Power System Stability Analysis Using Recursive Projection Method

Aysar Musa

Submitted to the
Institute of Graduate Studies and Research
in partial fulfillment of the requirements for the Degree of

Master of Science
in
Electrical and Electronic Engineering

Eastern Mediterranean University
January 2013
Gazimağusa, North Cyprus

Approval of the Institute of Graduate Studies and Research

Prof. Dr. Elvan Yılmaz
Director

I certify that this thesis satisfies the requirements as a thesis for the degree of Master of Science in Electrical and Electronic Engineering.

Prof. Dr. Aykut Hocanın
Chair, Department of Electrical and Electronic Engineering

We certify that we have read this thesis and that in our opinion it is fully adequate in scope and quality as a thesis for the degree of Master of Science in Electrical and Electronic Engineering.

Prof. Dr. Osman Kükrer
Supervisor

Examining Committee

1. Prof. Dr. Osman Kükrer
2. Prof. Dr. Runyi Yu
3. Asst. Prof. Dr. Suna Bolat

ABSTRACT

Stability represents significant criteria in power system operation. Stability analysis of power systems has been done by using an efficient numerical technique that is the recursive projection method (RPM). RPM analyzes the outputs of the time domain simulation code (TDSC) that is used to simulate the dynamics of a power system, to define a slow/unstable operating mode as a subspace of system's full space and applying Newton method to improve the convergence of system solution, while fixed-point iteration method is used in the supplement subspace of stable modes. The analysis is performed by detecting those eigenvalues of the state matrix with magnitudes greater than unity, and creating the corresponding orthonormal basis that participates in extending the solution's convergence. This leads to getting a more accurate and stable solution in power systems. When a perturbation occurs to the power system, applying RPM allows the numerical solution to reach its steady-state mode in a short time and without continuous oscillation. Verification of RPM's achievements has been performed on an example of 6-bus power system. The environment of this work is the Matlab program supported by the power system toolbox (PST).

Keywords: Power system analysis, recursive projection method, numerical integration methods.

ÖZ

Kararlılık güç sistemlerinin çalışması açısından önemli bir kriterdir. Bu çalışmada güç sistemlerinin kararlılık çözümlemesi, özyineli izdüşüm yöntemi (RPM) denen etkili bir sayısal yöntem kullanılarak yapılmıştır. RPM ilk önce güç sisteminin dinamiğinin benzetimi için kullanılan zaman erim benzetim yazılımının çıktılarını analiz eder ve yavaş veya kararsız çalışma kiplerini, sistem uzayının değişimsiz bir altuzayı olarak tanımlar. Bu değişimsiz altuzay üzerinde Newton yöntemi uygulanıp çözümün yakınsaması iyileştirilir. Kararlı kiplere ait tümleyen uzay üzerinde ise sabit-nokta dürümü uygulanmaya devam eder. Analiz, durum matrisinin özdeğerlerinin bulunmsı ve bunlardan genliği birden büyük olanlara karşılık gelen ve çözümün yakınsamasını sağlayacak olan tam dikgen temel oluşturarak yapılır. Bu yolla güç sistemlerinin analizinde kararlı ve doğruluğu yüksek olan bir çözüm elde edilir. Güç sisteminde bir hata oluştuğunda, RPM sayısal çözümün durağan duruma kısa zamanda ve salınımsız olarak erişmesini sağlar. RPM’in başarımı 6 baralı bir sistem üzerinde denenerek doğrulanmıştır.Çalışmalar Matlab ortamında, daha önce geliştirilmiş olan Güç Sistemleri Paketi kullanılarak yapılmıştır.

Anahtar kelimeler: Güç sistemleri kararlılığı, Özyineli İzdüşüm Yöntemi, Sayısal Çözüm Yöntemleri.

This dissertation is dedicated to my beloved wife and my parents for their love, devoting their time to support me. Further, I would like to dedicate this work to my uncle for his encouragement and endless support.

ACKNOWLEDGMENTS

Thank god for helping me and giving me the ability to complete my thesis. My deepest thanks and appreciation go to my supervisor; Prof. Dr. Osman Kükrer who supported me during my thesis, without him this work may not be possible.

I would like to thank the chairman of my department, Prof. Dr. Aykut Hocanın, advisor Assoc.Prof. Dr. Hasan Demirel, and all the staff of Electrical and Electronic Engineering Department for their guidance and support.

I would like to thank Eastern Mediterranean University for accepting me as a master student, and giving me the opportunity to be a member of graduate students of EMU.
My deepest thanks and gratitude for Thesis Monitoring Committee members, Prof. Dr. Osman Kükrer, Prof. Dr. Runyi Yu and Asst. Prof. Dr. Suna Bolat for their supervision and guidance to discuss my thesis.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS/ABBREVIATIONS

B Transmission line susceptance

e_i Internal bus voltage of the machine

f Frequency of the system

H Inertia constant

h Integration step length

J Moment of inertia of the machine

P Unstable/slow invariant subspace of ϕ_x

P_e Electrical power

P_m Mechanical power

$P_{\max}$ Maximum power transmitted

Q Orthogonal complement subspace of P

R_a Stator winding resistance of synchronous generator

S Machine rating

T_a Acceleration torque

T_e Electrical torque

T_{cc} Critical clearing time

T_{fc} Fault clearing time

T_m Mechanical torque

t_c Clearing fault time

t_f End simulation time

u Control parameters

U_γ Magnitude of eigenvalues

$X_d^{'}$ Direct axis subtransient reactance

X_T Impedance of the transmission line

x Dynamic state variables

y Instantaneous state variables

Z_p Orthonormal basis

θ Rotor angle of the machine

θ_c Fault clearing angle

θ_m Rotor angular site according to a constant axis

λ Eigenvalues of the state matrix

λ_k Eigenvalues of the continuous mode (physical) system

μ_k Eigenvalues of the discrete mode (integration) system

ϕ_x Jacobian matrix at steady state

ω_e Electrical angular velocity

ω_{eo} Nominal quantity of electrical angular velocity

ω_m Mechanical angular velocity

ω_{mo} Nominal quantity of Mechanical angular velocity

DAE Differential and algebraic equation

DCPS Dynamic Computation for Power Systems

EM Euler method

MEM Modified Euler method

ODE Ordinary differential equation

PST Power system toolbox

RPM Recursive projection method

TDSC Time domain simulation code

WECC Western Electricity Coordinating Council

Chapter 1

INTRODUCTION

In recent times, increasing transmission capability and the incessant extending of scale in interconnected power systems take power systems to extreme operating conditions. Some small perturbations occuring to the system may lead to fluctuations in voltage, frequency and loads. Therefore, stability criteria are one of the major factors which cause restriction in the capability of power transmission in the electrical power system [1].

Power system stability indicates the ability of a power system, for a certain initial operating condition to retrieve an equilibrium status after exposure to a disturbance. Hence, the stability criterion tries to preserve the integrity of a power system which means that the power system entirely stays intact without any tripping of loads or generators, excluding disconnecting of the faulting components or purposely tripped to maintain normal operation of the remaining system components. Stability is a procedure of equilibrium between opposing parameters; instability is produced when a disturbance occurs and causes sustained imbalance of opposing parameters [2].

Power system is a highly complex nonlinear dynamic system, and for modeling and analysis, it should be represented by a set of differential and algebraic equations DAEs. The precise stability analysis of a system entails itemized simulations utilizing DAEs which entirely model the system.

Historically, the stability problem has been attempted from 1920. At that time there were no computers and the computations were mainly done using hand calculations. In 1950, analogue computers were developed and used for simulating the power system stability problem. After six years a computer program for power system stability was developed mainly to analyze the tangent stability of a system. Over the years, another development took place to implement high response excitation systems, which resulted in increased capability of improving tangent stability of the system. But this application also resulted in a problem of weak damping of the system oscillation, and this problem has been overcome by implementing power system stabilizers [3].

During the years, power system stability has become a challenge for power engineers because of the large interconnected systems, and they were faced with various problems. One of these problems is modeling the system to get the correct assessment of power system stability, which needs correct development of a mathematical model to obtain approximate solution through numerical techniques; the mathematical model of a system is a set of nonlinear differential and algebraic equations DAEs. Also there is no availability to an accurate solution for DAEs [4].

Another problem is preserving synchronous operation of a system. The stability issue arises as a result of the dynamic response of the synchronous generators after a perturbation occurs, as power systems depend on these machines for electric power generation. So, an important condition which should be satisfied during operation of the system is that all the machines stay in synchronism. This side of stability is affected by the rotor angle dynamics and power-angle relation [2], [3].

The developments that occur in modern power systems lead to an increasing tendency to focus on effects of instability, which give the necessity of evolving new techniques to improve transient stability since it plays a significant role in preserving safety of power system operation. Power system transient phenomena play an important role in designing, developing and operating power systems. Investigating this phenomena gives important information on the machines in showing the ability to maintain their synchronism throughout wide unexpected perturbation such as various faults, losing main part of the load and power generation units [3], [5].

Plenty of studies have been devoted over the years to handle the problem of dynamic stability in power systems. Dynamic simulation should be used to analyze and solve the stability problem of the power system efficiently. In other words, stability simulation criteria depend on dynamic model derived [5], [6]. Transient stability simulation problem is sorted as step by step solution of differential-algebraic initial value problems. This solution allows reducing the interface error to a more acceptable level [7].

In the present time, power systems are being adjacent to their stability limits because of the environmental and economic restrictions. Maintaining a stable operation of a power system is consequently a very important issue and much concentration on the study of stability problems has been carried out [8]. Analysis methods that take dynamics of the components in the power system into account like small signal analysis, can efficiently enhance dynamic performance and augment power transmission of the system [4].

Small signal stability (or small-disturbance stability) is the ability of restoring the operation mode to its original mode or a new mode and maintain synchronism after a small disturbance. The problem is usually one of the insufficient damping of the system oscillations, which is caused by the lack of sufficient damping torque. Oscillations will appear between two or more generators, as soon as AC generators were operated in parallel [2], [9]. Small signal stability problems could be either local or global mode in nature. The first mode is related to the oscillations of generating units at a specific station in regard to the remainder of system. The second mode is related to the oscillations of many machines in one portion of the system versus machines in the other portions; these oscillations are named 'inter-area mode oscillation' as well. To analyze and design power systems, small signal stability is the most significant prerequisite, which consists of oscillation mode and mode form, correlativity analysis, stability area estimation and its sensitivity. Small signal stability has many approaches of analysis methods like eigen-structure analysis which is based on theoretical solution and time domain simulation based on numerical solution [10], [11].

There are various theoretical and numerical techniques used in power system stability studies. One of the theoretical techniques is based on Lyapunov’s stability theorem, which gives a basis for optimal dynamic stability design of power systems. It is applied to analyze and improve the stability of mathematical solutions of a dynamical system. One of the studies that have been carried out in [12] is applied by using Lyapunov’s direct method to get suitable and feasible investigation of the stability of systems with deviating argument of delay type. The ways of constructing Lyapunov functions for linear systems with fixed coefficients are already well-established. Although this technique gives satisfactory results, it is not efficient for large power systems and it will be complicated and difficult to handle for stability problems. Numerical methods are widely used since they can handle different types of dynamic models and sequences of events for complex power systems. In other words, they are applicable to analyze several forms of complex nonlinear phenomena [12], [13].

An interesting case is tracking the system trajectory and determining the tasks needed to recover and restore the system when imbalance is observed in load-generation. This needs to integrate the DAEs. For this reason, some techniques are developed and carried out step by step, integrating the DAEs of the system from the initial value to get dynamic response to perturbations. The importance of a dynamical simulation tool in power system transient analysis leads to the use of various kinds of numerical integration techniques like trapezoidal and Euler methods [6], [14].

The Trapezoidal method of variable step size integration is very efficient, and widely used. It is one of the good approaches in numerical integration techniques that is used to insert synthetic elements in the system which could impact the correct solution to a certain degree. Trapezoidal method is used in [14] to give an efficient solution for a boundary value problem by immediate calculation of a trajectory on the stability boundary that is designated as a critical trajectory. Therefore, in this study, the method depends basically on the calculation of the trajectory for assessment of stability [3], [6], [14].

The Euler method is used in [15] to support C/C++ software to solve the mathematical representation of power system dynamic equipment which includes synchronous generators, turbine-governors and exciters. This software is the Dynamic Computation for Power Systems (DCPS) software package, and it is applied to show the basic modeling and calculation ways that deal with power system dynamics and carry out power system transient stability analysis. WECC 9-bus system was used to check the impact of changing load demand on the critical clearing time T_{cc}. The outcome showed that an increase in the load demand leads to linearly decreasing T_{cc}. The transient stability is performed to check the response of the equipment for the three-phase fault case. During this case, critical fault clearing time T_{fc} is solved, and the system becomes unstable if T_{fc} becomes greater than T_{cc} [15].

Although this approach gives noticeable improvement in the stability solution, it suffers from some disadvantages that reduce its applicability, such as the lower accuracy and the synthetic numerical oscillations that are frequently encountered in switching events, and thus in discontinuities. Furthermore, this method by itself will fail at large step size integration and it will go to divergence. Another problem is the large number of DAEs in the mathematical model of a large power system. Therefore, the solution through those methods needs to be supported and developed by another technique [16].

Recursive projection method (RPM) is stabilization of an unstable numerical procedure; it is performed by calculating a projection onto the unstable subspace. RPM is applied to recognize and solve the unstable/slow invariant subspace by obtaining outcome information from a time domain simulation code TDSC [17]. Newton or special Newton iteration is carried out to improve convergence of the unstable/slowly-converging mode; in contrast, fixed-point iteration method of the TDSC is utilized and kept to evolve as the stable/fast-decaying mode of a system. During the numerical process, the projection is updated effectively, and resizing the dimension of the unstable subspace can be done; decrease or increase in the dimension can occur. It is notable that this method is quite efficient in the case of a small size of the unstable subspace compared to the size of full state system. Furthermore, RPM is effectively applicable to accelerate iterative actions in case of slow convergence caused by some tardily decaying modes [18].

RPM, which is originally proposed in [18], started to be applied recently in power system stability problems; it is receiving more attention because of its ability to enhance convergence and produce eigenvalues as a byproduct from TDSC. Besides that, it efficiently works by preserving numerical and modeling facilities of TDSCs, and saves in cost by cancelling additional programming costs [17].

To summarize, a major problem in the numerical stability analysis of power systems is the inadequacy of the existing time domain numerical simulation methods in predicting the dynamics of the system after a disturbance. The aim of this thesis is to apply the RPM procedure to stabilize the numerical simulation of the dynamics of system when a fault occurs, thus enabling an accurate prediction of system behavior in transients.

This thesis is organized as follows. In Chapter 1 the stability problems in power systems is surveyed and discussed. Chapter 2 deals with the mathematical formulation of the stability problem for single-and multi-machines systems, while Chapter 3 introduces the RPM method and its application to the numerical simulation of power system transients.

In Chapter 4, simulation results for a number of test cases are presented and discussed. Finally, in Chapter 5 some conclusions are drawn and suggestions for future work are made.

Chapter 2

STABILITY ANALYSIS OF POWER SYSTEM

2.1 Synchronous Machines

The main part in an electrical power system that deals with stability analysis is generator and its rotor part, since it plays a major role in system synchronism. Therefore, performing dynamic model and developing mathematical equations should be achieved to describe the dynamics of the rotor and its angle position.

The important thing is when the power system operates in the steady state mode it is subject to a perturbation. This leads to alteration in the voltage angles of the synchronous generators. This will produce an unbalance between the generation and load of the system and will create a new operating mode with different voltage angles.

The stability analysis determines the effect of disturbances on the behavior of synchronous machines in power systems. The perturbation may be small like changing in load demand; or large such as loss of generator, line, or a series of such disturbances. The modification that occurs from the initial (steady state) operating mode to the new operating mode is called the transient duration, and the behavior of the system during this time is called dynamic system performance. The main point in stability criteria is whether synchronous generators preserve their synchronism at the termination point of transient duration. If the oscillatory response of a power system over the transient duration is damped and the settling of the system occurs successfully to the new steady operating mode in a limited time, the system will be stable, and if not the system will be unstable [19].

The system contains inherent forces that tend to reduce oscillations when the system has ingrained forces, and these forces try to reduce the oscillations, this case known as asymptotic stability. This type of stability is targeted in the future of power system. The continuous oscillation is excluded from asymptotic stability, although this type of steady state oscillation may be considered stable in mathematical analysis. The concern is on the practical side, because this continuous oscillation may be undesirable to both of the feeding system and the costumer in electrical power system. The asymptotic stability also shows the workable characteristic for an agreeable operating state [3], [19].

The stability problem focuses on the behavior of synchronous machines after being subjected to a disturbance. Therefore, it is important to study the stability analysis of these machines by performing dynamic modeling and developing mathematical equations [20].

2.2 Swing Equation

A swing equation is significant in power system analysis for studying transient dynamic criteria and power oscillations in power systems. At steady-state operating mode, the relative location of the rotor axis and magnetic field axis is preserved, and the angle between those two axes is called power (or torque) angle. Over the perturbation period, the machine rotor will accelerate or decelerate according to the axis of rotation and (or rotating air gap) synchronously and the starting point of rotor motion proportionally. This motion can be represented by a mathematical equation which is called the swing equation [2], [19]. After this period, if the machine rotor tends to its synchronous speed, the generator will preserve its stability. If the perturbation does not produce any change in power, the rotor will go to its original location. If the perturbation produces a change in generation or load, the rotor will move to a new operating angle according to the proportional synchronously regenerating field.

In transient stability analysis, every generator will be represented by two state equations. After fault clearing, which could include separating fault line, the bus admittance matrix of the system will be recalculated to reflect the alteration in the system, as well as recalculating electrical power of ith generator [21]. The new bus admittance matrix could be reduced because of separating the faulted line. At post-fault case, performing the simulation will continue to define the stability of the system, and the output plotting will shows the direction of the system behavior to stable or unstable mode. Generally, the output solution will be for two swings to approve that there is existence or not of difference between two swings. If the angle between these two swings is fixed, the system is stable, if not the system is unstable [22].

At steady state operating mode, all the machines of the system rotate at a similar angular velocity, but when a perturbation occurs, one or some of the machines may accelerate or decelerate its angular velocity, this representing a risk on the power system that may cause loss of synchronism. This problem has affects system stability, and those machines that lose synchronism should be separated from the system. Otherwise, large damage may occur [23].

2.3 Swing Equation of Single Machine

Consider a system consists of a three phase synchronous generator with its prime mover, and this generator connected to an infinite bus. Machine model is shown in Figure (2.1) are the base of swing equation derivation and shows the electro-mechanical oscillations in power system [19], [24], [25].

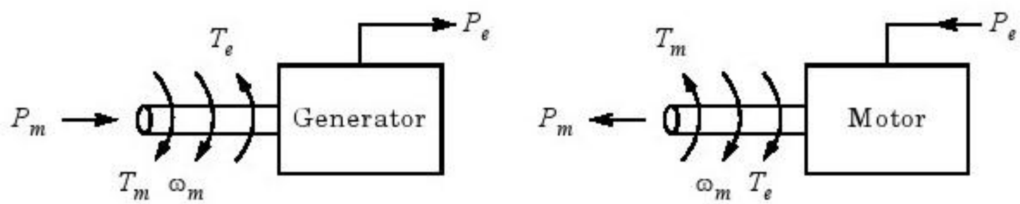

Figure 2.1: Power and torque components in synchronous machines

The symbol m refers to mechanical components and the symbol e refers to electrical components. In synchronous machines, mechanical torque T_m and electromagnetic torque T_e are produced, the first one from the prime mover and the second one from the machine itself. At steady state operation, T_m and T_e are equal and there is no producing of acceleration or deceleration torque. That is $T_m - T_e = 0$

At perturbation case, T_m will be greater than T_e and acceleration torque Ta will appear, which includes inertia produced from the inertia of the prime mover and the machine as shown below in Figure (2.1). This torque will cause accelerating of the machine.

The mathematical representation of rotor dynamics is described by using Newton's second law in the following differential equation [20], [24]

$$J\frac{d^2\theta_m}{dt^2} = T_m - T_e = T_a \tag{2.1}$$

where

J : Moment of inertia of the machine (kg.m^2)

θ_m : Rotor angular site according to a constant axis (rad)

T_m : Mechanical torque (N.m)

T_e : Electrical torque (N.m)

T_a : Net acceleration torque (N.m)

Multiplying both sides by angular velocity ω_m

$$\omega_m J\frac{d^2\theta_m}{dt^2} = P_m - P_e \tag{2.2}$$

$P_m = T_m\omega_m$ and $P_e = T_e\omega_e$ are the effective mechanical and electrical powers on the rotor

ω_m : Rotor angular speed (rad/s)

The formula (2.2) represents the angular acceleration expressed by mechanical angle, using electrical angle (2.2) becomes,

$$\frac{2}{p}\omega_m J\frac{d^2\theta_e}{dt^2} = P_m - P_e \tag{2.3}$$

By rearranging the left hand side the resultant equation will be

$$2\frac{2}{p\omega_m}\left(\frac{1}{2}\omega_m^2 J\right)\frac{d^2\theta_e}{dt^2} = P_m - P_e \tag{2.4}$$

The relationship between mechanical and electrical angular velocity ω_m, ω_e respectively of the rotor is

$$\omega_m = \frac{\omega_e}{\frac{p}{2}} \tag{2.5}$$

Dividing equation (2.4) by the rating of the machine (S), and using equation (2.5) give

$$\frac{2}{\omega_e} \frac{(\frac{1}{2}\omega_m^2 J)}{S} \frac{d^2\theta_e}{dt^2} = \frac{P_m - P_e}{S} \tag{2.6}$$

The electrical quantities are computed by per unit base, as well as moment of inertia J, and inertia constant H of the synchronous machine, given by

$$H = \frac{0.5J\omega_{m0}^2}{S} \tag{2.7}$$

The practical side of power systems shows that through perturbation, there is no considerable swerve of angular velocity from the nominal quantities ω_{m0}, ω_{e0}. After substituting equation (2.7) in equation (2.6) the resulting form of the equation will be

$$\frac{2H}{\omega_{e0}} \frac{d^2\theta_e}{dt^2} = P_m^{pu} - P_e^{pu} \tag{2.8}$$

The symbol pu refers to the mechanical and electrical power values are in $p.u.$ of the rating of machine. It is acceptable to choose the powers quantities in $p.u.$ at the same base of synchronous machine. Therefore, the final equation could be written as:

$$\frac{2H}{\omega_0} \frac{d^2\theta}{dt^2} = P_m - P_e \tag{2.9}$$

2.4 Power - Angle Relation

Assume a simple model of one synchronous generator connected to an infinite bus as shown in Figure (2.2). To simplify this model, classical model can be taken by substituting the generator with a fixed voltage behind a transient reactance. In such a system, there is a maximum power which could be transferred from the generator to the infinite bus. The relation between the generated electrical power and the rotor angle of the machine is shown by the following equation [20], [24], [25].

$$P_e = \frac{E_1 E_2}{X_T} \sin\theta = P_{\max} \sin\theta \tag{2.10}$$

where

c

$$P_{\max} = \frac{E_1 E_2}{X_T} \tag{2.11}$$

2.5 Equal-Area Criterion

Equal-area criterion is one of the graphical techniques developed to analyze system stability; the graph shown in Figure (2.3) explains the stored energy in the rotating body that confirms whether the machine can preserve stability after perturbation. This technique could be utilized after sharp disturbance occurred to the machine to predict the stability at a short time (speedily). It is useful for a system consisting one machine connected to an infinite bus or a two-machine system, but unacceptable for large systems. For a simple system consisting of one synchronous machine connected to an infinite bus, it can be presented by a classical model as shown in Figure (2.2)

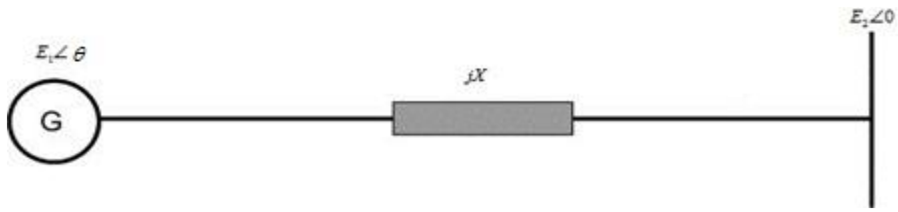

Figure 2.2: Single machine infinite bus system

From (2.9), the following formula can be obtained

$$\left(\frac{d\theta}{dt}\right)^2 = \frac{\omega_0}{H}\int_{\theta_0}^{\theta} P_a d\theta \qquad (2.12)$$

while $\frac{d\theta}{dt}$ is the relative speed according to the rotating base frame, its initial value is zero. After a perturbation, the machine accelerates and the value of P_a will be positive. With regard to stability, P_a should reverse its sign after $\frac{d\theta}{dt}$ reaches to zero. Therefore, P_a is a function of θ and has a various $P_a(\Theta) \leq 0$.

$$\int_{\theta_0}^{\theta} P_a d\theta = 0 \qquad (2.13)$$

and the limit of P_a

$$P_a(\theta_{\max}) = 0 \quad \text{and} \quad \int_{\theta_0}^{\theta_{\max}} P_a d\theta = 0 \qquad (2.14)$$

If the plot of P_a is a function of θ, the formula (2.14) can presents the area under the curve that limited by θ_0 and θ_{max}. The total area under the curve should be zero (the positive part equal the negative part); based on this it is called as the equal area criterion [20]. Using (2.11) and fixing P_m

$$P_e = \frac{E_1 E_2}{X}\sin\theta \qquad (2.15)$$

while X relies on the status of the system which may be pre-fault, fault or post-fault. The behavior of the system and the power angle corresponding to those three cases are shown in Figure (2.3). By using (2.14), it is clear seen that the stability is achieved with fault clearing angle up to θ_c, when the two areas A_1 and A_2 are equal. The system will be unstable if the fault clearing angle exceeds θ_c. The maximum value of fault clearing time that preserves the stability of the system is called critical fault clearing time T_{fc}, and the corresponding maximum rotor angle is θ_m. The equal area criterion also has the ability to calculate the maximum power that can be transferred through the system for a specific fault script, and to check the system if it is stable or unstable according to the given data and perturbation [20], [23], [24].

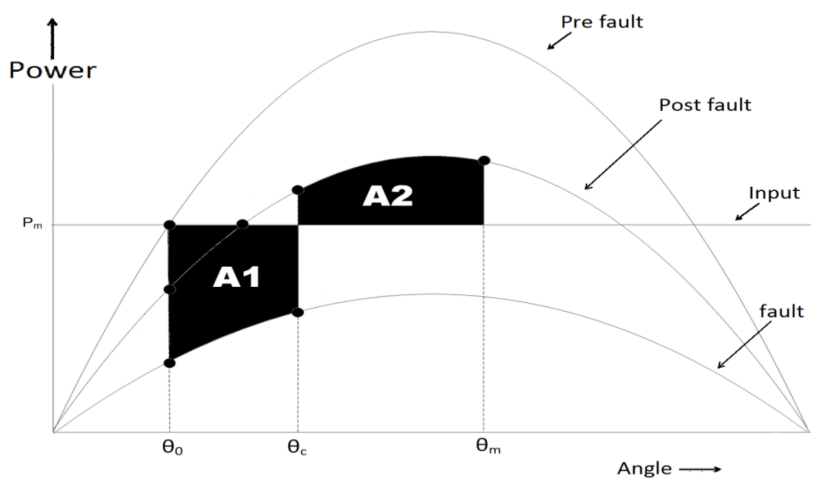

Figure 2.3: Equal area criterion of one machine with infinite bus system

2.6 Multi-Machine Power System

Considering a multi-machine system, the equation of the output power of its machine can be obtained by reducing this system and keeping only the internal machine buses

$$P_{ei} = \sum_{k=1}^{N} E_i E_k (G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik}) \qquad i = 1, 2, ...N \tag{2.16}$$

and the swing equations for synchronous machines are

$$\frac{2H_i}{\omega_0} \frac{d^2\theta_i}{dt^2} = P_{mi} - P_{ei} \qquad i = 1, 2, ...N \tag{2.17}$$

By using the trapezoidal method in (2.17)

$$\theta_i^n = -\frac{h^2\omega_0}{8H_i} P_{ei}^n + a_i^{n-1} \tag{2.18}$$

Where

$$a_i^{n-1} = \theta_i^{n-1} + h(\omega_i^{n-1} - \omega_0) + \frac{h^2\omega_0}{8H_i}(2P_{mi} - P_{ei}^{n-1}) \qquad i = 1, 2, ...N \tag{2.19}$$

From (2.16) and (2.19) it is acceptable to solve θ_i^n by applying Newton method, and the following linear equation is solved for $\Delta\theta$ to update θ.

$$\Delta a = J \Delta\theta \tag{2.20}$$

where J is the Jacobian matrix

Power system is a type of big complex systems including generating plants, transmission grids and distribution substations. To deal with such a system, efficient representation should be adopted to study the behavior of the system and stability should be analyzed as well. This can be achieved through mathematical representation by a set of differential and algebraic equations [20], [24], [25].

2.7 Differential Algebraic Equations

In studying the dynamics of a multi-machine power system, different types of stability problems can be modeled by a set of first-order parameterized differential equations in the form of

$$\dot{x} = f(x, y, p) \ , \ f : \mathbf{R}^{n+m+p} \rightarrow \mathbf{R}^{n} \tag{2.21}$$

and algebraic equations in the form of

$$0 = g(x, y, p), g : \mathbf{R}^{n+m+p} \rightarrow \mathbf{R}^{m} \tag{2.22}$$

$$x \in X \subset \mathbf{R}^{n} \quad , \quad y \in Y \subset \mathbf{R}^{m} \quad , \quad p \in P \subset \mathbf{R}^{p}$$

The dynamic state variables x and instantaneous state variables y are distinct in the state space $X \times Y$. The dynamic state variables are time dependent such as generator voltage and rotor phase, and the instantaneous states variables as bus voltages. The space P consists of system parameters (as equipment constants e.g. inductance) and operating parameters (such as generating units and loads). This type of DAEs is used vastly in numerical solutions and stability estimation in power systems, because it gives realistic solution for the system's parameters, and it is the easiest utility for sparse matrix approach as well. Since, it is not general possible to get analytical solution for this differential algebraic equations (DAEs), approximate numerical techniques are used. The numerical integration techniques use a stepwise procedure to get a set of solutions for dependent variables according to a set of solutions for the independent variables. Integration methods are based on two styles which are explicit or implicit and single-step or multi-step. Applying integration formula to the solved differential equations separately is called explicit method, while discretizing those differential equations and solving them together at the same time as a set is called implicit method. Although the implicit method is too complex it is very useful in numerical stability. Single-step method do not save previous step values and utilize it in the present step integration, contrary to multi-steps method which requires storing and using the information of the previous step to solve the present step integration [26], [27].

In power systems, time domain simulation code (TDSC) is a significant technique for dynamic analysis. It is used to study the transient behavior by applying numerical solution to DAEs of such a system. Power system grids consist of many generators, governors, transformers and various loads. To represent such a grid, each component of those devices requires differential and algebraic equations; consequently, the total number of differential and algebraic equations of a power system might be enormously large. The procedure of time domain simulation of power systems depends on step by step numerical integration of DAEs. For each corresponding step, there is a numerical error which can be calculated by local truncation error. To make the solution more accurate, either small step size integration or high order approximation should be used [16].

The numerical integration techniques have two classes; explicit and implicit methods. The explicit method depends on fixed-point iteration with active computation, but it has numerical stability problems in some cases. The implicit method based on solving nonlinear equations at each step is more accurately and stable but it is slow. The implicit method is commonly utilized in power system dynamic simulation, and wide range of work has been varied out to improve computational efficiency [27].

The complexity of the mathematical model will increase for larger scale power systems. It leads to decreased accuracy of the solution and numerical stability as well. Consequently, efficient methods should be used to treat these problems and solve DAEs for various types and scales of power systems such as Modified Euler method, Trapezoidal method and Runga-Kutta method.

2.8 Modified Euler's method

The main part of transient stability study is load flow calculation to get system conditions before a fault occurs. In power systems, it is important to solve first order differential equations (two for every machine) to get the alterations of both voltage angle and machine speed. For m machines, there are 2m simultaneous differential equations as shown in the following equations [28].

$$\frac{d\theta}{dt} = \omega_{i(t)} - 2\pi f \tag{2.23}$$

$$\frac{d\omega_i}{dt} = (\frac{\pi f}{H_i})(P_{mi} - P_{ei(t)}) \tag{2.24}$$

where, $i = 1, 2, ..., m$

By assuming the action of the governor is negligible, P_{mi} stay constant with $P_{mi} = P_{mi(0)}$.

In applications of Modified Euler's method, the initial appreciations of the voltage angle and machine speed is obtained at time $(t + \Delta t)$ from the following formula

$$\theta_{i(t+\Delta t)}^{(0)} = \theta_{i(i)}^{'} + \left. \frac{d\theta_i}{dt} \right|_{(t)} \Delta t \tag{2.25}$$

$$\omega_{i(t+\Delta t)}^{(0)} = \omega_{i(i)}^{'} + \left. \frac{d\omega_i}{dt} \right|_{(t)} \Delta t \tag{2.26}$$

Evaluating the derivatives from (2.23) and (2.24) with calculating $P_{ei(t)}$ allow calculating powers at time t. While $P_{ei(0)}$ is calculated at the moment just after occurrence the disturbance.

To obtain the second values, the derivatives at time $(t+\Delta t)$ should be evaluated. This step demand determines the initial values for the machine powers at time $(t+\Delta t)$. Getting these values of power are obtained by calculating the new components of the internal voltage according to the following formula

$$e_{i(t+\Delta t)}^{'(0)} = \left|E_i^{'}\right| \cos\theta_{i(t+\Delta t)}^{(0)} \tag{2.27}$$

$$f_{i(t+\Delta t)}^{'(0)} = \left|E_i^{'}\right| \sin\theta_{i(t+\Delta t)}^{(0)} \tag{2.28}$$

To get the solution of the network, the voltage at the internal machine buses should be fixed. By fixing the voltage at the internal machines buses, the solution of the network can be determined. At fault cases like a three phase fault on the bus n, the voltage E_n is fixed to be zero. The terminal currents of the machine are calculated by calculating both of bus and internal voltage as shown in the form

$$I_{ti(t+\Delta t)}^{(0)} = (E_{ti(t+\Delta t)}^{'(0)} - E_{ti(t+\Delta t)}^{(0)}).\frac{1}{r_{ei} + jx'_{di}} \tag{2.29}$$

for the machine powers

$$P_{ei(t+\Delta t)}^{(0)} = \mathrm{Re}\left|I_{ti(t+\Delta t)}^{(0)} (E_{ti(t+\Delta t)}^{'(0)})^{*}\right| \tag{2.30}$$

The second values of both voltage angle and machine speed are calculated by

$$\theta_{i(t+\Delta t)}^{(1)} = \theta_{i(t)}^{(1)} + \left(\frac{\left.\frac{d\theta}{dt}\right|_{(t)} + \left.\frac{d\theta}{dt}\right|_{(t+\Delta t)}}{2} \right) \Delta t \tag{2.31}$$

$$\omega_{i(t+\Delta t)}^{(1)} = \omega_{i(t)}^{(1)} + \left(\frac{\left.\frac{d\omega}{dt}\right|_{(t)} + \left.\frac{d\omega}{dt}\right|_{(t+\Delta t)}}{2} \right) \Delta t \tag{2.32}$$

$$\left.\frac{d\theta_i}{dt}\right|_{(t+\Delta t)} = \omega_{i(t+\Delta t)}^{(0)} - 2\pi f \tag{2.33}$$

$$\left.\frac{d\omega_i}{dt}\right|_{(t+\Delta t)} = \frac{\pi f}{H_i} (P_{mi} - P_{ei(t+\Delta t)}^{(0)}) \tag{2.34}$$

The final values of the voltage for the machine buses at time $(t+\Delta t)$ are

$$e_{i(t+\Delta t)}^{'(1)} = \left|E_i^{'}\right| \cos \theta_{i(t+\Delta t)}^{(1)} \tag{2.35}$$

$$f_{i(t+\Delta t)}^{'(1)} = \left|E_i^{'}\right| \sin \theta_{i(t+\Delta t)}^{(1)} \tag{2.36}$$

Then the solution of DAEs of the network is performed again to get final system voltages at time $(t+\Delta t)$. The procedure is repeated till it reaches to the ultimate value $T_{\max}$ defined in the study of such a network.

Modified Euler's method plays a big role in the calculations of transient stability of power systems. It can show the behavior of the machines and the performance of the system. The solution is evaluated by a series of estimates for internal machine bus voltage and machine speed at the next time step, and repeating this operation allows evaluating line currents and swinging impedances for preselected lines [28].

Chapter 3

RECURSIVE PROJECTION METHOD RPM

3.1 Definition of RPM

Recursive projection method (RPM) is one of the main new techniques to enhance the stability criteria in power systems. This method is used to stabilize the iterative procedures of solving nonlinear systems. At each step of iterative procedure, RPM takes the output information from time domain simulation code (TDSC) to define the slow/unstable invariant subspace of the full state-space of the system. The full state-space consists of two invariant subspaces; fast/stable subspace and slow/unstable subspace. Fixed-point iteration is applied to solve the first one, while Newton method is applied to improve the convergence of the second one [17], [29].

Since power system is a complex nonlinear dynamic system and can be modeled by a set of nonlinear differential and algebraic equations DAEs, RPM is investigated in predicting power system steady state. By taking ordinary differential equation (ODE) of the system

$$\dot{x} = F(x,u) \tag{3.1}$$

where $x \in R^N$ symbolizes to state variables, and $u \in R^S$ symbolizes control parameters. The derivation of Fixed-point iteration method can be done by defining the initial states $x^{(0)}$ and control parameters u, that is

$$x^{(v+1)} = \phi(x^{(v)}, u) \tag{3.2}$$

The behavior of solving the iteration (3.2) depends on the dominant eigenvalues of the Jacobian matrix ϕ_x at steady state $x^*(u)$, and those eigenvalues control the convergence of this iteration by their magnitude (largest magnitude), i.e. whether they lie outside or inside of the unit disk.

$$U_\gamma = \{|z| \leq 1-\gamma\} \quad , \gamma > 0 \tag{3.3}$$

This can be shown by perturbing the solution around its steady state as

$$x^{(v)} = x^* + \delta^{(v)} \tag{3.4}$$

substituting (3.4) in (3.2) and using the first two terms of the Taylor series expansion

$$x^{(v)} = x^* + \delta^{(v+1)} = \phi\left(x^* + \delta^{(v)}, u\right) \approx \phi\left(x^*, u\right) + \phi_x\left(x^*, u\right)\delta^{(v)} = x^* + \phi_x\left(x^*, u\right)\delta^{(v)} \tag{3.5}$$

from which we obtain

$$\delta^{(v+1)} = \phi_x\left(x^*, u\right)\delta^{(v)} \tag{3.6}$$

The scheme is convergent and close to steady state operation mode if all the eigenvalues have a magnitude less than one (located inside the unit disk), and it diverges if any of those eigenvalues have magnitudes greater than one (located outside the unit disk). In addition, the scheme is slowly convergent if any of those eigenvalues has a magnitude close to the boundary. The first case represents a stable scheme and the system operates in the steady state mode, but the second and third cases represent unstable and critical scheme and new technique should be applied to solve such problems. RPM is able to handle these cases; it can retrieve the convergence of the second case and improve convergence of the third case [17].

3.2 Basic Procedure of RPM

The slow convergent or the divergent schemes is the result of from some eigenvalues approaching (or leaving) the unit disk. The main idea is finding an eigenspace corresponding to the unstable scheme, which is achieved effectively by recursive projection method, utilizing iterates from fixed-point iteration [17], [18], [29].

Consider a system has N eigenvalues, and some of those eigenvalues are located outside the unit disk:

$$|\lambda_1| \geq .... \geq |\lambda_k| > 1-\gamma \geq |\lambda_{k+1}| \geq .... \geq |\lambda_N| \tag{3.7}$$

where k is the number of eigenvalues which lie outside the unit disk.

The Jacobian matrix ϕ_x of the system, which has range space R^N, can be written as the direct sum of two subspaces; P which is unstable/slow invariant subspace of ϕ_x and has eigenvalues $\{\lambda_u\}_1^k$, and Q which is the orthogonal complement of P

$$R^N = P \oplus Q = PR^N \oplus QR^N \tag{3.8}$$

Since the stabilization procedure needs to find the projectors P and Q, it is important to find an orthonormal basis for the subspace P. This basis is represented by $Z_p \in R^{N \times k}$, and satisfies $Z_p^T Z_p = I_k \in R^{k \times k}$. The orthogonal projector of R^N onto subspace P is $Z_p Z_p^T$, and $Z_q Z_q^T = I - Z_p Z_p^T$ is the complement orthogonal projector of R^N onto subspace Q. The state variables of the system $x \in R^N$ can be written as

$$p = Z_p Z_p^T x \in P \quad , \qquad q = (I - Z_p Z_p^T) x \in Q \tag{3.9}$$

by applying the projectors to (3.2), the fixed-point iterations for p and q can be rewritten as

$$p^{(v+1)} = f(p^{(v)}, q^{(v)}, u) \equiv Z_p Z_p^T \phi(p^{(v)} + q^{(v)}, u) \tag{3.10}$$

$$q^{(v+1)} = g(p^{(v)}, q^{(v)}, u) \equiv (I - Z_p Z_p^T)\, \phi(p^{(v)} + q^{(v)}, u) \tag{3.11}$$

According to *Lemma 1*, the procedure will work with the steady state eigenvalues located all inside the unit disk U_γ and the Jacobian matrix g_q is

$$g_q^* = (I - Z_p Z_p^T)\, \phi_x(x^*, u)\, (I - Z_p Z_p^T) \tag{3.12}$$

and the iteration (3.11) is locally convergent in the proximity of steady state.

By using RPM procedure, the convergence of iteration (3.2) is enhanced by applying the Newton method on the subspace P. At the same time fixed-point iteration scheme is maintained on subspace Q . The procedure is performed according to the following steps:

by defining the initial states

$$p^{(0)} = Z_p Z_p^T x^{(0)}(u) \quad , \quad q^{(0)} = (I - Z_p Z_p^T) x^{(0)}(u) \tag{3.13}$$

and updating p and q iteratively gives

$$p^{(v+1)} = p^{(v)} + (I - f_p^{(v)})^{-1} \; (f(p^{(v)}, q^{(v)}, u) - p^{(v)}) \tag{3.14}$$

$$q^{(v+1)} = g(p^{(v)}, q^{(v)}, u) \tag{3.15}$$

The iterations (3.14) and (3.15) continues until achieving the convergence at

$$x^*(u) = x^{(v+1)}(u) = p^{(v+1)} + q^{(v+1)} \tag{3.16}$$

The Newton method is applied to the unstable/slow modes by solving (3.5a) for the steady state. That is the following equation is solved

$$p = f(p, q, u) \quad \Rightarrow \quad F(p) = p - f(p, q, u) = 0 \tag{3.17}$$

Newton's method for iteratively solving nonlinear equations is applied to (3.11) to get

$$p^{(v+1)} = p^{(v)} - F_p^{-1}\left(p^{(v)}\right)\left(p^{(v)} - f(p^{(v)}, q, u)\right) \tag{3.18}$$

where the Jacobian matrix is given by

$$F_p(p)=\frac{\partial F}{\partial p}=I-\frac{\partial f}{\partial p}=I-f_p(p,q,u) \tag{3.19}$$

Substitution of (3.19) in (3.18) gives (3.14).

3.3 Computational Properties

To get effective computations, the term $(I-f_p^{(v)})^{-1}$ requires one formation instead of updating at each step. The variables $z \in R^k$ are inserted to represent $p \in P$:

$$z \equiv Z_p^T p = Z_p^T x \;\;, \quad p = Z_p x \quad , \quad x = Z_p z + q \tag{3.20}$$

which corresponds to a change of coordinates. It is noteworthy that the Jacobian matrix related to the Newton part changes its dimensions through performing the RPM procedure and decreases $N \times N$ to $k \times k$, and the iteration (3.14) is

$$z^{(v+1)} = z^{(v)} + (I - Z_p^T \phi_x Z_p)^{-1} \left[Z_p^T \phi(z^{(v)}, q^{(v)}, u) - z^{(v)} \right] \tag{3.21}$$

To obtain (3.21), first (3.18) is multiplied by Z_p^T and the substitutions

$p = Z_p z,\ f(p,q,u) = Z_p Z_p^T \phi(p+q,u)$ are made to give

$$\begin{aligned} z^{(v+1)} &= z^{(v)} + Z_p^T (I - f_p^{(v)})^{-1} Z_p \left[Z_p^T \phi(z^{(v)}, q^{(v)}, u) - z^{(v)} \right] \\ &= z^{(v)} + (Z_p^T Z_p - Z_p^T f_p^{(v)} Z_p)^{-1} \left[Z_p^T \phi(z^{(v)}, q^{(v)}, u) - z^{(v)} \right] \\ &= z^{(v)} + (I_m - Z_p^T \phi_x Z_p)^{-1} \left[Z_p^T \phi(z^{(v)}, q^{(v)}, u) - z^{(v)} \right] \end{aligned} \tag{3.22}$$

where the last line follows from $f_p(p,q,u) = Z_p Z_p^T \phi_x(p+q,u)$.

It is not necessary to have the system equations in explicit form. This is because the required computations can be performed implicitly using the information provided by the TDSC. As a result, the Jacobian matrix also cannot be found in explicit form. Matrix-vector products are produced from various methods of approximations [17], [18], [29].

One of those approximations is shown below. Since the orthonormal basis for the subspace P are obtained. That is,

$$Z_p = [Z_{p1}, ..., Z_{pk}] \in R^{N \times k} \tag{3.23}$$

and for epsilon value ε > 0, approximating the i^{th} column of $\phi_x Z_p$ can be achieved by the form

$$\phi_x Z_{pi} \approx [\phi(x + \varepsilon Z_{pi}, u) - \phi(x, u)] \; / \; \varepsilon \quad , \qquad i = 1, 2, ..., k \tag{3.24}$$

Chapter 4

SIMULATIONS AND RESULTS

The large increase in power demand and preserving the power system stability issue pose significant problems. To handle such problems and apply efficient techniques that enhance system stability at various types of operating modes, numerical simulation should be used. Numerical simulation provides an interactional environment with thousands of dependable and precise built-in mathematical functions. These functions give solutions to a wide domain of mathematical problems comprising matrix algebra, differential equations, non-linear systems, and other numerous kinds of scientific computations. In numerical simulations, Matlab software is commonly used which has been strengthened by the effectual SIMULINK program. SIMULINK is a diagrammatic program utilized to simulate dynamic systems. Besides, it gives the possibility of simulating linear and nonlinear systems readily and efficiently [25]. Since the dynamical modeling of power systems is represented by a set of non-linear differential equations, it is suitable to use such a program to analyze and study power system stability problems.

4.1 Case Study

In this study, the numerical simulation has been done by using power system toolbox PST of Matlab [21]. PST contains a set of M-files developed to aid in exemplary power system analysis. It supports the user to perform all power system analysis like power flow solution and transient stability. The power system network chosen as a case study which consists of three generators and six buses is shown in Figure (4.1).

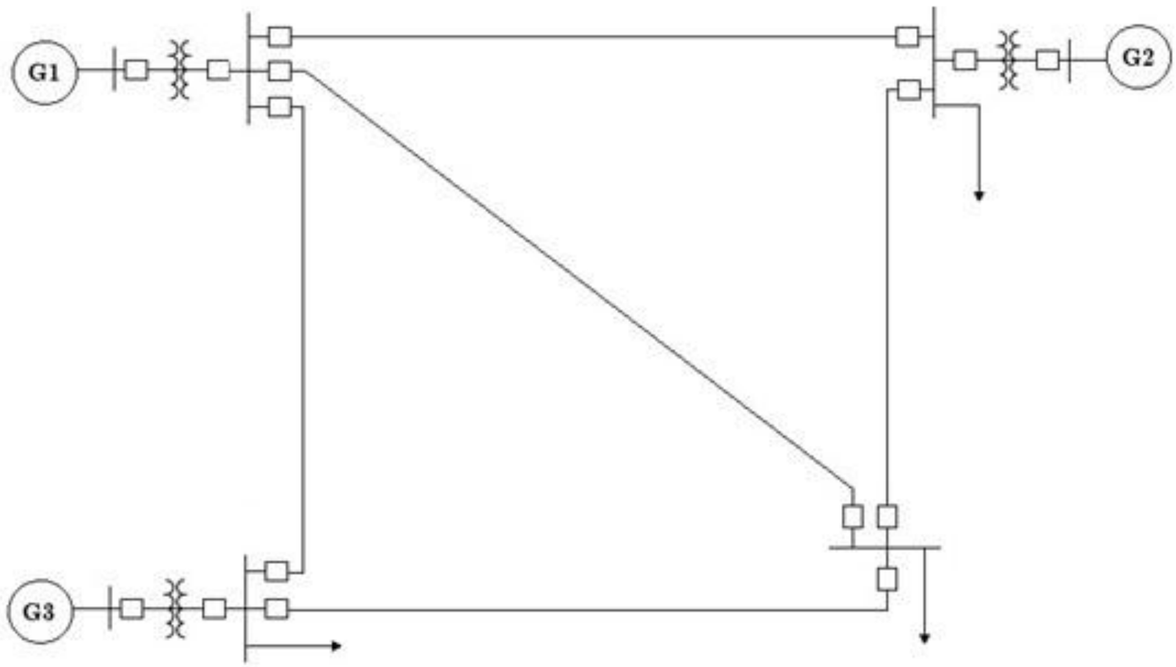

Figure 4.1: three generators and six buses power system

Here, bus1 is the slack bus with specific voltage $V1 = 1.06\angle 0^{\circ}$, and all the values of resistance, reactance, capacitance, and voltage magnitude are in per unit and the power base is chosen as 100 MVA. The power system installation is defined by two certain matrices for bus and line that are utilized in the load flow computation. Solving load flow is important to determine the operating conditions that are used in modeling the dynamic devices. Required data is shown in the following tables which include the data set for generators, loads, buses, and lines.

Table 1: Line data

LINE DATA				
Bus No.	Bus No.	R PU	X PU	½ B PU
1	4	0.035	0.225	0.0065
1	5	0.025	0.105	0.0045
1	6	0.040	0.215	0.0055
2	4	0.000	0.035	0.0000
3	5	0.000	0.042	0.0000
4	6	0.028	0.125	0.0035
5	6	0.026	0.175	0.0300

Table 2: Load data

LOAD DATA		
Bus	Load	
No.	MW	Mvar
1	0.00	0.00
2	0.00	0.00
3	0.00	0.00
4	100.00	70.00
5	90.00	30.00
6	160.00	110.00

Table 3: Generators data

GENERATORS DATA				
Bus	Voltage	Generation	Mvar Limits	
No.	Mag.	MW	Min.	Max.
1	1.06			
2	1.04	150.00	0.00	140.00
3	1.03	100.00	0.00	90.00

Table 4: Machines data

MACHINES DATA			
Gen. No.	R_a PU	$X_d^{'}$ PU	H Sec.
1	0.00	0.20	20.00
2	0.00	0.15	4.00
3	0.00	0.25	5.00

4.2 Transient Stability Simulation Using ode32 Solver

The first step in the numerical simulation of stability is implementing the power flow solution by using the **lfnewton** code. This code is based on the Newton method in order to execute load flow and calculate the physical values like active and reactive power of the system. Besides, **trstab** is used with **lfnewton** to analyze the system after being subjected to various types of fault, and calculate the new bus admittance matrix. When a fault occurs in the system, the protection relays directly indicates the fault and sends a trip signal to the circuit breaker to separate the faulted part.

When the system is exposed to a fault, the simulation is divided into three cases; pre-fault, during fault, and post-fault. The fault may cause a change in the operation mode according to the voltage values and machine angles as explained in Chapter 2. The aim of this numerical simulation is to enhance and preserve the stability of the system through those three steps.

In this dissertation, a fault of three-phase is applied on the line (5-6) close to bus (6). Separating this line by disconnecting the circuit breakers of the ends of same line, the fault will be cleared. To carry out transient stability analysis for a specific clearing time t_c and final simulation time t_f , **trstab** code is ran, which is based on the **ode32** solver of Matlab. The first machine is a reference for the other two machines with respect to speed and angle. For t_c = 0.4 s and t_f =5 s the output simulation in Figure (4.2a) shows the oscillations in the angles of both machines. In this case, for $t_c \leq$ 0.4 the oscillation will continue for a certain time with decreasing amplitude before reaching to the steady state. This is illustrated by choosing t_f =30 s as shown in Figure (4.2b).

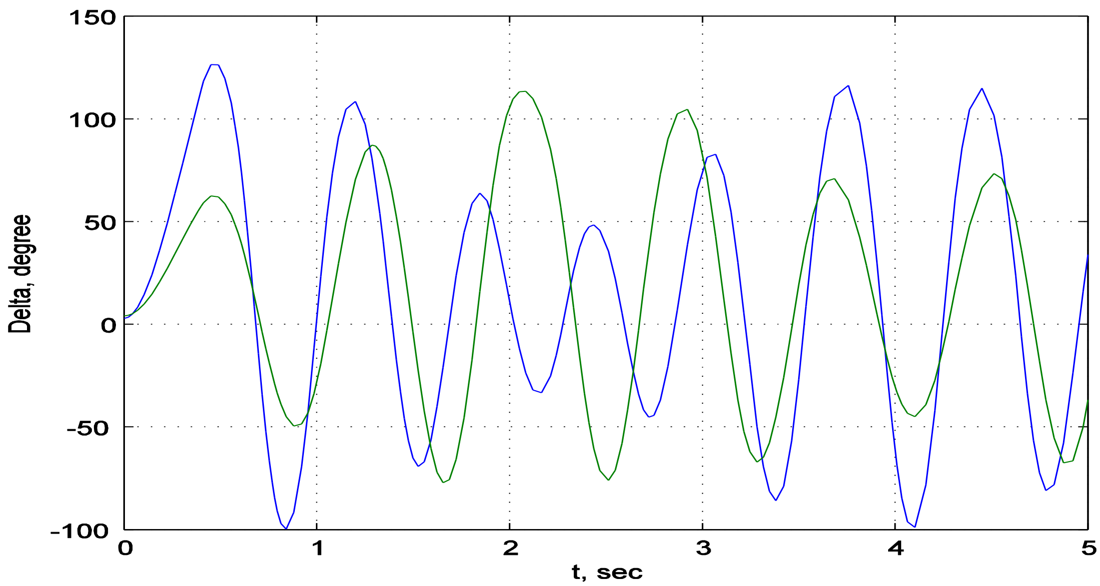

Figure 4.2a: Phase angle difference for both 2nd and 3rd machine at t_f =5 s

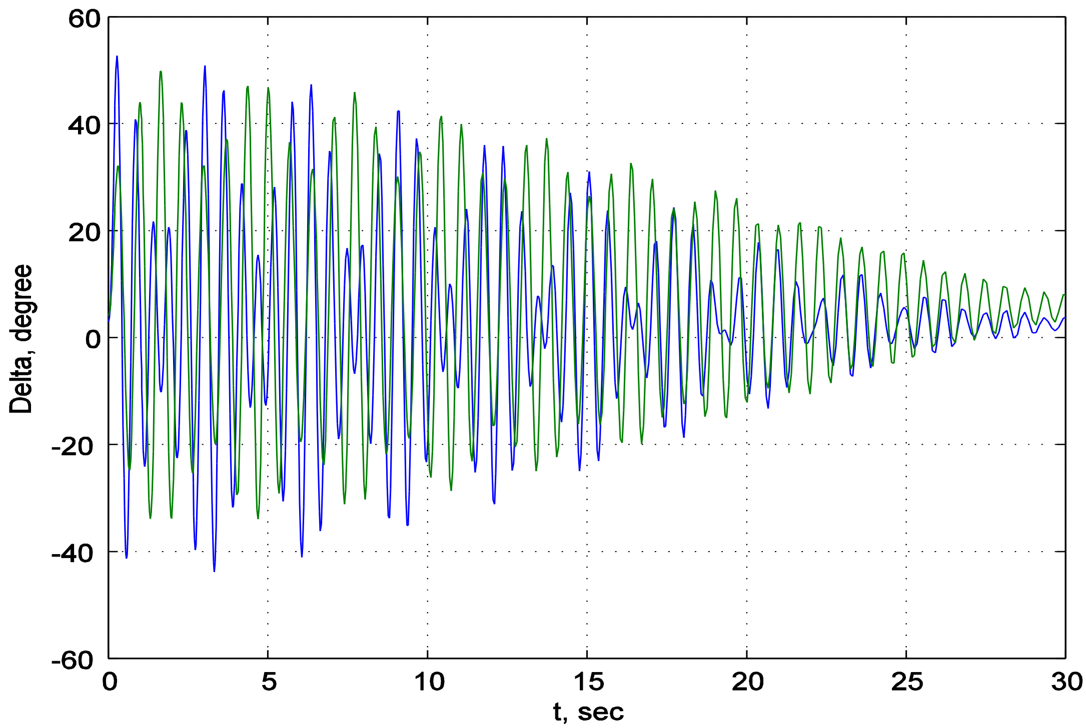

Figure 4.2b: Phase angle difference for both 2nd and 3rd machine at =30 s

To choose best conditions in this code like decreasing the step size to give a solution with optimal level of accuracy, it still gives unstable solution with oscillation that was taken as an indication of error in the calculations as shown in Figure 4.2c.

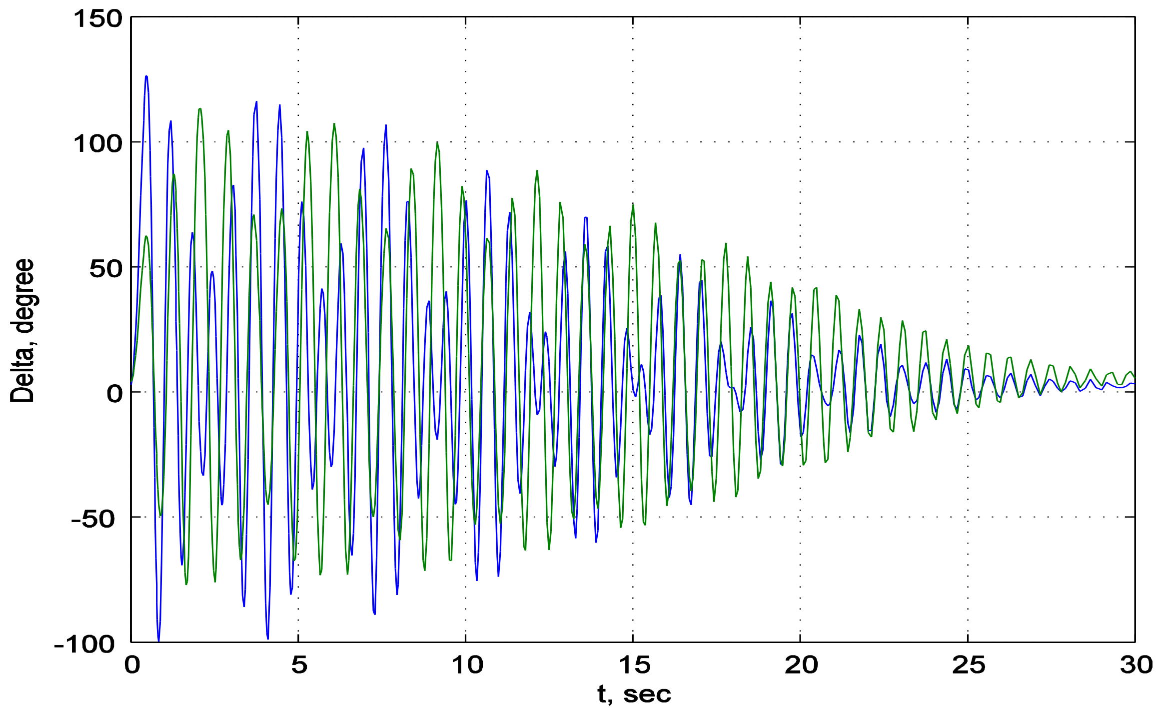

Figure 4.2c: Phase angle difference at best conditions for both 2nd and 3rd machine at t_f =30s

Here, the steady-state solutions for both the 2nd and 3rd machine angles are nearly 2.8° and 6° respectively. The solution given by **trstab** for this case shows that the system will continue oscillating for a specific time after the fault is cleared. Duration of this time is not short; it may lead to a sequence of other faults and may cause damage to the system's switchgear. Although steady-state is achieved after such a long time, the angles still contain small continuous oscillations. Therefore, if this simulation result is taken to be reliable, extra work should be done to force the system to return to the steady-state mode in shorter time, and achieve steady-state without any continuous oscillation as well. On the other hand, the oscillations in the steady state are actually an indication that this simulation result is not very reliable. Thus, **trstab** is not that efficient by itself in all the situations, and the accuracy should be improved by applying an efficient and active technique, which is the RPM procedure. Also, **trstab** is based on the ordinary differential equation solver **ode32,** which solves the system for a specified span of time, and RPM requires the solution at each time step in the code. Therefore, **ode32** should be replaced by another solver, which is the Modified Euler Method (MEM), so that RPM can be easily applied.

4.3 Application of Modified Euler Method

To understand MEM well, it is better to start with the Euler Method (EM). EM is characterized as the easiest algorithm in the domain of numerical solution of differential equations, applicable to first order equations. Although EM gives a solution with least accuracy, it generates a foundation for comprehending more developed methods. Consider the following differential equation

$$\frac{dx}{dt} = p(t)\ x \tag{4.1}$$

where $p(t)$ is a known function. According to EM, the derivative in the differential equation can be approximated by

$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} \cong \frac{x(t+\Delta t) - x(t)}{\Delta t} = p(t)x \tag{4.2}$$

where Δt is the step size

by rearranging (4.2)

$$x(t+\Delta t) = x(t) + p(t)\ x(t)\Delta t \tag{4.3}$$

by using the iterative procedure (4.3) will be

$$x(t_{k+1}) = x(t_k) + p(t_k)x(t_k)\Delta t \tag{4.4}$$

where $t_{k+1} = t_k + \Delta t$

Euler method can be sum up for the general form $\dot{x} = f(t, x)$ as,

$$x(t_{k+1}) = x(t_k) + \Delta t. f[t_k, x(t_k)] \tag{4.5}$$

EM presumes that variables of the equation are constant during the step size Δt . So, it is not applicable in all the situations, and has a deficiency in accuracy at a case of fast changing of variables. To enhance the method, additional update should be applied to the right-hand side of the equation. The update uses the average of the solutions for two contiguous steps as below

$$x(t_{k+1}) = x(t_k) + \frac{\Delta t}{2}(f_k + f_{k+1}) \tag{4.6}$$

where $f_k = f(t_k, x(t_k))$ and $f_{k+1} = f(t_{k+1}, x(t_{k+1}))$

Thus, utilize EM to get $x(t_{k+1})$ that allows executing predictor-error method, and MEM procedure is a combination of the two steps: Euler predictor and predictor-error. That is

Euler predictor $$y_{k+1} = x_k + h. f(t_k, x_k) \tag{4.7}$$

Predictor-error $$x_{k+1} = x_k + \frac{h}{2}.[f(t_k, x_k) + f(t_{k+1}, y_{k+1})] \tag{4.8}$$

4.4 Finding the Jacobian Matrix

The RPM procedure requires the computation of the Jacobian matrix of the difference equations (see (3.2)) that result from the application of the particular numerical solution technique to the continuous-time differential equations (DE). In general, the DE's are quite complicated and usually very difficult to obtain explicitly. To see how the Jacobian matrix of the discretized equations of a power system can be indirectly obtained through application of a numerical procedure, we consider a linear second order system with the equations

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= a_1 x_1 + a_2 x_2 + u \end{aligned} \tag{4.9}$$

which can be represented as $\dot{x}(t) = F\left(x(t)\right)$ where $x = [x_1, x_2]^T$ and

$$F(x) = \begin{bmatrix} x_2 \\ a_1 x_1 + a_2 x_2 + u \end{bmatrix} \tag{4.10}$$

Suppose the numerical solution technique applied to (4.7) results in the discrete-domain equations

$$x_{k+1} = \phi(x_k) = \begin{bmatrix} \phi_1(x_k) \\ \phi_2(x_k) \end{bmatrix} \tag{4.11}$$

The Jacobian matrix of the function vector ϕ is given by

$$\phi_x = \begin{bmatrix} \dfrac{\partial \phi_1}{\partial x_1} & \dfrac{\partial \phi_1}{\partial x_2} \\ \dfrac{\partial \phi_2}{\partial x_1} & \dfrac{\partial \phi_2}{\partial x_2} \end{bmatrix}$$

This matrix can be evaluated numerically by using the following procedure:

$$\phi_x v_n = \frac{1}{\varepsilon}[\phi(x + \varepsilon v_n) - \phi(x)] \qquad n = 1, 2 \tag{4.12}$$

where v_n are the vectors,

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \;,\; v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

For the Modified Euler method the function vector ϕ is written as

$$\phi(x) = x + \frac{1}{2}h[F(x) + F(x + hF(x))]$$
$$= x + \frac{1}{2}h[F(x) + F(\hat{x})] \quad (4.13)$$

where

$$\hat{x} = x + hF(x) = \begin{bmatrix} x_1 + hx_2 \\ x + h(a_1x_1 + a_2x_2 + u) \end{bmatrix} \quad (4.14)$$

evaluation of (4.10) at the predictor solution

$$F(\hat{x}) = \begin{bmatrix} (1 + ha_2)x_2 + ha_1x_1 + hu \\ a_1(x_1 + hx_2) + a_2(x_2 + ha_1x_1 + ha_2x_2 + hu) + u \end{bmatrix} \quad (4.15)$$

$$= \begin{bmatrix} (1 + ha_2)x_2 + ha_1x_1 + hu \\ (a_1 + ha_1a_2)x_1 + (a_1h + a_2 + ha_2^2)x_2 + (1 + ha_2)u \end{bmatrix} \quad (4.16)$$

$$\phi(x) = \begin{bmatrix} x_1 + \frac{1}{2}h(x_2 + (1 + ha_2)x_2 + ha_1x_1 + hu) \\ x_2 + \frac{1}{2}h(a_1x_1 + a_2x_2 + u + (a_1 + ha_1a_2)x_1 + (a_1h + a_2 + ha_2^2)x_2 + (1 + ha_2)u) \end{bmatrix} \quad (4.17)$$

finally, the Jacobian matrix can be obtained as

$$\phi_x(x) = \begin{bmatrix} 1 + \frac{1}{2}h^2a_1 & \frac{1}{2}h(1 + ha_2) \\ \frac{1}{2}h(2a_1 + ha_1a_2) & 1 + \frac{1}{2}h(2a_2 + a_1h + a_2^2h) \end{bmatrix} \quad (4.18)$$

The numerical procedure in (4.10) was applied to the system in (4.7) and the computed Jacobian matrix was compared with the theoretical result (4.11), with an excellent match.

4.5 Transient Stability Simulation Using modeu Solver

After using MEM (modeu) solver instead of **ode32** solver in **trstab**, running **lfnewton** and new **trstab** (which was called **eigv**) for a three phase fault at the same position (at bus 6 between lines 5 and 6) the following results are obtained:

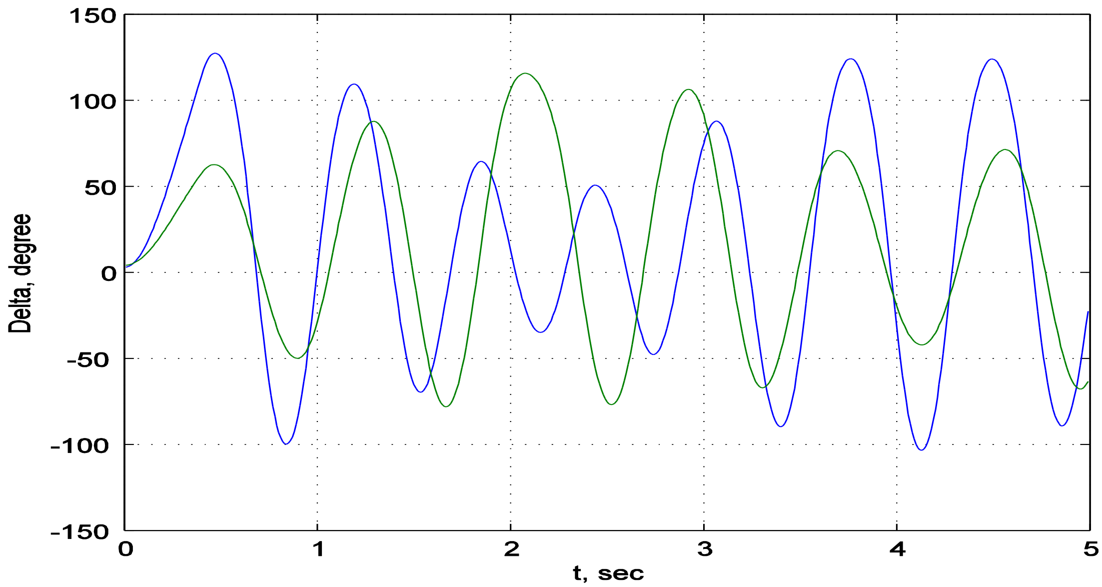

Figure 4.3: Phase angle difference for both 2nd and 3rd machine with MEM at h = 0.01

The similarity of the outputs in Figures (4.2 a) and (4.3) proves that the new code **eigv** is working correctly. In this case study, the system contains six state variables; two for each machine, which are rotor's speed and angle. Finding the Jacobian matrix allows calculating eigenvalues and eigenvectors.

4.6 Transformation Region of Stability

When the differential equations of a continuous-time system are transformed to discrete form by the numerical integration algorithm, the modes of the system will be transformed according to this relationship

$$\mu_k = e^{h\lambda_k} \tag{4.19}$$

where h integration step size,

$\mu_k \in \sigma(\phi_x)$ eigenvalues of the discrete mode (integration) system

$\lambda_k \in \sigma(F_x)$ eigenvalues of the continuous mode (physical) system

The transformation results in the conversion of the stability zone from the left half part of the complex plane for F_x to the inner region of the unit disk for ϕ_x. Therewith, applying numerical techniques to solve ODEs is imprecise. The effective relation is tightly associated to the integration scheme. For example, this case study deals with the Modified Euler Method, and for this method,

$$\hat{x}^{(\nu+1)} = x^{(\nu)} + hF(x^{(\nu)}, u) \tag{4.20}$$

$$x^{(\nu+1)} = x^{(\nu)} + h[F(x^{(\nu)}, u) + F(\hat{x}^{(\nu+1)}, u)]/2 \tag{4.21}$$

$$\equiv \phi(x^{(\nu)}, u)$$

$$\mu_k = [(1 + h\lambda_k)^2 + 1]/2 \tag{4.22}$$

4.7 Performing RPM

After calculating the eigenvalues, it should be checked whether their magnitudes are less-equal-larger than unity. Then the critical values that cause the unstable behavior of the numerical algorithm is separated, and the corresponding eigenvectors to create a new basis is found which consequently improves the numerical stability. The result for the three-phase fault case is shown in Figure (4.4). The operating condition is the same as before, that is applying a three phase fault between the lines 5 and 6, t_c =0.4 s and t_f =5 s. The time integration step h is adjustable, and in this case is chosen as h=0.01. After running the program the magnitudes of the eigenvalues are determined as,

λ = [1.0000 1.0000 1.0000 1.0000 0.9999 1.0001] , n =6

As shown in λ, the number of critical eigenvalues are k =5. According to the RPM procedure, it will produce a matrix of orthonormal basis Z_{po} corresponding to those critical eigenvalues. The produced Z_{po} is

$Z_{po} = [$

-0.0006 + 0.0000i	-0.0038 + 0.0000i	-0.0742 - 0.5645i	0.0000 + 0.4350i	-0.0001 + 0.6975i
-0.0006 + 0.0000i	-0.0038 + 0.0000i	-0.0756 - 0.5755i	0.0000 + 0.3880i	0.0001 - 0.7159i
-0.0006 + 0.0000i	-0.0038 + 0.0000i	-0.0758 - 0.5771i	-0.0001 - 0.8125i	-0.0000 + 0.0316i
0.0972 - 0.0000i	0.2980 - 0.0001i	-0.0013 - 0.0099i	0.0000 + 0.0029i	-0.0000 + 0.0053i
0.5658 - 0.0000i	-0.8014 + 0.0002i	0.0004 + 0.0031i	0.0000 + 0.0006i	-0.0000 + 0.0011i
-0.8188 + 0.0000i	-0.5185 + 0.0001i	0.0003 + 0.0021i	0.0000 + 0.0007i	-0.0000 + 0.0014i]

After calculating the solution at time step k, the Jacobian matrix, and Z_{po} are substituted in the main equations of the RPM algorithm to give the corrected solution at time step $k+1$. This solution improves the convergence of the solution and enhances the ability to evaluate system stability as shown in Figure (4.4)

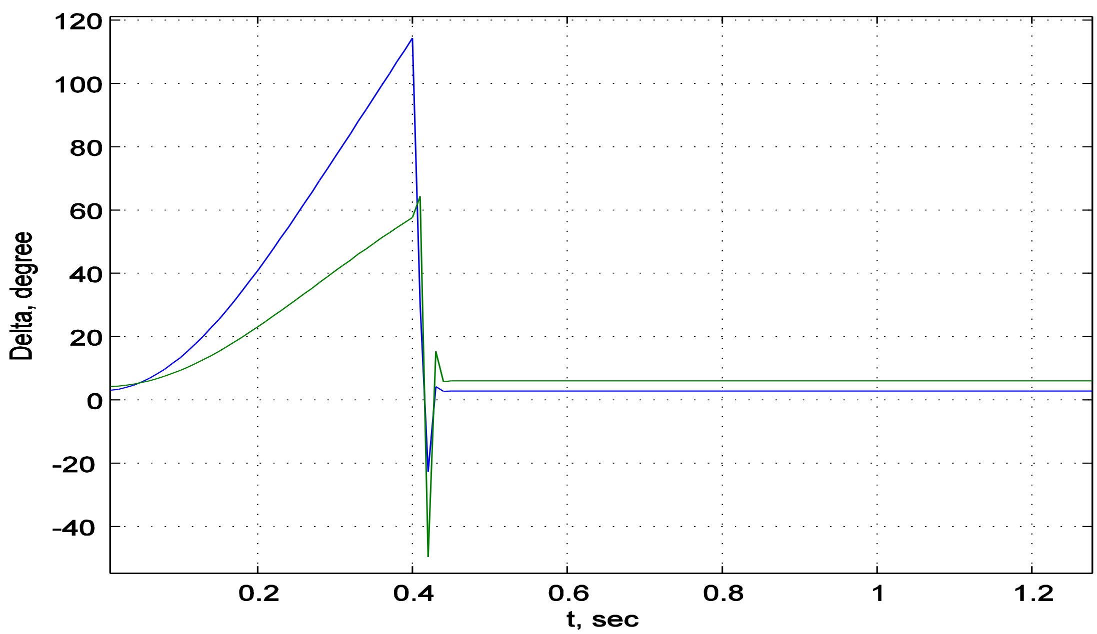

Figure 4.4: Phase angle difference for both 2nd and 3rd machine by RPM

To prove that RPM works efficiently, can check the steady state solutions in both Figure (4.2) and Figure (4.4) to show that the values of the two machine angles are almost equal. In addition, RPM reaches the steady-state mode in a very short time 0.45 s comparing with the previous case at 30 s, without any oscillation. The comparison between classical method and RPM is shown in Figure (4.5 a-b).

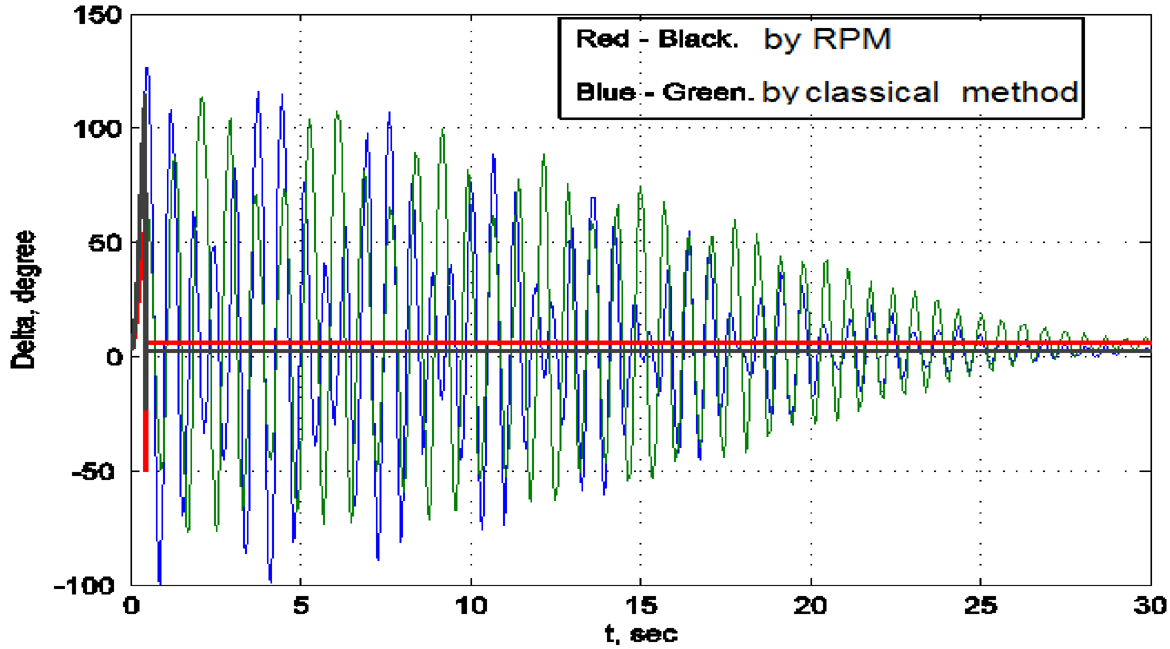

Figure 4.5a: Comparison between classical method and RPM

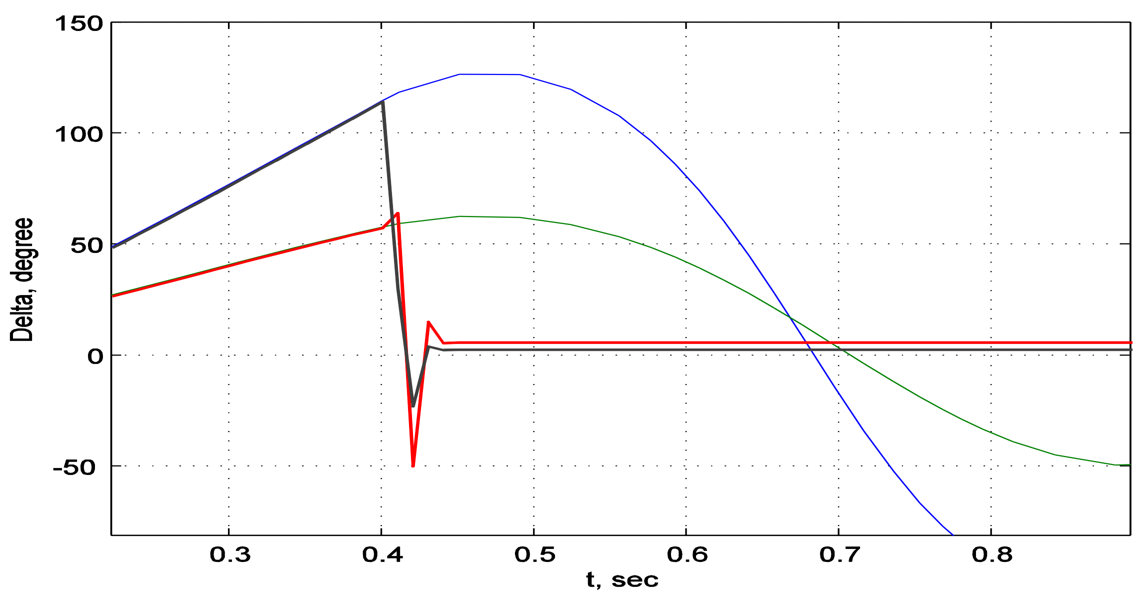

Figure 4.5 b: Comparison between classical method and RPM, showing initial part

We have taken a large simulation end-time to see the achievement of RPM clearly, as shown in Figure (4.6). The numerical solution given by **trstab** continues to oscillate in the steady-state, whereas it is perfectly stable with RPM.

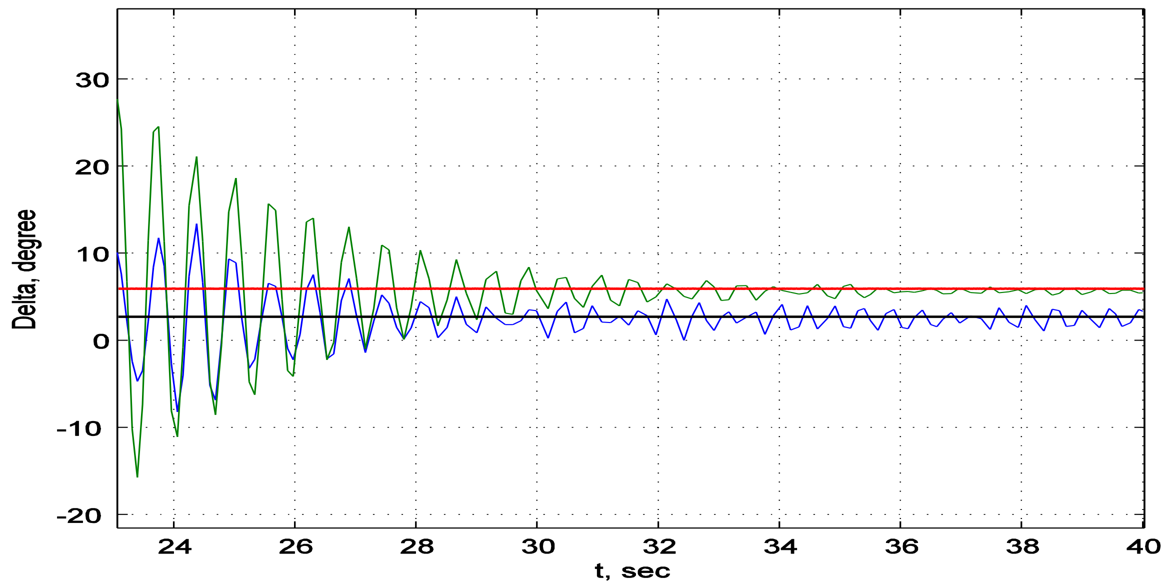

Figure 4.6: Steady-state solution of the system by classical method and RPM

One of the best achievements of RPM is performing the solution and reaching steady-state mode with a time shorter than that in classical methods; this leads to having reliability and efficiency in the response of system simulations. For a step size h=0.15, the solution for classical method will give unstable solution, while RPM will preserve the stable solution as shown in Figure (4.7). Note that the large simulation time here is for observing the behavior of stability analysis in power system simulation, not for an actual application in real time. In practice, the time t_{cc} and the steady-state period depend on many conditions like the size of fault, and the ability of the switchgear and the grid to handle such a fault and pass it successfully.

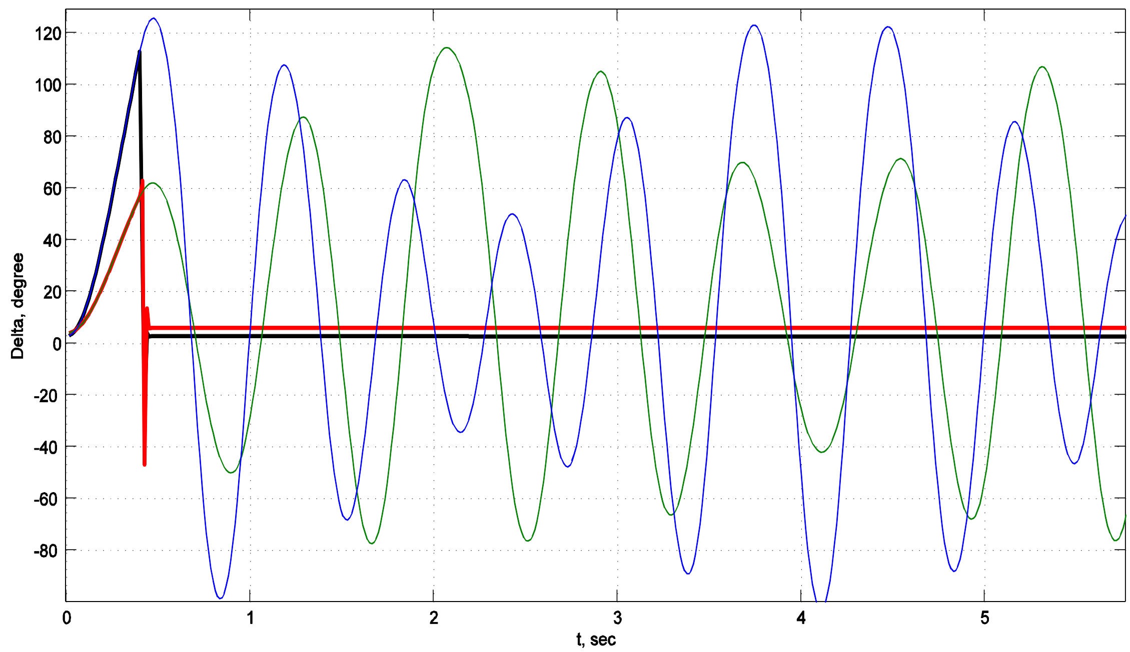

Figure 4.7: Comparison between classical method and RPM for h =1.5

4.8 Dynamic Simulation Using Power System Toolbox (PST)

Application of RPM at the beginning of this study was carried out by using the power system toolbox PST [30]. But it was later abandoned because of the difficulties in using the transient simulation code **s_simu** in conjunction with the RPM procedure. PST allows modeling machines and control systems, and establishes active modeling to state variables for designing damping controllers. These models are coded as m-files Matlab functions. One of those m-files is **s_simu** which has driving functions for transient stability analysis. The code **s_simu** needs only input data to prepare a good environment for analyzing and solving the system dynamical equations. First of all, this code was used and many operating tests have been done on the 9-bus WECC (or WSCC) power system [23] to see the behavior of the system; showing when and how the system can regain its steady state mode after subjected to different types of faults. This study is stopped in this thesis when trying to apply the RPM algorithm with **s_simu**, because it contains complex steps and it is difficult to connect RPM to it. Therefore, **trstab** m-file is used instead of **s_simu** and the complete work is implemented successfully.

Chapter 5

CONCLUSION

In this study, the power system stability problem is considered, and effective ideas are introduced for efficient analysis of power system behavior for various types of operating conditions. It has focused on transient stability under specific disturbances. The main criterion in power system stability, namely the equal area criterion, is also demonstrated in this study. Mathematical representation of power systems is performed to obtain differential and algebraic equations that have been solved numerically. In this work, MEM solver is used because it gives simple and accurate solution, and it is based on time stepping that allows application of the RPM algorithm easier than using the ODE solver, which is based on a time span solution.

An effective numerical technique is presented, which is the RPM algorithm, it is a new approach used in power systems recently to improve stability analysis. RPM is a stabilization algorithm that has the ability to expand the convergence domain of fixed point iteration schemes. In other words, RPM evaluates subspaces for the case of divergent iterations and correcting those iterations by applying Newton's method, whereas on the supplement, fixed point iteration is preserved to evolve the convergent iterations. A test example of a 6-bus power system is considered to show the achievements of the RPM technique and Modified Euler Method in the Matlab environment. Besides, numerical simulation studies performed on such a system have shown the success of this technique.

At each time step, the Matlab code calculates the Jacobian matrix and finds its eigenvalues. After that it compares the magnitudes of those eigenvalues, separating the larger ones and creating a corresponding ortho-normal basis which helps the code to find the correct solution and stabilize the numerical system.

The RPM procedure represents a useful algorithm in power system stability analysis that can handle all the operating situations, stable, slow-decaying and unstable modes, effectively and with high accuracy. As a result, RPM forces power system simulation to reach the steady-state mode in a very short time compared with conventional procedures, and without erroneous oscillations.

Future work that may be suggested is the application of the RPM algorithm to larger power systems, and for the case of unstable invariant subspaces with large dimensions. Theoretical work should also be carried out on this method to understand the level of accuracy improvement.

REFERENCES

[1] Han, X., Tian, N., et.al., “Small signal stability analysis on power system considering load characteristics,” *Power and Energy Engineering Conf.*, 2009, pp.1-4.

[2] Grigsby, L.L., *Electric Power Engineering Handbook*: *Power system dynamic and Stability*. New York: Taylor & Francis, 2006.

[3] Kundur, P., *Power System Stability and Control*. California: McGraw-Hill, 1993.

[4] Su, Y.C., Cheng, S.J., et.al., “Power system dynamic stability analysis and stability type discrimination,” *International Conf. on Power System Technology*, pp.1-6, 2006.

[5] Zhao, X., Zhou, J., et.al., “Dynamic modeling and transient stability simulation of a synchronized generators in power systems,” *International Conf. on Power System Technology*, pp.1-6, 2006.

[6] Wang, K., Crow, M.L., “Numerical simulation of stochastic differential algebraic equations for power system transient stability with random loads,” *IEEE Power and Energy Society General Meeting*, pp.1-8, 2011.

[7] Rudnick, H., Hughes, F.M., et.al., “A discrete model approach to power system transient performance simulation,” *American Control Conf.*, pp.846-852, 1984.

[8] Chakrabarti, S., “Notes on power system voltage stability”, http://home.iitk.ac.in/~saikatc/EE632_files/VS_SC.pdf, retrieved on Jun 2011.

[9] Yuanzhang, S., Lixin, W., et.al., “A review on analysis and control of small signal stability of power systems with large scale integration of wind power,” *International Conf. on Power System*, pp.1-6, 2010.

[10] Gang, W., Hua, Y., et.al., "Power system small signal stability analysis based on character identification of time domain simulation," *International Conf. on Advanced Power System Automation and Protection*, pp.1444-1448, 2011.

[11] Han, X., Tian, N., et.al., "Small signal stability analysis on power system considering load characteristics," *Power and Energy Engineering Conf.*, pp.1-4, 2009.

[12] Pai, M.A., Sauer, P.W., "Stability analysis of power systems by lyapunov's direct method," *IEEE Control System Magazine*, pp.23-27, 1989.

[13] Davy, R.J., Hiskens, I.A., "Lyapunov functions for multimachine power systems with dynamic loads," *IEEE Trans. on Circuits and Systems-I: Fundamental Theory and Applications*, vol.44, No.9, pp.796-812, 1997.

[14] Lopes, B.I.L., de Souza, A.C.Z., "Handling voltage studies under a dynamic system model," *IEEE Power Tech*, Russia, pp.1-6, 2005.

[15] Hashim, N., Sallehhudin, A.A., et.al., "Modeling of power system dynamic devices incorporated in dynamic computation for power systems(DCPS) for transient stability analysis," *International Electric Machines & Drivers Conf.* (IEMDC), pp.647-652, 2011.

[16] Meliopoulos, A.P., Stefopoulos, G.K., et.al., "Improved numerical integration method for power/power electronic system based on three-point collection," *IEEE Conf. on Decision and Control, and European Control Conf.*, pp.6780-6787, 2005.

[17] Fan, M.H., Ajjarapu, V., et.al., "RPM-based approach to extract power system steady state and small signal stability information from the time-domain simulation," *IEEE Trans. on Power Syst.*, 26, (1), pp. 261–269, 2011.

[18] Shroff, G.M., Keller, H.B., "Stabilization of unstable procedure: the recursive projection method," *SIAM J. Numer. Anal.*, vol.30, no. 4, pp.1099-1120, Aug. 1993.

[19] Panda, P.C., “Power system stability using Matlab”, http://ethesis.nitrkl.ac.in/277/, retrieved on July 2012.

[20] Pal, M.K., “Lecture notes on power system stability”, http://www.mkpalconsulting.com/files/stabilitybook.pdf, retrieved in May2012.

[21] Saadat, H., *Power system analysis*. New York: McGraw-Hill, 1999.

[22] Andersson, G., “Modeling and analysis of electric power systems”, http://www.eeh.ee.ethz.ch/uploads/tx_ethstudies/modelling_hs08_script_02.pdf, retrieved on August 2012.

[23] Singh, B., Sharma, N.K., et.al., “Enhancement of voltage stability by coordinated control of multiple FACTS controllers in multi-machine power system environments,” *International Conf. on Sustainable Energy and Intelligent Systems* (SEISCON), pp. 18-25, 2011.

[24] Anderson, P.M., Fouad, A.A., *Power system control and stability*. Iowa: The Iowa State Univ. Press, 1977.

[25] Saadat, H., *Power system analysis*. New York: McGraw-Hill, 1999.

[26] Venkatasubramanian, V., Zaborszky, J., et.al., “A stability theory of differential-algebraic systems such as the power system,”. *IEEE International Symposium*, vol.5, pp. 2517-2520, 1992.

[27] Sanchez-Gasca, J. J., D'Aquila, R., et.al., “Variable time step, implicit integration for extended term power system dynamic simulation,” *IEEE Power Industry Computer Applications Conf.*, Salt Lake City, USA, pp.183-189, 1995.

[28] Nimje, A.A., Panigrahi, C.K., “Transient stability analysis using modified Euler’s iterative technique,” *International Conf. on power systems* ICPS’90, pp.1-4, 2009.

[29] Fan, M.H., Wang, C.S.,et.al., “Power system equilibrium tracing and bifurcation detection based on the continuation of the recursive projection method,” *Generation, Transmission & Distribution, IET*, vol. 6, Issue 3, pp.199-206, 2012.

[30] Chow, J.H., Cheung, K. W., “A toolbox for power system dynamics and control engineering education and research,” *IEEE Trans. Power Syst*., vol.7, no.4, pp.1559–1564, 1992.