ABSTRACT

Producing new products and representing the outstanding ones in a right time in a competitive market is considered to be a vital factor for each firm. To gain the customer's confidence in a specific product, each manufacturer tries to meet all the customer's demands when representing a product.

This goal is not likely to be achieved unless the product represents all those special and demanded features which are supposed to satisfy customers' needs. Selecting the ideal design for a product is always considered to be a complex procedure since increase in number of attributes makes this selection more complicated. The ultimate success and popularity of the product is heavily depended on the selection methods used by manufacturers in order to assess the various designs. That is the reason which has captured the focus of researches to represent a solution to make the procedure easier and more accurate. To choose the most outstanding solution, this study uses Modified Data Envelopment Analysis. The attributes of products are categorized in two different levels as beneficial (output) and non-beneficial (input). Unlike the previous used methodologies this study is using a model which concentrates on performance attributes and not designers' preferences.

Keyword: product design, inputs and outputs, beneficial attribute, modified data envelopment analysis

ÖZ

Yeni ürünler üretmek ve seçkinlerini rekabetçi olan pazara doğru zamanda sunmak her firma için çok önemli bir faktör olarak kabul edilir. Belirli bir üründe müşterinin güvenini kazanmak için, üreticiler müşterilerinin tüm taleplerini karşılamaya çalışır. Ürün, müşterilerin memnuniyetine, ihtiyaçlarına ve taleplerine karşılık verinceye kadar bu hedefe ulaşmak mümkün değildir. Ürün için ideal tasarımı seçmek her zaman karmaşık bir süreç olarak Kabul edilir ve ürünün niteliklerini arttırmak işi daha da karmaşık bir hale sokar. Ürünün nihai başarı ve popüleritesi, üreticilerin bir çok tasarım içerisindeki tercih yöntemine bağlıdır. Bu sebepten dolayı, yöntemlerin daha kolay ve kesin hale getirilip sunulması, araştırmaların odak noktası haline gelmiştir. Bu çalışma, en seçkin çözüme ulaşmak için, Değiştirilmiş Veri Zarflama Analizini kullanır. Ürünün nitelikleri, Faydalı (çıkış) ve Faydasız (giriş) olarak iki kategoriye ayrılır.

Bu çalışma, daha önce kullanılmış metodolojilerin aksine tasarımcıların tercihlerine göre değil, fayda sağlayacak performans modeline göre konsantre olmuştur.

Anahtar Kelime: Ürün Tasarımı, Girişler Ve Çıkışlar, Yarar Niteliği, Değiştirilmiş Veri Zarflama Analizi

To my father and mother

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NOTATION

DMUs Decision making units

DMU o DMU under consideration

E Vector of ones

M Number of inputs (non-beneficial variables)

n Number of DMUs (Total number of alternatives)

s Number of outputs (beneficial variables)

s^- Input excesses

s^+ Output short falls

u Vector of weights for outputs

v Vector of weights for inputs

X Matrix of inputs

Y Matrix of outputs

x o Vector of inputs of the DMU under consideration

y o Vector of outputs of the DMU under consideration

θ Variable representing the reduction in input variables to reach the best DMU

λ Vector of reference variables to the best alternative for the DMUo

Chapter 1

INTRODUCTION

1.1 Background

Since the markets in all countries are competitive, it is important to all firms to represent new products to market to stay profitable in the market. Changes in economic conditions, technology and more importantly customer's demands, lead firms to immediately, apply these factors to their new products, for them their product to be competitive. Failure or success of firms in representing new products depends on the design, which they choose for their new product (Rao 2007). It would be possible to choose the perfect design for the new product if firms apply different approaches and considering vital and important factors in their design (Besharati *et. al* 2006).

There have been several approaches introduced to select the perfect design. Thompson (1991) represented the "Alternative Methodology for Evaluation of Design" for the first time. Hsiuo (1998) described a new method on selecting the perfect design by applying fuzzy selection method. Calantone (1999) used "Analytical Hierarchy Process (AHP)" as a method to choose the new product. Besharati *et.al* (2006) entered a new factor to the model as customers' expectations. Rao (2007) used "Graph Theory Matrix Approach (GTMA)" to obtain the best design.

1.2 Statement of Purpose

Since the markets in all countries are competitive, it is important to all firms to represent new products to market to stay profitable in the market. This study tries to employ "Modified Data Envelopment Analysis Models" to select the best design among several other designs. The important difference between DEA and other approaches is that, the designer will not be in need of specifying the preferences of the product; however, it uses the features of the product. This makes the method to prevent choosing the proper weigh for each product which itself is a complicated task to perform.

1.3 Assumptions

This study assumes that in all the DMUs, similar inputs are used to generate similar outputs. All the data is non-negative. The essential DEA models are used to evaluate all those observed DMUs, which are input or output oriented. Hence, input and output oriented DEA models are the focus of the study.

1.4 Main Aims of the Study

The aims of this study are following:

1) Evaluation and selection of the perfect design of power electronic device by using Modified DEA

2) Determining the differences between DEA and Modified DEA models

3) Practical use of DEA models in industry

Chapter 2

THEORETICAL BACKGROUND

2.1 DEA Background

Data Envelopment Analysis (DEA) is theoretical framework to assess the efficiency. DEA is non-parametric linear programming technique, which, is usually being used to assess the efficiency of systems and making a practical frontier. In other words, DMUs with several inputs and outputs are the result of input-output data.

Two estimation models of DEA are called CCR (Charnes, Cooper and Rhodes CCR model) and BCC (Banker, Charnes and Cooper BCC model). DEA is considered to be the best approach in identifying of those units which have either the maximum output (which are generated from the inputs) or the minimum input (which generates the outputs). Charnes *et all* (1980) showed that difficulties could arise if the value of a weight in an operator equals to zero. Considering an efficient DMU as a non-efficient one is one of those difficulties. They showed that using Non-Archimedean Epsilon (ε) as the lower bound as values for weights, could be a solution to overcome the problem. Ali *et al* (1994) according to Ali (1993) found a solution to find (ε) but Mehrabian *et al* (1998) on Ali *et al* (1994) conclusion which described being bounded for multiplying and unbounded for envelopment, showed that the solution for both CCR and BBC models is inefficient. They suggested a confidence range for (ε). Envelopment Analysis have been introduced by Besent *et al*(1988) for the first time.

They assessed the approach for the concept of CCR and tried to focus on those hyper-planes, which make the production frontier. Charnes *et al.* (1991) came by a new perspective for allocating the correct values to weights. They showed that the data vectors of values of weights are similar to normal vector of production frontier. For efficient DMUs, estimating the absolute return to scale is done by Banker *et al.* (1988) according to Banker (1984,1986) and Teral (1988). The aim was to find a proper calculation solution to calculate the rest of those hyper-plans, which are considered to make the bounded production frontier.

2.2 Decision Making Units

Nowadays it is important to organizations to know what factors are used to compare them with other similar organizations. For instance, comparing a faculty of a university with similar faculties of other universities or comparing a branch of a bank with other branches across a country. In the above examples, each university or bank is considered a system and each faculty or a branch of a bank is considered as a decision-making unit.

Hence, system refers to summation of decision-making units.

In a system, assume that the decision making unit of level j, use the inputs as $x_{ij}(i = 1,2,3,\ldots\ldots,m)$ to generate the outputs as $y_{rj}(r = 1,2,3,\ldots\ldots,s)$. Figure 2.1 illustrates this concept.

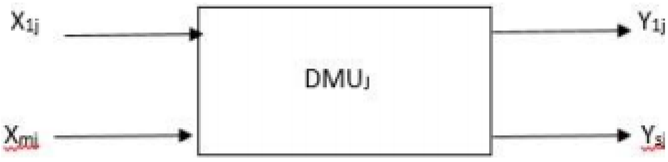

Fig 2.1. Relation between inputs and outputs

2.3 Efficiency

Imagine a unit, which consumes x as input and generates y as output. The equation is as following:

$$Efficiency = \frac{output}{input} = \frac{y}{x} \tag{2-1}$$

This concept will not cause any difficulties when DMU has single input and output. However, if there are several inputs and outputs, if values of outputs are u_r and cost of inputs are v_i, measuring the efficiency rate of the DMU at level j will be as following which is called economical efficiency.

$$Efficiency = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \tag{2-2}$$

However, values of outputs and cost of inputs are not always available. Hence, in this case data envelopment analysis should be used. In other words, in data envelopment analysis mathematical model is used to measure the efficiency rate.

2.4 Production Function

To measure the efficiency rate, production function is needed. Production function is an equation, which, results in the maximum outputs for whatever combination of inputs. Usually the function is not available. In practice, numbers of observations are available which could be used to estimate the theoretical production function. It has to be mentioned that this function is being used in microeconomics a lot because by estimating it, realizing the efficiency of a DMU is calculable.

In microeconomics, there are two approaches to estimate the production function:

1) Parametric Approaches

2) Non-Parametric Approaches

In parametric approaches, the production function is available and efficiency could be easily calculated via it. However, non-parametric approach is opposite. Data envelopment analysis is a non-parametric approach. DEA estimates the efficiency of each DMU by employing the observed resources as inputs and outputs, without taking into consideration of their weights. Moreover, it also identifies the inefficient resources in inefficient DMUs. In contrast to parametric approach, DEA focuses on each single observation instead of estimating the parameters. It also estimates a production function based on the observations without considering a default function. Hence, the outstanding features of DEA are accordingly:

1) There are no limitations in number of inputs and outputs.

2) There is no need to adopt production function shape.

3) There is no need to obtain the costs of inputs and outputs to measure the efficiency rate

2.5 CCR Model

2.5.1 Production Possibility Set (PPS)

As it has been mentioned previously, due to some reasons the production function is not always available. Hence, a set should be constructed as production possibility set (PPS) which is considered to be of part production function as a limited boundary.

Based on the production technology, Production function of PPS is an estimated boundary which, has the above features.

Assume that there are n decision-making units and it is desired to assess DMU_p $(p \in \{1,2,3\ldots\ldots,n\})$ with $x_{1p}\ldots.x_{mp}$ used as inputs which generate $y_{1p}\ldots y_{sp}$. Input vector of DMUj is shown by $x_j = \left(x_{1j}\ldots x_{mj}\right)^t$ and the output vector is shown as $y_j = \left(y_{1j}\ldots y_{sj}\right)^t$ and imagine that $x_j \geq 0$ and $x_j \neq 0$ and also $y_j \geq 0$ and $y_j \neq 0$. PPS, which, is assigned to T, is defined as following:

$$T = \{\left(x, y\right)|non-negative\,vector\,of\;x\,is\,able\,to\,generate\,non-negative\,vector\,of\;y\}$$

As it has been mentioned earlier, the production function is a function, which generates the maximum output for whatever amount of input is used. Hence, if the production function is available it is accepted as the efficiency frontier and each unit could be evaluated according to it. Figure 2.2 illustrates both production function and PPS.

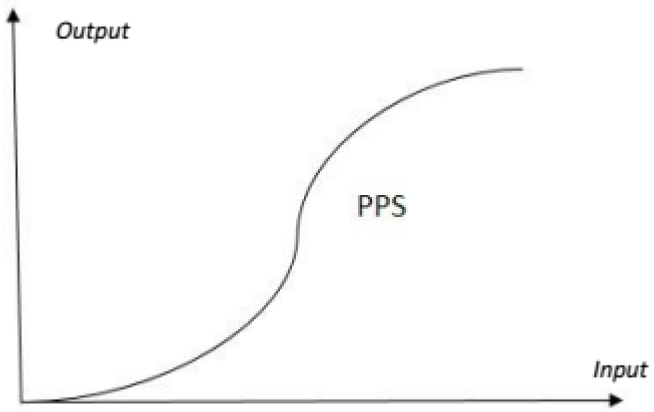

Fig 2.2. production function and PPS.

Since, the production function is not available; PPS will not be available either. So the following principles are assumed for PPS and according to them, the set T should be considered in a way, which would be true in the following principles:

1) Observation

$$(x_j, y_j) \in T \qquad j \in 1 \ldots n$$

2) Constant Return to Scale

$$\forall x \forall y \forall t > 0, (x, y) \in T \rightarrow (tx, ty) \in T$$

3) Monotonicity

$$\forall (x, y) \in T \rightarrow \forall \bar{x}, \bar{y} : (\bar{x} \geq x, y \geq \bar{y}) \rightarrow (\bar{x}, y) \in T$$

4) Convexity

$$(x, y) \in T \qquad (\bar{x}, \bar{y}) \in T$$
$$\forall \lambda \in [0,1] \rightarrow \lambda (x, y) + (1-\lambda)(\bar{x}, \bar{y}) \in T$$

5) Minimization and extrapolation

The smallest PPS, which is true for the previous 4 principles

Hence, the determined PPS according to the previous principles is as following which is known as T_c:

$$T_c = \left\{ (x, y) | x \geq \sum_{j=1}^{n} \lambda_j x_j,\ y \geq \sum_{j=1}^{n} \lambda_j y_j, \lambda_j \geq 0,\ j = 1 \ldots n \right\} \qquad (2\text{-}3)$$

T_c is a convex cone, which includes all the DMUs. Figure 2.3 shows PPS of CCR.

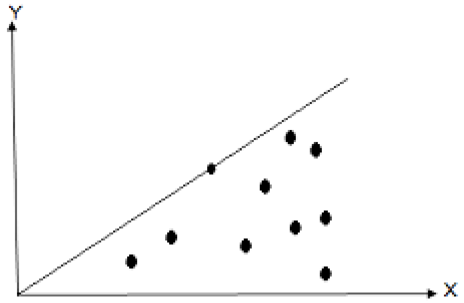

Figure 2.3. PPS of T_c

2.6 CCR Model According to PPS

Assume that DMUj $\left(j \in \{1,2...n\}\right)$ is n different DMUs which by using $x_j \left(j=1...n\right)$ as input vector generates $y_j \left(j=1...n\right)$ as output vector. Assume that $\mathrm{DMU_p}$ $\left(p \in \{1,2...n\}\right)$ is needed to be assessed. Now if costs of output of $\mathrm{DMU_p}$ are $u_1...u_s$ and the input costs are $v_1...v_s$, the following fraction is maximum if:

$$I_p = \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}} \qquad (2\text{-}4)$$

Now other remaining decision-making units will be treated the same. But if the output costs are very huge and costs of inputs are petit, I_p would be infinite. To overcome the issue, the limitation is needed to be applied. $I_j \leq 1 \;\; j=1...m$ …m. So by applying this limitation CCR model for evaluating DMUp is as following:

$$\begin{aligned} & Max \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}} \\ & s.t \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}} \leq 1 \\ & u_r \geq 0 \\ & v_i \geq 0 \end{aligned} \qquad (2\text{-}5)$$

In the above model, inputs or outputs could have costs equal to zero which might represents the efficient DMU as an inefficient one.

Let us assume both $\mathrm{DMU_a}$ and $\mathrm{DMU_b}$ with m inputs and s outputs. x_{ia} $\left(i=1...m\right)$ as inputs of unit a and y_{ra} $\left(r=1...s\right)$ as outputs of unit a and also x_{ib} $\left(i=1...m\right)$ as inputs of unit b and y_{rb} $\left(r=1...s\right)$ as outputs of unit b. DMUa is prior to DMUb if $\begin{pmatrix} -x_a \\ y_a \end{pmatrix} \geq \begin{pmatrix} -x_b \\ y_b \end{pmatrix}$ and at least one of the elements be unequal and greater than others. In CCR, there are three forms to convert inefficient DMU to an efficient one.

1) Decreasing the input

2) Increasing the output

3) Decreasing the input and increasing the output

These kinds of conversions are shown in the following figures.

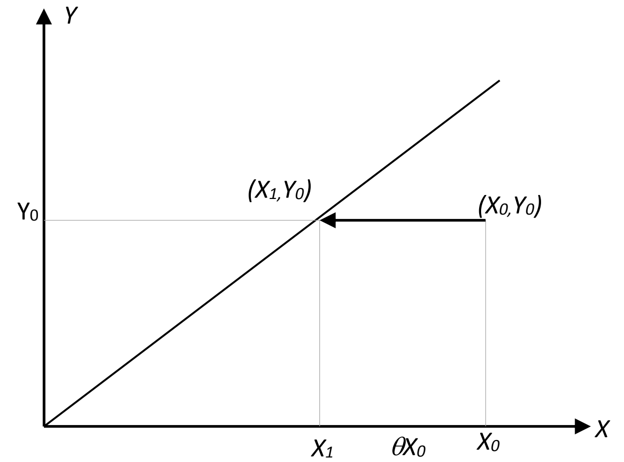

Figure 2.4. Decreasing the Input

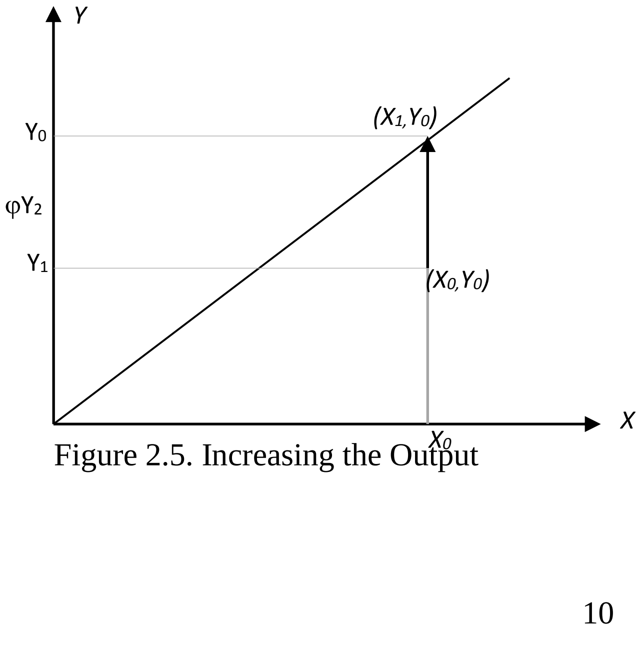

Figure 2.5. Increasing the Output

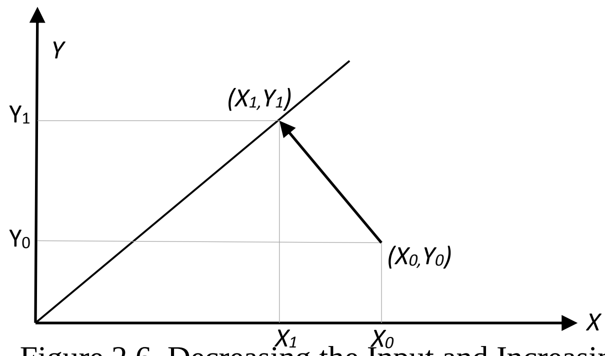

Figure 2.6. Decreasing the Input and Increasing the Output

So for the figure 2.4 PPS is as following

$$\begin{array}{ll} Min & \theta \\ s.t & (\theta x_0, y_0) \\ & \theta\, free \end{array} \tag{2-6}$$

$$\begin{array}{ll} Min & \theta \\ s.t & \sum_{j=1}^{n} \lambda_j x_j \le \theta x_0 \\ & \sum_{j=1}^{n} \lambda_j y_j \le \theta y_0 \\ & \theta\, free \end{array} \tag{2-7}$$

Imagine DMU_0 is one of the DMUs being assessed. The following multiplier model which is duality envelopment (2-7), is used to calculate the weights of inputs v_i and weights of outputs u_i.

$$\begin{array}{ll} Max & \theta = \sum_{r=1}^{s} u_r y_{r0} \\ s.t & \sum_{i=1}^{m} v_i x_{i0} = 1 \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \\ & v_i, u_r \ge 0 \end{array} \tag{2-8}$$

The vectored form of the above linear programming is as following:

$$
\begin{aligned}
&Max \quad \theta = uy_0 \\
&s.t \quad vx_0 = 1 \\
&\quad uy_j - vx_j \le 0 \\
&\quad v, \quad u \ge 0
\end{aligned}
\qquad (2-9)
$$

Now according to the conditions of equation (2-8), the linear programming of it could be rewritten by using a real variable θ and non-negative variables $\lambda_j \ge 0$ represented as following:

$$
\begin{aligned}
&Min \quad \theta \\
&st. \quad -\sum_{j=1}^{n} \lambda_j x_j + \theta x_0 \ge 0 \\
&\quad \sum_{j=1}^{n} \lambda_j y_j \ge y_0 \\
&\quad \lambda_j \ge 0 \\
&\quad \theta \, free
\end{aligned}
\qquad (2\text{-}10)
$$

The vectored form of the above linear programming is as following:

$$
\begin{aligned}
&Min \quad \theta \\
&st. \quad \theta \mathrm{x}_0 - x\lambda \ge 0 \\
&\quad \lambda y - y_0 \ge 0 \\
&\quad \lambda \ge 0 \\
&\quad \theta \, free
\end{aligned}
\qquad (2\text{-}11)
$$

For $\lambda = (\lambda_1 ... \lambda_n)$, the first constraint of (2-9) and duality (2-10) are corresponding and the variables are shown in the following table:

Table 2.1. Corresponding Primal and Dual CCR

(2-9) Variables	(2-) constraint	Variables(2-10)	(2-9) constraint
$V \ge 0$	10	$\theta \ge 0$	$Vx_0=1$
$u \ge 0$	$\theta x_0 - \lambda x_j \ge 0$	$\lambda_j \ge 0$	$-Vx_j - Vy_j \le 0$
	$\lambda y_j - y_0 \ge 0$		

Feasible answer of model (2-3) is $\theta = 1, \lambda_0 = 1,\ \lambda_j = 0\ (j \neq 0)$. Hence the optimum amount of θ is shown as θ^* which is not bigger than 1. In other words, since the final constrains of (2-3) is non-zero and $y_0 \geq 0$ and $y_0 \neq 0$, results λ to be non-zero. Hence according to (2-3) θ must be greater than zero and to consider all, the result will be $0 \leq \theta^* \leq 1$. According to the previous assumptions, if $\theta^* \leq 1$, then $(\lambda x, \lambda y)$ would be as $(\theta x_0, y_0)$ and accordingly the slack variable of $s^- \in R^m$ and $s^+ \in R^s$ will be generated. The slack vectors are as following:

$$s^- = \theta x_0 - \lambda x_j$$
$$s^+ = \lambda y_j - y_0$$

For $s^- \geq 0$ and $s^+ \geq 0$, feasible answer (θ, λ) from (2-10) is resulted. Then for all $j = 1...n$ model (2-10) could be rewritten as:

$$\begin{aligned}
&Min && \theta \\
&st. && \theta x_0 - y_0 + s^- = 0 \\
& && \lambda \mathrm{y}_j - y_0 + s^+ = 0 \\
& && \lambda \geq 0 \\
& && s^- \geq 0 \\
& && s^+ \geq 0 \\
& && \theta\ free
\end{aligned} \tag{2-12}$$

1) DMU$_0$ in CCR is efficient if the optimum $(\theta^*, \lambda^*, s^{-*}, s^{+*})$ is true for (2-12) and $\theta^* = 1$ and all the auxiliary variables are equal to zero.

2) DMU$_0$ in CCR is in weak form of efficiency if, in the above optimum answer $\theta^* = 1$ and all the auxiliary variables are not zero and $s^- \geq 0$ and $s^+ \geq 0$.

3) DMU$_0$ in CCR is inefficient, if $\theta^* \neq 1$.

2.7 Non-Archimedean Epsilon

To start the section an example is used.

Example: assume two DMUs with two inputs and one output. The information related to them is represented in table 2.2.

Table 2.2. Information of Example

DMU	**B**	**A**
x_1	2	2
x_2	5	6
y_1	1	1

Now the original model of CCR will be employed on the above DMUs.

B

$$\begin{aligned} Max \quad & u \\ st. \quad & 2v_1 + 6v_2 = 1 \\ & u - 2v_1 - 6v_2 \le 0 \\ & u - 2v_1 - 5v_2 \le 0 \end{aligned}$$

A

$$\begin{aligned} Max \quad & u \\ st. \quad & 2v_1 + 5v_2 = 1 \\ & u - 2v_1 - 6v_2 \le 0 \\ & u - 2v_1 - 5v_2 \le 0 \end{aligned}$$

Both above linear programming equations, generate equal answer as $v_2^* = 0$ and $v_1^* = 0.5$ and $u^* = 1$. Since the objective function for both equations is equal to one, It could comprehended that both DMS (A and B) are efficient. However, since both outputs are equal to one, hence DMU_B is inefficient. The reason for that is, $v_2^* = 0$ which states that DMU_B uses more amount of input with respect to DMU_A in other words, the second input doesn’t have any influence on efficiency. So according to linear programming Both DMUs are efficient and even if a number of DMUs were added to the system, the same result would be achieved. Ali *et al.* (1993) used non-Archimedean Epsilon to overcome the mentioned errors. For non-negative constrains in CCR, they used $v_i \geq \varepsilon$ and $u_r \geq \varepsilon$ instead of $v_i \geq 0$ and $u_r \geq 0$. As the result, the new CCR model is as following:

$$\begin{aligned}
&Max \qquad \theta = \sum_{r=1}^{s} u_r y_{r0} \\
&s.t \qquad \sum_{i=1}^{m} v_i x_{i0} = 1 \\
&\qquad\qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \\
&\qquad\qquad v_i \geq 1\varepsilon, \quad u_r \geq 1\varepsilon
\end{aligned} \qquad (2-13)$$

Duality for of the above equation is as follow:

$$\begin{aligned}
&Max \qquad \theta - \left(\sum_{i=1}^{m} s_i + \sum_{r=1}^{s} s_r \right) \\
&st. \qquad \sum_{j=1}^{n} \lambda_j x_j + s_i^{-} = \theta x_0 \\
&\qquad\qquad \sum_{j=1}^{n} \lambda_j y_j + s_r^{+} = y_0 \\
&\qquad\qquad \lambda_j, s_i^{-}, s_r^{+} \geq 0
\end{aligned} \qquad (2\text{-}14)$$

According to the theoretical framework to prevent the weights to become zero, ε was allocated as a lower boundary. Ali *et al*.(1994) according Ali(1993,1994) suggested,

An upper bound for ε in a way which multiplier part be feasible and envelopment part being bounded. However, Mehrabian *et al.* (1998) illustrated in an example that the work of Ali(1994) is not true. They suggested a procedure to determine the confidence interval for ε. The confidence interval for ε is an interval which, for any value of ε for both multiplier and envelopment part, all DEA models are bounded. They also suggested a linear programming equation to identify the proper interval for ε.

For each observed DMU, the following equation shall be solved.

$$
\begin{aligned}
&Max \quad && \varepsilon_j \\
&st. && \sum_{i=1}^{m} v_i x_{i0} = 1 \\
& && \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \\
& && v_i \geq 1\varepsilon \quad u_r \geq 1\varepsilon
\end{aligned}
\tag{2-15}
$$

The above linear programming equation is feasible. Assume ε_j^* is the optimum answer for (2-15). ε_j^* shows the maximum value of ε in the feasible area of CCR model. Now ε^* is defined as following:

$$\varepsilon^* = Max\left\{\varepsilon_1^* ... \varepsilon_n^*\right\} \tag{2-16}$$

In the end the value of ε^* is used as a lower bound for all the weights in all the DMUs in CCR/ε model. Mehrabian et al. (1998) showed that $\left[0, \varepsilon^*\right]$ is a confidence interval for ε.

Chapter 3

METHODOLOGY

3.1 Modified CCR Model through Facet Analysis

This chapter discusses the CCR model through facet analysis. Facet analysis is the focus of hyper-plans on PPS frontier. As it has been mentioned earlier, frontier boarder estimates production function in input-output space. For the original DEA models, those hype-plans, which generate PPS on efficient DMUs, construct the structure of efficient frontier. Facet analysis enhances us to obtain information about those hyper-plans.

The following sections will focus on those hyper-plans, which construct and relocate the efficient frontier. This movement should occur according to PPS perspectives. Furthermore, this chapter describes Modified CCR model through facet analysis. In other words, when CCR model is used on number of observations without considering the ε, for DMUs on the weak frontier and efficient DMUs, efficiency rate will be used. However, level of true efficiency for the efficiency of weak DMUs and those being compared to them is not calculable via CCR/ ε.

3.2 Facet Analysis

Bessent et al (1988), Chang *et al.* (1991, 1995) have proved facet analysis for the first time. They entered this concept in CCR model. For a polyhedral with n dimensions, facet analysis is described as a district, which has degree of freedom of n-1. In other words, a district with n-1 dimensions, in a district with n dimensions, is a facet with *n-1* dimensions for linear polyhedral hyper-plans. Facet analysis in DEA creates an equation with algebraic and geometric perspectives. In other words, facet analysis for T_c, revises the relation between feasible districts of (2-2) and (2-6). Return to scale is used to implement the concepts of facet analysis. For those DMUs being analyzed such as (x_0, y_0), facet analysis shows that return to scale is the feasible answer for (2-2) and (2-6). These answers are normal vectors for generating PPS hyper-plans.

3.2.1 Return to Scale

The equation which is used in economics is describes in details in Figure 3.1.

Function of $y = f(x)$ on the top of the figure is production function which states that for each x, y in the mentioned function is maximum. This result is technical efficiency. Hence, coordinates of p, which is inside the PPS is not included in the concepts that this study focuses on. Only those coordinates, which are on the production frontier, are important to us. Two figures are shown in following. In the second figure, average production behavior $\left(a.p = y/x\right)$ and the final productivity $\left(m.p = \partial y/\partial x\right)$ are defined. The confluence point of these two lines is shown by x_0. Now in the first figure, average corresponding y/x for the slope of the line from center of coordinates to y and $\partial y/\partial x$ will be have the same coordinates as x_0.

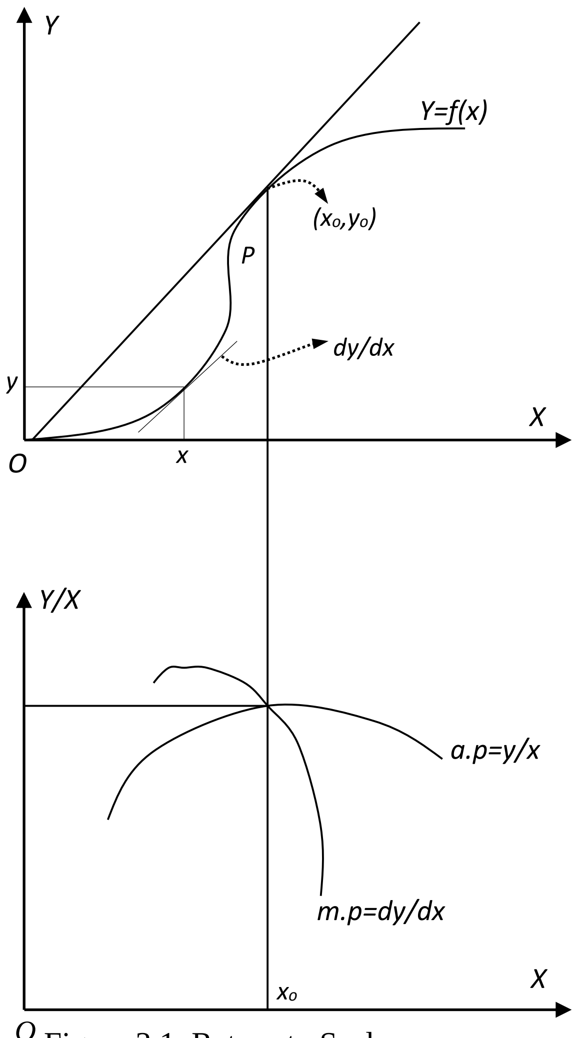

Figure 3.1. Return to Scale

As it is shown in the figure, slope of the line is increasing when moving from x to x_0. In this case, it is said that return to scale is increasing. Later on when the slope of lines start to decrease, return to scale starts to decrease accordingly and for x_0 the return to scale is constant. In a similar approach, it is assumed that $\left(m.p = {\partial y}/{\partial x}\right)$ increases as x increases till reaches to a point on $f\left(x\right)$ and in this point turns and after that it decreases.

As it has been shown in the above figure, according to $a.p$ and $m.p$ curves, left edge of $m.p$ is above $a.p$, which states that on the left side of x_0, outputs change with more pace than inputs. Now, on the right side of x_0, right edge of $m.p$ is below $a.p$ which states that the reverse position for inouts occur.

In economics books, Return to Scale (RTS) is defined as a value for just outputs. Bessent (1988) developed this concept for multiple outputs. In cases which there are several in and outputs, RTS cause changes on production under the influence of production factors. From the mathematical point of view, RTS for multiple in and outputs is defined as following:

Definition 3-1:

Assume, $(x_0, y_0) \in T(T_c)$, and for the constant value of $\beta \geq 0$ the equation will be:

$$\beta(\alpha) = Max\{\beta \mid (\alpha x_0, \beta y_0) \in T\}, \quad \gamma = \lim_{\alpha \to 1} \frac{\beta(\alpha) - 1}{\alpha - 1}$$

$(x_0, y_0) : \gamma = 1$ Return to scale is constant

$(x_0, y_0) : \gamma > 1$ Return to scale is increasing

$(x_0, y_0) : \gamma < 1$ Return to scale is decreasing

Now to extend the concepts, produced hyper-planes are needed which are generalized as following:

H_0 hyper plan with $m+s$ dimension on input-output space includes (x_0, y_0). The inclusion is formulized as following:

$$\mathrm{H_0 : u(y-y_0)-v(x,x_0)=0} \tag{3-1}$$

In the above formula, $u \in R^s$ and $v \in R^m$ are vector coefficients. Now u_0 will be defined as:

$$u_0 = vx_0 - uy_0 \tag{3-2}$$

As the result, the final hyper-plan (3-1) is as following:

$$uy_0 - vx + u_0 = 0 \tag{3-3}$$

In general, a hyper-plan divides the space in to two half-spaces. If H_0 hyper-plan, which contains PPS, is located in one of these two half-spaces, then the generated hyper-plan with PPS is at (x_0, y_0). The reason is the contact point of generated hyper-plan and PPS is at (x_0, y_0). The following equation is applied for dependent DMUS, for all those coordinates (x, y) which are allocated to PPS:

$$uy - vx + u_0 \le 0$$

From the definition of PPS and the mentioned features, a hyper-plan is generated which could be shown as $u > 0$ and $v > 0$, since PPS is only located on one segment of hyper-plan. Furthermore, if a linear equation is multiplied to a non-zero number, then the result will be the same. So to estimate the features of resources the following constrain is established.

$$\text{"} \ \forall x_0 = 1 \text{"} \tag{3-4}$$

PPS is generated from the observed DMUS and $uy - vx + u_0 = 0$. The above discussion, re-express the relation between $u \in R^s$ and $v \in R^m$ with weights of observed DMUs. Now in CCR model for efficient DMUs such as (x_0, y_0) from (3-2) (3-3) (3-4): $uy_0 = 1$.

Since $u_0 = 0$ hence the hyper-plan of $uy - vx = 0$ is a hyper-plan for PPS at (x_0, y_0) with normal vector $(\ -v,\ \ u\ )$, which passes from the center of coordinates.

3.2.2 Facet analysis on CCR

In input-output space, all the efficient frontier hyper-plans pass from the center of coordinates and efficient DMUs followed and tracked in them. An efficient DMU such as (x_0, y_0) in CCR, is considered in (3-3) with the optimal solution of v^* and u^*. So:

$$\sum_{r=1}^{s} u_r y_{r0} = 1 = \sum_{i=1}^{m} v_i x_{i0}$$
$$\sum_{r=1}^{s} u_r^* y_{r0} - \sum_{i=1}^{m} v_i x_{i0} = 0$$

As it **w**as shown in the previous section, the hyper-plan of

$$\sum_{r=1}^{s} u_r^* y_{r0} - \sum_{i=1}^{m} v_i x_{i0} = 0$$

In input-output space is a hyper-plan which passes from the center of coordinates and creates T_c at (x_0, y_0) and $(\ -v^*,\ \ u^*\ )$ is its normal vector. Hence, (x_0, y_0) is efficient in CCR model. When $\theta^* = 1$, optimal value of (3-3) is equal to one. A DMU has a weak efficiency in CCR model if, for *r* or *i* $v_i^* = 0$ or $u_i^* = 0$ and s_i^- or s_r^+ are non-zero and non-negative. So those hyper-plans which pass from DMUs with weak efficient, have at least a zero parameter in their normal vector of $(\ -v^*,\ \ u^*\ )$.

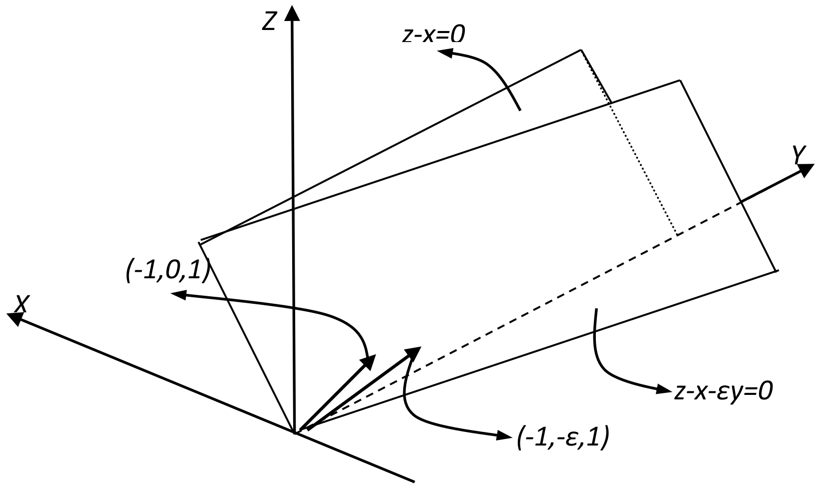

Figure 3.2. Having at least a zero parameter in normal vector

Figure 3.2 in 3 dimensional spaces describes that, if normal vector of a hyper-plan becomes equal to zero by one of its factors, this hyper-plan is parallel with its corresponding axis. These hyper-plans construct weak frontier and called weak frontier hyper-plans. So they are parallel with the last axis of inputs or outputs. As it has been mentioned for separating the efficient DMUs, non-Archimedean Epsilon used as lower bound. In fact Epsilon interferes with normal vectors and doesn't allow the weak efficient hyper-plans to be constructed. Figure 3.3 illustrates the above discussion.

Changes in efficiency of weak efficient DMUs and those DMUs being compared to them, is dependent on Epsilon. Choosing the right value for Epsilon influences the efficiency of DMUs.

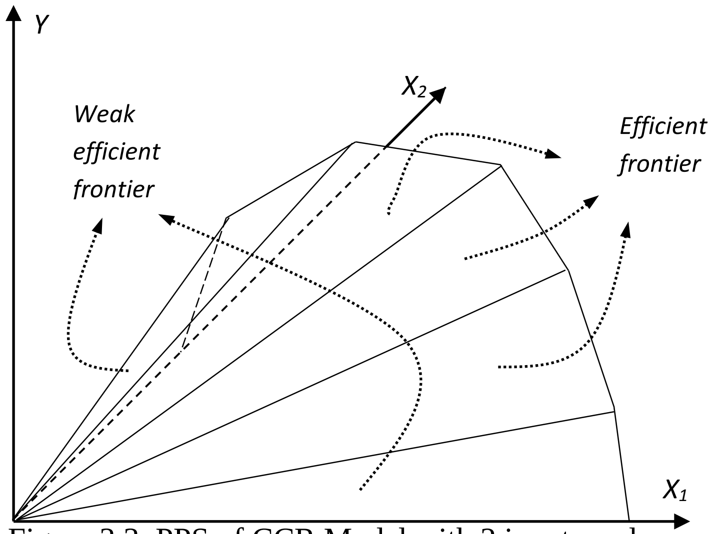

Figure 3.3. PPS of CCR Model with 2 inputs and an output.

Figure 3.3 illustrates PPS of CCR Model with 2 inputs and an output and shows the weak frontier. The goal in the following part is to modify the hyper-plans of T_c. by assessing the correct definitions of efficiency, calculation for CCR starts. This modification should be done in a way, which keeps the features of T_c and decrease the errors efficiency of weak efficient DMUs and those DDMUS being compared to them.

To make this happen, to relocate the weak frontier of hype-plans, a change between modified hyper-plans and the mentioned ones is needed. This clears that T_c could also be defined from the interface of those efficient DMUs, which creates hyper-plans and pass from the center of coordinates. For correspondence between each efficient DMU, too many hyper-plans exist which apply to the above conditions. This states that a normal vector and a point on T_c determine a hyper-plan. Those T_c hyper-plans, which pass from efficient DMUs and center of coordinates, are defined via their normal vector. In other word, it is observable that weights vector $\left(-v, \ u \right)$ could evaluate

T_c hyper-plans in a way which normal vector does. CCR model is able to find those weight elements such as u and v, which are used to evaluate DMUs, substantially. Hence, it is observable that correspondence between each DMU in CCR model could be used to determine the normal vector of T_c hyper-plans. Now, an interval is defined for the variables of this normal vector for each efficient DMU. This range determines a form, which keeps the features of T_c. This interval is used for obtaining those eligible hyper-plans which, could be replaced with weak frontier hyper-plans.

For an efficient DMU (x_0, y_0) for each *i=1…m* ، *r=1…s* ، v_i and u_r, the following linear programming is considered.

$$\begin{aligned} &Max \quad u_r && (3-5) \\ &s.t \quad UY_o + u_o = 1 \\ &\quad UY_j - VX_j + u_o \le 0 \;\; for \;\; j = 1,\dots,n \\ &\quad VX_o = 1 \\ &\quad U \ge 0, V \ge 0 \\ &\quad u_o \;\; free \end{aligned}$$

$$\begin{aligned} &Min \quad u_r && (3-6) \\ &s.t \quad UY_o + u_o = 1 \\ &\quad UY_j - VX_j + u_0 \le 0 \;\; for \;\; j = 1,\dots,n \\ &\quad VX_o = 1 \\ &\quad U \ge 0, V \ge 0 \\ &\quad u_o \;\; free \end{aligned}$$

$$\begin{aligned} &Max \quad v_i && (3-7) \\ &s.t \quad UY_o + u_o = 1 \\ &\quad UY_j - VX_j + u_o \le 0 \;\; for \;\; j = 1,\dots,n \\ &\quad VX_o = 1 \\ &\quad U \ge 0, V \ge 0 \\ &\quad u_o \;\; free \end{aligned}$$

$$\begin{aligned} &Min \quad v_i && (3-8) \\ &s.t \quad UY_o + u_o = 1 \\ &\quad UY_j - VX_j + u_0 \le 0 \;\; for \;\; j = 1,\dots,n \\ &\quad VX_o = 1 \\ &\quad U \ge 0, V \ge 0 \\ &\quad u_o \;\; free \end{aligned}$$

Assume, u_r^+ and u_r^- are the optimal values for (3-5) and (3-6), respectively and also v_i^+ and v_i^- are the optimal values for (3-7) and (3-8), respectively:

$$\varepsilon_r^+ = Min \left\{ u_r^+ \text{ for efficient DMU} \right\}$$

$$\varepsilon_r^- = Max \left\{ u_r^- \text{ for efficient DMUs} \right\}$$

$$\varepsilon_i^+ = Min\left\{v_i^+ \text{ for efficient DMUs}\right\}$$

$$\varepsilon_i^- = Max\left\{v_i^- \text{ for efficient DMUs}\right\}$$

Definition 3.2:

For efficient DMU (x_0, y_0) in CCR model, each hyper-plan

$(\ -v^*,\ \ u^*\ )$, is as the same as the normal vector which applies in the following inequality and is the accepted hyper-plans for T_c.

$$\varepsilon_r^- \le u_r^* \le \varepsilon_r^+ \quad \forall r = 1,2,\ldots,s$$

$$\varepsilon_i^- \le v_i^* \le \varepsilon_i^+ \quad \forall i = 1,2,\ldots,m$$

3.3 Facet analysis on CCR model

3.3.1 Modified CCR Model

In this section CCR model will be modified with facet analysis. As it has been mentioned in previous section, when CCR model is used on a set of DMUs without non-Archimedean Epsilon, the efficiency for those DMUs on weak frontier, and also for those DMUs being compared to this frontier, could not accurately be calculated. Epsilon function separates, weak DMUs out of efficient DMUs. But CCR/ε model is not able to do so. When unique values are used for Epsilon as lower bound, in fact the zero element of normal vector from the weak frontier hyper-plan causes interferes. So if a proper value is calculated for ε a hyper-plan from the weak frontier will be relocated.

It is known that when a variable of normal vector from weak hyper-plan is zero, there exist i or r which $u_r = 0$ and $v_i = 0$. So it is clear that having Epsilon could prevent the normal vector of these hyper-plans. Because of Epsilon, normal vector of weak frontier is not allowed to move towards zero. It means that $\left(s_i^-, \ s_r^+ \right)$ could be non-zero. According to CCR for the movement of weak frontier, it should be known that DMUs are supposed to be placed in the interface of efficient and weak frontiers. Figure (3-4) illustrates some of these DMUs with two inputs-one output, and one input- two outputs.

So to use this modification in CCR, these DMUs should be recognized from the other DMUs. To do so, according to the concepts on CCR efficiency, the following linear programming should be considered for efficient DMUs.

The above linear programming could be applied for the observed DMUs. However, it is infeasible for inefficient DMUs. Efficient DMUS in a situation where the optimal solution of (3-9) for them is non-zero, are those which could be placed on the interface of efficient.

$$\begin{aligned} & Max && \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{m} s_r^+ \\ & st. && x_{i0} - \sum_{j=1}^{n} \lambda_j x_{ij} - S_i^- = 0 \\ & && y_{r0} - \sum_{j=1}^{n} \lambda_j y_{rj} + S_r^+ = 0 \\ & && \lambda_j, \ \ S_i^-, \ \ S_r^+ \geq 0 \end{aligned} \tag{3-9}$$

frontier and weak efficient hyper-plan. Assume that the set of these DMUs i β s. (Figure 3.4)

Now the following linear programming equations should be solved for those DMUS, which are in β.

$$
\begin{aligned}
&Max && v_i \\
&st. && \sum_{i=1}^{m} v_i x_{i0} = 1 \\
& && \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \; j = 1, \;\; 2 \;\; \dots \;\; n \\
& && v_i \ge 0 \; u_i \ge 0
\end{aligned} \tag{3-10}
$$

And

$$
\begin{aligned}
&Max && \mathrm{u}_i \\
&st. && \sum_{i=1}^{m} v_i x_{i0} = 1 \\
& && \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \; j = 1, \;\; 2 \;\; \dots \;\; n \\
& && v_i \ge 0 \; u_i \ge 0
\end{aligned} \tag{3-11}
$$

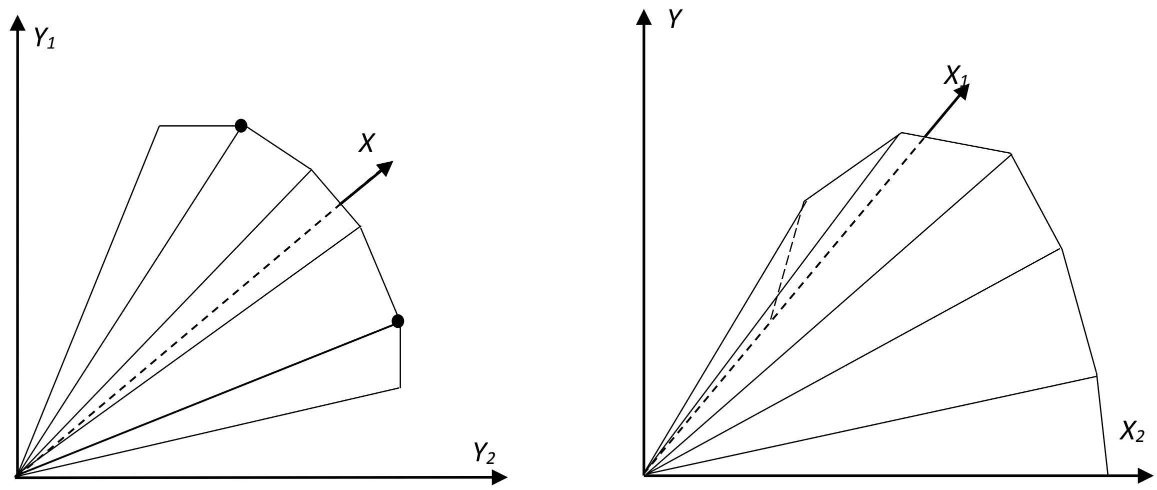

Figure (3.4). Members of β for T_c in two forms.

Assume that the optimal solutions in (3-10) and (3-11) are shown by u_r^+ and v_r^+ respectively. To decrease the number of calculations, it is suggested that, linear programming (3-10) and (3-11) be solved only for v_i and u_r with similar indices and for the situation where $s_i^- > 0$ and $s_r^+ > 0$ have the same optimal solution as (3-9).

For each *r=1….s* and *i= 1…m*, assume,

$$\epsilon_r = min\{u_r^+ | DMU \;\in B\} \; \forall r = 1,2 \dots s \quad (12 - 3)$$

$\epsilon_i = min\{v_i^+ | DMU \in B\} \, \forall i = 1,2 \dots m \quad (13-3)$

Now according to (3-12) and (3-13), CCR model is modified as following:

$$
\begin{aligned}
&Max && u_r \\
&st. && \sum_{i=1}^{m} v_i x_{i0} = 1 \\
& && \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \; j = 1, \; 2 \; \dots \; n \\
& && v_i \ge \varepsilon_i \; u_r \ge \varepsilon_r \qquad (3\text{-}14)
\end{aligned}
$$

By referring to the concepts of ε_r and ε_i and applying them as the lower bound for each weight in CCR model, acceptable hyper-plans will be generated and weak frontier hyper-plans will be relocated. This relocation in MCCR prevents unbound envelopment and feasibility in multiplier perspectives.

Chapter 4

ANALYSIS AND RESULTS

4.1 Selection of Best Product Design using DEA

Among many similar units, the most effective units can be identified using DEA method. The units can be banks, educational organizations, hospitals etc. The units under evaluation, commonly, take a major number of inputs and convert to a large number of outputs. When there are multiple inputs and multiple outputs making the evaluation of the most complex, DEA is an excellent method to use. In the existing problem, a product design selection has been assumed. Table.1 characterizes 14 designs alternatives debated by Besharati et al. (2006) for Power Electronic Device. Particularly, there are no inputs and outputs in this problem that can be used to find the best product design or unit. Considered problem has certain attributes like junction temperature, manufacturing cost and thermal cycles to failure.

Table4.1. Electronic Power Attributes

Attributes / DMUs	JT (input)	MC (input)	CF (output)
1	126	85	220
2	105	99	380
3	138	65	140
4	140	60	130
5	147	52	106
6	116	88	270
7	112	92	320
8	132	75	170
9	122	85	235
10	135	62	150
11	115	73	333
12	100	145	434
13	102	173	443
14	123	64	292

Hence as to use DEA model in this case, it is necessary to specify inputs and outputs virtually. In table one it can be noticed that number of cycles to fail is a beneficial feature so with higher values for CF performance of the product is much better. The Junction Temperature and Manufacturing Cost are unbeneficial features. In other words it is better to minimize values of these features for the same output (CF) Hence this problem present as two inputs (JT and MC) and one output (CF) in the DEA model. After normalizing the data of the Table 4-1 with respect to the single output i.e., the number of cycles to failure (CF), the data is presented in the Table 4-2.

Table 4.2. Normalized Data of Electronic Power Devise

Attributes / DMUs	JT (input)	MC (input)	CF (output)
1	0.5727	0.3864	1
2	0.2763	0.2605	1
3	0.9857	0.4643	1
4	1.0769	0.4615	1
5	1.3868	0.4906	1
6	0.4296	0.3259	1
7	0.35	0.2875	1
8	0.7765	0.4412	1
9	0.5191	0.3617	1
10	0.9	0.4133	1
11	0.345	0.22	1
12	0.23	0.334	1
13	0.23	0.3897	1
14	0.421	0.22	1

4.2 Implementing the Solution

Now by using CCR model and solving linear problem for all DMUs, the efficient DMUs will be achieved. There is one sample solved linear program for DMU_3 by using Lingo software.

$$
\begin{aligned}
&Max \quad u \\
&st. \quad 0.9857v_1 + 0.4643v_2 = 1 \\
&\qquad u - 0.5727v_1 - 0.3864v_2 \le 0 \\
&\qquad u - 0.2763v_1 - 0.2605v_2 \le 0 \\
&\qquad u - 0.9857v_1 - 0.4643v_2 \le 0 \\
&\qquad u - 1.0769v_1 - 0.4615v_2 \le 0 \\
&\qquad u - 1.3868v_1 - 0.4906v_2 \le 0 \\
&\qquad u - 0.4296v_1 - 0.3259v_2 \le 0 \\
&\qquad u - 0.3500v_1 - 0.2875v_2 \le 0 \\
&\qquad u - 0.7765v_1 - 0.4412v_2 \le 0 \\
&\qquad u - 0.5191v_1 - 0.3617v_2 \le 0 \\
&\qquad u - 0.9000v_1 - 0.4133v_2 \le 0 \\
&\qquad u - 0.3450v_1 - 0.2200v_2 \le 0 \\
&\qquad u - 0.2300v_1 - 0.3340v_2 \le 0 \\
&\qquad u - 0.2300v_1 - 0.3897v_2 \le 0 \\
&\qquad u - 0.4210v_1 - 0.2200v_2 \le 0 \\
&\qquad u \ge 0, \quad v_1 \ge 0 \quad v_2 \ge 0 \\
&\qquad u^* = 0.4738 \qquad v_1^* = 0.0 \qquad v_2^* = 2.1538
\end{aligned}
$$

As it has been observed, since the lower bound is considered zero for v_1 and v_2, as the result v_1^* and v_2^* for some of DMUs such as DMU_{13} and DMU_{14} is calculated to be zero. however the efficiency of these DMUs (U*) is equal to one and as it has been mentioned in previous chapter of this very study, this number is the representative OF WEAK efficiency. Hence, efficient scores have to be estimated for them. Table 4-3 presents the CCR model for all DMUs.

Table 4.3. CCR Model Results

DMUs	**U***	**V_1***	**V_2***
1	0.5848	0.8142	1.3812
2	1	2.2702	1.4304
3	0.4738	0	2.1538
4	0.4767	0	2.1665
5	0.4484	0	2.0383
6	0.731	1.0179	1.7266
7	0.8573	1.1938	2.025
8	0.4986	0	2.2665
9	0.6341	0.8816	1.4954
10	0.5323	0	2.4195
11	1	1.3924	2.3619
12	1	2.2707	1.4304
13	1	4.3478	0
14	1	0	4.5455

According to what had been discussed on modified CCR model, set of DMUs which have the best efficient among all DMUs and by using (3-10) and (3-11) calculate V_1, V_2 and U_1 is assumed.

For evaluating amount of $\mathcal{E}_r$ and $\mathcal{E}_i$ use (3-12) and (3-13)

$$\varepsilon_{i\,1} = 0.4210$$

$$\varepsilon_{i\,2} = 0.3897$$

$$\varepsilon_r = 0$$

Now by using the calculated Epsilon and equation (3-14), new results are calculated for DMU_3.

$$
\begin{array}{ll}
Max & u \\
st. & 0.9857v_1 + 0.4643v_2 = 1 \\
& u - 0.5727v_1 - 0.3864v_2 \le 0 \\
& u - 0.2763v_1 - 0.2605v_2 \le 0 \\
& u - 0.9857v_1 - 0.4643v_2 \le 0 \\
& u - 1.0769v_1 - 0.4615v_2 \le 0 \\
& u - 1.3868v_1 - 0.4906v_2 \le 0 \\
& u - 0.4296v_1 - 0.3259v_2 \le 0 \\
& u - 0.3500v_1 - 0.2875v_2 \le 0 \\
& u - 0.7765v_1 - 0.4412v_2 \le 0 \\
& u - 0.5191v_1 - 0.3617v_2 \le 0 \\
& u - 0.9000v_1 - 0.4133v_2 \le 0 \\
& u - 0.3450v_1 - 0.2200v_2 \le 0 \\
& u - 0.2300v_1 - 0.3340v_2 \le 0 \\
& u - 0.2300v_1 - 0.3897v_2 \le 0 \\
& u - 0.4210v_1 - 0.2200v_2 \le 0 \\
& u \ge 0, \quad v_1 \ge 0.4210 \quad v_2 \ge 0.3897 \\
& u^* = 0.4647 \qquad v_1^* = 0.0744 \qquad v_2^* = 1.9958
\end{array}
$$

Table 4-4 shows recalculate the modified CCR model with new Ɛ to find weak efficient DMUs.

Table4.4. Modified CCR Model Results

DMUs	**U^***	**V_1^***	**V_2^***
1	0.5848	0.8142	1.3812
2	1	2.2702	1.4304
3	0.4230	0.4210	1.2629
4	0.4058	0.4210	1.1844
5	0.3318	0.4210	0.8482
6	0.731	1.0178	1.7266
7	0.8573	1.1937	2.0249
8	0.4808	0.4210	1.5255
9	0.6340	0.8828	1.4976
10	0.4758	0.4210	1.5027
11	1	0.4210	3.8852
12	1	3.7819	0.3897
13	0.9782	3.6875	0.3897
14	0.9680	0.4210	3.7398

By comparing table 4-3 and table 4-4 some differences in values of u^*, v_1^* and v_2^* could be noticed (Table 4-5). These differences are caused by using value of Ɛ as lower boundaries, the weak efficient boundaries are moved and as mentioned before by replacing these boundaries, the optimal value of DMU's which are on these boundaries and also the DMU's which are comparing with them will be change.

Table 4.5. Comparison of the Classic and Modified CCR

DMU	**θ^* CCR**	**θ^* Modified CCR**
1	0.5848	0.5848
2	1	1
3	0.4738	0.4230
4	0.4767	0.4058
5	0.4484	0.3318
6	0.7310	0.731
7	0.8573	0.8573
8	0.4986	0.4808
9	0.6341	0.6340
10	0.5323	0.4758
11	1	1
12	1	1
13	1	0.9782
14	1	0.9680

Chapter 5

CONCLUSION AND DISCUSSION

5.1 Conclusion and Discussion

As it has been seen earlier, some of DMUs are located on frontier and some others are not. Not all DMUs make the efficient frontier and it is obvious that some of them are likely to construct the weak frontier. Now by implementing the modified CCR model on theses DMUs, results will be the shift of them from weak frontier to efficient frontier.

As an example, in fig 5.1 DMU_2 Is located on efficient frontier and DMU_{13} and DMU_{14} are located on weak frontier. DMU 12 and 11 are also located on efficient frontier and weak frontier interface. By using Epsilon as the lower bound and movement in weak frontier, the efficiency of those inefficient DMUs will change. As an instance, if a straight line is drawn from the center of coordinates to DMU_3, it will cross the CCR model at point of A'. it also crosses the efficient frontier in MCCR at A''. length of AA'' in MCCR is larger than length of AA' in CCR. The efficiency for that is calculated as follow:

CCR model: $\theta_A^{*} = \frac{OA^{'}}{OA}$

MCCR model: $\theta_A^{**} = \frac{OA^{''}}{OA}$

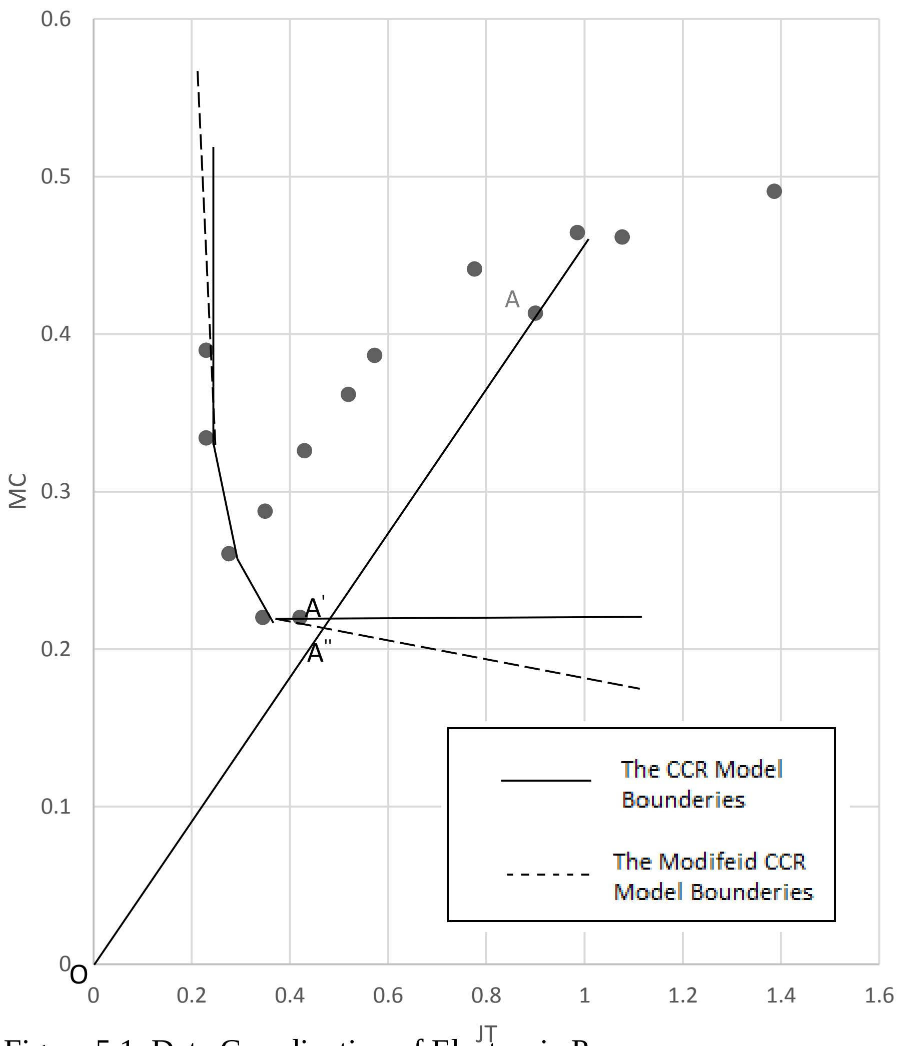

Figure 5.1. Data Coordination of Electronic Power

Results show that score of efficiency in MCCR is greater than the one for CCR. According to table 4.5 for DMU_3, this score is certified.

The focus of this study was on a specific case study. To obtain more comprehensive results that are more comprehensive it is suggested for further studies to select projects and cases with more DMUs.

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APPENDIX

DMU 1 CCR Model

Variable	Value	Reduced Cost
U	0.5847705	0.000000
V_1	0.8142335	0.000000
V_2	1.381181	0.000000

Row	Slack or Surplus	Dual Price
1	0.5847705	1.000000
2	0.4152295	0.000000
3	0.000000	0.1470445
4	0.8564965	0.000000
5	0.9294928	0.000000
6	1.222016	0.000000
7	0.2151513	0.000000
8	0.9730091E-01	0.000000
9	0.6568591	0.000000
10	0.3374714	0.000000
11	0.7188820	0.000000
12	0.000000	0.8529555
13	0.6381781E-01	0.000000
14	0.1407496	0.000000
15	0.6188175E-01	0.000000
16	0.000000	0.5847705

DMU2 CCR Model

Variable	Value	Reduced Cost
U	1.000000	0.000000
V_1	2.270677	0.000000
V_2	1.430372	0.000000

Row	Slack or Surplus	Dual Price
1	1.000000	1.000000
2	0.8531125	0.000000
3	0.000000	1.000000
4	1.895062	0.000000
5	2.105409	0.000000
6	2.850715	0.000000
7	0.4416411	0.000000
8	0.2059689	0.000000
9	1.394261	0.000000
10	0.6960740	0.000000
11	1.634782	0.000000
12	0.9806544E-01	0.000000
13	0.000000	0.000000
14	0.7967172E-01	0.000000
15	0.2706369	0.000000
16	0.000000	1.000000

DMU3 CCR Model

Variable	Value	Reduced Cost
U	0.4738316	0.000000
V_1	0.000000	0.1205395
V_2	2.153780	0.000000

Row	Slack or Surplus	Dual Price
1	0.4738316	1.000000
2	0.3583890	0.000000
3	0.8722809E-01	0.000000
4	0.5261684	0.000000
5	0.5201378	0.000000
6	0.5828128	0.000000
7	0.2280853	0.000000
8	0.1453801	0.000000
9	0.4764161	0.000000
10	0.3051906	0.000000
11	0.4163257	0.000000
12	0.000000	1.000000
13	0.2455309	0.000000
14	0.3654964	0.000000
15	0.000000	0.000000
16	0.000000	0.4738316

DMU4 CCR Model

Variable	Value	Reduced Cost
U	0.4767064	0.000000
V_1	0.000000	0.9236511E-01
V_2	2.166847	0.000000

Row	Slack or Surplus	Dual Price
1	0.4767064	1.000000
2	0.3605634	0.000000
3	0.8775731E-01	0.000000
4	0.5293608	0.000000
5	0.5232936	0.000000
6	0.5863489	0.000000
7	0.2294691	0.000000
8	0.1462622	0.000000
9	0.4793066	0.000000
10	0.3070423	0.000000
11	0.4188516	0.000000
12	0.000000	0.000000
13	0.2470206	0.000000
14	0.3677140	0.000000
15	0.000000	1.000000
16	0.000000	0.4767064

DMU5 CCR Model

Variable	Value	Reduced Cost
U	0.4484305	0.000000
V_1	0.000000	0.2008834
V_2	2.038320	0.000000

Row	Slack or Surplus	Dual Price
1	0.4484305	1.000000
2	0.3391765	0.000000
3	0.8255198E-01	0.000000
4	0.4979617	0.000000
5	0.4922544	0.000000
6	0.5515695	0.000000
7	0.2158581	0.000000
8	0.1375866	0.000000
9	0.4508765	0.000000
10	0.2888300	0.000000
11	0.3940073	0.000000
12	0.000000	0.000000
13	0.2323685	0.000000
14	0.3459030	0.000000
15	0.000000	1.000000
16	0.000000	0.4484305

DMU6 CCR Model

Variable	Value	Reduced Cost
U	0.7310346	0.000000
V_1	1.017892	0.000000
V_2	1.726646	0.000000

Row	Slack or Surplus	Dual Price
1	0.7310346	1.000000
2	0.5190877	0.000000
3	0.000000	0.4504735
4	1.070725	0.000000
5	1.161980	0.000000
6	1.527670	0.000000
7	0.2689654	0.000000
8	0.1216380	0.000000
9	0.8211542	0.000000
10	0.4218806	0.000000
11	0.8986904	0.000000
12	0.000000	0.5495265
13	0.7978008E-01	0.000000
14	0.1759542	0.000000
15	0.7735976E-01	0.000000
16	0.000000	0.7310346

DMU7 CCR Model

Variable	Value	Reduced Cost
U	0.8573450	0.000000
V_1	1.193766	0.000000
V_2	2.024981	0.000000

Row	Slack or Surplus	Dual Price
1	0.8573450	1.000000
2	0.6087773	0.000000
3	0.000000	0.6539921
4	1.255729	0.000000
5	1.362750	0.000000
6	1.791625	0.000000
7	0.3154380	0.000000
8	0.1426550	0.000000
9	0.9630357	0.000000
10	0.4947744	0.000000
11	1.053969	0.000000
12	0.000000	0.3460079
13	0.9356472E-01	0.000000
14	0.2063561	0.000000
15	0.9072621E-01	0.000000
16	0.000000	0.8573450

DMU8 CCR Model

Variable	Value	Reduced Cost
U	0.4986401	0.000000
V_1	0.000000	0.000000
V_2	2.266546	0.000000

Row	Slack or Surplus	Dual Price
1	0.4986401	1.000000
2	0.3771532	0.000000
3	0.9179510E-01	0.000000
4	0.5537171	0.000000
5	0.5473708	0.000000
6	0.6133273	0.000000
7	0.2400272	0.000000
8	0.1529918	0.000000
9	0.5013599	0.000000
10	0.3211695	0.000000
11	0.4381233	0.000000
12	0.000000	0.4448156
13	0.2583862	0.000000
14	0.3846328	0.000000
15	0.000000	0.5551844
16	0.000000	0.4986401

DMU9 CCR Model

Variable	Value	Reduced Cost
U	0.6340749	0.000000
V_1	0.8828850	0.000000
V_2	1.497635	0.000000

Row	Slack or Surplus	Dual Price
1	0.6340749	1.000000
2	0.4502393	0.000000
3	0.000000	0.2307382
4	0.9287113	0.000000
5	1.007862	0.000000
6	1.325049	0.000000
7	0.2332916	0.000000
8	0.1055048	0.000000
9	0.7122416	0.000000
10	0.3659251	0.000000
11	0.7794939	0.000000
12	0.000000	0.7692618
13	0.6919856E-01	0.000000
14	0.1526168	0.000000
15	0.6709926E-01	0.000000
16	0.000000	0.6340749

DMU10 CCR Model

Variable	Value	Reduced Cost
U	0.5323010	0.000000
V_1	0.000000	0.5807089E-01
V_2	2.419550	0.000000

Row	Slack or Surplus	Dual Price
1	0.5323010	1.000000
2	0.4026131	0.000000
3	0.9799177E-01	0.000000
4	0.5910961	0.000000
5	0.5843213	0.000000
6	0.6547302	0.000000
7	0.2562303	0.000000
8	0.1633196	0.000000
9	0.5352045	0.000000
10	0.3428502	0.000000
11	0.4676990	0.000000
12	0.000000	0.000000
13	0.2758287	0.000000
14	0.4105976	0.000000
15	0.000000	1.000000
16	0.000000	0.5323010

DMU11 CCR Model

Variable	Value	Reduced Cost
U	1.000000	0.000000
V_1	0.000000	0.000000
V_2	4.545455	0.000000

Row	Slack or Surplus	Dual Price
1	1.000000	1.000000
2	0.7563636	0.000000
3	0.1840909	0.000000
4	1.110455	0.000000
5	1.097727	0.000000
6	1.230000	0.000000
7	0.4813636	0.000000
8	0.3068182	0.000000
9	1.005455	0.000000
10	0.6440909	0.000000
11	0.8786364	0.000000
12	0.000000	1.000000
13	0.5181818	0.000000
14	0.7713636	0.000000
15	0.000000	0.000000
16	0.000000	1.000000

DMU12 CCR Model

Variable	Value	Reduced Cost
U	1.000000	0.000000
V_1	4.347826	0.000000
V_2	0.000000	0.000000

Row	Slack or Surplus	Dual Price
1	1.000000	1.000000
2	1.490000	0.000000
3	0.2013043	0.000000
4	3.271739	0.000000
5	3.682174	0.000000
6	5.029565	0.000000
7	0.8678261	0.000000
8	0.5217391	0.000000
9	2.376087	0.000000
10	1.256957	0.000000
11	2.913043	0.000000
12	0.5000000	0.000000
13	0.000000	1.000000
14	0.000000	0.000000
15	0.8304348	0.000000
16	0.000000	1.000000

DMU13 CCR Model

Variable	Value	Reduced Cost
U	1.000000	0.000000
V_1	4.347826	0.000000
V_2	0.000000	0.000000

Row	Slack or Surplus	Dual Price
1	1.000000	1.000000
2	1.490000	0.000000
3	0.2013043	0.000000
4	3.271739	0.000000
5	3.682174	0.000000
6	5.029565	0.000000
7	0.8678261	0.000000
8	0.5217391	0.000000
9	2.376087	0.000000
10	1.256957	0.000000
11	2.913043	0.000000
12	0.5000000	0.000000
13	0.000000	0.000000
14	0.000000	1.000000
15	0.8304348	0.000000
16	0.000000	1.000000

DMU14 CCR Model

Variable	Value	Reduced Cost
U	1.000000	0.000000
V_1	0.000000	0.000000
V_2	4.545455	0.000000

Row	Slack or Surplus	Dual Price
1	1.000000	1.000000
2	0.7563636	0.000000
3	0.1840909	0.000000
4	1.110455	0.000000
5	1.097727	0.000000
6	1.230000	0.000000
7	0.4813636	0.000000
8	0.3068182	0.000000
9	1.005455	0.000000
10	0.6440909	0.000000
11	0.8786364	0.000000
12	0.000000	0.000000
13	0.5181818	0.000000
14	0.7713636	0.000000
15	0.000000	1.000000
16	0.000000	1.000000

DMU1 Modified CCR Model

Variable	Value	Reduced Cost
U	0.5847705	0.000000
V_1	0.8142335	0.000000
V_2	1.381181	0.000000

Row	Slack or Surplus	Dual Price
1	0.5847705	1.000000
2	0.4152295	0.000000
3	0.000000	0.1470445
4	0.8564965	0.000000
5	0.9294928	0.000000
6	1.222016	0.000000
7	0.2151513	0.000000
8	0.9730091E-01	0.000000
9	0.6568591	0.000000
10	0.3374714	0.000000
11	0.7188820	0.000000
12	0.000000	0.8529555
13	0.6381781E-01	0.000000
14	0.1407496	0.000000
15	0.6188175E-01	0.000000
16	0.000000	0.5847705
17	0.3932335	0.000000
18	0.9914813	0.000000
19	0.5847705	0.000000

DMU2 Modified CCR Model

Variable	Value	Reduced Cost
U	1.000000	0.000000
V_1	2.270677	0.000000
V_2	1.430372	0.000000

Row	Slack or Surplus	Dual Price
1	1.000000	1.000000
2	0.8531125	0.000000
3	0.000000	1.000000
4	1.895062	0.000000
5	2.105409	0.000000
6	2.850715	0.000000
7	0.4416411	0.000000
8	0.2059689	0.000000
9	1.394261	0.000000
10	0.6960740	0.000000
11	1.634782	0.000000
12	0.9806544E-01	0.000000
13	0.000000	0.000000
14	0.7967172E-01	0.000000
15	0.2706369	0.000000
16	0.000000	1.000000
17	1.849677	0.000000
18	1.040672	0.000000
19	1.000000	0.000000

DMU3 Modified CCR Model

Variable	Value	Reduced Cost
U	0.4230844	0.000000
V_1	0.4210000	0.000000
V_2	1.262907	0.000000

Row	Slack or Surplus	Dual Price
1	0.4230844	1.000000
2	0.3060093	0.000000
3	0.2222501E-01	0.000000
4	0.5769156	0.000000
5	0.6131218	0.000000
6	0.7803403	0.000000
7	0.1693584	0.000000
8	0.8735119E-01	0.000000
9	0.4610164	0.000000
10	0.2522500	0.000000
11	0.4777748	0.000000
12	0.000000	1.000000
13	0.9555634E-01	0.000000
14	0.1659002	0.000000
15	0.3199600E-01	0.000000
16	0.000000	0.4738316
17	0.000000	-0.1205395
18	0.8732065	0.000000
19	0.4230844	0.000000

DMU4 Modified CCR Model

Variable	Value	Reduced Cost
U	0.4058247	0.000000
V_1	0.4210000	0.000000
V_2	1.184453	0.000000

Row	Slack or Surplus	Dual Price
1	0.4058247	1.000000
2	0.2929547	0.000000
3	0.1904765E-01	0.000000
4	0.5577494	0.000000
5	0.5941753	0.000000
6	0.7591108	0.000000
7	0.1610502	0.000000
8	0.8205558E-01	0.000000
9	0.4436625	0.000000
10	0.2411331	0.000000
11	0.4626098	0.000000
12	0.000000	1.000000
13	0.8661265E-01	0.000000
14	0.1525867	0.000000
15	0.3199600E-01	0.000000
16	0.000000	0.4767064
17	0.000000	-0.1683651
18	0.7947531	0.000000
19	0.4058247	0.000000

DMU5 Modified CCR Model

Variable	Value	Reduced Cost
U	0.3318626	0.000000
V_1	0.4210000	0.000000
V_2	0.8482617	0.000000

Row	Slack or Surplus	Dual Price
1	0.3318626	1.000000
2	0.2370125	0.000000
3	0.5431900E-02	0.000000
4	0.4756178	0.000000
5	0.5129851	0.000000
6	0.6681374	0.000000
7	0.1254475	0.000000
8	0.5936267E-01	0.000000
9	0.3692970	0.000000
10	0.1934948	0.000000
11	0.3976240	0.000000
12	0.000000	1.000000
13	0.4828684E-01	0.000000
14	0.9553501E-01	0.000000
15	0.3199600E-01	0.000000
16	0.000000	0.4484305
17	0.000000	-0.2768834
18	0.4585617	0.000000
19	0.3318626	0.000000

DMU6 Modified CCR Model

Variable	Value	Reduced Cost
U	0.7310346	0.000000
V_1	1.017892	0.000000
V_2	1.726646	0.000000

Row	Slack or Surplus	Dual Price
1	0.7310346	1.000000
2	0.5190877	0.000000
3	0.000000	0.4504735
4	1.070725	0.000000
5	1.161980	0.000000
6	1.527670	0.000000
7	0.2689654	0.000000
8	0.1216380	0.000000
9	0.8211542	0.000000
10	0.4218806	0.000000
11	0.8986904	0.000000
12	0.000000	0.5495265
13	0.7978008E-01	0.000000
14	0.1759542	0.000000
15	0.7735976E-01	0.000000
16	0.000000	0.7310346
17	0.5968915	0.000000
18	1.336946	0.000000
19	0.7310346	0.000000

DMU7 Modified CCR Model

Variable	Value	Reduced Cost
U	0.8573450	0.000000
V_1	1.193766	0.000000
V_2	2.024981	0.000000

Row	Slack or Surplus	Dual Price
1	0.8573450	1.000000
2	0.6087773	0.000000
3	0.000000	0.6539921
4	1.255729	0.000000
5	1.362750	0.000000
6	1.791625	0.000000
7	0.3154380	0.000000
8	0.1426550	0.000000
9	0.9630357	0.000000
10	0.4947744	0.000000
11	1.053969	0.000000
12	0.000000	0.3460079
13	0.9356472E-01	0.000000
14	0.2063561	0.000000
15	0.9072621E-01	0.000000
16	0.000000	0.8573450
17	0.7727659	0.000000
18	1.635281	0.000000
19	0.8573450	0.000000

DMU8 Modified CCR Model

Variable	Value	Reduced Cost
U	0.4808764	0.000000
V_1	0.4210000	0.000000
V_2	1.525597	0.000000

Row	Slack or Surplus	Dual Price
1	0.4808764	1.000000
2	0.3497211	0.000000
3	0.3286399E-01	0.000000
4	0.6410909	0.000000
5	0.6765616	0.000000
6	0.8514244	0.000000
7	0.1971773	0.000000
8	0.1050828	0.000000
9	0.5191236	0.000000
10	0.2894732	0.000000
11	0.5285529	0.000000
12	0.000000	1.000000
13	0.1255031	0.000000
14	0.2104789	0.000000
15	0.3199600E-01	0.000000
16	0.000000	0.4986401
17	0.000000	-0.4219402E-01
18	1.135897	0.000000
19	0.4808764	0.000000

DMU9 Modified CCR Model

Variable	Value	Reduced Cost
U	0.6340749	0.000000
V_1	0.8828850	0.000000
V_2	1.497635	0.000000

Row	Slack or Surplus	Dual Price
1	0.6340749	1.000000
2	0.4502393	0.000000
3	0.000000	0.2307382
4	0.9287113	0.000000
5	1.007862	0.000000
6	1.325049	0.000000
7	0.2332916	0.000000
8	0.1055048	0.000000
9	0.7122416	0.000000
10	0.3659251	0.000000
11	0.7794939	0.000000
12	0.000000	0.7692618
13	0.6919856E-01	0.000000
14	0.1526168	0.000000
15	0.6709926E-01	0.000000
16	0.000000	0.6340749
17	0.4618850	0.000000
18	1.107935	0.000000
19	0.6340749	0.000000

DMU10 Modified CCR Model

Variable	Value	Reduced Cost
U	0.4758571	0.000000
V_1	0.4210000	0.000000
V_2	1.502782	0.000000

Row	Slack or Surplus	Dual Price
1	0.4758571	1.000000
2	0.3459247	0.000000
3	0.3193999E-01	0.000000
4	0.6355173	0.000000
5	0.6710519	0.000000
6	0.8452507	0.000000
7	0.1947613	0.000000
8	0.1035428	0.000000
9	0.5140770	0.000000
10	0.2862404	0.000000
11	0.5241429	0.000000
12	0.000000	1.000000
13	0.1229022	0.000000
14	0.2066072	0.000000
15	0.3199600E-01	0.000000
16	0.000000	0.5323010
17	0.000000	-0.1340709
18	1.113082	0.000000
19	0.4758571	0.000000

DMU11 Modified CCR Model

Variable	Value	Reduced Cost
U	1.000000	0.000000
V_1	0.4210000	0.000000
V_2	3.885250	0.000000

Row	Slack or Surplus	Dual Price
1	1.000000	1.000000
2	0.7423673	0.000000
3	0.1284299	0.000000
4	1.217554	0.000000
5	1.246418	0.000000
6	1.489946	0.000000
7	0.4470646	0.000000
8	0.2643594	0.000000
9	1.041079	0.000000
10	0.6238360	0.000000
11	0.9846738	0.000000
12	0.000000	1.000000
13	0.3945035	0.000000
14	0.6109119	0.000000
15	0.3199600E-01	0.000000
16	0.000000	1.000000
17	0.000000	0.000000
18	3.495550	0.000000
19	1.000000	0.000000

DMU 12 Modified CCR Model

Variable	Value	Reduced Cost
U	1.000000	0.000000
V_1	3.781914	0.000000
V_2	0.3897000	0.000000

Row	Slack or Surplus	Dual Price
1	1.000000	1.000000
2	1.316482	0.000000
3	0.1464597	0.000000
4	2.896668	0.000000
5	3.252590	0.000000
6	4.435945	0.000000
7	0.7517134	0.000000
8	0.4357086	0.000000
9	2.108592	0.000000
10	1.104146	0.000000
11	2.564786	0.000000
12	0.3904943	0.000000
13	0.000000	1.000000
14	0.2170629E-01	0.000000
15	0.6779198	0.000000
16	0.000000	1.000000
17	3.360914	0.000000
18	0.000000	0.000000
19	1.000000	0.000000

DMU13 Modified CCR Model

Variable	Value	Reduced Cost
U	0.9782937	0.000000
V_1	3.687539	0.000000
V_2	0.3897000	0.000000

Row	Slack or Surplus	Dual Price
1	0.9782937	1.000000
2	1.284140	0.000000
3	0.1420901	0.000000
4	2.825651	0.000000
5	3.172663	0.000000
6	4.326772	0.000000
7	0.7328762	0.000000
8	0.4243836	0.000000
9	2.057016	0.000000
10	1.076862	0.000000
11	2.501554	0.000000
12	0.3796412	0.000000
13	0.000000	1.000000
14	0.2170629E-01	0.000000
15	0.6598941	0.000000
16	0.000000	1.000000
17	3.266539	0.000000
18	0.000000	-0.5570000E-01
19	0.9782937	0.000000

DMU14 Modified CCR Model

Variable	Value	Reduced Cost
U	0.9680040	0.000000
V_1	0.4210000	0.000000
V_2	3.739814	0.000000

Row	Slack or Surplus	Dual Price
1	0.9680040	1.000000
2	0.7181667	0.000000
3	0.1225398	0.000000
4	1.182024	0.000000
5	1.211295	0.000000
6	1.450591	0.000000
7	0.4316629	0.000000
8	0.2545424	0.000000
9	1.008908	0.000000
10	0.6032277	0.000000
11	0.9565610	0.000000
12	0.000000	1.000000
13	0.3779238	0.000000
14	0.5862314	0.000000
15	0.3199600E-01	0.000000
16	0.000000	1.000000
17	0.000000	-0.7600000E-01
18	3.350114	0.000000
19	0.9680040	0.000000

Selection of Product Designs using Modified Data Envelopment Analysis

Pooya Haghani

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Approval of the Institute of Graduate Studies and Research

Prof. Dr. Elvan Yılmaz
Director

I certify that this thesis satisfies the requirements as a thesis for the degree of Master of Science in Mechanical Engineering.

Prof. Dr. Uğur Atikol
Chair, Department of Mechanical Engineering

We certify that we have read this thesis and that in our opinion it is fully adequate in scope and quality as a thesis for the degree of Master of Science in Mechanical Engineering.

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