Black Hole Evaporation

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ABSTRACT

The humankind perceives reality as the set of events of which are detectable by human senses. Although it is not possible to possess a full knowledge of everything, the human brain is capable of predicting the hidden constituents of the universe by performing algorithms on observations. Furthermore, the falsifiability of theoretical frameworks contributes to the flame of enthusiasm experienced by theorists. From this standpoint, this thesis focuses on the interconnection of gravitational phenomena and Planckian systems. The study examines the quantum nature of black holes, as well as the hypothetical astronomical objects commonly referred to as black branes or black strings within the context of Hawking radiation, which is the so-called *black hole radiation*. Since the background geometry is curved, it gives rise to an effective potential, which in turn results in a scattering process. Under this perspective, as an example, the linear stability of a $(2+1)$-dimensional Mandal-Sengupta-Wadia black hole is studied against small time-dependent perturbations. Subsequently, a $(4+1)$-dimensional dilatonic black string is considered and it is shown that there exists a resemblance between tachyonic particles and the fifth dimension, as the greybody factor evaluations only allowed for imaginary masses to be present.

As the final step, a particular $(3+1)$-dimensional curved spacetime that might lead to experimental studies is considered: $z = 2$ Lifshitz-like black brane (which is also counted as a black hole) with hyperscaling violation. To analyze its radiation, we first tackle the problem with the tools of general relativity and derive its complete analytical blackbody radiation. Then, a particular holographic model is studied with the purpose of deriving its analytical dissipative properties: $\eta \propto T^{3/2}$, $\sigma_{DC} \propto T^{3/2}$, and $\rho_{DC} \propto T^{-3/2}$ which are the shear viscosity, the DC-conductivity, and the

DC-resistivity, respectively. The aforementioned observables are achieved via the fluid/gravity correspondence, built upon the two-point correlation function $G_{O_+}(\omega,0) = -i\omega\left(r_+^4+\omega^2\right)/3r_+$. The metric dynamic critical exponent is originally chosen as $z=2$ in order for supporting superconducting fluctuations. However, this choice has also determined the characteristics of the dual model living on the three-dimensional boundary: a strongly-coupled, non-relativistic fluid exhibiting Lifshitz-like symmetry. Any possible confirmation of the theoretically-obtained dissipative parameters would act as a supplementary empirical evidence for the quantum properties of spacetime.

Keywords: Hawking Radiation, Greybody Factor, Decay Rate, Absorption, Evaporation, Fluid/Gravity Correspondence, Hyperscaling Violation, Strongly-Coupled Fluid.

ÖZ

İnsan beyni gerçekliği, duyularla tanımlanabilen olaylar zinciri olarak algılamaya eğimlidir. Evrendeki her şeyi tam anlamıyla bilmek mümkün olmasa da insan beyni yapılan gözlemler ile algoritmalar oluşturarak, evrendeki gizli bileşenleri tahmin etme yeteneğine sahiptir. Bunun yanında, teorik temellerin yanlışlanabilirliği, tüm teorik fizikçilerin deneyimlerinin ardındaki heyecana katkıda bulunmaktadır. Bu noktadan hareketle; bu tezde, kütleçekimsel etkileşimler ile Planck sistemlerinin bağlantılarına odaklanılmıştır. Bu çalışmada; kara delik, kara zar ve kara sicimlerin kuantum doğası, *kara delik ışınımı* olarak da bilinen Hawking radyasyonu başlığı altında ele alınmıştır. Arka plan geometrisinin eğriliğinden dolayı oluşan etkin potansiyel sonucunda, saçılma olayları meydana gelmektedir. Bu bilgiler ışığında; örnek olarak, $(2+1)$-boyutlu Mandal-Sengupta-Wadia kara deliğinin stabilitesi, zamana bağlı pertürbasyonlar aracılığıyla incelenmiştir. Daha sonra, $(4+1)$-boyutlu dilatonik kara sicim ele alınarak, takyonik tanecikler ile olan ilişkisi bulunmuştur.

Son olarak, gerçekçi $(3+1)$-boyutlu bir model olan ve deneysel çalışmalara da katkıda bulunabilecek yüksek ölçek ihlalli ve Lifshitz benzeri bir kara zar (aynı zamanda kara delik olarak da nitelendirilebilir) incelenmiştir. Öncelikle bu modelin radyasyonunu analiz edebilmek için genel görelilik prensipleri kullanılmış ve kara cisim ışıması ile ilgili parametrelerin tümü analitik olarak bulunmuştur. Daha sonra holografik modelin dağıtıcı özellikleri olan akışkanlık, DC-iletkenlik ve DC-direnç; $\eta \propto T^{3/2}$, $\sigma_{DC} \propto T^{3/2}$ ve $\rho_{DC} \propto T^{-3/2}$ olarak bulunmuştur. Teorik yöntemlerle hesaplanan bu gözlemsel parametreler, iki-nokta korelasyon fonksiyonu $G_{O_+}(\omega,0) = -i\omega\left(r_+^4+\omega^2\right)/3r_+$ kullanılarak, akışkan/kütleçekimi ilişkisi ile elde edilmiştir. Metrik dinamik kritik katsayısı, süperiletken etkileşimleri ele almak için

spesifik olarak $z = 2$ olarak seçilmiştir. Ancak bu tercih aynı zamanda üç boyutlu sınırda var olan dual modelin karakteristik özelliklerini inşa etmiştir; ki bu sınırlar da kuvvetli bağlı ve relativistik olmayan Lifshitz simetrisine sahip akışkanları içermektedir. Bu tezde teorik hesaplara dayanarak elde edilen parametereler gözlemlenildiği takdirde, uzay zamanın kuantum mekaniksel özelliklerinin de deneysel kanıtları desteklenmiş olacaktır.

Anahtar Kelimeler: Hawking Radyasyonu, Gri Cisim Faktörü, Bozunma Katsayısı, Emilim, Buharlaşma, Akışkan/Genelçekim İlişkisi, Yüksek Ölçek İhlali, Kuvvetli Bağlı Akışkan.

Dedicated to my grandmother Huriye Hüdaoğlu and my grandfather Ahmet Gürsel

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$\vec{\kappa}$	wave vector
$\dot{}$	derivative with respect to time-coordinate
$'$	derivative with respect to radial-coordinate
$\Box$	d'Alembertian
η	shear viscosity
Γ	decay rate
γ	greybody factor
σ_{abs}	absorption-cross section
σ_{DC}	DC-conductivity
ρ_{DC}	DC-resistivity
κ_s	surface gravity
(z, θ)	dynamic and hyperscaling violating exponents
d	total number of spatial coordinates
D	total number of spacetime coordinates
r_*	tortoise coordinate
r_+	outer event horizon
r_-	inner event horizon
s	entropy density
T_C	critical temperature
T_H	Hawking temperature

Chapter 1

INTRODUCTION

"The theory of gravitation deals with phenomena on a cosmic scale, whereas Yang Mills theory is concerned with the opposite end - the smallest scale conceivable. Someday the two will meet, when we come to grips with what is inside that perceived singularity we call black hole." - Kerson Huang

1.1 Thesis Framework

This thesis does not only examine the theoretical aspects of the scattering phenomena of black holes/branes, but also inspects their dual models, which are regarded as living on the boundary of the corresponding bulk model. Although there exist many studies discussing the holography between gravitational and quantum field theory models, few have actually focused on exact analytical methods for maintaining the observables of each scenario. Drawing upon theories of general relativity and holographic principle, this study provides information regarding perturbations of a $(2+1)$-dimensional Mandal-Sengupta-Wadia black hole, tachyonic evaporation of a $(4+1)$-dimensional dilatonic black string, wave dynamics of a $(3+1)$-dimensional black brane with hyperscaling violation and its dual observables living on the boundary. The dual system is analytically evaluated to hold a rather small shear viscosity to entropy density ratio, which suggests that the system under consideration is highly likely to be corresponding to a strongly-coupled, non-relativistic fluid in $(2+1)$ dimensions. The structure of the thesis goes as follows: Within this introduction, firstly the motivation behind the research will be provided. In what follows, some general information regarding certain gravitational phenomena will be explained very briefly. Chapter 2 emphasises the effect of small perturbations on

Mandal-Sengupta-Wadia black holes, whereas Chapter 3 covers the evaporation process of a $(4+1)$-dimensional black string. Generally speaking; any black hole, black brane or black string (let me call them continuum objects for simplicity) display dissipative properties, once they are disturbed from outside. This process is commonly referred to as the membrane paradigm [1, 2]. The membrane paradigm is suggested to be applied to black branes rather than black holes, as there exists no translational invariance within the horizons of black holes [3]. This is the reason why Lifshitz-like black branes are chosen in Chapter 4.

The membrane paradigm tends to reflect issues that are connected to the relationship between a D-dimensional astronomical object and the $(D-1)$-dimensional theory living on the boundary. A detailed study on the evaporation of continuum objects has directed us towards the following question: Can we possibly identify observational evidence for the traces of one physical law valid at some specific scale in another, which naively appears to be completely irrelevant? Consistent with previous research carried out by scientists expertised in different areas, this question can be answered with the aid of the fluid/gravity correspondence [4], which is a rather specific version of the holographic principle. During the 1990s, the ideas mainly constructed by Charles Thorn [5], Gerard ‘t Hooft [6], and Leonard Susskind [7] merged together beautifully and gave birth to the insight known as the ‘holographic principle’, which laid the foundations of finding an appropriate answer for the question of our concern. With the purpose of searching for a fulfilling answer, we have written two papers [8, 9] on hyperscaling violating Lifshitz-like black branes with $z = 2$ dynamic exponent. The first article examines the bulk properties of the chosen model, whereas in the second one, the holographic approach is adopted. Chapter 4 can be considered as a combination of these two studies and it is noteworthy to stress that these papers made it clear that there indeed exists a direct relation between general relativity and quantum mechanics.

Note that in this work, the natural units are considered, i.e. $c = G = k_B = \hbar = 1$.

1.2 Motivation

For centuries, humankind has been driven by the genuine curiosity about the logical explanations and reasonings of the phenomena occurring in the universe that we live in, and today, it still remains as an ongoing motivation behind an immense amount of discoveries made possible. The endeavours of having a complete understanding of the way the universe functions require establishing individual laws valid for different energy (or distance) regimes, which would in turn characterise the degrees of freedom of any arbitrary system from Planck scale to cosmic scale. Have you ever wondered what the word "reality" actually corresponds to? Is reality limited by human perception? If one imagines a hypothetical observer deprived of human perspicacity, would reality change its self-representation, or would the observer's conception narrow down the entire picture? Treating the Big Bang as the initial cause behind everything existing in today's universe, one could argue that laws belonging to different scales need to have a common root, albeit seeming to be completely different at first glance. Thus, it is highly probable for a unique and consistent theory of nature to be subsisting behind the scenes, which reduces to appropriate branches for the cases when specific constraints are applied on the systems and observers of concern. For a long time, physicists have been trying to construct this unified theory and quantum gravity, which retains its popularity up to today, is regarded as the best candidate proposed so far.

Very often, we find ourselves wondering how time can come to an end or how it was created at the first place. Once the creation of the universe as we know it is tried to be visualised, it is tempting to think of time as being the main focusing point. Nevertheless, time in a sense is being created or destroyed within the dense astronomical black structures. Our current knowledge addresses that once the black hole interior is of concern, time and space lose their characteristics. This perspective combined with the singularity point would lead one to conclude that rather than thinking of time as the main concept, one could rather concentrate on the combination of quantum and classical laws in the vicinity of black holes. Furthermore, string theory suggests that space and time themselves may emerge in the theory at large distances.

To be able to have a grisp on quantum gravity, one shall perhaps ask herself/himself of the mapping between quantum systems and classical models. Would it be misleading to treat our every day experiences as quantum mechanical phenomena under specific constraints? John Wheeler had long been possessing curiosity regarding the correspondence principle proposed by Niels Bohr in 1913. He had a strong feeling that studying semi-classical (or alternatively semi-quantum) analysis of scattering processes could exploit the question marks in his mind, which were mainly about the transition from quantum laws to classical laws [10]. The great majority of natural processes observable in our daily lives can be explained via laws of classical mechanics and special theory of relativity. Considering that our minds are wired in such a way to make sense of the phenomena we do experience in our daily lives, one can suggest that our basic instincts make it tempting to think in a Newtonian or Galilean perspective. However, it is beyond doubt that the complete picture needs to be way more complicated than the ones that human mind can grasp.

In 1905, the proposal of Einstein's postulates led to a revolutionary era where it would seem that we live in a region of spacetime in which everything we see, experience and think to be universal is only an approximation to all that has been happening through the entire universe. Just like the Newtonian and Galilean perspectives were shown to be approximations to general relativity, it would not be absurd to think that general relativity itself can be thought as an approximation as well. What we do observe may not always reflect what actually is taking place in the entire picture. Considering that we, as the observers, can be treated as the low energy limit of the overall picture, it would be of no surprise to view our perspectives as being equivalent to an approximation of the actual phenomena. To be more precise, it is beyond doubt that Pythagorean theorem correctly enables one to express the shortest distance between two points as a function of the infinitesimal coordinate displacements of concern, once the spacetime is flat. Nonetheless, when one wishes to conceptualise the entire scenario, in other words when one does not impose any constraints on the curvature of spacetime, the Pythagorean theorem can no longer be viewed as the correct equation relating the points to each other. In this case, the metric tensor $g_{\mu\nu}$ is added to the theory and any spacetime geometry can be described via

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu. \tag{1.1}$$

Equation (1.1) can be thought as the modified Pythagorean theorem, which guides us through the right direction. This is only one example stressing that in order to be equipped with a general interpretation for the laws of the universe, one should first make sure that s/he eliminates the limitations on the theory. Approaching the problem from this viewpoint enables one to see that the beauty and consistency of general relativity is not adequate for referring it to as *the complete theory of gravitation*, as there are constraints within the theory. Surely, this does not imply that Einstein's theory of general relativity is an inconsistent theory; in contrary, it is outstandingly successful when phenomena such as deflection of starlight, the perihelion shift and time dilation in a gravitational field are considered. However, it encounters problems when small scales are of interest; whence it seems to be valid under some constraints only. Thus, it would certainly be beneficial if one gets rid of the constraint on the *scale* and attempts to modify general relativity in such a way that it would automatically include laws of quantum mechanics as well.

A theory relating the astronomical scales to Planckian scales would also be enlightening for understanding the mechanism behind the atomic nuclei. In 1931, Dirac suggested that constructing a complete theory regarding the spooky behaviour of atomic nuclei would be a burdensome task, since it would force us to revise our fundamental understanding of nature. Dirac also stated that constructing a theory directly from the observations would exceed the human intelligence. Hence, he suggested the future theorists to search indirect ways of approaching the problem [11]. Dirac's perspective is supported by the studies conducted on holographic principle: As the low energy behaviour of strongly coupled systems seems to cause problems when attempted to be approached by quantum chromodynamics, they are studied from a different perspective, which is commonly referred to as the anti-de-Sitter/conformal field theory (AdS/CFT) correspondence [12, 13]. Recently, many physicists are working on AdS/CFT correspondence proposed by Maldacena [14], with the purpose of applying principles of holography to strongly coupled systems. The dissipative properties of the horizons of astronomical objects are investigated via membrane paradigm [1, 3] and the resemblance between laws of hydrodynamics and general relativity are compared.

Furthermore, chiral symmetry breaking and quark confinement are unresolved phenomena in low energy quantum chromodynamics, which await a consistent explanation. It is highly probable for the holographic principle to hold the answers for the ambiguous behaviour within these processes.

Currently, scientists are searching for ways of combining laws of high energy phenomena occurring at small scales and concepts of general relativity under the title quantum gravity and a construction, known as string theory, enable one to combine all these perspectives together. The physical mechanism behind interactions at short distances requires further examination, as there still exist open questions such as quark confinement and chiral symmetry breaking. In this era, quantum gravity, which still continues to challenge physicists, dominates the interactions.

The holographic principle which plays a vital role in gluing all the pieces together and maintaining the most basic perspective. This would imply that there needs to be a convincing scientific theory capable of describing phenomena occurring at the smallest scales of large objects moving very close to speed of light.

1.3 Discovery of (2+1)-Dimensional Black Holes

$(2+1)$-dimensional theories of general relativity play a requisite role in constructing a relatively simpler frame of mind for conceiving $(3+1)$-dimensional gravity. Emerging mainly during the 1980s, theorists had been attempting to assemble a $(2+1)$-dimensional theory of gravity with this purpose in mind; however, there seemed to exist an inevitable challenge along the way: $(2+1)$-dimensional spacetimes did not welcome black hole solutions [15]. In 1992; Banados, Teitelboim and Zerilli (BTZ) managed to overcome this obstacle by proposing a rotating black hole solution characterised by mass M, charge Q, and angular momentum J, as can be seen from the following metric [16]

$$ds^2 = -N^2 dt^2 + N^{-2} dr^2 + r^2 (N^{\phi} dt + d\phi)^2, \tag{1.2}$$

with $N^2 = -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}$ and $N^{\phi}(r) = -\frac{J}{2r^2}$. Note that l represents the radius, which is directly linked to the negative cosmological constant via $l^{-2} = -\Lambda$. Solution (1.2) is now widely known as BTZ black holes in $(2+1)$-dimensions. BTZ black holes can be treated as the starting point for $(2+1)$-dimensional gravity theories. Furthermore, it is also referred to as low energy string theory solution [17]. Low-dimensional gravity theories have been extensively studied by physicists such as Witten [18, 19], Rocek [20], Bardakçı et al [21], Chan and Mann [15], and many more. As desired, these studies are now considered as valuable sources for studying conceptual aspects of lower dimensional gravity.

1.4 Remarks on Scattering and Hawking Radiation

The scattering phenomena for atoms heavier than hydrogen atom dates back to 1933, when John Wheeler published his first solo-authored article entitled with "Theory of Dispersion and Absorption of Helium" [22]. This paper can be considered as the first study that reveals the connection between absorption and scattering [10]. To be equipped with a well-founded insight, it would be beneficial to first approach the problem from an ordinary, classical scenario. Assume that a system with mass M labelled as 'the target' is hit by another system with mass m and velocity $\vec{v}$. Let say that for simplicity, the target is treated as stationary. The incident system is said to be scattered if it keeps on moving by experiencing a change in momentum, i.e.

$$\exists \vec{P'} : \ for \, \forall \, \vec{P} \, \exists \, \Delta\vec{P} = \vec{P'} - \vec{P} \, \wedge \, \Delta\vec{P} \neq 0. \tag{1.3}$$

In other words, there needs to exist a final momentum $\vec{P'}$ such that for all initial momentum $\vec{P}$ values, this difference is non-zero, under the condition that the incident system keeps on moving. Now, one can relate the scattering process to the absorption probability by continuing the aforementioned thought experiment. Imagine the target is replaced with another system having a high absorption coefficient, such as an object made up of sponge. Then, it would be trivial to see that the absorption and scattering probabilities need to be related. If the incident system gets fully absorbed, then the observer would not detect any scattering. The exact opposite would be true as well: If no absorption takes place, there would be a 100% probability for the process of scattering. Therefore, one can conclude that gathering information on scattering amplitudes provides findings about the constituents and properties of the target system.

Adapting this thought experiment to the theory of astronomical black objects, one needs to get rid of the classical limit and introduce quantum fluctuations. The scenario is adapted as follows: Suppose that there exists a black hole with mass M with a strong gravitational influence due to its intense density. According to Hawking [23], a black hole radiates some energy off its surface. Hawking radiation can be thought as an outcome of quantum fluctuations in the vicinity of ultra-dense black objects such as black holes. It occurs as a consequence of the strong gravitational effects around the event horizon. A collection of particle-antiparticle pairs are produced continuously within the quantum foam by the boundary of the ultra-dense black object; and subsequently, one of the particles gets sucked by the black object, whereas the other escapes to infinity. Mathematically, this finite temperature can be defined as

$$T_H = \frac{\kappa_s}{2\pi}. \tag{1.4}$$

The radiation can technically be measured, if the black object is small enough. In fact, experimental attempts are being made for testing the theory of Hawking radiation in laboratory analogues of black holes [24]. But how about the full spectrum that a hypothetical observer would see? The classically black but quantum mechanically radiating objects are able to absorb only a portion of the waves approaching them from distant regions of spacetime. Likewise, they seem to be unable to emit the entire thermal radiation produced off their surface all the way to a hypothetical observer situated infinitely far away. Nevertheless, when the event horizon is of concern, it appears that the spectrum they emit is Planckian. The prime cause behind this paradox seems to be the fields in the background, including the spacetime itself. Since the background geometry possesses curvature due to the presence of the ultra-dense object and fields of concern, the curvature can be treated as a gravitational potential, which in turn enables a scattering process to take place. Hence, if one desires to comprehend the quantum properties of spacetime and its puzzling genesis, scattering phenomena within these regions would be a good point to focus on, as empirical evidence of any kind would also provide information regarding the quantum fluctuations giving rise to Hawking's elegant framework.

Thus, in brief, a hypothetical observer at spatial infinity would only be able to detect the scattered portion of the original Hawking radiation produced by the black hole. Consequently, the radiation quantities (greybody factor, absorption cross-section, and decay rate) give information about the target object; whence *black hole evaporation* is a key subject for understanding the linkage between quantum and classical mechanical laws. In other words, *black hole evaporation* opens the way for approaching a classical object with the tools of quantum mechanics. There exists a distinction in terms of their relevant scales, and yet quantum mechanical laws can be used to describe the phenomena considering them both. Moreover, it enables one to think delicately about the commonly heard phrase “nothing can escape from black holes”.

Admittedly, it is highly probable for the *black hole evaporation* to contain information regarding the unsolved remedies in many different areas of physics. For instance, in Ref. [25], it is shown that there exists a mapping between the superconducting phase in superconductors and a black hole close to its final state.

1.5 Aspects of Lifshitz-like Black Branes

In 1974, ’t Hooft made a proposition that for any gauge theory, there exists a dual string model in the large N limit [26]. Thus, the Lifshitz-like black brane of our concern may be linked not only to the $(2+1)$-dimensional field theory model that it corresponds to, but also to the associated one-spatial dimensional dual string theory. At this point, we shall emphasise that both the observational data and the theoretical framework of the Veneziano model [27] indicate the likely possibility of having an underlying string structure to hadronic matter [28]. Once these relations are investigated, one cannot go without noticing the relevance of the theory of magnetic monopoles and the hadronic matter. In 1974, Mandelstam [29] proposed a model in which the experimentally required confinement condition was satisfied. During his work, he combined the Nielsen-Olesen interpretation of ST [30] and Nambu’s idea [31] of treating quarks as the carriers of magnetic charges.

Dirac proposed that if one can figure out why electrons and protons exhibit different properties, s/he would automatically recognise the reason behind the differences between electricity and magnetism [11]. The non-Abelian massless monopoles in low energy effective action of supersymmetric theories seem to possess an active role in low energy behavior of quantum chromodynamics. Furthermore, the ground state of quantum chromodynamics can be treated as a dual superconductor [28–30, 32]. Therefore, in this thesis, it is suggested that the transport coefficients of the Lifshitz-like black brane of our concern is highly likely carrying information about the magnetic monopoles, superconductors and low energy behavior of quantum chromodynamics.

The dynamic scaling exponent of the model is chosen to be $z = 2$ so as to support superconducting fluctuations [33]. Furthermore, theories with $z = 2$ scaling describe multicritical points in certain liquid crystals and have been shown to arise at quantum critical points in toy models of the cuprate superconductors [34]. On the other hand, the spatial dimension of our bulk spacetime is adopted as $d = 3$. The reason behind this specific choice is to shift the perception of a three-spatial dimensional reality created by our minds (as a direct consequence of observing the macroscopic world) to a $(2+1)$-dimensional holographic scenario in which the two cases exhibit common properties. In Ref. [6], 't Hooft claims that to be able to construct a consistent quantum gravity model, the observable degrees of freedom should be described as if they were Boolean variables defined on a two-dimensional lattice, which also coincides with our specific choice of dimensionality. In Refs. [35–38] these phenomena are explained via numerical simulations; however, there seems to be a gap in literature for exact analytical approaches, as the usual perturbative methods are not applicable in this regime.

In fluid/gravity correspondence, the dynamic critical exponent z and the hyperscaling violating factor θ play a vital role in both characterizing the properties of the bulk model and determining the scaling behavior of the observables in the dual scenario. On the gravitational side, besides being subject to an overall hyperscaling violation factor, the metric also encounters a temporal anisotropy due to quantum critical phenomena. Such Lifshitz-like spacetimes correspond to the dual models, which experience continuous phase transitions [39].

Additionally, it would be beneficial to stress that in Ref. [40], an exact solution for the Mott problem has been maintained, and moreover, it is shown that black holes are good candidates for revealing information regarding the superconductivity. In Ref. [41], an interesting solid state approach to black hole thermodynamics is maintained. In particular, the framework of the proposals made in Ref. [41] suggests a mapping between quantum physics of black holes and thermodynamic properties of superconductors. The duality between the two perspectives can be tabulated as follows:

Table 1.1: The mapping between black hole and solid state physics cases.

Black hole case	**Superconducting case**
Speed of light	Fermi velocity
Black hole temperature	T_C of superconducting condensate
Event horizon	Metal-superconductor interface
Schwarzschild radius	Coherence length
Quantum state of a black hole	Bardeen, Cooper and Schrieffer (BCS)
Black hole evaporation	Andreev reflection processes
Hayden and Preskill's information mirror	Entanglement swapping
Traversable Einstein-Rosen bridge	Crossed Andreev reflections

As can be seen from Table 1.1, the black hole temperature of a concerned bulk-gravitational model has a corresponding dual analogy: the critical temperature of the superconducting condensate in a particular condensed matter system. The desire of interpreting the relationship between some particular gravitational structures and condensed matter systems such as superconductors has resulted in the greybody factor evaluation for $(3+1)$-dimensional non-Abelian charged Lifshitz-like black branes with $z = 2$ hyperscaling violation. Moreover, $z = 2$ models correlate gauge/string theory correspondence and quantum mechanical systems in condensed matter physics. The similarities between two phenomena are presented elaborately in [42].

Chapter 2

STABILITY OF MANDAL-SENGUPTA-WADIA BLACK HOLES

2.1 Preliminary Remarks

A black hole can retain its name as long as it preserves its stability, just like any other physical object nominated for a specific name by the humankind. The concept of 'black hole stability' was first addressed by Regge and Wheeler [43], followed by Zerilli, [44]; and since then, a plentiful amount of studies have been conducted on this issue, some of which can be found in Refs. [45–55]. Regge and Wheeler pursued the problem from a pedagogical point of view, which is of no surprise, as Wheeler has a reputation for his elegant and yet 'easy to grasp' explanations regarding advanced topics. The intense curiosity driven by Wheeler and Regge enabled them to visualise a Schwarzschild black hole like a sphere of water held together by gravitational forces. They viewed the black hole from this perspective so as to be able to possess a solid understanding by having a comparison with a familiar concept from our daily lives. Equipped with this notion, they aimed to achieve a basic intuition on the possible scenarios that could take place provided that the black hole were subject to a small perturbation. Their interpretation goes as follows: Assume that a system in equilibrium is disturbed by an external effect via being subject to a small perturbation. If the initially small disturbance happens to grow exponentially in time, the system is said to be 'unstable', whereas for the stability to be maintained, it needs to be only oscillating around the equilibrium [43]. Keeping this intuition in mind, this chapter [1] will be concerned with the effect of small spacetime-dependent perturbations acting on a $(2+1)$-dimensional electrically charged Mandal-Sengupta-Wadia black hole. There exist three subsections within this chapter: In Sec. 2.2, the black hole structure will be mentioned briefly, whereas Sec. 2.3 is reserved for the stability analysis of the concerned model. And finally, the results are summarised throughout the last section; Sec 2.4.

2.2 Geometrical Structure

Before one starts discussing the black hole structure of interest, it would be beneficial to introduce the action that includes information regarding the dynamics of the system. As already stated throughout the introduction, the Einstein-Maxwell-Dilaton action has many implications in string theory. However, in this chapter, the subject will be examined from a relativist's perspective only. For applications of this action in string theory, one may refer to [57–60] and the references therein.

The Einstein-Maxwell-Dilaton action in $(2+1)$-dimensions can be expressed as [15]

$$S_{EMD} = \int d^3x\sqrt{-g}\left[R - \frac{B}{2}\left(\nabla\phi\right)^2 - \exp\left(-4a\phi\right)F_{\mu\nu}F^{\mu\nu} + 2\exp\left(b\phi\right)\Lambda\right]. \tag{2.1}$$

In this action, Λ is the cosmological constant, ϕ and $F_{\mu\nu}$ stand for the dilaton and Maxwell fields, respectively. Furthermore, a, b and B are dimensionless constants where a and b represent the coupling of dilaton with the Maxwell field and the cosmological constant, respectively [15]. These constants play a key role in determining the black hole structure. Applying variations in the metric, gauge and

[1] This chapter is based upon the article entitled "Linear Stability of Mandal-Sengupta-Wadia Black Holes" [56].

dilaton leads to

$$R_{\mu\nu} = \frac{B}{2}\nabla_{\mu}\phi\nabla_{\nu}\phi + \exp\left(-4a\phi\right)\left(-g_{\mu\nu}F^2 + 2F_{\mu}^{\alpha}F_{\nu\alpha}\right) - 2g_{\mu\nu}\exp\left(b\phi\right)\Lambda, \tag{2.2}$$

$$\nabla^{\mu}\left(\exp\left(-4a\phi\right)F_{\mu\nu}\right) = 0, \tag{2.3}$$

and

$$\frac{B}{2}\left(\nabla^{\mu}\nabla_{\mu}\phi\right) + 2a\exp\left(-4a\phi\right)F^2 + b\exp\left(b\phi\right)\Lambda = 0. \tag{2.4}$$

The Einstein-Maxwell-Dilaton action provides a solution which corresponds to charged static dilaton black holes. These black holes are allowed to hold magnetic or electric charges, whilst in this chapter, the electric case will be inspected only. Equations (2.2), (2.3), and (2.4) represent the equations of motion of the theory. Since the background geometry of interest is going to be the one under the influence of a charged Mandal-Sengupta-Wadia black hole, one needs to apply the conditions $b = 4a = \frac{B}{2} = 4$ and $\phi_0(r) = -\frac{1}{4}\ln\left(\frac{r}{\beta}\right)$ $\{\beta$:constant$\}$ on the constants found in Eqs. (2.2), (2.3) and (2.4). Note that $\phi_0(r)$ represents the static dilaton field. As desired, these specific choices give rise to [15]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + \beta r d\theta^2, \tag{2.5}$$

with $f(r) = 8\Lambda\beta r - 2m\sqrt{r} + 8Q^2$ and Q represents the electric charge belonging to the electromagnetic vector potential. The metric function $f(r)$ can alternatively be presented in the form

$$f(r) = 8\Lambda\beta\left(\sqrt{r} - r_+\right)\left(\sqrt{r} - r_-\right), \tag{2.6}$$

where r_+ stands for the outer and r_- for the inner horizon. The horizons can compactly be defined as

$$r_{\pm} = \frac{m \pm \sqrt{m^2 - 64\Lambda\beta Q^2}}{8\Lambda\beta}. \tag{2.7}$$

The non-zero Maxwell tensor components are $F_{tr} = -F_{rt} = e^{4\phi}\frac{Q}{\sqrt{\beta r}} = \frac{Q\sqrt{\beta}}{r^{\frac{3}{2}}}$ with $A_\mu = 2Q\sqrt{\frac{\beta}{r}}\delta^t_\mu$. The condition $m \geqslant 8Q\sqrt{\Lambda\beta}$ needs to be satified in order for having a black hole solution. Clearly, if one wishes to examine the case in the absence of electric charge, Eq.(2.7) needs to be subdivided as $r_+ = \frac{m}{4\Lambda\beta}$ and $r_- = 0$.

The general definition [61]

$$T_{H(general)} = \frac{1}{4\pi}\frac{d}{dr}(-g_{tt})\sqrt{-g^{tt}g^{rr}}\Bigg|_{\sqrt{r}=r_+}, \tag{2.8}$$

gives birth to the unique Hawking temperature expression for the charged black hole of our interest which reads

$$T_{H(charged)} = \frac{8\Lambda\beta r_+ - m}{4\pi r_+} = \frac{\sqrt{m^2 - 64\Lambda\beta Q^2}}{4\pi r_+}. \tag{2.9}$$

In order to maintain a real-valued observable temperature, one needs to fulfill the requirement $m \geqslant 8Q\sqrt{\Lambda\beta}$. For the case with no Maxwell field within the model, Eq. (2.9) becomes

$$T_{H(neutral)} = \frac{m}{4\pi r_+} = \frac{\Lambda\beta}{\pi}. \tag{2.10}$$

Note that the Hawking temperature of the uncharged model comes out to be positive iff $\Lambda\beta > 0$ is satisfied. For systems with negative temperature, the reader is referred to [62].

2.3 Linear Stability Analysis

Having discussed the model's geometrical properties and the static solution, one can now move on to the effect of small perturbations on the system. Therefore, this section is reserved for inspecting the linear stability of an electrically charged static Mandal-Sengupta-Wadia black hole in $(2+1)$-dimensions. The perturbed field equations derived from action (2.1) lead to the solution

$$ds^2 = -\exp(2\gamma)dt^2 + \exp(2\alpha)dr^2 + \exp(2\eta)d\theta^2, \tag{2.11}$$

$$F_{tr} = q\exp(\alpha + \gamma - \eta + 4\phi), \tag{2.12}$$

in which γ, α, and η are (t,r)-dependent metric functions and $q = Q$. Going back to Eq. (2.2), one can evaluate the non-vanishing components of the Ricci tensor, which results in

$$R_{tr} = R_{rt} = 4\dot{\phi}\phi', \tag{2.13}$$

and

$$R_{\theta}^{\theta} = -2\left[\Lambda - q^2\exp(-2\eta)\right]\exp(4\phi). \tag{2.14}$$

Once γ, α, η, and ϕ are treated as perturbed versions of the static background fields $\gamma_0(r)$, $\alpha_0(r)$, $\eta_0(r)$, and $\phi_0(r)$, one can express

$$\gamma \equiv \gamma(r,t) = \gamma_0(r) + \delta\gamma, \tag{2.15}$$

$$\alpha \equiv \alpha(r,t) = \alpha_0(r) + \delta\alpha, \tag{2.16}$$

$$\eta \equiv \eta(r,t) = \eta_0(r) + \delta\eta, \tag{2.17}$$

and

$$\phi = \phi(r,t) = \phi_0(r) + \delta\phi. \tag{2.18}$$

The perturbations $\delta\gamma$, $\delta\alpha$, $\delta\eta$, and $\delta\phi$ can be regarded as 'very small', provided that $\delta\gamma =\in \gamma_1(r,t)$, $\delta\alpha =\in \alpha_1(r,t)$, $\delta\eta =\in \eta_1(r,t)$ and $\delta\phi =\in \phi_1(r,t)$ with $\in << 1$. Comparing Eq. (2.5) with Eq. (2.11) and setting $\eta_1(r,t) = 0$ results in $\exp(2\eta) = \beta r$. Then, the non-zero Ricci components (2.13) and (2.14) turn out to be

$$R_{tr} = \dot{\alpha}\eta', \tag{2.19}$$

$$R_\theta^\theta = \left[\eta'(\alpha' - \gamma') - (\eta')^2 - \eta''\right]\exp(-2\alpha), \tag{2.20}$$

and the Klein-Gordon equation (2.4) evolves into

$$\exp(-2\alpha)\left[\phi'' - \phi'(\alpha' - \gamma' - \eta')\right] - \exp(-2\gamma)\left[\ddot{\phi} + \dot{\phi}(\dot{\alpha} - \dot{\gamma})\right] +$$

$$\exp(4\phi)\left[-\Lambda - q^2\exp(-2\eta)\right] = 0. \tag{2.21}$$

From this point onwards, the main task is to linearize both the field and the Klein-Gordon equations. When small perturbations are of concern, the linearized equations will enable one to investigate the effect of the associated disturbance on the geometry, as the first-order terms are dominant. To be more precise, taking Eqs. (2.15-2.18) as well as the gauge $\eta_1(r,t) = 0$ into account and equating Eqs. (2.13) and (2.14) with Eqs. (2.19) and (2.20), we obtain the following set of equations:

$$\dot{\alpha}_1 + 2\dot{\phi}_1 = 0, \tag{2.22}$$

$$4\left(\Lambda\beta r - Q^2\right)(\alpha_1 + 2\phi_1) + r\left(4\Lambda\beta r - m\sqrt{r} + 4Q^2\right)(\alpha_1' - \gamma_1') = 0, \tag{2.23}$$

$$4\left(\Lambda\beta r-Q^2\right)(\alpha_1+2\phi_1)+r\left(4\Lambda\beta r-m\sqrt{r}+4Q^2\right)(\alpha_1^{'}-\gamma_1^{'})+4r^2\phi_1^{''}\left(4\Lambda\beta r-m\sqrt{r}+4Q^2\right)$$

$$-\frac{r^2}{(4\Lambda\beta r-m\sqrt{r}+4Q^2)}\ddot{\phi}_1+4r(6\Lambda\beta r-m\sqrt{r}+2Q^2)\phi_1^{'}=0, \tag{2.24}$$

One may notice that Eq. (2.23) is included within Eq. (2.24). Consequently,

$$\phi_1^{''}+\left[\frac{6\Lambda\beta r-m\sqrt{r}+2Q^2}{r(4\Lambda\beta r-m\sqrt{r}+4Q^2)}\right]\phi_1^{'}-\frac{\ddot{\phi}_1}{(8\Lambda\beta r-2m\sqrt{r}+8Q^2)^2}=0. \tag{2.25}$$

If one applies Fourier transformation with respect to time, s/he obtains

$$\phi_1(r,t)=\phi_1(r)\exp(-ikt), \tag{2.26}$$

with k representing the frequency. Thus, one can rewrite Eq. (2.25) in the form of an effective Klein-Gordon equation

$$\phi_1^{''}(r)+h\phi_1^{'}(r)-j\phi_1(r)=0, \tag{2.27}$$

in which h and j read

$$h=\frac{12\Lambda\beta r-2m\sqrt{r}+4Q^2}{rf}, \tag{2.28}$$

and

$$j=\frac{-k^2}{f^2}. \tag{2.29}$$

As aforementioned, the stability was first introduced into the theory of black holes by Regge and Wheeler and throughout their analysis, they had checked the behaviour of the associated one-dimensional Schrödinger-like equation, with the aid of introducing a new variable which is now known as the tortoise coordinate. By definition, the tortoise coordinate can be found from the metric function via the relation [63]

$$r_* = \int \frac{dr}{f}. \tag{2.30}$$

For the case of our concern, it reduces to

$$r_* = \frac{1}{4\Lambda\beta\,(r_+ - r_-)} \ln\left(\frac{(\sqrt{r} - r_+)^{r_+}}{(\sqrt{r} - r_-)^{r_-}}\right) \tag{2.31}$$

The range $r_+ < \sqrt{r} < \infty$ is analogous to $-\infty < u < \infty$, as $\sqrt{r} \to r_+$ leads to $r_* \to -\infty$. As one gets closer to the black hole, the radial coordinate alters more slowly with the tortoise coordinate due to $\frac{dr}{dr_*} \to 0$. Therefore, the main interest will be the region that satisfies $r > r_+$. In other words, the tortoise coordinate parametrizes the entire region outside the black hole [64]. From Eq. (2.30) it can be seen that for the extremal case where $r_+ = r_-$, the tortoise coordinate can no longer be defined. Subsequently, the effective Klein-Gordon equation (2.27) can be expressed in terms of the tortoise coordinate as

$$\frac{d^2\phi_1(r_*)}{dr_*^2} + X\frac{d\phi_1(r_*)}{dr_*} + k^2\phi_1(r_*) = 0, \tag{2.32}$$

in which

$$X = 4\Lambda\beta - \frac{m}{\sqrt{r}} + \frac{4Q^2}{r}. \tag{2.33}$$

Substituting

$$\phi_1\,(r_*) = \mathcal{R}(r_*) r^{-\frac{1}{4}}, \tag{2.34}$$

in Eq. (2.32) brings about the desired Schrödinger-like one-dimensional wave equation

$$-\frac{d^2\mathcal{R}}{du^2}+\left[V_{eff}(r)-k^2\right]\mathcal{R}(u)=0, \tag{2.35}$$

in which $V_{eff}(r)$ represents the effective potential

$$V_{eff}(r)=\frac{f\Big(f+4(\sqrt{r}m-8Q^2)\Big)}{16r^2}. \tag{2.36}$$

Thus, having evaluated the effective potential, one can now check whether the static Mandal-Sengupta-Wadia black hole is stable or not. The physical insight behind checking the stability goes as follows: A bound state may be present iff V_{eff} is negative. If there exists a bound state, an unstable mode should be presenting as well. Thus, it would be enough to check whether the effective potential admits any negative solutions [64]. Recall that for the solution to be in the form of a black hole, there existed a condition: $m \geqslant 8Q\sqrt{\Lambda\beta}$. Hence, it can be recognised that the effective potential is positive definite, implying that static electrically charged Mandal-Sengupta-Wadia black holes are linearly stable for s-mode perturbations.

2.4 Comments and Discussions

Throughout this chapter, non-rotating and time-independent electrically charged Mandal-Sengupta-Wadia black holes are checked for their stability. In order for achieving so, infinitesimally small (t,r)-perturbations are applied to the black hole, thereby influencing the dilaton and the metric fields. The perturbed equations are then linearized and reduced to one-dimensional Schrodinger-like equation via applying the necessary constraints and introducing the tortoise coordinate. The analysis used is a semi-analytical method which can also be accessed in Refs. [65, 66] and is built upon the Fubini-Sturm theorem [67]. The results obtained in this work came out to be consistent with Ref. [68] where it was stated that Mandal-Sengupta-Wadia black holes are stable under small time-dependent perturbations. In conclusion, by checking the sign of the effective potential under the black hole condition, it was observed that the black hole of concern is linearly stable.

Chapter 3

EVAPORATION OF (4+1)-DIMENSIONAL BLACK STRINGS

3.1 Prologue

This chapter will be covering how particles evaporating off a five-dimensional black string derived from Einstein-Yang-Mills-Born-Infeld-Dilaton action behave, as they propagate through the spacetime of concern [2] . In addition, it will be shown that particles ejected as a consequence of Hawking radiation are compelled to be tachyons, i.e. particles holding imaginary mass values. The reasons behind this limitation will be clarified in the forthcoming sections.

It is of no doubt that our current literature is filled with studies conducted on the wave dynamics of black holes. Yet, the analytical approaches to greybody factors of higher dimensional black strings do not appear to be that many. For some references on this concern, one may check [70–72]. One of the possible reasons behind this scantiness could be the severity arising in the theory due to the stringy structure in $D > 4$ dimensions. Briefly speaking, further research is needed to be carried out on five-dimensional black strings and their corresponding evaporation process. In this chapter, the black string of concern includes a dilatonic field and the propagation of massive tachyons will be examined under the concerned geometry.

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[2] This chapter is based upon the article named "Absorption Cross Section and Decay Rate of Dilatonic Black Strings" [69].

3.2 Background Geometry

In this section, the geometrical properties of five-dimensional dilatonic black strings will be investigated briefly. As the starting point, let us introduce the action of the theory which goes as [73]

$$I_{EYMBID} = -\frac{1}{16\pi G_{(D)}} \int_{\mathcal{M}} d^d x \sqrt{-g} \left[\mathcal{R} - \frac{4\left(\nabla\psi\right)^2}{D-2} + 4\chi^2 e^{-b\psi} \left(1 - \sqrt{1 + \frac{Fe^{2b}}{2\chi^2}} \right) \right]. \tag{3.1}$$

Here, ψ represents the dilaton field, χ is the Born-Infeld parameter and $b = -\frac{4}{d-2}\alpha$ where $\alpha = \frac{1}{\sqrt{d-1}}$ is the dilaton parameter. Furthermore, $G_{(D)}$ stands for the D-dimensional Newtonian constant and it can be related to $\left(G_{(4)}\right)$ as follows:

$$G_{(D)} = G_{(4)} L^{D-4}. \tag{3.2}$$

However, as stated in introduction, we will consider $G_{(D)} = 1$. One shall record that L is regarded as the upper limit of the compact coordinate, i.e. $\left(\int_0^L dz = L\right)$. Furthermore, $\mathcal{R}$ stands for the Ricci scalar and $F = F^{(\bar{a})}_{\lambda\rho} F^{(\bar{a})\lambda\rho}$ where the two-form Yang-Mills field is given by

$$F^{(a)} = dA^{(\bar{a})} + \frac{1}{2\sigma} C^{(\bar{a})}_{(\bar{b})(\bar{c})} \left(A^{(\bar{b})} \wedge A^{(\bar{c})} \right), \tag{3.3}$$

with $C^{(\bar{a})}_{(\bar{b})(\bar{c})}$ and σ being structure and coupling constants, respectively. The Yang-Mills potential $A^{(a)}$is defined by following the Wu-Yang ansatz [74]

$$A^{(\bar{a})} = \frac{Q}{r^2} \left(x_i dx_j - x_j dx_i \right), \tag{3.4}$$

$$r^2 = \sum_{i=1}^{d-1} x_i^2,\ 2 \leq j+1 \leq i \leq d-1, 1 \leq \bar{a} \leq (d-1)(d-2)/2, \tag{3.5}$$

in which Q denotes the Yang-Mills charge. The dilatonic field is in the form expressed below:

$$\psi = -\frac{(d-2)}{2}\frac{\alpha \ln r}{\alpha^2+1}. \tag{3.6}$$

Having mentioned the dynamics of the model, let us now introduce the background geometry whose infinitesimal interval can be achieved from [73]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{rf(r)} + \beta r dz^2 + d\theta^2 + \sin^2\theta d\phi^2. \tag{3.7}$$

Note that $f(r) = r - r_+$ and $\beta = \frac{4Q^2}{3}$. As in the previous chapter, r_+ indicates the outer horizon which obeys the relation

$$r_+^{\frac{d(d-2)+2}{d}} = \frac{32}{L^{d-4}}\left(\frac{Q^2 d}{d-1}\right)^{\frac{d-2}{2}}, \tag{3.8}$$

The theory of our choice possesses $D = 5$, or similarly $d = 4$,thereby reducing Eq.(3.8) to

$$r_+ = 4\left(\frac{4Q^2}{3}\right)^{\frac{2}{5}} = 4.488Q^{4/5}. \tag{3.9}$$

In order to find Hawking temperature, one first needs to evaluate the surface gravity. Recall Hawking radiation definition Eq.(1.4):

$$T_H = \frac{\kappa_s}{2\pi}. \tag{3.10}$$

The surface gravity can then be found via [75]

$$\kappa_s = \nabla_\mu \Upsilon_\mu \nabla^\mu \Upsilon^\mu, \tag{3.11}$$

where

$$\Upsilon^{\mu} = [1,0,0,0,0] \tag{3.12}$$

is the timelike Killing vector. In this case,the surface gravity can be derived by using

$$\kappa_s = \left.\frac{\sqrt{r}df/dr}{2}\right|_{r=r_+} = \frac{\sqrt{r_+}}{2}. \tag{3.13}$$

Finally, the Hawking temperature reads

$$T_H = \frac{\sqrt{r_+}}{4\pi}. \tag{3.14}$$

If one compares Eq. (3.14) with what has been obtained in Ref. [73], s/he would notice the two results do not match. This is because in Ref. [73], the spacetime geometry is assumed to be symmetric. Nevertheless, from Eq. (3.7) it can be seen $g_{tt} \neq \frac{1}{g_{rr}}$.

3.3 Wave Equation of a Massive Scalar Tachyonic Field

The purpose of this section is to find an exact solution for the Klein-Gordon equation considered for massive tachyons and investigate its behaviour for the cases when $r \to r_+$ and $r \to \infty$. Klein-Gordon equation can be presented as

$$\left[\Box - (i\mu)^2\right]\Psi(t,\mathbf{r}) = \left[\Box + \mu^2\right]\Psi(t,\mathbf{r}) = 0. \tag{3.15}$$

Considering the ansatz

$$\Psi(t,\mathbf{r}) = R(r)Y_l^m(\theta,\phi)e^{ikz}e^{-i\omega t}, \tag{3.16}$$

with $Y_l^m(\theta,\phi)$ representing the spherical harmonics and k being constant, the radial part of Eq.(3.15) reduces to

$$rf\frac{R''}{R'}+\frac{R'}{R}\left(f+rf'\right)+\frac{\omega^2}{f}-\frac{k^2}{\beta r}+\mu^2-\lambda=0, \tag{3.17}$$

where $\lambda = l(l+1)$.

Eq.(3.17) can be regarded as a generator leading to a familiar equation derived by Euler [76]: the hypergeometric differential equation, under the requirement that the essential conditions are satisfied. The hypergeometric series, which is the generalised version of the geometric series

$$1+x+x^2+..., \tag{3.18}$$

can be defined as [77]

$$1+\frac{ab}{c}x+\frac{a(a+1)b(b+1)}{2c(c+1)}x^2+..., \tag{3.19}$$

with the constant parameters a, b and c. The hypergeometric differential equation emerges in a wide range of physical models. To be more specific, from the flow of compressible fluids to the Schodinger equation for a symmetrical top molecule [77], hypergeometric functions appear withing the theory. Thus, one needs to hold a firm understanding on these concepts. In the most general case, the hypergeometric differential equation is expressed in the form

$$x(1-x)\frac{d^2X}{dx^2}+\left[c-(1-a+b)x\right]\frac{dX}{dx}+abX=0. \tag{3.20}$$

which possesses the general solution

$$X(x) = A\,{}_2F_1\left(a,b;c;x\right) + Bx^{1-c}\,{}_2F_1\left(a-c+1,b-c+1;2-c;x\right), \qquad (3.21)$$

as long as $1-c \leq 0$. In order for making use of the hypergeometric function, the variable of our case is changed via $r = r_+ - zr_+$, after multiplying Eq.(3.17) by $r\beta f(r)R(r)$. Then, one can write

$$z(1-z)R'' + (1-2z)R' + \left[\frac{\omega^2}{zr_+} + \frac{k^2}{\beta(1-z)r_+} - \mu^2 + \lambda\right]R = 0. \qquad (3.22)$$

Eq.(4.20) can be demonstrated as a hypergeometric differential equation under the condition

$$\left[\frac{\omega^2}{zr_+} + \frac{k^2}{\beta(1-z)r_+} - \mu^2 + \lambda\right] = \frac{A^2}{z} - \frac{B^2}{1-z} + C, \qquad (3.23)$$

in which

$$\begin{aligned} A &= -\frac{\omega}{2\kappa}, \\ B &= \frac{ik}{2\kappa\sqrt{\beta}}, \\ C &= \lambda - \mu^2. \end{aligned} \qquad (3.24)$$

In general, a hypergeometric differential equation has its solution in the following form:

$$R = \xi_1\left(-z\right)^{iA}\left(1-z\right)^{-B}F\left(a,b;c;z\right) + \xi_2\left(-z\right)^{-iA}\left(1-z\right)^{-B}F\left(\alpha,\varsigma;\eta;z\right). \qquad (3.25)$$

The relevant constants a,b and c obey the relations

$$a = \frac{1}{2}\left(1+\sqrt{1+4C}\right) + iA - B,$$

$$b = \frac{1}{2}\left(1 - \sqrt{1+4C}\right) + iA - B, \tag{3.26}$$

$$c = 1 + 2iA.$$

whereas α, ς and η read

$$\alpha = a - c + 1, \tag{3.27}$$

$$\varsigma = b - c + 1, \tag{3.28}$$

and

$$\eta = 2 - c.$$

Let us visualise what might possibly be occurring during the evaporation process of the black string. The strong gravitational effects due to the presence of the astronomical object of our concern, a pair of virtual particles can pop in and out of existence, spontaneously. Our interpretation goes as follows: Shortly after the creation of the virtual particles, one of the pairs is pulled into the black string, while in the mean time, its pair follows the exact opposite path. This would imply that when one pair moves towards the center of the black string, the other travels toward spatial infinity. In that case, for $r \to r_+$, one should only reckon with the radial solution directed into the horizon, or in other words, the purely ingoing solution. Therefore, the constant ς_1 in Eq.(4.21) needs to be set to zero for achieving logical consistency. Mathematically speaking, the radial solution needs to be in the form

$$R = \xi_2 \left(-z\right)^{-iA} \left(1-z\right)^{-B} F\left(\alpha, \varsigma; \gamma; z\right). \tag{3.29}$$

It should not go without saying that there exists another physical constraint that is required to be applied on the radial solution. Based on the analytical evaluations carried out,it has been seen that $\sqrt{1+4C}$ present in constants (3.22) is obliged to be imaginary, otherwise the greybody factor turns out to diverge.Hence, one needs to set

$$4\mu^2 > 4\lambda - 1, \tag{3.30}$$

so as to achieve

$$\sqrt{1+4C} = i\tau, \tag{3.31}$$

with

$$\tau = \sqrt{4\mu^2 - 4\lambda - 1}, \tau \in \mathbb{R}. \tag{3.32}$$

Having discussed the necessary physical conditions on the radial solution, let us now further examine its behaviour at spatial infinity and in the near horizon regime. For $r \to r_+$ (or alternatively for $z \to 0$),we are only left with

$$R_{NH} = \xi_2 \left(-z\right)^{-iA}. \tag{3.33}$$

If one wishes to deal with the full solution rather than the radial one only, s/he can express

$$\psi_{NH} = \xi_2 e^{-i\omega(\widehat{r}_* + t)}. \tag{3.34}$$

Recall that the tortoise coordinate is defined as

$$r_* = \int \frac{dr}{\sqrt{r}f} \tag{3.35}$$

which enables one to write

$$\widehat{r}_* = \lim_{r \to r_+} r_* \simeq \frac{\ln(-z)}{\sqrt{r_+}} = \frac{1}{2\kappa} \ln(-z) \implies z = -e^{2\kappa \widehat{r}_*}. \tag{3.36}$$

Now, the behaviour of the radial function at spatial infinity can be checked. In the asymptotic region where $r \to \infty$ or $z \to \infty$, one could take advantage of the inverse transformation property of hypergeometric functions which goes as [78]

$$F(\alpha, \varsigma; \eta; z) = (-z)^{-\alpha} \frac{\Gamma(\eta)\Gamma(\varsigma - \alpha)}{\Gamma(\varsigma)\Gamma(\eta - \alpha)} F(\alpha, \alpha + 1 - \eta; \alpha + 1 - \varsigma; 1/z) + \\ (-z)^{-\varsigma} \frac{\Gamma(\eta)\Gamma(\alpha - \varsigma)}{\Gamma(\alpha)\Gamma(\eta - \varsigma)} F(\varsigma, \varsigma + 1 - \eta; \varsigma + 1 - \alpha; 1/z). \tag{3.37}$$

This property plays a crucial role in obtaining an analytical expression for the asymptotic wave function. With the aid of Eq.(3.37), the radial function becomes

$$\Phi_{SI} \backsimeq \xi_2 (-z)^{-iA-B} (-z)^{-\alpha} \frac{\Gamma(\eta)\Gamma(\varsigma - \alpha)}{\Gamma(\varsigma)\Gamma(\eta - \alpha)} + \xi_2 (-z)^{-iA-B} (-z)^{-\varsigma} \frac{\Gamma(\eta)\Gamma(\alpha - \varsigma)}{\Gamma(\alpha)\Gamma(\eta - \varsigma)}. \tag{3.38}$$

Although Eq.(3.38) represents the behaviour of the radial function at an infinite distance away from the black string correctly, it can be presented in a more elaborate form. Applying the simplifications

$$-iA - B - \alpha = -\frac{1}{2}(1 + i\tau), \tag{3.39}$$

and

$$-iA-B-\varsigma=-\frac{1}{2}\left(1-i\tau\right), \tag{3.40}$$

together with substituting $x=-z$, the asymptotic solution yields

$$\Phi_{SI}=\frac{1}{\sqrt{x}}\left[\xi_2 x^{-\frac{i\tau}{2}}\frac{\Gamma(\eta)\Gamma(\varsigma-\alpha)}{\Gamma(\varsigma)\Gamma(\eta-\alpha)}+\xi_2 x^{\frac{i\tau}{2}}\frac{\Gamma(\eta)\Gamma(\alpha-\varsigma)}{\Gamma(\alpha)\Gamma(\eta-\varsigma)}\right]. \tag{3.41}$$

One can now apply the definition of the tortoise coordinate so as to be able to express Φ_{SI} as a function of $\widehat{r}_*$. Then,

$$\widehat{r}_*=\lim_{r\to\infty}r_*\simeq-\frac{2}{\sqrt{r}}, \tag{3.42}$$

where

$$x=r-r_+ \quad x|_{r\to\infty}\simeq r=4e^{-2\widehat{r}_*}. \tag{3.43}$$

Furthermore, $\widehat{r}_*=\ln r_*$. In this case, the radial function becomes

$$\Phi_{SI}=\frac{1}{\sqrt{r}}\left[\Lambda_1 e^{i\widehat{r}_*\tau}+\Lambda_2 e^{-i\widehat{r}_*\tau}\right], \tag{3.44}$$

in which

$$\Lambda_1=2^{-i\tau}\xi_2\frac{\Gamma(\eta)\Gamma(\varsigma-\alpha)}{\Gamma(\varsigma)\Gamma(\eta-\alpha)}, \tag{3.45}$$

and

$$\Lambda_2=2^{-i\tau}\xi_2\frac{\Gamma(\eta)\Gamma(\alpha-\varsigma)}{\Gamma(\alpha)\Gamma(\eta-\varsigma)}. \tag{3.46}$$

Finally, the wave-function for $r \to \infty$ takes the form

$$\psi_{SI} = \frac{1}{\sqrt{r}} \left[\Lambda_1 e^{i(\widehat{r}_* \tau - \omega t)} + \Lambda_2 e^{-i(\widehat{r}_* \tau + \omega t)} \right]. \tag{3.47}$$

3.4 Evaporation of Dilatonic Black String

3.4.1 Evaluation of Flux for $r \to r_+$ and $r \to \infty$

This subsection carries significance in the sense that the outcome for greybody factor is directly linked to the wavefunction forms in two regions, namely at spatial infinity and within the near horizon region. As the wavefunctions will then be used to calculate the associated flux values, one can recognize why the wavefunction forms are the key concepts to focus on here. In the light of this information, let us first evaluate the flux in the vicinity of horizon. By definition [61]

$$\digamma_{NH} = \frac{A_{BH}}{2i} \left(\overline{\psi}_{NH} \partial r_* \psi_{NH} - \psi_{NH} \partial r_* \overline{\psi}_{NH} \right). \tag{3.48}$$

For our case, Eq.(3.48) takes the form

$$\digamma_{NH} = -4\pi\beta \left| \xi_2 \right|^2 r_+. \tag{3.49}$$

Nonetheless,in the asymptotic region, the flux can be calculated by using

$$\digamma_{SI} = \frac{A_{BH}}{2i} \left(\overline{\psi}_{SI} \partial r_* \psi_{SI} - \psi_{SI} \partial r_* \overline{\psi}_{SI} \right), \tag{3.50}$$

in which

$$\psi_{SI} = \frac{r_*}{2} \left[\Lambda_2 e^{-i(\widehat{r}_* \tau + \omega t)} \right]. \tag{3.51}$$

and $\overline{\psi}$ is the complex conjugate of Eq.(3.51). The flux in this regime reduces to

$$\mathcal{F}_{SI} = -4\pi\beta\,|\Lambda_2|^2\,\tau. \tag{3.52}$$

it would be useful to recall that if the mass of the particles were chosen to be real, it would be necessary to replace Eq.(3.51) with $\psi_{SI} \rightarrow \frac{r_*}{2}\left[\Lambda_2 e^{\widehat{r}_*\tau - i\omega t}\right]$. Nonetheless, this specific choice would be problematic, as the greybody factor evaluation would not be possible in that case.

3.4.2 Greybody Factor

In the most general case, the greybody factor belonging to any black astronomical object can be calculated via [70]

$$\gamma^{l,k} = \frac{\mathcal{F}_{NH}}{\mathcal{F}_{SI}}. \tag{3.53}$$

For the specific five-dimensional dilatonic black string of our concern, Eq.(4.50) becomes

$$\gamma^{l,k} = \frac{|\xi_2|^2\,r_+}{|\Lambda_2|^2\,\tau}. \tag{3.54}$$

Substituting Eq.(3.46) into Eq.(3.54) and using [79]

$$|\Gamma(iy)|^2 = \frac{\pi}{y\sinh(\pi y)}, \tag{3.55}$$

$$|\Gamma(1+iy)|^2 = \frac{\pi y}{\sinh(\pi y)}, \tag{3.56}$$

and

$$\left|\Gamma(\frac{1}{2}+iy)\right|^2 = \frac{\pi}{\cosh(\pi y)}, \tag{3.57}$$

the greybody factor attains

$$\gamma^{l,k} = \frac{\kappa r_+}{\omega}\left(e^{\frac{2\pi\omega}{\kappa}} - 1\right)\Xi, \tag{3.58}$$

with

$$\Xi = \frac{e^{2\pi\tau} - 1}{\left[e^{\pi\left(\tau + \frac{\omega}{\kappa} - \frac{k}{\kappa\sqrt{\beta}}\right)} + 1\right]\left[e^{\pi\left(\tau + \frac{\omega}{\kappa} + \frac{k}{\kappa\sqrt{\beta}}\right)} + 1\right]}. \tag{3.59}$$

The graphical analysis of Eq.(3.58) can be viewed in Figure 3.1.

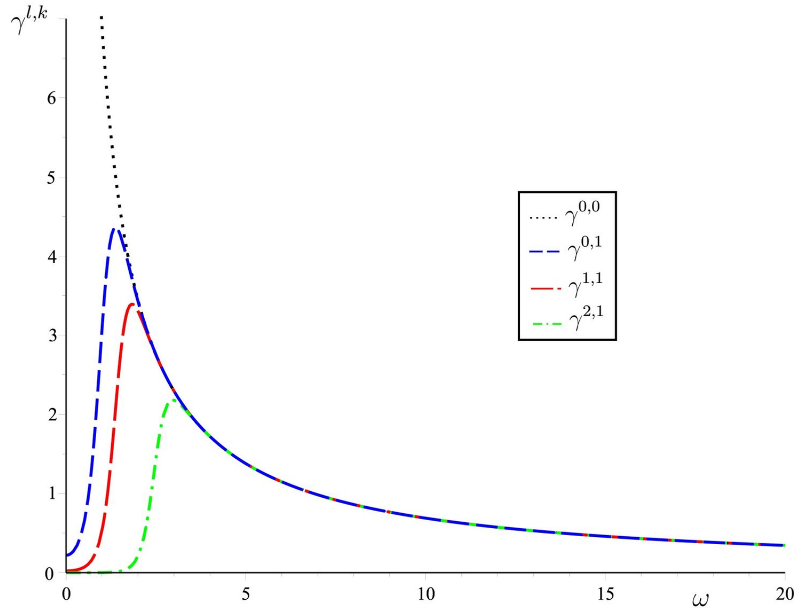

Figure 3.1: Plots of the greybody factor versus frequency for different l and k values. The plots obey the relation (3.58) and the configuration of the dilatonic five-dimensional black string goes as follows: $\mu = 3$ and $Q = 0.2$.

Having a closer look at the behaviour of the greybody factor for different l and k values, one can comment that the $l = k = 0$ case is the one exhibiting an odd behaviour in the long distance (low frequency) era. The greybody factor in this regime seems to possess a divergent behaviour unlike the others. On the other hand, once the short distance (high energy) era is of concern, the values that l and k take seem to be irrelevant. Lastly, the peak values experience a fall, as l values are raised. Although Figure 3.1 is plotted by using $\mu = 3$ and $Q = 0.2$, the overall trends came out to be the same, once the parameters were changed.

3.4.3 Absorption Cross-Section

In arbitrary D-dimensions, the optical theorem originated from the partial wave expansion reduces to [80]

$$\sigma^{l,k} = \frac{4\pi\,(l+1)^2}{\omega^3}\gamma^{l,k}, \tag{3.60}$$

which represents the absorption cross-section of a five-dimensional black string. For the model of our choice, it takes the form

$$\sigma^{l,k} = \frac{4\pi\,(l+1)^2\,\kappa r_+}{\omega^4}\left(e^{\frac{2\pi\omega}{\kappa}} - 1\right)\Xi. \tag{3.61}$$

which can in turn be used for evaluating the total cross section via [81]

$$\sigma_{abs}^{Total} = \sum_{l=0}^{\infty}\sigma^{l,k}. \tag{3.62}$$

The dependence of absorption cross-section on l, k and ω is illustrated graphically in Figure 3.2. The cross-section of the model possesses a similar behaviour as the greybody factor, in the sense that within short distances (for $\omega \to \infty$), the trends are the same regardless of the values that k and l take.

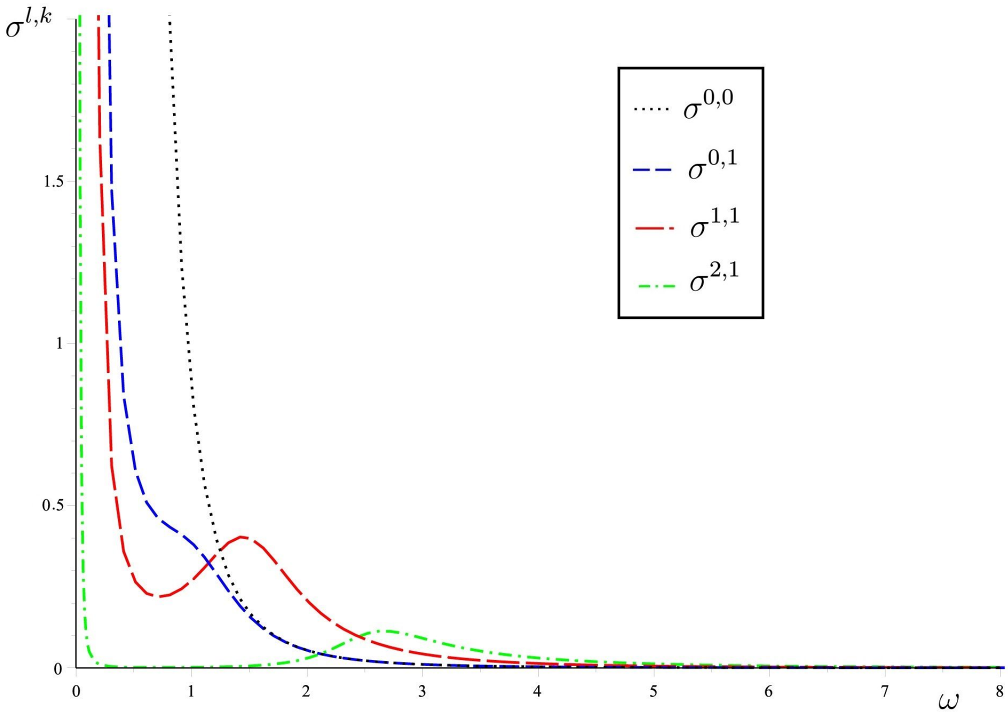

Figure 3.2: Plots of the absorption cross-section versus frequency. The plots are governed by Eq.(3.61). The configuration of the dilatonic five-dimensional black string is as follows: $\mu = 3$ and $Q = 0.2$.

For $\omega \to 0$, however, all curves tend to infinity. Lastly, as can be seen from metric (3.7), the black string of our interest is non-rotating implying there does not exist superradiance [63]. This behaviour can be confirmed from the graph by noticing the plots do not attain any negative cross section values.

3.4.4 Decay Rate

As the final step, the decay rate of the black string of concern will be obtained analytically by using [82]

$$\Gamma_{DR}^{l,k} = \frac{\sigma^{l,k}}{e^{\frac{2\pi\omega}{\kappa}} - 1} = \frac{4\pi\,(l+1)^2\,\kappa r_+}{\omega^4}\Xi. \tag{3.63}$$

In Figure 3.3, the behaviour of decay rate is presented for various energy values. From the figure, it can be noticed that the decay rate vanish in the high energy era, whereas it exhibits a divergent behaviour for $\omega \to 0$.

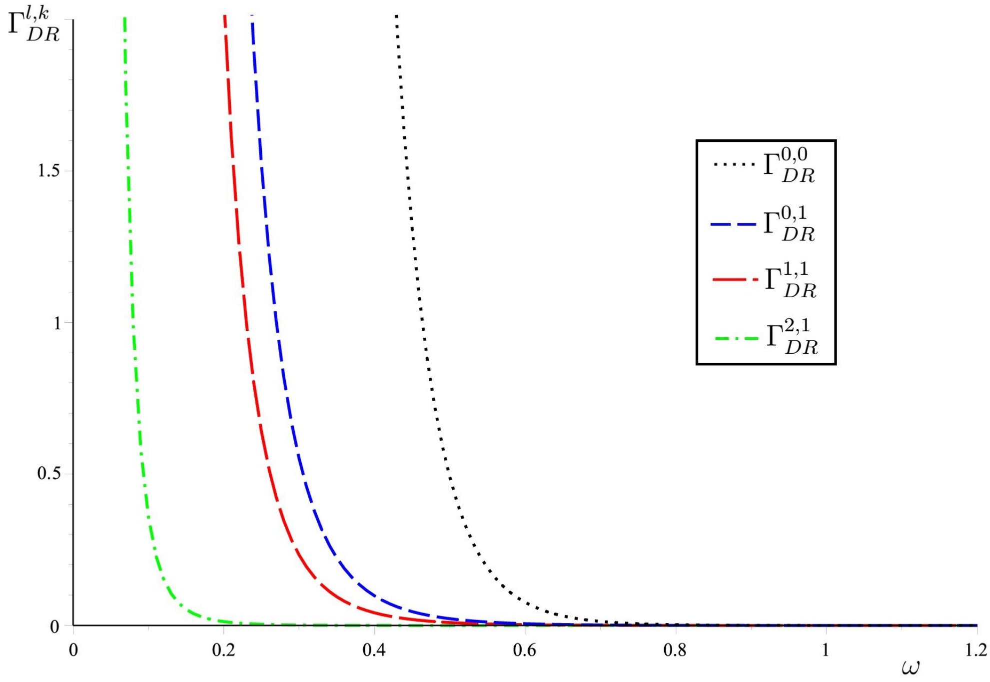

Figure 3.3: Plots of the decay rate versus frequency. The plots are governed by Eq.(3.63). The configuration of the dilatonic five-dimensional black string is chosen as follows: $\mu = 3$ and $Q = 0.2$.

There does not seem to exist unique and distinctive behaviour for different l and k values. However, it can be noted that for ascending l values, the divergence of decay rate occurs at smaller ω values.

Chapter 4

(3+1)-DIMENSIONAL LIFSHITZ-LIKE BLACK BRANE HOLOGRAPHY

4.1 Applications of (3+1)-Dimensional Hyperscaling Violating Theories with z=2 and $\theta = -1$

As had already been stated in the introduction, the exponents z and θ occupy a central role both in the structure of the astronomical object and in the dissipative properties of the holographic model. When there exists hyperscaling violation within the theory, the scale invariance is broken into covariance, which in turn gives birth to a power law scaling of the thermodynamic parameters, compared to the ones belonging to a conformal field theory [83]. Since the main focusing structure of this chapter [3] consists of a model desired to match with the ordinary intuition of spacetime as experienced by the humankind, the total dimensionality will be chosen as $D = 4$. The dynamical exponent, on the other hand, is allowed to possess a wide range of values and one shall stress that each z value has its own implications in not only general relativity, but also for condensed matter systems. In this work, $z = 2$ is assigned to this exponent and there exist several reasons behind this specific choice, some of which will now be explained briefly.

It is worthy to point out that systems holding $z = 2$ are proved to exhibit the properties

[3] This chapter is based upon the articles entitled “Greybody Factors of Holographic Superconductors with $z = 2$ Lifshitz Scaling” [8] and “Holographic Dissipative Properties of Non-relativistic Black Branes with Hyperscaling Violation" [9].

of a superconductor [33] which indicates it is highly likely for these structures to have a great many of applications not only in physics, but also in other fields of science. Superconductors can be noticed in our daily lives as mechanicsms that help to minimize the energy consumed by human beings. They play a crucial role in the latest technology for transportation and make it possible for one to travel a relatively large distance in a rather short period via eliminating friction. Some examples for uses of superconductors can be seen in Ref. [84]. Superconductors are also mandatory if one wishes to reconstruct the conditions for high-energy physics with the purpose of figuring out what precisely has happened during the Big Bang. The experimental verifications of the open questions such as the unified theory of everything, cosmic inflation, cosmological problem, existence of magnetic monopoles and so on are all being tested and pursued within a particle accelerator. However, without the use of a superconductor, gathering observational evidences does not seem to be possible, at least with the current technology. Last but not least, all living things on Earth seem to have the chance to benefit from superconductors, under the threat of a health condition (up to the extent that is cared enough by the humanity). Superconductors are found in magnetic resonance imaging scanners, whence they can be used for diagnosis of certain diseases for both humans and other living beings. For instance, in veterinary medicine, one can check for existence of a respiratory cyst of a dog [85] or scan her/his brain to see whether there exists the condition known as the hereditary polioencephalomyelopathy [86]. In brief, the choices of $z = 2$ and $D = 4$ are not random, but rather compulsory in order for being able to explore a strongly coupled system perceivable by human intuition, whose bulk geometry can dispel some ambiguous characteristics of these systems. This can, in turn, contribute to challenging phenomena such as figuring out how to build high-temperature superconductors, relating aspects of magnetic monopoles to other seemingly-irrelevant theories, getting a fulfilling insight on the strange behaviour of second order phase transitions in liquid crystals or other systems and so on. This can be carried out by using the main tools of the holographic principle. The exact opposite scenario would also be of advantage: For the cases when some specific astronomical objects are desired to be investigated further, their dual models (which may, for instance, be as easily accessible as a liquid crystal) could in principle be adequate to interprete the concerned astronomical property. Surely, this could be possible iff the mapping between the two (or more) theories is precise enough, supported by an extensive range of examples and experimental evidences. Here, we mainly aim for contributing to the examples regarding the realisation of holographic principle in nature.

This chapter consists of the following structure: Sect. 4.2 includes details on the propagation of a massless scalar field in a non-Abelian, electrically charged Lifshitz-like black brane with hyperscaling violation. Accordingly, the greybody factor, absorption cross-section and decay rate of the concerned brane is maintained by means of flux evaluations. In Sec. 4.3, the bulk results are linked to the boundary model and dissipative properties of the event horizon is investigated.

4.2 Scalar Field Propagation in Lifshitz-like Black Branes

4.2.1 Properties of the Bulk Model

The evolution of the dynamics of a system is encoded in the affiliated Lagrangian [87], thus providing the Lagrangian form of concern would be a good starting point.

$$\mathcal{L} = \frac{N}{\lambda} tr(\partial^{\mu}\Phi\partial_{\mu}\Phi + ...), \tag{4.1}$$

where the fields of the theory, i.e. Φ_k, are large $N \times N$ matrices and the associated interactions can be expressed via [88]

$$O = tr(\Phi_{k_1}\Phi_{k_2}...\Phi_{k_m}), \tag{4.2}$$

at which $k \in \mathbb{N}$. The arbitrary constant λ in the denominator of Eq.(4.1) represents the 't Hooft coupling and it carries a vital importance when it comes to determining whether a system is coupled weakly or strongly.If the system under consideration is strongly coupled, λ is obliged to be large. For the model of interest within this chapter, one shall imagine a situation where the dilatonic field ϕ is coupled to not only the gravitational field and the cosmological constant Λ, but also to the Maxwell $\mathcal{A}$ and N $SU(2)$ Yang-Mills A_k^a fields $(a = 1,2,...,N)$. The dynamics of such a system can be mathematically expressed via the Lagrangian [89]

$$\mathcal{L} = \sqrt{-g}\left[R - V(\phi) - \frac{1}{2}(\partial\phi)^2 - \sum_{k=1}^{N}\frac{1}{4g_k^2}e^{\lambda\phi}F_k^2 - \frac{1}{4}e^{\lambda\phi}F_{\mu\nu}^a F^{a\mu\nu}\right] \tag{4.3}$$

where $\Lambda = -[D(z-1)^2 + z - 1]$, $\mathcal{A} = (\varphi_0 + qr)dt$ (φ_0 represents the gauge parameter), $F_{\mu\nu}^a = \partial_\mu A_\upsilon^a - \partial_\upsilon A_\mu^a + \varepsilon^{abc}A_\mu^b A_\upsilon^c$, $V(\phi) = \Lambda e^{-\lambda\phi}$. Moreover, R implies the usual Ricci curvature and g_k determines the coupling strength to the Maxwell field. For the $spin-0$ dilatonic field, the following ansatz is established:

$$\phi = \frac{\theta}{\lambda}\log r. \tag{4.4}$$

From Eq.(4.4), one can comment that the hyperscaling violating parameter is introduced as a consequence of the presence of the dilaton. Thus, the exponent θ of the condensed matter system on the boundary determines the strength of the dilatonic field in the bulk, and vice versa. The holography suggests the action of the bulk theory in $(d+1)$-dimensions has a direct influence on the observables of the boundary, which live in d-dimensions. The field equations of Lagrangian (4.3) inholds a family of solutions which can compactly be presented as ,

$$ds^2 = r^{\theta}\left(-r^{2z}f(r)dt^2 + \frac{dr^2}{r^2 f(r)} + r^2\sum_{i=1}^{D-2} dx_i^2\right), \tag{4.5}$$

in which the metric function reads

$$f(r) = 1 - \frac{q^2}{2\,(z-1)\,r^{2(z-1)}}, \tag{4.6}$$

with q being the parameter determining the electric charge of the brane via $Q = \frac{\omega}{16\pi}q$ and $\theta = \frac{2}{D-2}[z-(D-1)]$. These solutions are commonly referred to as hyperscaling violating Lifshitz-like black branes carrying non-Abelian electric charges.

For $D = 4$ and $z = 2$, the violation exponent comes out to be $\theta = -1$, and metric (4.5) reduces to

$$ds^2 = -N(r)dt^2 + \frac{dr^2}{N(r)} + r\sum_{i=1}^{2} dx_i^2, \tag{4.7}$$

where $N(r) = r^3 f(r)$. As a result, the metric function becomes $f(r) = 1 - \frac{q^2}{2r^2}$. From Eq.(4.7), one can notice that the spacetime of concern is non-relativistic. In literature, a vast number of studies on non-relativistic backgrounds can be found, amongst which some can be accessed from Refs. [90–95]. The surface gravity of the model can be written as

$$\kappa_s = \frac{1}{2}\frac{dN(r)}{dr}\bigg|_{r=r_+} = r_+^2, \tag{4.8}$$

in which the event horizon is $r_+ = q/\sqrt{2}$. Substituting Eq.(4.8) into definition (1.4), the Hawking temperature becomes

$$T_H = \frac{r_+^2}{2\pi} = \frac{q^2}{4\pi}. \tag{4.9}$$

4.2.2 Klein Gordon Equation

Let us now inspect how scalar particles with no mass propagate through the model of our concern. To achieve this, one needs to introduce the massless Klein-Gordon equation, which can be demonstrated as

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\upsilon}\partial_\nu\varphi\right) = 0. \tag{4.10}$$

The separation of variables enables one to express the wavefunction in the form [96]

$$\varphi(t,\overrightarrow{x}) = \Phi(r)e^{i\vec{\kappa}.\vec{x}}e^{-i\omega t}, \tag{4.11}$$

where $\vec{\kappa}$ and $\vec{x}$ stand for the wave and spatial vectors, respectively. Moreover, one can get an idea about the energy of the emitted radiation from the frequency, ω. For the case when no specific values are assigned to the exponents, i.e. for metric (4.5), Eq. (4.10) transforms into

$$\frac{d}{dr}\left[f(r)r^{2+\tilde{\eta}-\theta}\frac{d\Phi}{dr}\right] + \frac{1}{r^{2+\theta-\tilde{\eta}}}\left(\frac{\omega^2}{r^{2(z-1)}f(r)} - \kappa^2\right)\Phi(r) = 0\,, \tag{4.12}$$

when only the radial part is of concern. Note that $\tilde{\eta} = \frac{\theta D}{2} + z + D - 3$.

If one wishes to evaluate the effective potential of the theory, tortoise coordinate would need to be introduced. Suppose that

$$\Phi(r) = \mathcal{F}(r)r^{-\xi}, \tag{4.13}$$

in which $\xi = \frac{(D-2)(2+\theta)}{4}$. Then, the tortoise coordinate r_* reads

$$r_* = \int r^{-(1+z)} \frac{dr}{f(r)}, \tag{4.14}$$

which results in

$$\frac{d^2 \mathcal{F}(r_*)}{dr_*^2} - V_{eff} \mathcal{F}(r_*) = -\omega^2 \mathcal{F}(r_*). \tag{4.15}$$

Consequently, the effective potential is found to be

$$V_{eff}(r) = r^{2(z-1)} f(r) \left[\frac{q^2}{2} \xi r^{3-z} + \xi(\xi + z) r^2 f(r) + \kappa^2 \right]. \tag{4.16}$$

Due to the reasons specified in the previous section, the differential equation will be solved for $z = 2$, $D = 4$ and $\theta = -1$. At this point, it would be worthy to emphasise that these black branes are stable. A detailed analysis for the stability will be provided in the upcoming subsection.

Going back to the wave-function analysis, the radial differential equation (4.12) with specific exponents turn into

$$N(r) \frac{d^2\Phi}{dr^2} + (4r^2 - 2r_+^2) \frac{d\Phi}{dr} + \left(\frac{\omega^2}{N(r)} - \frac{\kappa^2}{r} \right) \Phi(r) = 0. \tag{4.17}$$

Let us introduce a new variable $\tilde{z}$ such that $\tilde{z} = r^{-2}(r^2 - r_+^2)$. Combining this with the ansatz

$$\Phi(\tilde{z}) = \tilde{z}^{\alpha} (1 - \tilde{z})^{\beta} G(\tilde{z}), \tag{4.18}$$

with $\beta = 3/2$ results in

$$\tilde{z}(1-\tilde{z})\frac{d^2G}{d\tilde{z}^2} + \left(1 - \frac{7\tilde{z}}{2} - \frac{i\omega(1-\tilde{z})}{r_+^2}\right)\frac{dG}{d\tilde{z}} + \left[\frac{5i\omega - 6r_+^2 - \kappa^2}{4r_+^2}\right]G = 0, \tag{4.19}$$

Checking the resemblance between Eq. (4.19) and the general hypergeometric differential equation (3.20) [79]

$$\tilde{z}(1-\tilde{z})\frac{d^2G}{d\tilde{z}^2} + \left[c - (1+a+b)\,z\right]\frac{dG}{d\tilde{z}} - abG = 0 \tag{4.20}$$

yields

$$G(\tilde{z}) = C_1\,{}_2F_1\left(a,b;c;\tilde{z}\right) + C_2\tilde{z}^{1-c}\,{}_2F_1\left(a-c+1,b-c+1;2-c;\tilde{z}\right), \tag{4.21}$$

where

$$a = \alpha + \frac{5}{4} \mp \frac{\sqrt{\kappa_s^2 - 4\kappa_s\kappa^2 - 4\omega^2}}{4\kappa_s}, \tag{4.22}$$

$$b = \alpha + \frac{5}{4} \pm \frac{\sqrt{\kappa_s^2 - 4\kappa_s\kappa^2 - 4\omega^2}}{4\kappa_s}, \tag{4.23}$$

$$c = 1 + 2\alpha. \tag{4.24}$$

Note that $\alpha = \pm\frac{i\omega}{2\kappa_s}$. In this study, we pick

$$\alpha = -(i\omega/2\kappa_s),$$

$$a = \frac{5}{4} - \frac{i}{2\kappa_s}\left(\omega + \widehat{\omega}\right), \tag{4.25}$$

$$b = \frac{5}{4} - \frac{i}{2\kappa_s}\left(\omega - \widehat{\omega}\right), \tag{4.26}$$

by establishing

$$\widehat{\omega} = \sqrt{\omega^2 + \kappa_s\left(\kappa^2 - \frac{\kappa_s}{4}\right)}, \tag{4.27}$$

for simplicity. Consequently, Eq. (4.24) takes the form

$$c = 1 - \frac{i\omega}{\kappa_s}. \tag{4.28}$$

Substituting Eq.(4.21) into the ansatz (4.18)

$$\Phi(\tilde{z}) = \tilde{z}^{\alpha}(1-\tilde{z})^{\beta}\left[C_1\,{}_2F_1\left(a,b;c;\tilde{z}\right) + C_2\,\tilde{z}^{1-c}\,{}_2F_1\left(a-c+1,b-c+1;2-c;\tilde{z}\right)\right] \tag{4.29}$$

is obtained. As previously mentioned, the examination of the radial solution for two specific regions (for $r \to r_+$ and $r \to \infty$) is rather important, in case one desires to obtain analytical solutions for the radiation parameters of $(z = 2, \theta = -1)$ black branes. Hence, let us explore the radial behaviour for different points of spacetime.

4.2.2.1 Near Horizon Solution

The first step will be checking the behaviour of the wavefunction in the near horizon era. For $\tilde{z} \to 0$, ${}_2F_1\left(a,b,c;0\right) = 1$, leading to

$$\Phi_{NH} = C_1 e^{\alpha \ln \tilde{z}} + C_2 e^{-\alpha \ln \tilde{z}}. \tag{4.30}$$

One shall first introduce the tortoise coordinate in this region, which can be denoted as

$$r_{*(NH)} = \frac{\ln\sqrt{1-\tilde{z}} - 1}{2r_+^2}. \tag{4.31}$$

Based on the Hawking radiation intuition, one needs to impose the constraint $C_2 = 0$ in order for the theory to admit waves approaching the horizon only. This is because one of the virtual particles will fall into the black brane, whilst the other reaches to infinity. Then, the general solution becomes

$$\Phi(\tilde{z}) = C_1 \tilde{z}^{\alpha} (1-\tilde{z})^{\beta} \, {}_2F_1\left(a, b; c; \tilde{z}\right), \tag{4.32}$$

Setting C_2 to zero and using $\alpha = -(i\omega/2\kappa_s)$, one obtains

$$\Phi_{NH} = C_1 e^{-i\omega \frac{\ln \tilde{z}}{2r_+^2}} = \widetilde{C_1} e^{-i\omega r_{*(NH)}}. \tag{4.33}$$

for the near horizon region, with $\widetilde{C_1} = C_1 e^{\omega\pi/2r_+^2}$. Inevitably,the wave function (4.11) reads

$$\varphi_{NH} = C_1 e^{-i\omega\left(t + \frac{\ln \tilde{z}}{2r_+^2}\right)} = \widetilde{C_1} e^{-i\omega r_{NH}^*} e^{-i\omega t}. \tag{4.34}$$

4.2.2.2 Asymptotic Behaviour

Amongst a numerous ways for calculating the flux, the method followed in Ref. [97] will be followed here. Thus, firstly, the behaviour of the wave function for $r \to \infty$ needs to be examined delicately. For the evaluation of the outgoing flux, one may use

$$\digamma_{SI} = \frac{\sqrt{-g} g^{rr}}{2i} (\Phi_{SI}^* \partial_r \Phi_{SI} - \Phi_{SI} \partial_r \Phi_{SI}^*). \tag{4.35}$$

which implies that in order for obtaining φ_{SI}, the asymptotic behaviour of hypergeometric solution should be studied. For $\tilde{z} \to \infty$, the general solution (4.32) becomes

$$\Phi_{SI}(\tilde{z}) = C_1 \tilde{z}^{\alpha} \Big[A_1 (1-\tilde{z})^{\beta} \, {}_2F_1\left(a, b; a+b-c+1; 1-\tilde{z}\right) + A_2\left(\tilde{z}\right) \, {}_2F_1\left(c-a, c-b; c-a-b+1; 1-\tilde{z}\right) \Big]. \tag{4.36}$$

At this point, one can derive benefit from the inverse transformation property obeyed by hypergeometric functions, which indicates that in general

$$
{}_2F_1(a,b,c;u) = A_1\, {}_2F_1(a,b,a+b-c+1;1-u) \tag{4.37}
$$

$$
+A_2(1-u)^{c-a-b}\, {}_2F_1(c-a,c-b,c-a-b+1;1-u). \tag{4.38}
$$

Record that the constants are expressed in terms of gamma functions as follows.

$$
A_1 = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}, \tag{4.39}
$$

$$
A_2 = \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}. \tag{4.40}
$$

In brief, for $r \to \infty$, the radial solution (4.36) exhibits the behaviour

$$
\Phi_{SI} = C_1 \left[A_1 \left(\frac{r_+}{r} \right)^3 + A_2 \right]. \tag{4.41}
$$

Our main interest is the low energy greybody factor, since the Lagrangian involves strong coupling, and furthermore, $\beta \in \mathbb{R}$ turns it into a severe challenge to apprehend between the ingoing and outgoing fluxes [97]. Therefore, in the asymptotic region, Eq. (4.17) comes up to

$$
\frac{d^2\Phi}{dr^2} + \frac{4}{r}\frac{d\Phi}{dr} = 0. \tag{4.42}
$$

The second-order Cauchy-Euler equation (4.42) admits the solution

$$
\Phi_{SI} = D_1 + \frac{D_2}{r^3}. \tag{4.43}
$$

As the final step, Eq. (4.41) can be compared to Eq. (4.43) in order to obtain the arbitrary constants. They come out as $D_1 = A_2 C_1$ and $D_2 = A_1 C_1 r_+^3$. Ultimately, the asymptotic flux (4.35) reads

$$\mathcal{F}_{SI} = 3\left(|D_{out}|^2 - |D_{in}|^2\right), \tag{4.44}$$

where

$$D_{out} = \frac{D_1 + iD_2}{2}, \tag{4.45}$$

and

$$D_{in} = \frac{D_1 - iD_2}{2}. \tag{4.46}$$

4.2.3 Quasinormal Modes and Stability Check

Throughout this subsection, the stability and quasinormal mode analysis will be carried out for the $(2,-1)$ Lifshitz-like black branes by considering spin-0 perturbations. As already stated in introduction, when a black astronomical object is externally disturbed, let say due to a particle crossing the event horizon, the object radiates energy. In the region $r > r_+$, there exists an effective potential, which can analytically be derived by inspecting the Zerilli & Regge-Wheeler equations. Note that the effective potential will always be produced in the same form, as long as the astronomical black object of concern is not changed. When this barrier is of interest, the composition of test fields perturbing the black object becomes irrelevant. The effective potential is in such a form that it experiences an exponential decay for $r \to r_+$ $(r_* \to -\infty)$ and $r \to \infty$ $(r_* \to \infty)$

In this regard, quasinormal modes can be treated as electromagnetic or gravitational perturbations of astronomical spacetimes [98]. During quasinormal inspection, one needs to consider how effective potential behaves.

From Eq. (4.16), one can see that the effective potential of the four-dimensional non-Abelian charged Lifshitz-like black brane with $(2,-1)$ metric exponents yields

$$V_{eff}(r) = N(r)\left(\frac{5r}{4} - \frac{r_+^2}{4r} + \frac{\kappa^2}{r}\right). \tag{4.47}$$

One can notice that $\lim_{r\to\infty} V_{eff}(r) \to \infty$. Thus, quasinormal modes of concern obey the required boundary conditions which states that the spin 0 field φ is obliged to be purely ingoing in the vicinity of event horizon; and furthermore, it needs to vanish asymptotically far away (a similar case can be recognised in Ref. [99]).

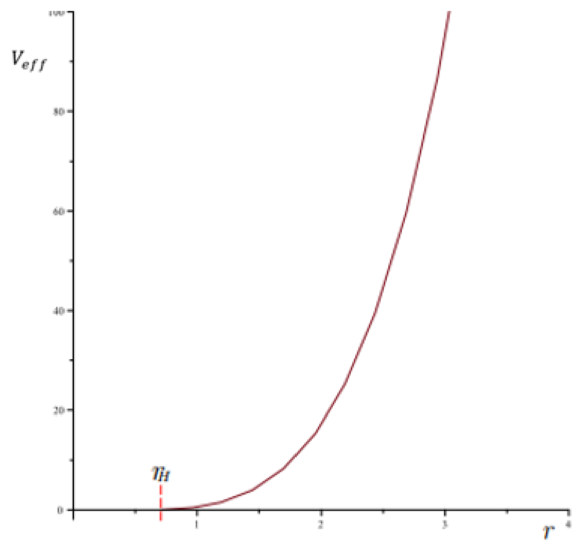

Figure 4.1: The behaviour of effective potential under the choice $q = 1$ and $\kappa = 0$.

As the asymptotic behaviour of the radial function had already been determined in Eq. (4.36), one can express

$$\begin{aligned} \Phi_{SI}(\tilde{z}) &\approx C_1 A_1 (1-\tilde{z})^{\beta} + C_1 A_2, \\ &\cong C_1 A_2 = C_1 \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}. \end{aligned} \tag{4.48}$$

This indicates that Eq.(4.48) should be forced to vanish for $\tilde{z} \to 1$. The asymptotic wave function can vanish if and only if $a=-n$ or $b=-n$ for $n=0,1,2,...$ The latter provides us with the precise quasinormal spectrum which goes as

$$\omega=-i\frac{q^2(n+1)(2n+3)+\kappa^2}{5+4n}. \tag{4.49}$$

From Eq.(4.49) one can notice that the over damping of the system is assured, as the quasinormal modes are found to be purely negative and imaginary. Hence, the four-dimensional electrically charged Lifshitz-like black brane with $z=2$ dynamic exponent is shown to be stable under massless spin 0 perturbations.

4.2.4 Radiation Parameters

This subsection is reserved for maintaining exact results for greybody factor, absorption cross-section and decay rate of the thermal radiation emitted by the black brane (4.7)

Based on the discussions in section 1.4, the greybody factor can be found via [97]

$$\gamma=1-\mathfrak{R}=\frac{2i(\mathfrak{D}-\mathfrak{D}^*)}{\mathfrak{D}\mathfrak{D}^*+i(\mathfrak{D}-\mathfrak{D}^*)+1}, \tag{4.50}$$

in which $\mathfrak{R}=|D_{out}|^2/|D_{in}|^2$ and $\mathfrak{D}=D_1/D_2$. To be more specific, one can write

$$\mathfrak{D}=\frac{3}{8}\frac{\Gamma\left(-\frac{1}{4}-iX\right)\Gamma\left(-\frac{1}{4}-iY\right)}{\Gamma\left(\frac{5}{4}-iY\right)\Gamma\left(\frac{5}{4}-iX\right)r_+^3}, \tag{4.51}$$

which yields

$$\mathfrak{D}\mathfrak{D}^*=\frac{2304}{\pi^4 r_+^6}\left[\Gamma(3/4)\right]^8\prod_{n=0}^{\infty}\frac{\varepsilon_n}{\left[1+\left(\frac{Y}{n-1/4}\right)^2\right]\left[1+\left(\frac{X}{n-1/4}\right)^2\right]}, \tag{4.52}$$

$$\mathfrak{D}-\mathfrak{D}^*=\frac{24}{\pi^2 r_+^3}\,\Xi\left[\Gamma(3/4)\right]^4\prod_{n=0}^{\infty}\varepsilon_n. \tag{4.53}$$

The expressions above are maintained in these neat forms by using

$$X = \frac{\omega - \hat{\omega}}{2r_+^2}, \tag{4.54}$$

$$Y = \frac{\omega + \hat{\omega}}{2r_+^2}, \tag{4.55}$$

$$\varepsilon_n = \left[1 + \left(\frac{Y}{n+5/4}\right)^2\right]\left[1 + \left(\frac{X}{n+5/4}\right)^2\right], \tag{4.56}$$

and

$$\Xi = \frac{(\sin\theta_1 \sin\theta_2 - \sin\theta_3 \sin\theta_4)}{\sin\theta_1 \sin\theta_2 \sin\theta_3 \sin\theta_4}. \tag{4.57}$$

Moreover, the angles of interest are $\theta_1 = \pi\left(\frac{5}{4} - iX\right)$, $\theta_2 = \pi\left(\frac{5}{4} - iY\right)$, $\theta_3 = \pi\left(\frac{5}{4} + iY\right)$ and $\theta_4 = \pi\left(\frac{5}{4} + iX\right)$. One of the key relations which helped us during our analytical steps goes as

$$\frac{\Gamma(x+iy)\Gamma(x-iy)}{[\Gamma(x)]^2} = \prod_{n=0}^{\infty}\left[1 + \left(\frac{y}{x+n}\right)^2\right]^{-1}, \tag{4.58}$$

together with the reflection formula

$$\frac{1}{\Gamma(Z)}\frac{1}{\Gamma(1-Z)} = \frac{\sin\pi Z}{\pi}, \tag{4.59}$$

where $Z \in \mathbb{C}$ [100]. Armoured by the properties above, the final form of the greybody factor is stated as

$$\gamma = \frac{2i\,\Xi\prod_{n=0}^{\infty}\varepsilon_n}{\frac{96\,[\Gamma(3/4)]^4}{\pi^2 r_+^3}\prod_{n=0}^{\infty}\frac{\varepsilon_n}{\left[1+\left(\frac{Y}{n-1/4}\right)^2\right]\left[1+\left(\frac{X}{n-1/4}\right)^2\right]} + i\,\Xi\prod_{n=0}^{\infty}\varepsilon_n + \frac{\pi^2 r_+^3}{24[\Gamma(3/4)]^4}}. \tag{4.60}$$

After achieving an exact solution for the greybody factor, one can now calculate the absorption cross-section which results in [101]

$$\sigma_{abs}=\sum_{l=0}^{\infty}\frac{i\pi}{\omega^2}\frac{2(2l+1)\;\Xi\;\prod_{n=0}^{\infty}\varepsilon_n}{\frac{96\;[\Gamma(3/4)]^4}{\pi^2 r_+^3}\prod_{n=0}^{\infty}\frac{\varepsilon_n}{\left[1+\left(\frac{Y}{n-1/4}\right)^2\right]\left[1+\left(\frac{X}{n-1/4}\right)^2\right]}+i\;\Xi\;\prod_{n=0}^{\infty}\varepsilon_n+\frac{\pi^2 r_+^3}{24[\Gamma(3/4)]^4}}. \tag{4.61}$$

Finally, the decay rate of the associated black brane can be written as

$$\Gamma=\frac{i\;\Xi\;\prod_{n=0}^{\infty}\varepsilon_n\;d^3k}{4\pi^3(e^{\omega/T_H}-1)\left[\frac{96\;[\Gamma(3/4)]^4}{\pi^2 r_+^3}\prod_{n=0}^{\infty}\frac{\varepsilon_n}{\left[1+\left(\frac{Y}{n-1/4}\right)^2\right]\left[1+\left(\frac{X}{n-1/4}\right)^2\right]}+i\;\Xi\prod_{n=0}^{\infty}\varepsilon_n+\frac{\pi^2 r_+^3}{24[\Gamma(3/4)]^4}\right]}. \tag{4.62}$$

4.3 Duality Between Bulk Observables and Strongly Coupled Systems

4.3.1 Some Key Aspects

Throughout the preceding sections, the bulk characteristics such as quasinormal modes, greybody factor, absorption cross-section and decay rate were explored via analytical methods. Now, in this section, the main focusing point will be the associated holographic model. In the dual picture, the model is built on the boundary of the bulk spacetime located at $r\to\infty$. Therefore, the fields present in the gravitational picture are mapped onto the dual operators of the holographic field theory of two-spatial boundary dimensions. In the astronomical picture, the field was picked to be in the massless spin 0 form, or in other words, as $\varphi(t,r,\vec{x})=\Phi(r)\,e^{i\vec{\kappa}.\vec{x}}\,e^{-i\omega t}$. This implies that Φ of bulk theory will represent O_Φ on the boundary, where O_Φ represents a well-defined boundary value for Φ.

Quasinormal modes (4.49) can be used to maintain the diffusion constant of the fluctuating horizon via membrane paradigm. The membrane paradigm suggests that small oscillations of a stretched horizon possess resemblance with the diffusive properties of a conserved charge in simple fluids [1,93,102]. To rephrase, a dispersion relation of the form $\omega = -i\mathcal{D}q^2$ puts forward the existence of diffusion of a conserved charge [102]. Comparing the dispersion relation with Eq. (4.49), one can write

$$\mathcal{D} = \frac{(n+1)(2n+3)}{5+4n}, \tag{4.63}$$

in which $\mathcal{D}$ represents the shear mode diffusion constant [103]. The diffusion constant is rather crucial in fluid/gravity duality, since it can be used to derive the ratio of shear viscosity to entropy density, denoted by η/s. Note that for $n = 0$, i.e. for the fundamental quasinormal mode, the diffusion constant becomes $\mathcal{D} = 3/5$. Recall the geometrical structure of black brane (4.5)

$$ds^2 = r^\theta \left(-r^{2z} f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 \sum_{i=1}^{2} dx_i^2 \right). \tag{4.64}$$

Applying $r \to 1/\tilde{r}$ and $\theta \to -\tilde{\theta}$ yields

$$d\tilde{s}^2 = \tilde{r}^{\tilde{\theta}} \left(-\frac{f(\tilde{r})}{\tilde{r}^{2z}} dt^2 + \frac{d\tilde{r}^2}{\tilde{r}^2 f(\tilde{r})} + \sum_{i=1}^{2} \frac{dx_i^2}{\tilde{r}^2} \right). \tag{4.65}$$

Metric (4.65) is in the same form as the family of non-relativistic branes mentioned in [104]. Before we start the analysis for η/s based on the universal relation derived by Kolekar, Mukherjee and Narayan, recall that for the model of our interest; $d = D-1 = 3$, $z = 2$ and $\tilde{\theta} = 1$. These specific choices obey the null energy conditions

$$(d-1-\tilde{\theta})((d-1)(z-1)-\tilde{\theta}) \geqslant 0, \qquad (z-1)(d-1+z-\tilde{\theta}) \geqslant 0. \tag{4.66}$$

In their work [104], the authors proposed that there exists a universal relation for all hyperscaling violating theories which goes as

$$\frac{\eta}{s} = \frac{d-z+1}{4\pi}\,\mathcal{D}\,\tilde{r}_H^{2-z}. \tag{4.67}$$

Thus, based on their proposal, if one can evaluate the diffusion constant from the dispersion relation of the astronomical black object, s/he can use tools of membrane paradigm to get the viscosity-to-entropy ratio of the holographic fluid model. Furthermore, it would be beneficial to stress that this ratio has played a vital role in experimental verification of AdS/CFT correspondence.

Going back to Eq. (4.67), although the authors have suggested that this relation would act as a universal law for theories based on background (4.65), in another study of theirs [93], they stated that the situation may as well have slight differences in the vicinity of a charge. Since η/s ratio evaluation via holographic principle is still a pending research question, we wanted to evaluate this ratio both from the *universal* relation (4.67) - as claimed by the authors- and from another method that will become apparent in the upcoming sections.

For the brane-fluid system of our concern, substituting $d=3$, $z=2$ and $\mathcal{D}=3/5$ into Eq. (4.67) leads to

$$\frac{\eta}{s} = \frac{3}{10\pi}. \tag{4.68}$$

Note that the so-called universal Kovtun-Son-Starinets bound seems to be satisfied, i.e. $\eta/s \geqslant \frac{1}{4\pi}$ [102]. From this result, one can conclude the following: If this value can experimentally be verified, it would be implied that the model under consideration would be carrying information regarding both $(3+1)$-dimensional

Lifshitz-like black branes and strongly coupled, non relativistic three-dimensional fluids. Moreover, any empirical evidence for Eq. (4.68) would confirm that the formulation proposed by Kolekar, Mukherjee and Narayan is also valid for charged hyperscaling violating Lifshitz-like backgrounds as well. It is also noteworthy addressing that the Kovtun-Son-Starinets bound is suspected to be an inherent property of semi-classical gravitational theory [102].

Let us now describe the framework developed by Gubser-Klebanov-Polyakov-Witten (GKPW) [105]. GKPW method states that under the consideration of an infinitesimal distance ε away from the boundary of the bulk spacetime, the equations of motion remain the same, albeit the perturbations in the action. As a result, the ultraviolet divergence is avoided and imposing $\varepsilon \to 0$ gives birth to a well-defined boundary value for Φ. Accordingly, the flux factor is defined as [106]

$$\mathcal{F}(\vec{\kappa},\omega) = \lim_{r\to\varepsilon} \sqrt{g}\, g^{rr}\Phi(r)\, \partial_r\Phi(r), \tag{4.69}$$

representing the momentum-space two-point correlation function. Alternatively,

$$\mathcal{F}(\vec{\kappa},\omega) = \langle O_\Phi(\vec{\kappa},\omega)\; O_\Phi(-\vec{\kappa},-\omega)\rangle\,. \tag{4.70}$$

One may check Refs. [106–108] for further regards. The two-point correlation function plays a key role in a wide range of experiments such as particle physics experiments including scattering processes and the transitions between states; and furthermore, it benefits from non-relativistic perturbation theory [109]. Consequently, the differential cross-section is achieved by using

$$d\sigma = \frac{1}{\mathcal{F}}\left|\mathcal{M}\right|^2 d\tilde{\Phi}, \tag{4.71}$$

where $\mathcal{M}$ and $d\tilde{\Phi}$ respectively stand for the matrix element and the phase factor.

4.3.2 Holographic Approach: Transport Coefficients of the Dual Model

4.3.2.1 Shear Viscosity

The shear viscosity of the holographic model can be evaluated via [105, 110, 111]

$$\eta = -\lim_{\omega\to 0}\frac{1}{\omega}Im\left[G_{O_+}(\omega,0)\right], \tag{4.72}$$

which is the so-called Kubo formula. Eq.(4.72) represents the linear response of an object after being subject to infinitesimal perturbations. Note that G_{O_+} stands for the two-point correlation function which can be calculated by inspecting how the radial function, namely Eq. (4.17), behaves for $r\to\infty$. Based on the relation $G_{O_+} = D_2/D_1$ [112], the retarded Green's function becomes

$$G_{O_+}(\omega,0) = \frac{8}{3}\frac{\Gamma\left(\frac{5}{4}-i\tilde{Y}\right)}{\Gamma\left(-\frac{1}{4}-i\tilde{Y}\right)}\frac{\Gamma\left(\frac{5}{4}-i\tilde{X}\right)}{\Gamma\left(-\frac{1}{4}-i\tilde{X}\right)}r_+^3, \tag{4.73}$$

in which $\tilde{X} = X|_{\kappa\to 0} = \frac{\omega-\sqrt{\omega^2-r_+^4/4}}{2r_+^2}$ and $\tilde{Y} = Y|_{\kappa\to 0} = \frac{\omega+\sqrt{\omega^2-r_+^4/4}}{2r_+^2}$.

After this point, one needs to take advantage of some mathematical tricks, in order for achieving analytical results. With this purpose in mind, let us express the ratio of complex Gamma functions as

$$\frac{\Gamma(b+iy)}{\Gamma(a+iy)} = \frac{\Gamma(b+iy)}{\Gamma(1-a+iy)}\frac{\Gamma(1-a+iy)}{\Gamma(a+iy)}\frac{\Gamma(a-iy)}{\Gamma(a-iy)}, \tag{4.74}$$

where $a,b\in\mathbb{R}$. As can be noticed from Eq. (4.74), both the numerator and the denominator are multiplied by $\Gamma(1-a+iy)\Gamma(a-iy)$. This simple trick have led to intriguing conclusions: It enabled us to find exact expressions for the dissipative properties of the dual model. Record that the complex Gamma functions obey the relations [79, 113]

$$\Gamma(1-a+iy)\,\Gamma(a-iy)=\frac{\pi}{sin\left[\pi\left(a-iy\right)\right]}, \tag{4.75}$$

and

$$\left|\Gamma(a+iy)\right|^2=\Gamma(a-iy)\,\Gamma(a+iy), \tag{4.76}$$

which in turn result in

$$\frac{\Gamma(b+iy)}{\Gamma(a+iy)}=\frac{1}{\left|\Gamma(a+iy)\right|^2}\frac{\Gamma(b+iy)}{\Gamma(1-a+iy)}\frac{\pi}{sin\left[\pi\left(a-iy\right)\right]}. \tag{4.77}$$

To be more specific, Eq. (4.77) can now be used to evaluate the two-point correlation function. The associated ratios take the form

$$\frac{\Gamma\left(\frac{5}{4}-i\tilde{X}\right)}{\Gamma\left(-\frac{1}{4}-i\tilde{X}\right)}=\frac{1}{\left|\Gamma\left(-\frac{1}{4}-i\tilde{X}\right)\right|^2}\frac{\pi}{sin\left[\pi\left(-\frac{1}{4}+i\tilde{X}\right)\right]}, \tag{4.78}$$

and

$$\frac{\Gamma\left(\frac{5}{4}-i\tilde{Y}\right)}{\Gamma\left(-\frac{1}{4}-i\tilde{Y}\right)}=\frac{1}{\left|\Gamma\left(-\frac{1}{4}-i\tilde{Y}\right)\right|^2}\frac{\pi}{sin\left[\pi\left(-\frac{1}{4}+i\tilde{Y}\right)\right]}. \tag{4.79}$$

Equipped with the tools of fluid/gravity correspondence, the region of interest will be chosen as the one where low energy is of concern, i.e the condition $w << r_+$ needs to be imposed. The main reason behind this is because the dual system will be considered to be living on the boundary, at which the bulk-gravitational model exhibits properties of hydrodynamics equations. Then, $\tilde{X}$ and $\tilde{Y}$ become

$$\begin{aligned} \tilde{X} &\sim \frac{\omega}{2r_+^2} - \frac{i}{4}, \\ \tilde{Y} &\sim \frac{\omega}{2r_+^2} + \frac{i}{4}. \end{aligned}, \tag{4.80}$$

respectively. Substituting the low energy behavior of $\tilde{X}$ and $\tilde{Y}$ into Eqs. (4.78) and (4.79) together with the commonly referred relation $z\Gamma(z) = \Gamma(1+z)$, the modulus squared terms can be replaced by

$$\left|\Gamma\left(-\frac{1}{4} - i\tilde{X}\right)\right|^2 = \frac{4r_+^2}{r_+^4 + \omega^2}\left|\Gamma\left(\frac{1}{2} - \frac{i\omega}{2r_+^2}\right)\right|^2, \tag{4.81}$$

and

$$\left|\Gamma\left(-\frac{1}{4} - i\tilde{Y}\right)\right|^2 = \left|\Gamma\left(-\frac{i\omega}{2r_+^2}\right)\right|^2. \tag{4.82}$$

Taking the relations [113]

$$|\Gamma(ib)|^2 = \frac{\pi}{b\sinh(\pi b)}, \tag{4.83}$$

and

$$\left|\Gamma\left(\frac{1}{2} + ib\right)\right|^2 = \frac{\pi}{\cosh(\pi b)}, \tag{4.84}$$

into account, gamma functions (4.78) and (4.79) finally evolve into rather simple forms that go as

$$\begin{aligned} \frac{\Gamma\left(\frac{5}{4} - i\tilde{X}\right)}{\Gamma\left(\frac{-1}{4} - i\tilde{X}\right)} &\sim \frac{i\left(r_+^4 + \omega^2\right)}{4r_+^2}\coth\left(\frac{\pi\omega}{2r_+^2}\right), \\ \frac{\Gamma\left(\frac{5}{4} - i\tilde{Y}\right)}{\Gamma\left(\frac{-1}{4} - i\tilde{Y}\right)} &\sim -\frac{\omega}{2r_+^2}\tanh\left(\frac{\pi\omega}{2r_+^2}\right). \end{aligned} \tag{4.85}$$

Recall that for our case, $b = -\omega/2r_H^2$. Plugging the results obtained in Eq. (4.85) in Eq. (4.73), the Green's function becomes

$$G_{O_+}(\omega, 0) = -\frac{i\omega\left(r_+^4 + \omega^2\right)}{3r_+}. \tag{4.86}$$

Ultimately, applying the condition $\omega \to 0$, one of the most significant observables of the dual model comes out as

$$\eta = \frac{r_+^3}{3}. \tag{4.87}$$

For the reasons that will become apparent, it is preferable to express the observables in terms of temperature. Thus, recalling $r_H = \sqrt{2\pi T}$, the shear viscosity can alternatively be written as

$$\eta = \zeta T^{3/2}, \tag{4.88}$$

in which $\zeta = \frac{2\sqrt{2\pi^3}}{3}$. Eq. (4.88) can be investigated for observing how the specific choice of metric exponents influenced the observables of the holographic image. Below, one can find the graphical illustration of the shear viscosity as a function of temperature.

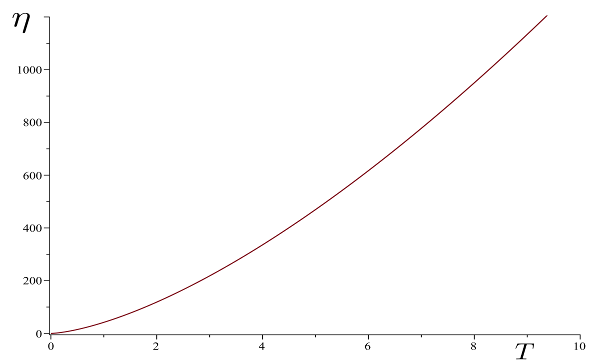

Figure 4.2: The shear viscosity as a function of temperature

From Eq.(4.88) one can comment that the dual theory lives in an effective dimension, which in general reads $d_{eff} = d_b - \theta$. Here, $d_b = 2$, as it represents the number of spatial dimensions on the boundary. For the case concerned here, i.e. for $(2,-1)$, the effective theory seems to live in three dimensions. The results of the analytical methods followed throughout this work suggest that for general (z,θ), the shear viscosity is directly proportional to a scaled temperature which goes as $\eta \propto T^{(d_b-\theta)/z}$. It is noteworthy to stress that our findings match with the ones in literature [104, 114–117].

4.3.2.2 DC-Conductivity

Another observable of the dual model is the so-called DC-conductivity. It can be derived from optical conductivity

$$\sigma^{ij}(\omega) = -\frac{1}{i\omega}\langle J^i(\omega) J^j(\omega)\rangle \tag{4.89}$$

which is the main bridge between the four-dimensional Lifshitz-like brane and the holographic $(2+1)$-dimensional fluid. Eq. (4.89) is also known as Kubo's formula in the literature and it can be viewed as the key function to be evaluated. Note that J^i denotes the current operator [118, 119]. The zero-frequency limit of Eq. (4.89) results in DC-conductvity, or mathematically speaking

$$\sigma_{DC} = \lim_{\omega\to 0}\sigma^{ij}(\omega). \tag{4.90}$$

For our case, Eq. (4.90) reduces to

$$\sigma_{DC} = \frac{e^2}{3}(2\pi)^{3/2}T^{3/2}, \tag{4.91}$$

where e is the charge of an electron. The DC-conductivity of the non-relativistic, strongly coupled fluid of our concern can be presented graphically as shown below.

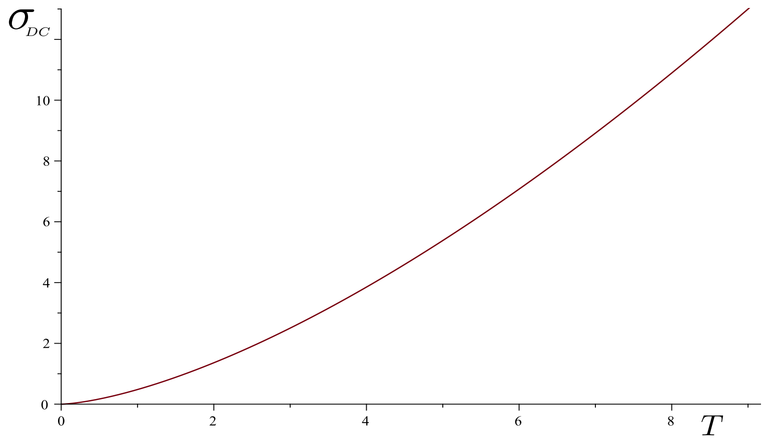

Figure 4.3: The graph of analytical DC-conductivity versus temperature

4.3.2.3 DC-Resistivity

The DC-resistivity of a strongly-coupled, non-relativistic fluid supporting superconducting fluctuations carries a vital importance, as its graphical representation can provide some conceptual findings. The DC-resistivity can simply be maintained via

$$\rho_{DC} = \frac{1}{\sigma_{DC}} = \frac{3}{e^2}(2\pi)^{-3/2}T^{-3/2}. \tag{4.92}$$

As can be seen from Eq. (4.92), the resistivity of the model exhibits an untrivial behaviour which deserves further attention. To be able to comment further, let us plot a graph presenting how resistivity behaves as a function of temperature.

From the graphical representation provided above, one can notice that there exists a sharp decrease in resistivity, around a non-zero temperature point. It is highly probable for this point to represent the critical temperature where a second-order phase transition takes place. As the bulk metric was chosen in such a way to support superconducting

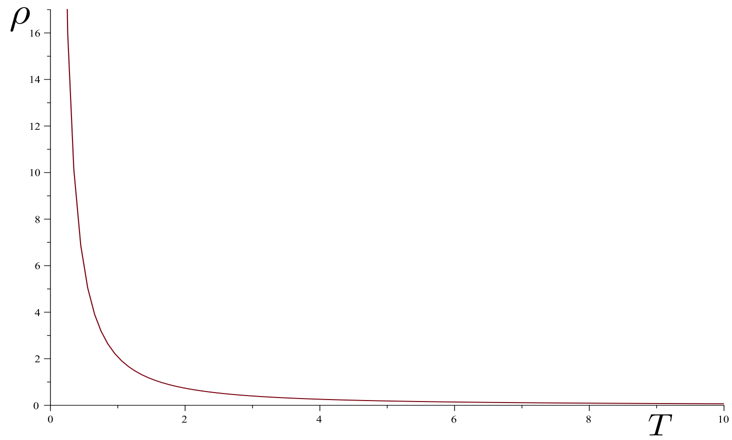

Figure 4.4: The graphical illustration of analytical DC-resistivity as a function of temperature

fluctuations, this is one of the key consequences belonging to this section.

In Ref. [118], one can have an access to a similar diagram. The authors first used analytical evaluations to figure out the holographic DC-conductivity of a system with arbitrary (z,θ), and subsequently discussed the behavior of the DC-resistivity for a non-relativistic system via numerical methods. Their analysis indicates that the DC-resistivity possesses different scaling behavior for different temperature regimes; namely, for $T \leq T_{critical}$ and $T > T_{critical}$. In our case, the holographic DC-resistivity behaves as $\rho_{DC} \propto T^{-3/2}$ which seems to be only a small portion of a broader picture. For studies on second order superfluid and superconducting phase transitions, one may refer to Refs. [120, 121].

Chapter 5

CONCLUSION

The key outcomes obtained within this thesis can be summarised as follows: As the starting point, a wide variety of gravitational interactions in the vicinity of $(2+1)$-dimensional Mandal-Sengupta-Wadia black holes and $(4+1)$-dimensional dilatonic black strings were investigated. Different dimensionalities and continuum objects were picked strategically, with the purpose of constructing a firm understanding on the semi-classical aspects of gravity for different scenarios. The effective potential of the Mandal-Sengupta-Wadia black hole was evaluated to be positive definite, whence assuring the linear stability. Besides, for the $(4+1)$-dimensional case, the analysis has shown that there exists a resemblance between tachyonic particles and the fifth dimension, as the greybody factor of the scattering process only allowed for imaginary masses to be present.

For the $(4+1)$-dimensional black string of concern, one of the key results obtained was that the greybody factor of the concerned object can be evaluated analytically, if and only if the mass is chosen to be imaginary. This suggests that there exists a resemblance between the fifth-dimension and the tachyons. As the last step, for the $(3+1)$-dimensional brane model, the propagation of massless scalar particles are analysed via Klein-Gordon equation; subsequently giving rise to analytical expressions for the absorption cross-section, decay rate, and greybody factor of the model. In what follows, the dual model is found to represent a strongly-coupled, non-relativistic fluid displaying Lifshitz-type symmetry. Furthermore, the analytical expressions obtained for the radiation parameters are linked to the theory living on the boundary of the bulk theory, giving rise to a relationship between a one-less dimensional theory and the gravitational theory of interest. Although the results obtained are purely theoretical, there exist strong indication that the findings are subject to experimentation, as the specific model of choice supports superconducting fluctuations. Moreover, the brane of concern has many implications in both string theory and condensed matter systems, as it is a solution of Einstein-Yang-Mills-Maxwell theory.

The main motivation behind this thesis was inspecting a holographic system where not only the bulk theory could be handled delicately, but also the dissipative observables of the dual scenario would be explored via the tools of a very substantial concept: the fluid/gravity correspondence. As aforementioned, one of the main tasks was maintaining information regarding strongly-coupled systems of the four-dimensional nature, as we perceive it. Having followed analytical methods, the two-point correlation function was found as $G_{O_+}(\omega,0) = -i\omega\left(r_+^4+\omega^2\right)/3r_+$, which resulted in $\eta \propto T^{3/2}$, $\sigma_{DC} \propto T^{3/2}$, and $\rho_{DC} \propto T^{-3/2}$. Note that these observables obey the general relation; i.e. $\eta \propto T^{(d_b-\theta)/z}$. These parameters correspond to the observables of the strongly-coupled, non-relativistic fluid in three dimensions. Although it is out of scope of this thesis, it would be inspiring to extend the study and check whether the dual model corresponds to a high-temperature superconductor. This can be achieved via experimental tools, and moreover, any possible confirmation of the theoretically-obtained dissipative parameters would act as a supplementary empirical evidence for the quantum properties of spacetime.

Furthermore, the analysis suggests that there exists a second order phase transition around some critical temperature within the theory. This outcome appears to be rather interesting,as the black brane of concern was originally chosen in such a way to support superconducting fluctuations, or in other words, the dynamic metric exponent was chosen as $z = 2$. The Lifshitz-like solution admitted by the Einstein-Yang-Mills-Dilaton action constrained the hyperscaling violation factor to be $\theta = -1$. Therefore, the holographic system under these conditions seems to be encrypted with a wealth of information not only on the theoretical aspects of the underlying string models, but also the ambiguous behaviour during the second order phase transitions of superconducting systems, which still remain as a peculiar phenomenon in experimental physics. The results obtained in this work can lead to interesting phenomena: the bulk spacetime includes Yang Mills and dilatonic fields, and from the perspective of an experimentalist, it contains information about superconducting phase transitions.

In this thesis, the effect of small perturbations on a $(2+1)$-dimensional Mandal-Sengupta-Wadia black hole, the tachyonic evaporation of a $(4+1)$-dimensional dilatonic black string, and the wave dynamics of a $(3+1)$-dimensional black brane with hyperscaling violation are investigated. Furthermore, in what follows, the dual observables living on the boundary of the $(3+1)$-dimensional brane are evaluated via linear response theory. Strictly speaking, different aspects of three and $(4+1)$-dimensional objects are first studied with the purpose of getting equipped with bulk-gravitational insights concerning the most dense objects of the universe. In order for not being limited to $(3+1)$-dimensions only, the theories were chosen to have different dimensionality. The concepts covered during the first three chapters are subsequently applied in Chapter 4, which is related to hyperscaling violating Lifshitz-like black branes in four dimensions.

For future references, it would be intriguing to extend the studies inspected in this thesis by checking the relevance of the findings with the theory of magnetic monopoles and D-branes from string theory and check whether it can provide us with any useful information regarding concepts like quark confinement and chiral symmetry breaking.

I would like to finalise the thesis with the words of Neil deGrasse Tyson: “We are all connected; to each other, biologically; to the earth, chemically. To the rest of the universe, atomically. I look up at the night sky, and I know that, yes, we are part of this universe, but perhaps more important than these facts is that the universe is in us. When I reflect on that fact, I look up—many people feel small, because they’re small and the universe is big, but I feel big, because my atoms came from those stars. We are not figuratively, but literally stardust. We are stardust brought to life, then empowered by the universe to figure itself out and we have only just begun. I know that the molecules in my body are traceable to phenomena in the cosmos. After all, what nobler thought can one cherish than that *the universe lives within us all*? The more of us that feel the universe and the connections within, the better off we will be in this world.”

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