

# N-Dimensional non-abelian dilatonic, stable black holes and their Born-Infeld extension

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## Abstract

We find large classes of non-asymptotically flat Einstein-Yang-Mills-Dilaton (EYMD) and Einstein-Yang-Mills-Born-Infeld-Dilaton (EYMBID) black holes in N-dimensional spherically symmetric spacetime expressed in terms of the quasilocal mass. Extension of the dilatonic YM solution to N-dimensions has been possible by employing the generalized Wu-Yang ansatz. Another metric ansatz, which aided in finding exact solutions is the functional dependence of the radius function on the dilaton field. These classes of black holes are stable against linear radial perturbations. In the limit of vanishing dilaton we obtain Bertotti-Robinson (BR) type metrics with the topology of  $AdS_2 \times S^{N-2}$ . Since connection can be established between dilaton and a scalar field of Brans-Dicke (BD) type we obtain black hole solutions also in the Brans-Dicke-Yang-Mills (BDYM) theory as well.

## I. INTRODUCTION

Recently we have found black hole solutions in the Einstein-Yang-Mills (EYM) theory by extending the Wu-Yang ansatz to higher dimensions [1, 2]. Both the YM charge and the dimensionality of the space time played crucial roles to determine the features of such black holes. It was found long ago, within the context of Dirac monopole theory that Wu-Yang ansatz solves the static, spherically symmetric YM equations for  $N = 4$  dimensional flat space time[3]. The  $SO(3)$  gauge structure was derived from the abelian electromagnetic (em) potential such that the internal and space time indices were mixed together in the potential. By a similar analogy we extend this idea to - nowadays fashionable- $N$  dimensional space times where  $SO(N - 1)$  is obtained through a non-abelian gauge transformation from the em potential within the static, spherically symmetric metric ansatz. The YM gauge potential is chosen to depend only on the angular variables and therefore they become independent of time ( $t$ ) and the radial coordinate ( $r$ ). Upon this choice the YM potential becomes magnetic type and by virtue of the metric ansatz the YM equations are easily satisfied. Such a choice renders the duality principle to be automatically absent in the theory. We note that by invoking the Birkhoff's theorem of general relativity  $t$  and  $r$  can be interchanged appropriately in the metric, while the YM potential preserves its form. The fact that the solutions obtained by this procedure pertain to genuine non-abelian character is obvious from comparison with the other known exact EYM solutions . The EYM solutions obtained by other ansaetze[4, 5, 6, 7, 8, 9] constructed directly from the non-abelian character and those obtained by our generalized Wu-Yang ansatz [1, 2] are the same. We admit, however, that although our method yields exact solutions it is restricted to spherical symmetry alone. Their solutions, on the other hand [4, 5, 6, 7], apply to less symmetric cases which at best can be expressed in infinite series, and in certain limit, such as vanishing of a function, they coincide with ours. Among other types, particle-like [8] and magnetic monopole [9] solutions are discussed in even dimensions. To make a comparison between EM and EYM solutions we refer to the different  $r$  powers in the solutions found so far. Specifically, the logarithmic term in the metric for  $N = 5$ , EYM theory, for instance, is not encountered in the  $N = 5$ , EM theory [1]. For  $N = 4$  it was verified on physical grounds that although the metric remained unchanged, the geodesics particles felt the non-abelian charges [10]. We note that Ref. [10] constitutes the proper reference to be consulted in obtaining a YM solution from

an EM solution, which is stated as a theorem therein. Our study shows that the distinction between the abelian and non-abelian contributions becomes more transparent for  $N > 4$ . Let us note that throughout this paper by the non-Abelian field we imply YM field whose higher dimensional version is obtained by the generalized Wu-Yang ansatz.

It is well-known that in general relativity the field equations admit solutions which, unlike the localized black holes can have different properties. From this token we cite the cosmological solutions of de-Sitter ( $dS$ )/ Anti de-Sitter ( $AdS$ ), the conformally flat and Bertotti-Robinson (BR) type solutions [11, 12], beside others in higher dimensions. In the EM theory the conformally flat metric in  $N = 4$  is uniquely the BR metric whose topology is  $AdS_2 \times S^2$ . This extends to higher dimensions as  $AdS_2 \times S^{N-2}$  which is no more conformally flat. The  $N = 4$ , BR solution can be obtained from the extremal Reissner-Nordstrom (RN) black hole solution through a limiting process. The latter represents a supersymmetric soliton solution to connect different vacua of supergravity. For this reason the BR geometry can be interpreted as a "throat" region between two asymptotically flat space times. Also, since the source is pure homogenous electromagnetic (em) field it is called an "em universe", which is free of singularities. Its high degree of symmetry and singularity free properties make BR space time attractive from both the string and supergravity theory points of view. We recall that even for a satisfactory shell model interpretation of an elementary particle, BR space time is proposed as a core candidate [13]. All these aspects ( and more), we believe, justifies to make further studies on the BR space times, in particular for  $N > 4$ , which incorporates YM fields instead of the em fields.

In this paper we obtain new non-asymptotically flat dilatonic black hole solutions and study their stability against linear perturbations[14]. Remarkably, they turn out to be stable against such perturbations. To obtain such metrics we start with a general ansatz metric in the Einstein-Yang-Mills-Dilaton (EYMD) theory. Our ansatz is of BR type instead of the RN type so that in the limit of zero dilaton instead of [1, 2], we obtain BR type metrics. This leads us to a particular class of dilatonic solutions coupled with the YM field. As expected, dilaton brings severe restrictions on the space time which possesses singularities in general. In the limit of zero dilaton we obtain a two parametric (i.e.  $Q$  and  $C$ ) solution which contains the BR solution as a subclass. The second parameter, which is labeled as  $C$ , in a particular limit can be shown to correspond to the quasilocal mass. Thus, keeping both  $Q \neq 0 \neq C$  and a non-zero dilaton gives us an asymptotically non-flat black hole model.

The case  $C = 0$  (without dilaton), yields a metric which is analogous to the BR metric[15].

Next, we extend our action to include the non-Abelian Born-Infeld (BI) interaction which we phrase as Einstein-Yang-Mills-Born-Infeld-Dilaton (EYMBID) theory. As it is well-known string / supergravity motivated non-linear electrodynamics due to Born and Infeld [16] received much attention in recent years. Originally it was devised to eliminate divergences due to point charges, which recovers the linear Maxwell's electrodynamics in a particular limit (i.e.  $\beta \rightarrow \infty$ ). Now it is believed that BI action will provide significant contributions for the deep rooted problems of quantum gravity. The BI action contains invariants in special combinations under a square root term in analogy with the string theory Lagrangian. Since our aim in this paper is to use non-Abelian fields instead of the em field we shall employ the YM field which by our choice will be magnetic type. Some of the solutions that we find for the EYMBID theory represent non-asymptotically flat black holes. Unfortunately for an arbitrary dilatonic parameter the solutions become untractable. One particular class of solutions on which we shall elaborate will be again the BR type solutions for a vanishing dilaton. We explore the possibility of finding conformally flat space time by choosing particular BI parameter  $\beta$ .

After studying black holes in the dilatonic theory we proceed to establish connection with the Brans-Dicke (BD) scalar field through a conformal transformation and explore black holes in the latter as well. Coupling of BD scalar field with YM field follows under the similar line of consideration.

The organization of the paper is as follows. In Sec. *II* we introduce the EYMD gravity, its field equations, their solutions and investigate their stability. The Born-Infeld (BI) extension follows in Sec. *III*. Sec. *IV* confines black holes in the Brans-Dicke-YM theory. The paper is completed with conclusion in Sec. *V*.

## II. FIELD EQUATIONS AND THE METRIC ANSATZ FOR EYMD GRAVITY

The  $N (= n + 1)$ –dimensional action in the EYMD theory is given by ( $G = 1$ )

$$I = -\frac{1}{16\pi} \int_{\mathcal{M}} d^{n+1}x \sqrt{-g} \left( R - \frac{4}{n-1} (\nabla\Phi)^2 + \mathcal{L}(\Phi) \right) - \frac{1}{8\pi} \int_{\partial\mathcal{M}} d^n x \sqrt{-h} K, \quad (1)$$

$$\mathcal{L}(\Phi) = -e^{-4\alpha\Phi/(n-1)} \mathbf{Tr}(F_{\lambda\sigma}^{(a)} F^{(a)\lambda\sigma}),$$

where

$$\mathbf{Tr}(\cdot) = \sum_{a=1}^{(n)(n-1)/2} (\cdot), \quad (2)$$

$\Phi$  refers to the dilaton scalar potential (we should comment that in this work we are interested in a spherical symmetric dilatonic potential, i.e.  $\Phi = \Phi(r)$ ) and  $\alpha$  denotes the dilaton parameter while the second term is the surface integral with its induced metric  $h_{ij}$  and trace  $K$  of its extrinsic curvature. Herein  $R$  is the usual Ricci scalar and  $\mathbf{F}^{(a)} = F_{\mu\nu}^{(a)} dx^\mu \wedge dx^\nu$  are the YM field 2-forms (with  $\wedge$  indicating the wedge product) which are given by [1, 2]

$$\mathbf{F}^{(a)} = d\mathbf{A}^{(a)} + \frac{1}{2\sigma} C_{(b)(c)}^{(a)} \mathbf{A}^{(b)} \wedge \mathbf{A}^{(c)} \quad (3)$$

with structure constants  $C_{(b)(c)}^{(a)}$  (see Appendix A) while  $\sigma$  is a coupling constant and  $\mathbf{A}^{(a)} = A_\mu^{(a)} dx^\mu$  are the potential 1-forms. Our choice of YM potential  $\mathbf{A}^{(a)}$  follows from the higher dimensional Wu-Yang ansatz [1, 2] where  $\sigma$  is expressed in terms of the YM charge. Variations of the action with respect to the gravitational field  $g_{\mu\nu}$  and the scalar field  $\Phi$  lead, respectively to the EYMD field equations

$$R_{\mu\nu} = \frac{4}{n-1} \partial_\mu \Phi \partial_\nu \Phi + 2e^{-4\alpha\Phi/(n-1)} \left[ \mathbf{Tr} \left( F_{\mu\lambda}^{(a)} F_\nu^{(a)\lambda} \right) - \frac{1}{2(n-1)} \mathbf{Tr} \left( F_{\lambda\sigma}^{(a)} F^{(a)\lambda\sigma} \right) g_{\mu\nu} \right], \quad (4)$$

$$\nabla^2 \Phi = -\frac{1}{2} \alpha e^{-4\alpha\Phi/(n-1)} \mathbf{Tr} (F_{\lambda\sigma}^{(a)} F^{(a)\lambda\sigma}), \quad (5)$$

where  $R_{\mu\nu}$  is the Ricci tensor. Variation with respect to the gauge potentials  $\mathbf{A}^{(a)}$  yields the YM equations

$$d \left( e^{-4\alpha\Phi/(n-1)} \star \mathbf{F}^{(a)} \right) + \frac{1}{\sigma} C_{(b)(c)}^{(a)} e^{-4\alpha\Phi/(n-1)} \mathbf{A}^{(b)} \wedge \star \mathbf{F}^{(c)} = 0 \quad (6)$$

in which the hodge star  $\star$  means duality. In the next section we shall present solutions to the foregoing equations in  $N$ -dimension. Wherever it is necessary we shall supplement our discussion by resorting to the particular case  $N = 5$ . Let us remark that for  $N = 4$  case since the YM field becomes gauge equivalent to the em field the metrics are still of RN/BR, therefore we shall ignore the case  $N = 4$ .

### A. N-dimensional solution

In  $N (= n + 1)$  –dimensions, we choose a spherically symmetric metric ansatz

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + h(r)^2 d\Omega_{n-1}^2, \quad (7)$$

where

$$d\Omega_{n-1}^2 = d\theta_1^2 + \sum_{i=2}^{n-2} \prod_{j=1}^{i-1} \sin^2 \theta_j d\theta_i^2, \quad 0 \leq \theta_{n-1} \leq 2\pi, \quad 0 \leq \theta_{k \neq n-1} \leq \pi. \quad (8)$$

while  $f(r)$  and  $h(r)$  are two functions to be determined. Our gauge potential ansatz is [1, 2]

$$\mathbf{A}^{(a)} = \frac{Q}{r^2} C_{(i)(j)}^{(a)} x^i dx^j, \quad Q = \text{YM magnetic charge}, \quad r^2 = \sum_{i=1}^n x_i^2, \quad (9)$$

$$2 \leq j+1 \leq i \leq n, \quad \text{and} \quad 1 \leq a \leq n(n-1)/2,$$

$$x_1 = r \cos \theta_{n-1} \sin \theta_{n-2} \dots \sin \theta_1, \quad x_2 = r \sin \theta_{n-1} \sin \theta_{n-2} \dots \sin \theta_1,$$

$$x_3 = r \cos \theta_{n-2} \sin \theta_{n-3} \dots \sin \theta_1, \quad x_4 = r \sin \theta_{n-2} \sin \theta_{n-3} \dots \sin \theta_1,$$

...

$$x_n = r \cos \theta_1.$$

We note that the structure constant  $C_{ij}^a$  are found similar to the case  $N = 5$  as described in Appendix A. The YM equations (6) are satisfied and the field equations become

$$\nabla^2 \Phi = -\frac{1}{2} \alpha e^{-4\alpha\Phi/(n-1)} \mathbf{Tr}(F_{\lambda\sigma}^{(a)} F^{(a)\lambda\sigma}) \quad (10)$$

$$R_{tt} = \frac{e^{-4\alpha\Phi/(n-1)} f}{(n-1)} \mathbf{Tr}(F_{\lambda\sigma}^{(a)} F^{(a)\lambda\sigma}) \quad (11)$$

$$R_{rr} = \frac{4(\Phi')^2}{(n-1)} - \frac{e^{-4\alpha\Phi/(n-1)}}{(n-1)f} \mathbf{Tr}(F_{\lambda\sigma}^{(a)} F^{(a)\lambda\sigma}) \quad (12)$$

$$R_{\theta_i\theta_i} = \frac{2(n-2)Q^2 e^{-4\alpha\Phi/(n-1)}}{h^2} - \frac{h^2 e^{-4\alpha\Phi/(n-1)}}{(n-1)} \mathbf{Tr}(F_{\lambda\sigma}^{(a)} F^{(a)\lambda\sigma}), \quad (13)$$

in which we note that the remaining angular Ricci parts add no new conditions. A proper ansatz for  $h(r)$  now is

$$h(r) = A e^{-2\alpha\Phi/(n-1)} \quad (14)$$

$$(A = \text{constant})$$

which, after knowing

$$\mathbf{Tr}(F_{\lambda\sigma}^{(a)} F^{(a)\lambda\sigma}) = \frac{(n-1)(n-2)Q^2}{h^4} \quad (15)$$

and eliminating  $f(r)$  from Eq.s (11) and (12) one gets

$$\Phi = -\frac{(n-1)}{2} \frac{\alpha \ln r}{\alpha^2 + 1}. \quad (16)$$

Upon substitution of  $\Phi$  and  $h(r)$  into the Eq.s (10)-(13) we get three new equations

$$(n-1) \left[ r (\alpha^2 + 1) f' + ((n-2)\alpha^2 - 1) f \right] - \left( \frac{(n-1)(n-2)Q^2}{A^4} \right) (\alpha^2 + 1)^2 r^{\left(\frac{2}{\alpha^2+1}\right)} = 0 \quad (17)$$

$$(n-1) \left[ r (\alpha^2 + 1) f'' + (n-1)\alpha^2 f' \right] - 2 \left( \frac{(n-1)(n-2)Q^2}{A^4} \right) (\alpha^2 + 1) r^{\left(-\frac{\alpha^2-1}{\alpha^2+1}\right)} = 0 \quad (18)$$

$$\begin{aligned} & (\alpha^2 + 1)^2 (n-2) (Q^2 - A^2) r^2 + \\ & A^4 \alpha^2 (\alpha^2 + 1) f' r^{\left(\frac{3\alpha^2+1}{\alpha^2+1}\right)} + \alpha^2 ((n-2)\alpha^2 - 1) A^4 f r^{\left(\frac{2\alpha^2}{\alpha^2+1}\right)} = 0. \end{aligned} \quad (19)$$

Eq. (17) yields the integral for  $f(r)$

$$f(r) = \Xi \left( 1 - \left( \frac{r_+}{r} \right)^{\frac{(n-2)\alpha^2+1}{\alpha^2+1}} \right) r^{\frac{2}{\alpha^2+1}}, \quad (20)$$

$$\Xi = \frac{(n-2)}{((n-2)\alpha^2 + 1) Q^2} \quad (21)$$

and the equations (18) and (19) imply that  $A$  must satisfy the following constraint

$$A^2 = Q^2 (\alpha^2 + 1). \quad (22)$$

One may notice that, with the solution (20), (7) becomes a non-asymptotically flat metric and therefore the ADM mass can not be defined. Following the quasilocal mass formalism introduced by Brown and York [17] it is known that, a spherically symmetric  $N$ -dimensional metric solution as

$$ds^2 = -F(R)^2 dt^2 + \frac{dR^2}{G(R)^2} + R^2 d\Omega_{N-2}^2, \quad (23)$$

admits a quasilocal mass  $M_{QL}$  defined by [18, 19]

$$M_{QL} = \frac{N-2}{2} R_B^{N-3} F(R_B) (G_{ref}(R_B) - G(R_B)). \quad (24)$$

Here  $G_{ref}(R)$  is an arbitrary reference function, which guarantees having zero quasilocal mass once the matter source is turned off and  $R_B$  is the radius of the spacelike hypersurface boundary. Applying this formalism to the solution (20), one obtains the horizon  $r_+$  in terms of  $M_{QL}$  as

$$r_+ = \left( \frac{4(\alpha^2 + 1) M_{QL}}{(n-1) \Xi \alpha^2 A^{n-1}} \right). \quad (25)$$

Having the radius of horizon, one may use the usual definition of the Hawking temperature to calculate

$$T_H = \frac{1}{4\pi} |f'(r_+)| = \frac{\Xi}{4\pi} \frac{[(n-2)\alpha^2 + 1]}{(\alpha^2 + 1)} (r_+)^{\gamma} \quad (26)$$

where  $\Xi$  and  $r_+$  are given above and  $\gamma = \frac{1-\alpha^2}{1+\alpha^2}$ .

In order to see the singularity of the spacetime we calculate the scalar invariants, which are tedious for general  $N$ , for this reason we restrict ourselves to the case  $N = 5$  alone. The scalar invariants for  $N = 5$  are as follows

$$R = \frac{\omega_1}{r \frac{4\alpha^2+1}{\alpha^2+1}} + \frac{\sigma_1}{r \frac{2\alpha^2}{\alpha^2+1}}, \quad (27)$$

$$R_{\mu\nu}R^{\mu\nu} = \frac{\omega_2}{r \frac{6\alpha^2+1}{\alpha^2+1}} + \frac{\omega_3}{r^2 \frac{4\alpha^2+1}{\alpha^2+1}} + \frac{\sigma_2}{r \frac{4\alpha^2}{\alpha^2+1}}, \quad (28)$$

$$R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} = \frac{\omega_4}{r \frac{6\alpha^2+1}{\alpha^2+1}} + \frac{\omega_5}{r^2 \frac{4\alpha^2+1}{\alpha^2+1}} + \frac{\sigma_3}{r \frac{4\alpha^2}{\alpha^2+1}} \quad (29)$$

where  $\omega_i$  and  $\sigma_i$  are some constants and

$$\begin{aligned} \lim_{\alpha \rightarrow 0} \omega_i &= 0, & \lim_{\alpha \rightarrow 0} \sigma_1 &= \frac{2}{Q^2}, \\ \lim_{\alpha \rightarrow 0} \sigma_2 &= \frac{20}{Q^4}, & \lim_{\alpha \rightarrow 0} \sigma_3 &= \frac{33}{Q^4}. \end{aligned} \quad (30)$$

These results show that, for non-zero dilaton field (i.e.  $\alpha \neq 0$ ), the origin is singular whereas for  $\alpha = 0$  (as a limit), we have a regular spacetime. Although these results have been found for  $N = 5$ , it is our belief that for a general  $N > 5$  these behaviors do not show much difference.

### 1. Linear dilaton

Setting  $\alpha = 1$ , gives the linear dilaton solution (20) as

$$f(r) = \frac{(n-2)}{(n-1)Q^2} \left( 1 - \left( \frac{r_+}{r} \right)^{\frac{n-1}{2}} \right) r, \quad h(r)^2 = 2Q^2 r \quad (31)$$

$$r_+ = \left( \frac{2^{\frac{7-n}{2}} M_{QL}}{(n-2)(|Q|)^{n-3}} \right). \quad (32)$$

One can use the standard way to find the high frequency limit of Hawking temperature at the horizon, which means that

$$T_H = \frac{1}{4\pi} |f'(r_+)| = \frac{(n-2)}{8\pi Q^2}. \quad (33)$$



Furthermore,  $M_{QL}$  is an integration constant which is identified as quasilocal mass, so one may set this constant to be zero to get the line element

$$ds^2 = -\Xi r dt^2 + \frac{dr^2}{\Xi r} + 2Q^2 r d\Omega_{n-1}^2, \quad (34)$$

$$\Xi = \frac{(n-2)}{(n-1)Q^2}. \quad (35)$$

By a simple transformation  $r = e^{\Xi\rho}$  this line element transforms into

$$ds^2 = \Xi e^{\Xi\rho} \left( -dt^2 + d\rho^2 + \frac{2(n-1)Q^4}{(n-2)} d\Omega_{n-1}^2 \right) \quad (36)$$

which represents a conformal  $M_2 \times S_{n-1}$  space time with the radius of  $S_{n-1}$  equal to  $\sqrt{\frac{2(n-1)}{(n-2)}}Q^2$ .

## 2. BR limit of the solution

In the zero dilaton limit  $\alpha = 0$ , we express our metric function in the form of

$$f(r) = \Xi_{\circ} (r - r_+) r, \quad \Xi_{\circ} = \frac{(n-2)}{Q^2}, \quad (37)$$

$$h^2 = A_{\circ}^2 = Q^2. \quad (38)$$

In  $N(=n+1)$ -dimensions we also set  $r_+ = 0$ ,  $r = \frac{1}{\rho}$  and  $\tau = \Xi_{\circ} t$ , to transform the metric (7) into

$$ds^2 = \frac{Q^2}{(n-2)} \left( \frac{-d\tau^2 + d\rho^2}{\rho^2} + (n-2) d\Omega_{n-1}^2 \right). \quad (39)$$

This is in the BR form with the topological structure  $AdS_2 \times S^{n-1}$ , where the radius of the  $S^{n-1}$  sphere is  $\sqrt{n-2}$ .

## 3. $AdS_2 \times S^{N-2}$ topology for $0 < \alpha < 1$

In this section we shall show that, the general solution given in Eq. (20), for some specific values for  $0 < \alpha < 1$ , may also represent a conformally flat space time. To this end, we set  $r_+ = 0$ , and apply the following transformation

$$r = \left( \Xi \frac{1 - \alpha^2}{1 + \alpha^2 \rho} \right)^{-\frac{1+\alpha^2}{1-\alpha^2}}, \quad (40)$$

$$\Xi = \frac{(n-2)}{((n-2)\alpha^2 + 1)Q^2}, \quad (41)$$

to get

$$ds^2 = (\Xi)^{-\frac{1+\alpha^2}{1-\alpha^2}} \left( \frac{1-\alpha^2}{1+\alpha^2} \right)^{-\frac{2}{1-\alpha^2}} \rho^{-\frac{2\alpha^2}{1-\alpha^2}} \left( \frac{-d\tau^2 + d\rho^2}{\rho^2} + \Xi A^2 \left( \frac{1-\alpha^2}{1+\alpha^2} \right)^2 d\Omega_{n-1}^2 \right). \quad (42)$$

To have a conformally flat space time, we impose  $\Xi A^2 \left( \frac{1-\alpha^2}{1+\alpha^2} \right)^2$  to be one, i.e.

$$\frac{(n-2)(1-\alpha^2)^2}{((n-2)\alpha^2+1)(\alpha^2+1)} = 1 \quad (43)$$

and therefore yields,  $\alpha^2 = \frac{n-3}{3n-5}$ . The line element (42) takes the form of a conformally flat space time, namely

$$ds^2 = a(\rho) \left( \frac{-d\tau^2 + d\rho^2}{\rho^2} + d\Omega_{n-1}^2 \right), \quad (44)$$

$$a(\rho) = 2^{\frac{3n-5}{n-1}} (n-2) \left( \frac{Q^2}{3n-5} \right)^{\frac{2n-2}{n-1}} (n-1)^{\frac{n-3}{n-1}} \rho^{-\frac{2\alpha^2}{1-\alpha^2}}. \quad (45)$$

## B. Linear Stability of the EYMD black holes

In this chapter we follow a similar method used by Yazadjiev [14] to investigate the stability of the possible EYMD black hole solutions, introduced previously, in terms of a linear radial perturbation. Although this method is applicable to any dimensions we confine ourselves to the five-dimensional black hole case given by Eq. (7). To do so we assume that our dilatonic scalar field  $\Phi(r)$  changes into  $\Phi(r) + \psi(t, r)$ , in which  $\psi(t, r)$  is very weak compared to the original dilaton field and we call it the perturbed term. As a result we choose our perturbed metric as

$$ds^2 = -f(r) e^{\Gamma(t,r)} dt^2 + e^{\chi(t,r)} \frac{dr^2}{f(r)} + h(r)^2 d\Omega_3^2. \quad (46)$$

One should notice that, since our gauge potentials are magnetic, the YM equations (6) are satisfied. The linearized version of the field equations (10-13) plus one extra term of  $R_{tr}$  are given by

$$R_{tr} : \frac{3}{2} \frac{\chi_t(t,r) h'(r)}{h(r)} = \frac{4}{3} \partial_r \Phi(r) \partial_t \psi(t,r) \quad (47)$$

$$\nabla_\circ^2 \psi - \chi \nabla_\circ^2 \Phi + \frac{1}{2} (\Gamma - \chi)_r \Phi' f = \frac{4\alpha^2 e^{\frac{4}{3}\alpha\Phi}}{Q^2 (\alpha^2 + 1)^2} \psi \quad (48)$$

$$R_{\theta\theta} : (2 - R_{\circ\theta\theta}) \chi - \frac{1}{2} h h' f (\Gamma - \chi)_r = \frac{8\alpha}{3(\alpha^2 + 1)} \psi \quad (49)$$

in which a lower index  $\circ$  represents the quantity in the unperturbed metric. First equation in this set implies

$$\chi(t, r) = -\frac{4}{3\alpha}\psi(t, r) \quad (50)$$

which after making substitutions in the two latter equations and eliminating the  $(\Gamma - \chi)_r$  one finds

$$\nabla_{\circ}^2\psi(t, r) - U(r)\psi(t, r) = 0 \quad (51)$$

where

$$U(r) = \frac{4e^{\frac{4}{3}\alpha\Phi}}{Q^2(1+\alpha^2)} = \frac{4}{Q^2(1+\alpha^2)r^{\frac{2\alpha^2}{1+\alpha^2}}}. \quad (52)$$

To get these results we have implicitly used the constraint (22) on A. Again by imposing the same constraint, one can show that  $U(r)$  is positive. It is not difficult to apply the separation method on (51) to get

$$\psi(t, r) = e^{\pm\epsilon t}\zeta(r), \quad \nabla_{\circ}^2\zeta(r) - U_{eff}(r)\zeta(r) = 0, \quad U_{eff}(r) = \left(\frac{\epsilon^2}{f} + U(r)\right), \quad (53)$$

where  $\epsilon$  is a constant. Since  $U_{eff}(r)$  is positive one can easily show that, for any real value for  $\epsilon$  there exists a solution for  $\zeta(r)$  which is not bounded. In other words by the linear perturbation our black hole solution is stable for any value of  $\epsilon$ . As a limit of this proof, one may set  $\alpha = 0$ , which recovers the BR case.

We remark that with little addition this method can be easily extended to any higher dimensions. This implies that the N-dimensional EYMD black holes are stable under the linear perturbation.

### III. FIELD EQUATIONS AND THE METRIC ANSATZ FOR EYMBID GRAVITY

The  $N (= n + 1)$ -dimensional action in the EYMBI-D theory is given by ( $G = 1$ )

$$I = -\frac{1}{16\pi} \int_{\mathcal{M}} d^{n+1}x \sqrt{-g} \left( R - \frac{4}{n-1} (\nabla\Phi)^2 + L(\mathbf{F}, \Phi) \right) - \frac{1}{8\pi} \int_{\partial\mathcal{M}} d^n x \sqrt{-h} K, \quad (54)$$

$$L(\mathbf{F}, \Phi) = 4\beta^2 e^{4\alpha\Phi/(n-1)} \left( 1 - \sqrt{1 + \frac{\mathbf{Tr}(F_{\lambda\sigma}^{(a)} F^{(a)\lambda\sigma}) e^{-8\alpha\Phi/(n-1)}}{2\beta^2}} \right) = \quad (55)$$

$$4\beta^2 e^{4\alpha\Phi/(n-1)} \mathcal{L}(X),$$

where

$$\mathcal{L}(X) = 1 - \sqrt{1 + X}, \quad X = \frac{\mathbf{Tr}(F_{\lambda\sigma}^{(a)} F^{(a)\lambda\sigma}) e^{-8\alpha\Phi/(n-1)}}{2\beta^2}, \quad \mathbf{Tr}(\cdot) = \sum_{a=1}^{n(n-1)/2} (\cdot), \quad (56)$$

while the rest of the parameters are defined as before. Variations of the EYMBID action with respect to the gravitational field  $g_{\mu\nu}$  and the scalar field  $\Phi$  lead respectively to the correspondence EYMBID field equations

$$R_{\mu\nu} = \frac{4}{n-1} \partial_\mu \Phi \partial_\nu \Phi - 4e^{-4\alpha\Phi/(n-1)} \left( \text{Tr} \left( F_{\mu\lambda}^{(a)} F_\nu^{(a)\lambda} \right) \partial_X \mathcal{L}(X) \right) + \quad (57)$$

$$\frac{4\beta^2}{n-1} e^{4\alpha\Phi/(n-1)} \mathcal{K}(X) g_{\mu\nu},$$

$$\nabla^2 \Phi = 2\alpha\beta^2 e^{4\alpha\Phi/(n-1)} \mathcal{K}(X), \quad (58)$$

where we have abbreviated

$$\mathcal{K}(X) = 2X \partial_X \mathcal{L}(X) - \mathcal{L}(X) \quad (59)$$

$$(\partial_X \mathcal{L}(X) = -\frac{1}{\sqrt{1+X}}).$$

Variation with respect to the gauge potentials  $\mathbf{A}^{(a)}$  yields the new relevant YM equations

$$\mathbf{d} \left( e^{-4\alpha\Phi/(n-1)} \star \mathbf{F}^{(a)} \partial_X \mathcal{L}(X) \right) + \frac{1}{\sigma} C_{(b)(c)}^{(a)} e^{-4\alpha\Phi/(n-1)} \partial_X \mathcal{L}(X) \mathbf{A}^{(b)} \wedge \star \mathbf{F}^{(c)} = 0. \quad (60)$$

It is remarkable to observe that the field equations (57-59) in the limit of  $\beta \rightarrow \infty$ , reduce to the Eq.s (4-6), which are the field equations for the EYMD theory. Also in the limit of  $\beta \rightarrow 0$ , Eq.s (57-59) give

$$R_{\mu\nu} = \frac{4}{n-1} \partial_\mu \Phi \partial_\nu \Phi, \quad (61)$$

$$\nabla^2 \Phi = 0 \quad (62)$$

which refer to the gravity coupled with a massless scalar field.

### A. N-dimensional solution

In  $N (= n + 1)$  –dimensions, we again, adopt the metric ansatz (7) and our YM potentials are given by Eq. (9). N-dimensional YM equations (60) are satisfied while the field equations

imply the following set of four equations

$$\nabla^2 \Phi = 2\alpha\beta^2 e^{4\alpha\Phi/(n-1)} \mathcal{K}(X) \quad (63)$$

$$R_{tt} = -\frac{4\beta^2 e^{4\alpha\Phi/(n-1)} f}{(n-1)} \mathcal{K}(X) \quad (64)$$

$$R_{rr} = \frac{4(\Phi')^2}{(n-1)} + \frac{4\beta^2 e^{4\alpha\Phi/(n-1)}}{(n-1)f} \mathcal{K}(X) \quad (65)$$

$$R_{\theta_i\theta_i} = \frac{-4(n-2)Q^2 e^{-4\alpha\Phi/(n-1)}}{h^2} \partial_X \mathcal{L} + \frac{4h^2\beta^2 e^{4\alpha\Phi/(n-1)}}{(n-1)} \mathcal{K}(X). \quad (66)$$

in which  $X$  is defined by (56). We use the same ansatz for  $h(r)$  as Eq. (14) which gives

$$X = \frac{(n-1)(n-2)Q^2}{2\beta^2 A^4} \quad (67)$$

and therefore, after eliminating  $f(r)$  from Eq.s (64) and (65), leads to (16). Upon substitution of  $\Phi$  and  $h(r)$  into the Eq.s (63)-(66) we find the following equations

$$(n-1) [r(\alpha^2+1)f' + ((n-2)\alpha^2-1)f] + 4\beta^2 \mathcal{K}(X) (\alpha^2+1)^2 r^{\left(\frac{2}{\alpha^2+1}\right)} = 0 \quad (68)$$

$$(n-1) [r(\alpha^2+1)f'' + (n-1)\alpha^2 f'] + 8\beta^2 \mathcal{K}(X) (\alpha^2+1) r^{\left(-\frac{\alpha^2-1}{\alpha^2+1}\right)} = 0 \quad (69)$$

$$(\alpha^2+1)^2 (4\beta^2 A^4 \mathcal{K}(X) - (4Q^2 \partial_X \mathcal{L} + A^2)(n-1)(n-2)) r^2 + \quad (70)$$

$$(n-1) A^4 \alpha^2 (\alpha^2+1) f' r^{\left(\frac{3\alpha^2+1}{\alpha^2+1}\right)} + (n-1) \alpha^2 ((n-2)\alpha^2-1) A^4 f r^{\left(\frac{2\alpha^2}{\alpha^2+1}\right)} = 0.$$

Eq. (68) yields the integral for  $f(r)$

$$f(r) = \Xi \left( 1 - \left( \frac{r_+}{r} \right)^{\frac{(n-2)\alpha^2+1}{\alpha^2+1}} \right) r^{\frac{2}{\alpha^2+1}}, \quad (71)$$

$$\Xi = -\frac{4\beta^2 (\alpha^2+1)^2 \mathcal{K}(X)}{(n-1)((n-2)\alpha^2+1)} \quad (72)$$

in which  $r_+$  is an integration constant connected to the quasi local mass i.e.,

$$r_+ = \left( \frac{4(\alpha^2+1)M_{QL}}{(n-1)\Xi\alpha^2 A^{n-1}} \right) \quad (73)$$

and  $\mathcal{K}(X)$  is abbreviated as in (59). This solution satisfies Eq. (69), but from Eq. (70)  $A$  must satisfy the constraint

$$4\mathcal{K}(X)\beta^2 A^4 (\alpha^2-1) + (n-1)(n-2)(4Q^2 \partial_X \mathcal{L} + A^2) = 0. \quad (74)$$

1. *Linear dilaton*

In the linear dilaton case i.e.,  $\alpha = 1$ , Eq. (71) yields

$$f(r) = \Xi \left( 1 - \left( \frac{r_+}{r} \right)^{\frac{(n-2)+1}{2}} \right) r, \quad h(r) = A\sqrt{r}, \quad r_+ = \left( \frac{8M_{QL}}{(n-1)\Xi A^{n-1}} \right) \quad (75)$$

in which

$$A^2 = 2Q^2 \sqrt{1 - \frac{Q_{cri}^2}{Q^2}}, \quad \Xi = \frac{2(n-2)}{(n-1)Q_{cri}^2} \left( 1 - \sqrt{1 - \frac{Q_{cri}^2}{Q^2}} \right) \quad (76)$$

where

$$Q_{cri}^2 = \frac{(n-1)(n-2)}{8\beta^2} \quad (77)$$

and  $Q^2 \geq Q_{cri}^2$ .

In this case one may set  $\Xi = A = 1$  to get

$$ds^2 = - \left( 1 - \left( \frac{r_+}{r} \right)^{\frac{(n-2)+1}{2}} \right) r dt^2 + \frac{1}{\left( 1 - \left( \frac{r_+}{r} \right)^{\frac{(n-2)+1}{2}} \right) r} dr^2 + r d\Omega_{n-1}^2. \quad (78)$$

2. *BR limit of the solution*

In the zero dilaton limit  $\alpha = 0$ , we express our metric functions (71) in the form

$$f(r) = \Xi_o (r - r_+) r, \quad \Xi_o = \frac{8\beta^2 (n-2)}{(n-1)(n-2) + 8\beta^2 Q^2}, \quad (79)$$

$$h^2 = A_o^2 = Q^2 - \frac{(n-1)(n-2)}{8\beta^2}. \quad (80)$$

In  $N(=n+1)$ -dimensions we also set  $r_+ = 0$ ,  $r = \frac{1}{\rho}$  and  $\tau = \Xi_o t$ , to transform the metric (7) into

$$ds^2 = \frac{1}{\Xi_o} \left( \frac{-d\tau^2 + d\rho^2}{\rho^2} + \Xi_o A_o^2 d\Omega_{n-1}^2 \right). \quad (81)$$

This is in the BR form with the topological structure  $AdS_2 \times S^{N-2}$ , where the radius of the sphere is  $\sqrt{\Xi_o} A_o$ . It can be shown that

$$\Xi_o A_o^2 = (n-2) \left( \frac{8\beta^2 Q^2 - (n-1)(n-2)}{(n-1)(n-2) + 8\beta^2 Q^2} \right) \quad (82)$$

which, in the limit of  $\beta \rightarrow \infty$ , becomes

$$\lim_{\beta \rightarrow \infty} \Xi_o A_o^2 = (n-2) \quad (83)$$

such that, the solution (81) becomes the BR type solution of EYMD theory (see Eq. (39)). We set now  $\Xi_\circ A_\circ^2 = 1$ , to obtain a conformally flat metric. This claims that

$$(n-2) \left( \frac{8\beta^2 Q^2 - (n-1)(n-2)}{(n-1)(n-2) + 8\beta^2 Q^2} \right) = 1 \quad (84)$$

and consequently we find

$$\beta^2 = \frac{(n-1)^2 (n-2)}{8Q^2 (n-3)}, \quad (85)$$

$$ds^2 = \frac{2Q^2}{(n-1)} \left( \frac{-d\tau^2 + d\rho^2}{\rho^2} + d\Omega_3^2 \right). \quad (86)$$

This particular choice of  $\beta$  casts the EYMBI metric into a conformally flat form with the topology of  $AdS_2 \times S^3$

### 3. $AdS_2 \times S^{N-2}$ topology for $0 < \alpha < 1$

As one may show, for  $0 < \alpha < 1$  and  $r_+ = 0$ , a similar transformation as (40), here also leads to the line element

$$ds^2 = (\Xi)^{-\frac{1+\alpha^2}{1-\alpha^2}} \left( \frac{1-\alpha^2}{1+\alpha^2} \right)^{-\frac{2}{1-\alpha^2}} \rho^{-\frac{2\alpha^2}{1-\alpha^2}} \left( \frac{-d\tau^2 + d\rho^2}{\rho^2} + \Xi A^2 \left( \frac{1-\alpha^2}{1+\alpha^2} \right)^2 d\Omega_{n-1}^2 \right). \quad (87)$$

Again we set  $\Xi A^2 \left( \frac{1-\alpha^2}{1+\alpha^2} \right)^2 = 1$  which gives the conformally flat line element

$$ds^2 = a(\rho) \left( \frac{-d\tau^2 + d\rho^2}{\rho^2} + d\Omega_{n-1}^2 \right), \quad (88)$$

with

$$a(\rho) = (\Xi)^{-\frac{1+\alpha^2}{1-\alpha^2}} \left( \frac{1-\alpha^2}{1+\alpha^2} \right)^{-\frac{2}{1-\alpha^2}} \rho^{-\frac{2\alpha^2}{1-\alpha^2}}. \quad (89)$$

## B. Linear Stability of the EYMBID black holes

Similar to the proof given in Sec. (II.B), here also we study the stability of the possible black holes in EYMBID theory which undergoes a linear perturbation. Again we give a detailed study for the 5-dimensional black holes which is extendible to any higher dimensions. Our perturbed metric is same as we adapted in Eq. (46). The linearized field equations plus

the extra term of  $R_{tr}$  are given now by

$$R_{tr} : \frac{(n-1)\chi_t(t,r)h'(r)}{2h(r)} = \frac{4}{3}\partial_r\Phi(r)\partial_t\psi(t,r) \quad (90)$$

$$\nabla_{\circ}^2\psi - \chi\nabla_{\circ}^2\Phi + \frac{1}{2}(\Gamma - \chi)_r\Phi'f = -\frac{8}{(n-1)}\alpha^2\beta^2e^{\frac{4}{(n-1)}\alpha\Phi}(\mathcal{L}(X_{\circ}) + 4X_{\circ}^2\partial_{X_{\circ}}^2\mathcal{L}(X_{\circ}))\psi \quad (91)$$

$$R_{\theta\theta} : (2 - R_{\circ\theta\theta})\chi - \frac{1}{2}hh'f(\Gamma - \chi)_r = \frac{16}{9}\alpha A^2\beta^2(2X_{\circ}\partial_{X_{\circ}}\mathcal{L}(X_{\circ}) - \mathcal{L}(X_{\circ}))\psi \quad (92)$$

in which our conventions are as before. The first equation in this set implies that

$$\chi(t,r) = -\frac{4}{3\alpha}\psi(t,r) \quad (93)$$

which, after we make substitutions in the two latter equations and eliminating the  $(\Gamma - \chi)_r$  we find

$$\nabla_{\circ}^2\psi(t,r) - U(r)\psi(t,r) = 0 \quad (94)$$

where

$$U(r) = \frac{8}{3}\beta^2e^{\frac{4}{3}\alpha\Phi}[\mathcal{L}(X_{\circ}) - 2X_{\circ}\partial_{X_{\circ}}\mathcal{L}(X_{\circ}) - \alpha^2(\mathcal{L}(X_{\circ}) + 4X_{\circ}^2\partial_{X_{\circ}}^2\mathcal{L}(X_{\circ}))]. \quad (95)$$

To get these results we have implicitly used the constraint (74) on  $A$ . Again by imposing the same constraint, one can show that  $U(r)$  is positive definite. We follow the separation method to get

$$\psi(t,r) = e^{\pm\epsilon t}\zeta(r), \quad \nabla_{\circ}^2\zeta(r) - U_{eff}(r)\zeta(r) = 0, \quad U_{eff}(r) = \left(\frac{\epsilon^2}{f} + U(r)\right), \quad (96)$$

where  $\epsilon$  is a constant. Here also the fact that  $U_{eff}(r) > 0$  can be justified which implies in turn that the system is stable. For  $\beta \rightarrow \infty$  this reduces to the case of EYMD black hole solution whose stability was already verified before.

#### IV. BLACK HOLES IN THE BDYM THEORY

In  $N (= n + 1)$  –dimensions we write the Brans-Dicke-Yang-Mills (BDYM) action as

$$I = -\frac{1}{16\pi} \int_{\mathcal{M}} d^{n+1}x \sqrt{-g} \left( \phi R - \frac{\omega}{\phi} (\nabla\phi)^2 + \mathcal{L}_m \right) - \frac{1}{8\pi} \int_{\partial\mathcal{M}} d^n x \sqrt{-h} K, \quad (97)$$

$$\mathcal{L}_m = -\mathbf{Tr}(F_{\lambda\sigma}^{(a)} F^{(a)\lambda\sigma}),$$



in which  $\omega$  is the coupling constant, and  $\phi$  stands for the BD scalar field with the dimensions  $G^{-1}$  ( $G$  is the  $N$ -dimensional Newtonian constant [21]). Variation of the BDYM's action with respect to the  $g_{\mu\nu}$  gives

$$\phi G_{\mu\nu} = \frac{\omega}{\phi} \left( \nabla_{\mu}\phi\nabla_{\nu}\phi - \frac{1}{2}g_{\mu\nu}(\nabla\phi)^2 \right) + 2 \left( \mathbf{Tr} \left( F_{\mu\lambda}^{(a)} F_{\nu}^{(a)\lambda} \right) - \frac{1}{4}g_{\mu\nu} \mathbf{Tr} \left( F_{\lambda\sigma}^{(a)} F^{(a)\lambda\sigma} \right) \right) + \quad (98)$$

$$\nabla_{\mu}\nabla_{\nu}\phi - g_{\mu\nu}\nabla^2\phi,$$

while variation of the action with respect to the scalar field  $\phi$  and the gauge potentials  $\mathbf{A}^{(a)}$  yields

$$\nabla^2\phi = -\frac{n-3}{2[(n-1)\omega+n]} \mathbf{Tr} \left( F_{\lambda\sigma}^{(a)} F^{(a)\lambda\sigma} \right), \quad (99)$$

and

$$\mathbf{d} \left( \star \mathbf{F}^{(a)} \right) + \frac{1}{\sigma} C_{(b)(c)}^{(a)} \mathbf{A}^{(b)} \wedge \star \mathbf{F}^{(c)} = 0, \quad (100)$$

respectively.

We follow now the routine process to transform BDYM action into the EYMD action[21]. For this purpose, one can use a conformal transformation (variables with a caret  $\hat{\cdot}$  denote those in the Einstein frame)

$$\hat{g}_{\mu\nu} = \phi^{\frac{2}{n-1}} g_{\mu\nu} \quad \text{and} \quad \hat{\Phi} = \frac{(n-3)}{4\hat{\alpha}} \ln \phi. \quad (101)$$

This transforms (97) into

$$\hat{I} = -\frac{1}{16\pi} \int_{\mathcal{M}} d^{n+1}x \sqrt{-\hat{g}} \left( \hat{R} - \frac{4}{n-1} \left( \hat{\nabla}\hat{\Phi} \right)^2 - e^{-4\hat{\alpha}\hat{\Phi}/(n-1)} \mathbf{Tr} \left( \hat{F}_{\lambda\sigma}^{(a)} \hat{F}^{(a)\lambda\sigma} \right) \right) - \frac{1}{8\pi} \int_{\partial\mathcal{M}} d^n x \sqrt{-\hat{h}} \hat{K}, \quad (102)$$

where

$$\hat{\alpha} = \frac{n-3}{2\sqrt{(n-1)\omega+n}}. \quad (103)$$

This transformed action is similar to the EYMD action given by (1). Variation of this action with respect to the  $\hat{g}_{\mu\nu}$ ,  $\hat{\Phi}$  and  $\hat{\mathbf{A}}^{(a)}$  gives

$$\hat{R}_{\mu\nu} = \frac{4}{n-1} \hat{\partial}_{\mu}\hat{\Phi}\hat{\partial}_{\nu}\hat{\Phi} + 2e^{-4\hat{\alpha}\hat{\Phi}/(n-1)} \left[ \mathbf{Tr} \left( \hat{F}_{\mu\lambda}^{(a)} \hat{F}_{\nu}^{(a)\lambda} \right) - \frac{1}{2(n-1)} \mathbf{Tr} \left( \hat{F}_{\lambda\sigma}^{(a)} \hat{F}^{(a)\lambda\sigma} \right) \hat{g}_{\mu\nu} \right], \quad (104)$$

$$\hat{\nabla}^2\hat{\Phi} = -\frac{1}{2}\hat{\alpha}e^{-4\hat{\alpha}\hat{\Phi}/(n-1)} \mathbf{Tr}(\hat{F}_{\lambda\sigma}^{(a)} \hat{F}^{(a)\lambda\sigma}), \quad (105)$$

$$\mathbf{d} \left( e^{-4\hat{\alpha}\hat{\Phi}/(n-1)\star\hat{\mathbf{F}}^{(a)}} \right) + \frac{1}{\sigma} C_{(b)(c)}^{(a)} e^{-4\hat{\alpha}\hat{\Phi}/(n-1)} \hat{\mathbf{A}}^{(b)} \wedge \star \hat{\mathbf{F}}^{(c)} = 0. \quad (106)$$

It is not difficult to conclude that, if we find a solution to the latter equations, by an inverse transformation, we can find the solutions of the related equations of the BDYM theory. In other words if  $(\hat{g}_{\mu\nu}, \hat{\Phi}, \hat{\mathbf{F}}^{(a)})$  is a solution of the latter equations, then

$$(g_{\mu\nu}, \phi, \mathbf{F}^{(a)}) = \left( \exp \left( -\frac{8\hat{\alpha}}{(n-1)(n-3)} \hat{\Phi} \right) \hat{g}_{\mu\nu}, \exp \left( \frac{4\hat{\alpha}}{(n-3)} \hat{\Phi} \right), \hat{\mathbf{F}}^{(a)} \right) \quad (107)$$

is a solution of (98-100) and vice versa.

One may call  $(g_{\mu\nu}, \phi, \mathbf{F}^{(a)})$ , the reference solution and  $(\hat{g}_{\mu\nu}, \hat{\Phi}, \hat{\mathbf{F}}^{(a)})$  the target solution. Hence our solution in EYMD would be the target solution i.e.

$$d\hat{s}^2 = -\hat{f}(r) dt^2 + \frac{dr^2}{\hat{f}(r)} + \hat{h}(r)^2 d\Omega_{n-1}^2, \quad (108)$$

where

$$\begin{aligned} \hat{f}(r) &= \hat{\Xi} \left( 1 - \left( \frac{\hat{r}_+}{r} \right)^{\frac{(n-2)\hat{\alpha}^2+1}{\hat{\alpha}^2+1}} \right) r^{\frac{2}{\hat{\alpha}^2+1}}, \quad \hat{h}(r) = \hat{A} e^{-2\hat{\alpha}\hat{\Phi}/(n-1)}, \\ \hat{\Xi} &= \frac{(n-2)}{((n-2)\hat{\alpha}^2+1)\hat{Q}^2}, \quad \hat{\Phi} = -\frac{(n-1)}{2} \frac{\hat{\alpha} \ln r}{\hat{\alpha}^2+1}, \quad \hat{A}^2 = \hat{Q}^2 (\hat{\alpha}^2+1), \\ \hat{r}_+ &= \left( \frac{4(\hat{\alpha}^2+1)\hat{M}_{QL}}{(n-1)\hat{\Xi}\hat{\alpha}^2\hat{A}^{n-1}} \right). \end{aligned} \quad (109)$$

Our reference solution would read now

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + h(r)^2 d\Omega_{n-1}^2, \quad (110)$$

in which

$$f(r) = \hat{\Xi} \left( 1 - \left( \frac{\hat{r}_+}{r} \right)^{\frac{(n-2)\hat{\alpha}^2+1}{\hat{\alpha}^2+1}} \right) r^{\frac{2(n-3)+4\hat{\alpha}^2}{(n-3)(\hat{\alpha}^2+1)}}, \quad h(r) = \hat{A} e^{-\frac{2\hat{\alpha}\hat{\Phi}(n+1)}{(n-1)(n-3)}} = \hat{A} r^{\frac{\hat{\alpha}^2(n+1)}{(\hat{\alpha}^2+1)(n-3)}}, \quad (111)$$

$$\phi = r^{\frac{-2(n-1)\hat{\alpha}^2}{(n-3)(\hat{\alpha}^2+1)}}, \quad \text{and } \mathbf{F}^{(a)} = \hat{\mathbf{F}}^{(a)} = \mathbf{d}\hat{\mathbf{A}}^{(a)} + \frac{1}{2\sigma} C_{(b)(c)}^{(a)} \hat{\mathbf{A}}^{(b)} \wedge \hat{\mathbf{A}}^{(c)} \quad (112)$$

where the YM potential is same as (9) with the new charge  $\hat{Q}$ . Herein one can find the Hawking temperature of the BDYM-black hole at the event horizon as

$$T_H = \frac{\hat{\Xi} [(n-2)\hat{\alpha}^2+1]}{4\pi(\hat{\alpha}^2+1)} (\hat{r}_+)^{\left( -\frac{(n-3)(\hat{\alpha}^2-1)-4\hat{\alpha}^2}{(\hat{\alpha}^2+1)(n-3)} \right)} \quad (113)$$

where  $\hat{r}_+$  is the radius of the event horizon.

## V. CONCLUSION

A simple class of spherically symmetric solutions to the EYMD equations is obtained in any dimensions. Magnetic type Wu-Yang ansatz played a crucial role in extending the solution to N-dimension. For the non-zero dilaton the space time possesses singularity, representing a non-asymptotically flat black hole solution expressed in terms of the quasilocal mass. Particular case of a linear dilatonic black hole is singled out as a specific case. Hawking temperature for all cases has been computed which are distinct from the EMD temperatures [22]. Stability against linear perturbations for these dilatonic metrics is proved. It has been shown that the extremal limit in the vanishing dilaton, results in the higher dimensional BR space times for the YM field. With the common topology of  $AdS_2 \times S^{N-2}$  for both theories, while the radius of  $S^{N-2}$  for the Maxwell case is  $(N - 3)$ , it becomes  $(N - 3)^{1/2}$  in the YM case. As a final contribution in the paper we apply a conformal transformation to derive black hole solutions in the Brans-Dicke-YM theory. It is our belief that these YMBR metrics, beside the dilatonic ones, will be useful in the string/supergravity theory as much as the EMBR metrics are.

**Acknowledgement 1** *We thank the anonymous referee for valuable and constructive suggestions.*

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## VI. APPENDIX A

We work on a group of proper rotations in  $(N - 1)$ –dimensions,  $SO(N - 1)$ , which forms a  $\frac{(N-1)(N-2)}{2}$  (i.e.,  $\binom{N-1}{2}$ )–parameter Lie group whose infinitesimal generators

are given by:

$$\begin{aligned}
L_1 &= x_2 \partial_{x_1} - x_1 \partial_{x_2} \\
L_2 &= x_3 \partial_{x_1} - x_1 \partial_{x_3} \\
L_3 &= x_3 \partial_{x_2} - x_2 \partial_{x_3} \\
L_4 &= x_4 \partial_{x_1} - x_1 \partial_{x_4} \\
L_5 &= x_4 \partial_{x_2} - x_2 \partial_{x_4} \\
L_6 &= x_4 \partial_{x_3} - x_3 \partial_{x_4} \\
&\dots
\end{aligned} \tag{A-1}$$

These operators satisfy commutation relations of the form

$$[L_i, L_j] = C_{(i)(j)}^{(k)} L_k, \tag{A-2}$$

where the  $C_{(i)(j)}^{(k)}$  are the structure constants. As an example we check

$$\begin{aligned}
[L_1, L_2] &= C_{(1)(2)}^{(3)} L_3 = L_3, \\
&\rightarrow C_{(1)(2)}^{(3)} = 1.
\end{aligned} \tag{A-3}$$

This can be done for all other combinations and the only 24 non zero terms are:

$$\begin{aligned}
C_{(2)(3)}^{(1)} &= C_{(4)(5)}^{(1)} = -C_{(3)(2)}^{(1)} = -C_{(5)(4)}^{(1)} = 1 \\
C_{(3)(1)}^{(2)} &= C_{(4)(6)}^{(2)} = -C_{(1)(3)}^{(2)} = -C_{(6)(4)}^{(2)} = 1 \\
C_{(1)(2)}^{(3)} &= C_{(5)(6)}^{(3)} = -C_{(2)(1)}^{(3)} = -C_{(6)(5)}^{(3)} = 1 \\
C_{(5)(1)}^{(4)} &= C_{(6)(2)}^{(4)} = -C_{(1)(5)}^{(4)} = -C_{(2)(6)}^{(4)} = 1 \\
C_{(1)(4)}^{(5)} &= C_{(6)(3)}^{(5)} = -C_{(4)(1)}^{(5)} = -C_{(3)(6)}^{(5)} = 1 \\
C_{(2)(4)}^{(6)} &= C_{(3)(5)}^{(6)} = -C_{(4)(2)}^{(6)} = -C_{(5)(3)}^{(6)} = 1
\end{aligned} \tag{A-4}$$

By a similar, routine procedure we can obtain the coefficients in any higher dimensions. For  $N = 6$ , for example, we have 40 non-zero coefficients, which we shall not elaborate.