Connectivity Analysis of Large-Scale Wireless Ad Hoc Networks with Heterogeneous Nodes

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Submitted to the
Institute of Graduate Studies and Research
in partial fulfillment of the requirements for the Degree of

Master of Science
in
Electrical and Electronic Engineering

Eastern Mediterranean University
January 2010
Gazimağusa, North Cyprus

Approval of the Institute of Graduate Studies and Research

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ABSTRACT

Research in large-scale ad hoc and wireless sensor networks (WSNs) has developed rapidly due to the large number of applications in environmental monitoring, structure health monitoring, contaminant detection, industrial process control, and military target tracking. The nodes in WSNs communicate in a multi-hop fashion to deliver the sensed data to a unit called the sink. Communication requires network connectivity that might not be possible all of the time due to sensor deployment strategy or node failure. Hence, connectivity can be considered as an essential requirement in WSNs, because without having a connected network, not all of the nodes will be reachable to transmit information.

This thesis has studied the problem of evaluating the connectivity in WSNs for a given number and given transmission ranges of nodes. Comparisons are carried out to evaluate the connectivity by using two different deployment strategies: deployment of homogeneous sensor nodes (in terms of transmission range) and heterogeneous sensor nodes. In particular, the connectivity of a WSN network with two types of nodes is analyzed.

A distinct feature of the study presented is the modeling of the network as a directed graph. Phase transition behavior of connectivity that is observed in homogeneous deployments is also apparent in networks with heterogeneous deployments where the networks are modeled as directed graphs. It is established through a large set of simulations that networks with homogeneous node deployments provide higher connectivity for a given power budget. These results are consistent with previous related findings that use undirected graphs for modeling networks.

Keywords: Ad Hoc Networks, Wireless Sensor Networks, Network Connectivity, Phase Transitions.

ÖZ

Tasarsız ve kablosuz algılayıcı ağlar (WSNs) hakkında araştırma, uygulama alanlarının genişliğinden dolayı hızla gelişmiştir. Bu uygulama alanları arasında çevre gözlemleme, sağlık durumu takibi, atık madde sezimi, endüstriyel süreç denetlemesi ve askeri hedef izleme bulunmaktadır. Algılayıcı ağlardaki düğümler çoklu aktarımlarla algılanan veriyi hedefe iletmektedir. İletişim için bağlantısallık gerekmektedir. Düğümlerin çalışmamasından veya yerleştirilmelerinden dolayı her zaman bağlantısallık sağlamak olanaklı değildir. Bu yüzden bağlantısallık telsiz algılayıcı ağların başarımlarını değerlendirmek için etkili bir ölçüttür.

Bu tezde, belirli bir sayıda düğüm ve bunların iletim uzaklığı ile elde edilen bağlantısallık oranları incelenmiştir. İki ana düğüm yerleştirme stratejisi olan tektürel ve çoktürel algılayıcı yerleştirme stratejileri karşılaştırılmıştır. Çoktürel yerleştirilmiş bir ağda iki farklı düğüm çeşidi kullanarak bağlantısallığı araştırılmıştır.

Çalışmanın özgün yanlarından biri algılayıcı ağların yönlendirilmiş çizge ile modellenmesidir. Tektürel yerleştirilmiş ağlarda görülen evre geçişleri, çoktürel yerleştirilmiş ağlarda da görülmektedir. Benzetimler (sabit bir güç oranı için), tektürel algılayıcı yerleştirme stratejisinin daha yüksek bağlantısallık sağladığını göstermektedir. Bu da daha önce yönsüz çizge ile yapılan modellemelerle elde edilen sonuçlarla uyuşmaktadır.

Anahtar Kelimeler: Tasarsız ağlar, Telsiz algılayıcı ağlar, Bağlantısallık, Evre geçişleri.

To My Parents, My Sisters and My Little Brother

ACKNOWLEDGEMENTS

First and foremost I wish to thank and express my appreciation to my supervisor and my teacher, Assoc. Prof. Dr. DOĞU ARİFLER for his effort, enthusiasm and support and giving me the freedom and time to work in this interesting topic during supervision of my thesis.

In particular I wish to thank my co-supervisor Assoc. Prof. Dr. AYKUT HOCANIN for offered expert advice and encouragement all of the time.

I would like to thank them for all their advice, insightful discussions and brilliant ideas that opened and showed the roadmap towards a successful work which was very useful in understanding the topic with more details. Without their support I would not have been able to complete this work.

I would like to express my special thanks to my parents, my sisters and also my little brother, for their love, trust, understanding, and every kind of support not only throughout my thesis but also throughout my life.

Finally I would like to express my appreciation to my best friends for their endless support and offering continuous encouragement during this time.

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LIST OF SYMBOLS/ABBREVIATIONS

A	Subregion of *B*.
B	Finite region.
$\lvert B \rvert$	Area of region *B*.
$D_i(R)$	Transmission disk of sensor a_i .
E	Edge.
G	Graph.
K	Constant of proportionality.
K_λ	2D homogeneous Poisson point process.
L	Side-length of the field.
N	Number of nodes.
P	Path between two nodes.
P_r	Received power.
P_{tx}	Transmitting power.
R	Range of large transmission range nodes.
R_c	Arbitrary communication range.
R_s	Arbitrary sensor range.
S	Sensing range.
V	Volts.
V	Vertex.
W	White noise power at the receiver.

a_i, a_j	Arbitrary sensor modes.
d	Transmitter-Receiver separation distance.
$l(d)$	Path loss.
mW	milli-Watt.
$n1$	Nodes of type 1 that have large transmission range.
$n2$	Nodes of type 2 that have small transmission range.
p	Probability of having (LTR) nodes.
r	Range of small transmission range nodes.
r_h	Range of homogeneous nodes.
s.e.	Standard Error.
$\mathbf{x}$	Random point.
α	Path loss exponent.
γ	Threshold.
λ	Sensor density.
μW	Micro-Watt.
$\mathbb{R}^2$	Two-Dimensional plane.
ζ_i, ζ_j	Location of sensors a_i and a_j.
$\lvert \zeta_i - \zeta_j \rvert$	Euclidean distance between the centers of transmission disks.
2D	Two-Dimensional.
Ah	Ampere-hour.
BGL	Boost Graph Library.
CDMA	Code Division Multiple Access.
GHz	Giga Hertz.

Kbps	Kilo bit per second.
KB	Kilo Byte.
LTR	Large Transmission Range.
LWIMs	Low Power Wireless Integrated Micro sensors.
MAC	Medium Access Control.
Mbps	Mega bit per second.
MB	Mega Byte.
MHz	Mega Hertz.
SNR	Signal-to-Noise Ratio.
STR	Small Transmission Range.
WINS	Wireless Integrated Network Sensors.
WSN	Wireless Sensor Network.

Chapter 1

INTRODUCTION

1.1 Wireless Ad Hoc and Sensor Networks

Ad hoc networks consist of nodes that communicate with each other wirelessly without the need for a fixed infrastructure. This differs from the traditional cellular or wireless local area networks that require a base station to mediate communication among nodes. One of the fields which have gained a growing interest by researchers in the last few years is wireless sensor networks (WSNs) [1]. WSNs are special kinds of ad hoc networks that have hundreds or thousands of nodes. Advances in technology and microelectronics have allowed WSNs to develop enormously. WSNs are autonomous and self-organizing systems that can communicate with each other using wireless technology.

The large number of sensor nodes which are deeply embedded into the physical environment can be used for environmental monitoring, structure health monitoring, contaminant detection, industrial process control, and military target tracking [1], [2]. These nodes have the capability to gather the required information from the environment and transmit the collected data to the fusion center. An example of sensor network configuration is illustrated in figure 1. Generally, the objective of a sensor network is to detect events of interests and send the aggregated data from all the nodes to a unit called the sink.

WSNs consist of hundreds or even thousands of small, cheap, and battery powered devices that can be deployed densely in the desired area called the deployment region. Each of these nodes is equipped with a processing unit, memory, sensing, power unit, and a short range radio transceiver unit, as for example illustrated in figure 2. The nodes communicate with each other in a multihop manner and every node can work as a router to forward the sensed data to the required destination.

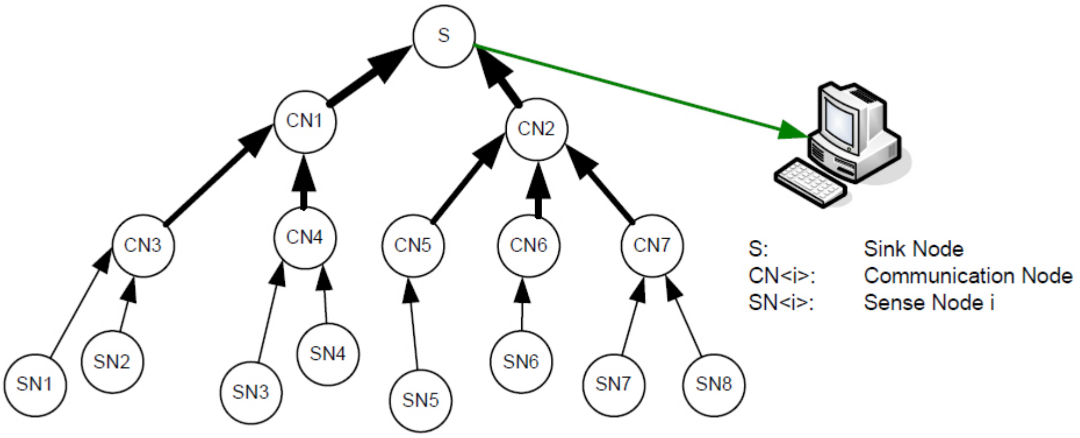


Figure 1: A wireless sensor network with tree topology, [3].

In a WSN, where the nodes have the ability for both sensing and communicating, coverage and connectivity issues are of primary concern [4], [5], [6]. Furthermore, WSN nodes are characterized by strict power and memory constraints, and are usually deployed in large numbers. For these reasons WSN nodes are different from any other traditional wireless devices. The resource constraint and power consumption are the most important issues in WSNs especially since as the physical size of nodes decreases, the energy capacity decreases as well [7].

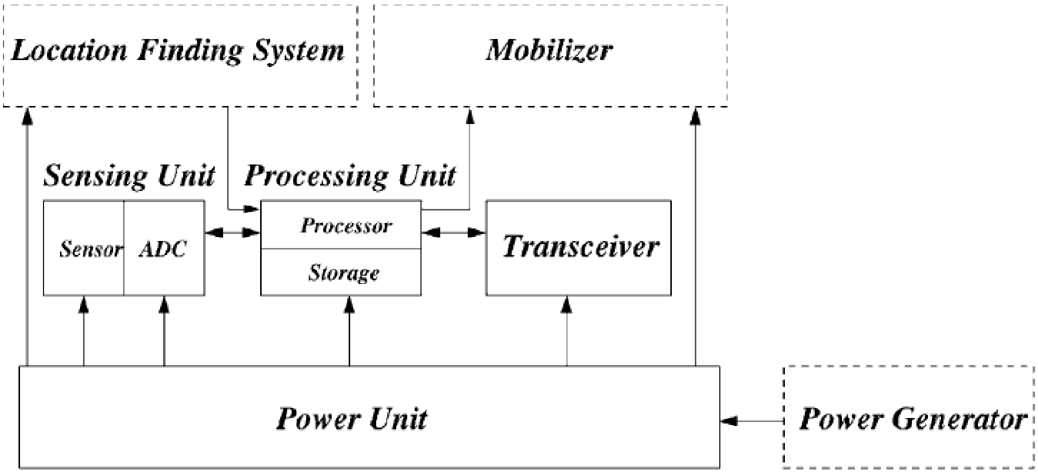


Figure 2: The general architecture of a sensor node, [1].

As mentioned earlier, the sensor node is a battery-powered microelectronic device. It can be equipped with a limited energy source (< 0.5 Ah, ~1.2 V). Transmission and reception, signal processing and idle listening are the main reasons for power consumption. However, the power consumed during transmission is the greatest portion of energy consumption of any node [1]. The energy required to transmit a packet of data over a distance *d* is larger for a larger value of *d*. Also, the energy consumed for transmitting a single bit of information is approximately same as that required for processing a thousand operations in a sensor node [8].

Communicating with a neighbor or close by node is less costly than communicating with nodes that are situated at farther distances. Because of that, the main challenge in sensor network design is energy conservation. Also, because the nodes are deployed in the deployment area are left unattended for long time, serious problems can be faced in the sensor network if a node is not functioning properly. Unfortunately during this time, any recharging or repairing or even the protection is not an available option. Hence the main focus of wireless sensor network design is to let the network perform its job in a perfect manner using the least possible amount of energy without affecting or breaking the connectivity of the nodes or decreasing the coverage of the deployed region.

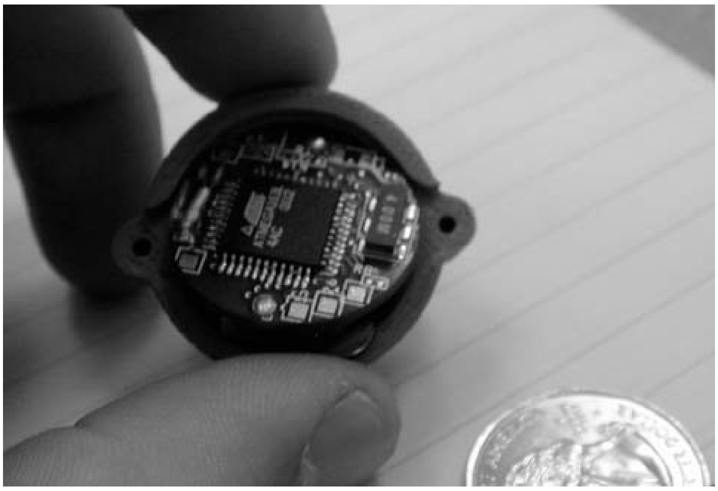


Figure 3: A wireless sensor network device, reproduced from [7].

In general, when people think about the wireless devices they think about laptops with an 802.11 interface or cell phones. These devices cost hundreds or thousands of dollars and they rely on complex and extensive infrastructure support. WSN nodes, on the other hand, are low cost small embedded devices as shown in figure 3 and do not rely on any pre-existing infrastructure, and future plan is to let the cost of this device to be less than $1 [7].

In 1996, UCLA and Rockwell science center produced the Low Power Wireless Integrated Micro sensors (LWIMs). This node supports over 100 Kbps wireless communications at a range of 10 meters using only 1 mW transmitter [9]. The Wireless Integrated Network Sensors (WINS) were produced later as the second generation in 1998 by same team. Each contained an Intel embedded processor and 4MB flash memory with 1MB RAM. Radio transmission supports 100 Kbps with capability to adjust the power consumption from 1 to 100 mW in active status and 0.8 mW in sleeping status. figure 4-a shows this model. Research efforts and new technology emerged later with a new cheaper model that had smaller size with less power consumption. For example, WeC node in the family of Mote products by UC Berkeley was released in 1999 (figure 4-b). It was built with 4 MHz Atmel microcontroller with 512 bytes of RAM and 8 KB of flash memory. Power consumption was decreased to $15\ m\mathrm{W}$ in active power state and $45\ \mu\mathrm{W}$ in sleep power state. This model also had a simple radio and supported $10\ \mathrm{Kbps}$ of data transmission with $36\ m\mathrm{W}$ transmitting power and $9\ m\mathrm{W}$ receiving power [9].

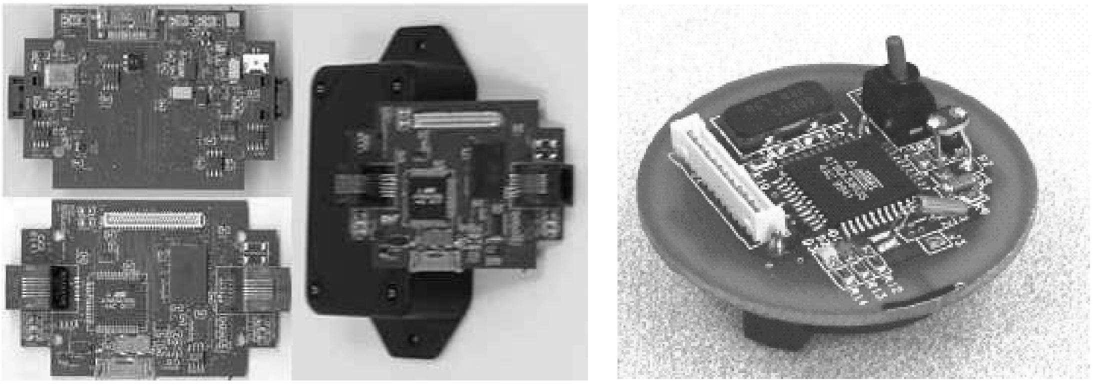

(a) (b)

Figure 4: models of wireless sensor nodes. (a) WINS sensor node. (b) WeC sensor node (motes family), reproduced from [9].

The need to find an alternate method of energy conservation for WSN or developing a technology which helps to extend the life time of a node is definitely important, since the network life time and its operation for several months or years depends on the power supply. For this reason, there are some ways that might help to improve energy storage or enhance power consumption technology:

- Find a method that helps to distribute power to the nodes in a better manner;
- Increase the energy density of the power resource or the battery;
- Discover a technology that help and enable a node to generate its own power by itself.

It is worth to mention that the new communication technologies such as Bluetooth [10] and ZigBee [11] have made a revolutionary change in wireless data communication.

The Bluetooth technology focuses on connectivity between large packet devices such as laptops and cell phones with data transmission up to 1Mbps and operating at 2.4 GHz. It is designed for low power consumption with three classes of radio transmissions: class 3 radios have a range of up to 1 meter, class 2 radios commonly used for mobile devices with a range of 10 meters, and class 1 radios have a range of 100 meters [10].

ZigBee technology focuses on providing high efficient connectivity between small packet devices with a transmission rate up to 250 Kbps and operates on 2.4 GHz. It is developed to meet the requirements of wireless networking among numerous low power devices such as sensor nodes. In fact, it is the most promising technology for ultra low power applications especially for wireless sensors nodes.

ZigBee devices can function for months or years with a small battery due to their low power output. Also, these devices in general have 50 meters of maximum range typical of WSN operation [11].

1.2 Problem Description

This thesis analyzes connectivity in wireless sensor networks. Connectivity is a graph theoretic concept that may be used to analyze whether the nodes can communicate with each other in a wireless ad hoc network. While coverage of a network represents the quality of service (e.g., surveillance) provided by the network, connectivity is used to assess the ability of transmitting and sharing information with other nodes in the network. In this work, connectivity is defined as the fraction of the nodes in the largest connected component in the network. The notions of connectivity and connected component will be made precise in Chapter 2.

The work in this thesis will be largely based on wireless sensor network settings. While connectivity does not imply coverage or vice versa, previous work proved that if transmission radius of a node is at least twice the sensing range, the network is connected provided that the coverage is guaranteed [12]. We will say that the network is made up of *homogeneous* nodes if they have identical transmission/sensing properties. Related work in the study of connectivity/coverage of wireless ad hoc networks has mainly focused on analyzing connectivity with homogeneous nodes due to analytical tractability.

The work in this thesis will involve the use of computer simulations in MATLAB to analyze improvements or deteriorations in connectivity when using heterogeneous nodes. We will say that the network is made up of *heterogeneous* nodes if they do not have identical transmission/sensing properties. In particular, the effects of having two types of nodes, one type with a small and the other type with a larger transmission range, on connectivity are investigated in detail for different transmission ranges and different number of nodes. As explained in the previous section, power consumption is an important constraint in WSNs. As such, when analyzing connectivity properties, power requirements of deployment strategies will also be taken into account. The findings of this work are expected to have an impact on the design and implementation of wireless ad hoc and sensor networks.

1.3 Related Work

In this section, existing work on connectivity and coverage in sensor networks is summarized. Gilbert [13] was the first to model a network of short-range wireless stations as a random graph. In his work, he modeled the locations of stations as a spatial Poisson point process. The locations of stations constituted the vertices or points of the graph. A pair of points was joined by an edge if the pair was separated by a distance less than R, which was called the range of the station. He showed that there existed a critical value for the expected number of points in a circle of radius R beyond which the number of stations belonging to a connected component was comparable to the total number of stations. If that number, on the other hand, was below the critical value, the network provided only local communication.

Cheng and Robertazzi [14] investigated how far a node's message percolates for Poisson distributed nodes on an infinitely large area. Philips et al. [15] showed that in order to have a connected network the expected number of neighboring nodes must grow logarithmically with the area of the network. They also showed the existence of a critical value for the expected number of nearest neighbors of a node to have an infinite connected component in an infinite area. Gupta and Kumar [16] studied asymptotic connectivity (i.e., when the number of nodes is large) and showed that the critical transmission range is $O\left(\sqrt{\frac{\ln n}{n}}\right)$.

Xing et al. [12] studied the coverage and connectivity and showed that if the radius of the transmission range of the nodes is at least twice the radius of their sensing range then the wireless sensor nodes are connected provided that sensing coverage is guaranteed. In another words, if $R_c \geq 2R_s$, where R_c is the communication range and

R_s is the sensing range, coverage implies connectivity. Dousse et al. [17] studied both pure ad hoc and hybrid networks and showed that the introduction of a sparse network of base stations helped significantly to improve the connectivity of the network. Also, they showed at low spatial density of nodes bottlenecks were unavoidable. Towsley and Liu [18] investigated the properties of large scale sensors networks and showed that the coverage of a sensor network exhibits different behaviors for different network configuration and parameters. Percolation based methods for studying the coverage and connectivity of wireless sensor networks were proposed in [6] and [17]. Miorandi and Altman [19] developed an analytical procedure for node isolation probability calculation in ad hoc networks in the presence of channel randomness for nodes distributed according to a Poisson point process. These results are used for obtaining an approximation of the connectivity features for very dense networks. A notable finding was that given a mean transmit power constraint, connectivity could not be improved by means of any random transmit power selection scheme. In [20], three types of connectivity were considered: full coverage with connectivity, partial coverage with connectivity and constrained coverage with connectivity. Two simple network topologies were shown to satisfy the constrained coverage with connectivity criterion. Dong et al. [21] studied the minimum density of nodes needed to achieve a connected wireless network with respect to Rayleigh fading channels. According to obtained results, fading effect degraded the connectivity of the wireless senor network.

Bettstetter [22] derived an approximation for the probability that a network (with border effects eliminated) is connected when the number of nodes N is large and the probability that the minimum node degree is one or more is almost 1:

$$P(\text{connected}) \cong P(\text{min. degree} \geq 1) = (1 - \mathrm{e}^{-\lambda\pi r^2}) \tag{1.1}$$

The critical transmission range is then given by

$$r \cong \sqrt{\frac{-\ln(1 - p_c^{\ 1/N})}{\lambda\pi}} \tag{1.2}$$

where p_c is the probability that the network is connected and λ is the density of nodes.

The work closest to the one described in this thesis is by Bettstetter [23]. He investigated the connectivity of randomly distributed wireless multihop networks. He considered nodes with homogeneous range assignments as well as heterogeneous range assignments. However, he considered two points as connected if there was a two-way communication between the points (i.e., the constructed graph for the network was undirected).

1.4 Contributions and Organization

This thesis has the following contributions:

1) First, the existing results on the connectivity of networks with homogeneous nodes are verified through simulations.

2) Then, the connectivity of networks with two types of nodes is analyzed. A distinct feature of the study is the modeling of the network as a directed graph. Phase transition behavior of connectivity is also apparent in networks that are modeled as directed graphs.

3) Finally, it is established through a large set of simulations that networks with homogeneous node deployments provide higher connectivity for a given power

budget compared to networks with heterogeneous nodes. These results are consistent with previous related findings, especially with those reported in [23].

The rest of the thesis is organized as follows. Chapter 2 describes models and methods employed in this thesis. Chapter 3 presents and analyzes the results of simulations. Chapter 4 concludes the thesis.

Chapter 2

MODELS AND METHODS

2.1 Graph Theory Background

First, several definitions from the graph theory will be introduced. The descriptions follow closely to those in [24]. A graph is a pair $G = (V, E)$ of sets where the elements of V are the vertices (or nodes, or points) of the graph, and the elements of E are its edges. An edge is usually written as xy where x and y are end vertices. Two vertices x and y are adjacent, or neighbors, if xy is an edge. A vertex with no neighbors is said to be isolated. A graph is said to be directed if the edges connecting the vertices are one-way; otherwise, the graph is undirected.

A path is a non-empty graph $P = (V, E)$ with

$$V = \{x_0, x_1, \ldots, x_k\} \qquad E = \{x_0x_1, x_1x_2, \ldots, x_{k-1}x_k\}$$

A non-empty graph G is called connected if any two of its vertices are linked by a path in G. Every connected graph contains a (spanning) depth-first search tree, with any given vertex as its root. Hence, the connectivity of a graph can be determined by running the depth-first search algorithm. Time complexity of the depth-first search algorithm is $O(V + E)$. A graph that is not connected is made up of connected components. The connected component with the largest number of nodes is called the largest connected component.

2.2 Point Processes

In the theory of point processes [25], Poisson point process is the most fundamental. A stationary homogeneous spatial Poisson point process satisfies the following conditions:

1. The number of points occurring within a finite region B is a random variable following a Poisson distribution with mean $\lambda|B|$ for some positive constant λ where $|B|$ is the area of B.

2. Given the total number of points n occurring in B, the locations of the n points represent an independent random sample of n locations, where each location is equally likely to be chosen in the area.

Hence, the restriction of a stationary Poisson point process to a region B under the condition that B has n points yields a Binomial point process with n points which is described next.

We say that a random point $\mathbf{x}$ is uniformly distributed in a region B if

$$P(\mathbf{x} \in A) = \frac{|A|}{|B|} \tag{2.1}$$

for all regions A in B. Also, n independent uniformly distributed random points can be superposed to form a binomial point process of n points:

$$P(\mathbf{x}_1 \in A_1, \ldots, \mathbf{x}_\mathrm{n} \in A_n) = \frac{|A_1| \cdots |A_n|}{|B|^n} \tag{2.2}$$

where A_i are subregions of B. The intensity of the binomial point process is denoted by λ. The intensity is given by

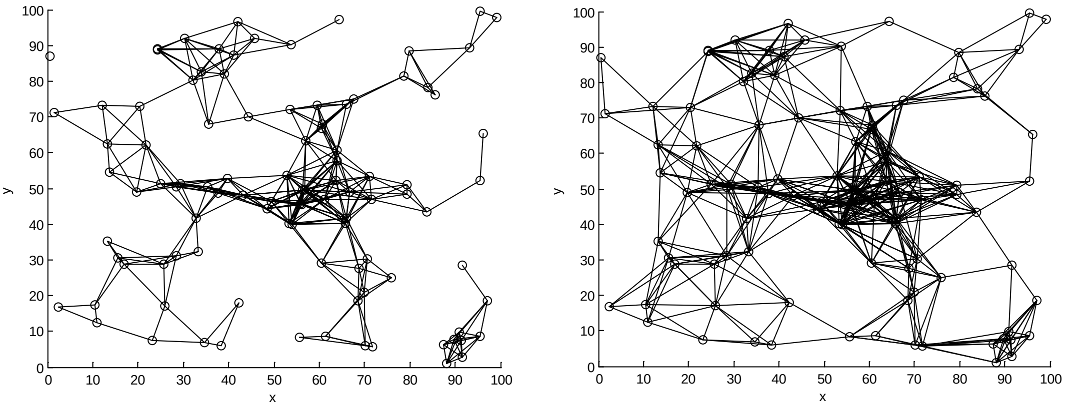
$$\lambda = \frac{n}{|B|} \tag{2.3}$$

The simulation of a binomial point process is trivial. One simulates a random point uniformly over a given region and the binomial point process is obtained by superposing *n* independent random points. A random point uniformly distributed on $[0,1]^2$ is obtained by uniformly generating the x-coordinate in $[0,1]$ and uniformly generating the y-coordinate in $[0,1]$

2.3 Random Geometric Graphs

A random geometric graph [2] can be defined as follows: A set of nodes or points is randomly scattered over a region according to some probability distribution. If two points separated by a distance less than a certain specified value (called the range), the points are connected by an edge.

Of particular interest in this work is the set of points distributed according to Binomial point process in a given region. In figure 5, the connectivity graph of a binomial point process with 100 points in a 100-by-100 region is illustrated. In figure 5-a, the range of nodes is 15 *m*, and in figure 5-b the range is 20 *m*. The graph in figure 5-a consists of connected components, whereas the graph in figure 5-b is fully connected.



(a) (b)

Figure 5: Connectivity graph of binomial point process, $N = 100,\ L = 100\ m$.
(a) Range = 15 *m*. (b) Range = 20 *m*.

2.4 Radio Propagation and Path Loss Model

The communication range or communication radius of a node is defined as the distance r which the node can communicate with all the nodes within this distance. The signal to noise ratio (*SNR*) at a receiver at a distance *d* from the transmitter is given by

$$SNR(d) = \frac{P_{tx} l(d)}{W} \tag{2.4}$$

where P_{tx} is the transmit power and W is the white noise power at the receiver. The function $l(d)$ is the path loss which is assumed to be proportional to $\left(\frac{1}{d}\right)^{\alpha}$ or

$$l(d) = Kd^{-\alpha} \tag{2.5}$$

where $K > 0$ is the constant of proportionality and α is the path loss exponent. Path loss exponent is usually assumed to be between 2 and 5. The value of α depends on the environment. In free space, it is close to 2 and in urban places, it is approximately equal to 3. The exponent will be larger when more obstructions are present.

Given a threshold γ, called the receiver sensitivity, the communication between a transmitter and a receiver at a distance d is successful if

$$\frac{P_{tx} l(d)}{W} \geq \gamma. \tag{2.6}$$

With $l(d) = Kd^{-\alpha}$, the communication is successful if

$$d \geq \left(\frac{KP_{tx}}{W\gamma} \right)^{1/\alpha} \tag{2.7}$$

Thus one can select the communication range

$$r = \left(\frac{KP_{tx}}{W\gamma} \right)^{1/\alpha} \tag{2.8}$$

for a given transmit power and receiver sensitivity. A detailed account of radio propagation and path loss models can be found in [26].

In this work, the effect of interference is ignored: It is assumed that there is either a Code Division Multiple Access (CDMA) scheme with perfectly orthogonal codes in effect or there is an efficient Medium Access Control (MAC) protocol at work that mediates access to the wireless channel.

The model described in this section is a simple communication model variously termed as the Boolean model, the disk model or the Gilbert model. It is a first-order approximation of communication with isotropic radio signals. In a network modeled as a graph, we will assume that there is an edge if the distance between the nodes is less than or equal to *r*.

2.5 The Toroidal Distance Metric

In spatial point process analysis, the effect of borders on simulation results may be evaluated by using a “wrap-around” or cyclic distance measure called the toroidal distance [22]. The basic idea is to create a torus so that a point on the edge of the region is considered “close” to nodes near the opposite edge of the region. This can be visualized by considering figure 6. The toroidal distance between two points $p_i=(x_i, y_i)$ and $p_j=(x_j, y_j)$ may then be easily calculated by using the following formula:

$$\begin{aligned} d_T(p_i, p_j) = \min\{ & \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}, \\ & \sqrt{(x_i - x_j + x_{\max})^2 + (y_i - y_j)^2}, \sqrt{(x_i - x_j)^2 + (y_i - y_j + y_{\max})^2}, \\ & \sqrt{(x_i - x_j - x_{\max})^2 + (y_i - y_j)^2}, \sqrt{(x_i - x_j)^2 + (y_i - y_j - y_{\max})^2}, \\ & \sqrt{(x_i - x_j + x_{\max})^2 + (y_i - y_j + y_{\max})^2}, \sqrt{(x_i - x_j + x_{\max})^2 + (y_i - y_j - y_{\max})^2}, \\ & \sqrt{(x_i - x_j - x_{\max})^2 + (y_i - y_j + y_{\max})^2}, \sqrt{(x_i - x_j - x_{\max})^2 + (y_i - y_j - y_{\max})^2}\} \end{aligned} \tag{2.9}$$

The toroidal distance measure enables us to evaluate simulation results as if the nodes were deployed in an infinitely large region.

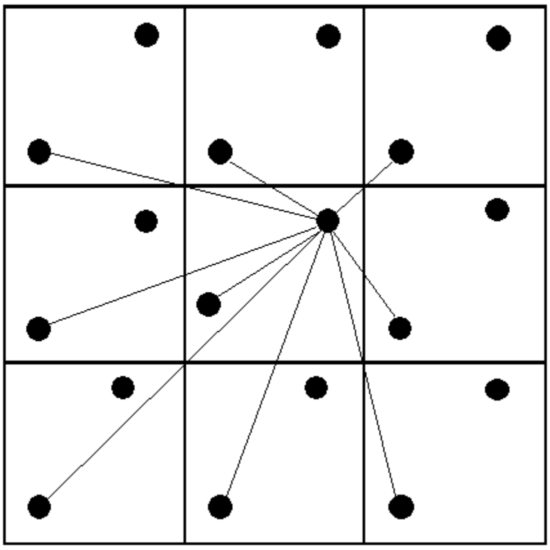


Figure 6: Calculation of the toroidal distance. The width and height of each rectangle is $x_{\max}$ and $y_{\max}$, respectively.

Chapter 3

SIMULATIONS AND ANALYSIS

3.1 Simulation Model and Assumptions

Simulations are used to analyze connectivity of homogeneous and heterogeneous WSNs. The simulations are written in Matlab. Graph algorithms make use of the Boost Graph Library (Matlab BGL) toolbox [27].

We will assume that N nodes are deployed in an $L \times L$ area. The node density will then be equal to:

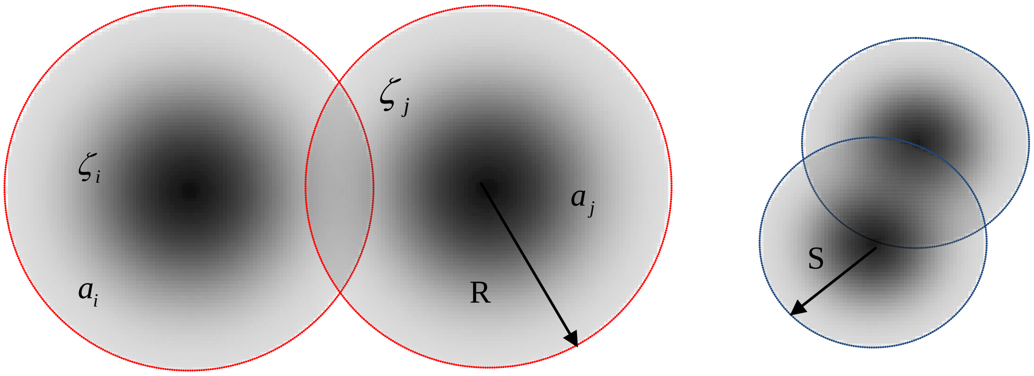
$$\lambda = \frac{N}{L^2} \tag{3.1}$$

Some modeling assumptions are outlined below.

Let $K_\lambda = \{\zeta_i : i \geq 1\}$ be a two dimensional homogeneous Poisson point process of density λ where ζ_i represents the location of the sensor a_i. Let $K_\lambda(L)$ be a random variable representing the number of points in an area $A = L \times L$. The probability that there are N points inside A is computed as:

$$p(K_\lambda(L) = N) = \frac{\lambda^N |A|^N e^{-\lambda|A|}}{N!} \tag{3.2}$$

For all $N \geq 0$ where $|A|$ is the size of A's area.



a. Communication range *R*　　　　b. Sensing range *S*

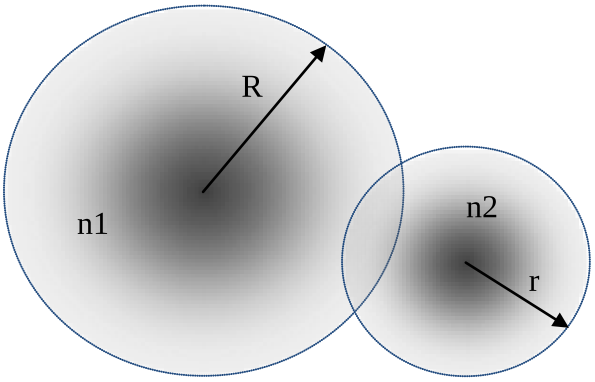
Figure 7: Graphical illustration of sensors nodes.
(a) Communicating sensors. (b) Collaborating sensors.

The transmission range of a sensor a_i is a disk of radius *R* centered at ζ_i and defined by: $D_i(R) = \{\zeta \in \mathbb{R}^2 : |\zeta_i - \zeta| \leq R\}$ where $D_i(R)$ is the transmission disk of sensor a_i which is centered at ζ_i (location of a_i).

Two sensors a_i and a_j are connected if and only if the Euclidean distance between the centers of their transmission disks satisfies $|\zeta_i - \zeta_j| \leq R$. (Figure 7).

It is worth noting that the radius of sensing disks of the sensors and the radius of their transmission are totally different. If we assume the radius of sensing disk is S and transmission range is R then they related to each other by $R = \beta \times S$ where $\beta \geq 2$. [12].

The network setup in the simulation scenario will be configured by using two types of nodes; the first type with a Small Transmission Range (STR) and the second type with a Larger Transmission Range (LTR). However, using nodes that have larger transmission range will consume more power than the short range sensors.



(a) Large transmission range (LTR) (b) Small transmission range (STR)

Figure 8: Illustration of transmission ranges for wireless sensor nodes.

Two deployment strategies will be tested: Homogeneous and heterogeneous (in terms of transmission range). As in figure 8, heterogeneous nodes (type n1) will have a range R (large transmission range), and a range r for the nodes of type $n2$ (small transmission range).

Homogeneous deployment strategy will have sensor nodes that have a range defined by r_h as shown in the figure 9.

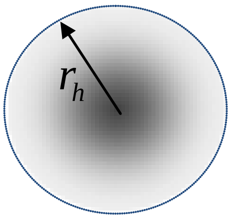


Figure 9: Homogeneous sensor node with transmission range.

For simulations, we considered radio model described in Section 2.4. Path loss exponent α is set to 3. We deploy 100, 200, 300, 400 and 500 nodes randomly and study the impact of placing such number of nodes in the deployment region on the connectivity properties. The simulation parameters that were used to obtain the results are listed in Table 1.

Table 1: Parameters applied in Matlab simulations.

Parameter	Notation	Value
Large Transmission Range node	R	(Test 1) R = 2-40 m (Test 2) R = 6-40 m
Small Transmission Range node	r	(Test 1) r = 2 m (Test 2) r = 6 m
Percentage of LTR nodes	p	{0.05, 0.10, 0.20, 0.60}
Number of deployed nodes	N	{100, 200,… , 500}
Path loss exponent	α	3
Side length of area	L	100 m
Range of homogeneous nodes	r_h	{2, 3,…, 20} m

The simulation experiments conducted focused on the connectivity of heterogeneous wireless networks. Comparisons were made between wireless sensors networks with homogeneous and heterogeneous nodes. All reported results were obtained by averaging results of 20 simulations. The sufficiency of 20 runs is further discussed in Appendix A.

Connectivity is analyzed by finding the connected components of a graph formed by the network of nodes. Connectivity is defined as the number of nodes in the largest connected component divided by the total number of nodes. Connectivity values close to 100% are desirable.

A heterogeneous network is created by designating a given proportion p of nodes as LTR nodes. The communication range in heterogeneous case is R for Large Transmission Range (LTR) nodes and r for Small Transmission Range (STR) nodes. In homogeneous case**,** we set the range of nodes to be r_h .

As mentioned, sensors are deployed in a random manner with node density λ in a square field with dimension (L x L) which is (10,000 m^2). Node density λ can be defined as number of sensor nodes in unit area. Figure 10 shows a sample of a hundred wireless sensor nodes has been separated randomly in the field.

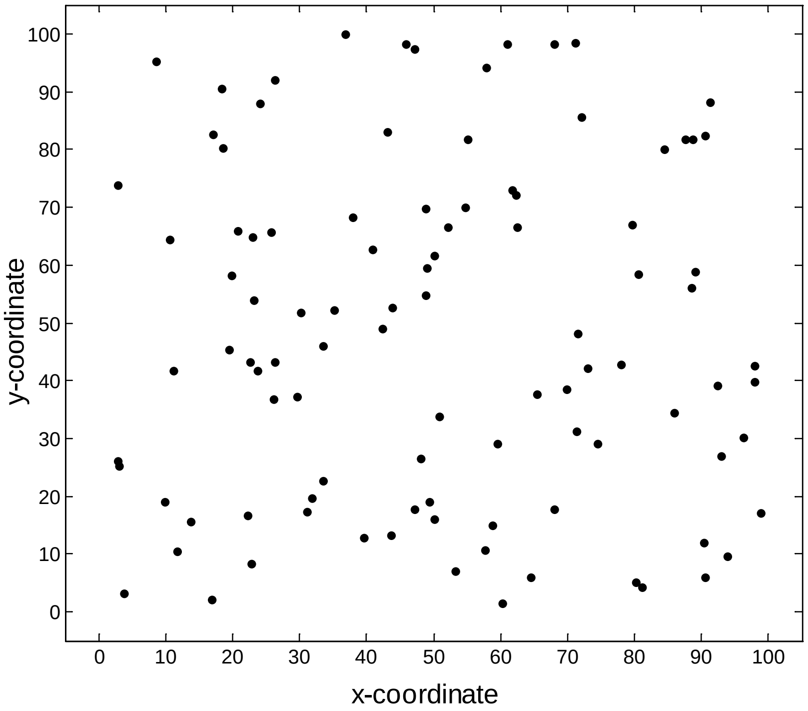


Figure 10: 100 sensor nodes randomly distributed in a 100 *m* by 100 *m* field.

3.2 Homogeneous Deployment Results

Figure 11 shows the connectivity results for wireless sensor network simulations with homogeneous nodes. The connectivity is higher if more nodes are deployed. The range of nodes r_h has its influence as well. The connectivity rises up when the range of the nodes increases. The connectivity reaches 100% when $r_h \geq 8$ meters for N=500. The connectivity of the network increases in a sharp manner after some point. This is called a phase transition: small fragmented and disconnected clusters (suddenly) become a single large connected component.

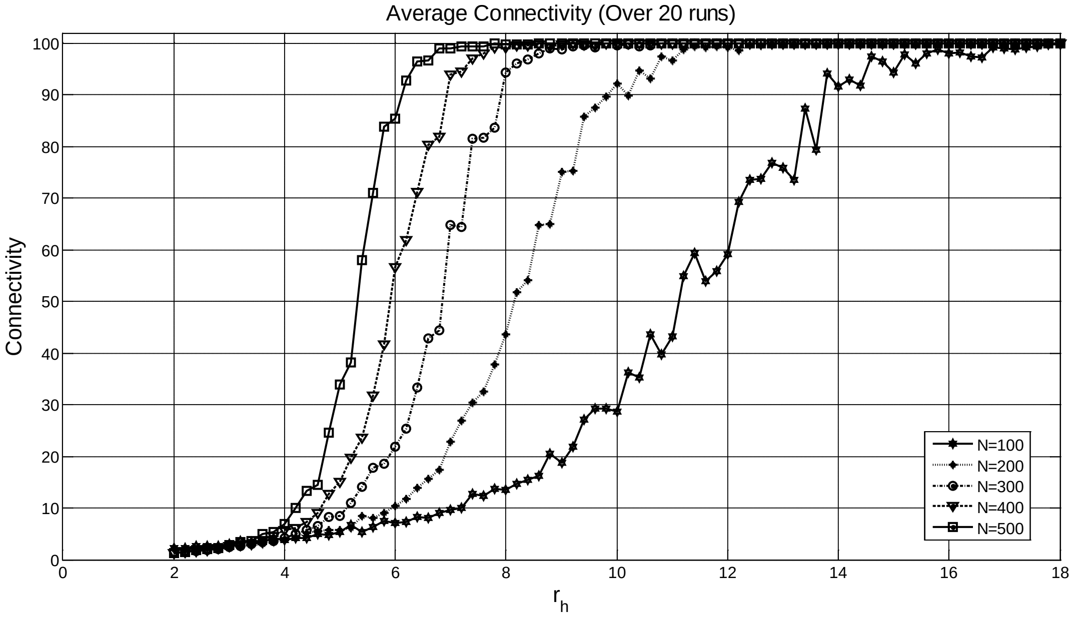


Figure 11: Network connectivity with homogeneous wireless sensor nodes.

Figure 12 shows the results of homogeneous deployment strategy with a transmission range starting from 2 until 16 meters with the number of nodes N = 100, 200, 300, 400 and 500, but this time using the toroidal distance metric in (2.9). The results show that the connectivity reaches to almost 100% when $r_h \geq 7$ meters for N=500. Sharper transitions to full connectivity are apparent compared to figure 11.

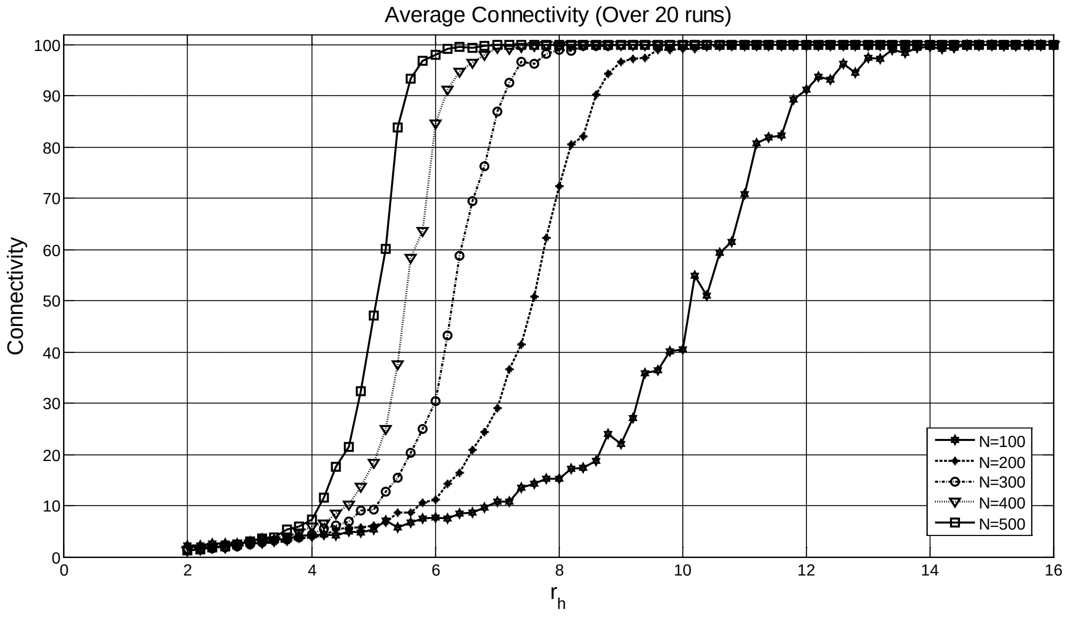


Figure 12: Network connectivity with homogeneous nodes using the toroidal distance metric.

The results obtained by simulations are consistent with previous findings. The critical transmission range threshold is apparent from the presented graphs.

The critical transmission ranges for 99% connectivity with Bettstetter's approximation obtained using (1.2) and the simulation results for homogeneous node deployments presented in this work are compared in Table 2. The reported values differ by at most 22%.

Table 2: Comparisons of critical transmission range for connectivity obtained via simulations and by using approximation (1.2), [23].

Number of Nodes (N)	r (m) (Approximation using (1.2))	r (m) (Simulation results)	r (m) (Simulation results with toroidal distance metric)
100	17.2	17.4	14.2
200	12.6	11.4	10
300	10.5	9.2	8.6
400	9.2	7.8	7.2
500	8.3	7	6.2

3.3 Heterogeneous Deployment Results

In the heterogeneous deployment scenario, nodes of type *n*1 and *n*2 with different transmission ranges were deployed in the given environment. The transmission range of *n*1 type is *R*, and *r* for type *n*2.

The following analyses will give us insight about the strategy we should prefer in deploying the nodes. We are interested in the connectivity of all of *N* deployed nodes and the extent of power consumption with different values of α and different probabilities p of having LTR nodes.

To evaluate the strategy, power requirement will be taken into account. The power consumed by the homogeneous deployment is proportional to r_h^α. The power consumed by the two-type heterogeneous deployment is proportional to $(1-p)r^\alpha + pR^\alpha$. The ratio of the power consumption in the heterogeneous case to the one in homogeneous case is given by

$$Power\ ratio = \frac{\left[(1-p)r^\alpha + pR^\alpha\right]}{\left(r_h^\alpha\right)} \tag{3.3}$$

figure 13 shows power ratio using transmission range r= 2 m for STR nodes and from R= 2 m to R= 40 m for LTR nodes, and homogeneous transmission range r_h = 10 m with path loss exponent set to α = 3.

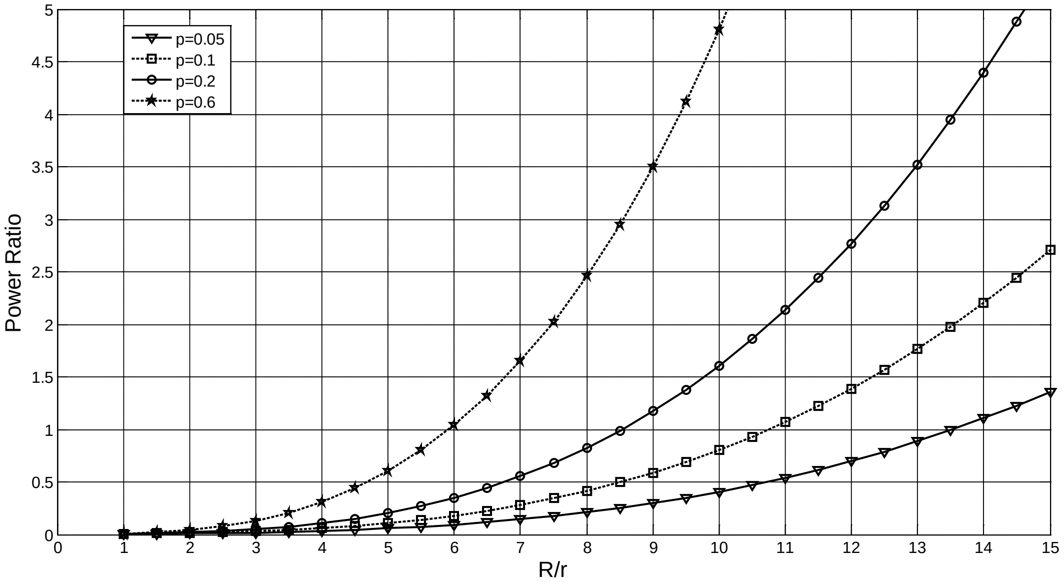


Figure 13: Power ratio, α = 3, r_h= 10 m, r = 2 m, R = 2-40 m.

The power ratio increases rapidly when the ranges of *r* and *R* are increased from 2 *m* in figure 13 to 6 *m* in figure 14.

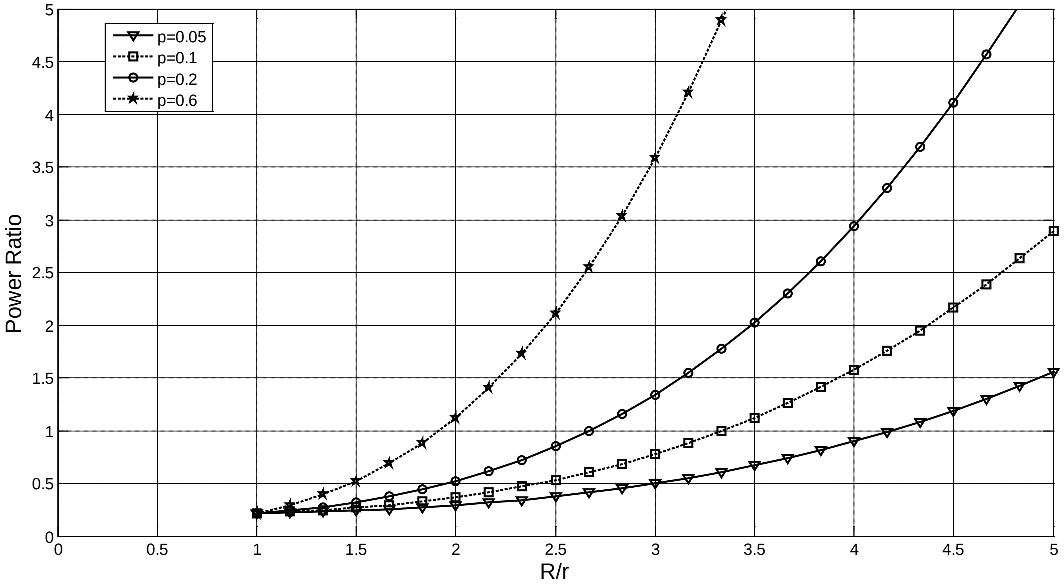


Figure 14: Power consumption ratio, α = 3, r_h =10 *m*, *r* = 6 *m*, *R* =6-40 *m*.

The effect of path loss exponent can be exhibited by comparing figure 14 and 15. When the path loss exponent is increased to 4, the power ratio curves increase more rapidly.

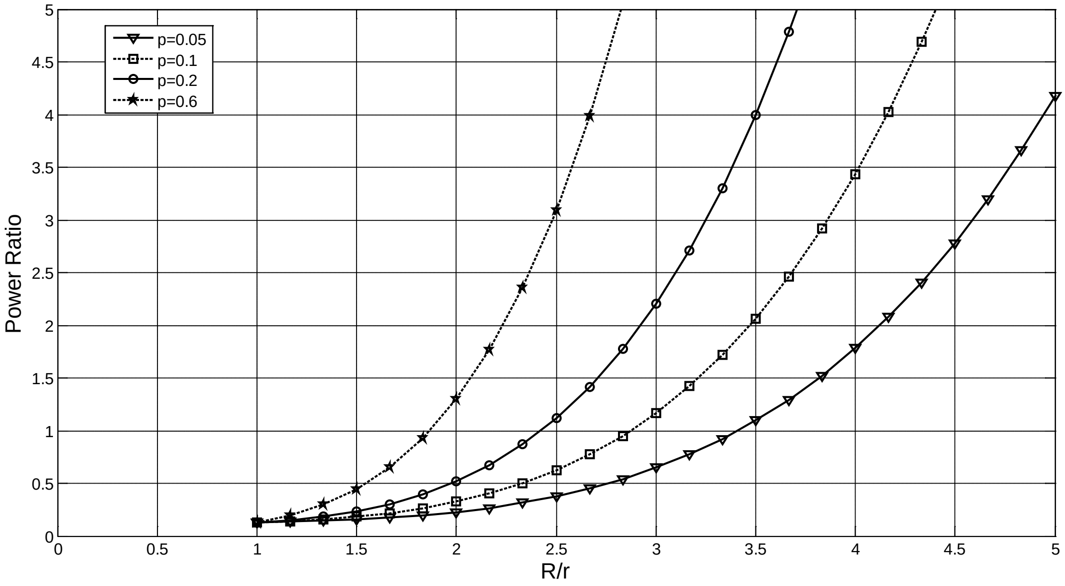


Figure 15: Power consumption ratio, α = 4, r_h= 10 *m*, *r* = 6 *m*, *R* = 6-40 *m*.

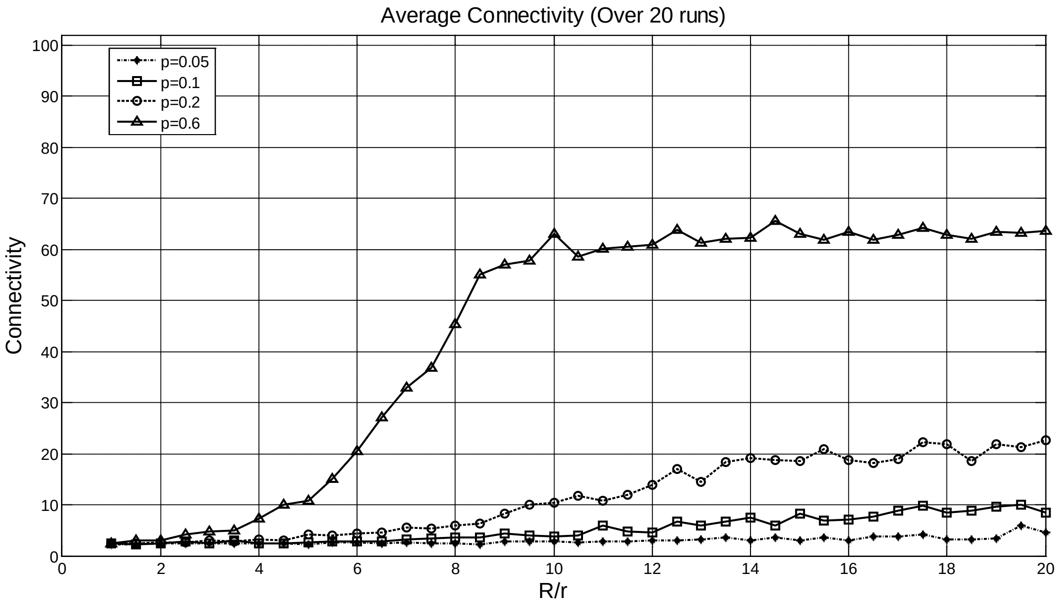
Connectivity of homogeneous and heterogeneous deployment strategies can be compared by identifying the transmission parameters for which the powers consumed by the strategies are equal (i.e., the power ratio is equal to 1). Table 3 summarizes identified *R*/*r* in the heterogeneous cases using figure 13 and 14.

Table 3: The ratio *R*/*r* for a power ratio 1.

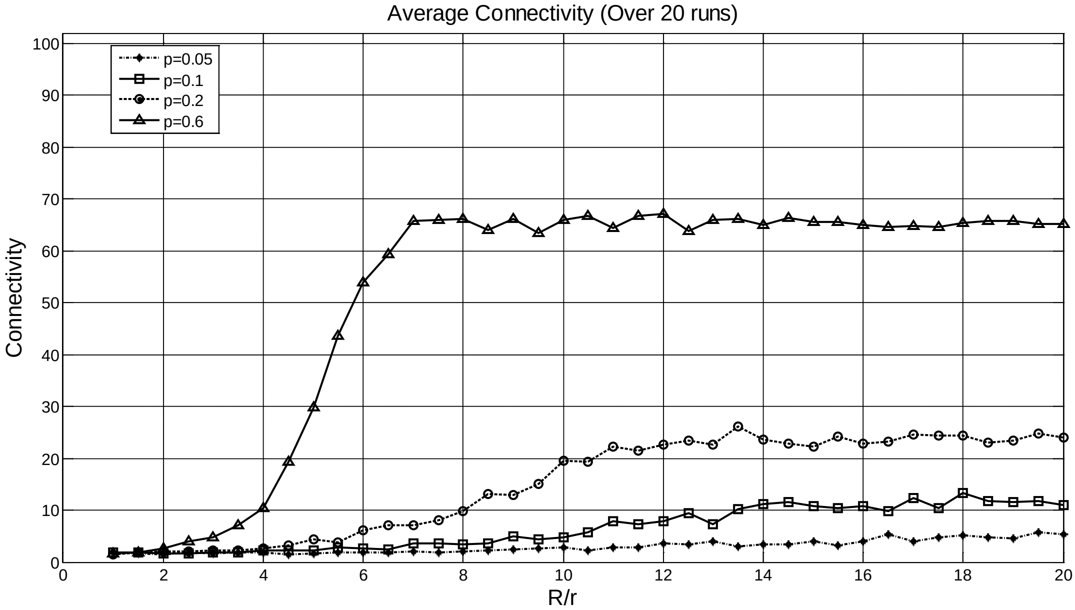
Probability of LTR / Ranges (m)	*p*= 0.05	*p*= 0.10	*p*= 0.20	*p*= 0.60
r = 2 *R* = 2-40	*R*/*r* = 13.5	*R*/*r* = 10.6	*R*/*r* = 8.5	*R*/*r* = 6
r = 6 R = 6-40	*R*/*r* = 4.2	*R*/*r* = 3.4	*R*/*r* = 2.7	*R*/*r* = 1.9

Figure 16 shows the network connectivity versus the ratio *R*/*r* for different values of *N*. It is once again noted that the connectivity remains close to zero until a certain threshold is achieved. It may also be concluded that an essential condition to achieve high connectivity degree with two different types of nodes is that both ranges must be higher than a certain critical range.

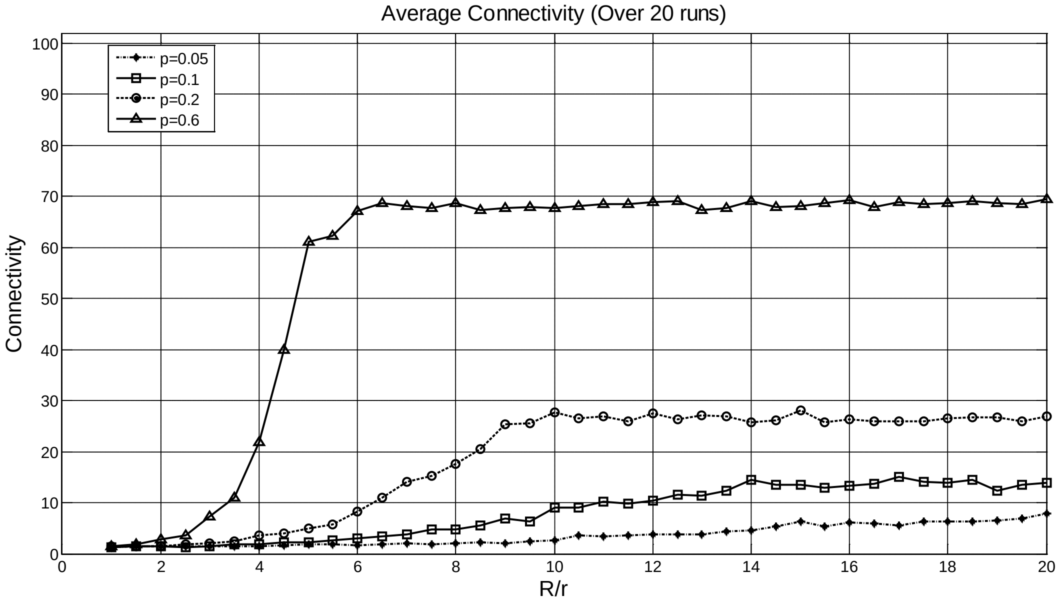
The critical range is the required range to connect the network nodes with each other. If one of the ranges is less than the critical range, it is impossible to achieve a connected network even if the other range is very high. Similar conclusion was also reached in [23] for networks that were modeled as undirected graphs.


Average Connectivity (Over 20 runs)
p=0.05
p=0.1
p=0.2
p=0.6
Connectivity
R/r
0
10
20
30
40
50
60
70
80
90
100
0
2
4
6
8
10
12
14
16
18
20

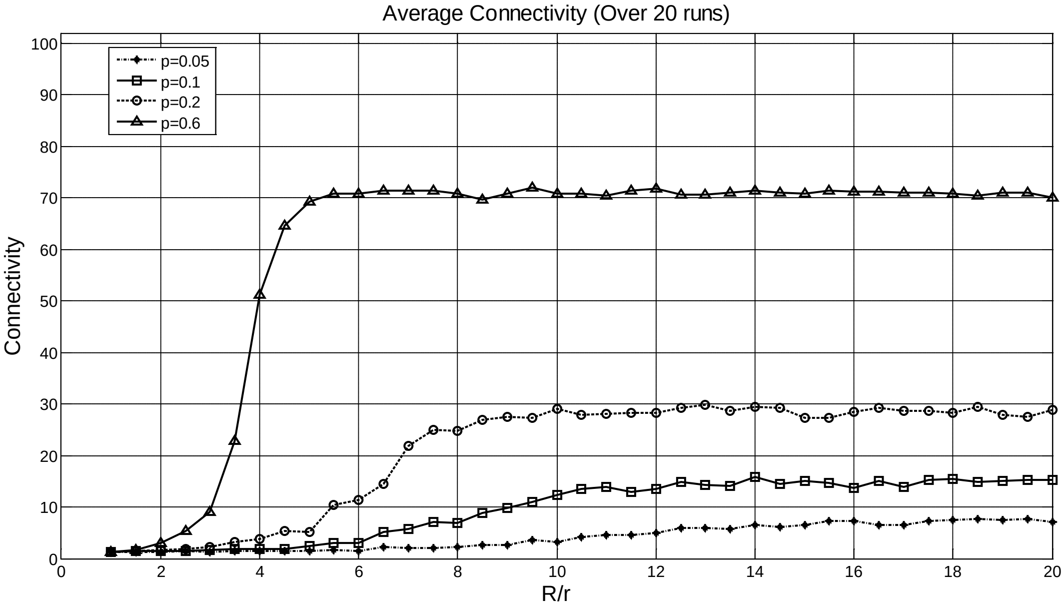
(a) $N = 100$


Average Connectivity (Over 20 runs)
p=0.05
p=0.1
p=0.2
p=0.6
Connectivity
R/r
0
10
20
30
40
50
60
70
80
90
100
0
2
4
6
8
10
12
14
16
18
20

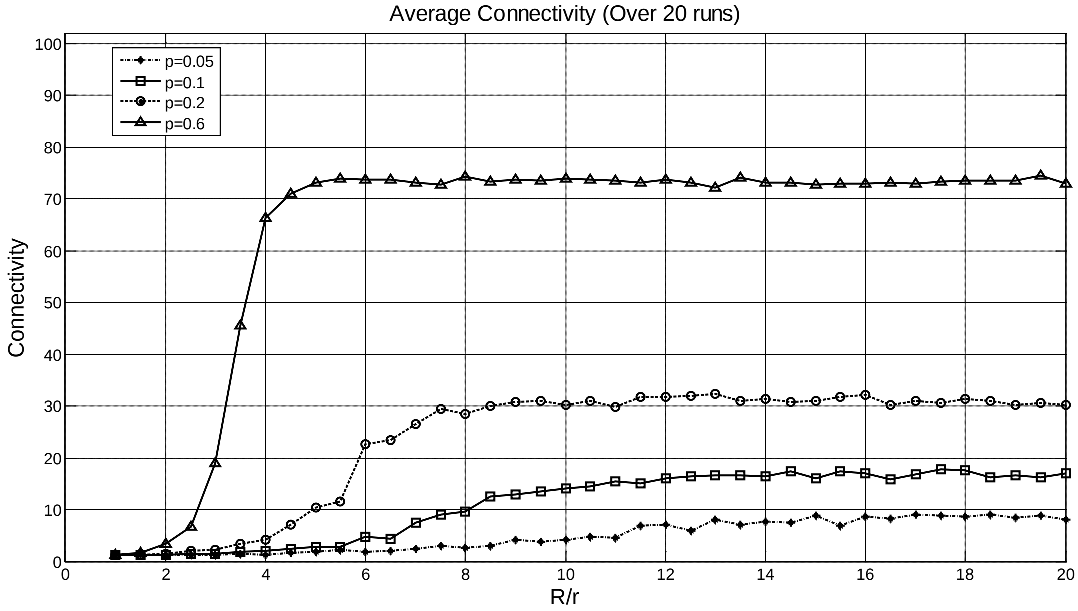
(b) $N = 200$


Average Connectivity (Over 20 runs)
p=0.05
p=0.1
p=0.2
p=0.6
Connectivity
R/r
0
10
20
30
40
50
60
70
80
90
100
0
2
4
6
8
10
12
14
16
18
20

(c) $N = 300$


Average Connectivity (Over 20 runs)
p=0.05
p=0.1
p=0.2
p=0.6
Connectivity
R/r
0
10
20
30
40
50
60
70
80
90
100
0
2
4
6
8
10
12
14
16
18
20

(d) $N = 400$



(e) $N = 500$

Figure 16: Network connectivity when R = 2-40 m, r = 2 m.

(a) N =100 (b) N =200 (c) N =300 (d) N = 400 (e) N = 500

The degree of connectivity improves when we make the LTR nodes to be 60% of total deployed nodes, but it still does not promise 100% connectivity as it is obvious in figure 16.

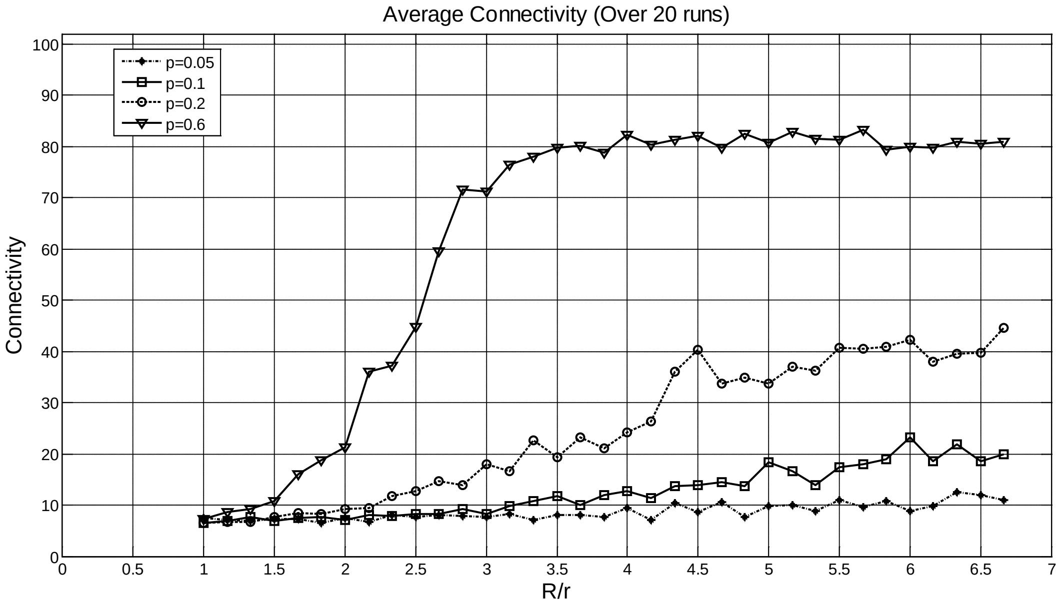
Although 500 nodes were used in the field, connectivity is still below the desired level and it is barely 30% when the probability of LTR nodes is 0.20. After certain ratio of R/r, the connectivity curve levels off for the considered probabilities (0.05, 0.10, 0.20 and 0.60) with number of nodes set to 100, 200, 300, 400, and 500.

Table 4 on the next page summarizes the results of connectivity comparing with homogeneous results when the ranges are r = 2 m and R = 2-40 m.

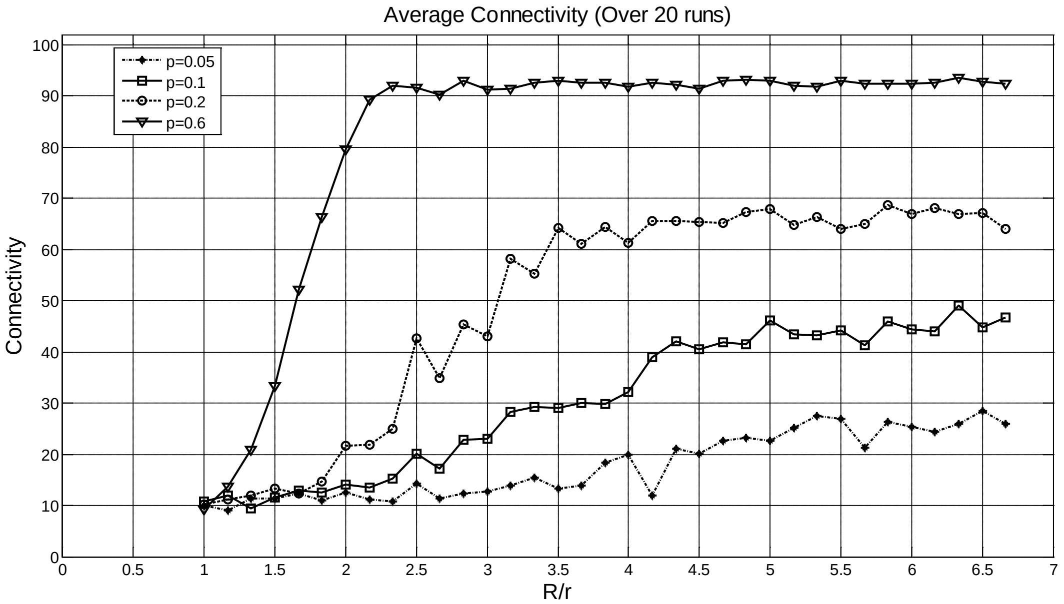
Table 4: Network connectivity for *R* = 2-40 *m*, *r* = 2 *m*.

No. of nodes	Homogeneous Connectivity when $r_h = 10$	Prob. of LTR models	p 0.05	p 0.10	p 0.20	p 0.60
		Ratio R/r	13.5	10.6	8.5	6
		Hetrogeneous Connectivity				
$N = 100$	29%	$R = 2-40,\ \ r = 2$	04%	05%	07%	17%
$N = 200$	91%	$R = 2-40,\ \ r = 2$	04%	07%	13%	52%
$N = 300$	99.3%	$R = 2-40,\ \ r = 2$	06%	10%	22%	68%
$N = 400$	99.6%	$R = 2-40,\ \ r = 2$	07%	13%	28%	71%
$N = 500$	99.9%	$R = 2-40,\ \ r = 2$	08%	17%	31%	73%

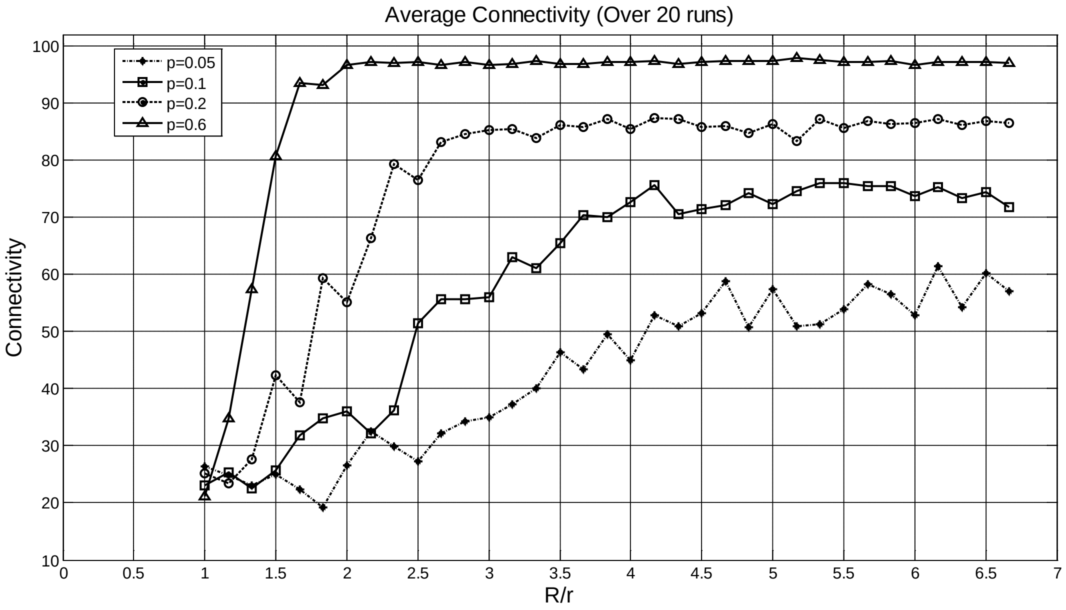
The results in figure 16 that have small range *r* =2 *m* and the results of figure 17 with *r* = 6 *m* show similar behavior.



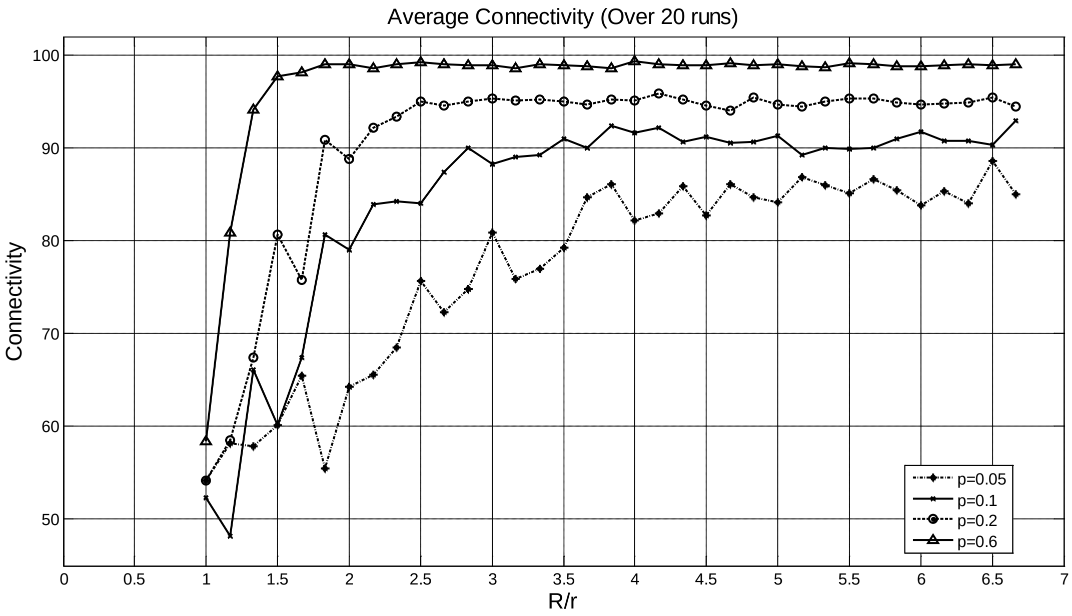
(a) N =100


Average Connectivity (Over 20 runs)
Connectivity
R/r
p=0.05
p=0.1
p=0.2
p=0.6
0
10
20
30
40
50
60
70
80
90
100
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7

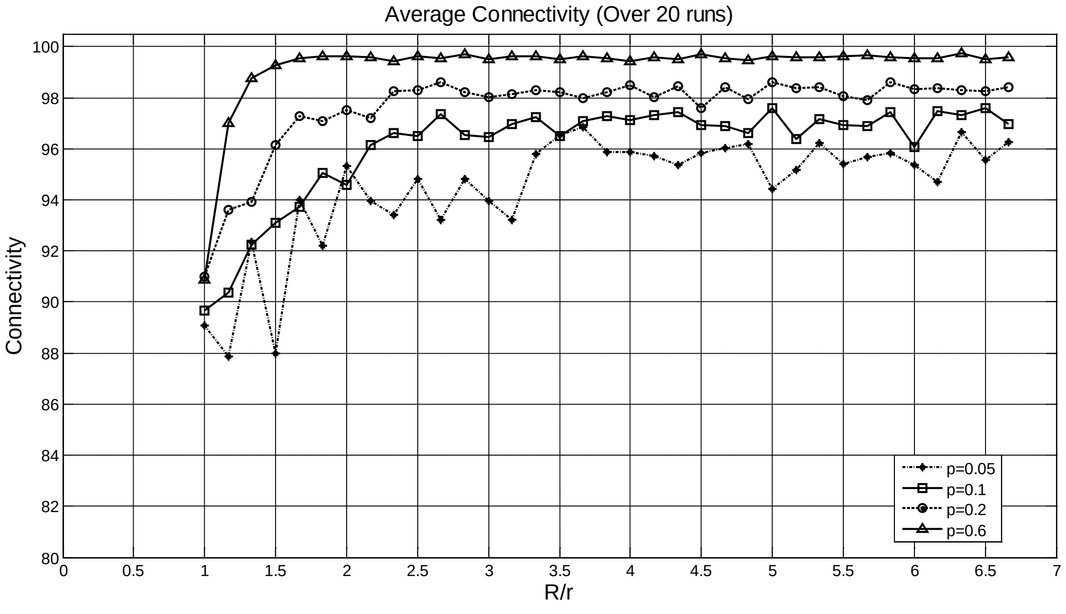
(b) N =200


Average Connectivity (Over 20 runs)
Connectivity
R/r
p=0.05
p=0.1
p=0.2
p=0.6
10
20
30
40
50
60
70
80
90
100
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7

(c) N =300



(d) *N* = 400



(e) *N* = 500

Figure 17: Network connectivity when R = 6-40 *m*, r = 6 *m*.

(a) N =100 (b) N =200 (c) N =300 (d) N =400 (e) N =500

Table 5 summarizes results for the heterogeneous deployment case when the ranges are *r* = 6 *m* and *R*=6-40 *m*.

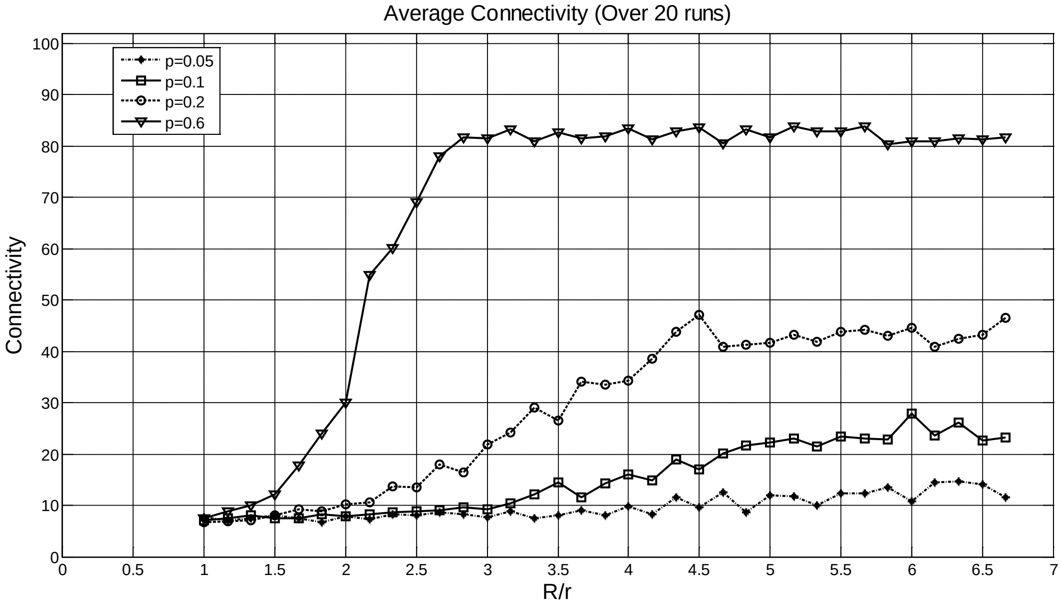
Table 5: Network connectivity for *R* = 6-40 *m*, *r* = 6 *m*.

No. of nodes	Homogeneous Connectivity when $r_h = 10$	Prob. of LTR / models	p 0.05	p 0.10	p 0.20	p 0.60
		Ratio R/r	4.2	3.4	2.7	1.9
		Hetrogeneous Connectivity				
$N = 100$	29%	$R = 6-40,\ \ r = 6$	07%	09%	13%	21%
$N = 200$	91%	$R = 6-40,\ \ r = 6$	15%	24%	38%	75%
$N = 300$	99.3%	$R = 6-40,\ \ r = 6$	48%	68%	78%	96%
$N = 400$	99.6%	$R = 6-40,\ \ r = 6$	83%	88%	94%	98%
$N = 500$	99.9%	$R = 6-40,\ \ r = 6$	95%	96.5%	97.7%	99.5%

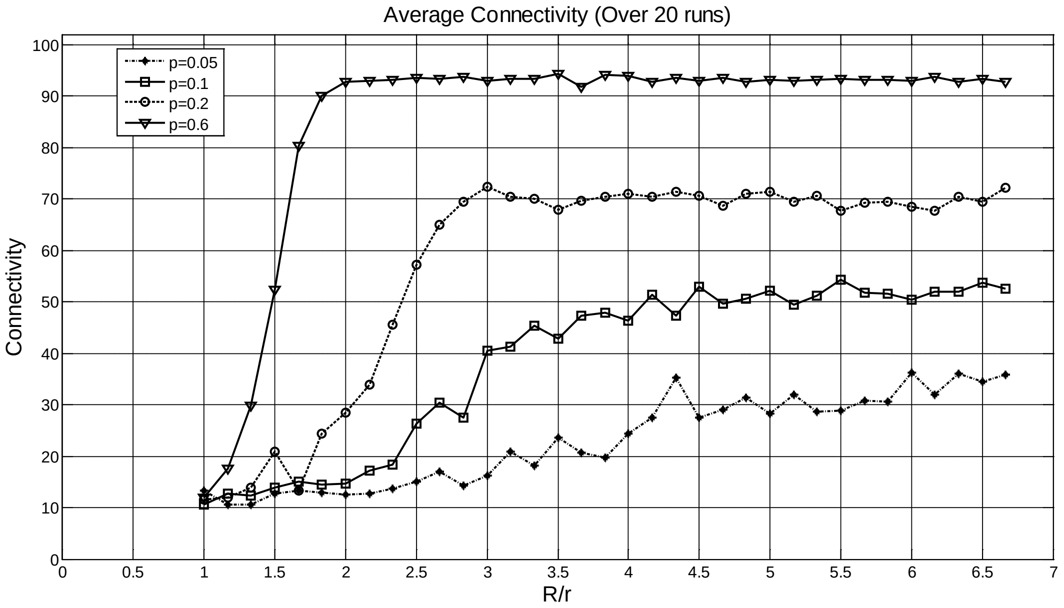
It may be concluded from all the results and the comparisons presented that homogeneous deployment strategey has better performance in terms of network connectivity. It is noted that there is a huge differenece between the numbers that represent the connectivity of homogeneous and heterogeneous deployment case when the range *r* and *R* was starting with 2 *m*. The second test (*r* and *R* starting with 6 *m*) yields higher connectivity results with large number of nodes. The observations can be explained as follow:

The results of the first test were obtained by using a lower tranmission range of 2 *m* for *r* and starting from 2 until 40 *m* for *R*. As a result, a lot of nodes were isolated due to insufficient transmission ranges especially the nodes that have range *r*. The nodes with range *r* seems to be below the critical range, and for this reason it was impossible to obtain a connected network. Both node types must exceed a certain critical range that is required to have a connected network. These observations obtained using directed graph models are consistent with those of Bettstetter [23] who analyzed networks using undirected graphs.

The connectivity results of heterogeneous deployments using the toroidal distance metric are illustrated in figure 18.



(a) N =100


Average Connectivity (Over 20 runs)
Connectivity
R/r
p=0.05
p=0.1
p=0.2
p=0.6
0
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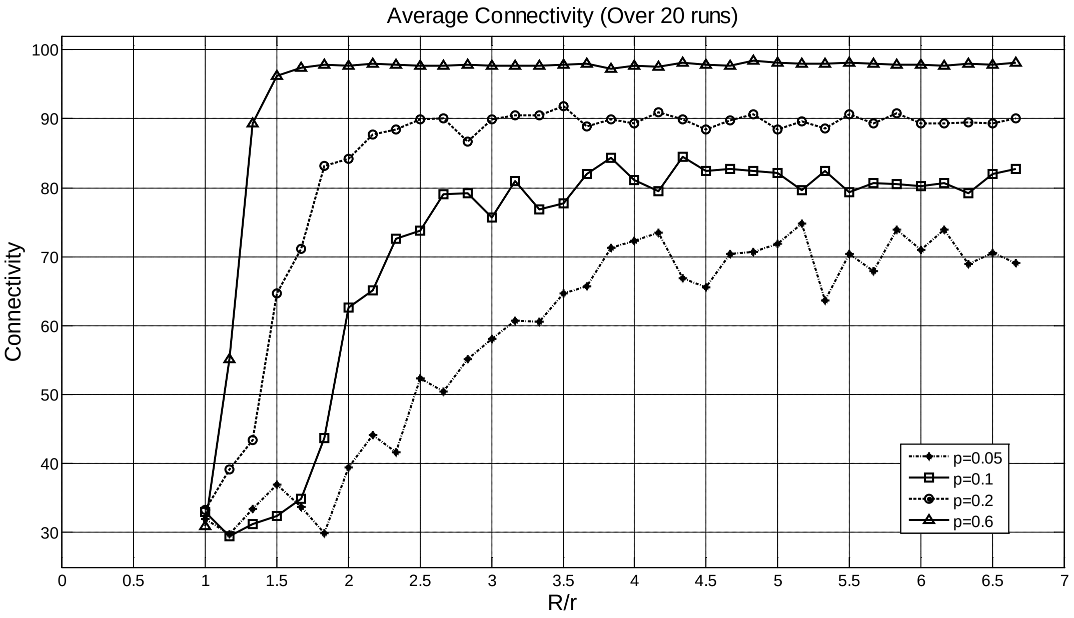
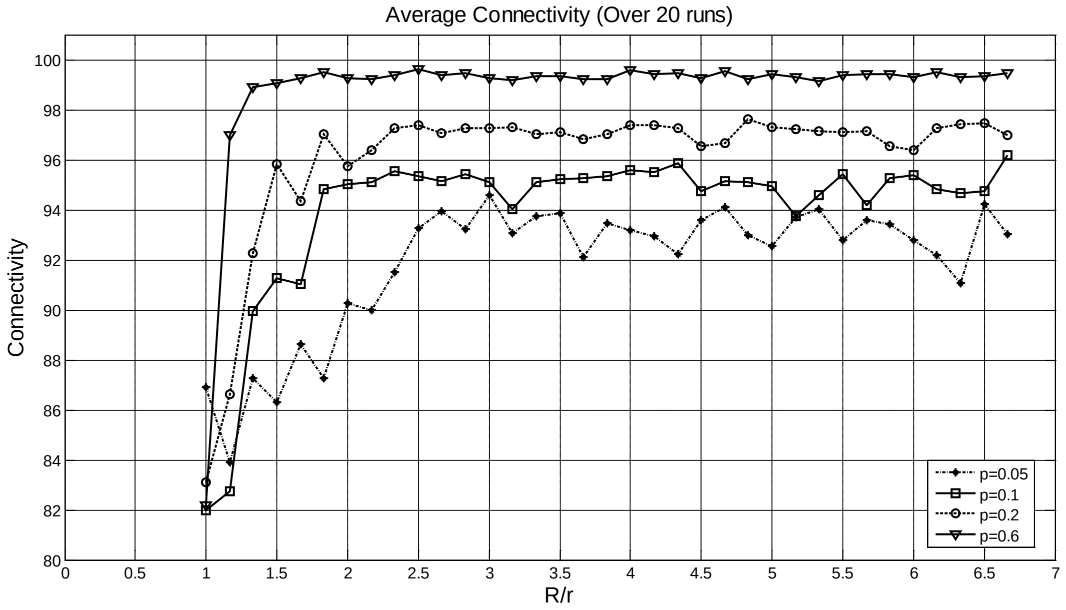
40
50
60
70
80
90
100
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7

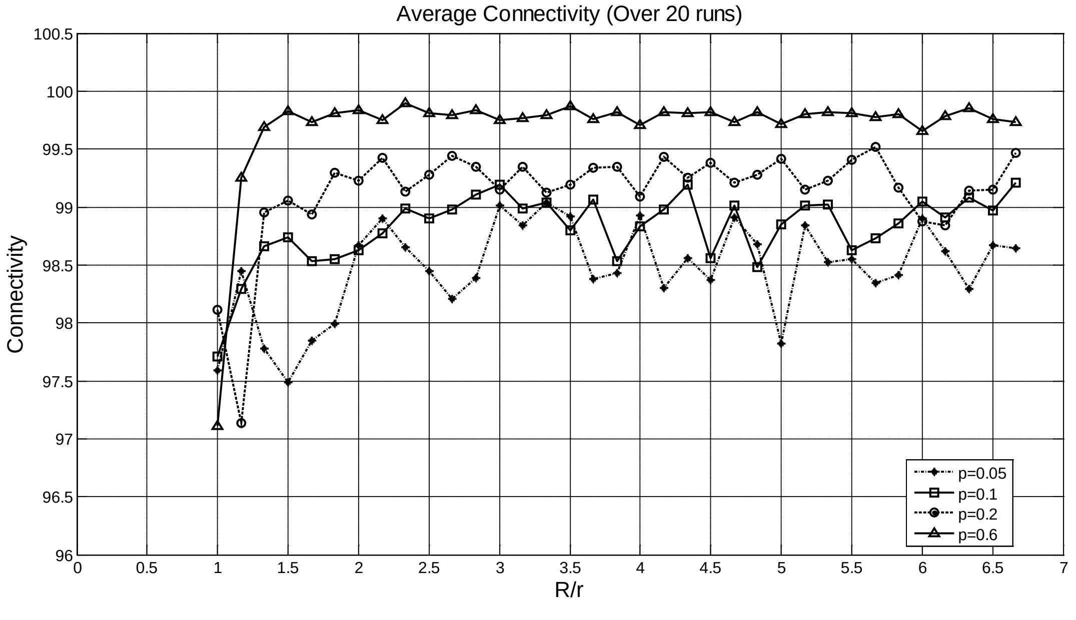
(b) N =200


Average Connectivity (Over 20 runs)
Connectivity
R/r
p=0.05
p=0.1
p=0.2
p=0.6
30
40
50
60
70
80
90
100
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7

(c) N =300



(d) *N* =400



(e) *N* =500

Figure 18: Network connectivity using the toroidal metric distance when *R* = 6-40 *m*, *r* = 6 *m*. (a) *N* =100 (b) *N* =200 (c) *N* =300 (d) *N* =400 (e) *N* =500

The figures demonstrate that border effects decrease the connectivity of the network, and this observation is obvious if we compare the previous results of homogeneous and heterogeneous strategies with the results using the toroidal distance metric. For example, in homogeneous case, we observed 29% connectivity when $N = 100$ in the finite area, while we observed 40% connectivity without border effects when r_h was equal to 10 *m*.

In heterogeneous case, we observed 34% connectivity when LTR probability was 0.20 and $R/r = 5$ with 100 nodes, whereas we had 41% connectivity for the same LTR probability, R/r, and the number of nodes.

Chapter 4

CONCLUSIONS

In this thesis, connectivity in large-scale wireless ad hoc networks such as wireless sensor networks is analyzed. Two deployment strategies, homogeneous and heterogeneous, are evaluated to determine the one with higher connectivity. The power consumption in both configurations is taken into consideration. The simulation results show that connectivity in wireless sensor networks with homogeneous node deployments have better connectivity performance than wireless sensor networks with heterogeneous nodes.

Specific contributions of this thesis involved the following. First, existing results on the connectivity of networks made up of homogeneous nodes were verified through simulations. The phase transition behavior observed in previous work was also observed in the conducted simulations. The connectivity of a network with two types of nodes was analyzed. The modeling of the network as a directed graph differed from previous analyses that used undirected graphs. Phase transition behavior of connectivity was also apparent in networks that are modeled as directed graphs.

Finally, it is established through a large set of simulations that networks with homogeneous node deployments provide higher connectivity for a given power budget. These results are consistent with previous related findings that studied connectivity properties of wireless networks.

Future research may involve developing the directed graph model for the network for further investigation of the connectivity properties of networks. In particular, statistical physics literature [28] on directed percolation must be surveyed in order to find connections between the results of network connectivity and directed percolation. In addition, better models for power consumption and power constraint must be included in the analysis in order to gain insight on the advantages of having a particular deployment strategy for wireless sensor networks.

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APPENDICES

Appendix A: Analysis of the Standard Error in Connectivity Results

The connectivity values presented are obtained by averaging results over 20 simulation runs. In order to evaluate the accuracy of the point estimate of connectivity, we calculate the standard error (s.e.) of the estimate by using the usual formula:

$$s.e. = \frac{s}{\sqrt{K}} \tag{A.1}$$

where s is the sample standard deviation of K connectivity data values obtained from simulations. The average of K values is a “good” estimate and the number of simulation runs is sufficient for that estimate if the s.e. is “small.” We will heuristically assume that the s.e. of connectivity is small if it is less than 5%. The sufficiency of the runs will be determined by considering the maximum s.e. of all values obtained in a given analysis. For instance, for the homogeneous node deployments with 100, 200, 300, 400, and 500 nodes, the maximum s.e. with 20 runs was 4.8%. With 50 runs, the maximum s.e. was 2.9%. The plots in Figures 19 and 20 illustrate the connectivity results in homogeneous node deployments with associated error bars obtained using s.e.-based confidence interval calculations.

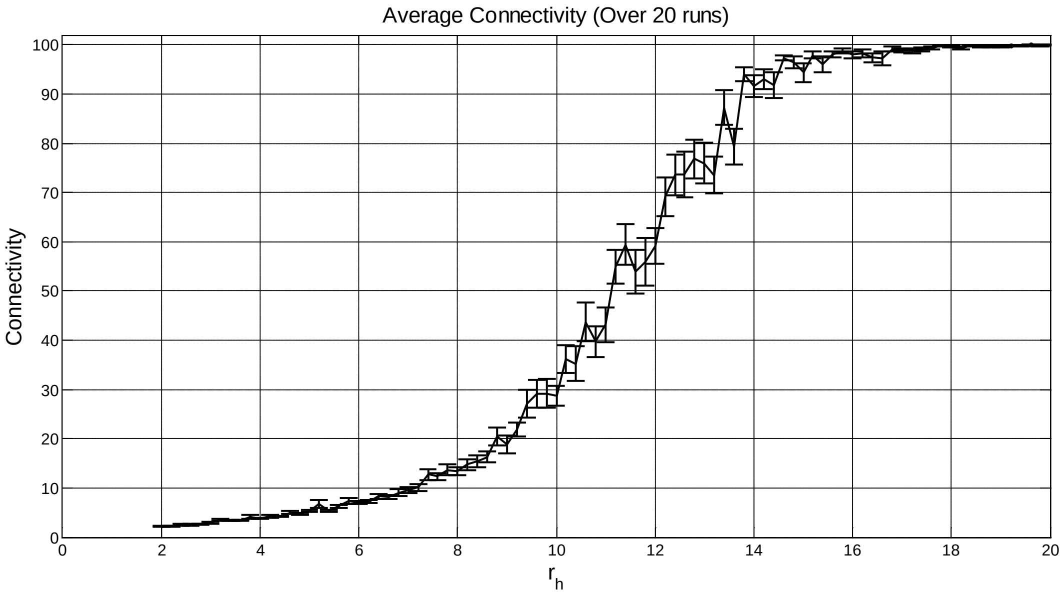

Figure 19: Connectivity of networks with homogeneous node deployments with s.e. confidence intervals for 20 runs.

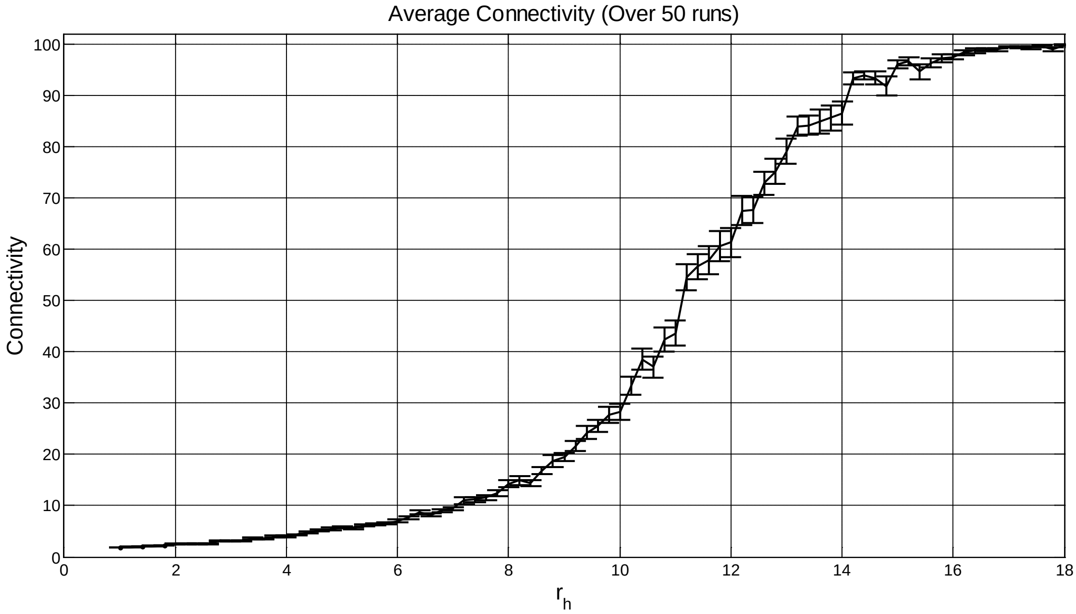

Figure 20: Connectivity of networks with homogeneous node deployments with s.e. confidence intervals for 50 runs

Appendix B: Program Codes

Appendix B.1: Power Ratio Calculation

Matlab M-file name : powratio

```
% homogeneous =xx
xx=10;
rr=6;
RR=6:1:40;

p1=0.05;
p2=0.10;
p3=0.20;
p4=0.60;

alpha=3;

power1=((1-p1)*rr^alpha+p1.*RR.^alpha)./(xx^alpha);
power2=((1-p2)*rr^alpha+p2.*RR.^alpha)./(xx^alpha);
power3=((1-p3)*rr^alpha+p3.*RR.^alpha)./(xx^alpha);
power4=((1-p4)*rr^alpha+p4.*RR.^alpha)./(xx^alpha);
plot(RR/rr,power1,'r-v',RR/rr,power2,'g-s',RR/rr,power3,'b-
o',RR/rr,power4,'k-p')
grid
xlabel('R/r'),ylabel('Power Ratio')
legend(['p=' num2str(p1)], ['p=' num2str(p2)], ['p='
num2str(p3)],['p=' num2str(p4)]);
axis([0 10 0 11])
```

Appendix B.2: Connectivity Calculation of Wireless Sensors with Homogeneous Nodes.

Matlab M-file name : conn_sim_ho

```
% Simulates a network of homogeneous nodes

clear all;
clc
% No. of iterations per scenario
NO_ITER = 30;

r = 2:0.2:20; % Range
N = 100:100:500; % No. of nodes

% Environment parameters
L=100; % Side length of square deployment area
% ran = sqrt (log(N)/(N))

% C is the connectivity matrix
% Dimension 1: Iteration, Dimension 2: r, Dimension 3: N
% here we generate a matrix with dimension ( no. of iteration [row]
* R [column] )with
% generating new matrix according to length of (N)
C = zeros(NO_ITER, length(r), length(N));

for NCnt=1:length(N)
    for rCnt = 1:length(r)
        disp(['N= ' num2str(N(NCnt)) ', r= ' num2str(r(rCnt))]);
        for simCnt = 1:NO_ITER

            locations = L*rand(N(NCnt),2);
            ranges=ones(N(NCnt),1)*r(rCnt);
%    here we generate matrix of ones,its dimension equal to
[length(N)*length(r)]for every iteration

            % Construct the adjacency matrix
            adj_mat=zeros(N(NCnt));
            for i=1:N(NCnt)
                for j=1:N(NCnt)
                    if  (i == j)
                        adj_mat(i,j) = 0;
%                     elseif ( comp_dist(locations(i,:),
locations(j,:)) <= ranges(i) )
                      elseif ( dist_toro(locations(i,:),
locations(j,:),L,L) <= ranges(i) )

                        adj_mat(i,j) = 1;
                    else
                        adj_mat(i,j) = 0;
                    end
                end
            end

            % Make a sparse matrix out of the adjacency matrix
```

```
            Q = sparse(adj_mat);
            % Component index (ci) and sizes of components
            [ci sizes] = components(Q);
            connectivity = (max(sizes)/N(NCnt))*100; % Size of the
largest cluster divided by N

            C(simCnt,rCnt,NCnt) = connectivity;

        end % simCnt
    end % rCnt
end % NCnt

plot(r,mean(C(:,:,1)),'m--h',r,mean(C(:,:,2)),'k-
*',r,mean(C(:,:,3)),'b--o',r,mean(C(:,:,4)),'r-
v',r,mean(C(:,:,5)),'g--s')
grid
xlabel('rh'),ylabel('Connectivity'), title(['Average Connectivity
(Over ', num2str(NO_ITER), ' runs)'])
legend(['N=' num2str(N(1))], ['N=' num2str(N(2))], ['N='
num2str(N(3))], ['N=' num2str(N(4))], ['N=' num2str(N(5))]);
axis([0 22 0 102])
```

Appendix B.3: Connectivity Calculation of Wireless Sensors with Heterogeneous Nodes.

Matlab M-file name : conn_sim

```
% Simulates a network of heterogeneous nodes

clear all;
clc;
% No. of iterations per scenario
NO_ITER = 20;

% R: Range of more powerful nodes
% r: Range of less powerful nodes
% p: Percentage of powerful nodes
R = 6:1:40;
r = 6;
p = 0.05:0.05:0.60;

% Environment parameters
N=400; % No. of nodes
L=100; % Side length of square deployment area

% C is the connectivity matrix
% Dimension 1: Iteration, Dimension 2: R, Dimension 3: p
C = zeros(NO_ITER, length(R), length(p));
%  new matrix will be generated here with dimension(iteration
%  as[row]*ranges[R])and according to length of P the matrix will
repeat
%  and genereate
for pCnt=1:length(p)
    for RCnt = 1:length(R)
        disp(['p= ' num2str(p(pCnt)) ', R= ' num2str(R(RCnt))]);
        for simCnt = 1:NO_ITER

            locations = L*rand(N,2);
            ranges=zeros(N,1);

            % Distribute ranges according to p
            for i=1:N
                u=rand;
%           here (u) will generate random number equal to(N)
                if (u<p(pCnt))
                    ranges(i)=R(RCnt);
%                     here we get the distribution of node according
to
%                     their percentage and their range
                else
                    ranges(i)=r;
                end
            end
            %else
                % Homogeneous case
            %    ranges=ones(N,1)*r;
            %end
```

```
            % Construct the adjacency matrix and compute distance
            adj_mat=zeros(N);
            for i=1:N
                for j=1:N
                    if  (i == j)
                        adj_mat(i,j) = 0;
                    elseif ( comp_dist(locations(i,:), 
locations(j,:)) <= ranges(i) )

%       elseif ( dist_toro(locations(i,:), locations(j,:),L,L) <= 
ranges(i) ) % toroidal distance

                        adj_mat(i,j) = 1;
                    else
                        adj_mat(i,j) = 0;
                    end
                end
            end

            % Make a sparse matrix out of the adjacency matrix
            Q = sparse(adj_mat);
            % Component index (ci) and sizes of components
            [ci sizes] = components(Q);
            connectivity = (max(sizes)/N)*100; % Size of the largest 
cluster divided by N

            C(simCnt,RCnt,pCnt) = connectivity;

        end % simCnt
    end % RCnt
end % pCnt

plot(R./r,mean(C(:,:,1)),'r-*',R./r,mean(C(:,:,2)),'g-
x',R./r,mean(C(:,:,4)),'b-o',R./r,mean(C(:,:,12)),'k-^')
grid
xlabel('R/r'),ylabel('Connectivity'), title(['Average Connectivity 
(Over ', num2str(NO_ITER), ' runs)',])
legend(['p=' num2str(p(1))], ['p=' num2str(p(2))], ['p=' 
num2str(p(4))],['p=' num2str(p(12))]);
axis([0 10 0 102])
```

Appendix B.4: Distance Calculation Functions

Matlab M-file name : comp_dist

```
function D = comp_dist(n1, n2)
D = sqrt((n1(1)-n2(1))^2 + (n1(2)-n2(2))^2);
```

Appendix B.5: Toroidal Distance Metric Function

Matlab M-file name : dist_toro

```
function d=dist_toro(x1,x2,XMAX,YMAX)
% Distance between two points x1 and x2
% XMAX and YMAX are the maximum X and Y coordinates of the plane
D1=sqrt(sum((x1-x2).^2));
D2=sqrt(sum((x1+[XMAX 0]-x2).^2));
D3=sqrt(sum((x1+[-XMAX 0]-x2).^2));
D4=sqrt(sum((x1+[0 YMAX]-x2).^2));
D5=sqrt(sum((x1+[0 -YMAX]-x2).^2));
D6=sqrt(sum((x1+[XMAX YMAX]-x2).^2));
D7=sqrt(sum((x1+[XMAX -YMAX]-x2).^2));
D8=sqrt(sum((x1+[-XMAX YMAX]-x2).^2));
D9=sqrt(sum((x1+[-XMAX -YMAX]-x2).^2));
d=min([D1 D2 D3 D4 D5 D6 D7 D8 D9]);
```