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Effect of the Born–Infeld parameter in higher dimensional Hawking radiation

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ABSTRACT

We show in detail that the Hawking temperature calculated from the surface gravity is in agreement with the result of exact semi-classical radiation spectrum for higher dimensional linear dilaton black holes in various theories. We extend the method derived first by Clément–Fabris–Marques for 4-dimensional linear dilaton black hole solutions to the higher dimensions in theories such as Einstein–Maxwell dilaton, Einstein–Yang–Mills dilaton and Einstein–Yang–Mills–Born–Infeld dilaton. Similar to the Clément–Fabris–Marques results, it is proved that whenever an analytic solution is available to the massless scalar wave equation in the background of higher dimensional massive linear dilaton black holes, an exact computation of the radiation spectrum leads to the Hawking temperature T_H in the high frequency regime. The significance of the dimensionality on the value of T_H is shown, explicitly. For a chosen dimension, we demonstrate how higher dimensional linear dilaton black holes interpolate between the black hole solutions with Yang–Mills and electromagnetic fields by altering the Born–Infeld parameter in aspect of measurable quantity T_H. Finally, we explain the reason of, why massless higher dimensional linear dilaton black holes cannot radiate.

1. Introduction

Although today there are several methods to compute the Hawking radiation (see for instance [1–6], and references therein), it still attracts interest to consider alternative derivations. On the other hand, none of them is completely conclusive. Nevertheless, the most direct is Hawking's original study [1], which computes the Bogoliubov coefficients between in and out states for a realistic collapsing black hole. The most significant remark on this study is that a black hole can emit particles from its event horizon with a temperature proportional to its surface gravity. Another elegant contribution was made to the Hawking radiation by Unruh [7]. He showed that it is possible to obtain the same Hawking temperature T_H, when the collapse is replaced by appropriate boundary conditions on the horizon of an eternal black hole. Instead of computing the Bogoliubov coefficients in order to obtain the black hole radiation, one may alternatively compute the reflection and transmission coefficients of an incident wave by the black hole. This method works best if the wave equation can be solved, exactly. From now on, we designate this method with "semi-classical radiation spectrum method" and abbreviate it as *SCRSM*.

Recently, Clément et al. [8] have studied the *SCRSM* for a class of non-asymptotically flat charged massive linear dilaton black holes. The metric of the associated linear dilaton black holes is a solution to the Einstein–Maxwell dilaton (EMD) theory in 4-dimensions. It is shown that in the high frequency regime, the *SCRSM* for massive black holes yield the same temperature with the surface gravity method. Their result for a massless black hole is in agreement with the fact that a massless object cannot radiate.

In this Letter, we shall extend the application of *SCRSM* to linear dilaton black hole solutions in Einstein–Maxwell dilaton (EMD) [9], Einstein–Yang–Mills dilaton (EYMD) [10] and Einstein–Yang–Mills-Born–Infeld dilaton (EYMBID) [11] theories in higher dimensions. The spacetimes describing these black holes are charged, dilatonic and non-asymptotically flat. First, we introduce a generic line-element of higher dimensional linear dilaton black holes in which the metric functions are apt for the EMD, EYMD and EYMBID theories, where the latter two are presented recently [10,11]. Next, we consider the statistical T_H of the massive linear dilaton black holes computed by using the surface gravity and discuss their evaporation processes. According to the Stefan's law, we show that higher dimensional linear dilaton black holes evaporate in an infinite time. In the meantime, during the evaporation process, the Hawking temperature remains constant for a given dimension. Besides this, the constant value of T_H increases with the dimensionality N. We then apply the *SCRSM* to the massive linear dilaton black holes and show that this computation exactly matches with the statistical T_H in the high frequency regime. Finally, we answer the question, why the massless extreme black holes do not radiate, by establishing a connection between our work and [8].

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The organization of the our Letter is as follows. In Section 2, we review briefly the higher dimensional linear dilaton black hole solutions in the EMD, EYMD and EYMBID theories. In Section 3, the evaporation of these black holes are discussed according to the Stefan's law. Section 4 is devoted to the analytical computation of the T_H via the *SCRSM* for the massive higher dimensional linear dilaton black holes. We plot some graphs to compare the results acquired from each theory. We draw our conclusions in Section 5.

2. Higher dimensional linear dilaton black holes in EMD, EYMD and EYMBID theories

The metric ansatz for static spherically symmetric solutions representing N-dimensional ($N \geqslant 4$) linear dilaton black holes can be introduced by

$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + h^2\,d\Omega_{N-2}^2, \tag{1}$$

where f and h are only functions of r and the spherical line element is

$$d\Omega_{N-2}^2 = d\theta_1^2 + \sum_{i=2}^{N-2}\prod_{j=1}^{i-1}\sin^2\theta_j\,d\theta_i^2, \tag{2}$$

in which $0 \leqslant \theta_k \leqslant \pi$ with $k = 1, \dots, N-3$, and $0 \leqslant \theta_{N-2} \leqslant 2\pi$. Here, we set a proper ansatz for the metric functions h as

$$h = Ae^{-\frac{2\alpha\Phi}{N-2}}, \tag{3}$$

where Φ is the dilaton field, α is the dilaton parameter and A is a coefficient to be determined for the respective theory. In the present Letter, dilaton parameter α for linear dilaton black holes is chosen by

$$e^{-\frac{2\alpha\Phi}{N-2}} = \sqrt{r} \rightarrow h = A\sqrt{r}. \tag{4}$$

The field equations, which are obtained from the action of the theory together with metric (1) suggest that the general form of the metric function f is

$$f = \Sigma r\left[1 - \left(\frac{r_+}{r}\right)^{\frac{N-2}{2}}\right], \tag{5}$$

where Σ is another coefficient to be determined for each theory. From now on, r_+ will be interpreted as the event horizon of the black hole. By following the mass definition for the non-asymptotically flat black holes, the so-called quasi-local mass M introduced by Brown and York [12], one can see that the horizon r_+ is related to the mass M and the dimension N through

$$r_+ = \left[\frac{8M}{(N-2)\Sigma A^{N-2}}\right]^{\frac{2}{N-2}}. \tag{6}$$

Higher dimensional linear dilaton black holes to the EMD theory was found long time ago by Chan et al. [9]. The solution is obtained from the following N-dimensional EMD action

$$I = \frac{-1}{16\pi}\int d^Nx\sqrt{-g}\left(R - \frac{4}{N-2}(\nabla\Phi)^2 - e^{-\frac{4\alpha\Phi}{N-2}}F^2\right), \tag{7}$$

where $F^2 = F_{\mu\nu}F^{\mu\nu}$ for the Maxwell field. The coefficients A and Σ that give the correct metric functions (4) and (5) through the action for the EMD theory are given by [9]

$$\Sigma \rightarrow \Sigma_{\text{EMD}} = \frac{4}{\gamma^2}\left(\frac{N-3}{N-2}\right)^2 \quad \text{and} \quad A \rightarrow A_{\text{EMD}} = \gamma, \tag{8}$$

where γ is a constant.

Besides the higher dimensional linear dilaton black hole solutions to the EMD theory, new N-dimensional linear dilaton black hole solutions to the EYMD and EYMBID theories are considered in the literature [10,11]. The actions are

$$\begin{aligned}\mathbf{I} = &-\frac{1}{16\pi}\int_{\mathcal{M}} d^Nx\sqrt{-g}\left[R - \frac{4}{N-2}(\nabla\Phi)^2 + \mathcal{L}(\Phi)\right]\\ &-\frac{1}{8\pi}\int_{\partial\mathcal{M}} d^{N-1}x\sqrt{-h}K,\end{aligned} \tag{9}$$

and

$$\begin{aligned}\mathbf{I} = &-\frac{1}{16\pi}\int_{\mathcal{M}} d^Nx\sqrt{-g}\left[R - \frac{4}{N-2}(\nabla\Phi)^2 + \mathcal{L}(\mathbf{F},\Phi)\right]\\ &-\frac{1}{8\pi}\int_{\partial\mathcal{M}} d^{N-1}x\sqrt{-h}K,\end{aligned} \tag{10}$$

which describe the EYMD and EYMBID theories, respectively. Here,

$$\mathcal{L}(\Phi) = e^{-\frac{4\alpha\Phi}{N-2}}\operatorname{Tr}\left(F_{\lambda\sigma}^{(a)}F^{(a)\lambda\sigma}\right), \tag{11}$$

$$\mathcal{L}(\mathbf{F},\Phi) = 4\beta^2 e^{\frac{4\alpha\Phi}{N-2}}\left(1 - \sqrt{1 + \frac{e^{-\frac{8\alpha\Phi}{N-2}}\operatorname{Tr}(F_{\lambda\sigma}^{(a)}F^{(a)\lambda\sigma})}{2\beta^2}}\right), \tag{12}$$

in which

$$\operatorname{Tr}(.) = \sum_{a=1}^{(N-1)(N-2)/2}(.). \tag{13}$$

In the actions (9) and (10) R is the usual curvature scalar, $\mathbf{F}^{(a)} = F_{\mu\nu}^{(a)}\,dx^\mu \wedge dx^\nu$ stands for the Yang–Mills (YM) 2-forms and β denotes the Born–Infeld parameter. The second term in the actions (9) and (10) is the surface integral with its induced metric h_{ij} and trace K of its extrinsic curvature. It is found that the corresponding coefficients to the metric functions (4) and (5) of the N-dimensional linear dilaton black hole solutions to the EYMD theory [10] are

$$\Sigma \rightarrow \Sigma_{\text{EYMD}} = \frac{(N-3)}{(N-2)Q^2} \quad \text{and} \quad A \rightarrow A_{\text{EYMD}} = \sqrt{2}Q, \tag{14}$$

and to the EYMBID theory [11] obtained as follows

$$\begin{aligned}&\Sigma \rightarrow \Sigma_{\text{EYMBID}} = \frac{2(N-3)}{(N-2)Q_c^2}\left[1 - \sqrt{1 - \frac{Q_c^2}{Q^2}}\right] \quad \text{and}\\ &A \rightarrow A_{\text{EYMBID}} = \sqrt{2}Q\left(1 - \frac{Q_c^2}{Q^2}\right)^{\frac{1}{4}}.\end{aligned} \tag{15}$$

Here Q is known as YM charge and Q_c is the critical value of YM charge in which $Q^2 > Q_c^2$ guarantees the existence of the metric in the EYMBID theory. The value of the Q_c^2 is given as

$$Q_c^2 = \frac{(N-2)(N-3)}{8\beta^2}. \tag{16}$$

3. Evaporation of higher dimensional linear dilaton black holes

It can be seen from the metric function (5) that for $r_+ > 0$, the horizon at $r = r_+$ hides the null singularity at $r = 0$. On the other hand, in the extreme case $r_+ = 0$ metric (1) still exhibits the features of the black holes. Since the central singularity $r = 0$ is null and marginally trapped, it prevents outgoing signals to reach external observers. Using the conventional definition of the statistical Hawking temperature [13], we get

$$T_H = \frac{1}{4\pi}f'(r_+) = \frac{(N-2)}{8\pi}\Sigma. \tag{17}$$

One can immediately observe that T_H is constant for an arbitrary dimension N and increases with the dimensionality of the spacetime. As we learned from the black body radiation, radiating objects loose mass in accordance with the Stefan's law [8]. Therefore while a black hole radiates, it should also loose from its mass. According to Stefan's law, we should first calculate the surface area of the black hole (1). The horizon area S_H is found as

$$S_H = \frac{2\pi^{\frac{N-1}{2}}}{\Gamma(\frac{N-1}{2})} h^{N-2}, \tag{18}$$

where $\Gamma(z)$ stands for the gamma function. After assuming that only neutral quanta are radiated, Stefan's law admits the following time-dependent horizon solutions

$$r_+(t) = \begin{cases} \exp(-\frac{1}{2\gamma^6}(\frac{N-3}{N-2})^3\mu(t)), & \text{EMD}, \\ \exp(-\frac{1}{2^7 Q^6}\mu(t)), & \text{EYMD}, \\ \exp(-\frac{1}{2^4 Q^6(1+\alpha)^3}\mu(t)), & \text{EYMBID} \end{cases} \tag{19}$$

where

$$\mu(t) = \frac{\sigma(N-3)^3 \pi^{(\frac{N-9}{2})}}{(N-2)\Gamma(\frac{N-1}{2})}(t-t_0) \quad \text{and}$$

$$0 < \alpha = \sqrt{1 - \frac{(N-2)(N-3)}{8\beta^2 Q^2}} \leqslant 1, \tag{20}$$

in which σ is Stefan's constant, and t_0 is an integration constant. From the results (19), we remark that T_H is constant with decreasing mass for a chosen dimension N, and the black holes reach to their extreme states $r_+ = 0$ in an infinite time. Namely, the required time to evaporate each black hole is infinite.

4. Calculation of T_H via *SCRSM*

Following the *SCRSM* [8], we now derive a more precise expression for the temperature of the higher dimensional linear dilaton black holes (1). To this end, we should first study the wave scattering in such spacetimes (1) with Eqs. (4) and (5). Contrary to the several black hole cases, here the massless wave equation

$$\Box \Psi = 0, \tag{21}$$

admits an exact solution in the spacetimes (1). The Laplacian operator on a N-dimensional metric is given by

$$\Box = \frac{1}{\sqrt{-g}} \partial_\upsilon \left(\sqrt{-g}\partial^\upsilon\right), \tag{22}$$

where υ runs from 1 to N. One may consider a separable solution as

$$\Psi = R(r) e^{-i\omega t} Y_l(\Omega_{N-2}), \tag{23}$$

in which $Y_l(\Omega_{N-2})$ is the eigenfunction of $(N-2)$-dimensional Laplace–Beltrami operator ∇^2_{N-2} with the eigenvalue $-l(l+N-3)$ [14]. After substituting harmonic eigenmodes (23) into the wave equation (21) and making a straightforward calculation, one obtains the radial equation:

$$h^{2-N}\left[\partial_r h^{N-2} f \partial_r R(r)\right] + \left[\frac{\omega^2}{f} - \frac{l(l+N-3)}{h^2}\right] R(r) = 0. \tag{24}$$

After changing the independent variable and the parameters as

$$y = 1 - \left(\frac{r}{r_+}\right)^{(\frac{N-2}{2})}, \qquad \tilde{\lambda}^2 = \frac{4}{(N-2)^2 \Sigma A^2} l(l+N-3),$$
$$\tilde{\omega} = \varepsilon\omega, \tag{25}$$

where

$$\varepsilon = \frac{2}{(N-2)\Sigma}, \tag{26}$$

one transforms the radial equation (24) into the following hypergeometric equation

$$\partial_y\left[y(y-1)\partial_y R(y)\right] + \left(\tilde{\omega}^2 \frac{y-1}{y} - \tilde{\lambda}^2\right) R(y) = 0. \tag{27}$$

Further, letting

$$\tilde{\Lambda} = 2ik, \tag{28}$$

where k is

$$k = \sqrt{\tilde{\omega}^2 - \tilde{\lambda}^2 - \frac{1}{4}} \tag{29}$$

(throughout the Letter we assume that k has a real value), we can obtain the general solution of (27) as follows

$$R(y) = C_1(-y)^{i\tilde{\omega}} F\left(\frac{1}{2} + i(\tilde{\omega}+k), \frac{1}{2} + i(\tilde{\omega}-k), 1+2i\tilde{\omega}; y\right)$$
$$+ C_2(-y)^{-i\tilde{\omega}} F\left(\frac{1}{2} + i(-\tilde{\omega}+k), \frac{1}{2} - i(\tilde{\omega}+k), 1-2i\tilde{\omega}; y\right). \tag{30}$$

Thus, the solution (30) leads to the general solution of Eq. (24) as

$$\begin{aligned} R(\rho) &= C_1\left(\frac{\rho-\tau}{\tau}\right)^{i\tilde{\omega}} F\left(\frac{1}{2}+i(\tilde{\omega}+k), \frac{1}{2}+i(\tilde{\omega}-k), 1+2i\tilde{\omega}; \frac{\tau-\rho}{\tau}\right) \\ &\quad + C_2\left(\frac{\rho-\tau}{\tau}\right)^{-i\tilde{\omega}} \\ &\quad \times F\left(\frac{1}{2}+i(-\tilde{\omega}+k), \frac{1}{2}-i(\tilde{\omega}+k), 1-2i\tilde{\omega}; \frac{\tau-\rho}{\tau}\right), \end{aligned} \tag{31}$$

in which

$$\rho = (r)^{(\frac{N-2}{2})}, \qquad \tau = (r_+)^{(\frac{N-2}{2})}. \tag{32}$$

Letting

$$\frac{\rho-\tau}{\tau} = e^{x/\varepsilon}, \tag{33}$$

one gets the behavior of the partial wave near the horizon ($r \to r_+$) as

$$\Psi \simeq C_1 e^{i\omega(x-t)} + C_2 e^{-i\omega(x-t)}, \tag{34}$$

where C_1 and C_2 are the amplitudes of the near-horizon outgoing and ingoing waves.

Now, we shall use the one of the special features of the hypergeometric functions in which it leads us to obtain the asymptotic behavior of the partial wave. The feature is nothing but a transformation of the hypergeometric functions of argument y in (31) to the hypergeometric functions of argument $1/y$. The relevant transformation is given by [15]

$$\begin{aligned} F(a,b;c;y) &= \frac{\Gamma(c)\Gamma(b-a)}{\Gamma(b)\Gamma(c-a)}(-y)^{-a} F(a, a+1-c; a+1-b; 1/y) \\ &\quad + \frac{\Gamma(c)\Gamma(a-b)}{\Gamma(a)\Gamma(c-b)}(-y)^{-b} F(b, b+1-c; b+1-a; 1/y). \end{aligned} \tag{35}$$

This transformation yields the partial wave near spatial infinity as

$$\Psi \simeq \left(\frac{r}{r_+}\right)^{\frac{2-N}{4}} \left\{B_1 \exp\left[i\left(\frac{k}{\varepsilon}x - \omega t\right)\right] + B_2 \exp\left[-i\left(\frac{k}{\varepsilon}x + \omega t\right)\right]\right\}, \tag{36}$$

where B_1 and B_2 denote the amplitudes of the asymptotic outgoing and ingoing waves, respectively. After a straightforward calculation, one may derive the relations between B_1, B_2 and C_1, C_2 as follows

$$B_1 = C_1 \frac{\Gamma(\hat{c})\Gamma(\hat{a}-\hat{b})}{\Gamma(\hat{a})\Gamma(\hat{c}-\hat{b})} + C_2 \frac{\Gamma(2-\hat{c})\Gamma(\hat{a}-\hat{b})}{\Gamma(\hat{a}-\hat{c}+1)\Gamma(1-\hat{b})},$$
$$B_2 = C_1 \frac{\Gamma(\hat{c})\Gamma(\hat{b}-\hat{a})}{\Gamma(\hat{b})\Gamma(\hat{c}-\hat{a})} + C_2 \frac{\Gamma(2-\hat{c})\Gamma(\hat{b}-\hat{a})}{\Gamma(\hat{b}-\hat{c}+1)\Gamma(1-\hat{a})}, \qquad (37)$$

where

$$\hat{a} = \frac{1}{2} + i(\tilde{\omega}+k), \qquad \hat{b} = \frac{1}{2} + i(\tilde{\omega}-k), \qquad \hat{c} = 1 + 2i\tilde{\omega}. \qquad (38)$$

The coefficient of reflection by the black hole is calculated by virtue of the fact that outgoing mode must be absent at the spatial infinity. This is because the Hawking radiation is considered as the inverse scattering by the black hole. Briefly $B_1 = 0$ and it naturally leads to

$$R = \frac{|C_1|^2}{|C_2|^2} = \frac{|\Gamma(\hat{a})^2|^2}{|\Gamma(\hat{a}-\hat{c}+1)^2|^2}, \qquad (39)$$

which is equivalent to

$$R = \frac{\cosh^2 \pi(k-\tilde{\omega})}{\cosh^2 \pi(k+\tilde{\omega})}. \qquad (40)$$

Thus the resulting radiation spectrum is

$$\left(e^{\frac{\omega}{T_H}} - 1\right)^{-1} = \frac{R}{1-R} = \frac{\cosh^2 \pi(k-\tilde{\omega})}{\cosh^2 \pi(k+\tilde{\omega}) - \cosh^2 \pi(k-\tilde{\omega})}. \qquad (41)$$

From here one may easily read the temperature

$$T_H = \frac{\omega}{2\ln\left[\frac{\cosh \pi(k+\tilde{\omega})}{\cosh \pi(k-\tilde{\omega})}\right]}, \qquad (42)$$

and for high frequencies $k \simeq \tilde{\omega} = \frac{2}{(N-2)\Sigma}\omega$, Eq. (41) reduces to

$$T_H \simeq \lim_{\omega \to \text{large value}} \frac{\omega}{2\ln\left[\frac{\cosh \pi(k+\tilde{\omega})}{\cosh \pi(k-\tilde{\omega})}\right]} \simeq \frac{\omega}{2\ln(\cosh 2\pi\tilde{\omega})}$$
$$\simeq \frac{N-2}{8\pi}\Sigma \qquad (43)$$

which is nothing but the statistical Hawking temperature (17), which we obtained before.

We plot T_H (42) versus frequency ω for each theory with $N = 5$, and display all graphs in Fig. 1. As it can be seen from the Fig. 1, in the high limits of the Born–Infeld parameter β, the thermal behavior of the linear dilaton black holes in the EYMBID theory exhibits similar behavior to the EYMD theory. For a particular choice of β, it is possible to see the common behaviors in thermal manner for the linear dilaton black holes in the EYMBID and EMD theories. So we can deduce that in a special range of the Born–Infeld parameter β, the linear dilaton black holes in the EYMBID theory interpolate thermally between the black holes in the EYMD and EMD theories. However, for $\omega \to \infty$, T_H reduces to the almost same constant value for each theory. The next figure, Fig. 2 is to examine T_H versus dimension N within the high frequency regime. According to Eq. (43), Fig. 2 represents T_H increasing linearly with N for the linear dilaton black holes in the EMD and EYMD theories, it increases parabolically in the EYMBID case. On the other hand, the similar behavior, where at the high limits of the Born–Infeld parameter β the behavior of the T_H in the EYMBID theory is almost close to the behavior of T_H in the EYMD theory, is also observed in Fig. 2 as highlighted in Fig. 1. On the other hand, in the EYMBID theory T_H takes limited real values depending on the Born–Infeld parameter β through the

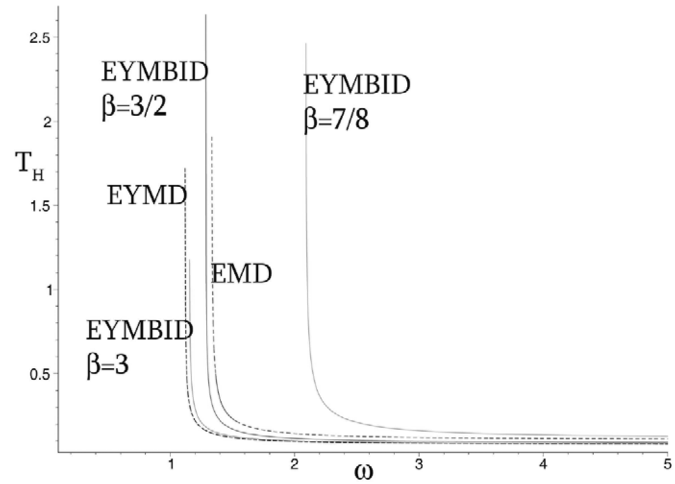

Fig. 1. Hawking temperature T_H as a function of ω in 5D. The relation is given by (42). Different line styles belong to different theories: Dotted lines represent the EMD, dashed lines represent the EYMD and solid lines correspond to the EYMBID. The physical parameters in (42) are chosen as follows: $l = 1$, $Q = 1$ and $\gamma = \sqrt{2}$.

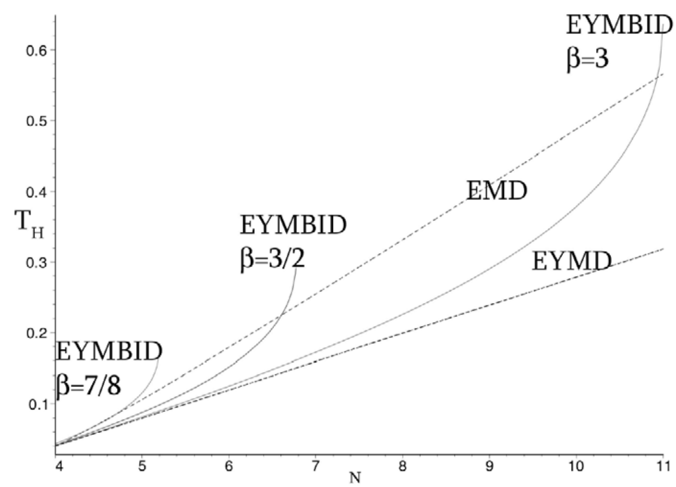

Fig. 2. A plot of the high frequency limits of Hawking temperatures T_H versus the dimension number N of the spacetime (1). Eq. (43) or Eq. (17) governs the plots. Different line styles belong to different theories: Dotted lines represent the EMD, dashed lines represent the EYMD and solid lines correspond to the EYMBID. The physical parameters in (43) are chosen as follows: $l = 1$, $Q = 1$ and $\gamma = \sqrt{2}$.

dimension N. In the EYMBID theory, T_H is real as long as the dimension N satisfies the condition

$$\frac{(N-2)(N-3)}{8Q^2} < \beta. \qquad (44)$$

If one studies the case $r_+ = 0$ (i.e. the case of extreme massless black holes), the above analysis for computing the Hawking radiation fails. In [8], it is successfully shown that the wave scattering problem in the extreme four-dimensional linear dilaton black holes in the EMD theory reduces to the propagation of eigenmodes of a free Klein–Gordon field in two-dimensional Minkowski spacetime with an effective mass. Conclusively, there is no reflection, so that the extreme linear dilaton black holes cannot radiate, although their surface gravities remain finite. Since setting $r_+ = 0$ reduces metric (1) to a conformal product $M_2 \times S^{N-2}$ of a two-dimensional Minkowski spacetime with the $(N-2)$-sphere of constant radius, the same interpretation is valid also for the extreme higher dimensional linear dilaton black holes in the EMD, EYMD and EYMBID theories. In summary, the massless higher dimensional linear dilaton black holes in the EMD, EYMD and EYMBID theories cannot radiate as well.

5. Conclusion

In this Letter, we have effectively utilized the *SCRSM* to derive the Hawking temperature T_H for massive, higher-dimensional ($N \geqslant 4$), linear dilatonic black holes in the EMD, EYMD and EYMBID theories. To do this, first we have attempted to solve the massless scalar wave equation, exactly. Exact solution of the wave equation plays a crucial role in deriving a more precise result of the temperature of those non-asymptotically flat black holes. After finding the solution in terms of the hypergeometric functions and using their intriguing features, we have demonstrated that in the high frequency regime, the results of *SCRSM* agree with the temperature obtained from the surface gravity for all considered theories.

One of the main results obtained from the Stefan's law is that as in the case $N = 4$ [8] the linear dilaton black holes evaporate in an infinite time, for $N \geqslant 5$ as well. The figures of T_H have some important results which are summarized as follows: (i) When the dimension N is fixed, the behavior of T_H versus frequency ω in the EYMBID theory exhibits similar behavior of the T_H in the EYMD theory with large β. (ii) From the thermal point of view, for a special range of β the linear dilaton black hole solutions to the EYMBID theory interpolate between the black hole solutions to the EMD and EYMD theories. (iii) Contrary to the EMD and EYMD theories, in the EYMBID theory, at high frequency regime, T_H increases with N parabolically rather than linearly. (iv) In the EYMBID theory, T_H is real unless the condition $\frac{(N-2)(N-3)}{8Q^2} < \beta$ is violated.

We also verify that contrary to the non-zero values of their surface gravity the massless, extreme higher dimensional linear dilaton black holes do not radiate. Finally, we remark that since our dilatonic black holes are conformally related to the Brans–Dicke black holes [16] our results can be extended to the latter theory as well.

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