Evaluation of Surrogate Assisted Differential Evolution Algorithm for Single-Objective Numerical Optimization

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Submitted to the
Institute of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of

Master of Science
in
Computer Engineering

Eastern Mediterranean University
September 2021
Gazimağusa, North Cyprus

Approval of the Institute of Graduate Studies and Research

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ABSTRACT

Hard optimization problems are solved successfully using nature inspired metaheuristics. However, in many cases of practical optimization problems, also called black-box problems, the evaluation of the objective function is main cause of high demand of computational resources. In the solution of these problems, objective function landscape is modeled mathematically, called a surrogate model which consist of replacing the objective function by an equivalent mathematical model, to reduce the computational evaluation time of the fitness function.

The differential evolution (DE) algorithm is implemented with 4 strategies called rand/1, rand/2, best/2 and rand to best/1 to optimize the benchmark functions listed CEC2017 competition with dimensions D=10 and D=30. CEC2017 benchmark set is composed of 30 different functions with different degree of complexities. Locations of optimal solutions for these functions is supposed to be unknown and that's why they are called black box functions. A surrogate model called the quadratic response surface model (QRSM) is used with Latin hyper square sampling strategy to replace objective function evaluations of benchmark functions. QRMS is used with DE for the solution of CEC2017 benchmark problems for the purpose of evaluating the performance of the surrogate assisted DE algorithm in terms of solution quality and runtime complexity.

Experimental results obtained from the 4 different DE and DE+QRSM strategies illustrated that the rand/1 DE strategy was generally the best strategy in speed and accuracy for both dimensions D=10 and D=30. Also, the results generated by DE and

DE+QRSM are compared with each other. As illustrated in tables of experimental evaluations, DE is found more accurate in majority of benchmark functions but it is slower generally. Also, a comparative study is done with other published algorithms such as L-SHADE, JSO, DISH, L-SHADE-LBR, JSO-LBR and DISH-LBR. Results obtained by these competitors are compared to only the best DE strategy, which is rand/1, employed within DE and DE+QRSM. The rand/1 strategy implemented within DE function was quit robust and performed better than other algorithms in many cases for D=10, but when implemented within DE+QRSM it becomes the worst one. For D=30 the rand/1 strategy loosed of its performance and was classified before the last position. Its rank is around of 80% when implemented within DE but it stays in last position with DE+QRSM algorithm.

Keywords: Black-box Optimization, Differential Evolution, Quadratic Response Surface Model, Surrogate Assisted Optimization.

ÖZ

Zor optimizasyon problemleri, doğadan ilham alan meta-sezgisel yöntemler kullanılarak başarılı bir şekilde çözülür. Bununla birlikte, kara kutu problemleri olarak da adlandırılan birçok pratik optimizasyon probleminde, amaç fonksiyonunun değerlendirilmesi, hesaplama kaynaklarının yüksek talebinin ana nedenidir. Bu problemlerin çözümünde, amaç fonksiyonunun hesaplamalı değerlendirme süresini azaltmak için amaç fonksiyonuna eşdeğer bir matematiksel model kullanılır. Amaç fonksiyonunun vekil model olarak adlandırılan bir matematiksel model ile değiştirilmesi sonucunda hesaplama süresinin kısaltılması amaçlanır.

Diferansiyel evrim (DE) algoritması, CEC2017 optimizasyon yarışmasında listelenen D=10 ve D=30 boyutlarındaki kıyaslama fonksiyonlarını optimize etmek için rand/1, rand/2, best/2 ve rand to best/1 olarak adlandırılan 4 strateji ile uygulanmıştır. CEC2017 kıyaslama seti, farklı karmaşıklık derecelerine sahip 30 farklı fonksiyondan oluşmaktadır. Bu fonksiyonlar için optimal çözümlerin yerleri bilinmez ve bu nedenle kara kutu fonksiyonları olarak adlandırılırlar. İkinci dereceden yanıt yüzeyi modeli (QRSM) olarak adlandırılan bir vekil model, kıyaslama işlevlerinin amaç işlevi değerlendirmelerinin yerini almak için Latin hiper kare örnekleme stratejisiyle birlikte kullanılır. QRMS, vekil destekli DE algoritmasının performansını çözüm kalitesi ve çalışma zamanı karmaşıklığı açısından değerlendirmek amacıyla CEC2017 kıyaslama problemlerinin çözümü için DE ile birlikte kullanılmıştır.

Dört farklı DE ve DE+QRSM stratejisinden elde edilen deneysel sonuçlar, Rand/1 DE stratejisinin hem D=10 hem de D=30 boyutları için genellikle hız ve doğruluk

açısından en iyi strateji olduğunu göstermiştir. Ayrıca DE ve DE+QRSM tarafından üretilen sonuçlar birbirleriyle karşılaştırılmıştır. Deneysel değerlendirme tablolarında gösterildiği gibi, DE kıyaslama fonksiyonlarının çoğunda daha başarılı bulunmuştur, ancak genel olarak daha yavaştır. Ayrıca L-SHADE, JSO, DISH, L-SHADE-LBR, JSO-LBR ve DISH-LBR gibi yayınlanmış diğer algoritmalarla karşılaştırmalı bir çalışma yapılmıştır. Bu rakipler tarafından elde edilen sonuçlar, yalnızca DE ve DE+QRSM içinde kullanılan en iyi DE stratejisiyle, rand/1, karşılaştırılmıştır. DE algoritması içinde uygulanan rand/1 stratejisi, D=10 için birçok durumda güvenilir çıktı ve diğer algoritmalardan daha iyi performans gösterdi, ancak DE+QRSM içinde uygulandığında en kötüsü sıralamada yer aldı. D=30 için rand/1 stratejisi başarımını kaybetti ve son pozisyondan önce sınıflandırıldı. DE içinde uygulandığında rankı %80 civarındadır ancak DE+QRSM algoritması ile son pozisyonda kalmaktadır.

Anahtar Kelimeler: Kara-kutu Optimizasyonu, Diferansiyel Evrim, Kuadratik Tepki Yüzey Modeli, Vekil Destekli Optimizasyon.

DEDICATION

I would like to dedicate this work to my family, especially my mother Jamila and to my good friends who supported me.

ACKNOWLEDGMENT

I would thank Assoc. Prof. Dr. Adnan Acan for his help and guidance, Asst. Prof. Dr. Ahmet Ünveren for teaching us metaheuristics course and to be a member of the jury also many thanks to Asst. Prof. Dr. Mehtap Köse Ulukök for also being a member of the jury.

Many thanks to my friend John, who gives me many advises and supported me.

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INTRODUCTION

1.1 Introduction

In real life there are many problems that need to be optimized and these problems belong to many fields of engineering and science. They can have very high computational complexities growing in the order of exponential or factorial functions and they become very hard to solve optimally when the size of the problems increase. With currently available technologies, optimal solutions of some hard problems may take years and often the computational resources become inadequate for their solutions. Metaheuristics are invented to optimize the response time but sacrifices should be done from an accuracy side. They don't require hard computations like computation of derivatives and they find the local optimum quickly. Also, a near optimal solution is found rapidly and in many cases it is capable to escape locally optimal solutions. However, the convergence to the global optimum is hard to control which means that obtaining a global optimum is not guaranty. Also, the standardization of a metaheuristic is almost impossible, that means its success is problem dependent. Differential evolution algorithm [1] is one of the most famous algorithms used to solve continues optimization problems and it was widely used in many fields and outcomes with satisfying results. Differential evolution has many strategies [2] [3] that can be used which are rand/1, rand/2, best/2 and rand to best/1.

The differential evolution algorithm is used to optimize the problem based on the fitness function, however some objective functions can be also a crucial problem because of their computational complexity, and to evaluate them they can take hours or even days. A solution for this problem is to use surrogate models [4] which are mathematical models used to approximate other mathematical functions. To build a surrogate model, samples should be constructed by a design of experiments method such us Latin hyper square [5]. After that the model and these samples are going to be used together with the objective function to obtain the model. Finally, the fitness function is approximated in the differential evolution algorithm by the built surrogate model. The used surrogate model is called quadratic response surface model[4] which is one of the most popular surrogate models, and it is based on a polynomial equation.

This thesis is organized as follows: the first chapter is an introductory chapter aiming to introduce the problem and introduces a literature review on how people have tried to solve it. In the second chapter, the employed algorithm together with the associated surrogate are introduced in details. In this respect, differential evolution algorithm with its different strategies, Latin hyper square sampling method and quadratic response surface model are presented with their fundamental characteristics. In the third chapter, the different results obtained through a number of experimental works are illustrated, analyzed and comparatively evaluated against published work in literature [6]. The last chapter contains conclusions and description of future work perspectives.

1.2 The State of Arts

In the last decades, to solve practical optimization problems in diverse area such as engineering, business and science [7] [8] [9] [10], various algorithms are developed. These algorithms are classified as metaheuristics. Several algorithms have been developed with inspiration from nature to approximate an acceptable solution or even a global optimal one. For example, Particle Swarm Optimization (PSO) [11] inspired from migration of bird flocks, Teaching-Learning-Based Optimization (TLBO) [12] inspired from influence of a teacher on learners and interaction between learners, Genetic Algorithm (GA) [13] inspired from the natural selection (survival of the fittest), and Differential Evolution (DE) [14] inspired from (GA) and Evolutionary Algorithms (EA). Metaheuristics have achieved a great efficiency in the solution of difficult nonlinear optimization problems with high dimensionality and when the analytical derivation of gradient is either not available or it is hard to obtain [15]. In most of benchmarks or functions published so far, metaheuristics methods outperformed the convolutional gradient-based mathematical programming methods [16].

After the remarkable evolution of the engineering industry, complex design and optimization problems which may take long run times to be solved become new challenges to metaheuristics [17] [18]. To obtain the global optimal solution, metaheuristics need to evaluate a complex objective function hundreds of thousand times which sometimes take minutes, hours or even days [19]. Also, there are many real world application problems which involve a large number of variables ($d>30$) and complex calculations due to the increase of the search space. In such a large-scale dimension, a fitness function needs to be evaluated large number of times to extract an approximate of a global optimum [20]. Due to high complexity, it is usually hard to locate a globally optimal through the fitness function evaluations because of convergence characteristics of metaheuristic algorithms. While a local optimal solution is easy to locate, extraction of a globally optimal solution may take asymptotically large execution time.

Surrogate models [21] are mathematical models that are able to predict functional value of a set of sampled points with a negligible error rate. The main benefits of surrogate models is reduction of computational cost of the underlying objective function and hence reducing the design cycle length [22] [23] [24]. Some of the most widely used surrogate models in literature are Radial Basis Functions (RBF) [25], Artificial Neural Networks (ANN) [26], Kriging [27] (or Gaussian Process (GP)), and Quadratic Response Surface Model(QRSM) [28]. In the last 3 decades the Surrogate Based Optimization (SBO) has been applied into simulation-based design for the solution of black-box expensive optimization problems. The Kriging model was used by Jones et al. [22] in their Efficient Global Optimization method (EGO). To solve specific benchmark problems, various EGO algorithms were proposed [23] [24] [29]. Based on results published in literature, the Kriging model was not very efficient to predict objective function values when the dimension of the problems is large ($d>30$). That's why EGO becomes inefficient or even infeasible in practice [30]. Regis and Shoemaker [31] developed a stochastic RBF method. This method was good to predict objective function values for nonconvex problems with high computational cost, but becomes also inefficient when $d>30$. Some varieties of Surrogate-Assisted Meta-heuristic Algorithms (SAMAs) [32] such as Surrogate-Assisted Evolutionary Algorithms (SAEAs) [33]. In this class of approaches surrogate models such as polynomial models, Kriging models and neural networks and evolutionary algorithms are used for solving problems such as dynamic optimization and constrained optimization problems. An example of algorithm based on this approach is RS-MOEA [34]. RS-MOEA were used to solve Multi-objective Optimization Problems (MOP) by replacing the objective functions by the Kriging model, and Radial Grid Projection (RGP) were used to visualize the high dimension space. RS-MOEA gave the best performance for DTLZ2, DTLZ5 for DTLZ benchmark problems and acceptable results for MAF benchmark problems. Also, RS-MOEA was tested on WFG benchmark problems and absolutely achieved the best results. Surrogate-Assisted Swarm Intelligence Algorithms (SASIAs) [35], in this class of approaches researchers tried to use Particle Swarm Optimization (PSO) and the surrogate model Generalized Regression Neural Network (GRNN) and they came out with a new algorithm called MUGPSO. MUGPSO was tested on 10 benchmark functions named CEC 2005 [36] and gave a solution that is close to optimal one for F1, F7, F8, F9 and F10 but not for others. SAMA have a great sampling efficiency because it is based on the population approach of meta-heuristics [37]. For the solution of medium dimension expensive optimization problems, the two researchers Liu et al. [38] proposed an algorithm named GP-assisted evolutionary algorithm (GPEME) which is using Gaussian Process Modeling (GP) and Differential Evolution Algorithm (DE). This algorithm was tested on 14 benchmark functions and find a good solutions for almost of functions except F4, F5, F13 and F14. Another algorithm called OPUS was proposed by Regis [39] using PSO and RBFs to solve black box complex functions. PSO-RBF was tested on 12 different functions the experiments have shown that OPUS-RBF is an excellent alternative for expensive black-box optimization**.** Moreover, two inventors Sun et al. [40] introduced a new algorithm (SA-COSO). SA-COSO was using two metaheuristics the first one is PSO and the second one is Social Learning-based PSO (SL-PSO) to cooperatively explore the global optima, [41]. That algorithm gave accurate results for 1000 objective functions with dimensions 50 and 100. According to [40] the algorithm SA-COSO gave the best value cost, for Ellipsoid function with dimension 100, but it stills far away from the global optimal cost which has the value 0.

Many algorithms were developed using multiple surrogate models with meta-heuristics for the resolution of Expensive Optimization Problems (EOPs). Among those algorithms there is Surrogate-Assisted Evolutionary Algorithms with an Uncertainty Grouping based infill Criterion (SAEA-UGC) [42]. SAEA-UGC is using EA and a weighted average surrogate models Kriging, RBF and Polynomial Response Surface (PR). For the global search the weighted average model was used and for the local search only RBF was used. The second algorithm is Committee-based Active Learning for Surrogate-Assisted PSO (CAL-SAPSO) [37]. CAL-SAPSO is using PSO as a metaheuristic and models Kriging, RBF and PR for global and local search. The last algorithm is Offline Data-Driven Evolutionary Algorithm optimization using selective Surrogate Ensembles (DDEA-SE) [43]. DDEA-SE uses an EA with a weighted average RBF surrogate models which means that different RBF surrogate models will be trained using different set of samples and to each model a weight will be attributed based on its accuracy. According to experiments [42], SAEA-UGC did a remarkable improvement, it outperforms both CAL-SAPSO and DDEA-SE on majority of test problems.

According to some researches [44], to solve engineering problems with high complexity researchers tried to adapt one or more surrogate models to their problems and to choose each time the best model. Some of members of that community instead of using an individual surrogate model they used many ones simultaneously by attributing a weight to each one of them. The use of more than one surrogate model reduces the effect of the mediocre surrogate model that involves the robustness predictions improvement. There is two ways of using surrogate models the first way is to use a single surrogate model by choosing the best one from a pool of surrogate models or to use a weighted average surrogate model. Applying an average of a weighted surrogate models have showed better performance than a single one.

Chapter 2

SURROGATE OPTIMIZATION WITH DIFFERENTIAL EVOLUTION AND QUADRATIC RESPONSE SURFACE MODEL

Differential Evolution (DE) is a very powerful algorithm used to solve continuous optimization problems. It was reputed by its fastness and accuracy during this last decade and was widely and frequently used by many researchers. In many real-life optimization problems, evaluation of the cost function for a potential solution is highly computationally expensive [45] which makes the optimization procedures useless particularly for real-time applications. Surrogate based optimization is a proposal to overcome this problem where computationally hard objective functions are modeled with easier to evaluate mathematical models. Among the most famous surrogate models, Quadratic Response Surface Model (QRSM). QRSM is an efficient and simple to implement model. In this study, DE and QRSM were used together to find acceptable solutions in a shorter evaluation time for complex objective functions [20].

2.1 Differential Evolution Algorithm (DE)

Differential Evolution is a meta-heuristic which was invented in 1995 by K. Price et R. Storn [1]. It was inspired from Genetic Algorithms (GAs) and Evolutionary Algorithms (EAs) that we apply crossover and mutation on a population of individuals to change the structure of chromosomes. One can see here a fruitful combination of the mutation and crossover concepts. DE has a target to solve continuous optimization problems with nonlinear constraints. During the last ten years research has proved that DE is a very effective algorithm used in real-life optimization problems as well as for the solution of difficult benchmarks problems. Among those real-life problems polynomial approximation, scheduling of satellite tasks, image processing and neural networks model training and adjustment of fuzzy functions are some examples. An algorithmic description of the DE algorithm is as follow. Detailed explanation of components of this algorithm are given in the following subsections.

2.1.1 Algorithm

According to reference [1], the DE algorithm is written below:

Begin

Step 0: /* Population initialization */

For $i = 1$ to $population\_size$

For $j = 1$ to var_num

$population_{ij} = d_j^L + rand(0,1) * \left(d_j^U - d_j^L\right)$;

End for;

End for ;

For $iterations = 1$ to $max\_iterations$

For $j = 1$ to $population\_size$

Step 1: /* Mutation (Perturbation) */

For $i = 1$ to var_num

$perturbation_i = p_{1i} + F_1 * (p_{2i} - p_{3i})$;

End for;

Step 2: /* Crossover (Recombination) */

$index = rand(1, var\_num)$;

For $i = 1$ to var_num

If($rand(0,1) \leq CR$ or $i == index$)

$offspring_i = perturbation_i$;

Else

$offspring_i = population_{ji}$;

End if ;

End for ;

Step 3: /* Selection */

/* Minimization case */

If($f(offspring) < f(population_j)$)

$Population_j = offspring$;

End if;

End for;

End for;

Step 4: /* Local optimal solution or global optimum */

$local\_or\_Global\_optimal\_solution = population_1$;

For $j = 2$ to $population\_size$

If $f(population_j) < f(local\_or\_Global\_optimal\_solution)$

$local\_or\_Global\_optimal\_solution = population_j$;

End if ;

End for ;

return $local\_or\_Global\_optimal\_solution$;

End ;

Figure 1: DE Pseudocode

Where, $population\_size$ is the number of population solutions, $max\_iterations$: is how many times DE will iterate, var_num: is the total number of the decision variables, d^l is the lowest value that a decision variable can take, d^u is the highest value that a decision variable can take, f is the objective function, $F_1$is the differential weight and it must be in [0,1] and, CR is the crossover rate and it is in [0,1].

2.1.2 Algorithm Description

2.1.2.1 Step 0: Population Initialization

This step consists of the random initialization of a specific number of chromosomes. The use of this equation,

$$population_{ij} = d_j^L + rand(0,1) * \left(d_j^U - d_j^L\right); \tag{1}$$

will ensure that the values of decision variables constituting each chromosome will be between d^U and d^L.

2.1.2.2 Step 1: Mutation (Perturbation)

In this step a perturbation vector is created which is also called a donor. Differences of vectors are calculated, from where the name differential is inspired. There are many variants of mutation [2] [3], four commonly used ones are implemented in this thesis study. These variants are rand/1, rand/2, best/2 and rand to best/1, as they are named in literature.

2.1.2.2.1 The Rand/1 Mutation Operator

Three different vectors are selected randomly from the population namely p_1, p_2 and p_3. Then, the perturbation vector is calculated by using the following equation:

$$perturbation_i = p_{1i} + F_1 * (p_{2i} - p_{3i}) \tag{2}$$

where p_1, p_2, p_3 and $population_j$ must be mutually different.

2.1.2.2.2 The Rand/2 Mutation Operator

Firstly, a selection of Five different vectors named p_1, p_2, p_3, $p_4$and p_5 from the population is done. Then, the perturbation vector is calculated by using the following equation:

$$perturbation_i = p_{1i} + F_1 * (p_{2i} - p_{3i} + \ p_{4i} - p_{5i}\ ) \quad (3)$$

where p_1, p_2, p_3, p_4, p_5 and $population_j$ must be mutually different.

2.1.2.2.3 The Best/2 Mutation Operator

Four different vectors are randomly selected from the population namely $p_2, p_3, p_4$and p_5, then the best chromosome is also selected from the population (it means the chromosome which has the minimal cost). After, the perturbation vector is calculated by using this equation:

$$perturbation_i = population_{best\ i} + F_1 * (p_{2i} - p_{3i} + \ p_{4i} - p_{5i}\ ) \quad (4)$$

p_2, p_3, p_4, p_5 and $population_j$ must be mutually different.

2.1.2.2.4 The Variant Rand to Best/1 Mutation Operator

In this mutation operator, a selection of three different vectors namely p_1, p_2 and p_3 is done randomly from the population. The best chromosome is also selected from the population means and the perturbation vector is calculated by using the following equation:

$$perturbation_i = p_{1i} + F_1 * (p_{2i} - p_{3i}) + F_2 * (population_{best\ i} - p_{1i}); \quad (5)$$

where p_2, p_3, p_4, p_5 and $population_j$ must be mutually different. F_2 is also called differential weight and it must be in [0,1].

2.1.2.3 Step 2: Crossover (Recombination) Operation

In Recombination operation, the $offspring_i$ also named perturbed vector is calculated as follow: an index named $index$ is selected randomly in $[1, var\_num]$ according to the following equation:

$$index = rand(1, var\_num); \tag{6}$$

Then, Through a loop starting from 1 to var_num a random number in [0,1] is generated each time, if that number is smaller or equal to the crossover rate CR, or the index of the decision variable constituting the chromosome is equal to $index$, than the $offspring_i$ value will be equal to the component value of the perturbation vector $perturbation_i$ as follow:

$$offspring_i = perturbation_i\ ; \tag{7}$$

Otherwise, the component value of the $population_{ji}$ will be assigned to the $offspring_i$ as follow:

$$offspring_i = population_{ji}\ ; \tag{8}$$

2.1.2.4 Step 3: Selection

In selection step the $poplation_j$ which also represents the parent is replaced by its $offspring$ if that one has a smaller cost than its parent according to the equation mentioned bellow:

$$Population_j = offspring; \tag{9}$$

If the $offspring$ cost is bigger than the cost of its parent the population will not undergo any change.

2.1.2.5 Step 4: Local Optimal Solution or Global Optimum

The chromosome with the minimal cost is selected from the population of solutions. That solution could be a local optimal solution which is an acceptable solution, or it will be a global optimum.

A global optimum is a solution of the problem that has smaller or equal objective function cost than all other solutions in the search space.

$$\forall s \in S$$

$$f(s^*) \leq f(s) \tag{10}$$

Where, f is the objective function, S is the set of solutions, s is a solution from S, and s^* is the global optimum.

A local optimal solution of the problem is a solution that has an objective function cost smaller or equal than some of solutions in the search space and bigger than some other solutions in the search space.

2.2 Surrogate Models

Surrogate models are mathematical methods to model high cost mathematical functions, in optimization problems to reduce the temporal complexity. There are many surrogate models such as Quadratic Response Surface Model (QRSM), Kriging, Radial Basis Function (RBF) and Inverse Distance Weighting (IDW). We construct a surrogate model by using samples from the design space. Those samples are obtained by using Design Of Experiments methods (DOE), as an example we can cite Latin Hyper Square (LHS). After the construction of our surrogate model we can combine it with our meta-heuristic DE to search for the global optimal solution. The surrogate model will replace the fitness function due to its high calculation cost, and in the end a good approximation of that objective function is obtained. In this thesis, LHS is used as a sampling method and QRSM as a surrogate method.

2.2.1 Latin Hyper Square Sampling (LHS)

Latin Hyper Square is a randomized sampling Design of Experiments (DOE) method. It has as a target to select samples points that covers as much as possible the design space. The idea of LHS is to select samples randomly with a specific distribution such as uniform. The selected samples mustn't have any intersection with any other sample according to rows and columns. In that case, the information provided by the limited number of samples to build the surrogate model will be maximized. In LHS, the range of each decision variable will be divided in disjoint sub intervals with equal probability, and from each interval a sample point will be picked randomly [5].

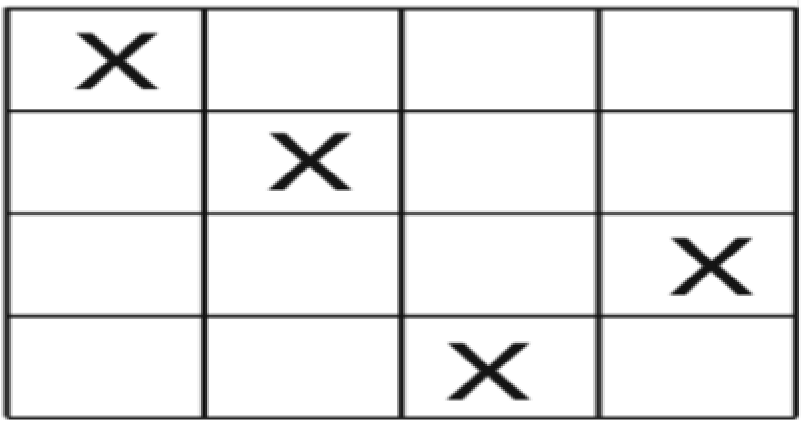

Figure 2: Latin Hyper Square (Two Dimension)

An example of a use of LHS:

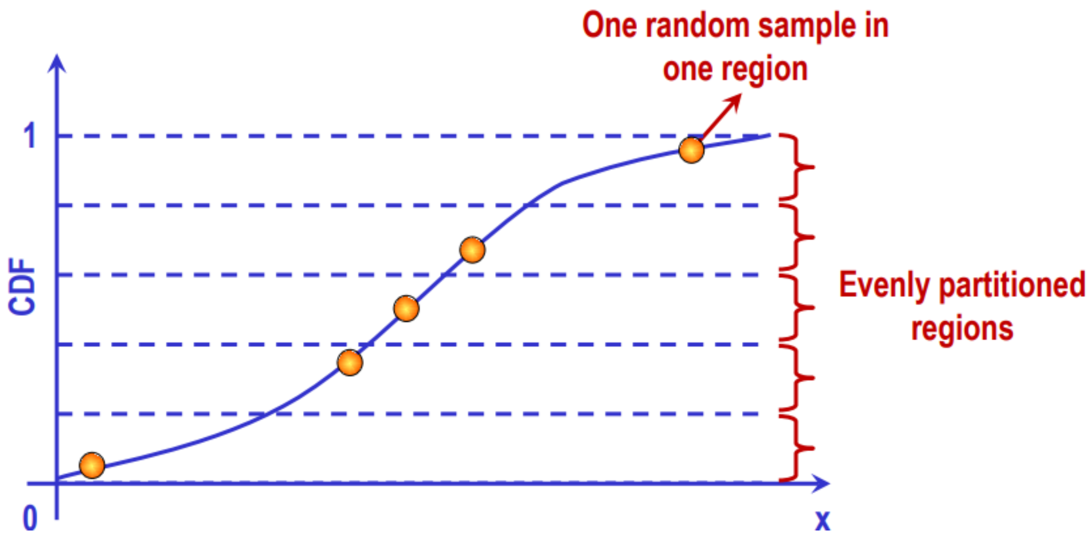

Figure 3: Example Latin Hyper Square (One Dimension)

The cumulative probability distribution curve is divided in equal intervals. After, a sample is taken randomly from each interval. In Figure 2 the curve is divided in 5 equal intervals and 5 samples are taken from each interval randomly. Just one sample can be taken by interval.

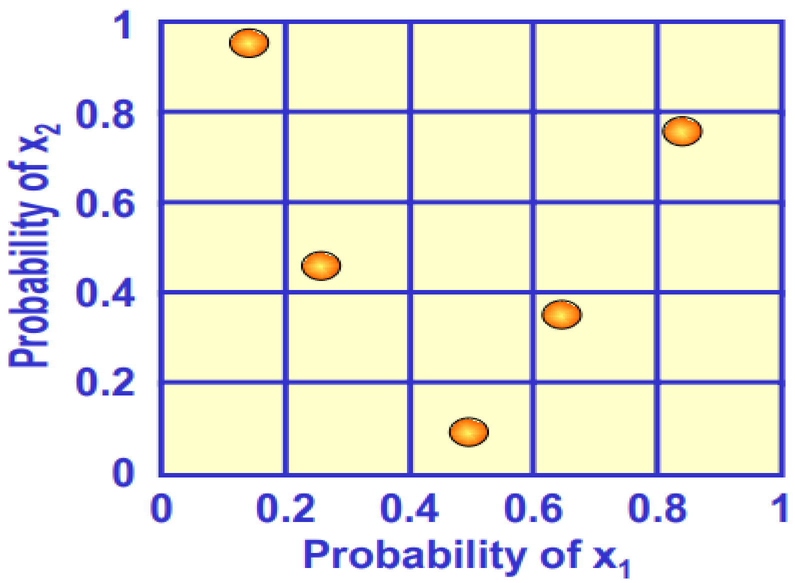

Figure 4: Example Latin Hyper Square (Two Dimension)

The two variables X_1 and X_2 must be independent. A one dimensional LHS samples are generated for X_1 and X_2 as shown in figure 2. The generated LHS samples are randomly combined to two-dimensional pairs in the way that one sample must be in each row and each column in the Latin matrix.

2.2.2 Quadratic Response Surface Model (QRSM)

2.2.2.1 Definition

QRSM is a quadratic polynomial approximation of the fitness function [4]. It can be described as follow:

$$Y_{surrogate}(X) = Y(X) + \varepsilon,\ X \in R^n \tag{11}$$

Where,

$Y(X)$: is the objective function cost of untried points,

$Y_{surrogate}(X)$: is the quadratic polynomial approximation of untried points, and

ε: is a random error normally distributed, $N(0, 6\ square)$.

2.2.2.2 Algorithm

Step 0: Generation of samples X using LHS. (The number of samples must be bigger than P).

Where,

$$P = (n + 1) * (n + 2)/2 \tag{12}$$

and, n is the dimension of the sample vector.

Step 1: Calculate the cost Y_s of samples X_s using the objective function f.

$$Y_s = f(X_s) \tag{13}$$

Where,

$$Y_s = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_m \end{bmatrix} \tag{14}$$

$$X_s = [X^1, X^2, \ldots, X^m] \tag{15}$$

and, m is the total number of samples.

Step 2: Calculation of the matrix U.

$$U = \begin{bmatrix} 1 & {X_1}^1 \ldots {X_n}^1 & \left({X_1}^1\right)^2 \ldots \left({X_n}^1\right)^2 & {X_1}^1 * {X_2}^1 \;\; {X_1}^1 * {X_3}^1 \ldots {X_{n-1}}^1 * {X_n}^1 \\ 1 & {X_1}^2 \ldots {X_n}^2 & \left({X_1}^2\right)^2 \ldots \left({X_n}^2\right)^2 & {X_1}^2 * {X_2}^2 \;\; {X_1}^2 * {X_3}^2 \ldots {X_{n-1}}^2 * {X_n}^2 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & {X_1}^m \ldots {X_n}^m & \left({X_1}^m\right)^2 \ldots \left({X_n}^m\right)^2 & {X_1}^m * {X_2}^m \;\; {X_1}^m * {X_3}^m \ldots {X_{n-1}}^m * {X_n}^m \end{bmatrix}$$

(16)

Step 3: Calculation of the coefficients β (It will be a vector of P rows)

$$\beta = (U^T * U)^{-1} * U^T * Y_s \tag{17}$$

Where,

$$\beta = [\beta_0 \quad \beta_1 \quad \cdots \quad \beta_n \quad \beta_{11} \quad \cdots \quad \beta_{nn} \quad \beta_{12} \quad \cdots \quad \beta_{n-1n}] \tag{18}$$

Step 4: Calculation of the surrogate cost of the untried points (predicted cost)

$$Y_{surrogate}(X^k) = \beta_0 + \sum_{i=1}^{n} \beta_i * X_i^k + \sum_{i=1}^{n} \beta_{ii} * \left(X_i^k\right)^2 + \sum_{i=1}^{n-1} \sum_{j>i} \beta_{ij} * X_i^k * X_j^k$$

(19)

Chapter 3

RESULTS AND INTERPRETATION

Four variants of DE were used to optimize the CEC 2017 benchmark functions. They are rand/1, rand/2, best/2 and rand to best/1. Ten runs were done for each strategy with dimensions 10 and 30. The decision variables domain is in [-100,100] and the maximal number of iterations for dimension 10 and dimension 30 are 1000 and 3000 respectively. The population size is 100, while the differential weights F_1 and F_2 are equal to 0.5, with a crossover rate CR of 0.9. Firstly, the DE algorithm used the objective function for experiments, after which a surrogate model called QRSM was used to model the different objective functions to reduce the execution time. The method used for sampling to construct the QRSM model is a DOE method called LHS. The number of samples for dimension 10 problem is 300 and the number of samples for dimension 30 problem is 600. The different experiments were done on computers with heterogeneous processors.

According to the reference [46], the description of the 30 benchmark functions is below:

Table 1: Summary of the CEC'17 Test Functions

	No	Functions	Fi*=Fi(x*)
Unimodal Functions	1	Shifted and Rotated Bent Cigar Function	100
	2	Shifted and Rotated Zakharov Function	200
Simple Multimodal Functions	3	Shifted and Rotated Rosenbrock's Function	300
	4	Shifted and Rotated Rastrigin's Function	400
	5	Shifted and Rotated Expanded Scaffer's F6 Function	500
	6	Shifted and Rotated Lunacek Bi_Rastrigin Function	600
	7	Shifted and Rotated Non-Continuous Rastrigin's Function	700
	8	Shifted and Rotated Levy Function	800
	9	Shifted and Rotated Schwefel's Function	900
Hybrid Functions	10	Hybrid Function 1 (N=3)	1000
	11	Hybrid Function 2 (N=3)	1100
	12	Hybrid Function 3 (N=3)	1200
	13	Hybrid Function 4 (N=4)	1300
	14	Hybrid Function 5 (N=4)	1400
	15	Hybrid Function 6 (N=4)	1500
	16	Hybrid Function 6 (N=5)	1600
	17	Hybrid Function 6 (N=5)	1700
	18	Hybrid Function 6 (N=5)	1800
	19	Hybrid Function 6 (N=6)	1900
Composition Functions	20	Composition Function 1 (N=3)	2000
	21	Composition Function 2 (N=3)	2100
	22	Composition Function 3 (N=4)	2200
	23	Composition Function 4 (N=4)	2300
	24	Composition Function 5 (N=5)	2400
	25	Composition Function 6 (N=5)	2500
	26	Composition Function 7 (N=6)	2600
	27	Composition Function 8 (N=6)	2700
	28	Composition Function 9 (N=6)	2800
	29	Composition Function 10 (N=3)	2900
	30	Composition Function 11 (N=3)	3000
Search Range: [-100,100]D			

3.1 Results and Interpretations

Table 2: DE Results for Dimension 10 and Variant Rand/1

		functions									
		f1		f2		f3		f4		f5	
D=10, DE/rand/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	100.00	16.69	200.00	17.35	300.00	7.18	400.00	8.63	522.70	7.83
	2	100.00	16.96	200.00	14.24	300.00	7.11	400.00	8.58	523.45	7.80
	3	100.00	18.16	200.00	17.44	300.00	7.16	400.00	8.67	527.22	7.82
	4	100.00	17.64	200.00	14.24	300.00	7.16	400.00	8.60	517.19	7.80
	5	100.00	15.74	200.00	17.11	300.00	7.19	400.00	8.60	523.41	7.82
	6	100.00	16.79	200.00	13.62	300.00	7.21	400.00	8.52	524.58	7.88
	7	100.00	17.47	200.00	14.46	300.00	7.16	400.00	8.58	527.20	7.82
	8	100.00	17.89	200.00	16.88	300.00	7.14	400.00	8.53	524.90	7.82
	9	100.00	16.10	200.00	16.08	300.00	7.13	400.00	8.55	526.48	7.78
	10	100.00	19.03	200.00	16.41	300.00	7.14	400.00	8.66	526.17	7.82
	Max	100.00	19.03	200.00	17.44	300.00	7.21	400.00	8.67	527.22	7.88
	Min	100.00	15.74	200.00	13.62	300.00	7.11	400.00	8.52	517.19	7.78
	Av	100.00	17.27	200.00	15.74	300.00	7.16	400.00	8.59	523.98	7.82
	Std	0.00	1.09	0.00	1.51	0.00	0.03	0.00	0.05	3.39	0.03

		functions									
		f6		f7		f8		f9		f10	
D=10, DE/rand/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	600.00	14.34	736.23	11.98	824.02	20.17	900.00	15.51	1250.94	18.39
	2	600.00	14.27	738.18	11.87	821.32	20.11	900.00	15.48	1287.63	18.56
	3	600.00	14.68	729.03	11.90	822.13	20.12	900.00	15.52	1456.13	18.22
	4	600.00	14.29	738.83	11.87	826.97	20.12	900.00	15.48	1730.12	18.08
	5	600.00	14.29	733.02	11.87	822.71	20.01	900.00	15.49	1310.05	18.16
	6	600.00	14.29	727.87	11.89	825.98	19.94	900.00	15.40	1408.68	18.58
	7	600.00	14.84	735.00	11.89	826.28	20.06	900.00	15.51	1377.47	18.61
	8	600.00	14.24	732.65	11.89	827.83	20.01	900.00	15.40	1138.69	18.30
	9	600.00	14.23	734.33	11.87	819.26	19.98	900.00	15.41	1366.14	18.21
	10	600.00	14.24	739.60	11.89	817.59	20.14	900.00	15.38	1257.38	18.60
	Max	600.00	14.84	739.60	11.98	827.83	20.17	900.00	15.52	1730.12	18.61
	Min	600.00	14.23	727.87	11.87	817.59	19.94	900.00	15.38	1138.69	18.08
	Av	600.00	14.37	734.48	11.89	823.41	20.07	900.00	15.46	1358.32	18.37
	Std	0.00	0.21	3.96	0.03	3.42	0.08	0.00	0.05	159.18	0.20

		functions									
		f11		f12		f13		f14		f15	
D=10, DE/rand/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	1100.00	12.06	1200.42	14.02	1301.00	14.90	1400.01	14.70	1500.00	33.38
	2	1100.00	12.07	1200.62	14.16	1306.51	14.84	1400.00	14.63	1500.00	31.29
	3	1100.00	12.14	1200.21	13.87	1302.25	14.70	1400.99	14.60	1500.00	29.48
	4	1100.00	12.03	1200.21	13.82	1301.99	14.79	1400.00	14.60	1500.01	33.13
	5	1100.00	12.03	1211.59	14.15	1308.06	14.74	1400.02	14.59	1500.00	30.36
	6	1100.00	12.01	1200.42	14.09	1300.09	14.85	1400.00	14.59	1500.09	30.84
	7	1100.00	12.03	1200.42	13.92	1302.93	14.74	1400.00	14.54	1500.00	35.07
	8	1101.99	12.11	1200.00	13.70	1306.52	14.77	1400.00	14.55	1500.00	32.25
	9	1100.00	12.04	1200.00	13.53	1306.13	14.76	1400.00	14.52	1500.02	32.40
	10	1100.00	12.01	1200.21	13.77	1300.00	14.87	1400.00	14.51	1500.01	34.04
	Max	1101.99	12.14	1211.59	14.16	1308.06	14.90	1400.99	14.70	1500.09	35.07
	Min	1100.00	12.01	1200.00	13.53	1300.00	14.70	1400.00	14.51	1500.00	29.48
	Av	1100.20	12.05	1201.41	13.90	1303.55	14.80	1400.10	14.58	1500.01	32.23
	Std	0.63	0.04	3.58	0.21	2.98	0.07	0.31	0.06	0.03	1.74

		functions									
		f16		f17		f18		f19		f20	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
D=10, DE/rand/1	1	1600.00	46.86	1700.04	26.96	1800.47	15.71	1900.00	24.09	2000.30	28.25
	2	1600.22	40.09	1700.43	27.16	1800.50	15.65	1900.00	24.04	2000.00	28.69
	3	1600.43	41.65	1701.02	27.49	1800.00	15.57	1900.01	24.21	2001.37	28.22
	4	1600.44	49.86	1703.19	27.16	1800.01	15.55	1900.00	24.09	2000.21	28.91
	5	1600.43	45.32	1700.62	27.52	1800.01	15.58	1900.01	24.20	2000.70	28.53
	6	1600.02	44.51	1703.75	27.25	1800.01	15.60	1900.00	24.10	2000.00	28.74
	7	1600.43	45.58	1700.31	27.24	1800.00	15.55	1900.00	24.09	2010.48	28.28
	8	1600.42	48.44	1700.31	27.19	1800.02	15.55	1900.00	24.20	2000.38	28.59
	9	1600.22	50.22	1700.10	26.96	1800.18	15.66	1900.01	24.06	2010.84	28.42
	10	1600.01	45.86	1715.61	27.30	1800.00	15.52	1900.00	24.12	2000.06	28.55
	Max	1600.44	50.22	1715.61	27.52	1800.50	15.71	1900.01	24.21	2010.84	28.91
	Min	1600.00	40.09	1700.04	26.96	1800.00	15.52	1900.00	24.04	2000.00	28.22
	Av	1600.26	45.84	1702.54	27.22	1800.12	15.60	1900.00	24.12	2002.43	28.52
	Std	0.19	3.26	4.78	0.19	0.20	0.06	0.00	0.06	4.35	0.23

		functions									
		f21		f22		f23		f24		f25	
D=10, DE/rand/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	2200.00	28.75	2300.40	40.20	2602.87	47.89	2759.65	36.60	2897.74	41.79
	2	2200.00	28.47	2300.00	39.95	2618.76	47.89	2745.51	36.35	2897.74	41.43
	3	2327.18	28.59	2300.00	40.11	2616.11	47.85	2755.32	36.46	2945.82	41.29
	4	2200.00	28.72	2300.00	40.08	2615.82	47.89	2756.58	36.32	2898.01	41.34
	5	2200.00	28.64	2300.00	39.95	2619.37	47.89	2754.23	36.40	2898.01	41.42
	6	2200.00	28.47	2300.00	40.05	2609.73	47.95	2753.67	36.46	2898.01	41.22
	7	2329.24	28.61	2300.00	40.06	2610.79	47.88	2744.43	36.33	2898.01	41.28
	8	2200.00	28.53	2300.00	39.98	2616.24	47.74	2750.66	36.41	2898.01	41.39
	9	2200.00	28.55	2300.00	40.08	2610.81	47.92	2750.91	36.33	2898.01	41.25
	10	2329.33	28.55	2300.00	40.06	2608.58	47.89	2746.80	36.40	2897.74	41.36
	Max	2329.33	28.75	2300.40	40.20	2619.37	47.95	2759.65	36.60	2945.82	41.79
	Min	2200.00	28.47	2300.00	39.95	2602.87	47.74	2744.43	36.32	2897.74	41.22
	Av	2238.58	28.59	2300.04	40.05	2612.91	47.88	2751.78	36.40	2902.71	41.38
	Std	62.12	0.10	0.13	0.08	5.21	0.06	5.02	0.08	15.15	0.16

		functions									
		f26		f27		f28		f29		f30	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
D=10, DE/rand/1	1	2900.00	61.90	3089.52	65.80	3100.00	57.46	3129.43	64.38	3394.18	155.13
	2	2900.00	61.84	3089.01	65.83	3100.00	57.44	3139.99	63.99	3394.05	149.54
	3	2900.00	61.90	3089.52	65.50	3100.00	57.42	3134.40	64.04	3501.86	160.70
	4	2900.00	61.87	3089.31	65.94	3196.57	57.58	3138.45	64.01	3407.05	158.08
	5	2900.00	61.89	3089.31	65.88	3100.00	57.39	3141.94	63.94	3394.12	162.48
	6	2900.00	61.89	3089.52	65.55	3100.00	57.44	3141.30	63.98	3394.19	172.69
	7	2900.00	61.87	3089.31	65.72	3100.00	57.39	3133.57	64.09	3442.16	158.81
	8	2900.00	61.84	3088.28	65.68	3100.00	57.39	3146.05	64.13	3394.21	167.16
	9	2900.00	61.87	3089.52	65.61	3383.73	57.56	3141.90	63.93	3407.08	167.31
	10	2900.00	61.92	3089.52	65.60	3196.57	57.61	3143.06	64.07	3406.94	166.09
	Max	2900.00	61.92	3089.52	65.94	3383.73	57.61	3146.05	64.38	3501.86	172.69
	Min	2900.00	61.84	3088.28	65.50	3100.00	57.39	3129.43	63.93	3394.05	149.54
	Av	2900.00	61.88	3089.28	65.71	3147.69	57.47	3139.01	64.06	3413.59	161.80
	Std	0.00	0.03	0.39	0.15	92.14	0.08	5.08	0.13	34.36	6.79

Results illustrated in Table 2, for D=10 show that the optimal solutions are obtained in all 10 different runs for functions f1, f2, f3, f4, f6 and f9. Also, for functions f11, f14, f15 and f19, optimal solutions are obtained in almost all runs, except for a few number of runs and, for these cases where optimal solutions are not extracted, the obtained solutions are very close to the optimal ones. The standard deviation values for the functions mentioned above are either 0 or very close to 0, signifying that the employed DE strategy is quite robust for the above listed 10 benchmark functions.

For benchmark functions f12, f13, f16, f18 and f20, still a few number of optimal solutions are extracted while the rest of the results are very close to the optimal ones. Also, the standard deviation values for functions f16 and f18 are smaller than 1, but for functions f12, f13 and f20, their standard deviations increased, and are between 2.98 and 4.35. These are significantly small values, when compared to the fitness values of the corresponding functions.

Considering the benchmark functions f5, f7 and f8, the objective function values in the 10 different runs are above the optimal values with a difference smaller than 100. The standard deviation scores for functions f5 and f7 are still significantly small compared to objective function values, however the standard deviation score for function f10 is significantly large with value 159.18. These values indicate that the employed DE strategy is still robust for functions f5, f7 and f8 but the characteristics of function f10 make it hard to locate its optimal solution and due to this, the employed DE strategy was not successful in getting out of locally optimal solutions.

Benchmark function f17 is an interesting one because the employed DE strategy could not locate the optimal solution in all runs, however the obtained results are very close to optimal and the standard deviation of 10 runs is 4.78. This is again significantly small compared to the level optimal objective function value for this function.

Functions f21 and f22 are the other two difficult functions for the employed DE strategy where a difference of around 100 is observed from the optimal solutions. In function f22, a standard deviation score of 0.13 was obtained, indicating a deep locally optimal solution into which the employed DE strategy was stuck at almost all the times. For function f21, the standard deviation score of 61.12 was obtained, which indicates the presence of many locally optimal solutions around the optimal location.

For the remaining benchmark functions f23, f28, f29 and f30, the objective function values obtained in all of the 10 different runs are above the optimal cost with values more than 300. Except for functions f28 and f30, standard deviation scores are not very high compared to the levels of objective function values, but significantly large to indicate that they are really difficult problems for the employed DE strategy.

The CPU time for each run does not vary a lot for majority of functions except for f16 and f30 with a respective standard deviation values of 3.26 and 6.79.

Table 3: DE Results for Dimension 10 and Variant Rand/2

		functions									
		f1		f2		f3		f4		f5	
D=10, DE/rand/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	100.00	13.98	200.00	8.64	300.00	8.17	400.00	9.55	523.45	8.88
	2	100.00	14.62	200.00	8.00	300.00	8.21	400.00	9.55	528.46	8.89
	3	100.00	17.49	200.00	7.97	300.00	8.17	400.00	9.61	536.03	8.88
	4	100.00	14.49	200.00	8.02	300.00	8.14	400.00	9.52	521.73	8.88
	5	100.00	15.85	200.00	8.00	300.00	8.14	400.00	9.53	535.65	8.97
	6	100.00	19.03	200.00	7.96	300.00	8.16	400.00	9.52	532.23	8.88
	7	100.00	18.91	200.00	7.97	300.00	8.14	400.00	9.52	539.63	8.88
	8	100.00	16.19	200.00	8.49	300.00	8.14	400.00	9.52	535.92	8.86
	9	100.00	17.16	200.00	8.24	300.00	8.14	400.00	9.61	530.57	8.89
	10	100.00	20.81	200.00	7.97	300.00	8.24	400.00	9.52	536.43	8.89
	Max	100.00	20.81	200.00	8.64	300.00	8.24	400.00	9.61	539.63	8.97
	Min	100.00	13.98	200.00	7.96	300.00	8.14	400.00	9.52	521.73	8.86
	Av	100.00	16.85	200.00	8.13	300.00	8.17	400.00	9.54	532.01	8.89
	Std	0.00	2.25	0.00	0.25	0.00	0.03	0.00	0.04	5.93	0.03

		functions									
		f6		f7		f8		f9		f10	
D=10, DE/rand/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	600.02	14.27	746.36	13.38	840.26	21.14	900.00	16.46	1363.46	19.56
	2	600.01	14.29	736.73	13.21	834.98	21.03	900.00	16.44	1743.60	19.42
	3	600.01	14.21	749.20	13.12	834.96	20.92	900.00	16.35	1424.61	19.48
	4	600.01	14.23	751.14	13.12	835.42	20.90	900.00	16.33	1718.83	19.53
	5	600.01	14.23	747.14	13.23	834.28	21.00	900.00	16.35	1455.77	19.30
	6	600.01	14.32	741.13	13.12	835.60	20.86	900.00	16.47	1418.90	19.44
	7	600.02	14.26	746.81	13.12	833.63	20.94	900.00	16.38	1725.18	19.34
	8	600.01	14.26	750.95	13.10	828.10	20.94	900.00	16.36	1360.88	19.09
	9	600.01	14.23	748.89	13.12	822.03	20.87	900.00	16.46	1671.98	19.27
	10	600.03	14.35	752.32	13.18	838.97	21.00	900.00	16.36	1134.00	19.03
	Max	600.03	14.35	752.32	13.38	840.26	21.14	900.00	16.47	1743.60	19.56
	Min	600.01	14.21	736.73	13.10	822.03	20.86	900.00	16.33	1134.00	19.03
	Av	600.01	14.26	747.07	13.17	833.82	20.96	900.00	16.40	1501.72	19.35
	Std	0.01	0.05	4.83	0.09	5.25	0.08	0.00	0.05	203.76	0.18

		functions									
		f11		f12		f13		f14		f15	
D=10, DE/rand/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	1107.73	12.95	1411.26	14.59	1313.08	15.74	1426.34	16.01	1502.39	39.00
	2	1108.76	12.79	1454.78	14.57	1317.70	15.82	1418.95	16.05	1502.96	38.63
	3	1109.63	12.78	1327.56	14.52	1312.54	15.68	1424.66	15.97	1504.36	35.54
	4	1107.25	12.78	1358.52	14.51	1314.74	15.71	1428.57	15.97	1503.32	37.11
	5	1110.03	12.87	1354.27	14.49	1312.63	15.74	1425.94	16.01	1502.21	35.85
	6	1109.72	12.76	1360.47	14.52	1314.37	15.82	1429.10	15.99	1503.22	35.24
	7	1107.49	12.81	1377.29	14.54	1315.39	15.72	1424.73	15.97	1501.48	36.02
	8	1107.10	12.76	1404.02	14.51	1311.93	15.68	1424.22	15.97	1502.88	34.51
	9	1108.66	12.85	1427.46	14.52	1315.79	15.80	1426.85	16.10	1502.47	35.66
	10	1108.92	12.79	1308.30	14.51	1314.66	15.74	1425.09	15.96	1503.05	37.07
	Max	1110.03	12.95	1454.78	14.59	1317.70	15.82	1429.10	16.10	1504.36	39.00
	Min	1107.10	12.76	1308.30	14.49	1311.93	15.68	1418.95	15.96	1501.48	34.51
	Av	1108.53	12.81	1378.39	14.53	1314.28	15.75	1425.45	16.00	1502.83	36.46
	Std	1.08	0.06	45.68	0.03	1.77	0.05	2.81	0.04	0.77	1.46

			f16		f17		f18		f19		f20	
			functions									
D=10, DE/rand/2		Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
		1	1600.26	48.67	1731.47	29.22	1800.21	16.88	1901.02	24.90	2031.79	28.58
		2	1603.65	47.69	1733.59	29.64	1802.39	16.79	1900.92	24.87	2034.37	28.38
		3	1603.20	51.57	1736.84	29.17	1802.86	16.88	1901.02	24.80	2032.75	28.50
		4	1601.44	45.94	1725.25	29.53	1800.15	16.79	1901.13	24.90	2032.60	28.35
		5	1600.46	48.27	1722.05	28.77	1800.52	16.79	1900.80	24.76	2035.91	28.55
		6	1604.63	51.98	1738.48	29.25	1800.22	16.85	1900.95	24.79	2033.63	28.42
		7	1611.43	44.41	1725.52	29.13	1800.35	16.77	1901.01	24.90	2036.66	28.55
		8	1600.36	47.81	1725.66	29.14	1800.37	16.80	1900.80	24.76	2025.07	28.41
		9	1601.73	49.56	1728.70	29.08	1800.23	16.79	1900.92	24.87	2025.55	28.45
		10	1606.79	49.78	1722.51	29.52	1800.31	16.88	1900.62	24.79	2033.13	28.45
		Max	1611.43	51.98	1738.48	29.64	1802.86	16.88	1901.13	24.90	2036.66	28.58
		Min	1600.26	44.41	1722.05	28.77	1800.15	16.77	1900.62	24.76	2025.07	28.35
		Av	1603.39	48.57	1729.01	29.24	1800.76	16.82	1900.92	24.83	2032.15	28.46
		Std	3.53	2.33	5.83	0.26	0.99	0.05	0.15	0.06	3.90	0.08

		functions									
		f21		f22		f23		f24		f25	
D=10, DE/rand/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	2328.71	29.70	2303.66	41.01	2632.37	49.22	2758.56	37.42	2898.46	42.99
	2	2336.84	29.48	2302.84	41.04	2625.02	49.23	2753.85	37.16	2898.48	42.76
	3	2200.00	29.61	2301.93	40.98	2630.25	49.17	2759.60	37.22	2898.45	42.85
	4	2337.87	29.50	2302.67	40.93	2633.37	49.16	2762.78	37.25	2897.74	42.84
	5	2339.24	29.62	2301.90	41.00	2631.73	49.16	2764.47	37.14	2898.42	42.74
	6	2336.24	29.52	2301.90	41.00	2629.35	49.19	2762.02	37.24	2898.33	42.84
	7	2334.44	29.59	2302.98	40.89	2628.18	49.27	2754.19	37.14	2897.74	42.81
	8	2200.00	29.48	2303.34	41.00	2621.53	49.22	2761.38	37.22	2898.46	42.90
	9	2200.00	29.61	2302.71	40.95	2634.27	49.17	2759.89	37.24	2897.74	42.76
	10	2200.00	29.50	2302.66	40.90	2625.69	49.05	2754.06	37.13	2897.74	42.78
	Max	2339.24	29.70	2303.66	41.04	2634.27	49.27	2764.47	37.42	2898.48	42.99
	Min	2200.00	29.48	2301.90	40.89	2621.53	49.05	2753.85	37.13	2897.74	42.74
	Av	2281.33	29.56	2302.66	40.97	2629.18	49.18	2759.08	37.22	2898.16	42.83
	Std	70.06	0.08	0.60	0.05	4.09	0.06	3.87	0.09	0.36	0.08

		functions									
		f26		f27		f28		f29		f30	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
D=10, DE/rand/2	1	2900.00	64.77	3089.01	65.02	3100.00	56.74	3155.20	178.29	3399.96	161.31
	2	2900.00	64.65	3089.01	64.99	3100.00	56.63	3176.80	179.23	3403.78	158.47
	3	2900.00	64.44	3089.01	64.93	3100.00	56.53	3155.17	171.35	3399.70	156.89
	4	2900.00	64.49	3089.31	64.94	3100.00	56.58	3154.27	172.66	3403.58	156.09
	5	2900.00	64.51	3089.01	64.94	3100.00	56.58	3152.83	172.83	3152.83	172.83
	6	2900.00	64.52	3089.52	64.68	3100.00	56.60	3163.58	182.26	3405.47	158.89
	7	2900.00	64.33	3086.89	64.94	3411.82	56.69	3159.46	171.85	3400.45	168.37
	8	2900.00	64.41	3089.01	64.93	3100.00	56.63	3155.13	180.38	3404.66	164.02
	9	2900.00	64.44	3089.52	64.82	3100.00	56.58	3161.59	182.30	3435.47	168.90
	10	2900.00	64.43	3089.01	64.93	3100.00	56.58	3163.12	173.49	3398.76	149.04
	Max	2900.00	64.77	3089.52	65.02	3411.82	56.74	3176.80	182.30	3435.47	172.83
	Min	2900.00	64.33	3086.89	64.68	3100.00	56.53	3152.83	171.35	3152.83	149.04
	Av	2900.00	64.50	3088.93	64.91	3131.18	56.61	3159.72	176.47	3380.47	161.48
	Std	0.00	0.13	0.75	0.10	98.61	0.06	7.16	4.45	80.70	7.14

Results illustrated in Table 3, for D=10, show that the global optimal solution is obtained in all 10 different runs for functions f1, f2, f3, f4 and f9. The standard deviation is obviously equal to 0 which means that the employed DE strategy is quite robust for the above listed 5 benchmark functions.

For benchmark functions f6, f15, f16, f18 and f19, the different obtained solutions are very close to the global optimal solution with a negligible error value. The standard deviation value is less than 1, indicating that a very small variation exists between the different obtained cost values for the 10 runs, this is due to the robustness of the DE strategy used.

Considering the benchmark functions f5, f7, f8, f11, f13, f14, f16, f17 and f20, the objective function values in the 10 different runs are above the optimal values with a difference smaller than 100. The standard deviation for the different functions mentioned above increased but still not important with values less than 6. These values indicate that the employed DE strategy is still robust.

The last remaining benchmark functions f10, f12, f21, f22, f23, f24, f25, f26, f27, f28, f29 and f30 are difficult functions. The error values in the different runs are more than 100 and less than 800. The standard deviation of the function f26 is equal to 0 which means that this DE strategy is stuck in a local optimum. Also for f22, f25 and f27, the standard deviation is importantly small with values smaller than 1. For the 3 functions f23, f24 and f29, the standard deviation values increased but are still not important. The maximal reached value is equal to 7.16, this is caused by the robustness of the DE strategy. Functions f10, f12, f21, f28 and f30's standard deviations increased and reached important values, with the biggest value of 203.76 occurring in function f10, an indication that the DE strategy is not robust and cannot overcome the local optimal values.

The CPU time for each run does not vary a lot for majority of the functions except for f29 and f30. Their respective standard deviation values are 4.45 and 7.14 but this variation is not really important.

Table 4: DE Results for Dimension 10 and Variant Best/2

		functions									
		f1		f2		f3		f4		f5	
D=10, DE/best/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	100.00	313.16	200.00	360.32	300.00	394.42	400.00	581.79	526.27	456.93
	2	100.00	313.16	200.00	359.52	300.00	392.73	400.00	578.40	522.75	456.33
	3	100.00	311.75	200.00	359.46	300.00	393.36	400.00	579.26	532.14	457.77
	4	100.00	312.33	200.00	361.25	300.00	393.54	400.00	577.47	526.77	455.80
	5	100.00	309.69	200.00	360.21	300.00	392.33	400.00	579.67	532.02	455.63
	6	100.00	310.15	200.00	359.66	300.00	394.65	400.00	580.25	520.76	455.59
	7	100.00	312.11	200.00	360.77	300.00	394.39	400.00	580.34	525.55	457.07
	8	100.00	312.61	200.00	359.93	300.00	394.68	400.00	580.70	524.58	455.52
	9	100.00	312.25	200.00	359.33	300.00	393.42	400.00	581.01	521.95	456.32
	10	100.00	311.41	200.00	360.08	300.00	396.06	400.00	581.15	526.64	455.99
	Max	100.00	313.16	200.00	361.25	300.00	396.06	400.00	581.79	532.14	457.77
	Min	100.00	309.69	200.00	359.33	300.00	392.33	400.00	577.47	520.76	455.52
	Av	100.00	311.86	200.00	360.05	300.00	393.96	400.00	580.00	525.94	456.30
	Std	0.00	1.17	0.00	0.68	0.00	1.22	0.00	1.37	4.08	0.84

		functions									
		f6		f7		f8		f9		f10	
D=10, DE/best/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	600.00	1013.59	736.32	908.94	824.22	1702.19	900.00	1251.00	1263.95	1546.75
	2	600.00	1011.00	737.25	907.50	825.66	1692.44	900.00	1249.08	1800.02	1559.35
	3	600.00	1011.28	736.43	908.74	829.67	1704.20	900.00	1253.08	1842.69	1559.14
	4	600.00	1011.57	735.64	907.72	823.33	1696.68	900.00	1251.91	1947.06	1528.62
	5	600.00	1012.18	736.73	907.66	820.61	1696.78	900.00	1249.24	1030.11	1502.48
	6	600.00	1011.37	731.29	907.93	825.58	1706.60	900.00	1250.35	1895.31	1546.31
	7	600.00	1011.88	737.01	907.74	833.91	1705.89	900.00	1248.99	2263.17	1495.61
	8	600.00	1004.13	729.67	908.11	832.89	1695.15	900.00	1249.65	1255.67	1553.19
	9	600.00	1005.61	735.82	908.25	818.32	1692.67	900.00	1250.86	2041.58	1500.31
	10	600.00	1011.37	738.22	907.22	829.56	1698.66	900.09	1247.52	1275.76	1550.09
	Max	600.00	1013.59	738.22	908.94	833.91	1706.60	900.09	1253.08	2263.17	1559.35
	Min	600.00	1004.13	729.67	907.22	818.32	1692.44	900.00	1247.52	1030.11	1495.61
	Av	600.00	1010.40	735.44	907.98	826.38	1699.13	900.01	1250.17	1661.53	1534.19
	Std	0.00	3.02	2.74	0.54	5.09	5.28	0.03	1.61	416.99	25.49

		functions									
		f11		f12		f13		f14		f15	
D=10, DE/best/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	1100.99	913.37	1596.80	1098.37	1305.95	1129.32	1406.32	2983.39	1500.00	2575.09
	2	1101.99	911.26	1861.32	1030.01	1415.77	1131.85	1421.99	2995.77	1501.00	2607.98
	3	1100.99	909.95	1993.24	1098.56	1306.70	1129.18	1498.57	2988.25	1500.00	2611.96
	4	1100.00	910.34	2215.98	1028.97	1305.95	1129.31	1425.97	3000.82	1500.46	2615.12
	5	1123.60	914.82	1521.84	1063.29	1306.51	1129.63	1403.98	3027.96	1500.02	2684.47
	6	1100.00	911.67	1329.82	1063.52	1620.29	1132.63	1547.26	3032.25	1585.58	2665.28
	7	1100.00	910.16	1330.91	1066.69	1350.76	1129.38	1550.35	2988.14	1500.03	2588.76
	8	1100.99	912.75	1472.35	1065.71	1305.95	1128.84	1421.99	3070.35	1501.07	2578.85
	9	1100.00	912.06	1734.28	1028.58	1305.95	1129.04	1409.95	3021.97	1500.09	2636.71
	10	1100.00	910.72	1825.40	1064.57	1324.02	1131.23	1420.00	2995.77	1500.02	2662.09
	Max	1123.60	914.82	2215.98	1098.56	1620.29	1132.63	1550.35	3070.35	1585.58	2684.47
	Min	1100.00	909.95	1329.82	1028.58	1305.95	1128.84	1403.98	2983.39	1500.00	2575.09
	Av	1102.86	911.71	1688.19	1060.83	1354.78	1130.04	1450.64	3010.47	1508.83	2622.63
	Std	7.32	1.57	290.83	25.65	99.64	1.34	58.27	27.39	26.97	38.16

		functions									
		f16		f17		f18		f19		f20	
D=10, DE/best/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	1746.05	4557.62	1720.84	5658.87	1820.19	1353.20	1900.00	2085.55	2020.03	2290.83
	2	1600.93	4393.02	1721.00	5325.72	1836.36	1354.17	1921.48	2092.44	2020.31	2316.24
	3	1612.08	4460.71	1722.30	5490.46	1820.04	1355.35	1903.71	2100.54	2000.31	2311.95
	4	1600.42	4323.04	1725.53	5522.54	1820.01	1354.74	1902.34	2100.51	2021.65	2358.83
	5	1611.19	4218.08	1740.28	5441.99	1820.21	1356.12	1901.52	2098.95	2017.75	2314.60
	6	1780.07	4475.31	1720.33	5644.55	1800.00	1352.56	1902.34	2099.02	2260.78	2348.63
	7	1731.62	4478.01	1703.32	5461.44	1846.29	1352.54	1904.21	2095.92	2020.27	2329.27
	8	1611.89	4384.14	1717.09	5623.80	1820.11	1355.43	1901.60	2098.34	2020.35	2328.21
	9	1611.68	4255.04	1720.68	5542.37	1821.11	1352.90	1900.61	2103.33	2000.01	2285.34
	10	1949.42	4490.83	1720.64	5655.58	1820.09	1353.89	1900.50	2100.09	2020.31	2318.08
	Max	1949.42	4557.62	1740.28	5658.87	1846.29	1356.12	1921.48	2103.33	2260.78	2358.83
	Min	1600.42	4218.08	1703.32	5325.72	1800.00	1352.54	1900.00	2085.55	2000.01	2285.34
	Av	1685.54	4403.58	1721.20	5536.73	1822.44	1354.09	1903.83	2097.47	2040.18	2320.20
	Std	115.74	109.92	8.96	110.45	12.02	1.29	6.35	5.11	77.96	22.72

		functions									
		f21		f22		f23		f24		f25	
D=10, DE/best/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	2328.96	2689.02	2300.34	3687.91	2611.02	4467.46	2746.79	3333.12	2943.62	3831.67
	2	2326.68	2689.18	2302.31	3692.72	2613.11	4463.02	2758.51	3342.76	2898.15	3822.57
	3	2200.00	2684.81	2300.93	3690.28	2618.26	4478.32	2754.45	3346.35	2898.08	3832.82
	4	2327.01	2686.32	2300.00	3694.60	2611.51	4466.34	2737.46	3339.40	2943.46	3829.17
	5	2326.62	2689.68	2300.00	3697.71	2612.10	4433.35	2739.39	3349.47	2898.21	3838.03
	6	2326.59	2687.44	2222.05	3588.82	2622.58	4447.12	2754.91	3340.65	2898.30	3834.08
	7	2200.00	2685.20	2300.34	3691.25	2610.32	4437.84	2761.33	3344.19	2898.13	3825.46
	8	2327.63	2683.59	2300.64	3688.32	2614.88	4499.88	2764.51	3352.63	2897.74	3830.26
	9	2336.78	2687.44	2301.41	3688.13	2620.22	4473.13	2743.46	3347.36	2945.83	3829.14
	10	2338.34	2682.11	2300.40	3686.32	2609.74	4444.05	2744.14	3346.91	2898.14	3837.33
	Max	2338.34	2689.68	2302.31	3697.71	2622.58	4499.88	2764.51	3352.63	2945.83	3838.03
	Min	2200.00	2682.11	2222.05	3588.82	2609.74	4433.35	2737.46	3333.12	2897.74	3822.57
	Av	2303.86	2686.48	2292.84	3680.60	2614.37	4461.05	2750.50	3344.28	2911.97	3831.05
	Std	54.91	2.53	24.88	32.44	4.48	20.59	9.49	5.59	22.32	4.85

		functions									
		f26		f27		f28		f29		f30	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
D=10, DE/best2	1	2954.22	5919.88	3155.16	6144.07	3383.73	5616.10	3134.66	6349.08	3442.17	5679.58
	2	2975.73	5916.68	3089.71	6010.72	3411.82	5604.55	3154.64	6301.88	1.25E+06	5686.66
	3	2900.00	5816.11	3089.64	5972.33	3100.00	5594.82	3148.58	6343.30	3500.74	5683.30
	4	2900.00	5815.42	3089.71	6014.01	3196.57	5609.42	3137.26	6305.28	8.21E+05	5681.95
	5	2900.00	5816.00	3090.12	5944.03	3411.82	5612.34	3133.36	6316.04	3483.38	5678.56
	6	2900.00	5810.79	3161.60	6085.24	3411.82	5617.56	3131.12	6334.80	3394.01	5687.73
	7	2900.00	5809.43	3095.98	6013.06	3411.82	5611.18	3131.80	6319.91	1.25E+06	5685.55
	8	2900.00	5811.69	3140.77	6150.62	3411.82	5624.30	3131.35	6298.68	3406.94	5685.47
	9	2988.08	5880.61	3089.71	6004.87	3100.00	5584.43	3177.84	6331.35	3394.01	5686.94
	10	2900.00	5813.21	3090.33	6024.68	3100.00	5586.69	3138.90	6367.18	3394.02	5686.84
	Max	2988.08	5919.88	3161.60	6150.62	3411.82	5624.30	3177.84	6367.18	1.25E+06	5687.73
	Min	2900.00	5809.43	3089.64	5944.03	3100.00	5584.43	3131.12	6298.68	3394.01	5678.56
	Av	2921.80	5840.98	3109.27	6036.36	3293.94	5606.14	3141.95	6326.75	3.35E+05	5684.26
	Std	36.02	45.90	30.32	68.72	149.02	13.39	14.85	22.42	5.46E+05	3.26

Results illustrated in Table 4, for D=10, show that the optimal solutions are obtained in all 10 different runs for functions f1, f2, f3, f4 and f9 with a standard deviation of 0. This shows that the DE strategy used to solve the problems is very robust and it escaped from the local optimum.

For the benchmark functions f11, f15, f18 and f19, the optimal solution is obtained a few number of times due to the DE strategy's good behavior. For the other runs, there is a variation of the error and the obtained cost values are very close to the optimal solution, but in other cases, there is a remarkable difference but still not very important with a values less than 100. The standard deviation increases from a function to another and the maximal value is 26.97, corresponding to the function f15 which shows that the DE strategy is not enough good and cannot avoid local optimums.

For the benchmark functions f5, f7, f8, f17 and f20, the error values obtained are not really important. They are less than 50 except in run 6 for the function f20 where the error value is 260. The standard deviation value for the all functions does not have an important value except for f20 which is equal to 77.96. These values indicate that the employed DE strategy is partly robust for functions f5, f7, f8 and f17, but the characteristics of function f20 make it hard to locate its optimal solution and the employed DE strategy is not successful in getting out of locally optimal solutions.

For the benchmark function f22, the error value overpassed 100. The difference from the optimal solution is bigger than 200 for f21 and f29. For benchmark function f16, f23, f24 and f26, it overpassed 300. The value of the error overpassed 400 for f25 and f27 benchmark functions and 600 for f28. The standard deviation is not big for some functions specially for f23 and f24 which was smaller than 10 but for some functions it is significant and the maximal standard deviation value reached 149.02 in f28. These scored values show that there were many local optimums and the solved functions were hard enough to reach an error value up to 600 and more.

Functions f10 and f12 are two difficult functions for the employed DE strategy where a difference of around 1000 is observed from the optimal solutions. Standard deviation score of f10 is 416.99 and for f12 290.83 which indicates many locally optimal solutions around the optimal location.

The last function f30 is a quite hard function. It gave in some cases an error between 394.01 and 500.74, which is also important. Also in runs 2, 4 and 7, the error becomes huge and according to its standard deviation value of 546375.71, it can be observed that the employed DE strategy cannot escape from the local optimums in some runs. In runs 2, 4 and 7 it failed to optimize f30.

For some functions, such as f1, f2, f3, f4, f5, f6, f7, f8, f9, f11, f13, f18, f19, f21, f24, f25, f28 and f30, the standard deviation of CPU time fluctuates, but without a significant variation. For the remaining functions, there is a variation which its importance vary from a function to another and the biggest value is 109.92 for the function f16.

Table 5: DE Results for Dimension 10 and Variant Rand to Best/1

		functions									
		f1		f2		f3		f4		f5	
D=10, DE/rand to best/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	100.00	299.72	200.00	378.65	300.00	411.08	400.00	574.97	525.15	448.57
	2	100.00	304.92	200.00	391.27	300.00	409.75	400.00	573.60	526.90	447.88
	3	100.00	287.82	200.00	387.96	300.00	402.09	400.00	574.43	518.70	447.24
	4	100.00	287.70	200.00	379.74	300.00	402.50	400.00	573.52	525.45	447.44
	5	100.00	288.15	200.00	368.68	300.00	402.01	400.00	573.33	528.65	446.85
	6	100.00	287.87	200.00	367.74	300.00	403.01	400.00	573.16	518.47	447.07
	7	100.00	288.07	200.00	368.86	300.00	403.59	400.00	575.11	529.70	446.94
	8	100.00	288.51	200.00	368.57	300.00	402.26	400.00	574.01	530.85	447.27
	9	100.00	288.29	200.00	368.88	300.00	403.31	400.00	573.05	520.49	446.72
	10	100.00	288.09	200.00	368.44	300.00	403.70	400.00	573.66	528.33	447.10
	Max	100.00	304.92	200.00	391.27	300.00	411.08	400.00	575.11	530.85	448.57
	Min	100.00	287.70	200.00	367.74	300.00	402.01	400.00	573.05	518.47	446.72
	Av	100.00	290.91	200.00	374.88	300.00	404.33	400.00	573.89	525.27	447.31
	Std	0.00	6.14	0.00	8.95	0.00	3.28	0.00	0.73	4.55	0.55

		functions									
		f6		f7		f8		f9		f10	
D=10, DE/rand to best/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	600.00	1007.06	734.86	926.26	820.56	1706.20	900.00	1252.27	1163.56	1521.59
	2	600.00	1006.03	738.50	925.79	824.03	1691.35	900.00	1251.94	1190.85	1501.56
	3	600.00	1006.39	734.46	926.79	823.70	1698.46	900.00	1249.80	1577.60	1504.61
	4	600.00	1006.27	733.52	927.85	825.67	1683.22	900.00	1251.32	1370.82	1534.38
	5	600.00	1006.30	736.21	926.44	822.09	1690.16	900.00	1250.97	1320.33	1482.87
	6	600.00	1006.22	738.48	926.30	828.14	1692.30	900.00	1251.24	1270.33	1486.16
	7	600.00	1005.69	739.26	926.29	822.97	1688.13	900.00	1250.55	1424.56	1517.61
	8	600.00	1006.35	741.46	926.21	824.07	1700.64	900.00	1250.39	1262.60	1500.25
	9	600.00	1006.71	740.63	926.38	824.66	1660.55	900.00	1252.33	1409.25	1526.52
	10	600.00	1006.83	737.49	926.01	826.71	1692.58	900.00	1251.74	2113.31	1494.30
	Max	600.00	1007.06	741.46	927.85	828.14	1706.20	900.00	1252.33	2113.31	1534.38
	Min	600.00	1005.69	733.52	925.79	820.56	1660.55	900.00	1249.80	1163.56	1482.87
	Av	600.00	1006.39	737.49	926.43	824.26	1690.36	900.00	1251.25	1410.32	1506.98
	Std	0.00	0.40	2.67	0.56	2.20	12.37	0.00	0.84	275.38	17.37

		functions									
		f11		f12		f13		f14		f15	
D=10, DE/rand to best/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	1100.01	888.39	1200.21	1057.30	1306.70	1173.63	1400.00	1201.71	1500.00	1059.98
	2	1100.00	888.41	1200.00	1027.56	1307.48	1172.43	1401.71	1201.38	1500.00	1060.32
	3	1100.00	888.75	1200.00	1046.08	1304.25	1172.69	1400.02	1200.99	1500.00	1060.95
	4	1100.00	887.79	1212.28	1081.93	1304.04	1172.21	1400.00	1200.76	1500.38	1058.53
	5	1100.00	887.66	1212.28	1076.50	1306.14	1172.55	1400.00	1200.66	1500.00	1060.34
	6	1100.03	888.30	1200.21	1050.12	1305.95	1173.60	1400.00	1200.13	1500.33	1059.15
	7	1100.00	887.63	1200.42	1063.66	1305.95	1172.13	1408.14	1201.85	1500.01	1060.45
	8	1100.00	888.19	1334.65	1056.84	1307.74	1172.50	1400.03	1201.33	1500.00	1060.09
	9	1100.00	888.32	1200.62	1079.59	1305.95	1171.97	1400.00	1198.82	1500.00	1060.20
	10	1100.00	888.21	1319.95	1078.53	1306.31	1171.74	1403.25	1201.18	1500.01	1059.17
	Max	1100.03	888.75	1334.65	1081.93	1307.74	1173.63	1408.14	1201.85	1500.38	1060.95
	Min	1100.00	887.63	1200.00	1027.56	1304.04	1171.74	1400.00	1198.82	1500.00	1058.53
	Av	1100.00	888.17	1228.06	1061.81	1306.05	1172.54	1401.31	1200.88	1500.07	1059.92
	Std	0.01	0.36	52.65	17.71	1.19	0.63	2.63	0.88	0.15	0.74

D=10, DE/rand to best/1		functions									
		f16		f17		f18		f19		f20	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	1600.30	1533.86	1715.12	2278.86	1800.00	3731.20	1900.00	2096.64	2000.00	2298.18
	2	1600.22	1533.74	1723.95	2282.44	1800.01	3729.20	1900.00	2096.15	2000.87	2335.77
	3	1600.23	1531.07	1700.40	2287.57	1800.01	3722.61	1900.00	2094.84	2000.85	2330.67
	4	1600.24	1531.27	1713.56	2285.34	1800.00	3726.80	1900.02	2096.50	2002.26	2346.22
	5	1600.71	1526.94	1700.17	2256.18	1800.02	3715.80	1900.00	2092.35	2000.01	2308.25
	6	1600.45	1558.79	1700.47	2288.61	1800.01	3707.38	1900.01	2092.72	2000.84	2337.89
	7	1600.64	1592.61	1716.07	2313.45	1800.01	3703.54	1900.01	2095.00	2000.63	2331.61
	8	1600.23	1534.13	1700.42	2285.91	1800.01	3695.63	1900.00	2093.55	2004.65	2333.35
	9	1611.38	1533.32	1700.72	2289.69	1800.17	3690.95	1900.00	2092.24	2000.06	2311.44
	10	1600.50	1492.60	1700.00	2249.50	1800.00	3684.26	1900.02	2095.58	2012.18	2328.00
	Max	1611.38	1592.61	1723.95	2313.45	1800.17	3731.20	1900.02	2096.64	2012.18	2346.22
	Min	1600.22	1492.60	1700.00	2249.50	1800.00	3684.26	1900.00	2092.24	2000.00	2298.18
	Av	1601.49	1536.83	1707.09	2281.75	1800.02	3710.74	1900.01	2094.56	2002.24	2326.14
	Std	3.48	25.28	9.09	17.89	0.05	16.89	0.01	1.72	3.77	15.12

		functions									
		f21		f22		f23		f24		f25	
D=10, DE/rand to best/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	2331.73	2692.33	2300.35	11640.79	2617.11	4539.38	2753.86	3317.17	2898.01	3706.69
	2	2200.00	2690.64	2300.34	11683.63	2615.26	4535.95	2756.24	3313.77	2945.79	3709.80
	3	2322.76	2691.83	2300.00	11743.88	2610.47	4541.03	2746.76	3316.75	2943.37	3710.70
	4	2285.01	2696.32	2300.00	11738.65	2621.50	4535.54	2741.77	3315.13	2898.03	3708.42
	5	2200.00	2689.21	2300.00	11734.66	2612.75	4542.81	2747.89	3317.38	2898.01	3705.99
	6	2200.00	2690.78	2300.00	11789.09	2611.25	4535.93	2749.28	3317.44	2897.74	3706.19
	7	2329.27	2691.88	2300.00	11823.03	2611.37	4532.19	2755.47	3314.07	2898.04	3705.82
	8	2321.73	2692.41	2200.00	11433.81	2612.49	4537.04	2754.94	3319.34	2898.04	3708.41
	9	2326.45	2694.00	2300.00	11713.60	2618.00	4544.54	2747.96	3318.86	2898.01	3703.65
	10	2321.03	2688.58	2300.00	11777.22	2617.75	4544.71	2754.45	3316.19	2898.01	3707.64
	Max	2331.73	2696.32	2300.35	11823.03	2621.50	4544.71	2756.24	3319.34	2945.79	3710.70
	Min	2200.00	2688.58	2200.00	11433.81	2610.47	4532.19	2741.77	3313.77	2897.74	3703.65
	Av	2283.80	2691.80	2290.07	11707.84	2614.80	4538.91	2750.86	3316.61	2907.30	3707.33
	Std	59.25	2.24	31.65	109.47	3.68	4.26	4.81	1.86	19.66	2.09

		functions									
		f26		f27		f28		f29		f30	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
D=10, DE/rand to best/1	1	2900.00	5832.22	3089.31	6274.72	3100.00	5494.95	3132.30	6327.49	3394.33	5929.21
	2	2900.00	5827.56	3089.52	6241.57	3100.00	5492.42	3142.80	6329.01	3394.71	5928.16
	3	2900.00	5827.95	3089.01	6272.43	3100.00	5492.14	3139.98	6293.83	3394.27	5919.58
	4	2900.00	5831.38	3089.64	6229.54	3411.82	5513.82	3139.38	6316.96	3394.45	5918.88
	5	2900.00	5830.90	3089.52	6242.91	3100.00	5495.54	3132.58	6348.69	34412.92	5916.65
	6	2900.00	5828.54	3089.95	6223.30	3100.00	5497.74	3128.79	6326.18	3394.35	5919.24
	7	2900.00	5829.24	3089.52	6250.55	3100.00	5495.46	3140.86	6325.26	883843.84	5914.08
	8	2900.00	5834.30	3089.31	6272.18	3196.57	5510.86	3140.18	6322.02	3394.52	5922.23
	9	2900.00	5837.73	3089.52	6235.14	3100.00	5492.28	3131.51	6333.44	3394.45	5919.75
	10	2900.00	5832.28	3089.52	6249.15	3100.00	5490.28	3134.07	6356.14	3394.29	5921.19
	Max	2900.00	5837.73	3089.95	6274.72	3411.82	5513.82	3142.80	6356.14	883843.84	5929.21
	Min	2900.00	5827.56	3089.01	6223.30	3100.00	5490.28	3128.79	6293.83	3394.27	5914.08
	Av	2900.00	5831.21	3089.48	6249.15	3140.84	5497.55	3136.24	6327.90	94541.21	5920.90
	Std	0.00	3.15	0.25	18.48	99.93	8.12	4.89	16.95	277503.94	4.69

Results illustrated in Table 5, for D=10, show that optimal solutions are obtained in all 10 different runs for functions f1, f2, f3, f4, f6 and f9. Also, for functions f11, f14, f15 and f19, optimal solutions are obtained in almost all except for a few number of runs, and for cases where optimal solutions are not extracted, the obtained solutions are very close to the optimal ones. The standard deviation values for the functions mentioned above are 0 or very close to 0 except for f14 which is 2.63. The obtained values means that the employed DE strategy is quite robust for the above listed 10 benchmark functions.

Considering the benchmark functions f17, f18 and f20, a few optimal solutions are still extracted as in the previous functions, while the rest of the results are very close to the optimal ones. Also, the standard deviation values for functions f18 is smaller than 1, but for functions f17 and f20, their standard deviations increased, and are between 3.77 and 9.09, which are significantly small values when compared to the fitness values of the corresponding functions.

For the function f12, the optimal solution is obtained 2 times (in run 2 and run 3). In the other runs, the obtained solutions in some runs are very close to optimal solutions while in other ones, there is a difference which overpass 100. Also, the function f22 reached a global optimum in run 8, but in the other runs the solutions cost equal or slightly bigger than 2300. The standard deviation value is equal to 52.65 for f12 which is an important value that shows that the employed DE strategy is not robust enough and the algorithm gets stuck many times in different local optimums. The standard deviation for f22 is 31.65 this value was obtained because of the difference between the obtained optimal solution and the other obtained values, which show that the f22 is stuck in many runs in a deep local optimum.

For benchmark functions f13 and f16, the different obtained solutions are very close to the optimal solution with a negligible error value. The standard deviation value is equal 1.19 and 3.48 which means that the DE strategy is a robust one.

Considering the benchmark functions f5 and f7, the objective function values in the 10 different runs are above the optimal values with a difference smaller than 100. The standard deviation scores for functions f5 and f7 are still significantly small compared to objective function values. These values indicate that the employed DE strategy is still robust for functions f5 and f7.

For function f10, there is a difference between the optimal cost and the scored cost values that overpassed 100, also, function f21 and f29 overpassed 200, and functions f23, f24, f25, f26 and f27 and f28 overpassed 300. The standard deviation values of f26 and f27 are 0 and 0.25 respectively means that the DE strategy is stuck in a deep local optimum. The standard deviation values of f23 is 3.68 and 4.81 for f24 which is significantly small. The remaining standard deviation for the other functions are importantly significant and reached 275.38 which means that there are many local optimum and the DE used strategy failed to avoid them.

For the last function f30, in some runs the obtained costs have a difference of only a value more than 300 but in run5 and run7 was very huge with a standard deviation of 277503.94.This means that in some runs, the algorithm still stuck in a deep local optimum or it fails to optimize the function.

For some functions, such as f4, f5, f6, f7, f9, f11, f13, f14 and f15, the CPU standard deviation values is very close to 0, indicating that there is not a significant variation in time. For the functions f1, f2, f3, f8, f10, f12, f16, f17, f18, f19, f20, f21, f23, f24, f25, f26, f27, f28, 29 and f30, the standard deviation is between 1.72 and 25.28. It fluctuates but without a big variation on time. For function f22, its CPU standard deviation is equal to 109.47 that shows an important variation in time between the different experiments.

From a perspective of speed, according to results illustrated in Table 2, Table 3, Table 4 and Table 5 for D=10, the fastest DE strategy in the 10 runs for the function f1 is rand/2 with a CPU average equal to 16.85 after there is rand/1 with a CPU average equal to 17.27. In third position, there is rand to best/1 strategy with a CPU average equal to 390.91 and the slowest one is best/2 with a CPU average equal to 311.86.

For the function f2 the fastest strategy is rand/2 with a CPU average equal to 8.13. After, there is the strategy rand/1, its CPU average is 15.74. In third position, there is best/2 with a CPU average value equal to 360.05 and in last position, rand to best/1 its CPU average is 374.88.

The best strategy based on the execution time for f3 is rand/1 with a CPU average equal to 7.16. After it is rand/2 with a CPU average of 8.17. After, there is best/2 with a CPU average equal to 393.96 and the slowest strategy is rand to best/1 with a CPU average of 404.33.

With a CPU average equal to 8.59, the strategy rand/1 takes the first position in speed to optimize f4. In second position there is the strategy rand/2 with a CPU average equal to 9.54. In next position, there is the strategy rand to best/1 with a corresponding CPU average of 573.89, and the slowest strategy is best/2 with a CPU average equal to 580.00.

Considering the benchmark function f5, the fastest applied strategy is rand/1, having a CPU average of 7.82. In second position, there is the rand/2 strategy with a CPU average of 8.89. In third position, there is the rand to best/1 strategy, its CPU average is 447.31 and the slowest variant is best/2 with a CPU average equal to 456.30.

The best strategy based on the execution time for f6 is rand/2 with a CPU average equal to 14.26. Next to this is rand/1, with a CPU average of 14.37. Also, there is the rand to best/1 strategy whose CPU average equals to 1006.39. The slowest strategy is best/2 with a CPU average of 1010.40.

For the function f7, the fastest strategy is rand/1 with a CPU average that equals to 11.89. After, it is the strategy rand/2, its CPU average is 13.17. In third position is the best/2 strategy with a CPU average value equal to 907.98 and in last position, rand to best/1 its CPU average is 926.43.

Considering the benchmark function f8, the fastest applied strategy is rand/1, its CPU average is 20.07. In second position, there is the strategy rand/2, its CPU average is 20.96. In third position, there is the strategy rand to best/1, and its CPU average is 1690.36. The slowest variant is best/2 with a CPU average equal to 1699.13.

With a CPU average equal to 15.46, the strategy rand/1 takes the first position in speed to optimize f9. In second position, there is the strategy rand/2 with a CPU average equal to 16.40. In next position, there is the strategy best/2 its corresponding CPU average is 1250.17 while the slowest strategy is rand to best/1 with a CPU average equal to 1251.25.

The fastest DE strategy in the 10 runs for the function f10 is rand/1 with a CPU average equal to 18.37. After, there is rand/2 with a CPU average equal to 19.35. In the next position is rand to best/1 strategy, with a CPU average equal to 1506.98 and the slowest one is best/2 with a CPU average equal to 1534.19.

Considering the benchmark function f11, the fastest applied strategy is rand/1, its CPU average is 12.05. In second position, there is the strategy rand/2 its CPU average is 12.81. In third position, there is the strategy rand to best/1, its CPU average is 888.17 and the slowest variant is best/2 with a CPU average equal to 911.71.

With a CPU average equal to 13.90, the strategy rand/1 takes the first position in speed to optimize f12. In second position, there is the strategy rand/2 with a CPU average equal to 14.53. In next position, is the best/2 strategy its corresponding CPU average is 1060.83 and the slowest strategy is rand to best/1 with a CPU average equal to 1061.81.

The fastest DE strategy in the 10 runs for the function f13 is rand/1 with a CPU average equal to 14.80. After, there is rand/2 with a CPU average equal to 15.75. In next position, there is best/2 strategy with a CPU average equal to 1130.04 and the slowest one is rand to best/1 with a CPU average equal to 1172.54.

The best strategy based on the execution time for f14 is rand/1 with a CPU average equal to 14.58. After there is rand/2 with a CPU average equal to 16.00. Next is the rand to best/1 strategy with a CPU average equal to 1200.88, and the slowest strategy is best/2 with CPU average of 3010.47.

For the function f15, the fastest strategy is rand/1 with a CPU average equal to 32.23. After, there is the strategy rand/2 its CPU average is 36.46. In third position, is the rand to best/1 strategy with a CPU average of 1059.92, while in the last position is the best/2 strategy its CPU average is 2622.63.

Considering the benchmark function f16, the fastest applied strategy is rand/1 and its CPU average is 45.84. In second position, there is the strategy rand/2 its CPU average is 48.57. In third position is the strategy rand to best/1 with CPU average of 1536.83, and the slowest variant is best/2 with a CPU average equal to 4403.58.

With a CPU average equal to 27.22 the strategy rand/1 takes the first position in speed to optimize f17. In second position, there is the strategy rand/2 with a CPU average equal to 29.24. In next position, there is the strategy rand to best/1 its corresponding CPU average is 2281.75 and the slowest strategy is best/2 with a CPU average equal to 5536.73.

For the function f18, the rand/1 holds the fastest strategy position with a CPU average of 15.60. After, there is the strategy rand/2, and its CPU average is 16.82. In third position, there is best/2 with a CPU average value equal to 1354.09, while rand to best/1 with a CPU average of 3710.74 is the slowest.

The best strategy based on the execution time for f19 is rand/1, with a CPU average of 24.12. Next to this is the rand/2 strategy with a CPU average of 24.83, and this is followed by the rand to best/1 strategy with a CPU average equal to 2094.56. Finally, the slowest strategy is best/2, its CPU average is 2097.47.

With a CPU average equal to 28.46, the strategy rand/2 takes the first position in speed to optimize f20. In second position is the rand/1 strategy with a CPU average equal to 28.52. In the next position is the strategy best/2 with a CPU average of 2320.20. The slowest strategy is rand to best/1 with a CPU average equal to 2326.14.

Considering the benchmark function f21, the fastest applied strategy is rand/1, having a CPU average of 28.59. In second position, is the rand/2 strategy, its CPU average is 29.56. In the third position, is the strategy best/2 with a CPU average of 2686.48. The slowest variant is rand to best/1, with a CPU average of 2691.80.

The best strategy based on the execution time for f22 is the rand/1 strategy, with a CPU average equal to 40.05. After this is the rand/2 strategy with a CPU average of 40.97 in the second position. Next to this is the best/2 strategy with a CPU average equal to 3680.60. The slowest strategy is rand to best/1 with a CPU average of 11707.84.

With a CPU average equal to 47.88, the rand/1 strategy takes the first position in rapidity to optimize f23. In second position is the rand/2 strategy with a CPU average of 49.18. In next position, is the best/2 strategy, its corresponding CPU average is 4461.05 and the slowest strategy is rand to best/1 with a CPU average equal to 4538.91.

The fastest DE strategy in the 10 runs for the function f24 is rand/1 with a CPU average equal to 36.40. After it is the rand/2 strategy with a CPU average equal to 37.22. In the next position is rand to best/1 strategy with a CPU average of 3316.61, and the slowest one is best/2 with a CPU average equal to 3344.28.

With a CPU average equal to 41.38, the rand/1 strategy takes the first position in speed to optimize f25. Rand/2 strategy takes the second position, with a CPU average of 42.83. Subsequent to that is the rand to best/1 strategy with a corresponding CPU average of 3707.05, while the slowest strategy is best/2 with a CPU average equal to 3831.05.

For the function f26, the fastest is the rand/1 strategy with a CPU average of 61.88, after which is the rand/2 strategy, having a CPU average of 64.50. In third position, is the rand to best/1 with a CPU average value equal to 5831.21 and in last position, is best/2 strategy, its CPU average is 5840.98.

The best strategy based on the execution time for f27 is rand/2 with a CPU average equal to 64.91. After this is the rand/1 strategy with a CPU average of 65.71. After, there is best/2 with a CPU average equal to 6036.36 and the slowest strategy is rand to best/1, its CPU average is 6249.15.

The fastest DE strategy in the 10 runs for the function f28 is rand/2 with a CPU average equal to 56.61. Rand/1 strategy takes the second position with a CPU average equal to 57.47. In next position, is the rand to best/1 strategy with a CPU average equal to 5497.55, while the slowest one is best/2 with a CPU average equal to 5606.14.

The fastest applied strategy for f29 is rand/1 its CPU average is 64.06. In second position is the rand/2 strategy, its CPU average is 160.47. In third position is the best/2 strategy, its CPU average is 6326.75 and the slowest variant is rand to best/1 with a CPU average equal to 6327.90.

Considering the benchmark function f30, the fastest applied strategy is rand/2, its CPU average is 161.48. In second position is the rand/1 strategy, its CPU average is 161.80. Next is the best/2 strategy its CPU average is 5684.26 and the slowest variant is rand to best/1 with a CPU average equal to 5920.90.

From an accuracy point of view, according to results illustrated in Table 2, Table 3, Table 4 and Table 5 for D=10, for the different benchmark functions f1, f2, f3 and f4, the optimal solution is obtained by the 4 different strategies.

Considering the benchmark function f6, the optimal solution is obtained by the different strategies rand/1, best/2 and rand to best/1. The strategy rand/2 gives a cost average equal to 600.01.

Also, for the function f9, the optimal solution is obtained by 3 different strategies namely; rand/1, rand/2 and rand to best/1 but best/2 gives a cost average of 900.01.

For f26, three of the strategies which are rand/1, rand/2 and rand to best/1 give the same cost average which is 2900.00. A value which is obtained during the 10 different runs and the strategy best/2 scores a cost average of 2921.80.

The strategy rand/1 scores the best cost average with a value equal to 523.98 for f25. In second, position is the rand to best/1 strategy with a cost average value equal to

525.27. In third position is the best/2 strategy, with a cost average of 525.94 while the rand/2 strategy is in the fourth position with a cost average equal to 532.01.

The most accurate DE strategy in the 10 runs for the function f7 is rand/1 with a cost average equal to 734.48. After this comes the best/2 strategy with a cost average equal to 735.44. In next position is rand to best/1 strategy with a cost average equal to 737.49 and the least accurate one is rand/2 with a cost average equal to 747.07.

For the function f8, the best strategy according to accuracy is rand/1 with a cost average of 823.41. Next to it is the rand to best/1 strategy, its cost average is 824.26. Third is the best/2 strategy with a cost average equal to 826.38, and finally, rand/2 is the worst strategy with a cost average equal to 833.82.

The best strategy based on the optimality of the obtained solution for f10 is rand/1, and its cost average is 1358.32. Rand to best/1 strategy comes second, its cost average is 1410.32. Third is rand/2 strategy with a cost average equal to 1501.72 and the least accurate strategy is best/2 with a cost average of 1661.53.

For the function f11, the most accurate strategy is rand to best/1 with a cost average of 1100.00 which is equal to the optimal solution since the values in the 10 different runs are equal or very close to the optimal solution. Next is the rand/1 strategy, its cost average is 1100.20. In third position is the best/2 strategy, its cost average is equal to 1102.86 and in last position, is the rand/2 strategy with a cost average value equal to 1108.53.

The variant rand/1 is the most accurate strategy with a cost average value equal to 1201.41 for f12. The variant rand to best/1 is in the second position, its cost average is equal to 1228.06. In third position is the variant rand/2 with a cost average equal to 1378.39 and the worst accurate variant is best/2, its scored cost average 1688.19.

Considering the benchmark function f13, the most accurate applied strategy is rand/1 since its cost average is 1303.55. In second position is the strategy rand to best/1, its cost average is 1306.05. In third position is the rand/2 strategy, its cost average is 1314.28 and the least accurate variant is best/2, with a cost average equal to 1354.78.

Also, the best variant for the function f14 is rand/1, its scored cost average is equal to 1400.10. After it is the rand to best/1 strategy, with a cost average equal to 1401.31. Then, rand/2 with a cost equal to 1425.45 is in the third position, and the worst strategy is best/2 with a cost average of 1450.6.

For the function f15, the best strategy according to accuracy is rand/1 with a cost average of 1500.01. After, it is the strategy rand to best/1, its cost average is equal to 1500.07. Then, the strategy rand/2 with a cost average equal to 1502.83 takes the third position. Finally, best/2 is the worst strategy with a cost average equal to 1508.83.

Also, the best variant for the function f16 is rand/1, its scored cost average is equal to 1600.26. After there is rand to best/1 with a cost average equal to 1601.49. Then, rand/2 with a cost equal to 1603.39 and the worst strategy is best/2. It scores a cost average of 1685.54.

The variant rand/1 is the best strategy according to accuracy with a cost average value equal to 1702.54 for f17. The variant rand to best/1 is in the second position, its cost average is equal to 1707.09. In third position is the variant best/2 with a cost average equal to 1721.20 while the worst accurate variant is rand/2, its scored cost average is 1729.01.

For the function f18, the best strategy according to accuracy is rand to best/1 with a cost average of 1800.02. Next is the rand/1 strategy, its cost average is equal to 1800.12. Then, the rand/2 strategy with a cost average equal to 1800.76. Finally, best/2 is the worst strategy with a cost average equal to 1822.44.

The best strategy based on the optimality of the obtained solution for f19 is rand/1 with a cost average equal to 1900.00 which is the same as the optimal solution. This value is obtained because the costs obtained in the different 10 runs are equal or very close to the optimal solution. After it is the rand to best/1 strategy, its cost average is 1900.01. Next is rand/2 with a cost average equal to 1900.92 and the least accurate strategy is best/2, its cost average is 1903.83.

Considering the benchmark function f20, the most accurate applied strategy is rand to best/1 its cost average is 2002.24. In second position is the strategy rand/1, its cost average is 2002.43. In third position is the strategy rand/2, its cost average is 2032.15 and the least accurate variant is best/2 with a cost average equal to 2040.18.

Also, the best variant for the function f21 is rand/1, its scored cost average is 2238.58. The next best variant is rand/2 with a cost average equal to 2281.33. Third is the rand to best/1 strategy with a cost equal to 2283.80, and the worst strategy is best/2. It scores a cost average of 2303.86.

The most accurate DE strategy in the 10 runs for the function f22 is rand to best/1 with a cost average equal to 2290.07. After this is best/2 with a cost average equal to 2292.84. In next position is rand/1 strategy with a cost average equal to 2300.04 and the least accurate one is rand/2 with a cost average equal to 2302.66.

For the function f23, the best strategy according to accuracy is rand/1 with a cost average of 2612.91. Next is the strategy best/2, its cost average is equal to 2614.37. Then, the strategy rand to best/1 with a cost average equal to 2614.80 is third while rand/2 is the worst strategy with a cost average equal to 2629.18.

The best strategy based on the optimality of the obtained solution for f24 is best/2 with a cost average equal to 2750.50. After it is the rand to best/1 strategy, with a cost average of 2750.86. Rand/1 follows with a cost average equal to 2751.78, and the least accurate strategy is rand/2, its cost average is 2759.08.

Also, the best variant for the function f25 is rand/2, its scored cost average is equal to 2889.16. After it is rand/1 with a cost average equal to 2902.71, which is followed by the rand to best/1 strategy with a cost of 2907.30. The worst strategy is best/2, it scores a cost average of 2911.97.

For the function f27, the best strategy according to accuracy is rand/2 with a cost average of 3088.93. After it is the strategy rand/1, its cost average is equal to 3089.28.

Then, the strategy rand to best/1 with a cost average equal to 3089.48. Finally, best/2 is the worst strategy with a cost average equal to 3109.27.

The variant rand/2 is the best strategy according to accuracy with a cost average value equal to 3131.18 for f28, while the variant rand to best/1 is in the second position with a cost average of 3140.84. In third position is the rand/1 variant, having a cost average of 3147.69 and the worst accurate variant is best/2, its scored cost average 3293.94.

The strategy rand to best/1 scores the best cost average with a value equal to 3136.24 for f29. In second position is the rand/1 strategy with a cost average value equal to 3139.01. In third position is the strategy best/2 with a cost average of 3141.95 and the strategy rand/2 is in the fourth position with a cost average equal to 3159.72.

For the last benchmark function f30, the most accurate DE strategy in the 10 runs is rand/2 with a cost average of 3380.47. After it is the rand/1 strategy, with a cost average equal to 3413.59. Rand to best/1 strategy takes the third position with a very bad cost average of 94541.21 while the least accurate strategy is best/2 with a cost average which is extremely divergent from the optimal solution with a value of 334840.69.

The strategies rand/1 and rand/2 are extremely faster than strategies best/2 and rand to best/1. The strategy rand/1 is the fastest strategy for 23 benchmark functions which are f3, f4, f5, f7, f8, f9, f10, f11, f12, f13, f14, f15, f16, f17, f18, f19, f21, f22, f23, f24, f25, f26, and f29, while strategy rand/2 is the fastest strategy for 7 benchmark functions which are f1, f2 f6, f20, f27, f28 and f3. For the 16 benchmark functions f1, f4, f5, f6, f8, f10, f11, f14, f15, f16, f17, f19, f24, f25, f26, and f28, the variant best/2 is the slowest and for the 14 benchmark functions f2, f3, f7, f9, f12, f13, f18, f20, f21, f22, f23, f27, f29, and f30, the strategy rand to best/1 is the slowest.

The strategies rand/1 and rand to best/1 are the most accurate ones while rand/2 and best/2 strategies are the least accurate. For the functions f1, f2, f3 and f4, the optimal solution is obtained by all the strategies. The strategy rand/1 is the most accurate for these 16 functions which are f6, f9 and f19 with an optimal cost average. After there is f5, f7, f8, f12, f13, f14, f15, f16 and f17 with a cost average very close to the optimal cost. The last gropu of functions is f10, f21, f23 and f26 had an acceptable cost otherwise it is in second position for the majority of the remaining functions. Also the strategy rand to best/1 is the most accurate for 8 functions. These functions are f6 with an optimal cost average, f9 with also an optimal cost average, f11 with an optimal average because the obtained solutions are globally optimal or very close to optimality, f18, f20, f22, f26 and f29. In majority of time rand to best/1 is in second position in accuracy. For the strategy rand/2 and best/2 are the worst in accuracy side. The variant rand/2 is the best for 6 functions. These functions are f9 with an optimal cost average, f25, f26, f27, f28, f30. Also, it is the worst strategy for 10 functions which are f5, f6, f7, f8, f11, f17, f22, f23, f24 and f29. The worst accuracy for the 14 functions, f10, f12, f13, f14, f15, f16, f18, f19, f20, f21, f25, f27, f28 and f30.

From the perspective of speed and accuracy, rand/1 strategy is the best one. Rand/2 is faster than rand to best/1 but rand to best/1 is better generally in accuracy. The best/2 strategy is the worst in time response and accuracy.

Table 6: DE Results for Dimension 30 and Variant Rand/1

		functions									
		f1		f2		f3		f4		f5	
D=30, DE/rand/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	100.00	30.31	23051.02	35.68	605.07	40.93	458.56	54.57	684.29	40.73
	2	100.00	30.17	336.56	35.60	304.25	40.56	458.56	54.24	691.02	40.83
	3	100.00	30.11	248.35	35.32	335.40	40.62	458.56	54.18	675.45	40.72
	4	100.00	29.98	65060.29	35.44	506.62	40.64	458.56	54.23	680.33	40.56
	5	100.00	29.89	162971.55	35.51	315.35	40.48	458.56	54.23	678.87	40.72
	6	100.00	30.15	79438.84	35.32	356.97	40.61	458.56	54.27	681.77	40.56
	7	100.00	29.89	200.00	35.30	388.64	40.65	458.56	54.24	670.29	40.75
	8	100.00	30.25	12796764.55	35.33	311.86	40.50	458.56	54.29	679.78	40.75
	9	100.00	29.91	230.78	35.27	343.20	40.56	458.56	54.26	678.23	40.56
	10	100.00	30.19	651333.12	35.29	305.26	40.59	458.56	54.21	691.19	40.67
	Max	100.00	30.31	12796764.55	35.68	605.07	40.93	458.56	54.57	691.19	40.83
	Min	100.00	29.89	200.00	35.27	304.25	40.48	458.56	54.18	670.29	40.56
	Av	100.00	30.08	1377963.51	35.41	377.26	40.61	458.56	54.27	681.12	40.68
	Std	0.00	0.16	4017063.85	0.14	100.39	0.13	0.00	0.11	6.46	0.09

		functions									
		f6		f7		f8		f9		f10	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)s
D=30, DE/rand/1	1	600.00	101.60	901.95	69.62	976.79	167.40	900.00	250.69	6459.34	134.18
	2	600.00	101.53	896.30	68.84	969.39	167.26	900.00	261.11	7106.72	134.85
	3	600.00	101.51	915.65	68.83	980.78	167.14	900.00	255.33	7780.92	134.39
	4	600.00	101.29	910.90	68.84	995.38	166.25	900.00	257.64	8235.21	134.38
	5	600.00	101.53	917.12	68.83	977.75	166.75	900.00	264.97	8003.09	134.00
	6	600.00	101.37	908.73	68.98	977.86	166.92	900.00	267.78	7980.68	135.02
	7	600.00	101.48	878.65	68.78	980.06	167.08	900.00	269.44	8017.41	134.69
	8	600.00	101.23	906.94	68.94	995.96	166.41	900.00	252.02	7788.24	135.11
	9	600.00	101.53	904.80	68.91	984.71	166.87	900.00	267.60	7081.70	134.99
	10	600.00	101.40	887.95	68.87	976.52	166.80	900.00	262.11	7781.99	134.93
	Max	600.00	101.60	917.12	69.62	995.96	167.40	900.00	269.44	8235.21	135.11
	Min	600.00	101.23	878.65	68.78	969.39	166.25	900.00	250.69	6459.34	134.00
	Av	600.00	101.45	902.90	68.94	981.52	166.89	900.00	260.87	7623.53	134.65
	Std	0.00	0.12	12.20	0.25	8.40	0.36	0.00	6.73	557.17	0.39

		functions									
		f11		f12		f13		f14		f15	
D=30, DE/rand/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	1165.27	62.34	7498.08	80.89	1382.69	72.59	1486.04	76.72	1539.04	63.88
	2	1163.78	62.31	6381.91	79.33	1374.22	72.35	1470.47	76.38	1515.10	63.48
	3	1162.31	62.34	8400.54	80.37	1387.75	72.26	1477.07	76.39	1541.68	63.51
	4	1165.03	62.31	5582.18	80.17	1383.68	72.29	1478.08	76.42	1531.00	63.65
	5	1110.39	62.20	4304.94	80.09	1383.57	72.26	1480.01	76.42	1536.23	63.45
	6	1152.51	62.28	20835.57	80.36	1367.97	72.32	1473.31	76.36	1526.18	63.46
	7	1149.96	62.26	7200.41	80.31	1386.29	72.26	1476.07	76.46	1537.67	63.48
	8	1148.41	62.34	4288.11	80.15	1375.66	72.24	1477.90	76.47	1533.67	63.57
	9	1217.45	62.20	7220.30	80.36	1387.87	72.24	1475.78	76.39	1544.87	63.48
	10	1156.51	62.28	5874.43	79.73	1390.37	72.29	1476.55	76.42	1530.99	63.48
	Max	1217.45	62.34	20835.57	80.89	1390.37	72.59	1486.04	76.72	1544.87	63.88
	Min	1110.39	62.20	4288.11	79.33	1367.97	72.24	1470.47	76.36	1515.10	63.45
	Av	1159.16	62.28	7758.65	80.18	1382.01	72.31	1477.13	76.45	1533.64	63.54
	Std	26.10	0.05	4787.05	0.42	7.14	0.10	4.10	0.10	8.54	0.13

		functions									
		f16		f17		f18		f19		f20	
D=30, DE/rand/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	1671.03	111.26	1935.21	159.76	1834.49	68.94	1929.21	158.89	2049.32	370.71
	2	1803.49	109.40	1785.28	157.75	1837.57	69.11	1916.95	158.47	2054.29	373.22
	3	2923.32	108.92	1859.69	156.92	1837.60	68.72	1917.82	159.04	2055.71	378.46
	4	2049.86	109.08	1787.52	158.95	1834.33	68.53	1912.14	158.09	2030.82	376.52
	5	2043.31	108.97	1785.04	156.97	1841.21	68.69	1923.38	158.90	2062.83	368.37
	6	2526.64	108.95	1785.91	158.79	1842.45	68.72	1926.26	158.84	2065.46	366.93
	7	2767.31	109.19	1819.97	157.78	1836.74	68.64	1912.37	158.37	2050.76	358.02
	8	1630.18	109.51	1832.54	158.89	1836.50	68.69	1931.63	159.31	2052.77	361.86
	9	2202.61	109.14	1774.39	158.28	1834.43	68.75	1923.22	159.14	2054.48	366.76
	10	1889.31	108.48	1814.02	159.31	1822.94	68.58	1930.53	158.79	2049.02	372.87
	Max	2923.32	111.26	1935.21	159.76	1842.45	69.11	1931.63	159.31	2065.46	378.46
	Min	1630.18	108.48	1774.39	156.92	1822.94	68.53	1912.14	158.09	2030.82	358.02
	Av	2150.71	109.29	1817.96	158.34	1835.83	68.74	1922.35	158.78	2052.55	369.37
	Std	451.27	0.75	49.13	0.97	5.30	0.17	7.23	0.37	9.35	6.34

		functions									
		f21		f22		f23		f24		f25	
D=30, DE/rand/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	2465.47	382.48	2300.00	252.36	2825.21	289.40	2997.05	174.22	2886.74	214.03
	2	2462.26	393.15	2300.00	252.58	2817.31	289.38	2984.11	174.19	2886.71	213.80
	3	2471.26	401.36	2300.00	252.55	2821.87	288.73	2997.78	174.08	2886.79	213.72
	4	2453.07	393.61	8479.69	254.34	2821.48	289.46	3007.09	174.10	2886.74	213.80
	5	2472.83	386.88	2300.00	252.89	2831.63	289.40	3007.70	174.11	2886.75	213.91
	6	2473.99	389.10	2300.00	252.55	2840.99	290.07	2990.64	174.16	2886.75	214.20
	7	2463.58	386.70	2300.00	252.19	2821.95	289.97	2984.46	174.18	2886.71	214.80
	8	2485.34	392.67	2300.00	255.03	2832.04	289.27	2999.72	174.14	2886.77	214.45
	9	2460.42	402.34	2300.00	252.16	2840.11	289.51	2995.29	175.31	2886.79	214.39
	10	2476.23	389.82	2300.00	252.22	2829.52	289.66	3001.41	174.72	2886.75	214.22
	Max	2485.34	402.34	8479.69	255.03	2840.99	290.07	3007.70	175.31	2886.79	214.80
	Min	2453.07	382.48	2300.00	252.16	2817.31	288.73	2984.11	174.08	2886.71	213.72
	Av	2468.45	391.81	2917.97	252.89	2828.21	289.48	2996.53	174.32	2886.75	214.13
	Std	9.28	6.30	1954.19	0.99	8.07	0.37	8.22	0.39	0.03	0.35

		functions									
		f26		f27		f28		f29		f30	
D=30, DE/rand/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	5240.76	400.84	3193.45	413.04	3203.29	328.62	3901.91	366.99	5009.24	732.66
	2	5395.98	400.83	3186.28	409.58	3100.00	328.57	3397.93	367.18	5016.79	723.16
	3	5394.76	401.34	3181.54	410.53	3203.29	329.44	3406.37	368.15	5130.28	723.31
	4	5187.58	401.02	3199.55	411.87	3100.00	328.87	3393.40	366.57	5010.14	734.78
	5	5331.03	401.66	3203.86	411.00	3100.00	328.54	3419.69	366.84	5008.11	740.47
	6	5146.38	401.17	3203.83	410.05	3100.00	327.80	3373.84	366.24	5008.60	753.84
	7	5398.46	401.16	3191.00	411.75	3100.00	328.30	3374.34	365.56	5129.04	735.76
	8	5253.90	401.20	3196.88	412.12	3100.00	327.73	3420.06	366.84	5013.89	739.79
	9	5346.94	401.08	3196.27	413.95	3213.98	329.10	3404.94	366.35	5127.97	746.00
	10	5225.82	401.55	3195.70	409.57	3100.00	328.24	3405.92	365.96	5132.05	735.67
	Max	5398.46	401.66	3203.86	413.95	3213.98	329.44	3901.91	368.15	5132.05	753.84
	Min	5146.38	400.83	3181.54	409.57	3100.00	327.73	3373.84	365.56	5008.11	723.16
	Av	5292.16	401.18	3194.84	411.35	3132.05	328.52	3449.84	366.67	5058.61	736.54
	Std	92.98	0.27	7.14	1.47	51.69	0.54	159.64	0.72	61.36	9.36

Results illustrated in Table 6, for D=30, show that the optimal solutions are obtained in all 10 different runs for functions f1, f6 and f9. The standard deviation values for the functions mentioned above are 0 which means that the employed DE strategy is quite robust for the above listed 3 benchmark functions.

Considering the different benchmark functions f4, f13, f14, f15, f18, f19 and f20, there is an error less than 100 for the different 10 runs. Their standard deviation value is between 4.10 and 9.35 which is not important except for the function f4 which is equal to 0. This means that the algorithm still gives accurate results but it is stuck in the same local optimum for f4 that scored a cost of 458.56 and for the remaining functions it is stuck in different local optimum which have a close cost value.

The function f3 and f17 are special functions. Function f3 has a difference from the optimal solution sometimes less than 100 but sometimes it exceeds 300. For f17, its error in some runs is less than 100 and more than 200 in others. Their standard deviation is also important which is respectively equal to 49.13 and 100.39. It means that this DE strategy gets stuck in many different local optima.

For the functions f22, it is stuck in the local optimum with a cost 2300.00 for 9 runs and in the run 4 it obtains a very bad solution with a cost of 8479.69. Also the function f28 had the same local optimum for 7 runs which scores 3100, and for 2 runs it reached the same local optimum with a cost equal to 3203.29 and another local optimum which is not far from these different local optimum, its score is 3213.98. The standard deviation of the function f28 is acceptable with a value 51.69 but standard deviation of f22 is very important with a value 1954.19 due to the value obtained in run number 4.

The difference from the global optimum of these different benchmark functions which are f5, f7, f8, f11, f21, f25 and f27 is more than 100 and less than 500. This is acceptable for a big dimension such as 30. The standard deviation of function f25 is very close to 0 with a value 0.03 which means that the DE is stuck in the same local region. The standard deviation of the other functions is also not important, its value is between 6.46 and 26.10.

The functions f23 and f24 have a difference from the optimal solution which is bigger than 500 with a standard deviation slightly larger than 8. It can be inferred from this that the different obtained solutions are less accurate and the algorithm gets stuck in same local region.

The function f29 has an important variation between 473.84 and 1001.91in its error value. The majority of obtained values are different from the optimal solution by around 400. The standard deviation is 159.64 which is an important value.

It can be observed that benchmark functions f2, f10, f12, f26 and f30 have a big error value. For the function f2, the optimal solution is obtained 1 time and some solutions which are close to the optimal one. The standard deviation of f2 and f12 increased significantly. This is most noticeable in f2 which is equal to 4017063.85. The standard deviation of f10, f12 and f26 is not really very important the maximal scored value is equal to 557.17. The scored values shows that the above mentioned functions are difficult ones.

The CPU time of the different functions does not vary significantly. In most of the cases, the standard deviation is smaller than 1 while in other 5 cases (f9, f20, f21, f27 and f30), it varies between 1.47 and 9.36.

Table 7: DE Results for Dimension 30 and Variant Rand/2

		functions									
		f1		f2		f3		f4		f5	
D=30, DE/rand/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	4.94E+07	37.07	1.08E+23	38.39	72985.40	43.46	488.01	56.92	733.40	45.76
	2	3.62E+07	36.43	2.25E+24	38.50	84968.48	43.26	488.06	56.74	735.10	45.66
	3	2.97E+07	36.99	1.77E+23	38.55	62867.52	43.37	487.63	56.69	733.17	45.58
	4	9.55E+07	37.14	8.31E+23	38.42	80754.79	43.35	487.12	56.63	730.06	45.47
	5	5.44E+07	36.43	5.88E+23	38.44	82466.70	43.34	486.92	56.66	728.24	45.60
	6	7.12E+07	37.00	6.53E+22	38.36	66721.55	43.26	486.90	56.72	721.68	45.63
	7	5.83E+07	37.07	3.60E+21	38.49	66277.05	43.37	488.58	56.71	725.74	45.57
	8	3.77E+07	36.41	1.90E+24	38.45	87405.40	43.40	486.67	56.68	713.80	45.43
	9	3.53E+07	37.08	3.70E+23	38.38	99964.39	43.26	487.44	56.61	733.89	45.57
	10	5.32E+07	36.41	1.66E+23	38.49	85145.33	43.37	487.12	56.68	722.06	45.52
	Max	9.55E+07	37.14	2.25E+24	38.55	99964.39	43.46	488.58	56.92	735.10	45.76
	Min	2.97E+07	36.41	3.60E+21	38.36	62867.52	43.26	486.67	56.61	713.80	45.43
	Av	5.21E+07	36.80	6.46E+23	38.45	78955.66	43.34	487.44	56.70	727.72	45.58
	Std	1.98E+07	0.33	7.99E+23	0.06	11583.94	0.07	0.62	0.09	6.88	0.09

		functions									
		f6		f7		f8		f9		f10	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
D=30, DE/rand/2	1	610.61	103.58	990.69	72.07	1032.58	152.85	1310.46	113.35	7784.70	140.90
	2	614.19	103.23	958.30	72.09	1007.22	152.77	1408.28	113.30	7593.32	140.68
	3	613.51	103.40	953.69	71.95	1049.65	152.74	1417.81	113.33	8123.12	140.65
	4	617.09	103.27	969.58	71.98	1045.62	152.69	1241.87	113.32	8331.91	140.43
	5	613.90	103.26	986.06	71.99	988.50	152.77	1201.73	113.32	7842.91	140.65
	6	615.85	103.51	986.83	71.96	1030.79	152.65	1167.52	113.29	7923.71	140.56
	7	614.37	103.18	984.77	72.12	1050.69	152.82	1336.41	113.30	7892.86	140.57
	8	614.02	103.35	972.87	71.96	1021.72	152.76	1164.14	113.33	7664.08	140.54
	9	613.95	103.30	1017.08	71.99	1020.73	152.74	1356.08	113.27	7712.05	140.40
	10	615.87	103.37	966.55	72.01	987.45	152.79	1556.08	113.21	7286.08	140.68
	Max	617.09	103.58	1017.08	72.12	1050.69	152.85	1556.08	113.35	8331.91	140.90
	Min	610.61	103.18	953.69	71.95	987.45	152.65	1164.14	113.21	7286.08	140.40
	Av	614.33	103.34	978.64	72.01	1023.50	152.76	1316.04	113.30	7815.47	140.61
	Std	1.74	0.13	18.50	0.06	23.21	0.06	125.71	0.04	287.52	0.14

		functions									
		f11		f12		f13		f14		f15	
D=30, DE/rand/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	1233.30	66.04	3113812.07	84.22	1538.05	76.78	1514.18	78.23	1589.77	67.36
	2	1209.92	65.77	3479371.80	84.40	1489.27	76.60	1514.96	78.34	1599.25	67.19
	3	1196.80	65.72	3177374.32	84.01	1569.59	76.61	1521.13	78.30	1584.11	67.22
	4	1234.61	65.74	3040027.83	84.43	1470.64	76.75	1514.60	78.16	1593.05	67.22
	5	1192.98	65.83	2151456.43	84.47	1566.99	76.60	1522.41	78.17	1596.03	67.22
	6	1238.27	65.71	3485808.33	84.47	1501.99	76.61	1521.56	78.28	1604.12	67.22
	7	1194.05	65.66	3855257.76	84.52	1545.63	76.77	1515.54	78.19	1593.71	67.25
	8	1234.35	65.75	4302188.41	84.29	1503.83	76.57	1515.00	78.14	1584.70	67.31
	9	1202.96	65.82	4126962.06	84.04	1541.90	76.58	1520.21	78.27	1584.85	67.19
	10	1220.28	65.85	2544343.57	84.32	1536.49	76.71	1516.88	78.17	1587.11	67.25
	Max	1238.27	66.04	4302188.41	84.52	1569.59	76.78	1522.41	78.34	1604.12	67.36
	Min	1192.98	65.66	2151456.43	84.01	1470.64	76.57	1514.18	78.14	1584.11	67.19
	Av	1215.75	65.79	3327660.26	84.32	1526.44	76.66	1517.65	78.23	1591.67	67.24
	Std	18.51	0.10	671901.50	0.18	33.26	0.09	3.29	0.07	6.78	0.05

		functions									
		f16		f17		f18		f19		f20	
D=30, DE/rand/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	3170.53	113.13	2385.98	163.46	1869.46	70.86	1948.12	149.75	2297.58	155.95
	2	3209.82	112.90	2229.53	162.94	1884.54	70.50	1945.80	149.50	2395.00	156.97
	3	3273.81	112.85	2309.61	162.85	1885.01	70.39	1948.68	149.43	2582.03	156.03
	4	3168.76	112.60	2245.49	163.19	1883.75	70.47	1945.83	149.48	2123.32	155.72
	5	3057.91	112.91	2468.69	162.82	1872.13	70.45	1944.35	149.43	2133.81	155.74
	6	2824.91	112.96	2336.17	162.82	1880.68	70.43	1944.53	149.50	2171.24	155.70
	7	2739.58	112.82	2313.86	163.27	1895.98	70.45	1943.27	149.42	2714.31	156.08
	8	3300.97	113.33	2319.00	163.07	1888.67	70.47	1945.53	149.67	2128.08	156.45
	9	3115.09	113.02	2180.91	162.90	1868.26	70.54	1946.82	150.01	2724.80	155.88
	10	3210.80	112.74	2376.65	163.08	1882.63	70.47	1944.97	149.56	2630.92	156.00
	Max	3300.97	113.33	2468.69	163.46	1895.98	70.86	1948.68	150.01	2724.80	156.97
	Min	2739.58	112.60	2180.91	162.82	1868.26	70.39	1943.27	149.42	2123.32	155.70
	Av	3107.22	112.93	2316.59	163.04	1881.11	70.50	1945.79	149.57	2390.11	156.05
	Std	186.10	0.20	83.69	0.22	8.79	0.13	1.69	0.19	252.53	0.39

		functions									
		f21		f22		f23		f24		f25	
D=30, DE/rand/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	2494.29	158.67	2313.79	236.87	2858.09	667.78	3009.83	179.93	2888.24	212.99
	2	2511.73	158.37	9335.35	239.26	2858.46	650.80	3005.08	179.70	2888.67	212.52
	3	2510.14	158.65	9634.68	238.93	2862.19	645.59	3042.02	179.73	2888.61	212.77
	4	2509.71	158.47	8352.60	239.49	2871.55	629.32	3022.75	179.68	2887.69	212.64
	5	2505.29	158.39	9163.88	239.63	2861.80	653.36	3030.45	179.79	2887.98	212.66
	6	2512.35	158.33	2309.67	236.84	2869.41	640.42	3021.54	179.76	2888.25	212.74
	7	2523.43	158.93	2309.79	236.33	2874.75	669.88	3026.50	179.79	2887.89	212.68
	8	2508.99	158.45	9208.77	239.18	2862.42	659.10	3025.24	179.78	2888.15	212.63
	9	2519.23	158.51	9072.45	239.48	2882.99	644.03	3026.59	179.71	2887.57	212.71
	10	2525.15	158.36	9120.81	239.54	2867.00	647.47	3027.27	179.88	2887.80	212.57
	Max	2525.15	158.93	9634.68	239.63	2882.99	669.88	3042.02	179.93	2888.67	212.99
	Min	2494.29	158.33	2309.67	236.33	2858.09	629.32	3005.08	179.68	2887.57	212.52
	Av	2512.03	158.51	7082.18	238.56	2866.87	650.78	3023.73	179.78	2888.08	212.69
	Std	9.02	0.19	3307.72	1.32	7.93	12.39	10.31	0.08	0.37	0.13

		functions									
		f26		f27		f28		f29		f30	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
D=30, DE/rand/2	1	5764.07	411.28	3196.05	413.31	3218.73	333.36	4018.15	366.65	9505.09	803.94
	2	5633.78	410.74	3204.72	414.17	3227.27	333.23	4318.23	365.84	11705.77	791.13
	3	5908.68	411.19	3196.56	412.84	3221.30	333.16	3481.39	366.56	8444.74	810.19
	4	5808.43	410.61	3203.79	412.98	3215.53	333.23	3509.20	366.17	7785.26	811.84
	5	5736.90	410.86	3176.02	413.59	3214.48	332.94	4058.62	365.65	8976.45	810.27
	6	5915.61	410.77	3211.94	414.48	3223.01	333.12	4209.71	366.10	7362.01	832.56
	7	5790.84	410.91	3183.29	412.50	3225.50	333.20	4115.53	366.17	9109.79	821.38
	8	5765.09	410.66	3218.33	412.64	3216.25	333.12	3528.07	366.21	10030.97	821.50
	9	5967.78	410.86	3205.16	414.76	3220.78	333.16	4200.58	366.03	7794.00	821.19
	10	5834.15	410.77	3176.00	413.48	3219.61	333.19	4219.61	366.18	9307.26	820.19
	Max	5967.78	411.28	3218.33	414.76	3227.27	333.36	4318.23	366.65	11705.77	832.56
	Min	5633.78	410.61	3176.00	412.50	3214.48	332.94	3481.39	365.65	7362.01	791.13
	Av	5812.53	410.86	3197.19	413.47	3220.25	333.17	3965.91	366.15	9002.13	814.42
	Std	98.43	0.22	14.61	0.78	4.22	0.11	328.59	0.29	1276.61	11.55

Results illustrated in Table 7, for D=30, show that the optimal solutions are not obtained in any of the 10 different runs for any of the functions but there is a small difference which is less than 100 for the different benchmark functions f4, f5, f6, f15, f18 and f19. The standard deviation of the function f4 is equal to 0.62 and the standard deviation of f6 and f19 is close to 1. Also, the standard deviation of the function f5, f15 and f18 is between 6.78 and 8.79. These values mean that the DE strategy is robust but it is stuck in a specific local region.

DE showed an interesting behavior in the optimization of the function f11, in 3 runs, the error value is less than 100 but in the other 7 runs it is more than 100. Its standard deviation is 18.51 which means that DE was medially robust to reach some close optimal solutions but it lacks the capability to avoid the local optimum solutions.

Considering the different benchmark functions f7, f8, f9, f13, f14, f21, f23, f25 and f28, their error cost solution values are more than 100 and less or around 500. They have an important variation of the standard deviation value. The standard deviation of f25 is very small with a value equal to 0.37, this means that the obtained solutions are very close to each other and the algorithm is looking to a solution in a specific region. Also, for functions f14, f21 and f23, their standard deviation value do not exceed 10 but for the remaining functions it fluctuates and increases until it reaches a value of 125.71. This implies that it is hard to locate its optimal solution and the employed DE strategy was not successful in getting out of locally optimal solutions.

The function f26 has a difference of around 600 from the global optimum for the 10 different runs and its standard deviation is equal to 10.31. The value is not really important and it shows that the DE is stuck in a local region.

For the functions f17 and f20, their solutions cost error is between 480.91 and 768.69, and between 123.32 and 724.80 respectively. The standard deviations are respectively equal to 83.69 and 252.53, it shows that the algorithm is losing its accuracy and it does not escape from the different local optimal solutions.

The results obtained for the functions f1, f2, f3, f10, f12, f26, f29 and f30 show an important error value which vary from a function to another. The standard deviation of f1, f2, f3 and f12 is huge. This means that they are hard functions but the standard deviation of the other functions do not exceed 1276.61, which means that DE cannot escape local optimal solutions but with a bad accuracy.

The CPU time of the different functions does not vary significantly with a standard deviation smaller than 1 except for 3 of them which are f22, f23 and f30 with respective standard deviation values of 1.32, 12.39 and 11.55. This also shows that the variation is not really important.

Table 8: DE Results for Dimension 30 and Variant Best/2

		functions									
		f1		f2		f3		f4		f5	
D=30, DE/best/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	100.00	1375.94	200.00	2000.54	300.00	2338.74	458.56	3744.51	735.19	2494.52
	2	100.00	1378.27	200.00	1998.03	300.00	2334.93	458.56	3742.51	675.79	2467.90
	3	100.00	1380.87	200.00	1999.54	300.00	2337.19	458.56	3741.72	694.26	2484.27
	4	100.00	1379.17	200.00	1999.42	300.00	2338.67	464.12	3730.22	707.45	2488.61
	5	100.00	1375.90	200.00	1998.59	300.00	2335.24	403.99	3719.97	690.51	2497.53
	6	100.00	1379.47	200.00	1998.61	300.00	2337.25	458.56	3745.47	717.89	2484.80
	7	100.00	1377.68	200.00	1999.32	300.00	2336.22	458.56	3735.96	679.41	2479.76
	8	100.00	1380.11	200.00	1998.65	300.00	2335.58	458.56	3738.08	716.01	2487.55
	9	100.00	1375.60	200.00	1999.54	300.00	2333.79	400.00	3747.83	702.39	2493.32
	10	100.00	1375.93	200.00	1999.46	300.00	2338.35	403.99	3738.11	689.08	2473.63
	Max	100.00	1380.87	200.00	2000.54	300.00	2338.74	464.12	3747.83	735.19	2497.53
	Min	100.00	1375.60	200.00	1998.03	300.00	2333.79	400.00	3719.97	675.79	2467.90
	Av	100.00	1377.89	200.00	1999.17	300.00	2336.60	442.35	3738.44	700.80	2485.19
	Std	0.00	1.97	0.00	0.71	0.00	1.71	27.46	8.28	18.62	9.34

		functions									
		f6		f7		f8		f9		f10	
D=30, DE/best/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	600.04	8345.99	917.05	5284.72	978.90	13681.30	902.82	9070.18	7645.32	12037.74
	2	603.61	8339.58	918.16	5279.29	1016.15	13638.82	904.45	9072.66	8168.86	12012.79
	3	603.40	8330.53	942.83	5287.39	1008.38	13637.64	900.00	9086.47	3882.29	12124.30
	4	602.29	8335.60	954.25	5286.11	1018.77	13691.41	906.63	9077.03	7950.93	12254.00
	5	601.45	8324.81	949.78	5286.92	1015.33	13665.86	905.73	9067.26	8038.75	11919.90
	6	600.09	8347.82	946.91	5279.32	982.67	13683.36	904.36	9070.12	8339.39	12105.32
	7	600.02	8348.78	914.18	5285.39	1001.67	13694.94	907.64	9074.56	4551.64	12297.25
	8	604.46	8335.91	963.61	5278.04	1011.49	13601.76	900.63	9078.40	6948.66	11990.99
	9	600.78	8357.91	909.42	5284.36	1005.45	13653.89	901.18	9079.74	8047.26	11947.76
	10	600.31	8322.93	929.87	5279.15	1016.27	13612.02	902.27	9076.62	7412.08	12339.66
	Max	604.46	8357.91	963.61	5287.39	1018.77	13694.94	907.64	9086.47	8339.39	12339.66
	Min	600.02	8322.93	909.42	5278.04	978.90	13601.76	900.00	9067.26	3882.29	11919.90
	Av	601.65	8338.99	934.61	5283.07	1005.51	13656.10	903.57	9075.30	7098.52	12102.97
	Std	1.68	11.22	19.23	3.67	14.10	32.95	2.61	5.62	1579.26	148.91

		f11		f12		f13		f14		f15	
		functions									
D=30, DE/best/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	1253.17	12011.52	9988.30	6427.62	2545.56	6184.53	1533.41	5826.25	2674.71	4739.17
	2	1245.97	12292.07	2780.43	6411.52	1394.44	6208.32	1490.02	5821.52	1591.84	4750.85
	3	1282.94	12026.37	7963.08	6493.99	7819.69	6209.57	1637.04	5818.31	1839.68	4729.20
	4	1116.43	12011.97	45912.73	6580.14	11233.58	6175.21	1617.48	5830.88	1608.82	4752.71
	5	1291.80	12327.75	22166.89	6518.22	1420.70	6206.08	1768.59	5812.24	1822.86	4729.20
	6	1250.24	12047.54	8584.48	6551.26	3611.03	6172.40	1660.68	5813.91	1717.48	4729.12
	7	1233.61	12028.72	14112.98	6607.55	1758.40	6188.67	1481.79	5824.69	1606.56	4754.58
	8	1247.25	12018.22	43564.41	6492.78	2603.66	6203.44	1534.99	5825.47	1713.65	4750.34
	9	1292.39	12009.78	35903.55	6485.98	2282.86	6172.29	1654.27	5830.91	1696.73	4735.18
	10	1179.68	12211.54	6047.08	6390.14	2219.75	6174.30	1515.93	5836.18	1714.33	4751.54
	Max	1292.39	12327.75	45912.73	6607.55	11233.58	6209.57	1768.59	5836.18	2674.71	4754.58
	Min	1116.43	12009.78	2780.43	6390.14	1394.44	6172.29	1481.79	5812.24	1591.84	4729.12
	Av	1239.35	12098.55	19702.39	6495.92	3688.97	6189.48	1589.42	5824.04	1798.67	4742.19
	Std	54.35	126.86	16278.05	71.65	3244.62	15.90	92.74	7.67	319.16	10.85

		functions									
		f16		f17		f18		f19		f20	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
D=30, DE/best/2	1	2519.93	9631.13	1993.72	14195.83	1830.50	5063.45	2016.80	13437.44	2322.82	12507.15
	2	2959.46	9241.70	1773.08	14285.34	1838.07	5060.44	1930.14	13429.53	2742.56	12802.03
	3	2386.66	9403.09	1934.10	14308.47	1919.82	5064.71	1982.06	13422.54	2062.07	12699.06
	4	2430.22	9309.56	2007.91	14215.48	1878.87	5071.73	2099.01	13395.09	2688.83	12749.10
	5	1982.59	9393.71	1935.07	14224.22	1935.45	5064.07	1979.46	13435.45	2226.87	12653.87
	6	2910.68	9386.49	2109.02	14188.52	1889.23	5070.50	2072.12	13420.10	2184.54	12863.42
	7	1968.52	9405.27	1842.57	14210.38	1829.72	5066.38	1918.99	13430.02	2059.92	12653.29
	8	1763.98	9318.64	2236.59	14186.95	1827.14	5060.94	1940.35	13417.05	2103.55	12648.97
	9	2383.72	9309.36	2231.69	14394.29	1850.64	5062.19	2024.20	13434.70	2251.95	12654.65
	10	3101.27	9485.24	2061.47	14302.30	2107.46	5068.89	2050.41	13427.97	2224.35	12757.37
	Max	3101.27	9631.13	2236.59	14394.29	2107.46	5071.73	2099.01	13437.44	2742.56	12863.42
	Min	1763.98	9241.70	1773.08	14186.95	1827.14	5060.44	1918.99	13395.09	2059.92	12507.15
	Av	2440.70	9388.42	2012.52	14251.18	1890.69	5065.33	2001.35	13424.99	2286.75	12698.89
	Std	450.88	109.38	152.27	68.58	85.46	3.95	61.59	12.47	241.61	99.43

D=30, DE/best/2		functions									
		f21		f22		f23		f24		f25	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	2525.80	14467.11	8403.05	22258.07	2834.41	26641.45	3029.80	15692.78	2887.19	18922.19
	2	2517.58	14497.39	9238.27	21950.92	2842.84	26761.75	3018.58	15728.96	2886.98	18921.89
	3	2489.60	14478.39	9124.28	22117.71	2820.20	26677.70	2886.36	15648.96	2886.96	18871.80
	4	2515.28	14457.77	8680.71	22098.29	2709.83	26856.18	3002.40	15670.13	2888.66	18919.21
	5	2498.03	14474.27	2300.00	21707.10	2842.66	26775.68	3089.80	15698.07	2887.40	18876.14
	6	2481.21	14472.35	9126.99	22122.41	2813.10	26946.74	2991.32	15717.43	2887.09	18920.92
	7	2466.54	14484.90	2303.01	21579.42	2707.66	26872.28	3001.46	15706.40	2886.98	18895.53
	8	2469.93	14467.22	4994.44	22325.10	2867.22	27090.42	3005.00	15692.51	2886.89	18922.17
	9	2516.53	14483.51	2300.00	21692.58	2714.87	26837.41	3022.51	15688.93	2888.11	18936.59
	10	2497.08	14444.66	2303.40	21632.96	2816.72	26767.60	3030.16	15715.92	2886.97	18919.29
	Max	2525.80	14497.39	9238.27	22325.10	2867.22	27090.42	3089.80	15728.96	2888.66	18936.59
	Min	2466.54	14444.66	2300.00	21579.42	2707.66	26641.45	2886.36	15648.96	2886.89	18871.80
	Av	2497.76	14472.76	5877.42	21948.45	2796.95	26822.72	3007.74	15696.01	2887.32	18910.57
	Std	20.89	14.83	3309.25	274.58	61.49	130.68	50.70	23.59	0.59	21.72

		functions									
		f26		f27		f28		f29		f30	
D=30, DE/best/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	5523.56	39447.11	3220.87	38709.65	3213.98	31325.17	3487.32	35076.90	5266.92	29112.80
	2	4035.61	39629.65	3231.90	38926.91	3100.00	31150.31	3485.16	34895.54	6367.55	29084.83
	3	5365.48	39907.24	3209.69	39238.51	3203.29	31452.67	3481.18	35339.55	5665.50	29171.59
	4	5743.44	39561.60	3225.84	39318.69	3240.13	31541.16	3717.71	35103.08	5381.55	29126.67
	5	5441.85	39637.55	3227.22	39135.11	3266.64	31577.07	3439.52	35063.95	5016.29	29127.15
	6	5548.23	39879.38	3208.92	39160.17	3253.93	31444.08	3410.11	35144.75	5003.15	29148.65
	7	5821.16	39681.54	3207.28	39228.18	3100.00	31408.12	3358.29	35374.97	5456.24	29206.18
	8	5637.63	39831.84	3217.46	39342.17	3100.00	31402.56	3706.81	35317.67	5266.67	29177.13
	9	4447.15	39618.92	3246.93	39458.22	3100.00	31395.58	3626.31	35009.28	5127.60	29222.68
	10	5490.72	39351.86	3212.69	39385.06	3253.93	31520.33	3540.32	35113.10	6121.04	29225.84
	Max	5821.16	39907.24	3246.93	39458.22	3266.64	31577.07	3717.71	35374.97	6367.55	29225.84
	Min	4035.61	39351.86	3207.28	38709.65	3100.00	31150.31	3358.29	34895.54	5003.15	29084.83
	Av	5305.48	39654.67	3220.88	39190.27	3183.19	31421.70	3525.27	35143.88	5467.25	29160.35
	Std	585.03	180.13	12.46	225.43	73.97	121.93	121.98	154.62	459.38	48.33

Results illustrated in Table 8, for D=30, show that the optimal solutions are obtained in all 10 different runs for functions f1, f2 and f3. The standard deviation values for the functions mentioned above are 0, implying that the employed DE strategy is quite robust for the above listed 3 benchmark functions.

For the 2 functions f6 and f9, the error value is very small and one global optimal solution is obtained in the experiment number 3 of f9. Their standard deviations are enough small with the values 1.68 and 2.61 which shows that the algorithm was accurate enough even though it cannot escape from a specific local region.

The function f4 cost value has a difference of less than 100 from the global optimum cost. In experiment number 9, the optimal solution is obtained and in runs 5 and 10, a very close optimal solution was obtained with a cost value equal to 403.99. In most of the other runs, a specific solution cost of 458.56 is obtained. The standard deviation value is trivial with a value of 27.46. According to the obtained results, the DE is still stuck in majority of runs in a deep local optimum solution but in other cases it reaches other diverse local optimum solutions.

Considering the different benchmark functions f11, f14, f18 and f19 the error value is little bit above 100 and little bit below 100 for the different 10 experiments except for the run 10 of f18 the difference from the global optimum reached 307.46. The standard deviation value for the 4 functions mentioned above increased importantly and becomes between 54.35 and 92.74 that means the DE is felling down in different local optimum solutions.

An error value between 100 and 500 is obtained for the different benchmark functions f5, f8, f15, f21 and f28 with an interesting variation from a function to another. The function f8 is showing an interesting result. For example, in run 1, the error value is 1174 which is important, but in run 2 the error value decreased significantly to 91.84. For function f28, the cost value of 3100.00 is obtained 4 times. The standard deviation value for f5, f8, and f21 ranges between 14.10 and 20.89 but it increased a little bit for f28 to reach 73.97 and increased importantly for f15 to reach 319.16, implying that DE is resulting in different local optimum solutions.

For the function f7 the difference from the optimal cost is around 200, for f25 it is close to 400 and for f27 it is around 500. These functions are not very difficult functions. The standard deviation for f25 is smaller than 1, which means that the obtained solutions are very close to each other and DE is stuck in a local optimum. For the other functions, the standard deviation values are 12.46 and 19.23, which means that there are different local optimal solutions.

For the different functions f17, f20, f24 and f29, there is an important fluctuation between the results of the 10 different runs. Also, the standard deviation value increased significantly and reached a minimal value equal to 50.70 in f24 and a maximal one of 241.61 in f20. This shows that it was hard for the DE strategy to locate the global optimum and escape from the local optimal solutions.

The remaining benchmark functions which are f10, f12, f13, f16, f22, f26 and f30 are hard enough to ensure an inability of the DE strategy to detect the global optimum or to even find an acceptable local optimal solution. Exceptions exist for f16 in run 5, 7 and 8 and for f22 in run 5, 7, 9 and 10. The standard deviation values for f16, 26 and

30 are 450.88, 585.03 and 459.38 respectively, which is noticeably significant but for the remaining functions, it increased significantly until it exceeds 1500 and even reaches 16278.05 in f12.

There is a negligible variation of the CPU time for the benchmark functions f2. The CPU time variation increased slightly for f1, f3, f4, f5, f6, f7, f9, f14, f15, f18, f19 and f21. Also, it increased more for f8, f12, f13, f17, f24, f25 and f30. For the remaining functions the variation is important.

Table 9: DE Results for Dimension 30 and Variant Rand to Best/1

D=30, DE/rand to best/1

	functions									
	f1		f2		f3		f4		f5	
Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
1	100.00	1396.40	4.15E+08	1975.11	393.84	2349.87	458.56	3735.24	680.91	2533.13
2	100.00	1395.99	1.07E+07	1974.86	445.71	2352.56	458.56	3737.80	691.54	2535.67
3	100.00	1396.63	6.79E+05	1975.88	420.65	2352.25	458.56	3736.07	690.85	2533.74
4	100.00	1394.91	3.84E+12	1974.11	353.69	2352.04	458.56	3732.51	678.92	2532.57
5	100.00	1396.90	1.10E+08	1974.05	320.98	2350.36	458.56	3741.26	681.51	2528.79
6	100.00	1397.24	2.55E+07	1974.47	522.20	2355.66	458.56	3733.51	680.68	2526.59
7	100.00	1396.44	3.70E+10	1974.13	369.17	2348.70	458.56	3736.54	684.40	2537.54
8	100.00	1395.87	8.32E+06	1974.33	416.51	2355.21	458.56	3735.93	676.84	2531.40
9	100.00	1397.44	5.31E+05	1975.55	503.53	2355.69	458.56	3735.52	653.12	2532.43
10	100.00	1395.97	8.73E+08	1975.28	462.79	2348.56	458.56	3733.81	675.92	2536.26
Max	100.00	1397.44	3.84E+12	1975.88	522.20	2355.69	458.56	3741.26	691.54	2537.54
Min	100.00	1394.91	5.31E+05	1974.05	320.98	2348.56	458.56	3732.51	653.12	2526.59
Av	100.00	1396.38	3.87E+11	1974.78	420.91	2352.09	458.56	3735.82	679.47	2532.81
Std	0.00	0.74	1.21E+12	0.65	64.45	2.74	0.00	2.47	10.65	3.34

		functions									
		f6		f7		f8		f9		f10	
D=30, DE/rand to best/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	600.00	8991.35	922.93	5287.09	993.88	13440.53	900.00	9568.51	7257.64	12245.56
	2	600.00	8990.63	918.63	5277.17	970.87	13435.40	900.00	9554.03	7808.46	12141.51
	3	600.00	8990.93	916.27	5285.72	964.97	13389.49	900.00	9555.30	7460.83	12147.11
	4	600.00	8986.36	913.83	5277.17	969.47	13419.08	900.00	9554.91	7299.97	12188.70
	5	600.00	9007.03	921.33	5267.59	967.52	13468.32	900.00	9554.17	8006.63	12236.95
	6	600.00	8996.44	901.49	5269.71	984.11	13478.14	900.00	9558.34	8034.05	12081.50
	7	600.00	8999.28	914.47	5266.70	986.11	13430.69	900.00	9551.38	7194.27	12177.16
	8	600.00	8996.17	921.68	5271.93	964.91	13444.45	900.00	9554.76	8121.71	12144.46
	9	600.00	8993.19	903.58	5267.17	977.22	13515.46	900.00	9557.68	7901.49	12189.01
	10	600.00	8994.67	899.18	5274.52	980.18	13421.19	900.00	9555.20	8300.29	12130.04
	Max	600.00	9007.03	922.93	5287.09	993.88	13515.46	900.00	9568.51	8300.29	12245.56
	Min	600.00	8986.36	899.18	5266.70	964.91	13389.49	900.00	9551.38	7194.27	12081.50
	Av	600.00	8994.61	913.34	5274.48	975.92	13444.27	900.00	9556.43	7738.53	12168.20
	Std	0.00	5.69	8.82	7.38	9.96	35.34	0.00	4.66	401.38	49.82

		functions									
		f11		f12		f13		f14		f15	
D=30, DE/rand to best/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	1109.77	4605.71	3453.14	6409.58	1381.43	5646.38	1474.13	5854.95	1543.88	4831.66
	2	1213.20	4626.21	13462.36	6408.96	1383.28	5645.79	1487.21	5848.01	1535.70	4832.43
	3	1118.86	4609.33	5620.99	6430.56	1383.58	5643.69	1478.87	5846.45	1540.95	4831.97
	4	1142.99	4616.37	4497.47	6390.41	1386.67	5649.11	1486.74	5848.74	1536.95	4832.97
	5	1151.28	4614.90	29437.22	6415.28	1383.39	5649.94	1487.64	5838.09	1540.05	4833.43
	6	1157.79	4612.39	11668.92	6473.01	1373.23	5638.97	1480.67	5854.45	1539.88	4827.31
	7	1155.12	4609.61	10959.20	6434.00	1379.41	5635.46	1485.21	5855.06	1535.37	4831.29
	8	1216.56	4617.02	9245.66	6434.00	1375.80	5651.25	1475.53	5852.38	1539.13	4829.60
	9	1144.77	4605.66	30324.40	6395.37	1389.12	5643.29	1483.86	5847.01	1537.15	4837.48
	10	1145.56	4609.91	7439.20	6424.07	1389.71	5648.34	1481.94	5852.07	1543.62	4829.40
	Max	1216.56	4626.21	30324.40	6473.01	1389.71	5651.25	1487.64	5855.06	1543.88	4837.48
	Min	1109.77	4605.66	3453.14	6390.41	1373.23	5635.46	1474.13	5838.09	1535.37	4827.31
	Av	1155.59	4612.71	12610.86	6421.52	1382.56	5645.22	1482.18	5849.72	1539.27	4831.76
	Std	34.78	6.21	9648.58	23.67	5.35	5.01	4.83	5.25	3.02	2.74

		functions									
		f16		f17		f18		f19		f20	
D=30, DE/rand to best/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	2825.68	22019.00	1709.23	2347.72	1836.80	5082.86	1920.90	14293.98	2049.23	13058.13
	2	2821.32	22218.65	1708.58	2352.89	1837.70	5086.79	1921.29	14292.34	2074.87	13197.64
	3	2086.26	22082.86	1700.63	2380.95	1835.27	5080.38	1927.66	14324.59	2052.64	13070.34
	4	1982.50	22963.35	1702.20	2377.64	1837.28	5079.99	1924.87	14350.81	2056.37	13153.24
	5	2420.71	22178.05	1701.06	2357.67	1841.32	5081.05	1931.55	14361.92	2058.60	13044.15
	6	2483.58	22351.79	1717.19	2354.99	1836.47	5082.25	1922.52	14353.81	2063.62	13107.84
	7	2404.25	22159.65	1701.17	2351.23	1840.06	5086.05	1922.62	14360.80	2053.83	13160.32
	8	2541.16	21940.79	1700.31	2348.39	1835.23	5084.24	1931.12	14368.69	2060.56	13064.54
	9	2558.99	21697.41	1701.26	2383.85	1841.11	5083.74	1923.26	14364.24	2057.44	13089.25
	10	1855.61	21657.67	1700.63	2356.99	1840.21	5083.53	1928.94	14369.70	2051.58	13029.55
	Max	2825.68	22963.35	1717.19	2383.85	1841.32	5086.79	1931.55	14369.70	2074.87	13197.64
	Min	1855.61	21657.67	1700.31	2347.72	1835.23	5079.99	1920.90	14292.34	2049.23	13029.55
	Av	2398.01	22126.92	1704.23	2361.23	1838.15	5083.09	1925.47	14344.09	2057.87	13097.50
	Std	330.05	367.45	5.63	13.97	2.34	2.27	4.03	29.76	7.38	55.87

		functions									
		f21		f22		f23		f24		f25	
D=30, DE/rand to best/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	2452.06	13305.17	2300.00	21591.85	2826.88	26662.30	2985.38	16071.22	2886.76	19930.81
	2	2452.36	13298.99	2300.00	21613.67	2834.95	26632.37	3006.66	16074.16	2886.76	19922.05
	3	2457.58	13303.64	2300.00	21677.26	2826.95	26660.80	2970.65	16124.53	2886.75	19972.93
	4	2453.40	13360.08	2300.00	21672.77	2841.29	26656.86	2989.32	16118.62	2886.77	19980.23
	5	2475.05	13351.72	9162.92	21656.95	2841.19	26582.80	2999.37	16107.98	2886.77	19975.51
	6	2465.92	13333.45	2300.00	21596.09	2832.32	26636.61	2972.59	16116.98	2886.78	19944.48
	7	2474.17	13347.76	2300.00	21610.79	2831.35	26615.80	2986.42	16072.21	2886.75	19972.09
	8	2462.32	13344.39	2300.00	21596.08	2813.79	26616.92	2994.77	16098.01	2886.74	19978.22
	9	2452.37	13360.16	8281.83	21965.52	2821.11	26808.79	2979.11	16091.99	2886.76	19988.44
	10	2468.60	13349.24	2300.00	21599.04	2841.41	26724.06	2978.25	16090.41	2886.71	19972.84
	Max	2475.05	13360.16	9162.92	21965.52	2841.41	26808.79	3006.66	16124.53	2886.78	19988.44
	Min	2452.06	13298.99	2300.00	21591.85	2813.79	26582.80	2970.65	16071.22	2886.71	19922.05
	Av	2461.38	13335.46	3584.47	21658.00	2831.12	26659.73	2986.25	16096.61	2886.75	19963.76
	Std	9.14	23.96	2715.86	112.98	9.21	64.41	11.63	20.04	0.02	22.76

		functions									
		f26		f27		f28		f29		f30	
D=30, DE/rand to best/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	5242.64	38690.42	3195.12	40033.49	3100.00	30985.36	3633.88	34953.65	5105.64	29007.72
	2	5110.71	38821.43	3208.93	40051.48	3100.00	31032.12	3804.82	34943.77	5030.81	29069.85
	3	5213.22	38813.05	3202.64	40182.82	3100.00	30993.83	3459.11	35001.88	5043.04	29112.47
	4	5459.83	38826.70	3201.92	40293.08	3100.00	31015.73	3402.64	35123.50	5132.22	29095.79
	5	5009.89	38877.20	3196.52	40223.25	3100.00	30998.10	3479.50	35054.55	5011.81	29082.91
	6	5330.84	38832.38	3191.81	40326.65	3100.00	30971.07	3422.90	34916.91	5050.78	29078.65
	7	5267.29	38787.53	3182.32	39853.87	3203.29	31026.15	3448.33	34852.11	5008.54	29083.92
	8	5289.85	38782.77	3193.66	40183.67	3100.00	31017.68	3396.71	34985.03	5042.34	29083.16
	9	5225.08	38884.87	3202.57	39986.57	3100.00	30984.86	3592.20	34935.00	5017.92	29093.67
	10	5279.56	38824.05	3194.97	39855.34	3100.00	31012.73	3412.41	34954.75	5019.52	29115.73
	Max	5459.83	38884.87	3208.93	40326.65	3203.29	31032.12	3804.82	35123.50	5132.22	29115.73
	Min	5009.89	38690.42	3182.32	39853.87	3100.00	30971.07	3396.71	34852.11	5008.54	29007.72
	Av	5242.89	38814.04	3197.05	40099.02	3110.33	31003.76	3505.25	34972.12	5046.26	29082.39
	Std	121.06	54.39	7.38	169.24	32.66	20.05	132.33	75.31	41.27	29.95

Results illustrated in Table 9, for D=30, show that the optimal solutions are obtained in all 10 different runs for functions f1, f6 and f9. The standard deviation values for the functions mentioned above are 0, which means that the employed DE strategy is quite robust for the above listed 3 benchmark functions.

Considering the different benchmark functions f4, f13, f14, f15, f17, f18, f19 and f20, the difference of their cost solutions value from the global optimum cost is smaller than 100. For the function f4 the standard deviation value is 0, it means that the DE strategy is stuck in a deep local optimum and for the other functions their standard deviation value is between 2.34 and 7.38, which is not a significant value. It shows that the DE strategy is giving solutions which belong to the same local region from the global search space.

The function f22 is a special function due to the fact that it is giving a constant cost solution value of 2300.00 for 8 runs but for 2 runs the DE strategy is giving 2 solutions with an error value bigger than 6000. The standard deviation is equal to 9162.92, which is a big value. It is showing the big difference of the 2 solutions from 2300.00 which is the cost of a deep local optimum. Also, the function f11 has an error sometimes less than 100 while at other times, exceeds 100. The standard deviation is not really big with a value equal to 24.78. It is showing that in some cases the DE strategy is good enough to stay in the local region of the global optimum.

The function f3 is showing an interesting instability in the rand to best/1 strategy which sometimes result solutions with a cost which has a difference of less than 100, more than 100 and around 200. With a standard deviation that is equal to 64.45, it can be inferred that the DE strategy was not enough good to avoid the different local optimal solutions.

The difference of the cost of the different obtained solutions from the global optimum cost of f5 and f8 is little bit more than 100. Also the error value of f7 slightly exceeds 200, except in run 10 when it is very close to 200. The standard deviation values of the 3 mentioned functions are equal to 10.65, 8.82 and 9.96. These values are not big but they show that DE could not escape from the different local solutions which have a close cost.

The error value of f21, f25 and f28 slightly exceeds 300. The standard deviation of f25 is very close to 0 which means that DE is stuck in a deep local optimum. Also the function f28 is similar but its standard value is 32.66 due to the different value in run 7 from the 9 other runs.

For the functions f23, f24 and f27, the error value is around 500. There is an exception for f24 in run 2, the cost value exceeds the optimal cost with little bit more than 600. The standard deviation 9.21, 11.63 and 7.38 shows the weakness from the strategy to avoid local optimum solutions.

The variation between the different solutions obtained in the 10 runs of the benchmark functions f16 and f29 increased significantly. Also their standard deviation values becomes equal to 330.05 and 132.33, meaning that there is an important variation between the different obtained local solutions.

The DE strategy gives a very bad performance for the remaining functions due to their hardness. Their standard deviation values are very high with a maximal value in the excess of millions, which display the important instability of the strategy except for f10, f26 and f30 with respective standard deviation values equal to 401.38, 121.06 and 41.27. These values display that the DE strategy cannot avoid the different local optimal solutions.

There is a negligible variation of the CPU time for the benchmark functions f1and f2. The CPU time variation increased slightly for f3, f4, f5, f6, f7, f9, f11, f13, f14, f15, f17 and f18. For f10, f12, f19, f20, f21, f23, f24, f25, f26, f28, f29 and f30, the variation increased and for the remaining functions it becomes significant.

Two approaches can be used to compare the different strategies, using the CPU time to have an idea of the fastest DE strategy, and using the cost solution value to have an idea of the most accurate one.

From a point of view of the response time, according to results illustrated in Table 6, Table 7, Table 8 and Table 9, for D=30, for the benchmark functions f1, f3, f5, f6, f10, f14, f15, f16, f18, f24 and f27, the fastest strategy is rand/1. In second position there is rand/2, while in third position there is best/2, and finally, in fourth position there is rand to best/1. The CPU average values of each function according to methods are respectively for rand/1 equal to 30.01, 40.61, 40.68, 101.45, 134.65, 76.45, 63.54, 109.29, 174.32 and 411.35. For rand/2, the CPU averages are 36.80, 43.34, 45.58, 103.34, 140.61, 78.23, 67.24, 112.93, 70.50, 179.78 and 413.47. For best/2, they reach 1377.89, 2336.60, 2485.19, 8338.99, 12102.97, 5824.04, 4742.19, 9388.42, 5065.33, 15696.01 and 39190.27 and for rand to best/1 they score 1396.38, 2352.09, 2532.81, 8994.61, 12.168.20, 5849.72, 4831.76, 22126.92, 5083.09, 16096.61 and 40099.02.

Also, the strategy rand/1 is the fastest DE strategy for the following benchmark functions f2, f4, f7, f11, f12, f13, f17, f23, f26, f28 and f30 with a corresponding CPU average values equal to 35.41, 54.27, 68.94, 62.28, 80.18, 72.31, 158.34, 289.48, 401.18, 328.52 and 366.67. After this is the rand/2 strategy with CPU average values of 38.45, 56.70, 72.01, 65.79, 84.32, 76.66, 163.04, 650.78, 410.86, 333.17 and 814.42. Then, there is rand to best/1 and its CPU average values are (1974.78, 3735.82, 5274.48, 4612.71, 6421.52, 5645.22, 2361.23, 26659.73, 38814.04, 31003.76 and 29082.39. Finally, the slowest strategy is best/2 with CPU average values equal to 1999.17, 3738.44, 5283.07, 12098.55, 6495.92, 6189.48, 14251.18, 26822.72, 39654.67, 31421.70 and 29160.35.

According to the response time, the fastest strategy is rand/2. In second order, there is rand/1. In third order, there is the strategy best/2 and the slowest one is rand to best/1 for 4 benchmark functions. For each function the CPU average is attributed in the same order of strategies mentioned above. For f9, the average values are equal to 113.30, 260.87, 9075.30 and 9556.43. For f19 they are equal to 149.57, 158.78, 13424.99 and 14344.09. For f20 they score 156.05, 369.37, 12698.89 and 13097.50 and for f25, the obtained CPU average values are 212.69, 214.13, 18910.57 and 19963.76.

Also, the strategy rand/2 is the fastest one for the functions f8, f21, f22 and f29 with CPU average values of 152.76, 158.51, 238.56 and 366.15. The rand/1 is less fast with the CPU average values equal to 166.89, 391.81, 252.89 and 366.67. The rand to best/1 strategy comes in third position with CPU average values equal to 13444.27, 13335.46, 21658.00 and 34972.12. The worst strategy among the 4 used strategies is best/2 and it scored CPU averages of 13656.10, 14472.76, 21948.45 and 35143.88 for the 4 mentioned functions.

In some functions the different DE strategies were 100% accurate, except rand/2 which never gave an optimal cost average. For the benchmark function f1, the 3 strategies rand/1, best/2 and rand to best/1 results in an optimal solution for the 10 different runs except rand/2 which gives an average of 52094585.36. Also, for the function f2, the strategy best/2 gives an optimal average for both functions f2 and f3. In second position, there is rand/2 with a cost averages of 1377963.51 and 377.26. The strategy rand to best/1 is positioned in third. Its cost averages are equal to 3.87E+11 and 1974.78. The worst strategy is rand/2 with a cost averages of 6.46E+23 and 78955.66 which are very far from the optimal solution values. However the optimal solution is reached in the 10 runs by rand/1 and rand to best/1 in f6 and f9, the two strategies best/2 and rand/2 did not reach the optimal average. The best/2 strategy is better than rand/2 with cost averages equal to 601.65 and 903.57 and the cost average scored by rand/2 are 614.33 and 1316.04.

Considering the benchmark functions f7, f12, f13, f14, f15, f16, f18, f19, f20, f22, f27, f29 and f30, the strategy rand/1 was able to score the best cost averages with respective values 902.90, 7758.65, 1382.01, 1477.13, 1533.64, 2150.71, 1835.83, 1922.35, 2052.55, 2917.97, 3194.84, 3449.84 and 5058.61. The strategy rand to best/1 is the next best accurate one with cost averages equal to 913.34, 12610.86, 1382.56, 1482.18, 1539.27, 2398.01, 1838.15, 1925.47, 2057.87, 3584.47, 3197.05, 3505.25 and 5046.26. For the functions f7, f12, f16, f20, f22, f29 and f30, the most third accurate strategy is best/2 with cost strategies values of 934.61, 19702.39, 2440.70, 2286.75, 5877.42, 3525.27 and 5467.25. The least accurate strategy is rand/2 which scores 978.64, 3327660.26, 3107.22, 2390.11, 7082.18, 3965.91 and 9002.13, but for the functions f13, f14, f15, f18, f19 and f27, it is opposite. The strategy rand/2 with averages 1526.44, 1517.65, 1591.67, 1881.11, 1945.79 and 3197.19 is better than best/2.

The strategy rand to best/1 is the best strategy from an accuracy point of view and the second-best strategy is rand/1 for the different functions f5, f8, f11, f17, f21, f24 and f26. The rand to best/1 strategy has cost averages equal to 679.47, 975.92, 1155.59, 1704.23, 2461.38, 2986.25 and 5242.89 and the rand/1 strategy scores a cost average with different values equal to 681.12, 981.52, 1159.16, 1817.96, 2468.45, 2996.53 and 5292.16. For all the different functions with an exception of function f11, the strategy best/2 is worse than the already mentioned strategies but it still better than rand/2. The cost average values of best/2 are equal to 700.80, 1005.51, 1239.35, 2012.52, 2497.76, 3007.74 and 5305.48, and rand/2 strategy scores 727.72, 1023.50, 1215.75, 2316.59, 2512.03, 3023.73 and 5812.53. Also, for f28, rand to best/1 strategy is the most accurate with a cost average equal to 3110.33. The next strategy is best/2 with an average equal to 3183.19, after it is rand/1 with an average of 3132.05 while the worst one is rand/2 which scores 3220.25.

For the function f25, rand/1 and rand to best/1 are in the top of the DE strategies with a cost average of 2886.75. The direct less accurate strategy is best/2 with an average of 2887.32 and the worst strategy is rand/2 with an average value equal to 2888.08.

The best/2 strategy also is the best for a few number of functions which are f4, f10 and f23, with the respective scores 442.35, 7098.52 and 2796.95. Both strategies rand/1 and rand to best/1 are in second position with an average of 458.56 for f4. For f10 and f23, the best second strategy is rand/1 with averages of 7623.53 and 2828.21. In third position there is rand to best/1 which scores 7738.53 and 2831.12 and the worst strategy most of the time is rand/2 with the average values equal to 487.44, 7815.47 and 2866.87.

Generally, the best DE strategy is rand/1 which is the faster for 22 benchmark functions and in the second position is the strategy rand/2, it is the fastest for 8 functions. For both strategies best/2 and rand to best/1, they are the worst ones.

The strategy rand/1 is the most accurate strategy for 17 functions, rand to best/1 is the best one for 12 functions and best/2 is the best for a small number of functions which is equal to 3 functions. The strategy rand/2 is the worst strategy in almost functions.

The results obtained for the different strategies showed that rand/1 is the best strategy in accuracy and speed since it is faster most times. The strategy rand to best/1 was more accurate than rand/2 but it was much slower. The strategy best/2 could not really give good results from the point of view of accuracy and fastness.

Table 10: DE+QRSM Results for Dimension 10 and Variant Rand/1

		functions									
		f1		f2		f3		f4		f5	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
D=10, QRSM,DE/rand/1	1	100.00	11.22	4.57E+18	10.83	5.23E+08	11.06	3010.03	10.98	612.72	10.73
	2	100.00	10.86	4.57E+18	10.26	1.04E+07	10.59	3010.03	10.25	612.72	10.08
	3	100.00	10.90	4.57E+18	10.30	1.04E+07	10.47	3010.03	10.23	612.72	10.05
	4	100.00	10.87	4.57E+18	10.28	1.04E+07	10.50	3010.03	10.28	612.72	10.06
	5	100.00	10.87	4.57E+18	10.34	2.05E+08	10.51	3010.03	10.28	612.72	10.06
	6	100.00	10.84	4.57E+18	10.41	1.04E+07	10.59	3010.03	10.23	612.72	10.08
	7	100.00	10.83	4.57E+18	10.28	5.23E+08	10.42	3010.03	10.22	612.72	10.14
	8	100.00	10.78	4.57E+18	10.28	1.04E+07	10.48	3010.03	10.22	612.72	10.05
	9	100.00	10.83	4.57E+18	10.30	1.04E+07	10.50	3010.03	10.22	612.72	10.05
	10	100.00	10.78	4.57E+18	10.28	1.04E+07	10.47	3010.03	10.30	612.72	10.08
	Max	100.00	11.22	4.57E+18	10.83	5.23E+08	11.06	3010.03	10.98	612.72	10.73
	Min	100.00	10.78	4.57E+18	10.26	1.04E+07	10.42	3010.03	10.22	612.72	10.05
	Av	100.00	10.88	4.57E+18	10.36	1.32E+08	10.56	3010.03	10.32	612.72	10.14
	Std	0.00	0.13	1.79E+09	0.17	2.15E+08	0.18	0.00	0.23	0.00	0.21

		functions									
		f6		f7		f8		f9		f10	
D=10, QRSM,DE/rand/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	694.68	10.53	808.41	10.59	934.98	10.72	11908.42	11.19	4287.83	10.98
	2	694.68	10.19	808.41	10.12	934.98	10.03	11908.43	10.64	4287.83	10.12
	3	694.68	10.05	808.41	9.98	934.98	10.00	11908.43	10.72	5151.64	10.20
	4	694.68	10.03	808.41	10.09	934.98	10.00	11908.43	10.59	4287.83	10.11
	5	694.68	10.00	808.41	9.98	934.98	10.03	11908.43	10.61	4464.19	10.11
	6	694.68	10.05	808.41	9.97	934.98	10.00	11908.43	10.59	5151.64	10.23
	7	694.68	10.20	808.41	9.98	934.98	10.11	11908.43	10.61	4287.83	10.28
	8	711.02	10.20	808.41	9.97	934.98	10.03	11908.43	10.59	4287.83	10.14
	9	694.68	10.17	808.41	10.28	934.98	9.98	11908.43	10.59	5151.64	10.30
	10	694.68	10.11	808.41	10.41	934.98	10.00	11908.43	10.59	5151.64	10.25
	Max	711.02	10.53	808.41	10.59	934.98	10.72	11908.43	11.19	5151.64	10.98
	Min	694.68	10.00	808.41	9.97	934.98	9.98	11908.42	10.59	4287.83	10.11
	Av	696.31	10.15	808.41	10.14	934.98	10.09	11908.43	10.67	4650.99	10.27
	Std	5.17	0.15	0.00	0.22	0.00	0.22	0.00	0.18	434.22	0.26

		functions									
		f11		f12		f13		f14		f15	
D=10, QRSM,DE/rand/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	322181.36	10.89	2508.90	11.11	2398.99	10.97	1647.05	10.90	8240.88	11.11
	2	322181.32	10.37	2508.90	10.86	2398.98	10.30	1647.07	10.22	8240.87	10.61
	3	563754.68	10.31	2508.90	10.76	2398.97	10.26	1647.02	10.37	8240.84	10.53
	4	322181.31	10.26	2508.90	10.83	2398.98	10.23	1647.06	10.30	8240.96	10.62
	5	322181.36	10.26	2508.90	10.84	2398.99	10.23	1647.09	10.26	8240.96	10.61
	6	408912.46	10.26	2508.90	10.73	2398.97	10.25	1647.10	10.28	8240.96	10.59
	7	322181.33	10.26	2508.90	10.69	2398.99	10.34	1647.18	10.28	8240.94	10.62
	8	408912.45	10.33	2508.90	10.67	2398.98	10.26	1647.04	10.34	8240.98	10.61
	9	322181.32	10.26	2508.90	10.70	2398.98	10.25	1647.13	10.25	8240.79	10.61
	10	322181.30	10.25	2508.90	10.70	2398.98	10.26	1647.12	10.26	8240.89	10.61
	Max	563754.68	10.89	2508.90	11.11	2398.99	10.97	1647.18	10.90	8240.98	11.11
	Min	322181.30	10.25	2508.90	10.67	2398.97	10.23	1647.02	10.22	8240.79	10.53
	Av	363684.89	10.35	2508.90	10.79	2398.98	10.34	1647.09	10.35	8240.91	10.65
	Std	79005.49	0.19	0.00	0.13	0.01	0.22	0.05	0.20	0.06	0.16

		functions									
		f16		f17		f18		f19		f20	
D=10, QRSM,DE/rand/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	3397.53	10.70	2787.14	10.69	1835.48	11.23	5.79E+06	10.92	3296.83	10.58
	2	3397.53	10.26	2787.14	10.14	1835.81	10.65	2.00E+07	10.30	3296.83	10.14
	3	3397.53	10.23	2787.14	10.12	1834.93	10.69	2.00E+07	10.36	3296.83	10.08
	4	3397.53	10.19	2787.14	10.23	1834.99	10.69	2.00E+07	10.48	3296.83	10.06
	5	3397.53	10.19	2787.14	10.11	1835.37	10.69	2.00E+07	10.37	3296.83	10.11
	6	3397.53	10.28	2787.14	10.12	1835.37	10.69	5.79E+06	10.25	3296.83	10.14
	7	3397.53	10.19	2787.14	10.12	1835.19	10.69	2.00E+07	10.26	3296.83	10.08
	8	3397.53	10.19	2787.14	10.11	1835.39	10.67	2.00E+07	10.34	3296.83	10.06
	9	3397.53	10.17	2787.14	10.19	1835.04	10.67	5.79E+06	10.23	3296.83	10.03
	10	3397.53	10.17	2787.14	10.19	1834.62	10.67	2.00E+07	10.33	3296.83	10.09
	Max	3397.53	10.70	2787.14	10.69	1835.81	11.23	2.00E+07	10.92	3296.83	10.58
	Min	3397.53	10.17	2787.14	10.11	1834.62	10.65	5.79E+06	10.23	3296.83	10.03
	Av	3397.53	10.26	2787.14	10.20	1835.22	10.73	1.57E+07	10.38	3296.83	10.14
	Std	0.00	0.16	0.00	0.17	0.34	0.18	6.86E+06	0.20	0.00	0.16

D=10, QRSM,DE/rand/1	functions									
	f21		f22		f23		f24		f25	
Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
1	2735.68	11.06	5619.90	11.50	2995.75	11.12	3053.78	10.95	4078.22	11.04
2	2735.68	10.41	5619.90	10.83	2995.75	10.51	3053.78	10.36	4078.22	10.30
3	2735.68	10.41	5619.90	10.72	2995.75	10.42	3053.78	10.25	4078.22	10.33
4	2735.68	10.39	5619.90	10.70	2995.75	10.44	3053.78	10.22	4078.22	10.34
5	2735.68	10.41	5619.90	10.70	2995.75	10.42	3053.78	10.22	4078.22	10.31
6	2735.68	10.51	5619.90	10.70	2995.75	10.42	3053.78	10.20	4078.22	10.33
7	2735.68	10.37	5619.90	10.70	2995.75	10.51	3053.78	10.26	4078.22	10.44
8	2735.68	10.39	5619.90	10.70	2995.75	10.42	3053.78	10.36	4078.22	10.31
9	2735.68	10.41	5619.90	10.69	2995.75	10.41	3053.78	10.22	4078.22	10.33
10	2735.68	10.41	5619.90	10.70	2995.75	10.44	3053.78	10.20	4078.22	10.33
Max	2735.68	11.06	5619.90	11.50	2995.75	11.12	3053.78	10.95	4078.22	11.04
Min	2735.68	10.37	5619.90	10.69	2995.75	10.41	3053.78	10.20	4078.22	10.30
Av	2735.68	10.48	5619.90	10.79	2995.75	10.51	3053.78	10.32	4078.22	10.41
Std	0.00	0.21	0.00	0.25	0.00	0.22	0.00	0.23	0.00	0.23

		functions									
		f26		f27		f28		f29		f30	
D=10, QRSM,DE/rand/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	6233.08	10.84	3467.12	11.22	3834.09	11.15	8524.22	10.73	3.29E+09	11.23
	2	6233.08	10.34	3467.12	10.59	3834.09	10.53	8524.22	10.25	3.29E+09	10.59
	3	6233.08	10.26	3467.12	10.59	3834.09	10.55	9874.56	10.44	3.29E+09	10.61
	4	6233.08	10.23	3467.12	10.61	3834.09	10.42	8524.22	10.26	3.29E+09	10.61
	5	6233.08	10.26	3467.12	10.59	3834.09	10.42	8524.22	10.14	3.29E+09	10.59
	6	6233.08	10.30	3467.12	10.62	3834.09	10.41	9874.56	10.26	3.29E+09	10.61
	7	6233.08	10.25	3467.12	10.59	3834.09	10.41	9874.56	10.26	3.29E+09	10.61
	8	6233.08	10.33	3467.12	10.62	3834.09	10.41	9874.56	10.25	3.29E+09	10.58
	9	6233.08	10.25	3467.12	10.56	3834.09	10.50	8524.22	10.12	3.29E+09	10.61
	10	6233.08	10.25	3467.12	10.62	3834.09	10.47	8524.22	10.23	3.29E+09	10.59
	Max	6233.08	10.84	3467.12	11.22	3834.09	11.15	9874.56	10.73	3.29E+09	11.23
	Min	6233.08	10.23	3467.12	10.56	3834.09	10.41	8524.22	10.12	3.29E+09	10.58
	Av	6233.08	10.33	3467.12	10.66	3834.09	10.53	9064.36	10.30	3.29E+09	10.66
	Std	0.00	0.18	0.00	0.20	0.00	0.23	697.31	0.17	21.99	0.20

Results illustrated in Table 10, for a DE algorithm using the QRSM model with a problem dimension D=10 show that the optimal solutions are obtained in all 10 runs for the benchmark function f1. The standard deviation is equal to 0 meaning that the DE/rand/1 is a robust algorithm.

For the two benchmark functions f6 and f18, the DE strategy gives enough good results with an error value which does not exceed 100. The standard deviation for f6 is 5.16 according to this value and the solution values obtained in the 10 different runs the DE strategy is still accurate but it gets stuck in a deep local optimum for 9 runs. Also, the standard deviation for f18 is 0.34, which shows that there is not a big variation between the different obtained solutions, and the DE strategy is stuck in a specific local region.

A difference in the range]100, 200[exists between the solution cost value and the optimal solution cost for the 3 benchmark functions f5, f7 and f8. The standard deviation for the different functions mentioned above is equal to 0. this display the weakness of the strategy of avoiding a deep local optimum.

For f14, there is a difference from the optimal cost of only 247, and the standard deviation is equal to 0.05, which is caused by a deep local optimum where the DE strategy gets stuck even the small noticed variation is still negligible.

An error value bounded between 600 and 800 is obtained by the used DE strategy for the different benchmark functions f21, f23, f24 and f27. The standard deviation obtained for the 4 different functions is equal to 0 because the algorithm is stuck in a specific and unique local optimum.

Considering the 2 benchmark functions f17 and f28, their cost difference from the optimal solution is around 1000. Also, their standard deviation is equal to 0, which means that the used strategy is again stuck in a local optimum.

For the benchmark functions f12, f13, f16, f20 and f25, the error value is about 1000 and some hundreds. Their standard deviation is 0 for all of them except for f13 with a value of 0.01 which is very close to 0, that means that the strategy is stuck in one and only one local optimum.

The error value varies around thousands for the different benchmark functions f4, f9, f10, f15, f22, f26 and f29. The standard deviation of f4, f9, f22 and f26 is 0 and for f15 it is almost 0. It means that the DE strategy is stuck more in a specific local optimum. For the remaining functions, it increases significantly and becomes around hundreds and thousands which shows that the strategy lost its robustness and accuracy and get blocked in different local optimums, even if it is giving in the majority of runs a specific local optimum.

The error value increased very importantly and reached millions for the functions f2, f3, f19 and f30, which are very hard functions. The standard deviation is very big for f2, f13 and f19, that is showing the failure of the strategy to detect the global optimum and to escape from the different local optimum solutions. For f30, the standard deviation is small with a value equal to 21.99, which shows that DE using QRSM is searching for a solution in a specific local region.

There is a negligible variation in the CPU time for the all test functions with a standard deviation less than 1.

Table 11: DE+QRSM Results for Dimension 10 and Variant Rand/2

		functions									
		f1		f2		f3		f4		f5	
D=10, QRSM,DE/rand/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	100.00	11.75	3.88E+17	11.92	3.73E+06	12.11	1946.84	11.64	616.92	11.58
	2	100.00	11.25	3.88E+17	11.20	2.07E+05	11.58	1946.84	11.12	616.92	10.92
	3	100.00	11.22	3.88E+17	11.20	3.73E+06	11.50	1946.84	11.01	616.92	10.89
	4	100.00	11.17	3.88E+17	11.20	1.98E+07	11.54	1946.84	11.01	616.92	11.01
	5	100.00	11.17	3.88E+17	11.20	2.07E+05	11.54	1946.84	11.00	616.92	10.92
	6	100.00	11.19	9.17E+17	11.15	1.98E+07	11.54	1946.84	11.01	616.92	10.90
	7	100.00	11.17	3.88E+17	11.29	1.98E+07	11.51	1946.84	10.98	616.92	10.89
	8	100.00	11.26	3.88E+17	11.20	2.07E+05	11.54	1946.84	11.14	616.92	10.89
	9	100.00	11.19	3.88E+17	11.15	2.07E+05	11.51	1946.84	11.03	616.92	10.98
	10	100.00	11.15	3.88E+17	11.15	2.07E+05	11.54	1946.84	11.01	616.92	10.92
	Max	100.00	11.75	9.17E+17	11.92	1.98E+07	12.11	1946.84	11.64	616.92	11.58
	Min	100.00	11.15	3.88E+17	11.15	2.07E+05	11.50	1946.84	10.98	616.92	10.89
	Av	100.00	11.25	4.41E+17	11.27	6.77E+06	11.59	1946.84	11.10	616.92	10.99
	Std	0.00	0.18	1.67E+17	0.23	9.06E+06	0.18	0.00	0.20	0.00	0.21

		functions									
		f6		f7		f8		f9		f10	
D=10, QRSM,DE/rand/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	801.36	11.67	815.36	11.51	907.36	12.51	7142.78	11.84	3626.80	11.67
	2	801.36	11.03	815.36	10.92	907.36	11.89	7142.78	11.20	5252.83	11.22
	3	801.36	10.98	815.36	10.94	907.36	11.93	7142.78	11.31	5252.83	11.09
	4	801.36	11.01	815.36	10.94	907.36	11.84	7142.78	11.22	5252.83	11.15
	5	801.36	11.14	815.36	10.95	907.36	11.87	7142.78	11.22	5252.83	11.09
	6	801.36	10.98	815.36	11.00	907.36	11.89	7142.78	11.25	3626.80	11.03
	7	801.36	11.03	815.36	10.95	907.36	11.76	7142.78	11.23	3626.80	11.08
	8	801.36	11.03	815.36	10.90	907.36	11.79	7142.78	11.33	5252.83	11.20
	9	801.36	11.03	815.36	10.89	907.36	11.75	7142.78	11.22	3626.80	11.08
	10	801.36	10.98	815.36	10.94	907.36	11.78	7142.78	11.20	3626.80	11.04
	Max	801.36	11.67	815.36	11.51	907.36	12.51	7142.78	11.84	5252.83	11.67
	Min	801.36	10.98	815.36	10.89	907.36	11.75	7142.78	11.20	3626.80	11.03
	Av	801.36	11.09	815.36	10.99	907.36	11.90	7142.78	11.30	4439.82	11.16
	Std	0.00	0.21	0.00	0.18	0.00	0.22	0.00	0.19	856.99	0.19

		functions									
		f11		f12		f13		f14		f15	
D=10, QRSM,DE/rand/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	1.32E+06	12.42	1860.91	11.82	2080.59	12.45	1659.60	11.70	4289.48	11.93
	2	1.32E+06	11.70	1861.43	11.08	2080.54	11.67	1659.62	11.29	4289.47	11.26
	3	1.32E+06	11.65	1860.20	11.09	2080.54	11.67	1659.57	11.09	4289.46	11.29
	4	1.32E+06	11.65	1862.06	11.19	2080.54	11.64	1659.52	11.11	4289.43	11.25
	5	1.32E+06	11.68	1861.66	11.06	2080.55	11.65	1659.58	11.09	4289.41	11.28
	6	1.32E+06	11.67	1859.12	11.09	2080.53	11.65	1659.54	11.09	4289.49	11.34
	7	1.32E+06	11.65	1860.80	11.06	2080.51	11.65	1659.43	11.19	4289.44	11.25
	8	1.32E+06	11.65	1860.46	11.09	2080.55	11.65	1659.55	11.08	4289.46	11.26
	9	1.32E+06	11.65	1861.25	11.06	2080.57	11.65	1659.38	11.09	4289.49	11.26
	10	1.32E+06	11.67	1860.09	11.19	2080.51	11.67	1659.62	11.09	4289.51	11.25
	Max	1.32E+06	12.42	1862.06	11.82	2080.59	12.45	1659.62	11.70	4289.51	11.93
	Min	1.32E+06	11.65	1859.12	11.06	2080.51	11.64	1659.38	11.08	4289.41	11.25
	Av	1.32E+06	11.74	1860.80	11.17	2080.54	11.74	1659.54	11.18	4289.46	11.34
	Std	0.08	0.24	0.87	0.23	0.02	0.25	0.08	0.19	0.03	0.21

		functions									
		f16		f17		f18		f19		f20	
D=10, QRSM,DE/rand/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	3380.09	12.26	3162.89	11.67	1859.87	11.97	11627050.53	12.29	3332.92	11.75
	2	3601.40	11.64	3162.89	11.15	1859.83	11.19	11627032.77	11.78	3332.92	11.22
	3	3601.40	11.67	3162.89	11.17	1859.80	11.15	11627044.44	11.65	3332.92	11.08
	4	3601.40	11.65	3162.89	11.06	1859.82	11.15	11627054.02	11.62	3332.92	11.09
	5	3601.40	11.67	3162.89	11.14	1859.81	11.17	11627047.59	11.68	3332.92	11.11
	6	3601.40	11.67	3162.89	11.03	1859.85	11.22	11627043.56	11.61	3332.92	11.12
	7	3601.40	11.67	3162.89	11.03	1859.82	11.20	8177.65	11.67	3332.92	11.22
	8	3601.40	11.64	3162.89	11.03	1859.85	11.17	11627035.84	11.67	3332.92	11.06
	9	3601.40	11.65	3162.89	11.03	1859.83	11.17	11627051.28	11.64	3332.92	11.09
	10	3601.40	11.67	3162.89	11.03	1859.84	11.17	1374781.76	11.53	3332.92	11.09
	Max	3601.40	12.26	3162.89	11.67	1859.87	11.97	11627054.02	12.29	3332.92	11.75
	Min	3380.09	11.64	3162.89	11.03	1859.80	11.15	8177.65	11.53	3332.92	11.06
	Av	3579.27	11.72	3162.89	11.13	1859.83	11.26	9439931.94	11.71	3332.92	11.18
	Std	69.98	0.19	0.00	0.20	0.02	0.25	4622076.79	0.21	0.00	0.21

		functions									
		f21		f22		f23		f24		f25	
D=10, QRSM,DE/rand/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	2554.75	12.01	8073.18	12.34	2873.43	11.89	3026.32	11.76	3823.59	12.28
	2	2554.75	11.29	8073.18	11.70	2873.43	11.25	3005.33	11.29	3823.59	11.62
	3	2554.75	11.31	8073.18	11.68	2873.43	11.19	3026.32	11.11	3823.59	11.61
	4	2554.75	11.29	8073.18	11.68	2873.43	11.22	3026.32	11.14	3823.59	11.64
	5	2554.75	11.31	8073.18	11.68	2873.43	11.26	3005.33	11.14	3823.59	11.59
	6	2615.49	11.33	8073.18	11.70	2873.43	11.25	3026.32	11.12	3823.59	11.61
	7	2554.75	11.31	8073.18	11.70	2873.43	11.22	3026.32	11.09	3823.59	11.61
	8	2554.75	11.31	8073.18	11.67	2873.43	11.20	3026.32	11.11	3823.59	11.61
	9	2554.75	11.26	8073.18	11.68	2873.43	11.23	3026.32	11.09	3823.59	11.61
	10	2554.75	11.29	8073.18	11.68	2873.43	11.19	3026.32	11.22	3823.59	11.61
	Max	2615.49	12.01	8073.18	12.34	2873.43	11.89	3026.32	11.76	3823.59	12.28
	Min	2554.75	11.26	8073.18	11.67	2873.43	11.19	3005.33	11.09	3823.59	11.59
	Av	2560.82	11.37	8073.18	11.75	2873.43	11.29	3022.12	11.21	3823.59	11.68
	Std	19.21	0.23	0.00	0.21	0.00	0.21	8.85	0.21	0.00	0.21

		functions									
		f26		f27		f28		f29		f30	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
D=10, QRSM,DE/rand/2	1	6414.66	11.82	3171.47	11.72	3977.09	12.31	5921.52	11.97	9.38E+09	11.93
	2	6414.66	11.17	3171.47	11.23	3977.09	11.67	5921.52	11.14	9.38E+09	11.20
	3	7001.86	11.20	3171.47	11.19	3977.09	11.68	5921.52	11.12	9.38E+09	11.23
	4	7001.86	11.20	3171.47	11.19	3977.09	11.68	5921.52	11.11	9.38E+09	11.19
	5	7001.86	11.33	3171.47	11.29	3977.09	11.67	10510.63	11.17	9.38E+09	11.20
	6	7001.86	11.19	3171.47	11.22	3977.09	11.67	5921.52	11.23	9.38E+09	11.23
	7	6414.66	11.20	3171.47	11.19	3977.09	11.68	5921.52	11.11	9.38E+09	11.23
	8	7001.86	11.20	3171.47	11.17	3977.09	11.68	5921.52	11.09	9.38E+09	11.15
	9	7001.86	11.20	3171.47	11.19	3977.09	11.68	4531.35	11.25	9.38E+09	11.15
	10	6414.66	11.29	3171.47	11.26	3977.09	11.68	5921.52	11.14	9.38E+09	11.20
	Max	7001.86	11.82	3171.47	11.72	3977.09	12.31	10510.63	11.97	9.38E+09	11.93
	Min	6414.66	11.17	3171.47	11.17	3977.09	11.67	4531.35	11.09	9.38E+09	11.15
	Av	6766.98	11.28	3171.47	11.26	3977.09	11.74	6241.41	11.23	9.38E+09	11.27
	Std	303.23	0.20	0.00	0.16	0.00	0.20	1562.38	0.26	104.80	0.23

Results illustrated in Table 11, for a DE algorithm using the QRSM model with a problem dimension D=10, show that the optimal solutions are obtained in all 10 runs for the benchmark function f1. The standard deviation is equal to 0, an indication that the DE/rand/2 is a robust algorithm.

For the function f18, the error value is around 59 and the standard deviation is very close to 0. It means that the DE strategy is still accurate but it cannot escape from a deep local optimum.

The difference from the global optimum cost is more than 100 and less than 300 for the different functions f5, f6, f7, f8 and f14. All the different functions have a standard deviation equal to 0 except f14, which has a standard deviation of 0.08. These different values show that the DE/strategy with QRSM still giving good results but it is stuck in a deep local optimum.

The error value is included in the range of 400 and 800 for the functions f12, f13, f21, f23, f24 and f27. Their standard deviation varies from a function to another. The standard deviation of f23 and f27 is equal to 0, that means the algorithm is blocked in a deep local optimum. The standard deviation of f12 is close to 1 and the standard deviation of f13 is close to 0 which means that there are very small variations in the obtained solutions and the DE strategy is resulting in the same local solution with a very small variation. The standard deviation of f21 is more important with a value of 19.21. This is resulting from the only different solution in the 10 runs which is run number 6 but the others gave same local optimum. Also, the standard deviation of f24 is not really important, its value is 8.85, due to that, the DE strategy is still stuck in the same local optimal solution for 8 runs and obtained another local optimum for 2 runs.

One thousand and some hundreds is the difference from the global optimum cost obtained by the DE strategy using QRSM for the functions f4, f16, f17, f20, f25, and f28. The standard deviation of f16 is equal to 69.98. For 9 runs, the same cost is obtained for f16 except in run number 1. The standard deviation of other functions is 0. The obtained values show that the DE strategy cannot escape from the deep local optimum.

Considering the different functions f9, f10, f15, f22, f26 and f29, their obtained costs are different from the global optimum costs with only thousands. The standard deviation is 0 or very close to 0 for f9, f15 and f22, this means that the strategy is resulting in the same local optimum solution, but for the remaining functions it is giving different local optimums.

For the function f3, an error value of thousands and millions is obtained, while for f19, the error value is about millions except for run 6. The standard value for both functions is very high, which shows that the DE strategy is resulting in different local optimal solutions.

For the different benchmark functions, f2, f11, f30 are very hard functions with their error values getting to millions. The standard deviation of f11 is very close to 0 with a value 0.08 that means the DE strategy is stuck in a deep local optimum. For the remaining function, the standard deviation becomes very high and shows that the resulted local optimal solutions are different from each other.

The CPU time for the all the benchmark functions vary very slightly with a standard deviation less than 1.

Table 12: DE+QRSM Results for Dimension 10 and Variant Best/2

		functions									
		f1		f2		f3		f4		f5	
D=10, QRSM,DE/best/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	100.00	749.15	2.99E+17	743.52	6.22E+08	751.22	4454.79	740.01	602.50	709.80
	2	100.00	749.07	9.23E+18	741.29	3.87E+05	746.82	4454.79	739.62	602.50	710.18
	3	100.00	750.36	2.99E+17	742.52	6.22E+08	748.87	4454.79	737.37	602.50	710.80
	4	100.00	749.66	2.69E+19	749.93	6.22E+08	744.34	4454.79	737.23	602.50	710.68
	5	100.00	749.68	2.69E+19	750.66	6.22E+08	747.12	4454.79	737.79	602.50	710.23
	6	100.00	749.10	2.99E+17	740.02	1.03E+07	740.90	4454.79	739.15	602.50	709.52
	7	100.00	748.59	1.48E+17	737.12	3.87E+05	745.20	4454.79	739.74	602.50	709.60
	8	100.00	750.21	2.99E+17	741.13	1.03E+07	738.81	4454.79	738.82	602.50	709.37
	9	100.00	748.84	9.23E+18	741.46	7.15E+08	757.10	4454.79	738.27	602.50	709.41
	10	100.00	750.01	2.99E+17	740.19	1.03E+07	741.63	4454.79	738.68	602.50	709.84
	Max	100.00	750.36	2.69E+19	750.66	7.15E+08	757.10	4454.79	740.01	602.50	710.80
	Min	100.00	748.59	1.48E+17	737.12	3.87E+05	738.81	4454.79	737.23	602.50	709.37
	Av	100.00	749.47	7.39E+18	742.78	3.24E+08	746.20	4454.79	738.67	602.50	709.94
	Std	0.00	0.61	1.09E+19	4.31	3.36E+08	5.38	0.00	0.99	0.00	0.51

		functions									
		f6		f7		f8		f9		f10	
D=10, QRSM,DE/best/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	810.18	738.90	803.04	728.67	910.41	709.09	6928.18	767.98	5852.57	724.44
	2	810.18	736.22	803.04	727.76	910.41	708.51	6928.18	766.53	4745.46	725.81
	3	810.18	740.35	803.04	727.84	910.41	707.89	6928.18	767.98	4745.46	725.33
	4	930.96	735.67	803.04	727.81	910.41	707.79	6928.18	765.48	4822.45	746.23
	5	810.18	739.38	803.04	726.79	910.41	707.65	6928.18	765.31	4653.32	752.00
	6	930.96	739.07	803.04	727.99	910.41	708.40	6928.18	766.14	4653.32	744.50
	7	810.18	739.82	803.04	726.89	910.41	708.26	6928.18	764.01	5011.30	730.79
	8	810.18	734.92	803.04	727.74	910.41	708.68	6928.18	767.28	5852.57	725.44
	9	753.90	737.43	803.04	727.98	910.41	708.65	6928.18	763.48	5011.30	727.50
	10	810.18	736.81	803.04	727.53	910.41	708.07	6928.18	765.70	4745.46	724.92
	Max	930.96	740.35	803.04	728.67	910.41	709.09	6928.18	767.98	5852.57	752.00
	Min	753.90	734.92	803.04	726.79	910.41	707.65	6928.18	763.48	4653.32	724.44
	Av	828.71	737.86	803.04	727.70	910.41	708.30	6928.18	765.99	5009.32	732.69
	Std	56.68	1.90	0.00	0.54	0.00	0.45	0.00	1.52	461.81	10.59

		functions									
		f11		f12		f13		f14		f15	
D=10, QRSM,DE/best/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	8.73E+04	751.50	2490.69	793.11	2490.81	823.73	1622.92	1805.62	4104.91	2086.58
	2	1.27E+05	737.29	2490.85	792.53	2490.77	825.79	1773.94	1832.78	4104.90	2083.92
	3	1.33E+06	733.00	2490.64	796.46	2490.79	826.95	1773.97	1845.35	3710.72	2212.64
	4	1.33E+06	737.71	2158.89	797.32	2490.76	825.40	1622.94	1876.54	4104.89	2252.37
	5	9.70E+05	739.24	2158.50	790.66	2490.81	816.60	1773.98	1856.63	3710.75	2188.34
	6	9.70E+05	735.14	2490.85	797.68	2490.79	830.16	1622.85	1853.37	4104.91	2088.79
	7	2.15E+05	733.92	2157.86	799.05	2490.80	831.11	1773.88	1872.95	3710.76	2051.21
	8	1.33E+06	738.34	2490.75	792.24	2490.73	826.76	1773.99	1898.86	3710.73	2108.48
	9	8.73E+04	750.94	2159.06	796.71	2490.81	810.43	1622.92	1865.80	3710.75	2096.11
	10	2.15E+05	737.46	2490.53	792.28	2490.80	817.60	1622.83	1891.64	4104.90	2251.64
	Max	1.33E+06	751.50	2490.85	799.05	2490.81	831.11	1773.99	1898.86	4104.91	2252.37
	Min	8.73E+04	733.00	2157.86	790.66	2490.73	810.43	1622.83	1805.62	3710.72	2051.21
	Av	6.67E+05	739.46	2357.86	794.80	2490.79	823.45	1698.42	1859.95	3907.82	2142.01
	Std	5.65E+05	6.51	171.52	2.93	0.03	6.56	79.62	27.80	207.74	76.08

		functions									
		f16		f17		f18		f19		f20	
	Exp	Cos	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
D=10, QRSM,DE/best/2	1	3167.53	774.37	3396.10	730.85	1844.34	748.71	2.58E+05	733.38	3217.39	729.32
	2	3862.07	759.37	3396.10	727.93	1844.38	744.25	5.55E+06	738.01	3217.39	718.63
	3	3573.57	749.43	3451.52	730.18	1844.38	752.64	1.08E+05	748.27	3284.96	724.72
	4	3167.53	773.44	3075.17	723.81	1844.35	750.21	3.15E+05	738.12	3284.96	723.72
	5	3862.07	759.90	2746.01	730.68	1844.40	747.93	5.55E+06	739.55	3284.96	717.89
	6	3047.37	749.02	3075.17	726.23	1844.37	749.52	2.58E+05	728.70	3427.13	720.83
	7	3730.82	769.15	2763.98	730.52	1844.37	751.13	5.55E+06	741.96	3284.96	724.19
	8	3862.07	760.85	3291.58	729.88	1844.35	748.38	6.44E+06	747.01	3311.22	722.78
	9	3047.37	750.94	3451.52	730.23	1844.39	751.82	4.96E+06	758.82	3284.96	718.15
	10	3862.07	759.80	3075.17	724.89	1844.38	747.21	5.55E+06	744.42	3217.39	727.64
	Max	3862.07	774.37	3451.52	730.85	1844.40	752.64	6.44E+06	758.82	3427.13	729.32
	Min	3047.37	749.02	2746.01	723.81	1844.34	744.25	1.08E+05	728.70	3217.39	717.89
	Av	3518.24	760.63	3172.23	728.52	1844.37	749.18	3.45E+06	741.82	3281.53	722.79
	Std	366.75	9.32	268.14	2.64	0.02	2.46	2.79E+06	8.44	61.90	3.93

		functions									
		f21		f22		f23		f24		f25	
D=10, QRSM,DE/best/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	2577.25	763.16	5127.97	722.08	2843.28	745.95	2978.65	737.18	3791.91	758.18
	2	2577.25	761.39	5127.97	723.22	2843.28	745.68	2978.65	735.33	3791.91	758.09
	3	2577.25	763.31	5127.97	724.59	2843.28	746.20	2978.65	741.54	3791.91	757.71
	4	2718.01	757.88	5127.97	724.39	2843.28	744.06	2978.65	733.56	3791.91	757.67
	5	2577.25	760.93	5127.97	722.05	2843.28	743.91	2978.65	739.73	3791.91	757.76
	6	2577.25	763.58	5127.97	717.48	2843.28	747.26	2978.65	736.96	3791.91	755.84
	7	2577.25	764.45	5127.97	720.99	2843.28	744.84	2978.65	737.73	3791.91	755.42
	8	2577.25	764.94	5127.97	721.19	2843.28	742.44	2978.65	742.33	3791.91	755.86
	9	2565.34	770.24	5127.97	716.15	2843.28	746.12	2978.65	736.78	3791.91	755.53
	10	2565.34	770.57	5127.97	727.04	2843.28	743.53	2978.65	740.77	3791.91	758.45
	Max	2718.01	770.57	5127.97	727.04	2843.28	747.26	2978.65	742.33	3791.91	758.45
	Min	2565.34	757.88	5127.97	716.15	2843.28	742.44	2978.65	733.56	3791.91	755.42
	Av	2588.94	764.04	5127.97	721.92	2843.28	745.00	2978.65	738.19	3791.91	757.05
	Std	45.62	3.92	0.00	3.26	0.00	1.49	0.00	2.82	0.00	1.22

		functions									
		f26		f27		f28		f29		f30	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
D=10, QRSM,DE/best/2	1	5730.71	750.68	3174.23	775.95	3907.23	747.73	4205.62	830.22	3.01E+09	826.10
	2	5730.71	750.47	3174.23	774.06	3885.87	722.08	4205.62	833.39	3.01E+09	826.56
	3	5730.71	748.77	3174.23	774.42	3907.23	744.42	4205.62	834.45	3.01E+09	824.42
	4	5730.71	749.10	3174.23	769.71	4015.02	733.74	4205.62	834.43	3.01E+09	825.90
	5	5730.71	750.29	3174.23	766.40	3907.23	745.22	4205.62	835.63	3.01E+09	819.04
	6	5730.71	749.12	3174.23	778.26	4015.02	736.70	4375.12	843.56	3.01E+09	826.27
	7	5730.71	748.95	3174.23	774.44	4015.02	737.35	4878.55	827.96	3.01E+09	824.93
	8	5730.71	749.63	3174.23	776.40	4015.02	735.90	4205.62	836.20	3.01E+09	825.84
	9	5730.71	748.74	3174.23	777.40	3885.87	725.69	25501.62	835.59	3.01E+09	826.12
	10	5730.71	750.49	3174.23	774.65	3907.23	746.12	4205.62	831.75	3.01E+09	821.42
	Max	5730.71	750.68	3174.23	778.26	4015.02	747.73	25501.62	843.56	3.01E+09	826.56
	Min	5730.71	748.74	3174.23	766.40	3885.87	722.08	4205.62	827.96	3.01E+09	819.04
	Av	5730.71	749.62	3174.23	774.17	3946.07	737.49	6419.46	834.32	3.01E+09	824.66
	Std	0.00	0.78	0.00	3.59	59.91	8.70	6708.12	4.18	5.45	2.48

Results illustrated in Table 12, for a DE algorithm using the QRSM model with a problem dimension D=10, show that the optimal solutions are obtained in all 10 runs for the benchmark function f1. The DE/best/2 is a robust algorithm since the standard deviation is equal to 0.

The function f18 is an easy function because the DE/strategy using QRSM shows some robustness by giving solutions with error costs less than 100. Its standard deviation is 0.02 which is a negligible value. This value shows that the algorithm is still stuck in a specific local optimum.

Considering the benchmark functions f5, f6, f7 and f8, the difference from the optimal cost is bigger than 100 and less than 300. The standard deviation of f6 is 56.68 which confirms that the strategy cannot escape from local optimum solutions. On the flip-side, the remaining functions have a null standard deviation, which means that the strategy is stuck in a deep local optimum. Also, for the f14, the difference for 5 runs is around 222 and for the other 5 runs it is around 373. Its standard deviation increases and becomes 79.62 due to the 2 similar solutions obtained in the different 10 runs.

An error of 1000 and hundreds is obtained for the benchmark functions f12, f13, f16, f17, f20, f21, f23, f24, f25, f27, f28 and f29, with has an exception in run number 9 with an error value of thousands. The standard deviation of f13 is almost 0 and for f23, f24, f25 and f27, it is 0. This proves the blocking of the DE strategy in a deep local optimum. For the function f21 and f29, the standard deviation increased importantly because for f29, all the obtained solutions are same except one. For f21 there are 8 runs resulting in the same solution and the last two runs gave same solution and for f12 there are solutions with a cost of around 2490 and solutions with a cost of around 2158.

For the remaining functions, the standard deviation also increased, which shows that the DE strategy with QRSM could not avoid the different local optimal solutions.

For following functions f4, f9, f10, f15, f22 and f26, the error values are around thousands. The standard deviation for f10 and f15 is more than 200 for both functions. For f10 the algorithm is resulting in many different local optimal solutions but for f15 the algorithm is stuck in 2 deep local optimal solutions. For the other functions, the standard deviation is equal to 0 which means that the strategy is blocked in a deep local optimum.

A difference of thousands and millions from the global optimum cost for the different hard functions f3, f11 and f19. Their standard deviation is very important because the DE strategy cannot escape the different local optimal solutions. For f11, there are just 4 same solutions obtained in the 10 runs.

For the functions f2 and f30, the error value is about millions and the standard deviation for f2 is very significant because of a diversity of the local optimum solutions obtained by the strategy. For the function f30, the standard deviation of 5.45 is small because the algorithm is finding solutions in the same local space.

The CPU time variation for the benchmark functions f1, f4, f5, f7, f8 and f26 is negligible with a standard deviation less than 1. The standard deviation is more than 1 and less than 11 for the functions f2, f3, f6, f9, f10, f11, f12, f13, f16, f17, f18, f19, f20, f21, f22, f23, f24, f25, f27, f28, f29 and f30. This means that the variation increased but still not important. For the functions f14 and f15, the variation gets more significant, and can be observed from the respective standard deviation values of 27.80 and 76.08.

Table 13: DE+QRSM Results for Dimension 10 and Variant Rand to Best/1

		functions									
		f1		f2		f3		f4		f5	
D=10, QRSM,DE/rand to best/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	100.00	737.35	1.18E+20	738.45	3.24E+06	2300.34	4387.19	2081.40	638.83	2218.60
	2	100.00	734.92	1.18E+20	735.79	3.24E+06	2597.79	4387.19	1967.52	638.83	2194.79
	3	100.00	736.04	1.18E+20	738.99	2.57E+06	2561.05	4387.19	2138.59	638.83	2238.94
	4	100.00	734.55	1.18E+20	737.79	2.57E+06	2546.64	4387.19	1910.92	638.83	2230.86
	5	100.00	733.97	2.95E+19	732.80	2.37E+07	2569.70	4387.19	1703.80	638.83	2220.50
	6	100.00	737.15	1.18E+20	737.40	2.57E+06	2500.20	4387.19	1766.45	638.83	2220.50
	7	100.00	736.57	1.18E+20	737.54	2.57E+06	2616.70	4387.19	1784.76	638.83	2142.50
	8	100.00	738.07	6.15E+14	733.83	2.57E+06	2778.92	4387.19	2452.96	638.83	2138.10
	9	100.00	736.53	1.18E+20	738.63	2.37E+07	2804.23	4387.19	2592.81	638.83	2207.07
	10	100.00	737.90	1.18E+20	737.25	2.37E+07	2809.83	4387.19	2594.50	638.83	2246.99
	Max	100.00	738.07	1.18E+20	738.99	2.37E+07	2809.83	4387.19	2594.50	638.83	2246.99
	Min	100.00	733.97	6.15E+14	732.80	2.57E+06	2300.34	4387.19	1703.80	638.83	2138.10
	Av	100.00	736.31	9.76E+19	736.85	9.06E+06	2608.54	4387.19	2099.37	638.83	2205.89
	Std	0.00	1.42	4.42E+19	2.08	1.01E+07	157.18	0.00	339.07	0.00	37.61

		functions									
		f6		f7		f8		f9		f10	
D=10, QRSM,DE/rand to best/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	687.83	807.73	799.62	782.03	903.82	709.63	5362.95	732.35	4190.00	759.96
	2	687.83	808.41	799.62	780.38	903.82	708.63	5362.94	731.47	4414.04	755.82
	3	687.83	808.91	799.62	781.38	903.82	709.10	5362.95	730.32	4414.04	755.43
	4	687.83	808.58	799.62	781.16	903.82	708.29	5362.95	732.27	4414.04	755.78
	5	687.83	807.91	799.62	781.13	903.82	709.04	5362.95	732.18	4414.04	755.56
	6	687.83	807.98	799.62	781.25	903.82	708.14	5362.95	732.19	4414.04	754.59
	7	687.83	809.43	799.62	780.85	903.82	708.59	5362.95	732.33	4414.04	753.05
	8	687.83	809.93	799.62	780.93	903.82	708.57	5362.95	731.30	4414.04	754.26
	9	687.83	807.70	799.62	780.49	903.82	708.17	5362.95	730.85	4414.04	754.31
	10	687.83	808.27	799.62	780.05	903.82	708.92	5362.95	732.61	4414.04	741.04
	Max	687.83	809.93	799.62	782.03	903.82	709.63	5362.95	732.61	4414.04	759.96
	Min	687.83	807.70	799.62	780.05	903.82	708.14	5362.94	730.32	4190.00	741.04
	Av	687.83	808.48	799.62	780.96	903.82	708.71	5362.95	731.79	4391.64	753.98
	Std	0.00	0.74	0.00	0.57	0.00	0.47	0.00	0.76	70.85	4.90

		functions									
		f11		f12		f13		f14		f15	
D=10, QRSM,DE/rand to best/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	4.95E+05	748.15	2136.90	745.28	2554.87	727.74	1847.62	756.42	5406.63	745.40
	2	4.95E+05	747.48	2509.20	756.96	2554.82	731.57	1847.61	755.45	5406.70	731.77
	3	4.95E+05	747.45	2137.01	744.89	2554.85	728.13	1847.60	757.54	5406.68	730.74
	4	4.95E+05	746.95	2137.59	747.43	2554.79	731.24	1847.60	757.10	5406.67	730.68
	5	4.95E+05	747.67	2137.28	744.73	2554.85	727.20	1847.62	760.79	5406.64	736.71
	6	4.95E+05	748.31	2509.20	761.94	2554.80	725.83	1847.61	757.32	5406.71	737.84
	7	4.95E+05	747.46	2509.20	750.97	2554.82	731.36	1847.60	763.23	5406.65	761.80
	8	4.95E+05	748.66	2509.20	749.43	2554.85	728.38	1847.61	757.15	5406.60	736.14
	9	4.95E+05	748.10	2509.20	751.68	2554.81	732.05	1847.61	750.33	5406.68	732.96
	10	4.95E+05	747.92	2509.20	750.57	2554.87	726.04	1847.60	764.55	4418.24	740.47
	Max	4.95E+05	748.66	2509.20	761.94	2554.87	732.05	1847.62	764.55	5406.71	761.80
	Min	4.95E+05	746.95	2136.90	744.73	2554.79	725.83	1847.60	750.33	4418.24	730.68
	Av	4.95E+05	747.81	2360.40	750.39	2554.83	728.96	1847.61	757.99	5307.82	738.45
	Std	0.04	0.51	192.10	5.54	0.03	2.39	0.01	4.05	312.57	9.43

D=10, QRSM,DE/rand to best/1	functions									
	f16		f17		f18		f19		f20	
Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
1	4918.94	730.74	7513.31	723.83	1824.97	750.93	5.22E+06	736.62	3710.58	751.29
2	4918.94	729.37	7513.31	722.57	1824.98	750.40	5.22E+06	736.22	3710.58	747.12
3	4918.94	729.84	7513.31	722.57	1824.99	750.82	4.89E+04	736.82	3710.58	751.77
4	4918.94	730.08	7513.31	722.13	1824.99	749.93	5.22E+06	737.00	3710.58	750.74
5	4918.94	728.93	7513.31	720.32	1824.99	748.24	5.22E+06	734.41	3710.58	752.11
6	5014.36	727.29	7513.31	721.41	1824.99	749.62	5.22E+06	737.17	3710.58	748.59
7	4918.94	729.99	7513.31	721.50	1824.98	750.43	5.22E+06	735.59	3710.58	752.13
8	4918.94	730.76	7513.31	721.94	1824.99	748.62	5.22E+06	733.44	3710.58	752.49
9	4918.94	729.90	7513.31	722.10	1825.01	749.57	5.22E+06	736.04	3710.58	752.95
10	4918.94	731.07	7513.31	722.61	1824.96	748.41	5.22E+06	737.79	3710.58	751.69
Max	5014.36	731.07	7513.31	723.83	1825.01	750.93	5.22E+06	737.79	3710.58	752.95
Min	4918.94	727.29	7513.31	720.32	1824.96	748.24	4.89E+04	733.44	3710.58	747.12
Av	4928.48	729.80	7513.31	722.10	1824.99	749.70	4.70E+06	736.11	3710.58	751.09
Std	30.18	1.09	0.00	0.93	0.01	0.99	1.64E+06	1.33	0.00	1.84

		functions									
		f21		f22		f23		f24		f25	
D=10, QRSM,DE/rand to best/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	2663.93	758.94	6091.52	716.29	2995.75	757.98	2946.59	726.59	3774.50	736.89
	2	2566.62	757.76	6091.52	716.36	2995.75	755.22	2946.59	726.76	3774.50	738.24
	3	2599.17	747.87	6091.52	716.06	2995.75	757.20	2946.59	725.33	3774.50	736.82
	4	2663.93	757.54	6091.52	715.55	2995.75	754.65	2946.59	725.94	3774.50	736.54
	5	2599.17	758.02	6091.52	715.64	2995.75	757.09	2946.59	726.01	3774.50	736.81
	6	2599.17	753.98	6091.52	715.61	2995.75	756.31	2946.59	726.18	3774.50	737.93
	7	2773.32	753.64	6091.52	715.94	2995.75	756.01	2946.59	726.98	3774.50	737.21
	8	2599.17	752.35	6091.52	715.42	2995.75	756.53	2946.59	727.31	3774.50	735.79
	9	2599.17	748.48	6091.52	716.17	2995.75	755.00	2946.59	726.67	3774.50	736.84
	10	2599.17	748.48	6091.52	715.92	2995.75	754.22	2946.59	728.34	3774.50	736.39
	Max	2773.32	758.94	6091.52	716.36	2995.75	757.98	2946.59	728.34	3774.50	738.24
	Min	2566.62	747.87	6091.52	715.42	2995.75	754.22	2946.59	725.33	3774.50	735.79
	Av	2626.28	753.71	6091.52	715.89	2995.75	756.02	2946.59	726.61	3774.50	736.95
	Std	60.02	4.32	0.00	0.33	0.00	1.22	0.00	0.83	0.00	0.71

		functions									
		f26		f27		f28		f29		f30	
D=10, QRSM,DE/rand to best/1	Exp	Cost	CPU	Cost	CPU	Cost	CPU	Cost	CPU	Cost	CPU
	1	6365.22	732.81	3162.15	759.30	3883.73	769.40	4267.42	754.65	1.18E+10	734.98
	2	6646.64	730.13	3295.06	761.30	3883.73	768.32	4267.42	754.23	1.18E+10	732.99
	3	5617.75	726.81	3162.15	760.69	3883.73	769.40	4267.42	757.43	1.18E+10	735.62
	4	6365.22	730.44	3162.15	757.56	3883.73	768.82	4267.42	754.03	1.18E+10	733.06
	5	6582.11	730.54	3163.86	753.95	3883.73	768.02	4267.42	754.39	1.18E+10	731.50
	6	5617.75	726.75	3295.06	760.05	3883.73	768.99	4267.42	753.52	1.18E+10	731.13
	7	6365.22	730.54	3295.06	764.95	3883.73	768.30	4267.42	754.19	1.18E+10	733.02
	8	6365.22	730.82	3295.06	765.12	3883.73	768.43	4267.42	755.00	1.18E+10	731.01
	9	5617.75	726.45	3162.15	756.31	3883.73	769.76	4267.42	754.58	1.18E+10	733.22
	10	6365.22	731.40	3162.15	757.09	3883.73	768.40	4267.42	754.55	1.18E+10	731.97
	Max	6646.64	732.81	3295.06	765.12	3883.73	769.76	4267.42	757.43	1.18E+10	735.62
	Min	5617.75	726.45	3162.15	753.95	3883.73	768.02	4267.42	753.52	1.18E+10	731.01
	Av	6190.81	729.67	3215.49	759.63	3883.73	768.78	4267.42	754.66	1.18E+10	732.85
	Std	408.00	2.20	68.49	3.60	0.00	0.58	0.00	1.05	78.07	1.54

Results illustrated in Table 13, for a DE algorithm using the QRSM model with a problem dimension D=10, show that the optimal solutions are obtained in all 10 runs for the benchmark function f1. The standard deviation is equal to 0 with an implication that the DE/rand to best/1 is a robust algorithm.

The error value for f6, f7 and f18 is less than 100 while their standard deviation is equal or very close to 0, this means that the DE strategy is stuck in a deep local optimum.

The difference between the solutions cost and the global optimums cost is slightly higher than 100 for f5 and f8 and their standard deviation values are equal to 0 because of a block in the deep local optimum.

The error values for the benchmark functions f14, f21, f23, f24 and f27 in the different runs are between 400 and 700. The standard deviation for f14 is 0.01 and 0 for f23 and f24. This means that the same local optimum is found in the 10 different runs, but in f21, the standard deviation increases to 60.02 and 68.49 for f27 due to the weaknesses of the DE strategy in avoiding different local optimal solutions.

For the functions f12, f13, f20, f25, f28 and f29, there is a difference of a thousand and hundreds from the optimal cost. For the function f12, a standard deviation of 192.10 shows that the strategy cannot avoid the different local optimal solutions. For the remaining functions, the standard deviation is 0 except for f13 which is equal to 0.03 as the strategy is stuck in a deep local optimum.

Considering the benchmark functions f9, f10, f11, f15, f16, f17, f22 and f26, their obtained solutions have their error values in the thousands. The standard deviation of f9, f17 and f22 is 0 to confirm that the strategy is stuck in a global optimum. The standard deviation for f17 is very close to 0 due to a small variation in the different obtained solutions. For f10, f15, f16 and f26, the standard deviations increased significantly and the maximal value of 408 is attained by f26. For function f26 and f10 the strategy could not avoid local optimal solutions and for f15 and f16 all the runs with the exception of 1 have the same solution cost.

The functions f2, f3, f19 and f30 are very hard functions. The error values obtained from these functions are millions and even more except for f19 in run 3 which was about thousands. The standard deviation is very huge for f2 and f3, which means that the algorithm cannot avoid the different local optimal solutions. Also, the standard deviation is very big for f19 because of the solution obtained in run 3. The standard deviation for f30 is equal to 78.07 which shows that there are different local solutions with very close costs.

For the CPU time, for the benchmark functions f6, f7, f8, f9, f11, f17, f18, f22, f24, f25 and f28, there is a very small variation on time with a standard deviation smaller than 1. For f2, f10, f12, f13, f14, f15, f16, f19, f20, f21, f23, f26, f27, f29 and f30, the standard deviation increases to values bigger than 1 and smaller than 10, denoting that the CPU time increases more. For f5, the standard deviation reaches a value of 37.61 denoting a larger time variation. For f3 and f4, the variation becomes very important with respective standard deviation values equal to 157.18 and 339.07.

Two approaches can be employed in a quest to compare different strategies. Comparison can be done using the CPU time to have an idea about the fastest DE strategy, and also using the cost average solution value to have an idea about the most accurate one.

From a speed perspective, according to results illustrated in Table 10, Table 11, Table 12 and Table 13, by using QRSM model for D=10, considering all the benchmark functions, rand/1 strategy is the fastest one with a CPU average values equal to 10.88, 10.36, 10.56, 10.32, 10.14, 10.15, 10.14, 10.09, 10.67, 10.27, 10.35, 10.79, 10.34, 10.35, 10.65, 10.26, 10.20, 10.73, 10.38, 10.14, 10.48, 10.79, 10.51, 10.32, 10.41, 10.33, 10.66, 10.53, 10.30 and 10.66. The strategy rand/2 is the second fastest strategy for the 30 different functions. Its CPU average values are 11.25, 11.27, 11.59, 11.10, 10.99, 11.09, 10.99, 11.90, 11.30, 11.16, 11.74, 11.17, 11.74, 11.18, 11.34, 11.72, 11.13, 11.26, 11.71, 11.18, 11.37, 11.75, 11.29, 11.21, 11.68, 11.28, 11.26, 11.74, 11.23 and 11.27. For the two other strategies, there is an alternation. The strategy best/2 is in third position for the functions f3, f4, f5, f6, f7, f8, f10, f11, f18, f20, f23 and f28 with average CPU values of 746.20, 738.67, 709.94, 737.86, 727.70, 708.30, 739.46, 749.18, 722.79, 745.00 and 737.49, but it is the slowest for the functions f2, f3, f9, f12, f13, f14, f15, f16, f17, f19, f21, f22, f24, f25, f26, f27, f2 and f30 with average CPU values equal to 749.47, 742.78, 765.99, 765.99, 732.69, 794.80, 823.45, 1859.95, 2142.01, 760.63, 728.52, 741.82, 764.04, 721.92, 738.19, 757.05, 749.62, 774.17, 834.32 and 824.66. The strategy rand to best/1 is in third position for the functions, f1, f2, f9, f10, f12, f13, f14, f15, f16, f17, f19, f21, f22, f24, f25, f26, f27, f29 and f30 with average CPU values of 736.31, 736.85, 731.79, 753.98, 750.39, 728.96, 757.99, 738.45, 729.80, 722.10, 736.11, 753.71, 715.89, 726.61, 736.95, 729.67, 759.63, 754.66 and 732.85, but it is the slowest for the functions f3, f4, f5, f6, f7, f8, f11, f18, f20, f23 and f28 with a CPU average values equal to 2608.54, 2099.37, 2205.89, 808.48, 780.96, 708.71, 747.81, 749.70, 751.09 and 756.02.

From an accuracy point of view, according to results illustrated in Table 10, Table 11, Table 12 and Table 13, by using QRSM model, for D=10, for the benchmark function f1, all the different strategies give an average of 100 which is equal to the global optimal cost.

Considering the benchmark function f2, the most accurate strategy is rand/2 with a cost average of 4.41E+17, the next one in accuracy is rand/1 with an average of 4.57E+18. After which is the strategy best/2 with an average of 7.39E+18 while the least accurate one is rand to best/1 with an average value equal to 9.76E+19.

The strategy rand/2 scores the best cost average with a value equal to 6.77E+06 for f3. In second position is the strategy rand to best/1 with a cost average value of 9.06E+06. In third position is the strategy rand/1 with a cost average of 1.32E+08 and the strategy best/2 is in the fourth position with a cost average equal to 3.24E+08.

The most accurate DE strategy in the 10 runs for the function f4 is rand/2 with a cost average equal to 1946.84. After it is rand/1 with a cost average equal to 3010.03. In the next position is rand to best/1 strategy with a cost average equal to 4387.19 and the least accurate one is best/2 with a cost average equal to 4454.79

For the function f5, the best strategy according to accuracy is best/2 with a cost average of 602.50. After it is the strategy rand/1, its cost average is equal to 612.72. Next is the strategy rand/2, with a cost average equal to 616.92. Finally, rand to best/1 is the worst strategy with a cost average equal to 638.83.

The best strategy based on the optimality of the obtained solution for f6 is rand to best/1 with a cost average equal to 687.83. After it is rand/1, its cost average is 696.31. In third position is rand/2 with a cost average equal to 801.36 and the least accurate strategy is best/2 its cost average is 828.71.

For the function f7, the most accurate strategy is rand to best/1 with a cost average of 799.62. After it is the strategy best/2, its cost average is 803.04. In third position, there is rand/1, its cost average is equal to 808.41, and in last position is rand/2 with a cost average value equal to 815.36.

The variant rand to best/1 is the most accurate strategy with a cost average value equal to 903.82 for f8. The variant rand/2 is in the second position, its cost average is equal to 907.36. In third position is the variant best/2 with a cost average equal to 910.41 and the worst accurate variant is rand/1, its scored cost average 934.98.

Considering the benchmark function f9, the most accurate applied strategy is rand to best/1, its cost average is 5362.95. In second position is the strategy best/2, its cost average is 6928.18. In third position is the strategy rand/2, its cost average is 7142.78 and the least accurate variant is rand/1 with a cost average equal to 11908.43.

Also, the best variant for the function f10 is rand to best/1. Its scored cost average is equal to 4391.64. After it is rand/2 with a cost average equal to 4439.82. Then, rand/1 with a cost equal to 4650.99 and the worst strategy is best/2. It scores a cost average of 5009.32.

For the function f11, the best strategy according to accuracy is rand/1 with a cost average of 363684.89. After it is the strategy rand to best/1, its cost average is equal to 495305.04. Then, the strategy best/2 with a cost average equal to 666537.20. Finally, rand/2 is the worst strategy with a cost average equal to 1317999.78

Also, the best variant for the function f12 is rand/2, its scored cost average is equal to 1860.80. After it is best/2 with a cost average equal to 2357.86. Then, rand to best/1 with a cost equal to 2360.40 and the worst strategy is rand/1. It scores a cost average of 2508.90.

The variant rand/2 is the best strategy according to accuracy with a cost average value equal to 2080.54 for f13. The variant rand/1 is in the second position, its cost average is equal to 2398.98. In third position is the variant best/2 with a cost average equal to 2490.79 and the worst accurate variant is rand to best/1, its scored cost average 2554.83.

For the function f14, the best strategy according to accuracy is rand/1 with a cost average of 1647.09. After it is the strategy rand/2, its cost average is equal to 1659.54. Then, the strategy best/2 with a cost average equal to 1698.42. Finally, rand to best/1 is the worst strategy with a cost average equal to 1847.61.

The best strategy based on the optimality of the obtained solution for f15 is best/2 with a cost average equal to 3907.82. After there is rand/2, its cost average is 4289.46. After it is rand to best/1 with a cost average equal to 5307.82 while the least accurate strategy is rand/1, its cost average is 8240.91.

Considering the benchmark function f16, the most accurate applied strategy is rand/1, its cost average is 3397.53. In second position is the strategy best/2, its cost average is 3518.24. In third position is the strategy rand/2, its cost average is 3579.27 and the least accurate variant is rand to best/1 with a cost average equal to 4928.48

Also, the best variant for the function f17 is rand/1, its scored cost average is equal to 2787.14, which is followed by rand/2, with a cost average equal to 3162.89. Next is best/2 with a cost equal to 3172.23, and the worst strategy is rand to best/1. It scores a cost average of 7513.31.

The most accurate DE strategy in the 10 runs for the function f18 is rand to best/1 with a cost average equal to 1824.99. After it is rand/1 with a cost average equal to 1835.22. In next position is best/2 strategy with a cost average of 1844.37 while the least accurate one is rand/2 with a cost average equal to 1859.83.

For the function f19, the best strategy according to accuracy is best/2 with a cost average of 3.45E+06. After it is the strategy rand to best/1, its cost average is equal to 4.70E+06. Then, the strategy rand/2 with a cost average equal to 9.44E+06, and finally, rand/1 is the worst strategy with a cost average equal to 1.57E+07.

The best strategy based on the optimality of the obtained solution for f20 is best/2 with a cost average equal to 3281.53. After it is rand/1, its cost average is 3296.83, after which is rand/2 with a cost average equal to 3332.92 and the least accurate strategy is rand to best/1, its cost average is 3710.58.

Also, the best variant for the function f21 is rand/2, its scored cost average is equal to 2560.82. After it is best/2 with a cost average equal to 2588.94. Next to it is rand to best/1 with a cost equal to 2626.28, and the worst strategy is rand/1. It scores a cost average of 2735.68

For the function f22, the best strategy according to accuracy is best/2 with a cost average of 5127.97. After it is the strategy rand/1, its cost average is equal to 5619.90. Then, the strategy rand to best/1 with a cost average equal to 6091.52. Finally, rand/2 is the worst strategy with a cost average equal to 8073.18.

The variant best/2 is the best strategy according to accuracy with a cost average value of 2843.28 for f23. The variant rand/2 is in the second position, its cost average is equal to 2873.43. In third position is the variant rand/1 and rand to best/1 with a cost average equal to 2995.75.

The strategy rand to best/1 scores the best cost average with a value of 2946.59 for f24. In second position is the best/2 strategy with a cost average value equal to 2978.65. In third position there is the rand/2 strategy with a cost average of 3022.12 while the rand/1 strategy is in the fourth position with a cost average equal to 3053.78.

For the last benchmark function f25, the most accurate DE strategy in the 10 runs is rand to best/1 with a cost average equal to 3774.50. After it is best/2 with a cost average equal to 3791.91. In next position is the rand/2 strategy with cost average equal to

3823.59 and the least accurate strategy is rand/1 with an extremely deviant cost average from the optimal solution equal to 4078.22.

For the function f26, the best strategy according to accuracy is best/2 with a cost average of 5730.71. In second position is the rand to best/1 strategy with a cost average of 6190.81. Next is the strategy rand/1 with a cost average equal to 6233.08. Finally, rand/2 is the worst strategy with a cost average equal to 6766.98

The best strategy based on the optimality of the obtained solution for f27 is rand/2, with a cost average equal to 3171.47. Next is best/2, its cost average is 3174.23. After it is rand to best/1 with a cost average equal to 3215.49, while the least accurate strategy is rand/1, its cost average is 3467.12

Also, the best variant for the function f28 is rand/1, its scored cost average is equal to 3834.09. Next is rand to best/1 with a cost average equal to 3883.73. Then best/2, with a cost equal to 3946.07 and the worst strategy is rand/2. It scores a cost average of 3977.09.

For the last benchmark function f29, the most accurate DE strategy in the 10 runs is rand to best/1 with a cost average equal to 4267.42. In second position is rand/2 with a cost average equal to 6241.41. In next position is best/2 strategy with cost average equal to 6419.46 and the least accurate one is rand/1 with an extremely deviant cost average from the optimal solution equal to 9064.36.

For the last benchmark function, the most accurate DE strategy in the 10 runs is best/2 with a cost average of 3.01E+09. After it is the rand/1 strategy, with a cost average equal to 3.29E+09. In the next position is the rand/2 strategy with a cost average equal to 9.38E+09, while the least accurate one is rand to best/1 with an extremely divergent cost average from the optimal solution with a value of 1.18E+10.

The fastest strategy is rand/1, the second one is rand/2 after it is rand to best/1 and slowest one generally is best/2. The strategy best to rand/1 is the most accurate for 9 functions, the second most accurate for 5 functions, the third most accurate for 7 functions and the weakest one for 8 functions. The strategy best/2 is the most accurate for 8 functions, it is the second most accurate one for 7 functions, and third most accurate for 9 functions, and the weakest one for 5 functions. The strategy rand/2 is the most accurate for 7 functions, the second most accurate one for 8 functions, third most accurate for 9 functions and the weakest one for 5 functions. The strategy rand/1 is the most accurate for 5 functions, the second most accurate one for 9 functions, in third position for 6 functions and the weakest one for 9 functions. The different strategies are similar to each other from a point of view of accuracy, except rand/1 which is the worst one, but it is the fastest one. After it is rand/2, implying that the best choice is rand/2 as it brings a much needed balance between speed and accuracy.

Table 14: DE+QRSM Results for Dimension 30 and Variant Rand/1

		functions									
		f1		f2		f3		f4		f5	
D=30, QRSM, DE/rand/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	100.00	163.44	1.65E+66	172.10	2.90E+12	166.47	176847.12	165.84	1293.76	166.44
	2	100.00	162.68	1.65E+66	172.02	2.76E+13	164.57	171524.52	164.77	1293.76	166.06
	3	100.00	162.71	6.62E+60	172.16	1.74E+12	164.64	200725.07	164.69	1508.88	164.30
	4	100.00	162.77	1.19E+66	171.02	1.42E+11	165.16	171524.51	164.88	1293.76	165.56
	5	100.00	162.63	6.62E+60	171.96	4.04E+12	164.38	171524.51	164.97	1293.76	166.06
	6	100.00	162.66	1.65E+66	171.23	1.51E+14	164.97	220399.77	165.74	1508.88	164.78
	7	100.00	162.74	1.65E+66	170.65	1.66E+11	164.21	171524.51	164.89	1508.88	164.44
	8	100.00	162.63	1.65E+66	171.46	2.76E+13	165.00	155928.30	164.96	1293.76	165.81
	9	100.00	162.74	1.65E+66	171.60	1.43E+12	166.23	171524.51	165.56	1254.84	164.74
	10	100.00	162.71	1.19E+66	170.81	3.94E+12	166.13	171524.51	164.89	1254.84	164.74
	Max	100.00	163.44	1.65E+66	172.16	1.51E+14	166.47	220399.77	165.84	1508.88	166.44
	Min	100.00	162.63	6.62E+60	170.65	1.42E+11	164.21	155928.30	164.69	1254.84	164.30
	Av	100.00	162.77	1.23E+66	171.50	2.21E+13	165.18	178304.73	165.12	1350.51	165.29
	Std	0.00	0.24	6.75E+65	0.56	4.66E+13	0.82	18418.95	0.42	110.38	0.78

		functions									
		f6		f7		f8		f9		f10	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
D=30, QRSM, DE/rand/1	1	865.52	169.14	1047.76	162.01	1787.29	167.72	131789.21	165.21	11766.93	172.27
	2	980.90	168.40	1047.76	161.45	1787.29	166.69	139833.56	166.53	13100.72	168.36
	3	958.74	167.65	1047.76	161.10	1787.29	167.09	146911.20	165.33	11684.36	169.20
	4	865.52	165.19	1047.76	161.49	1787.29	166.84	131789.16	165.02	12721.98	168.50
	5	865.52	165.41	1047.76	161.41	1787.29	167.00	139833.59	166.64	11766.93	169.25
	6	837.33	170.28	1047.76	161.24	1493.55	166.48	139833.57	166.91	13257.80	167.33
	7	865.52	166.16	1047.76	161.52	1493.55	166.17	139833.54	166.38	13487.46	168.26
	8	992.02	166.38	1047.76	161.32	1493.55	166.55	87849.53	164.78	12829.79	171.59
	9	965.12	166.03	1047.76	161.34	1787.29	167.65	139833.59	166.69	12829.80	169.54
	10	923.58	175.69	1047.76	161.32	1562.22	166.03	87849.46	164.64	12876.40	168.09
	Max	992.02	175.69	1047.76	162.01	1787.29	167.72	146911.20	166.91	13487.46	172.27
	Min	837.33	165.19	1047.76	161.10	1493.55	166.03	87849.46	164.64	11684.36	167.33
	Av	911.98	168.03	1047.76	161.42	1676.66	166.82	128535.64	165.81	12632.22	169.24
	Std	58.22	3.17	0.00	0.24	144.19	0.56	21877.88	0.89	656.92	1.57

		functions									
		f11		f12		f13		f14		f15	
D=30, QRSM, DE/rand/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	6.24E+06	168.61	1.45E+04	166.14	6.07E+04	166.28	2001.76	170.37	2.90E+04	166.50
	2	1.39E+05	167.28	2.44E+04	165.10	7.69E+04	165.19	2033.49	168.79	1.33E+04	165.63
	3	2.01E+07	166.97	3.33E+04	165.16	3.68E+04	164.80	2022.54	168.73	1.09E+04	165.58
	4	3.71E+06	167.89	1.00E+04	165.83	7.69E+04	165.00	1970.56	168.51	2.11E+04	166.34
	5	6.24E+06	168.61	1.34E+04	165.53	1.15E+05	165.45	2005.90	168.29	2.11E+04	166.66
	6	4.60E+09	167.08	7.01E+03	165.75	3.68E+04	164.88	1961.35	168.57	2.90E+04	165.66
	7	3.71E+06	167.87	2.18E+04	165.28	3.68E+04	164.94	2005.90	168.08	5.30E+03	165.86
	8	3.71E+06	170.40	1.41E+04	165.75	3.68E+04	165.11	2056.67	169.09	1.09E+04	165.55
	9	4.95E+07	167.58	6.76E+03	165.69	1.15E+05	165.50	2005.90	168.11	2.11E+04	166.22
	10	8.59E+08	168.31	6.32E+03	166.08	3.68E+04	165.22	2056.35	168.93	2.90E+04	165.95
	Max	4.60E+09	170.40	3.33E+04	166.14	1.15E+05	166.28	2056.67	170.37	2.90E+04	166.66
	Min	139210.14	166.97	6.32E+03	165.10	3.68E+04	164.80	1961.35	168.08	5.30E+03	165.55
	Av	5.55E+08	168.06	1.52E+04	165.63	6.28E+04	165.24	2012.04	168.75	1.91E+04	165.99
	Std	1.45E+09	1.01	8.85E+03	0.36	3.21E+04	0.43	31.68	0.66	8.60E+03	0.41

		functions									
		f16		f17		f18		f19		f20	
D=30, QRSM, DE/rand/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	5.38E+04	165.97	8.63E+07	165.75	1945.27	164.00	2.41E+09	166.80	5297.32	174.83
	2	2.79E+04	166.34	8.73E+07	164.64	1945.31	163.60	3.14E+08	166.02	4869.11	170.42
	3	2.89E+04	165.17	4.55E+07	164.74	1945.11	163.08	2.41E+09	165.78	5670.20	168.98
	4	2.89E+04	164.88	5.76E+07	165.50	1945.25	163.40	2.07E+10	165.38	5720.36	170.65
	5	2.75E+04	164.92	3.80E+07	164.82	1945.35	163.41	1.68E+09	165.53	5670.20	169.67
	6	2.89E+04	164.94	5.76E+07	164.53	1945.27	163.47	2.41E+09	165.88	5041.97	170.40
	7	2.89E+04	164.57	1.46E+07	164.88	1945.26	163.30	3.14E+08	165.91	4916.10	169.89
	8	2.80E+04	165.30	5.76E+07	165.97	1945.24	163.35	2.06E+10	165.64	6013.86	169.89
	9	2.89E+04	164.41	1.92E+07	165.41	1945.46	163.40	2.66E+09	165.99	4840.87	174.18
	10	3.23E+04	165.35	1.92E+07	165.24	1945.22	163.44	3.14E+08	165.60	4790.07	169.45
	Max	5.38E+04	166.34	8.73E+07	165.97	1945.46	164.00	2.07E+10	166.80	6013.86	174.83
	Min	2.75E+04	164.41	1.46E+07	164.53	1945.11	163.08	3.14E+08	165.38	4790.07	168.98
	Av	3.14E+04	165.18	4.83E+07	165.15	1945.27	163.45	5.38E+09	165.85	5283.00	170.83
	Std	7.98E+03	0.60	2.62E+07	0.50	0.09	0.24	8.11E+09	0.39	450.68	2.00

		functions									
		f21		f22		f23		f24		f25	
D=30, QRSM, DE/rand/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	3820.51	166.08	13755.86	170.29	4087.26	170.63	3978.29	171.45	1.49E+05	166.14
	2	3561.97	165.74	16549.15	172.04	3922.57	168.65	3978.29	171.06	1.49E+05	165.81
	3	3561.97	167.08	14984.59	170.35	4446.97	169.45	3946.11	171.87	1.49E+05	166.66
	4	3820.51	166.14	13395.11	169.85	4087.26	168.73	3978.29	171.88	7.19E+04	165.49
	5	3686.05	165.77	14116.67	168.59	4087.26	170.46	3978.29	170.60	1.49E+05	165.69
	6	3927.59	167.72	14116.67	170.45	4011.23	171.01	3978.29	171.02	7.19E+04	165.91
	7	3735.19	167.16	16294.03	170.88	4087.26	169.32	3978.29	171.38	7.19E+04	165.44
	8	3561.97	166.38	13642.98	169.67	4089.34	169.99	3978.29	171.13	1.49E+05	167.16
	9	3735.19	166.55	14343.52	169.17	3991.07	168.75	3978.29	170.85	1.49E+05	165.61
	10	3671.43	166.66	15167.43	169.78	4517.40	172.80	3946.11	171.65	7.19E+04	165.39
	Max	3927.59	167.72	16549.15	172.04	4517.40	172.80	3978.29	171.88	1.49E+05	167.16
	Min	3561.97	165.74	13395.11	168.59	3922.57	168.65	3946.11	170.60	7.19E+04	165.39
	Av	3708.24	166.52	14636.60	170.11	4132.76	169.98	3971.85	171.29	1.18E+05	165.93
	Std	125.08	0.64	1092.33	0.95	193.18	1.30	13.57	0.43	3.98E+04	0.57

		functions									
		f26		f27		f28		f29		f30	
D=30, QRSM, DE/rand/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	28667.10	170.63	4160.65	167.95	31271.67	164.92	9.78E+06	167.65	1.42E+10	167.67
	2	29495.07	170.12	5721.47	168.37	43227.21	165.83	5.13E+06	166.11	1.42E+10	166.38
	3	29495.07	169.87	5721.47	166.73	28847.50	166.30	1.39E+06	165.69	1.42E+10	167.44
	4	13994.60	170.13	5096.54	167.30	47095.31	165.27	3.71E+08	166.84	1.42E+10	168.68
	5	13994.60	170.12	4843.06	166.55	48391.49	168.03	1.31E+08	166.77	5.36E+10	167.22
	6	16086.39	170.56	5172.08	165.91	14208.24	166.81	8.08E+06	166.78	1.42E+10	167.12
	7	13994.60	169.99	4730.64	166.86	18392.99	165.61	1.13E+09	167.34	1.42E+10	166.95
	8	20272.53	170.12	4730.64	166.98	43227.21	165.14	3.73E+04	167.42	1.42E+10	167.03
	9	16086.39	170.31	5698.28	167.59	31271.67	164.08	9.47E+06	166.33	5.36E+10	167.26
	10	13994.60	170.13	4730.64	166.72	24130.43	163.96	8.66E+06	166.59	5.36E+10	166.11
	Max	29495.07	170.63	5721.47	168.37	48391.49	168.03	1.13E+09	167.65	5.36E+10	168.68
	Min	13994.60	169.87	4160.65	165.91	14208.24	163.96	3.73E+04	165.69	1.42E+10	166.11
	Av	19608.10	170.20	5060.55	167.10	33006.37	165.60	1.67E+08	166.75	2.60E+10	167.19
	Std	6902.99	0.24	524.69	0.72	12079.13	1.23	3.58E+08	0.61	1.91E+10	0.70

Results illustrated in Table 14, for a DE algorithm using the QRSM model with a problem of dimension D=30, show that the optimal solutions are obtained in all 10 runs for the benchmark function f1. The standard deviation is equal to 0, that means the DE/rand/1 is a robust algorithm.

For the function f18 there is an error value around 145 and its standard deviation is equal to 0.09 which shows that the strategy is giving the same local optimal solution with a very small variation. A difference from the global optimum between 200 and 400 for the both functions f6 and f7. The standard deviation for f6 is significant with a value equal to 58.22 which means that the strategy cannot avoid the different local optimum but the standard deviation of f7 is null, it means that the strategy gets stuck in a deep local optimum.

Considering the benchmark functions f5, f8 and f14, the error value obtained is between 500 and 1000. The standard deviation which is equal to 110.38, 144.19 and 31.68 for them all is significant. This means that the strategy cannot escape from the different local optimal solutions but the cost 1293.76 is obtained for f5 in majority of runs.

The error value is about a thousand and hundreds for the different benchmark functions f21, f23 and f24. The standard deviation is about 100 for f21 and f23 because the strategy is resulting in many local optimal solutions but the standard deviation of f24 is equal to 13.57, because it obtained a solution cost equal to 3978.29 for 8 runs and a solution cost of 3946.11 for only 2 runs.

The function f27 has a lot of variations in its solutions. Sometimes the error value is about a thousand and hundreds while at other times, it is about thousands. The standard deviation is important and equal to 524.69 due to a big variation in the local optimal obtained solutions.

Considering the different benchmark functions f4, f9, f10, f12, f13, f15, f16, f20, f22, f25, f26 and f28, the difference from the global optimum cost is in the thousands. The standard deviation of the different functions is significant due to a major variation in the obtained local optimal solutions. For some functions like f16 and f25, the same local optimal solution is obtained in majority of runs such as 28922.57 and 148915.92.

For the function f11, the error value varies between thousands and millions and its standard deviation is significant, proving that the strategy could not escape the different local optimal solutions.

The remaining functions are hard functions and their obtained error values are in the millions, while their standard deviation is huge with an implication that the strategy cannot avoid the different local optimal solutions.

The CPU time does not vary importantly for the different functions. This is remarkable from their CPU standard deviation with values smaller than 1, but for f6, f10, f11, f20, f23 and f28, the variation increased slightly and their standard deviation are equal to 3.17, 1.57, 1.01, 2.00, 1.30 and 1.23.

Table 15: DE+QRSM Results for Dimension 30 and Variant Rand/2

		functions									
		f1		f2		f3		f4		f5	
D=30, QRSM, DE/rand/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	5.45E+07	167.25	6.36E+59	168.79	1.77E+14	173.57	1.41E+05	170.96	1341.88	170.59
	2	5.23E+07	166.62	6.36E+59	168.09	8.78E+13	170.46	2.30E+05	170.74	1513.16	170.07
	3	3.96E+07	166.80	9.01E+65	167.65	3.17E+14	169.82	2.39E+05	170.84	1341.89	170.56
	4	5.52E+07	166.52	1.20E+66	168.00	3.17E+14	169.99	2.30E+05	172.05	1513.16	170.35
	5	6.81E+07	166.75	5.71E+60	167.48	5.32E+10	171.13	1.98E+05	170.09	1513.16	170.23
	6	6.74E+07	166.70	3.72E+65	169.11	2.61E+11	170.35	2.00E+05	171.32	1341.89	169.93
	7	5.78E+07	166.58	1.19E+66	169.00	8.78E+13	169.92	1.41E+05	170.77	1341.89	169.95
	8	4.02E+07	166.45	3.85E+66	167.39	8.78E+13	169.93	2.30E+05	171.54	1341.89	169.98
	9	5.65E+07	166.64	1.19E+66	169.11	1.97E+12	172.32	2.30E+05	170.29	1341.89	169.93
	10	5.94E+07	166.67	7.74E+66	167.55	3.17E+14	171.55	2.30E+05	170.51	1341.89	169.92
	Max	6.81E+07	167.25	7.74E+66	169.11	3.17E+14	173.57	2.39E+05	172.05	1513.16	170.59
	Min	3.96E+07	166.45	6.36E+59	167.39	5.32E+10	169.82	1.41E+05	170.09	1341.88	169.92
	Av	5.51E+07	166.70	1.65E+66	168.22	1.39E+14	170.91	2.07E+05	170.91	1393.27	170.15
	Std	9.54E+06	0.22	2.42E+66	0.71	1.34E+14	1.25	3.74E+04	0.59	82.73	0.26

		functions									
		f6		f7		f8		f9		f10	
D=30, QRSM, DE/rand/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	862.38	173.13	1059.02	166.11	1659.50	174.78	1.52E+05	168.76	1.16E+04	170.82
	2	910.08	177.86	1060.59	165.27	1659.51	174.21	1.13E+05	168.76	1.39E+04	174.47
	3	882.70	172.51	1053.53	165.44	1659.50	174.27	1.65E+05	168.81	1.53E+04	173.46
	4	863.35	172.60	1041.34	165.47	1659.50	174.11	2.10E+05	168.72	1.24E+04	168.61
	5	863.35	172.55	1054.76	165.45	1659.51	174.14	1.13E+05	169.07	1.39E+04	172.83
	6	946.12	175.72	1050.44	165.28	1857.71	173.99	1.38E+05	169.79	1.34E+04	174.16
	7	863.35	172.80	1058.90	165.45	1857.71	174.16	1.13E+05	168.75	1.34E+04	172.24
	8	863.35	172.54	1033.92	165.36	1659.50	174.03	1.63E+05	171.77	1.21E+04	174.92
	9	862.38	174.60	1062.01	165.39	1659.50	174.16	1.15E+05	170.82	1.30E+04	173.43
	10	863.35	172.62	1053.54	165.22	1659.51	174.24	1.80E+05	170.46	1.31E+04	170.57
	Max	946.12	177.86	1062.01	166.11	1857.71	174.78	2.10E+05	171.77	1.53E+04	174.92
	Min	862.38	172.51	1033.92	165.22	1659.50	173.99	1.13E+05	168.72	1.16E+04	168.61
	Av	878.04	173.69	1052.80	165.45	1699.15	174.21	1.46E+05	169.57	1.32E+04	172.55
	Std	28.39	1.82	8.94	0.25	83.57	0.22	3.38E+04	1.10	1.04E+03	2.00

		functions									
		f11		f12		f13		f14		f15	
D=30, QRSM, DE/rand/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	4.80E+08	171.01	6.16E+06	171.04	24525.89	172.54	2009.02	172.32	3.15E+04	173.71
	2	3.82E+07	171.71	1.68E+06	170.07	24813.27	171.84	2006.39	171.59	8.85E+04	172.41
	3	4.80E+08	169.34	3.37E+06	170.26	19263.69	171.20	1929.30	171.77	5.15E+04	173.63
	4	1.32E+05	171.99	3.76E+06	170.04	56840.18	171.54	2001.86	171.48	3.16E+04	172.88
	5	4.80E+08	170.01	2.95E+06	170.24	19298.21	171.27	2005.51	171.49	3.15E+04	173.07
	6	4.80E+08	169.81	5.22E+06	170.10	20079.97	171.54	2005.97	171.34	2.11E+04	173.19
	7	1.77E+05	170.09	3.06E+06	170.20	19585.76	171.20	1967.15	171.88	2.27E+04	172.15
	8	4.80E+08	170.43	4.29E+06	170.18	20282.63	171.26	1929.26	171.77	3.68E+04	174.52
	9	4.80E+08	169.06	3.24E+06	170.10	20826.07	171.30	2006.92	171.80	2.26E+04	172.51
	10	4.80E+08	169.23	3.09E+06	170.24	25094.79	171.49	2007.00	171.91	3.16E+04	172.82
	Max	4.80E+08	171.99	6.16E+06	171.04	56840.18	172.54	2009.02	172.32	8.85E+04	174.52
	Min	1.32E+05	169.06	1.68E+06	170.04	19263.69	171.20	1929.26	171.34	2.11E+04	172.15
	Av	3.40E+08	170.27	3.68E+06	170.25	25061.05	171.52	1986.84	171.74	3.70E+04	173.09
	Std	2.26E+08	1.02	1.27E+06	0.29	11414.61	0.41	32.72	0.28	2.01E+04	0.71

		functions									
		f16		f17		f18		f19		f20	
D=30, QRSM, DE/rand/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	32010.09	170.65	4.37E+07	169.45	1997.38	172.58	4.13E+08	171.37	4675.16	173.72
	2	32010.09	170.03	6.15E+07	169.34	1945.26	171.82	1.38E+09	171.10	4612.82	170.62
	3	32010.09	170.49	6.15E+07	168.73	1943.51	171.88	4.13E+08	170.63	4675.16	170.73
	4	32010.09	169.95	1.90E+08	170.96	1949.77	171.98	4.13E+08	170.77	4587.39	173.13
	5	32010.09	170.12	6.15E+07	169.18	1969.25	172.24	4.13E+08	170.82	4950.53	173.16
	6	56523.36	170.60	1.83E+08	169.26	1954.68	171.98	4.13E+08	170.93	4675.16	171.13
	7	32010.09	170.10	1.05E+08	170.48	1970.99	171.98	4.13E+08	170.84	4969.45	172.83
	8	32010.09	170.06	1.90E+08	169.28	1988.70	171.73	4.13E+08	170.70	4920.28	173.15
	9	32010.09	170.13	4.37E+07	169.87	1936.46	171.91	4.13E+08	170.84	4675.16	173.36
	10	32010.09	169.95	3.82E+06	168.50	1909.61	171.99	7.87E+07	171.10	4675.16	172.44
	Max	56523.36	170.65	1.90E+08	170.96	1997.38	172.58	1.38E+09	171.37	4969.45	173.72
	Min	32010.09	169.95	3.82E+06	168.50	1909.61	171.73	7.87E+07	170.63	4587.39	170.62
	Av	34461.42	170.21	9.44E+07	169.50	1956.56	172.01	4.76E+08	170.91	4741.63	172.43
	Std	7751.78	0.27	6.91E+07	0.75	25.87	0.24	3.36E+08	0.22	145.27	1.16

D=30, QRSM, DE/rand/2	functions										
		f21		f22		f23		f24		f25	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	3640.90	168.65	1.41E+04	172.51	3984.42	174.07	3783.23	172.49	9.06E+04	169.73
	2	3640.90	168.43	1.61E+04	171.13	4075.72	172.57	3808.43	171.54	9.06E+04	168.76
	3	3965.38	169.18	1.47E+04	169.32	4309.82	175.42	3808.43	171.40	9.06E+04	168.78
	4	3640.90	171.84	1.44E+04	170.62	4075.72	173.07	3808.43	171.69	1.54E+05	168.42
	5	3812.16	168.06	1.36E+04	171.09	4092.80	174.57	3808.43	171.35	9.06E+04	169.07
	6	3746.84	168.98	1.36E+04	169.48	4203.04	177.30	3808.43	172.46	9.06E+04	168.72
	7	3458.70	168.01	1.33E+04	170.15	4309.82	173.89	3923.31	171.93	9.06E+04	168.81
	8	4127.75	168.03	1.39E+04	175.95	3984.42	173.49	3808.43	171.71	1.33E+05	169.03
	9	4127.75	167.98	1.42E+04	172.15	3925.53	175.39	3808.43	171.69	9.06E+04	168.79
	10	3640.90	168.82	1.53E+04	172.90	4343.12	173.35	3808.43	171.46	9.06E+04	168.87
	Max	4127.75	171.84	1.61E+04	175.95	4343.12	177.30	3923.31	172.49	1.54E+05	169.73
	Min	3458.70	167.98	1.33E+04	169.32	3925.53	172.57	3783.23	171.35	9.06E+04	168.42
	Av	3780.22	168.80	1.43E+04	171.53	4130.44	174.31	3817.40	171.77	1.01E+05	168.90
	Std	225.83	1.15	8.51E+02	1.97	151.62	1.41	38.05	0.41	2.29E+04	0.34

		functions									
		f26		f27		f28		f29		f30	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
D=30, QRSM, DE/rand/2	1	2.08E+04	170.74	4316.74	172.91	5.04E+04	174.49	5.73E+07	170.91	5.18E+10	171.73
	2	2.38E+04	168.81	5515.15	172.04	5.04E+04	173.01	1.84E+08	170.54	5.18E+10	171.30
	3	2.38E+04	169.09	4895.27	171.71	3.45E+04	172.10	1.89E+09	172.46	5.18E+10	171.18
	4	2.08E+04	169.78	4444.62	171.84	3.39E+04	172.57	1.84E+08	171.84	5.18E+10	171.29
	5	2.38E+04	169.43	4587.03	173.11	3.45E+04	172.18	8.99E+07	169.51	5.18E+10	171.04
	6	3.50E+04	171.51	4587.03	171.71	3.45E+04	172.57	1.89E+09	172.10	5.18E+10	171.24
	7	2.08E+04	170.38	4923.00	172.46	2.37E+04	174.60	1.89E+09	170.24	5.18E+10	170.99
	8	2.08E+04	169.20	4016.50	173.57	3.67E+04	174.47	2.10E+09	171.41	5.18E+10	170.93
	9	2.08E+04	169.01	4316.74	172.65	3.39E+04	171.77	2.10E+09	169.34	5.18E+10	171.07
	10	2.70E+04	170.85	4223.19	171.21	3.45E+04	172.19	1.92E+06	170.45	5.18E+10	171.07
	Max	2.08E+04	168.81	4016.50	171.21	2.37E+04	171.77	1.92E+06	169.34	5.18E+10	170.93
	Min	3.50E+04	171.51	5515.15	173.57	5.04E+04	174.60	2.10E+09	172.46	5.18E+10	171.73
	Av	2.37E+04	169.88	4582.53	172.32	3.67E+04	172.99	1.04E+09	170.88	5.18E+10	171.18
	Std	4.49E+03	0.93	433.76	0.74	8.05E+03	1.10	9.91E+08	1.06	12645.97	0.23

Results illustrated in Table 15, for a DE algorithm using the QRSM model with a problem dimension D=30, show that the error value for the functions f6, f7 and f18 belongs to the range of values bigger than 100 and smaller than 400. Their standard deviation is not very significant because the strategy rand/2 cannot avoid the different local optimal solutions, but their cost still close to each other. These functions are the easiest ones for this strategy but with reduced robustness.

The functions f8 and f14 are special functions. The error value of f8 is between 800 and 1058 and its standard deviation is equal to 83.57, which means that there are only 3 different obtained costs; the cost 1659.50 is obtained 5 times, the cost 1659.51 is obtained 3 times and the cost 1857.71 is obtained 2 times. Also, f14 has an error value bigger than 500 and smaller than 610 for the different 10 runs while its standard deviation is equal to 32.72. This value shows that the strategy cannot avoid the different local optimal solutions.

A difference of 1000 and hundreds from the global optimum cost for the benchmark functions f5, f21, f23 and f24. The standard deviation values of f21 and f23 are equal to 225.83 and 151.62 because there is an important variation in the obtained solutions, which means that the strategy results many local optimal solutions. For the functions f5, the standard deviation is equal to 82.73 because the same solution cost is obtained 7 times and the other cost is obtained 3 times. For f24, the standard deviation is equal to 38.05 because the same solution cost is obtained 8 times and the 2 other costs are different.

For the function f27 the difference of the solutions cost and the global optimum cost vary a lot from 1316.50 until 2815.15. Its standard deviation is also significant with a value equal to 433.76. This value displays that the strategy failed to find the global optimum and it always results a local optimum.

Considering the benchmark functions f9, f10, f13, f15, f16, f20, f22, f25, f26 and f28, the obtained error value during their optimization is about thousands which shows the difficulty of these functions. Their standard deviation is also very significant. It is in the thousands for the majority of them which means there are many local optimal solutions obtained by the strategy using QRSM, but there is some exceptions, such as f16, f20 and f22. For the function f16 the same local optimal cost were obtained 9 times which can be the same local optimal solution. For f20 the standard deviation is only 145.27 and same cost were obtained 5 times which refers to the same local solution. Also for f22, the standard deviation is equal to 850.81, but there are many different local optimal solutions.

The functions f1, f2, f3, f11, f12, f17, f19, f29 and f30 are very hard functions. Their error values are millions and their standard deviation are also huge, this means that the strategy could not stabilize, and the obtained local optimal solutions varies significantly, and are very far from the global optimum.

There are negligible CPU time variations for the functions f1, f2, f4, f5, f7, f8, f12, f13, f14, f15, f16, f17, f18, f19, f24, f25, f26, f27 and f30, with a CPU standard deviation less than 1. For the benchmark functions f3, f6, f9, f10, f11, f20, f21, f22, f23, f28 and f29, the variation increases slightly and becomes between 1 and 2.

Table 16: DE+QRSM Results for Dimension 30 and Variant Best/2

		functions									
		f1		f2		f3		f4		f5	
D=30, QRSM, DE/best/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	100.00	1.48E+04	7.42E+70	1.55E+04	3.43E+08	1.51E+04	1.67E+05	1.52E+04	1381.01	1.49E+04
	2	100.00	1.48E+04	7.42E+70	1.54E+04	4.01E+11	1.51E+04	1.21E+05	1.52E+04	1381.01	1.49E+04
	3	100.00	1.49E+04	6.36E+59	1.54E+04	1.83E+12	1.51E+04	1.67E+05	1.52E+04	1381.01	1.49E+04
	4	100.00	1.49E+04	7.42E+70	1.55E+04	1.07E+07	1.51E+04	1.40E+05	1.51E+04	1381.01	1.50E+04
	5	100.00	1.49E+04	5.64E+75	1.54E+04	2.64E+10	1.52E+04	8.78E+04	1.52E+04	1381.01	1.49E+04
	6	100.00	1.49E+04	7.43E+73	1.55E+04	7.01E+13	1.52E+04	6.86E+04	1.52E+04	1381.01	1.49E+04
	7	100.00	1.49E+04	1.19E+66	1.54E+04	4.39E+11	1.51E+04	1.17E+05	1.53E+04	1381.01	1.50E+04
	8	100.00	1.49E+04	9.64E+68	1.55E+04	4.04E+11	1.51E+04	1.42E+05	1.54E+04	1381.01	1.50E+04
	9	100.00	1.49E+04	1.19E+66	1.55E+04	3.04E+14	1.52E+04	1.05E+05	1.51E+04	1381.01	1.49E+04
	10	100.00	1.49E+04	3.20E+75	1.55E+04	7.46E+12	1.52E+04	1.55E+05	1.54E+04	1381.01	1.49E+04
	Max	100.00	1.49E+04	5.64E+75	1.55E+04	3.04E+14	1.52E+04	1.67E+05	1.54E+04	1381.01	1.50E+04
	Min	100.00	1.48E+04	6.36E+59	1.54E+04	1.07E+07	1.51E+04	6.86E+04	1.51E+04	1381.01	1.49E+04
	Av	100.00	1.49E+04	8.92E+74	1.55E+04	3.85E+13	1.51E+04	1.27E+05	1.52E+04	1381.01	1.49E+04
	Std	0.00	25.79	1.95E+75	44.37	9.58E+13	54.62	33377.21	105.54	0.00	47.59

D=30, QRSM, DE/best/2	functions										
		f6		f7		f8		f9		f10	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	933.33	15030.07	1045.44	14543.46	1635.90	15246.18	1.58E+05	14975.18	11906.39	15092.16
	2	931.87	15093.44	1045.44	14539.64	1856.75	15106.64	1.70E+05	15091.41	13413.23	15245.24
	3	936.56	14995.33	1045.44	14539.81	1635.90	15182.69	2.78E+05	14984.94	12828.85	15203.02
	4	954.44	15062.54	1045.44	14544.64	1635.90	15293.32	1.70E+05	15052.05	12961.20	15170.33
	5	927.58	15023.16	1045.44	14539.12	1635.90	15316.04	1.70E+05	15128.43	14583.77	15321.00
	6	933.33	15136.00	1045.44	14552.18	1856.75	15274.31	1.70E+05	15096.72	14505.67	15030.48
	7	933.33	15276.79	1045.44	14535.32	1592.93	15253.96	9.89E+04	14966.19	13203.63	15165.73
	8	889.00	15370.25	1045.44	14541.66	1887.82	15134.36	1.70E+05	15062.83	12865.26	15226.43
	9	1006.54	15276.68	1045.44	14537.95	1670.26	15379.51	2.78E+05	14969.11	13379.37	15143.28
	10	916.13	15333.31	1045.44	14544.96	1635.90	15332.37	1.70E+05	15124.48	14526.80	15146.42
	Max	1006.54	15370.25	1045.44	14552.18	1887.82	15379.51	2.78E+05	15128.43	14583.77	15321.00
	Min	889.00	14995.33	1045.44	14535.32	1592.93	15106.64	9.89E+04	14966.19	11906.39	15030.48
	Av	936.21	15159.76	1045.44	14541.87	1704.40	15251.94	1.84E+05	15045.13	13417.42	15174.41
	Std	29.87	140.92	0.00	4.73	114.07	87.46	5.46E+04	65.77	879.72	81.21

D=30, QRSM, DE/best/2	functions										
		f11		f12		f13		f14		f15	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	6.75E+07	14989.86	8123.10	14722.39	25196.44	14903.35	2189.45	15134.02	43667.67	15066.50
	2	7.09E+07	15042.57	32598.33	14669.82	28148.52	14818.88	2072.99	15183.47	66558.89	14985.77
	3	1.21E+06	14834.35	8230.31	14728.09	28148.28	14843.15	1901.50	15222.67	137752.21	14949.08
	4	1.45E+08	14972.91	16914.43	14750.42	35375.33	14827.19	1901.51	15342.23	137751.14	15134.62
	5	6.94E+06	14896.99	42795.08	14728.82	28148.58	15017.41	1901.49	15261.38	36418.46	15006.84
	6	1.74E+07	15085.16	5771.38	14814.71	28148.70	14914.27	1901.47	15228.79	59550.55	15006.75
	7	1.16E+08	14878.10	7023.75	14815.48	25287.26	14806.01	2147.04	15214.95	15856.23	15095.09
	8	2.29E+06	14904.94	6201.92	14768.90	48669.63	15012.60	2169.89	15161.21	23063.47	15012.27
	9	128939.48	15002.37	5348.84	14825.17	28148.15	14954.13	2169.84	15203.17	6169.66	15014.22
	10	3.46E+09	15028.62	15310.52	14796.13	28148.53	14893.10	1901.48	15236.65	73249.79	15160.26
	Max	3.46E+09	15085.16	42795.08	14825.17	48669.63	15017.41	2189.45	15342.23	137752.21	15160.26
	Min	128939.48	14834.35	5348.84	14669.82	25196.44	14806.01	1901.47	15134.02	6169.66	14949.08
	Av	3.89E+08	14963.59	14831.77	14761.99	30341.94	14899.01	2025.67	15218.85	60003.81	15043.14
	Std	1.08E+09	81.25	12902.22	50.87	7008.99	77.17	134.36	57.25	46359.64	68.23

D=30, QRSM, DE/best/2

	functions									
	f16		f17		f18		f19		f20	
Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
1	35089.76	15193.02	2.97E+06	14910.56	1900.94	14811.84	4.07E+09	15429.64	5135.81	15318.56
2	120535.78	15154.41	2.12E+07	14988.34	1937.54	14804.18	4.01E+08	15209.69	5006.41	15214.28
3	166914.23	15221.58	2.07E+07	14953.07	1900.99	14798.38	9.56E+09	14941.12	4335.03	15154.34
4	34031.13	15041.55	4.57E+08	14934.88	1901.00	14817.69	1.93E+08	15178.37	4941.59	15285.63
5	81042.99	15199.80	4.33E+07	14996.52	1918.55	14839.42	1.71E+09	15013.24	4223.54	15353.67
6	33552.53	15061.99	1.10E+08	14922.03	1932.93	14850.84	4.01E+08	15036.73	5481.95	15258.52
7	17088.42	15210.75	2.17E+07	14865.46	1901.01	14923.74	1.77E+08	15455.69	5719.57	15344.55
8	120535.79	15191.86	2.79E+08	14942.99	1918.29	14878.67	1.71E+09	15358.78	4788.72	15136.09
9	54546.80	15181.50	2.16E+07	15020.46	1900.99	14879.87	1.43E+08	15214.95	5083.49	15102.89
10	34031.13	15129.54	2.34E+08	15050.91	1868.28	14874.18	1.88E+08	15271.19	4802.96	15494.42
Max	166914.23	15221.58	4.57E+08	15050.91	1937.54	14923.74	9.56E+09	15455.69	5719.57	15494.42
Min	17088.42	15041.55	2.97E+06	14865.46	1868.28	14798.38	1.43E+08	14941.12	4223.54	15102.89
Av	69736.86	15158.60	1.21E+08	14958.52	1908.05	14847.88	1.86E+09	15210.94	4951.91	15266.30
Std	50326.11	62.45	1.53E+08	55.61	19.82	40.92	2.98E+09	175.29	457.57	119.19

		functions									
		f21		f22		f23		f24		f25	
D=30, QRSM, DE/best/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	3501.15	15355.63	15039.09	15665.82	4027.59	15359.53	3803.54	15767.58	95964.57	15089.29
	2	3755.26	15255.62	12936.10	15489.59	4109.11	15418.94	3913.31	15803.37	77559.73	15114.02
	3	3814.39	15186.34	13256.76	15647.91	4937.52	15473.07	3913.31	15738.16	71912.08	15113.45
	4	3648.00	15418.33	12936.10	15601.08	4775.21	15481.71	3703.83	15896.07	58544.06	15126.54
	5	3893.00	15382.87	14619.19	15677.07	4446.95	15753.53	3819.13	16071.08	97084.95	15068.87
	6	3799.46	15398.92	15313.86	15404.76	4051.30	15486.50	3765.22	15812.31	58544.06	15157.99
	7	3799.45	15317.04	15547.21	15794.56	4051.30	15536.03	3803.54	15463.68	58544.06	15154.92
	8	3564.50	15299.41	14916.78	15721.19	4054.81	15596.04	3692.57	15492.26	58544.06	15132.42
	9	3799.45	15300.58	15061.13	15640.69	4002.16	15621.53	3696.88	15517.72	1.39E+05	15148.96
	10	3755.26	15353.43	15353.97	15655.54	4109.11	15620.99	3803.35	15631.44	1.34E+05	15162.99
	Max	3893.00	15418.33	15547.21	15794.56	4937.52	15753.53	3913.31	16071.08	1.39E+05	15162.99
	Min	3501.15	15186.34	12936.10	15404.76	4002.16	15359.53	3692.57	15463.68	58544.06	15068.87
	Av	3732.99	15326.82	14498.02	15629.82	4256.50	15534.79	3791.47	15719.37	85002.18	15126.95
	Std	122.85	70.39	1039.17	111.19	342.31	115.30	80.27	193.98	30991.84	31.09

		functions									
		f26		f27		f28		f29		f30	
D=30, QRSM, DE/best/2	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	54484.65	15166.64	5145.04	15371.07	18938.74	15219.83	1.97E+07	14982.45	5.95E+10	15359.78
	2	65323.82	15176.23	4901.85	15634.94	65823.53	15265.98	4.44E+07	14954.72	5.95E+10	15556.01
	3	56267.89	15188.79	4882.64	15553.60	40221.93	15148.18	2.50E+08	14951.96	5.95E+10	15372.39
	4	22213.54	15085.36	4802.11	15383.40	19288.09	15081.44	5.53E+08	14889.27	5.95E+10	15266.12
	5	63470.07	15164.28	5295.34	15422.66	40221.93	15197.98	3.19E+08	14856.01	5.95E+10	15347.07
	6	43277.92	15188.55	4031.60	15560.73	65823.53	15313.10	1.07E+07	15050.70	5.95E+10	15340.33
	7	56267.89	15260.56	4893.32	15515.39	12673.75	15180.52	4.50E+07	14830.72	5.95E+10	15428.83
	8	54484.65	15224.93	4893.87	15508.31	24151.19	15132.03	1.44E+09	14990.78	5.95E+10	15595.87
	9	17523.32	15153.27	5223.30	15374.24	12078.83	15193.97	7.42E+08	14990.79	5.95E+10	15192.42
	10	56267.89	15232.80	4893.32	15374.24	44476.14	15242.53	3.40E+08	14797.74	5.95E+10	15646.51
	Max	65323.82	15260.56	5295.34	15634.94	65823.53	15313.10	1.44E+09	15050.70	5.95E+10	15646.51
	Min	17523.32	15085.36	4031.60	15371.07	12078.83	15081.44	1.07E+07	14797.74	5.95E+10	15192.42
	Av	48958.17	15184.14	4896.24	15469.86	34369.76	15197.56	3.76E+08	14929.51	5.95E+10	15410.53
	Std	16450.26	48.75	347.19	96.50	20239.97	67.55	4.47E+08	81.84	5.95E+10	146.42

Results illustrated in Table 16, for a DE algorithm using the QRSM model with a problem dimension D=30, show that the optimal solutions are obtained in all 10 runs for the benchmark function f1. The standard deviation is equal to 0, implying that the DE/best/2 is a robust algorithm.

The function f18 is an easy function for this strategy. In 9 runs, the error value exceeds the 100 slightly except for the run number 10 it is equal to 68.28 and its standard deviation is equal to 19.82. This shows that the strategy is still accurate, but it cannot avoid the different local optimal solutions.

For the functions f6 and f7, the error value is bigger than 200 and smaller than 450. The standard deviation of f6 is equal to 29.87. It means that the strategy gives as result many local optimal solutions but the standard deviation of f7 is equal to 0, which shows that the strategy gets stuck in a deep single local optimum.

Considering the different benchmark functions f5, f8 and f14 the error value obtained during their optimization is between 500 and 1100. The standard deviation of f5 is equal to 0 because the strategy gets stuck in a deep local optimum but for the remaining functions it increases importantly that shows the limitation of the used strategy to avoid the local optimal solutions.

For the functions f21, f23 and f24, there is a difference of a thousand and hundreds from the global optimum cost, and their standard deviation values is significant, which means that the strategy cannot avoid the local optimal solutions.

The benchmark function f27 is a special function, its error value vary between 1000 and hundreds or thousands. The standard variation is equal to 347.19 which shows a big variation of the different local optimal solutions obtained by the DE strategy.

During the optimization of the different benchmark functions f4, f9, f10, f12, f13, f15, f16, f20, f22, f25, f26 and f28 have error values of thousands. The standard deviation values of the functions mentioned above are about thousands except for f10, f20 and f22 which are equal to 879.72, 457.57 and 1039.17. These values show that the used strategy has a weakness in providing different local optimal solutions.

For the different remaining functions, the optimization is importantly hard and the error value is about millions and their standard deviation too. These values show that the strategy is not accurate at all and it gives many local optimums.

There is a remarkable variation of the CPU time for all the functions with a standard deviation more than 30 and in some functions it increases until it reaches a values slightly smaller than 200 except for the benchmark function f7 with a CPU standard deviation equal to 4.73.

Table 17: DE+QRSM Results for Dimension 30 and Variant Rand to Best/1

		functions									
		f1		f2		f3		f4		f5	
D=30, QRSM, DE/rand to best/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	100.00	16296.27	2.17E+67	16424.97	1.65E+09	14929.79	1.56E+05	15092.38	1461.23	14991.96
	2	100.00	16289.72	2.17E+67	16361.81	8.33E+11	14968.37	1.95E+05	15341.72	1461.23	15005.31
	3	100.00	16290.73	1.50E+68	16421.07	4.38E+13	15054.41	1.29E+05	15189.07	1461.23	15049.39
	4	100.00	16290.37	2.53E+66	16444.28	2.70E+12	14924.97	1.95E+05	15299.95	1461.23	15007.51
	5	100.00	16287.38	2.17E+67	16346.94	1.37E+11	15163.44	2.43E+05	15382.42	1461.23	15000.63
	6	100.00	16285.72	2.53E+66	16368.76	8.71E+13	15356.32	1.56E+05	15082.44	1461.23	15015.49
	7	100.00	16280.45	6.53E+63	16376.97	9.74E+12	15003.96	2.43E+05	15231.25	1461.23	14979.14
	8	100.00	16275.48	2.17E+67	16379.25	3.33E+06	14887.41	1.56E+05	15131.49	1461.23	14976.42
	9	100.00	16288.81	1.51E+65	16433.05	5.83E+12	14907.85	1.43E+05	15193.89	1461.23	15007.92
	10	100.00	16301.42	1.19E+66	16430.13	1.81E+12	14926.11	1.06E+05	15394.09	1461.23	14992.46
	Max	100.00	16301.42	1.50E+68	16444.28	8.71E+13	15356.32	2.43E+05	15394.09	1461.23	15049.39
	Min	100.00	16275.48	6.53E+63	16346.94	3.33E+06	14887.41	1.06E+05	15082.44	1461.23	14976.42
	Av	100.00	16288.64	2.43E+67	16398.72	1.52E+13	15012.26	1.72E+05	15233.87	1461.23	15002.62
	Std	0.00	7.31	4.53E+67	35.31	2.85E+13	146.57	4.60E+04	115.84	0.00	20.74

		functions									
		f6		f7		f8		f9		f10	
D=30, QRSM, DE/rand to best/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	920.42	15267.35	966.57	16097.85	1404.57	16201.06	1.29E+05	15191.47	12918.67	15175.48
	2	943.46	15309.13	966.57	16098.71	1404.57	16240.05	1.44E+05	14850.72	13376.59	15272.76
	3	874.16	15383.87	966.57	16124.00	1404.57	16227.13	1.52E+05	15001.60	11723.15	15574.87
	4	879.57	15303.64	966.57	16105.78	1404.57	16236.74	1.62E+05	15081.08	13187.24	15559.76
	5	943.46	15363.65	966.57	16096.07	1404.57	16259.94	1.44E+05	14948.83	13624.81	15067.56
	6	920.42	15341.53	966.57	16098.73	1404.57	16263.40	2.07E+05	15128.46	13684.23	15315.43
	7	967.85	16376.84	966.57	16092.31	1404.57	16252.62	1.20E+05	14983.51	14048.33	15089.91
	8	1009.60	15454.47	966.57	16100.16	1404.57	16261.14	1.44E+05	14918.95	12536.37	15081.85
	9	825.89	15336.74	966.57	16115.29	1404.57	16235.41	2.07E+05	15082.60	11665.64	15206.56
	10	956.39	15427.45	966.57	16102.59	1404.57	16228.50	1.20E+05	15005.39	14391.29	15274.09
	Max	1009.60	16376.84	966.57	16124.00	1404.57	16263.40	2.07E+05	15191.47	14391.29	15574.87
	Min	825.89	15267.35	966.57	16092.31	1404.57	16201.06	1.20E+05	14850.72	11665.64	15067.56
	Av	924.12	15456.47	966.57	16103.15	1404.57	16240.60	1.53E+05	15019.26	13115.63	15261.83
	Std	52.88	328.33	0.00	9.62	0.00	19.42	3.14E+04	102.54	917.05	182.61

		functions									
		f11		f12		f13		f14		f15	
D=30, QRSM, DE/rand to best/1	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	1.07E+06	15036.52	6643.87	14847.91	1.21E+05	15066.78	2108.53	16570.10	47278.10	15077.08
	2	4.18E+08	15004.25	12389.99	14872.48	1.21E+05	15096.06	2108.50	16564.08	17650.33	15161.24
	3	4.87E+06	15054.22	8149.50	14801.80	1.11E+05	15063.39	2108.43	16589.07	71333.16	15169.24
	4	259563.77	15018.15	10003.10	14845.71	1.11E+05	15087.11	2108.45	16629.68	42261.93	15131.36
	5	4.86E+06	15156.75	18636.83	14859.56	1.21E+05	15111.44	2108.49	16674.59	28989.37	15179.55
	6	4.86E+06	15162.89	6964.85	14866.66	1.21E+05	15141.00	2108.50	16671.92	17650.26	15181.42
	7	4.93E+06	15298.63	7857.89	14865.07	1.11E+05	15119.94	2108.52	16711.56	44532.02	15239.05
	8	4.89E+06	15123.61	23869.64	14890.14	1.21E+05	15151.91	2108.44	16629.22	42261.87	15203.86
	9	273125.44	15131.80	5653.88	14864.20	1.11E+05	15130.52	2108.52	16694.99	44946.19	15194.09
	10	259563.76	15094.61	23912.18	14812.93	1.11E+05	15156.46	2108.55	16744.85	28990.81	15162.58
	Max	4.18E+08	15298.63	23912.18	14890.14	1.21E+05	15156.46	2108.55	16744.85	71333.16	15239.05
	Min	259563.76	15004.25	5653.88	14801.80	1.11E+05	15063.39	2108.43	16564.08	17650.26	15077.08
	Av	4.45E+07	15108.14	12408.17	14852.65	1.16E+05	15112.46	2108.49	16648.01	38589.40	15169.95
	Std	1.31E+08	87.91	7114.99	27.01	5399.55	33.49	0.04	61.63	16026.13	43.43

D=30, QRSM, DE/rand to best/1	functions									
	f16		f17		f18		f19		f20	
Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
1	8550.47	14738.49	2.80E+08	15019.09	1880.25	14657.93	7.67E+09	15246.80	4507.98	15615.22
2	8550.47	14849.17	4.06E+07	14877.28	1880.15	14673.84	7.41E+09	15149.79	4704.30	15594.81
3	8550.47	14781.64	8.87E+07	15000.34	1880.02	14700.83	7.67E+09	15186.31	4390.06	15321.87
4	11570.63	14845.35	5.00E+08	14988.45	1880.20	14677.71	7.41E+09	15193.51	4606.84	15767.93
5	19637.59	14897.18	1.01E+07	14886.04	1880.68	14671.86	3.83E+09	15358.61	4615.07	15376.41
6	8550.47	14879.89	4.02E+08	15054.03	1880.14	14679.60	1.24E+09	15182.81	4615.07	15477.75
7	48101.88	14928.83	6.85E+06	15048.01	1880.18	14720.71	7.95E+09	15211.45	4615.07	15829.28
8	8550.47	14793.17	1.01E+07	14879.70	1904.00	14735.50	1.00E+09	15226.32	5172.76	15309.53
9	8550.47	14824.29	3.43E+07	14983.94	1880.39	14726.82	1.05E+10	15131.33	4615.07	15181.55
10	8550.47	14823.01	1.01E+07	15034.58	1880.21	14719.47	7.41E+09	15316.29	4615.07	15481.04
Max	48101.88	14928.83	5.00E+08	15054.03	1904.00	14735.50	1.05E+10	15358.61	5172.76	15829.28
Min	8550.47	14738.49	6.85E+06	14877.28	1880.02	14657.93	1.00E+09	15131.33	4390.06	15181.55
Av	13916.34	14836.10	1.38E+08	14977.15	1882.62	14696.43	6.21E+09	15220.32	4645.73	15495.54
Std	12509.70	56.91	1.85E+08	70.29	7.51	27.53	3.12E+09	70.97	203.24	207.58

D=30, QRSM, DE/rand to best/1		functions									
		f21		f22		f23		f24		f25	
	Exp	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)	Cost	CPU (s)
	1	4018.76	14969.37	12899.75	15233.93	3932.34	15160.77	3712.50	15608.20	1.52E+05	15721.00
	2	4093.23	15079.13	13352.01	15178.43	4191.77	15572.80	3712.50	15650.75	1.87E+05	15670.27
	3	3804.80	14932.23	14338.55	15187.73	4190.23	15436.56	3791.20	15472.23	1.52E+05	15697.38
	4	4018.76	15025.13	15371.94	15229.85	4070.46	15668.13	4038.59	15587.68	2.14E+05	15636.34
	5	3997.14	15134.31	14866.05	15893.54	4225.95	15692.81	3791.20	15507.86	2.14E+05	15726.69
	6	3997.34	15071.82	14172.65	15298.66	4280.54	15564.55	3712.50	15650.60	1.52E+05	15782.84
	7	3770.17	15064.83	13352.01	15216.31	4325.48	15267.83	3791.20	15597.81	1.52E+05	15791.84
	8	4084.74	15202.11	12552.98	15308.07	4164.58	15378.30	3712.50	15670.24	1.87E+05	15704.71
	9	3634.33	15287.90	12697.31	15700.88	4234.71	15407.38	3712.50	15596.95	1.13E+05	15774.06
	10	3886.60	15265.40	13352.01	15268.57	4325.48	15951.82	4038.59	15492.07	1.52E+05	15776.83
	Max	4093.23	15287.90	15371.94	15893.54	4325.48	15951.82	4038.59	15670.24	2.14E+05	15791.84
	Min	3634.33	14932.23	12552.98	15178.43	3932.34	15160.77	3712.50	15472.23	1.13E+05	15636.34
	Av	3930.59	15103.22	13695.52	15351.60	4194.15	15510.10	3801.33	15583.44	1.67E+05	15728.20
	Std	151.26	119.00	949.31	242.92	119.89	229.60	130.11	69.94	3.21E+04	52.55

		functions									
		f26		f27		f28		f29		f30	
D=30, QRSM, DE/rand to best/1	Exp	Cost	CPU	Cost	CPU	Cost	CPU	Cost	CPU	Cost	CPU
	1	29083.59	15342.79	6268.49	15255.71	50509.94	15060.76	6.07E+07	14820.47	1.85E+10	15034.36
	2	19238.62	15299.94	4404.94	15074.22	34917.04	15175.39	7.23E+07	14842.82	4.82E+10	14971.98
	3	23056.08	15528.76	4913.00	15060.94	21463.20	14997.62	6.69E+07	15062.38	4.82E+10	14958.76
	4	26552.13	15435.72	4361.13	15104.11	50509.94	15153.84	7.23E+07	14856.71	1.86E+10	15638.80
	5	24793.29	15481.54	4468.96	15088.42	62365.07	15153.44	7.23E+07	14974.72	4.82E+10	14981.32
	6	25105.90	15369.39	4285.40	15131.72	30229.67	15055.78	8.47E+07	14993.38	1.85E+10	15226.13
	7	24793.29	15399.87	4201.45	15144.89	43458.63	15056.41	2.56E+06	14954.86	4.82E+10	15047.23
	8	19237.86	15433.24	4361.13	15038.84	27558.82	15072.24	8.44E+06	14940.64	1.85E+10	15053.64
	9	36754.38	15435.50	6785.77	15208.63	21307.22	15002.21	1.23E+08	15157.24	1.85E+10	15058.39
	10	18374.14	15317.43	4361.13	15018.50	32099.93	15112.85	1.22E+07	15051.34	1.85E+10	15207.40
	Max	36754.38	15528.76	6785.77	15255.71	62365.07	15175.39	1.23E+08	15157.24	4.82E+10	15638.80
	Min	18374.14	15299.94	4201.45	15018.50	21307.22	14997.62	2.56E+06	14820.47	1.85E+10	14958.76
	Av	24698.93	15404.42	4841.14	15112.60	37441.95	15084.05	5.76E+07	14965.46	3.04E+10	15117.80
	Std	5482.47	72.81	916.65	74.69	13740.30	62.60	3.84E+07	107.00	1.53E+10	204.39

Results illustrated in Table 17, for a DE algorithm using the QRSM model with a problem dimension D=30, show that the optimal solutions are obtained in all 10 runs for the benchmark function f1. The standard deviation is equal to 0 that means the DE/rand to best/1 is a robust algorithm.

The function f18 is an easy function and the algorithm steal robust with an error value around 80 except for the run 8 it reaches 104. The standard deviation for this function is not really important with a value equal to 7.51. This value is due to the variation in run number 8 because in the all other runs the obtained solution costs is very close 1080 which means that the strategy is stuck in a deep local optimum.

For the benchmark function f6 the error value is between 225 and 409 but for the function f7 it does not vary and keeps a single error value equal to 266.57. The f6 standard deviation is equal to 52.88 which shows that the strategy cannot avoid the different local optimal solutions but for f7 the standard deviation is equal to 0 because the strategy gets blocked in a deep local optimum.

The error value of f5, f8 and f14 increased slightly and it becomes between 600 and 1000 which means that the difficulty of the functions increased too. Their standard deviation is equal to 0 except for f14 which is very close to 0 with a value equal to 0.04 which means that the strategy get stuck in a deep local optimum for the 3 different functions.

For the both functions f21 and f24 the difference from the global optimum cost is around 1000 and hundreds. Their standard deviation overpassed 100 which is an important value showing a variation in the different local optimal solutions obtained by the strategy.

For f23 and f27 the variation and the value of the error value increases and it reaches values more than 2000 in some runs. The standard deviation value is important for f23 it is equal to 119.89 and for f27, it is equal to 916.65 these values shows that strategy cannot again avoid local optimal solutions.

Considering the benchmark functions f4, f9, f10, f12, f13, f15, f16, f20, f22, f25, f26 and f28, during their optimization the difference from the global optimum cost increased remarkably and becomes about thousands. The standard deviation of the function is important, it is generally equal to thousands except for the functions f10, f20 and f22 which is around hundreds. This values shows that the strategy failed again to avoid the deep local optimum solution but in some functions such as f16 and f20 a same local optimum is obtained many times, 7 and 5 times.

A huge error value equal to millions is obtained for the benchmark functions f2, f3, f11, f17, f19, f29 and f30 which shows the hardness of the functions. Also, their standard deviation is very high equal to millions that display again the failure of the strategy to find an acceptable solution and avoid the different local optimal solutions.

There is a variation of the CPU time and its importance vary from a function to another. For the functions f1 and f7 is negligible this is shown by the standard deviation value which is smaller than 10. For the functions f2, f5, f8, f12, f13, f14, f15, f16, f17, f18, f19, f24, f25, f26, f27 and f28 it increases but still not important and their standard deviation does not reach 100. For the remaining functions the CPU time vary importantly and their standard deviation value is overpassing the 100 and in some cases it even overpasses the 200 but not with a big difference.

To compare the different strategies there is two ways, using the CPU time to have an idea about the fastest DE strategy and using the cost average solution value to have an idea about the most accurate one.

From a speed side, according to results illustrated in Table 14, Table 15, Table 16 and Table 17, by using QRSM model, for D=30, for 28 benchmark functions rand/1 strategy is the fastest one with a CPU average values equal to 162.77, 165.18, 165.12, 165.29, 168.03, 161.42, 166.82, 165.81, 169.24, 168.06, 165.63, 165.24, 168.75, 165.99, 165.18, 165.15, 163.45, 165.85, 170.83, 166.52, 170.11, 169.98, 171.29, 165.93, 167.10, 165.60, 166.75 and 167.19 but it is in second position for the 2 functions f2 and f26 with a CPU average equal to 171.50 and 170.20. The second fastest strategy is rand/2 for 28 functions too with a respective CPU averages equal to 166.70, 170.91, 170.91, 170.15, 173.69, 165.45, 174.21, 169.57, 172.55, 170.27, 170.25, 171.52, 171.74, 173.09, 170.21, 169.50, 172.01, 170.91, 172.43, 168.80, 171.53, 174.31, 171.77, 168.90, 172.32, 172.99, 170.88 and 171.18 but it is the best one for both function f2 and f26 with CPU averages equal to 168.22 and 169.88. For the remaining strategies they are alternating from a fastness point of view. The strategy best/2 is in third position for the benchmark functions f1, f2, f4, f5, f6, f7, f8, f10, f11, f12, f13, f14, f15, f17, f19, f20, f25, f26 and f29 with CPU averages equal to 14881.30, 15460.29, 15224.61, 14940.74, 15159.76, 14541.87, 15251.94, 15174.41, 14963.59, 14761.99, 14899.01, 15218.85, 15043.14, 14958.52, 15210.94, 15266.30, 15126.95, 15184.14 and 14929.51, but it is the slowest for the remaining benchmark functions with CPU average values equal to 15148.61, 15045.13, 15158.60, 14847.88, 15326.82, 15629.82, 15534.79, 15719.37, 15469.86, 15197.56 and 15410.53. The strategy rand to best/1 is opposite of best/2 it is in third position for the benchmark functions f3, f9, f16, f18, f21, f22, f23, f24, f27, f28 and f30 with a CPU average equal to 15012.26, 15019.26, 14836.10, 14696.43, 15103.22, 15351.60, 15510.10, 15583.44, 15112.60, 15084.05 and 15117.80 but the worst one for the remaining functions with CPU averages equal to 16288.64, 16398.72, 15233.87, 15002.62, 15456.47, 16103.15, 16240.60, 15261.83, 15108.14, 14852.65, 15112.46, 16648.01, 15169.95, 14977.15, 15220.32, 15495.54, 15728.20, 15404.42 and 14965.46.

From an accuracy side, according to results illustrated in Table 14, Table 15, Table 16 and Table 17, by using QRSM model, for D=30, the optimal solution for function f1 is obtained by 3 strategies except for the strategy rand/2 with a cost average of 5.51E+07.

The strategy rand/1 is the most accurate strategy for 9 benchmark functions which are f2, f5, f9, f15, f17, f21, f26, f28 and 30 with a cost average equal to 1.23E+66, 1350.51, 128535.64, 19062.47, 4.83E+07, 3708.24, 19608.10, 33006.37 and 2.60E+10. For the 6 functions f3, f6, f14, f16, f23 and f29 it becomes in second position with a cost average of 2.21E+13, 911.98, 2012.04, 31412.32, 4132.76, 1.67E+08. It is in third position for the 8 benchmark functions f4, f7, f8, f12, f13, f18, f19 and f25 with an average equal to 178304.73, 1047.76, 1676.66, 15157.73, 62848.50, 1945.27, 5.38E+09 and 118091.74. This strategy is the worst for the 6 remaining benchmark functions which are f10, f11, f20, f22, f24 and f27 with the respective cost averages equal to 12632.22, 5.55E+08, 5283.00, 14636.60, 3971.85 and 5060.55.

The strategy rand/2 is the best strategy for the 6 benchmark functions f6, f13, f14, f19, f23 and f27 their cost average are 878.04, 25061.05, 1986.84, 4.76E+08, 4130.44 and 4582.53. It is in second position for the different 11 benchmark functions f2, f8, f9, f10, f11, f15, f17, f20, f22, f25 and f26 with cost averages of 1.65E+66, 1699.15, 146006.13, 13209.54, 3.40E+08, 36951.66, 9.44E+07, 4741.63, 14328.52, 101203.77 and 23708.07. Also, for the 6 benchmark functions f5, f16, f21, f24, f28 and f30 it becomes in third position with an averages equal to 1393.27, 34461.42, 3780.22, 3817.40, 36711.82 and 5.18E+10. For the remaining 7 benchmark functions 5.51E+07, 1.39E+14, 207099.07, 1052.80, 3.68E+06, 1956.56 and 1.04E+09.

The strategy best/2 is the best strategy for only 3 benchmark functions which are f4, f24 and f25 with a cost averages of 127125.72, 3791.47 and 85002.18. This strategy is also the second best strategy for the 8 benchmark functions f5, f7, f12, f13, f18, f19, f21 and f28 with these respective cost averages values 1381.01, 1045.44, 14831.77, 30341.94, 1908.05, 1.86E+09, 3732.99 and 34369.76. For the 10 functions f2, f3, f10, f11, f14, f17, f20, f22, f27 and f29 with these averages 8.92E+74, 3.85E+13, 13417.42, 3.89E+08, 2025.67, 1.21E+08, 4951.91, 14498.02, 4896.24 and 3.76E+08 and it is the worst strategy for 8 benchmark functions which are f6, f8, f9, f15, f16, f23, f26 and f30 with these cost averages 936.21, 1704.40, 183651.09, 60003.81, 69736.86, 4256.50, 48958.17 and 5.95E+10.

The last DE strategy is rand to best/1 which is the most accurate for the majority of functions. These 11 functions are f3, f7, f8, f10, f11, f12, f16, f18, f20, f22 and f29 with these costs 1.52E+13, 966.57, 1404.57, 13115.63, 4.45E+07, 12408.17, 13916.34, 1882.62, 4645.73, 13695.52 and 5.76E+07. It is also in second position for only 2 functions f4 and f24 with a cost average of 172256.97 and 3801.33. For the 5 benchmark functions f6, f9, f15, f23 and f26 this strategy is in third position with a cost average values equal to 924.12, 152979.59, 38589.40, 4194.15 and 24698.93 but it is the least accurate strategy for the 9 remaining functions with a cost average values equal to 2.43E+67, 1461.23, 116046.13, 2108.49, 1.38E+08, 6.21E+09, 3930.59, 167318.73 and 37441.95.

The DE strategy rand/1 is the fastest for f28 functions in second position there is the strategy rand/2, after there is best/2 and the slowest one is rand to best for 19 functions. From an accuracy point of view both rand/1 is the best strategy, after there are rand to best/1 and rand/2 and the worst is best/2 strategy. The rand/1 strategy is the most accurate for 10 functions and the second most accurate for 6 functions also rand to best/2 is the most accurate for 12 functions and in second position for 2 functions in the opposite of rand/2 which is the best for only 6 functions but the second best one for 11 functions. The worst strategy is best/2 with only 4 functions in first position and 8 functions in second position. The strategy rand/1 is the best strategy because it is mostly accurate and also the fastest.

3.2 DE+Objective Function vs DE+QRSM

According to results in Tables 3.1.1 and 3.1.9, for D=10, for the DE strategy rand/1, when DE uses the objective function, it is more accurate than DE using QRSM for all benchmark functions except for f1 because the global optimum is found by both for the 10 different runs. The DE algorithm is faster than the DE using the fitness function for all functions except for 3 functions which are f3, f4 and f5.

For a dimension equal to 10, from Tables 3.1.2 and 3.1.10, the results shows that for the function f1 both DE rand/2 with objective function and DE rand/2 with QRSM finds the optimal solution for 10 different experiments but for the 29 other functions the strategy using the objective function is more accurate that the strategy using the surrogate model. By using the QRSM model the CPU time is improved except for 4 benchmark functions which are f2, f3, f4 and f5.

The results of D=10 from both Tables 3.1.3 and 3.1.11 shows that when the strategy best/2 is using the objective function it is more accurate for almost of function except for f1 because using QRSM also gives a global optimal solution for the 10 runs, but it loses in CPU time except for the benchmark functions f1, f2, f3, f4 and f5 where it is faster.

The different results mentioned in Table 3.1.4 and 3.1.12, for D=10 and the strategy rand to best/1 shows that for the benchmark function f1 both objective function and QRSM model find the global optimum for the 10 runs. For the other 29 functions using the objective function gives more accuracy than the QRSM model but using the surrogate model answers faster except for the 5 first functions.

According to the results in the Table 3.1.5 and 3.1.13, for a dimension equal to 30 the DE strategy rand/1 gives the global optimum for f1 during the 10 experiments when it uses the objective function or the QRSM model. For the remaining functions using the fitness function is better than the QRSM model because the results are closer to the global optimum. For the benchmark functions f1, f2, f3, f4, f5, f6, f7, f10, f11, f12, f13, f14, f16, f17, f18 and 19, using the fitness functions is faster than QRSM but the DE strategy with QRSM model is faster for the remaining functions.

For the strategy rand/2 according to results mentioned in Table 3.1.6 and 3.1.14, for D=30, the DE strategy using the objective function is more accurate for all functions and also faster for the benchmark functions from f1 to f21 than the DE strategy using QRSM model.

The results for strategy best/2 and D=30 mentioned in both Tables 3.1.7 and 3.1.15 shows that the global optimum is found by using the fitness function or QRSM during all experiments. By using the fitness function, the results are more accurate than using QRSM for all functions except in f12. It is also faster for the benchmark functions from f1 to f21 and the function f24.

The results in Table 3.1.8 and 3.1.16, for the strategy rand to best/1 and D=30 show that for f1, both using objective function or QRSM, the global optimum is always found. For the remaining functions using the objective functions is more accurate except for f12 and it is also faster for the benchmark functions from f1 to 15 and from f17 to f21.

3.3 Comparison with Other People Work

L-SHADE [47] algorithm is based on DE and SHADE [48]. For the step of mutation it is using the strategy Current-to-Pbest/I/BIN to produce the donor vector it consist to select one vector randomly, to use the current vector and to select one vector from a specific number of best vectors according to a p probability between [2/N, 0.2] where N is the population size. Also a vector is selected from the union of two sets which are the population and a memory set containing the parent which had a worse cost than their child. This 4 vectors should be different and they are used with F to find the value of the donor vector. This algorithm is also provided by a memory which contains the successful F and CR (F: mutation Factor, CR: crossover factor), which were generated in the iteration where the children overpassed their parents. These two factors are generated and updated by choosing randomly M_{CR} and M_F for each parent. M_{CR} and M_F are initialized to 0.5 and updated by using Lehmer mean after each generation if a best F and CR are stored in memory S_F and $S_{CR.}$ The all mentioned steps are from SHADE algorithm, L-SHADE just added a step which consist to minimize linearly the population size based on the number of fitness evaluations after the end of each generation.

JSO [49] is based in iL-SHADE [50] which is very similar to L-SHADE with some differences. In iL-SHADE the M_{CR} and M_F are initialized to 0.8 instead of 0.5. When they are picked randomly to calculate CR and F if the last element in the memory is chosen then its value will be changed to 0.9. F and CR are updated according to the generation number. To update M_{CR} and M_F a weighted Lehmar mean is used with an equally waited values of M_{CR} and M_F of the current generation. The last change is the value of p is calculated by using the maximal value of p the minimal value of p the maximal number and the current number of fitness evaluations. The only modification in iL-SHADE to obtain JSO is a new mutation strategy which is DE/current-to-p Best-w/1, in this algorithm a new crossover factor is used with F called $F_{W,}$ its value is calculated by using F and the number of evaluated fitness function in beginning it will make the best vector not very important and with time its importance will increase.

DISH [51] is based on JSO, the value of p is calculated by using p_{max}, p_{min} and the ratio of FES which is the number of evaluated fitness functions and MAXFES which is the maximal number of evaluated fitness functions. This ratio is called $FES_{ratio.}$ The update of F, F_w, M_{CR} and the linear reduction of the generation will also depend on FES_{ratio}.

During the calculation of MCR and MF the mean is needed to be calculated the Linear Bias Reduction LBR [6] is used in the calculation of mean to influence its nature by updating its parameters. This procedure will also influence on the value of F and CR through MCR and MF to make a balance between exploration and exploitation principle according to the need. This method was used with L-SHADE, JSO and DISH and new algorithms were obtained called L-SHADE-LBR, JSO-LBR and DISH-LBR.

Table 18: Error Values of L-SHADE, JSO, DISH, L-SHADE-LBR, JSO-LBR, DISH-LBR, DE/Rand/1 and DE/Rand/1+QRSM, D=10

Function	L-SHADE	JSO	DISH	L-SHADE-LBR	JSO-LBR	DISH-LBR	DE/rand/1	DE/rand/1+QRSM
f1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
f2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.57E+18
f3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.32E+08
f4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2610.00
f5	1.46	1.82	2.03	1.93	1.83	1.78	23.98	112.72
f6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	96.31
f7	11.65	12.06	12.03	12.08	12.16	12.02	34.48	108.41
f8	1.33	1.88	1.85	2.19	1.85	2.26	23.41	134.98
f9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.10E+04
f10	22.24	18.23	33.78	34.81	54.87	29.37	358.32	3650.99
f11	0.12	0.15	0.00	0.00	0.00	0.00	0.20	3.63E+05
f12	15.26	0.27	16.74	7.40	16.74	2.64	1.41	1308.90
f13	2.44	2.47	3.04	2.80	2.68	3.29	3.55	1098.98
f14	0.05	0.05	0.04	0.16	0.15	0.12	0.10	247.09
f15	0.13	0.12	0.34	0.24	0.29	0.24	0.01	6740.91
f16	0.45	0.36	0.54	0.72	0.51	0.55	0.26	1797.53
f17	0.15	0.14	0.93	0.46	0.59	0.54	4.48	1087.14
f18	0.22	0.14	0.20	0.22	0.59	0.23	0.12	35.22
f19	0.01	0.01	0.01	0.01	0.01	0.00	0.00	1.57E+07
f20	0.01	0.01	0.37	0.81	0.37	0.40	2.43	1296.83
f21	145.07	150.60	138.58	132.33	136.32	144.46	138.58	635.68
f22	100.06	100.01	100.00	100.01	100.01	100.03	100.04	3419.90
f23	301.09	302.48	301.07	301.02	295.59	301.40	312.91	695.75
f24	259.22	266.65	274.88	301.87	265.69	265.44	351.78	653.78
f25	423.76	409.51	409.51	401.51	405.97	406.86	402.71	1578.22
f26	300.00	294.12	300.00	300.00	300.00	300.00	300.00	3633.08
f27	389.39	389.23	389.42	389.47	389.40	389.45	389.28	767.12
f28	423.13	357.84	329.47	329.47	336.13	341.15	347.69	1034.09
f29	233.27	233.49	234.72	235.01	234.58	233.74	239.01	6164.36
f30	32442.80	394.51	16418.68	324409.30	16417.71	394.52	413.59	3.29E+09

A comparison study is done between 6 algorithms from the reference [6], DE/rand/1 and DE/rand/1+QRSM. The table below contains the error value for the different algorithms.

According to the Table 18, for D=10 the algorithms L-SHADE, JSO, DISH, L-SHADE-LBR, JSO-LBR, DISH-LBR, DE/rand/1 and DE/rand/1+QRSM gave an error value of 0 which means that the global optimum is obtained for f1. For the functions f2, f3, f4, f6 and f9 all algorithms gave the optimal solution except DE/rand/1+QRSM with an error rate very important, for some functions such as f2, f3 and f9, it decreased for f4 and becomes small for f6 with a value of 96.31. From f15, 16, f18 and f19 DE/rand/1 is the most accurate, with an error value 0 or very close to 0. The other algorithms doesn’t have a big difference in accuracy but DE/rand/1+QRSM has an important error value in majority of functions except for f18 and f14 with an error value of 35.22 and 247.09 it is classified as the worst optimizer. For the functions f5, f7, f8, f10, f23, f24 and f25 DE/rand/1 and DE/rand/1+QRSM are the worst algorithms but DE/rand/1 is better. The best algorithm for functions 5, 7, 8 and 24 is L-SHADE, for f10, JSO is the best, for f25 the best algorithm is L-SHADE-LBR and the algorithm JSO-LBR performed the best for f23. JSO is the most accurate for functions f26, f27 and 30 and DE/rand/1+QRSM is the worst with an important error value especially for f30 with an error value of 3.29E+09. Also, for the different functions f21, f22 and f28 DISH is in first position and as always DE/rand/1+QRSM is the worst. For the remaining function f29 QRSM is the worst and L-shade is the best.

For all functions the error value of DE/rand/1 is not really important and the DE/rand/1+QRSM overpass importantly other algorithms error functions.

Table 19: Error Values of L-SHADE, JSO, DISH, L-SHADE-LBR, JSO-LBR, DISH-LBR, DE/Rand/1 and DE/Rand/1+QRSM, D=30

Function	L-SHADE	JSO	DISH	L-SHADE-LBR	JSO-LBR	DISH-LBR	DE/rand/1	DE/rand/1+QRSM
f1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
f2	0.00	0.00	0.00	0.00	0.00	0.00	1.38E+06	1.23E+66
f3	0.00	0.00	0.00	0.00	0.00	0.00	77.26	2.21E+13
f4	54.33	58.56	58.56	58.56	58.67	58.56	58.56	1.78E+05
f5	7.09	5.98	8.57	7.14	8.17	7.30	181.12	850.51
f6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	311.98
f7	3.64	36.35	39.02	37.29	38.44	37.84	202.90	347.76
f8	7.63	5.95	8.61	7.34	8.46	7.32	181.52	876.66
f9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.28E+05
f10	1541.92	1588.64	1633.72	1552.08	1524.14	1568.43	6623.53	1.16E+04
f11	14.81	12.86	6.87	4.03	4.31	7.47	59.16	5.55E+08
f12	966.57	213.18	235.11	81.06	99.96	67.31	6558.65	1.40E+04
f13	15.07	17.47	16.16	15.52	16.11	16.84	82.01	6.15E+04
f14	21.34	21.37	21.95	21.72	21.95	22.03	77.13	612.04
f15	2.47	1.52	1.21	1.18	1.17	1.21	33.64	1.76E+04
f16	62.28	35.57	49.85	54.67	35.28	36.23	550.71	2.98E+04
f17	34.86	33.16	35.05	34.49	33.89	33.55	117.96	4.83E+07
f18	21.46	20.78	20.81	20.84	20.06	20.45	35.83	245.27
f19	5.08	5.46	5.01	4.82	4.08	4.43	22.35	5.38E+09
f20	33.23	30.11	29.57	29.73	27.04	26.80	52.55	3283.00
f21	206.87	206.40	208.71	208.35	208.56	208.07	368.45	1608.00
f22	100.00	100.00	100.00	100.00	100.00	100.00	717.97	1.24E+04
f23	348.59	348.33	350.99	350.53	350.72	349.82	528.21	1832.76
f24	423.77	425.15	426.26	426.35	425.92	426.22	596.53	1571.85
f25	386.75	386.70	386.70	386.70	386.70	386.69	386.75	9391.84
f26	924.55	925.48	932.72	956.12	931.31	937.16	2692.16	1.70E+04
f27	501.68	492.66	498.14	493.27	493.27	489.96	494.84	2360.55
f28	311.49	306.49	321.72	304.26	308.52	300.00	332.05	3.02E+04
f29	435.40	439.24	434.84	434.82	434.01	435.01	549.84	1.67E+08
f30	1994.91	1969.64	1975.66	1969.24	1972.83	1966.70	2058.61	2.60E+10

From the results in Table 19, for D=30, the error value of all algorithms is equal to 0 for f1. For all other benchmark functions DE/rand/1+QRSM is the least accurate, It is error value in majority of time has a big difference from the other algorithms error values, there is about 60% of the functions. DE/rand/1 is the algorithm directly before the last algorithm in accuracy for all functions except f1, f4 and f27. For these mentioned functions, it is in first position, in second position with JSO, DISH, L-SHADE-LBR and in fourth position. The error value of DE using rand/1 strategy is close to error values of other algorithms for majority of functions with a percentage of 70%. For the function f22 the 6 algorithms L-SHADE, JSO, DISH, L-SHADE-LBR, JSO-LBR and DISH-LBR gave the minimal error value of 100. For the different benchmark functions f4, f7, f13, f14, f24 and f26 the algorithm L-SHADE is the most performed one. In first position for the functions f5, f8, f17, f21 and f23, there is the algorithm JSO. L-SHADE-LBR is the best for also f11. JSO-LBR

Is the best for the functions f10, f15, f16, f18, f29 and DISH-LBR is the most accurate for f12, f20, f25, f27, f28 and f30.

Chapter 4

CONCLUSION

According to the obtained results for the algorithm DE with its different strategies which are rand/1, rand/2, best/2 and rand to best/1 the best strategy to use generally is rand/1 because in majority of functions it was the variant who had the best accuracy also it was the fastest one for both dimensions 10 and 30.

DE algorithm is affected by the size of the problem, when the dimension was equal to 10 it obtained in many cases the optimal solution and quit good solutions with a minimal error value. Also, the strategy rand/1 performed better than L-SHADE, JSO, DISH, L-SHADE-LBR, JSO-LBR, DISH-LBR in several instances or equivalently. However, for dimension 30 the performance decreased by around 70%, the strategy rand/1 wasn’t classified as best for any of the benchmark functions except for f1 comparing to the algorithms mentioned below, which shows the vital affection of the problem dimension in DE algorithm.

Using QRSM as a surrogate model can give interesting results and can sometimes give results close to the DE algorithm using objective functions but in majority of time the error value is very high. Comparing to the all DE strategies with objective functions and with L-SHADE, JSO, DISH, L-SHADE-LBR, JSO-LBR, DISH-LBR, using QRSM by DE gave the worst results for all dimensions, except for f12 when the strategies best/2 and rand to best/1 were used it performed better than DE with objective functions. DE + QRSM is faster than DE with objective functions in all functions for D=10 except for 2 functions. For D=30 when QRSM where used the CPU time becomes more important than when the objective function where used for more than 50% of the functions. There is an anomaly for some functions in CPU, it is caused by the use of heterogeneous computers.

For future research it is advised to use computers with equal processors power to obtained results doesn’t containing speed anomaly and to have a more important accuracy with surrogate models it is better to use weighted average surrogate models.

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