enlarging the assigned load for the machines that has makespan less than the maximum gained one and repeats the fitting process. This method works if the added load will make some changes in the fitted jobs.

5.4 Illustrative Examples

5.4.1 Makespan Problem on Single Machine (H_1)

The processing times of 10 jobs are below and the machine maintenance parameters are $a_s = 18,\ \beta = 2.5,\ \eta = 90,\ t_m = 5, t_r = 15,\ \dot{\tau} = 49.313$

i	1	2	3	4	5	6	7	8	9	10
P_i	54	50	49	45	39	33	25	22	18	13

Solution:

LS list in non-increasing order including the a_s is:

j	1	2	3	4	5	6	7	8	9	10	11
P_j	54	50	49	45	39	33	25	22	18	18	13

Iteration 1:

$a_s = 0,\ k = 1,\ P_1 = 54$ and $P_1 > \dot{\tau}$, then $s_1 = (54)$,

S = {54}

$a_s = 0, \quad k = 2$

Iteration 2:

LS is:

j	1	2	3	4	5	6	7	8	9	10
P_j	50	49	45	39	33	25	22	18	18	13

$P_1 = 50$ and $P_1 > \dot{\tau}$, then $s_2 = (50)$

S = {54, 50}

$a_s = 0, \qquad k = 3$

Iteration 3:

LS is:

j	1	2	3	4	5	6	7	8	9
P_j	49	45	39	33	25	22	18	18	13

$P_1 = 49$ and $P_1 < \dot{\tau}$, then $s_3 = (49, \ldots$

$a_s = 49, \qquad G = 49 + 0 = 49$ and $H = 49 + 13 = 62$

$$\left[\left(\frac{49.313}{90}\right)^{2.5} - \left(\frac{49}{90}\right)^{2.5}\right] < \left[\left(\frac{62}{90}\right)^{2.5} - \left(\frac{49.313}{90}\right)^{2.5}\right]$$

$0.0526 < 2.575$, then $s_3 = (49)$

S = {(54), (50), (49)}

$a_s = 0, \qquad k = 4$

Iteration 4:

LS is:

j	1	2	3	4	5	6	7	8
P_j	45	39	33	25	22	18	18	13

$P_1 = 45$ and $P_1 < \dot{\tau}$, then $s_4 = (45, \ldots$

$a_s = 45, \qquad G = 45 + 0 = 45$ and $H = 45 + 13 = 58$

$$\left[\left(\frac{49.313}{90}\right)^{2.5} - \left(\frac{45}{90}\right)^{2.5}\right] < \left[\left(\frac{58}{90}\right)^{2.5} - \left(\frac{49.313}{90}\right)^{2.5}\right]$$

$0.682 < 1.668$, then $s_4 = (45)$

S = {(54), (50), (49), (45)}

$a_s = 0, \qquad k = 5$

Iteration 5:

LS is:

j	1	2	3	4	5	6	7
P_j	39	33	25	22	18	18	13

$P_1 = 39$ and $P_1 < \dot{\tau}$, then $s_5 = (39, ...$

$a_s = 39$, $G = 39 + 0 = 39$ and $H = 39 + 13 = 52$

$$\left[\left(\frac{49.313}{90}\right)^{2.5} - \left(\frac{39}{90}\right)^{2.5}\right] < \left[\left(\frac{52}{90}\right)^{2.5} - \left(\frac{49.313}{90}\right)^{2.5}\right]$$

$1.48 > 0.473$, then $s_5 = (39, 13)$

S = {(54), (50), (49), (45), (39, 13)}

$a_s = 0$, $k = 6$

Iteration 6:

LS is:

j	1	2	3	4	5
P_j	33	25	22	18	18

$P_1 = 33$ and $P_1 < \dot{\tau}$, then $s_6 = (33, ...$

$a_s = 33$, $G = 33 + 0 = 33$ and $H = 33 + 18 = 51$

$$\left[\left(\frac{49.313}{90}\right)^{2.5} - \left(\frac{33}{90}\right)^{2.5}\right] < \left[\left(\frac{51}{90}\right)^{2.5} - \left(\frac{49.313}{90}\right)^{2.5}\right]$$

$2.112 > 0.293$, then $s_5 = (33, 18)$

S = {(54), (50), (49), (45), (39, 13), (33, 18)}

$a_s = 0$, $k = 7$

Iteration 7:

LS is:

j	1	2	3
P_j	25	22	18

$P_1 = 25$ and $P_1 < \dot{\tau}$, then $s_7 = (25, ...$

$a_s = 25, \qquad G = 25 + 22 = 47$ and $H = 25 + 22 + 18 = 65$

$\left[\left(\frac{49.313}{90}\right)^{2.5} - \left(\frac{47}{90}\right)^{2.5}\right] < \left[\left(\frac{65}{90}\right)^{2.5} - \left(\frac{49.313}{90}\right)^{2.5}\right]$

$0.38 > 3.32$, then $s_7 = (25, 22)$

S = {(54), (50), (49), (45), (39, 13), (33, 18), (25, 22)}

$$a_s = 0, \qquad k = 8$$

Iteration 8:

$RL = 18 < \dot{\tau}$, then $s_{st} = 45$

$$\left(\frac{t_m}{t_r}\right) + \left(\frac{RL}{\eta}\right)^{\beta} < \left(\frac{s_{st} + RL}{\eta}\right)^{\beta} - \left(\frac{s_{st}}{\eta}\right)^{\beta}$$

$$\frac{5}{15} + \left(\frac{18}{90}\right)^{2.5} > \left(\frac{45 + 18}{90}\right)^{2.5} - \left(\frac{45}{90}\right)^{2.5}$$

$$0.3512 > 0.2332$$

Thus,

S = {(54), (50), (49), (45, 18), (39, 13), (33, 18), (25, 22)}

Then, there are two solutions:

S_1 =	45	54	50	49	13	39	18	33	22	25

S_1 =	33	54	50	49	13	39	18	45	22	25

The result preventive maintenance plan in both cases is:

PM	0	1	1	1	1	0	1	0	1	0

In both cases the expected makespan (EMS) = 405.18. This value can be calculated for S_1 as follows:

$$EMS = 45 + \left[\left(\frac{45+18}{90}\right)^{2.5} - \left(\frac{18}{90}\right)^{2.5}\right] 15 + 5 + 54 + \left(\frac{54}{90}\right)^{2.5} 15 + 5 + 50 + \left(\frac{50}{90}\right)^{2.5} 15 + 5 + 49 + \left(\frac{49}{90}\right)^{2.5} 15 + 5 + 13 + 39 + \left(\frac{13+39}{90}\right)^{2.5} 15 + 5 + 18 + 33 + \left(\frac{18+33}{90}\right)^{2.5} 15 + 5 + 22 + 25 + \left(\frac{22+25}{90}\right)^{2.5} 15 = 405.18$$

If the production schedule and preventive maintenance planning are solved separately then, most probably the jobs which are available at time zero will be processed in SPT order. Accordingly, the corresponding preventive maintenance schedule is:

PM	0	0	0	1	0	1	0	1	0	1

Thus, the EMS = 462.1

5.4.2 Expected Makespan Problem on Single Machine (H_2)

For the previous example and using heuristic in section 5.2.5:

LS list in non-increasing order is:

j	1	2	3	4	5	6	7	8	9	10	11
P_j	54	50	49	45	39	33	25	22	18	18	13

S = {∅} and $A_s = 0$

Iteration 1:

$p_1 = 54 > \dot{\tau},$ then $s_2 = (54)$ and resort LS.

$S = \{(54)\},\ A_s = 0 + 54 + \left(\frac{54}{90}\right)^{2.5} 15 + 5 = 63.183$, do PM after the subset.

Iteration 2:

LS is:

j	1	2	3	4	5	6	7	8	9	10
P_j	50	49	45	39	33	25	22	18	18	13

$p_1 = 50 > \dot{\tau},$ then $s_3 = (50)$ and resort LS.

$S = \{(54), (50)\},\ A_s = 63.183 + 50 + \left(\frac{50}{90}\right)^{2.5} 15 + 5 = 121.634$, do PM after the subset.

Iteration 3:

LS is:

j	1	2	3	4	5	6	7	8	9
P_j	49	45	39	33	25	22	18	18	13

$p_1 = 49 < \dot{\tau},$ then $G = 0$ and $H = 13$

For G:

$RL = 45 + 39 + 33 + 25 + 22 + 18 + 13 = 195$

$$k = \frac{195 + 18}{49.313} = 4.32,\ \text{then}\ k_1 = 3 \text{ and } k_2 = 4$$

$k = k_1 = 3$

$$EMS_{k_1}^{G} = 121.634 + 49 + \left(\frac{49}{90}\right)^{2.5} 15 + 5 + 195 + 3 * 5 + 3\left(\frac{195+18}{4*90}\right)^{2.5} 15 + \left[\left(\frac{195+18}{4*90}\right)^{2.5} - \left(\frac{18}{90}\right)^{2.5}\right] 15 = 404.8$$

$k = k_2 = 4$

$$EMS_{k_2}^{G} = 121.634 + 49 + \left(\frac{49}{90}\right)^{2.5} 15 + 5 + 195 + 4 * 5 + 4\left(\frac{195+18}{5*90}\right)^{2.5} 15 + \left[\left(\frac{195+18}{5*90}\right)^{2.5} - \left(\frac{18}{90}\right)^{2.5}\right] 15 = 405.2065$$

For H:

$RL = 45 + 39 + 33 + 25 + 22 + 18 = 182$

$k = \frac{182 + 18}{49.313} = 4.05,$ then $k_1 = 3$ and $k_2 = 4$

$k = k_1 = 3$

$EMS_{k_1}^{H} = 121.634 + 49 + 13 + \left(\frac{62}{90}\right)^{2.5} 15 + 5 + 182 + 3 * 5 + 3\left(\frac{182+18}{4*90}\right)^{2.5} 15 +$

$\left[\left(\frac{182+18}{4*90}\right)^{2.5} - \left(\frac{18}{90}\right)^{2.5}\right] 15 = 405.076$

$k = k_2 = 4$

$EMS_{k_2}^{H} = 121.634 + 49 + 13 + \left(\frac{62}{90}\right)^{2.5} 15 + 5 + 182 + 4 * 5 + 4\left(\frac{182+18}{5*90}\right)^{2.5} 15 +$

$\left[\left(\frac{182+18}{5*90}\right)^{2.5} - \left(\frac{18}{90}\right)^{2.5}\right] 15 = 406.15$

$EMS_k = \min\{404.8, 405.2065, 405.076, 406.15\} = 404.8,$ then $s_4 = (54)$ and resort LS.

$S = \{(54), (50), (49)\},$ $A_s = 178.914$, do PM after the subset.

Iteration 4:

LS is:

j	1	2	3	4	5	6	7	8
P_j	45	39	33	25	22	18	18	13

$p_1 = 45 < \dot{\tau},$ then $G = 0$ and $H = 13$

For G:

$RL = 39 + 33 + 25 + 22 + 18 + 13 = 150$

$k = \frac{150 + 18}{49.313} = 3.4,$ then $k_1 = 2$ and $k_2 = 3$

$k = k_1 = 2$

$EMS_{k_1}^{G} = 178.914 + 45 + \left(\frac{45}{90}\right)^{2.5} 15 + 5 + 150 + 2 * 5 + 2\left(\frac{150+18}{3*90}\right)^{2.5} 15 +$

$\left[\left(\frac{150+18}{3*90}\right)^{2.5} - \left(\frac{18}{90}\right)^{2.5}\right] 15 = 405.04$

$k = k_2 = 3$

$EMS_{k_1}^{G} = 178.914 + 45 + \left(\frac{45}{90}\right)^{2.5} 15 + 5 + 150 + 3 * 5 + 3\left(\frac{150+18}{4*90}\right)^{2.5} 15 +$

$\left[\left(\frac{150+18}{4*90}\right)^{2.5} - \left(\frac{18}{90}\right)^{2.5}\right] 15 = 405.224$

For H:

$RL = 39 + 33 + 25 + 22 + 18 = 137$

$k = \dfrac{137 + 18}{49.313} = 3.4,$ then $k_1 = 2$ and $k_2 = 3$

$k = k_1 = 2$

$EMS_{k_1}^{H} = 178.914 + 45 + 13 + \left(\frac{58}{90}\right)^{2.5} 15 + 5 + 137 + 2 * 5 + 2\left(\frac{137+18}{3*90}\right)^{2.5} 15 +$

$\left[\left(\frac{137+18}{3*90}\right)^{2.5} - \left(\frac{18}{90}\right)^{2.5}\right] 15 = 404.8835$

$k = k_2 = 3$

$EMS_{k_2}^{H} = 178.914 + 45 + 13 + \left(\frac{58}{90}\right)^{2.5} 15 + 5 + 137 + 3 * 5 + 3\left(\frac{137+18}{4*90}\right)^{2.5} 15 +$

$\left[\left(\frac{137+18}{4*90}\right)^{2.5} - \left(\frac{18}{90}\right)^{2.5}\right] 15 = 405.9453$

$EMS_k = \min\{405.04, 405.224, 404.88, 405.945\} = 404.88,$ then $s_5 = (13, 45)$ and resort LS.

$S = \{(54), (50), (49), (13, 45)\},\ A_s = 246.9153$, do PM after the subset.

Iteration 5:

LS is:

j	1	2	3	4	5	6
P_j	39	33	25	22	18	18

$p_1 = 39 < \dot{\tau},$ then $G = 0$ and $H = 18$

For G:

$RL = 33 + 25 + 22 + 18 = 98$

$k = \dfrac{98 + 18}{49.313} = 2.35$, then $k_1 = 1$ and $k_2 = 2$

$k = k_1 = 1$

$EMS_{k_1}^{G} = 246.9153 + 39 + \left(\frac{39}{90}\right)^{2.5} 15 + 5 + 98 + 1 * 5 + \left(\frac{98+18}{2*90}\right)^{2.5} 15 +$

$\left[\left(\frac{98+18}{2*90}\right)^{2.5} - \left(\frac{18}{90}\right)^{2.5}\right] 15 = 405.503$

$k = k_2 = 2$

$EMS_{k_1}^{G} = 246.9153 + 39 + \left(\frac{39}{90}\right)^{2.5} 15 + 5 + 98 + 2 * 5 + 2\left(\frac{98+18}{3*90}\right)^{2.5} 15 +$

$\left[\left(\frac{98+18}{3*90}\right)^{2.5} - \left(\frac{18}{90}\right)^{2.5}\right] 15 = 405.9455$

For H:

The item in H is the dummy job ($a_0 = 18$)

$RL = 33 + 25 + 22 + 18 = 98$

$k = \dfrac{98}{49.313} = 1.987$, then $k_1 = 0 \; and \; k_2 = 1$

$k = k_1 = 0$

$EMS_{k_1}^{H} = 246.9153 + 39 + \left[\left(\frac{39+18}{90}\right)^{2.5} - \left(\frac{18}{90}\right)^{2.5}\right] 15 + 5 + 98 + \left(\frac{98}{90}\right)^{2.5} 15 =$

411.99

$k = k_2 = 1$

$EMS_{k_2}^{H} = 246.9153 + 39 + \left[\left(\frac{39+18}{90}\right)^{2.5} - \left(\frac{18}{90}\right)^{2.5}\right] 15 + 5 + 98 + 5 +$

$\left(\frac{98}{2*90}\right)^{2.5} 15 = 404.997$

$EMS_k = \min\{405.503, 405.95, 411.99, 404.997\} = 404.997$, then $s_1 = (45)$ and resort LS.

$S = \{(39), (54), (50), (49), (13, 45)\}$, $A_s = 295.4352$, do PM after the subset

Iteration 6:

LS is:

j	1	2	3	4
P_j	33	25	22	18

$p_1 = 33 < \dot{\tau}$, then $G = 0$ and $H = 18$

For G:

$RL = 25 + 22 + 18 = 65$

$k = \frac{65}{49.313} = 1.3$, then $k_1 = 0$ and $k_2 = 1$

$k = k_1 = 0$

$EMS_{k_1}^{G} = 295.4352 + 33 + \left(\frac{33}{90}\right)^{2.5} 15 + 5 + 65 + \left(\frac{65}{90}\right)^{2.5} 15 = 406.31$

$k = k_2 = 1$

$EMS_{k_1}^{G} = 295.4352 + 33 + \left(\frac{33}{90}\right)^{2.5} 15 + 5 + 65 + 5 + 2\left(\frac{65}{2*90}\right)^{2.5} 15 = 407.007$

For H:

$RL = 25 + 22 = 47$

$k = \frac{47}{49.313} = 0.95$, then $k = k_1 = 0$

$k = k_1 = 0$

$EMS_{k_1}^{H} = 295.4352 + 33 + 18 + \left(\frac{33+18}{90}\right)^{2.5} 15 + 5 + 47 + \left(\frac{47}{90}\right)^{2.5} 15 = 405.017$

$EMS_k = \min\{406.31, 407.007, 405.017\} = 405.017$, then $s_6 = (18{,}33)$ and do PM. Also, $s_7 = (22, 25)$ and no PM because it is the last subset.

$$S = \{(45), (54), (50), (49), (13, 45), (18, 33), (22, 25)\}$$

The corresponding PM plan is:

PM	0	1	1	1	1	0	1	0	1	0

The expected makespan is 405.017

5.4.3 Minimizing Expected Maximum Makespan on Multi Machines: Dummy Balance

In the following numerical example (see Table 5.3 and 5.4), for 43 jobs and 4 machines, the dummy balance that can be happen at a level greater than the optimal balance will be shown.

Table 5.3: Jobs processing times

j	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	Total
P_j	60	60	59	59	57	55	55	54	53	51	563
j	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	
P_j	50	50	49	49	49	47	45	45	43	43	470
j	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	
P_j	42	41	39	38	35	35	34	33	32	31	360
j	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	
P_j	29	27	25	25	24	22	21	19	18	17	277
j	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	
P_j	16	13	9								38
Grand Total (L)											***1658***

Table 5.4: Machines maintenance parameters

	M1	**M2**	**M3**	**M4**
β_i	2	2	2	2
η_i	90	80	60	40
a_0	0	0	0	0
t_{mi}	5	5	5	5
t_{ri}	15	15	15	15
τ^*_i	51.962	46.188	34.641	23.094

Whatever the method that has been used to distribute the jobs on the machines, Table 5.5 shows that there are a good balance between 540.6 and 541.8. Actually, it is a dummy balance where a better balance can be obtained at a lower level, Table 5.6 shows for the same problem another good balance between 525.26 and 527.23. The point here is how can be avoid the trap (dummy balance) and the answer is by starting from the lower bound of the problem to looking for a good or optimal solution. This can be noted when the distribution method respect the assigned load of each machine that determined by the lower possible bound of the problem (preemptive case).

5.4.4 Minimizing Expected Maximum Makespan on Parallel Machines: Exact Method (B&B)

The jobs are { 54, 50, 45, 39, 25, and 18 }. The machines maintenance parameters are:

$\{\beta_1 = 2.5,\ \eta_1 = 90,\ a_0^1 = 0,\ t_{m_1} = 5,\ t_{r_1} = 15,$ and $\dot{\tau}_1 = 49.313\}$ and;

$\{\beta_2 = 1.6,\ \eta_2 = 60,\ a_0^2 = 0,\ t_{m_2} = 5,\ t_{r_2} = 15,$ and $\dot{\tau}_2 = 41.553\}$

- **The optimal solution of the preemptive problem**

<u>For M₁:</u>

$$k^1 = \frac{a_0^1 + \sum_{j=1}^{n} P_j}{\dot{\tau}_1} = \frac{0 + 231}{49.313} = 4.68$$

$$k_1 = 3 \text{ or } k_2 = 4$$

If $k^1 = k_1 = 3$

$$T = \frac{a_0^1 + \sum_{j=1}^{n} P_j}{(k^1 + 1)} = \frac{0 + 231}{4} = 57.75$$

$EMS_{m1,231,3} = \sum_{j=1}^{n} P_j + k^1 t_{m1} + (k^1 + 1)\left(\frac{T}{\eta_1}\right)^{\beta_1} t_{r1} = 321 + 3 * 5 +$

$(3 + 1)\left(\frac{57.75}{90}\right)^{2.5} * 15 = 265.79$

If $k^1 = k_2 = 4$

$$T = \frac{a_0^1 + \sum_{j=1}^{n} P_j}{(k^1 + 1)} = \frac{0 + 231}{5} = 46.2$$

$EMS_{m1,231,4} = \sum_{j=1}^{n} P_j + k^1 t_{m1} + (k^1 + 1)\left(\frac{T}{\eta_1}\right)^{\beta_1} t_{r1} = 321 + 4 * 5 +$

$(4 + 1)\left(\frac{46.2}{90}\right)^{2.5} * 15 = 265.16$

$$\therefore EMS_{m1,231,k^1} = min\{265.79, 265.16\} = 265.16 = EMS_{m1,231,4}$$

For M_2

$$k^2 = \frac{a_0^2 + \sum_{j=1}^{n} P_j}{\dot{\tau}_2} = \frac{0 + 231}{41.553} = 5.559$$

$$k_1 = 4 \text{ or } k_2 = 5$$

If $k^2 = k_1 = 4$

$$EMS_{m2,231,4} = 300.37$$

If $k^2 = k_2 = 5$

$$EMS_{m2,231,5} = 300.25$$

$$\therefore EMS_{m2,231,k^2} = min\{300.37, 300.25\} = 300.25 = EMS_{m2,231,5}$$

Dividing the load on the two machines

$$\lambda_1 = \frac{EMS_{m1,231,4}}{EMS_{m1,231,4}} = 1, \qquad \lambda_2 = \frac{EMS_{m1,231,4}}{EMS_{m2,231,5}} = \frac{265.16}{300.25} = 0.8831$$

$$\lambda_t = \lambda_1 + \lambda_2 = 1.8831$$

$$\therefore l_1 = \frac{\sum_{j=1}^{n} P_j}{\lambda_t} * \lambda_1 = 122.67 \text{ and } l_2 = \frac{\sum_{j=1}^{n} P_j}{\lambda_t} * \lambda_2 = 108.33$$

Makespan on each machine depend on the assigned load

$$EMS_{m1,122.67,k^1} = 138.93 = EMS_{m1,122.67,2}$$

$$EMS_{m2,108.33,k^2} = 138.3 = EMS_{m2,108.33,2}$$

Table 5.5: Dummy Balance

M	l_i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
1	443.59	55	35	35	34	33	32	31	29	27	25	24	22	21	19	18	17	457
2	434.82	49	49	47	45	43	43	41	39	38	25	16	13					448
3	410.47	55	54	53	51	50	50	49	45	9								416
4	369.12	60	60	59	59	57	42											337
																	Total	1658
Expected Makespan for the assigned Load																		
M	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total	EMS
1	55.0	35.0	35.0	34.0	33.0	32.0	31.0	29.0	27.0	25.0	24.0	22.0	21.0	19.0	18.0	17.0	457.0	541.4
2	49.0	49.0	47.0	45.0	43.0	43.0	41.0	39.0	38.0	25.0	16.0	13.0	0.0	0.0	0.0	0.0	448.0	540.6
3	55.0	54.0	53.0	51.0	50.0	50.0	49.0	45.0	9.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	416.0	541.3
4	60.0	60.0	59.0	59.0	57.0	42.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	337.0	541.8

Table 5.6: A better balance

M	l_i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
1	443.59	60	60	59	59	57	55	55	39									444
2	434.82	54	53	51	50	50	49	49	49	31								436
3	410.47	47	45	45	43	43	42	41	38	35	32							411
4	369.12	35	34	33	29	27	25	25	24	22	21	19	18	17	16	13	9	367
																	Total	1658
Expected Makespan for the assigned Load																		
M	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total	EMS
1	60.0	60.0	59.0	59.0	57.0	55.0	55.0	39.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	444.0	525.26
2	54.0	53.0	51.0	50.0	50.0	49.0	49.0	49.0	31.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	436.0	526.37
3	47.0	45.0	45.0	43.0	43.0	42.0	41.0	38.0	35.0	32.0	0.0	0.0	0.0	0.0	0.0	0.0	411.0	527.23
4	35.0	34.0	33.0	29.0	27.0	25.0	25.0	24.0	22.0	21.0	19.0	18.0	17.0	16.0	13.0	9.0	367.0	526.11

Using Newton's method to adjust the loads (l_1, l_2)

$$f(l_1) = 2l_1 - L + k^1 t_{m1} - k^2 t_{m2} + (k^1+1)\left(\frac{l_1}{(k^1+1)\eta_1}\right)^{\beta_1} t_{r1} -$$

$$(k^2+1)\left(\frac{L-l_1}{(k^2+1)\eta_2}\right)^{\beta_2} t_{r2} = 0.6278$$

$$f'(l_1) = 2 + \frac{\beta_1 t_{r1}}{\eta_1}\left(\frac{l_1}{(k^1+1)\eta_1}\right)^{\beta_1-1} + \frac{\beta_2 t_{r2}}{\eta_2}\left(\frac{L-l_1}{(k^2+1)\eta_2}\right)^{\beta_2-1} = 2.4225$$

Then, the adjusted load is:

$$l_1(new) = l_1 - \frac{f(l_1)}{\acute{f}(l_1)} = 122.41 \quad \text{and} \quad l_2 = 108.59$$

Thus:

$$EMS_{m1,122.41,2} = EMS_{m2,108.59,2} = 138.64$$

The optimal solution of the preemptive problem is reached. The solution of the preemptive problem will be the lower bound (LB) for the non-preemptive problem.

Branch and bound and the solution of non-preemptive problem

Before starting the B&B, the upper bound should be described and the approximation method introduced in the next section will be used to define it. The feasible solution defined by the approximation method is in Table 5.7.

Table 5.7: A feasible solution generated by the approximation method

M \ j	1	2	3	4	l_i	EMS_{m1,l_i,k^i}
1	54	50	18	0.0	122.0	138.6260
2	45	39	25	0.0	109.0	139.6919

Then, the upper bound (UB) for the B&B is 139.6919. Also, it can be noted that moving just one unit load from M_2 to M_1 increases the maximum expected makespan on M_1 and it will be greater.

Node (0):

It is the optimal solution of the preemptive problem, but it is infeasible for the current non-preemptive problem. So the branch and bound method starts with this solution.

$l_1 = 122.41,\ l_2 = 108.59$

$EMS_{m1,122.41,2} = EMS_{m2,108.59,2} = 138.64$

Node (1):

The jobs are in LPT order (LS) and the assigned job on each machine will be in list of the assigned jobs A_s^i.

J_1 on M_1 and $l_1 = 122.41$

$$k^1 = \frac{a_0^1 + l_1}{\dot{\tau}_1} = \frac{0 + 122.41}{49.313} = 2.48$$

$$k_1 = 1\ and\ k_2 = 2$$

If $k^1 = k_1 = 1$

$T = \frac{a_0^1 + l_1}{(k^1+1)} = \frac{0+122.41}{2} = 61.205$

$T > P_1$, thus the PM is in the end of T. Hence,

$EMS_{m1,122.41,1} = A_s^1 + (l_1 - P_1) + k^1 t_{m1} + \left[\left(\frac{T}{\eta_1}\right)^{\beta_1} - \left(\frac{P_1}{\eta_1}\right)^{\beta_1}\right] t_{r1} + \left(\frac{T}{\eta_1}\right)^{\beta_1} t_{r1} =$

138.85

If $k^1 = k_2 = 2$, then

$$T = \frac{a_0^1 + l_1}{(k^1 + 1)} = \frac{0 + 122.41}{3} = 40.803$$

$T < P_1$, thus the PM is performed immediately after finishing J_1.

$$Rl = 122.41 - 54 = 68.41 \text{ and } a_1^1 = 0$$

$$k^1 = \frac{Rl}{\dot{\tau}_1} = \frac{68.41}{49.313} = 1.387$$

$$k_1 = 0 \text{ or } k_2 = 1$$

If $k^1 = k_1 = 0,$ then $T = 68.41$

$$EMS_{m1,122.41,0} = A_s^1 + t_{m1} + (l_1 - P_1) + k^1 t_{m1} + \left(\frac{l_1 - P_1}{\eta_1}\right)^{\beta_1} t_{r1} = 139.15$$

If $k^1 = k_2 = 1,$ then $T = 34.205$

$$EMS_{m1,122.41,1} = A_s^1 + t_{m1} + (l_1 - P_1) + k^1 t_{m1} + (k^1 + 1)\left(\frac{T}{\eta_1}\right)^{\beta_1} t_{r1} = 139.26$$

Then for node (1),

$$EMS_{m1,122.41,1} = 138.85 \text{ and } EMS_{m2,108.59,2} = 138.64$$

Adjustment:

At node (1), M_1 is mixed (preemptive and non-preemptive) and M_2 is still pure preemptive. So,

$$f(l_1) = A_s^1 + (l_1 - P_1) + k^1 t_{m1} + \left[\left(\frac{l_1}{(k^1+1)\eta_1}\right)^{\beta_1} - \left(\frac{P_1}{\eta_1}\right)^{\beta_1}\right] t_{r1} + \left(\frac{l_1}{(k^1+1)\eta_1}\right)^{\beta_1} t_{r1} -$$

$$(L - l_1) - k^2 t_{m2} - (k^2 + 1)\left(\frac{L - l_1}{(k^2+1)\eta_2}\right)^{\beta_2} t_{r2} = 0.2151$$

$$\grave{f}(l_1) = 2 + \frac{\beta_1 t_{r1}}{\eta_1}\left(\frac{l_1}{(k^1+1)\eta_1}\right)^{\beta_1 - 1} + \frac{\beta_2 t_{r2}}{\eta_2}\left(\frac{L - l_1}{(k^2+1)\eta_2}\right)^{\beta_2 - 1} = 2.529$$

$$l_1(new) = l_1 - \frac{f(l_1)}{\grave{f}(l_1)} = 122.325 \quad \text{and} \quad l_2 = 108.675$$

$$EMS_{m1,122.325,1} = 138.7468 = EMS_{m2,108.675,2}$$

<u>Node (2):</u>

J_1 on M_2 and there is no job assigned to M_1. With the same procedure in node (1):

$$EMS_{m1,122.325,1} = 138.8721 = EMS_{m2,108.675,2}$$

Node (3):

J_1 and J_2 on M_1 and there is no jobs assigned to M_2 yet.

$$EMS_{m1,122.211,0} = 138.895 = EMS_{m2,108.789,2}$$

It can be noted that all the branches that will propose J3, J4 and J5 on M_1 will be discarded because they will exceeds the UB value. This case can be noted later for J_4 on M_1 at node (8) and for J_4 on M_1 at node (10).

Node (4):

J_1 on $M_{1,}$ and J_2 on M_2.

$$EMS_{m1,122.5274,1} = 138.996 = EMS_{m2,108.473,1}$$

Node (6):

J_1 and J_2 on M_2 (Total load on M_2 is 104) and there is no jobs assigned to M_1 yet. The node is fathomed because as if any of the remaining jobs added to M_2, then the load of M_2 exceeds its load in the feasible solution and resulting a greater makespan. If the entire remaining load assigned to M_1, then its load becomes greater than the expected makespan of that one in the UB. See fathoming procedure, part c.

Node (9):

J_1 on M_1 and, J_2 and J_3 no $M_{2.}$ (Total load on M_2 is 95). The node is fathomed due to the same reasons in node (6).

Node (11):

J_1 and J_3 on M_2 (Total load on M_2 is 99) and, J_2 on M_1. The node is fathomed due to the same reasons in node (6).

Node (13):

J_1 and J_3 on M_1 (Total load on M_1 is 99) and, J_2 and J4 on M_2 (Total load on M_2 is 89).

The node is fathomed. Only one job of the remaining jobs can be on M_2 which J_6 (18). In this case, the total load on M_2 will be 107 and 124 on M_1. As it showed earlier with the feasible solution above; just one unit above the 122 units on M_1 will produce greater expected makespan of the one in the UB. So, the node is fathomed. Figure 5.6 shows the complete tree of the branch and bound problem.

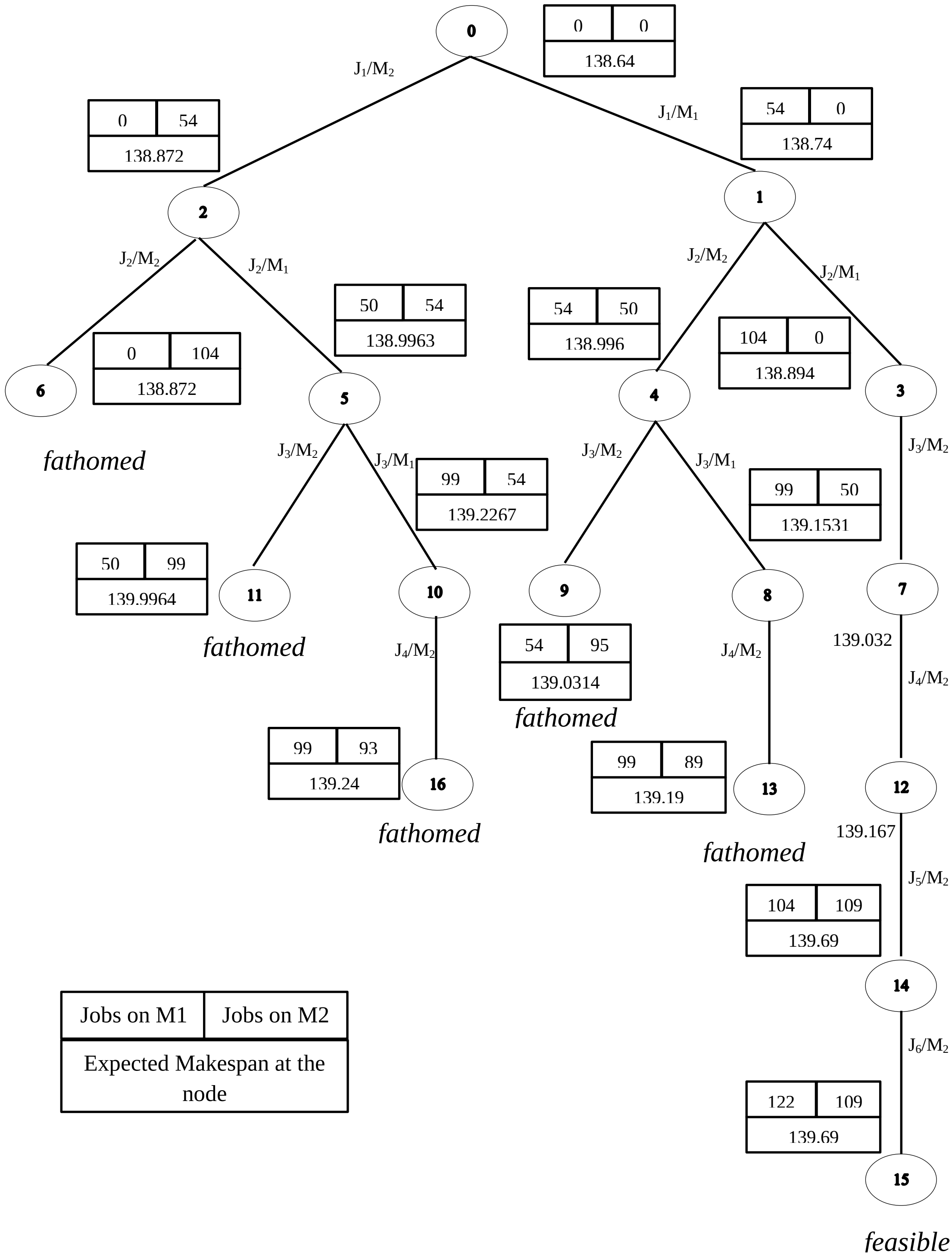

Figure 5.6: Branch and Bound three for example in 5.4.4

5.4.5 Minimizing Expected Maximum Makespan on Parallel Machines:

Approximation Method

The following is 44 jobs should be assigned on 4 machines in order minimizing the maximum expected makespan and the difference between maximum and minimum makespan.

Table 5.8: Jobs processing times

j	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**
P_j	70	68	67	66	65	65	64	63	62	60	650
j	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	
P_j	59	58	58	57	55	54	54	53	51	50	549
j	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	
P_j	49	47	46	45	43	43	41	40	38	37	429
j	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	
P_j	36	35	33	32	30	29	29	28	27	27	306
j	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	
P_j	23	23	20	20	0	0	0	0	0	0	86
Grand Total (L)											***2020***

Table 5.9: Machines maintenance parameters

	M_1	**M_2**	**M_3**	**M_4**
β_i	2.5	2	1.9	1.6
η_i	90	75	70	60
a_0	0	0	0	0
t_{mi}	5	5	5	5
t_{ri}	15	15	15	15
τ^*_i	49.313	43.301	41.502	41.553

Solution of the preemptive problem:

Machine I

$$k^1 = 40.9632 \text{ and } k_1 = 39, \quad k_2 = 40$$

If $k^1 = k_1 = 39$ then, T = 50.5

$$EMS_{m1,L,39} = 2356.506$$

If $k^1 = k_2 = 40$ then, T = 49.2683

$$EMS_{m1,L,40} = 2356.3602$$

Machine II

$$k^1 = 46.65 \text{ and } k_1 = 45, \quad k_2 = 46$$

If $k^1 = k_1 = 45$ then, T = 43.913

$$EMS_{m1,L,45} = 2481.545$$

If $k^1 = k_2 = 46$ then, T = 42.9787

$$EMS_{m1,L,46} = 2481.5121$$

Machine III

$$k^1 = 48.673 \text{ and } k_1 = 47, \quad k_2 = 48$$

If $k^1 = k_1 = 47$ then, T = 42.0833

$$EMS_{m1,L,47} = 2528.814$$

If $k^1 = k_2 = 48$ then, T = 41.2245

$$EMS_{m1,L,48} = 2528.7795$$

Machine IV

$$k^1 = 48.6122 \text{ and } k_1 = 47, \quad k_2 = 48$$

If $k^1 = k_1 = 47$ then, T = 42.0833

$$EMS_{m1,L,47} = 2663.194$$

If $k^1 = k_2 = 48$ then, T = 41.22449

$$EMS_{m1,L,48} = 2663.1752$$

Table 5.10 shows the solution summary of the preemptive problem.

Table 5.10: Solution summary of the preemptive problem

i	**Min EMS$_{mi,L,k}^{i}$**	**λ_i**	**l_i**	**Remark**
1	2356.3602	1	536.35291	λ_t = 3.76618 $l_i = \frac{L}{\lambda_t}\lambda_i$ $\forall\ i = 1, 2, \dots, m$
2	2481.5121	0.94967	509.30264	
3	2528.7795	0.93182	499.78286	
4	2663.1752	0.88479	474.56159	

<u>Assigned the load for each machine by fit algorithm:</u>

Table 5.11 shows the fitted load for each machine and their expected makespan. It is a feasible solution and to improve it there are two scenarios can be applied:

1. Enlarging the loads of the machines by equivalent (may not equals) values and repeat the fitting process. This scenario will not work in the current example because the shortage in load for machine one cannot occupy by any load next to 63 even with negative or positive allowances.
2. Switching two jobs or two groups of jobs between the machines has maximum expected makespan and the machine has minimum makespan. This process can be repeated until we reach to a satisfied feasible solution.

<u>Improving the current feasible solution:</u>

There are about 8.353 unit load shortage in machine 1 and about 9.438 unit load extra on machine 4. The extra load on Machine 4 resulting from the shortage on the other three machines not machine 1 only. The proposed procedure to balance the load is switching the load between M_1 and M_4 by 8 unit difference in the switched loads. This can be performed by moving the job with 64 value from M_1 to M_4 and the jobs with 33, 20 and 20 from M_4 to M_1, where (32+20+20)-64 = 8. Table 5.12 shows the status of the load and the expected makespan on each machine after switching process. It can be noted that the current feasible solution still can be improved.

More improvement:

The feasible solution can be improved by moving just one unit load from M_1 to M_3. Job with 65 unit loads from M_1 will be switched with jobs with 41 and 23 unit loads on M_3. Table 5.13 shows the best feasible solution can be reached by this method.

Table 5.11: Assigned the load to the machines using fit algorithm

M	l_i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
1	536.35	70	68	67	66	65	65	64	63									528
2	509.3	62	60	59	58	58	57	55	54	46								509
3	499.78	54	53	51	50	49	47	45	43	43	41	23						499
4	474.56	40	38	37	36	35	33	32	30	29	29	28	27	27	23	20	20	484
																	Total	2020
Expected makespan for the assigned Load																		
M	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**		EMS
1	70.0	68.0	67.0	66.0	65.0	65.0	64.0	63.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	528.0	618.4
2	62.0	60.0	59.0	58.0	58.0	57.0	55.0	54.0	46.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	509.0	626.2
3	54.0	53.0	51.0	50.0	49.0	47.0	45.0	43.0	43.0	41.0	23.0	0.0	0.0	0.0	0.0	0.0	499.0	623.4
4	40.0	38.0	37.0	36.0	35.0	33.0	32.0	30.0	29.0	29.0	28.0	27.0	27.0	23.0	20.0	20.0	484.0	636.4

Table 5.12: Improving the feasible solution

M	l_i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
1	536.35	70	68	67	66	65	65	63	32	20	20							536
2	509.3	62	60	59	58	58	57	55	54	46								509
3	499.78	54	53	51	50	49	47	45	43	43	41	23						499
4	474.56	64	40	38	37	36	35	33	30	29	29	28	27	27	23			476
																	Total	2020
Expected makespan for the assigned Load																		
M	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**		EMS
1	70.0	68.0	67.0	66.0	65.0	65.0	63.0	32.0	20.0	20.0	0.0	0.0	0.0	0.0	0.0	0.0	536.0	628.08
2	62.0	60.0	59.0	58.0	58.0	57.0	55.0	54.0	46.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	509.0	626.22
3	54.0	53.0	51.0	50.0	49.0	47.0	45.0	43.0	43.0	41.0	23.0	0.0	0.0	0.0	0.0	0.0	499.0	623.39
4	64.0	40.0	38.0	37.0	36.0	35.0	33.0	30.0	29.0	29.0	28.0	27.0	27.0	23.0	0.0	0.0	476.0	626.75

Table 5.13: Improving the feasible solution

M	l_i	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**Total**
1	536.35	70	68	67	66	65	63	41	32	23	20	20						535
2	509.3	62	60	59	58	58	57	55	54	46								509
3	499.78	65	54	53	51	50	49	47	45	43	43							500
4	474.56	64	40	38	37	36	35	33	30	29	29	28	27	27	23			476
																	Total	2020
Expected makespan for the assigned Load																		
M	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**		**EMS**
1	70.0	68.0	67.0	66.0	65.0	63.0	41.0	32.0	23.0	20.0	20.0	0.0	0.0	0.0	0.0	0.0	535.0	625.78
2	62.0	60.0	59.0	58.0	58.0	57.0	55.0	54.0	46.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	509.0	626.22
3	65.0	54.0	53.0	51.0	50.0	49.0	47.0	45.0	43.0	43.0	0.0	0.0	0.0	0.0	0.0	0.0	500.0	625.18
4	64.0	40.0	38.0	37.0	36.0	35.0	33.0	30.0	29.0	29.0	28.0	27.0	27.0	23.0	0.0	0.0	476.0	626.75

5.5 Summary

In order to minimize the maximum expected makespan on parallel machines, the expected makespan for a set of jobs on a single machine was studied and some of its properties were illustrated. Two heuristics methods were proposed for scheduling the production and preventive maintenance plan simultaneously. The first heuristics (H_1) minimizes the penalty value by choosing one of two proposed segments (G and H) where each segment containing one or more jobs according to the best time to perform the preventive maintenance ($\dot{\tau}$). The second heuristics (H_2), optimizes the proposed segments depending on the properties of the optimal solution of the preemptive problem. The proposed heuristics were examined using numerical problems with different sizes (16, 30 and 50 jobs) and the results shows a good performance comparing with the preemptive solution (lower bound) for both of them with simple preference for the H_2, see Table 5.14. H_1 does not need many calculations and it is very simple for use. H_2 most probably will produce better solution especially when the age of the machine at the beginning is more than zero.

Table 5.14: Performance comparison results for H_1 and H_2

Problem Size	H_1	H_2	$H_1 = H_2$
n = 16	30%	60%	10%
n = 30	50%	50%	---
n = 50	40%	50%	10%
Total (16, 30, 50)	40%	53.33%	6.67%

Thereafter, two methods were proposed to minimizing the expected maximum makespan on parallel machines which have different maintenance parameters. A branch and bound method for the exact solution is proposed in addition to a heuristic method based on a first fit algorithm for the large size problems. Both of the methods depend on the properties of the optimal solution of the preemptive problem.

Chapter 6

CONCLUSION AND RECOMMENDATIONS

6.1 Introduction

The system reliability is a key factor for each of the continuity of the production process according to its time table, the product quality and the on time delivery. Preventive maintenance activities consume some of the production time but it is improving the system reliability by reducing the failures. Therefore, the cooperation between them by coordinating their decisions will lead to increasing the system reliability, reducing the system unavailability time and increase of the production capacity. In this study, this problem has been considered for a single machine and muti-machine in parallel where the system reliability is monitoring according to the non-homogeneous Poisson process that has the Weibull rate function in their failure rate function. The preventive maintenance is perfect and restore the machine to as good as new condition while the repair due to sudden failures is minimal and just keep the machine in functional conditions (as bad as old). The problem has been studied for three performance measures, total expected completion time and minimum makespan for a single machine and minimizing the expected maximum makespan for parallel machines.

6.2 The Minimization of the Expected Total Completion Time on a Single Machine

In the case of a single machine with minimizing the expected total completion time, the problem is reduced from $n! \times 2^n$ cases, where $n!$ is the number of possible jobs sequences and 2^n is the number of possible preventive maintenance scenarios, to 2^n by proving that the global optimal solution of the problem has the jobs in SPT order. Moreover, depending on the convexity of the failure rate function and some other proven lemmas, a solution procedure to determining the optimal preventive maintenance scenario is provided. Accordingly, the problem is reduced to n iterations where n is the number of jobs.

Subsequently, the effect of the maintenance parameters on the decisions of the model has been investigated. The results show that in additional to their effect on the performance measure value they have an effect on the preventive maintenance decisions as well. The changes in the maintenance parameters may change the number of preventive maintenance times during the time horizon or changing the plan (preventive maintenance scenario) or both.

The integration of the production schedule and preventive maintenance planning leads to some saving in the performance measure value (expected total completion time) compared to the scheduling of them separately. This value depends on the problem size and the value of the jobs (length of the jobs processing times) where the jobs with long processing times causes more penalty due to delaying the maintenance times (the discussed case is non-preemptive) in the separate solution. The integrating solution may decrease this penalty by performing the preventive maintenance early. Also, if the maintenance parameters together produce a working environment with a high value of the failure function then, the integrating solution will have more saving in comparison with the separate solution.

6.3 Single Machine and Expected Makespan

To optimize the expected makespan on parallel machines, the expected makespan on a single machine should be investigated first. In the deterministic case of a single machine the makespan is the sum of the jobs' processing times independently from the sequence of the jobs. Unfortunately, this is not the case for the probabilistic case, where the expected makespan effected by both of the proposed preventive maintenance plan and the job sequence. Therefore, the possible solutions for the problem that should be investigated are $n! \times 2^n$. In this work, the optimal solution can be defined using the enumerative method. In spite of the problem difficulty, two heuristics have been provided to determine a near optimal solution or optimal solution in some cases. The first heuristics (H_1), is fast and provided the solution with less calculations effort than the second heuristics (H_2). H_1 considers the local optimal solution as the global optimal regardless the remaining unscheduled jobs. The solution provided by H_2 is most probably better than the H_1 solution when the machine age at the beginning of the time horizon is greater than zero, but it needs more calculations effort. H_2 cannot guarantee the optimality of its solution because it depends on the jobs combinations (segments) produced by the statements in G and H which may neglect some better solutions. A good saving in the expected makespan value can be obtained from integrating the production schedule and preventive maintenance plan on a single machine as shown in the numerical results and this saving could be more depending on the size of the problem and maintenance parameters.

6.4 Multi-Machine in Parallel and Maximum Expected Makespan

The problem is to assign *n* jobs on *m* machines where each job can be processed on any one of the available machines in order to minimizing the maximum expected makespan and keeping the difference between maximum and minimum makespan minimum as possible. In other words, it is the problem of distributing the given load on the available machines given that each machine must be used and the difference between the expected makespan on those machines is minimal. The maintenance parameters of each machine (β, η, t_m, t_r and a_0) may have different values. The problem is more complicated than its deterministic version when n is greater than $2m$. Otherwise, the problem can be solved optimally by LPT rule (for deterministic case).

First, the problem is assumed to be preemptive problem and the total load that can be assigned on each machine is obtained from the optimal preemptive solution. Second, the problem is solved as non-preemptive problem based on the properties of the optimal solution in the preemptive case. Exact and approximation methods are proposed to solve the non-preemptive problem and both of them are depending on the preemptive case solution.

The branch and bound method is the exact method provided to solve the problem optimally. All the possible assignment scenarios can be generated and the unpromising branches can be eliminated early. At the beginning, nodes represent infeasible solution where there are some jobs assigned as non-preemptive jobs and the rest of the load of the machine is preemptive. At each node the preemptive part of the load of the machines will be adjusted according to the Newton's method. The feasible solution is reached if the entire load at the node is non-preemptive and it can be optimal or gives a new upper bound. This method is hard in case of $m > 2$.

An approximation method based on the first fit algorithm is introduced also to determine a good solution which can be optimal. This method is suitable for the large size problems regardless the number of available machines. It depends on the properties of the optimal solution of the preemptive case which define the load on each machine. Therefore, using the first fit method, the longest jobs are assigned to the less malfunction machine if it is possible according to machine defined load. The solution provided by the first fit algorithm can be improved for a better solution if it is required.

Considering these two decisions separately (production and maintenance) will lead to balancing the total load (total jobs processing time) on the *m* available machines. Therefore, the load on the machines will increase the value of the expected maximum makespan and the difference between the maximum and minimum makespan values when the maintenance parameters of the machines are not equal.

Finally, it is clear that the integration of these two decisions gives an important saving in the completion times (see the numerical results for the illustrative example in 5.4.5).

6.5 Recommendations

The schedules of jobs and preventive maintenance are considered simultaneously, in spite of the difficulties caused by random machine failures. We still insist to keeping these considerations and improving the methods that aim to solve these models.

The performance measure or the second performance measure in the case of a bi-objective function is the maintenance cost.

By using dynamic programming an effective solution methods may be elaborated for a single machine as well.

The availability of the maintenance resources constraints in case of parallel machine may be investigated.

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