On The Use of Modified Data Envelopment Analysis Models for Product Line Selection

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ABSTRACT

The process of converting the raw materials to a final product in a factory is called production line. These processes include refinement, purification and assembly. Afterward, selection of best production line would be the next step which leads to determining the best product. Now to make the choice easier this thesis proposes a method based on Data Envelopment Analysis (DEA) for product line selection. DEA is a technique based on simple linear programming. This method is often used to measure the performance of Decision Making Units (DMUs) and to choose the most efficient ones which could generate multiple outputs via multiple inputs. DEA has been used in different sciences from mechanical and industrial engineering to economics and finance and results show that the accuracy of method is substantial. The method has not been applied to product line selection problem before, though. Hence this study tries to investigate the matter on product line selection problem by testing the DEA methodology. To generate the evidence in a quantitative manner, a real life sample is discussed and the results are argued. The results of this study are expected to be used by managers to make the correct decisions in order to achieve both maximum consumers' satisfaction and maximum profitability.

Keyword: Product Line Selection, Data Envelopment Analysis, Linear Programing

ÖZ

Bir fabrikada ham maddelerin nihai ürüne dönüştürülme işlemine üretim hattı denir. Bu işlemler arıtma, tesfiye ve sentez aşamalarıni içerir. Bir sonraki aşamada, en uygun ürün hattı seçimi için, en iyi ürünün belirlenmesi gerekir. Bunun belirlenmesini kolaylaştırmayı amaçlayarak, bu tez Veri Zarflama Analizi (Data Envelopment Analysis) yöntemini ürün hattı seçimi için önermektedir. Veri Zarflama Analizi (DEA) basit bir çizgisel programlama tekniğidir. Bu yöntem genellikle karar verme birimlerinin (DMUs) performansını ölçmek ve çoklu girişleri ve çıkışları göz önünde bulundurarak en verimli ürünü seçmek için kullanılır. DEA mekanik ve endüstriyel mühendislik, ekonomi ve finans gibi farklı bilim dallarında da kullanılıyor ve elde edilen sonuçlar yöntemin doğruluğunu kanıtlamaktadır. Bu çalışma daha önce ürün hattı seçim problemi üzerine uygulanmadığı için, DEA metodolojisi yardımı ile incelenir. Bu tezde gerçek bir ürün kullanılmakta ve sonuçlar tartışılmaktadır. Araştırma sonuçlarının maksimum tüketici memnuniyeti ve maksimum kar elde edilmesi amaçlanarak yöneticiler tarafından kullanılması beklenmektedir.

Anahtar Kelime: Ürün hattı seçimi, Veri zarflama analizi, Çizgisel programlama

To My Father and Mother

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I would never have been able to finish my dissertation without the guidance of my committee members, help from friends, and support from my family.

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NOTATION

DMUs Decision making units

DMU o DMU under consideration

E Vector of ones

M Number of inputs (non-beneficial variables)

n Number of DMUs (Total number of alternatives)

s Number of outputs (beneficial variables)

s^- Input excesses

s^+ Output short falls

u Vector of weights for outputs

v Vector of weights for inputs

X Matrix of inputs

Y Matrix of outputs

x o Vector of inputs of the DMU under consideration

y o Vector of outputs of the DMU under consideration

θ Variable representing the reduction in input variables to reach the best DMU

λ Vector of reference variables to the best alternative for the DMUo

Chapter 1

INTRODUCTION

1.1 Background

There are two main issues which every financial institute is faced in representing a new product to the market. The product should satisfy all the customers' needs, in one hand (Customers have variety of tastes as well as different needs), and the producing procedure on another hand are the issues. Financial problems and lack of resources in each part could be the reasons which a financial institute is not able to acquire several production lines for each specific product. Even if the facilities are sufficient, it won't be financially feasible. So the question here is how far a firm could go to cover these varieties.

MAUT is the model which has been developed recently by Thevenot, et al. (2006, 2007) based on multi-attribute theory, in product line selection. MAUT selects the best optimum among the varieties of tastes on the basis of two or more choose able variables. It has to be mentioned that in producing a variety of products by the mentioned model, the firm faces number of difficulties such as variety of inputs, technology of product, human resources, variety of output. Obviously, complexity and being time consuming to overcome the issues is another state of problem.

Hence to choose wisely and get the best results by avoiding the mentioned issues, there is a demand for another and of course better solution. Data Envelopment

Analysis (DEA) is considered being a better solution to choose the best optimum and to reach the maximum net profit. This study tries to utilize modified DEA to choose the best optimum and to reach the maximum net profit and to reduce the complexity of manufacturing a product and achieve the needed competitive advantage in the industry.

The future of each company relies on the decisions which are made in different situations. To estimate the economic growth of a firm, all the decisions should be evaluated periodically. Among a number of tools which are available, DEA is one of the best non parametric tools to evaluate the Decision making unit's performance. DEA derives of Data Envelopment Analysis which is used to evaluate the Decision making unit's performance. It contains several inputs and outputs.

1.2 Research Question

Previous studies used DEA model by considering the values of inputs and outputs to be equal or greater than zero. Since in real world zero could not be allocated for inputs, this study purposes the following questions:

1) Could output and input be equal to zero?
2) If they can't, how could Epsilon is estimated?

1.3 Statement of Purpose

To reach the maximum profit is the ultimate goal of each organization. This goal is assumed to be achieved by selecting the best product. On the other hand, the quality of each product is depended on the production functions. Hence, finding the existing relationship between production functions and market coverage for each product is extremely important. By knowing this relation perfectly the ultimate goal assumed to be reached. This study has chosen a real case study to implement and investigate the mentioned research questions.

1.4 Limitations

The current study has chosen 15 Decision Making Unit's (DMUs) to investigate the results. Although it is proved that the conclusions in this study is accurate, it is suggested to select cases with multiple inputs and outputs in order to have more comprehensive results.

1.5 Thesis Structure

The following thesis includes different sections: In the second section, a review of the existing literature on the subject is done; in section III, I the hypothesis is developed according to empirical evidences, data and methodology are explained; in section IV, sample research methodology is discussed; in Section V, comments on the empirical findings of the study followed by the conclusion.

Chapter 2

THEORETICAL CONSIDERATIONS AND EMPIRICAL STUDIES

Data envelopment analysis (DEA) is theoretical framework, which explains efficiency. DEA is a non-parametric, linear programing technique, which determines a practical production frontier for the desirable system efficiency. There have been two common models defined for DEA, CCR and BCC. One of the most efficient and well-known techniques to observe the minimum input and maximum output in a system is DEA. Charnes, Cooper and Rhodes (1980) showed that if one of the factors related to weight approaches to zero, it could be problematic. Showing the efficient DMUs as the inefficient ones could be the result of such a matter. They proved that by introducing a new element such as "ε" as the lower bound, the problem could be faced. Ali and Seiford (1993) found a solution to find the proper **ε** using the findings of Ali (1994). Mehrabian et al. (1998) proved that the finding of the previous mentioned study by Ali et al. (1993) could not be reliable since attaining an unbounded modification for both CCR and BCC models might be impossible. They defined **ε** not as a single number but as a certain number between intervals. Modification analysis was introduced by Benset et al. (1988) for the first time. Later on Jung et al. (1991, 1995) developed the idea. They investigated the concept of **ε** as a certain number between intervals for CCR and tried to focus on the hyper-planes, which show the production frontier.

Charnes et al. (1991) introduced a new sight for the weight factors. The investigated that data vector of weight factors are similar to normal vector of bounded production frontier. Banker et al. (1988) by using the findings of Banker (19684, 1986) and Teral (1988), tried to estimate the return to scale for efficient DMUs. Hence, they tried to find new solutions to calculate the remaining hyper-plans, which were supposed to form the bounded production frontier.

2.1 Production Function

2.1.1 Production

Production refers to all those direct changes, which cause the good to increase in desirability. One of the common kinds of changes is the change in materials. It means that the final product has a new and different shape with respect to the raw materials. For instance producing a vehicle out of other materials is a production. Even a simple change in usage of a product is a production. For instance transferring a good from inventory to sales department is a production. The other form of production is changes in time horizons. For example storing goods in inventory until the right moment of demand (when the demand increases for a specific good) is a change in time horizon.The result of the act of change in production is called product. Production resources are those material which being used to form a good.

2.1.2 Production Function

Production function expresses the relation among those production resources that a production unit uses (input) and the final services or goods produced (output) in a specific time horizon without considering the price of the good (Leftwitch 1975).

Production function equation is as follow:

$$y = f\ (x_1, x_2, \ldots . x_n) \qquad (2.1)$$

In the above equation y is the output and $x_1, x_2, \ldots . x_n$ are the inputs. A production unit could either increase or decrease the input resources to make changes in the output. It could also produce a completely new product by manipulating the amount of input resources. Hence, by an increase a sole input resources the output is expected to increase with in a specific amount. Production function with two inputs and an output is as follow:

$$y = f\ (x_1, x_2) \qquad (2.2)$$

In the above equation y is the output and x_1 is the first input (for instance human resources) and x_2 is the second input (for instance investment). If the investment is constant amount over a time period ($x_2 = 20$) the function is as follow:

$$y = f\ (x_1 | x_2 = 20) \qquad (2.3)$$

The graphical vector is as follow:

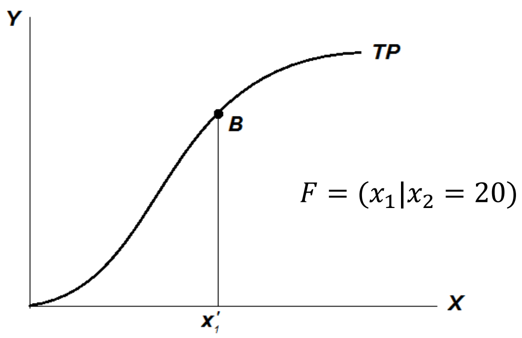

Figure 2.1. Total Production Curve

According to Fig. 2.1, the slope of the curve is ascending until B and passing the nod it is descending. The above curve is called total product function. The exact feature of the production function depends on the scale of productivity of the inputs in different levels. Productivity of production factors depends on the used technology.

For example, labor force, mechanical equipment and modern technology could increase the productivity. It has to be mentioned that these factors are necessary to increase the productivity, they are not enough though. A simultaneous effort of all units is also needed. To be more comprehensive, 14 units of products good could be made out of 3 units of human resources and 2 units of investments. By developments in technology and increase in productivity by keeping the input constant, output could be increased by 4 units which would be equal to 18 units. It is concluded that technology could make changes in the features for production function. Changes in the production function curve by adding technology is as following:

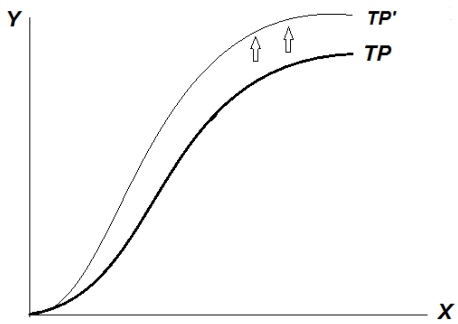

Figure 2.2. Total Reduction Curve- Technological Change

2.2 Diminishing Returns Rule

The decreasing return to scale rule states that by increasing in an output, holding other input constant, output is expected to behave in 3 different ways. At first, the output will have a rapid ascending behavior, later on, the increasing will occur with a slower pace, and finally, no matter how much the input changes, considerable change will not happen in output (Leftwich 1975).

2.3 Production Curve

Production curves include 3 different models.

- Total product curve
- Average product curve
- Marginal product curve

2.3.1 Total Product Curve

Product curve which has been introduced earlier in the present section, illustrates the output of the production factors. This curve is also known as total product curve called TP (Fig 2.3). Total product curve for the first steps is concaved with ascending slope. This segment of the curve illustrates the outcome of a number of variable units in corporation with a number of constant units without the expected efficiency. In other words, scattered units of resources apply a constant unit of production, although the efficiency will not increase. Increase in input gradually until a certain level such as B, will lead to increase in output. In other words by adding more labor force the output will increase in amount. At point B the return to scale will behave decreasing. As the result by increasing in input, the output will increase with a slower pace.

When $L1$ units of labor force are used by one unit of investment, total product amount will be maximum. The following figure shows that by increasing the labor force more than $L1$ leads the output to decrease in amount.

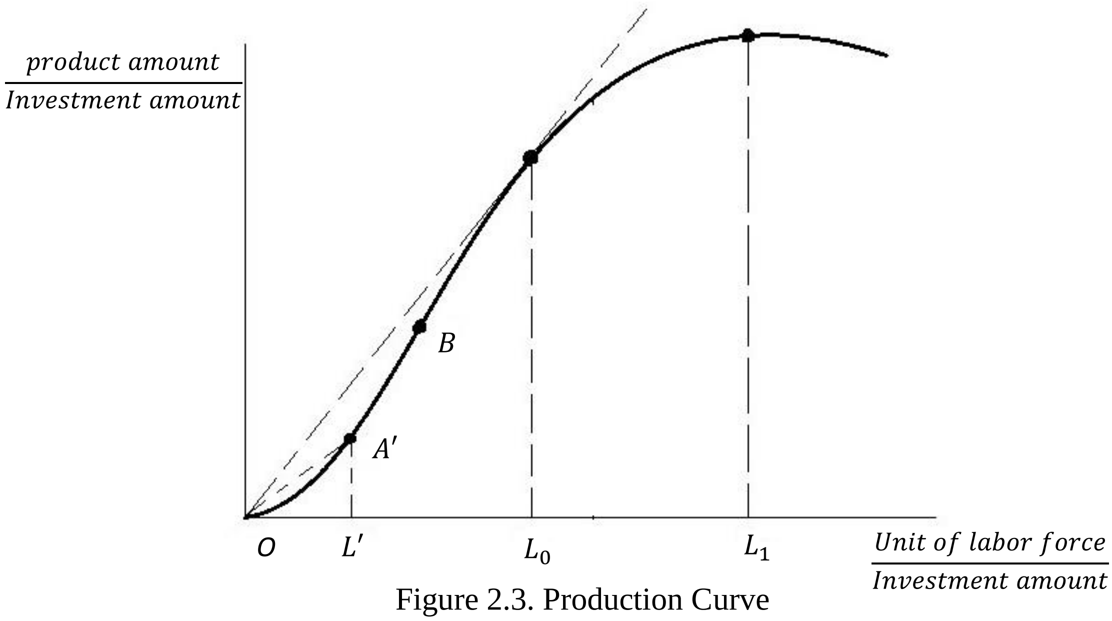

Figure 2.3. Production Curve

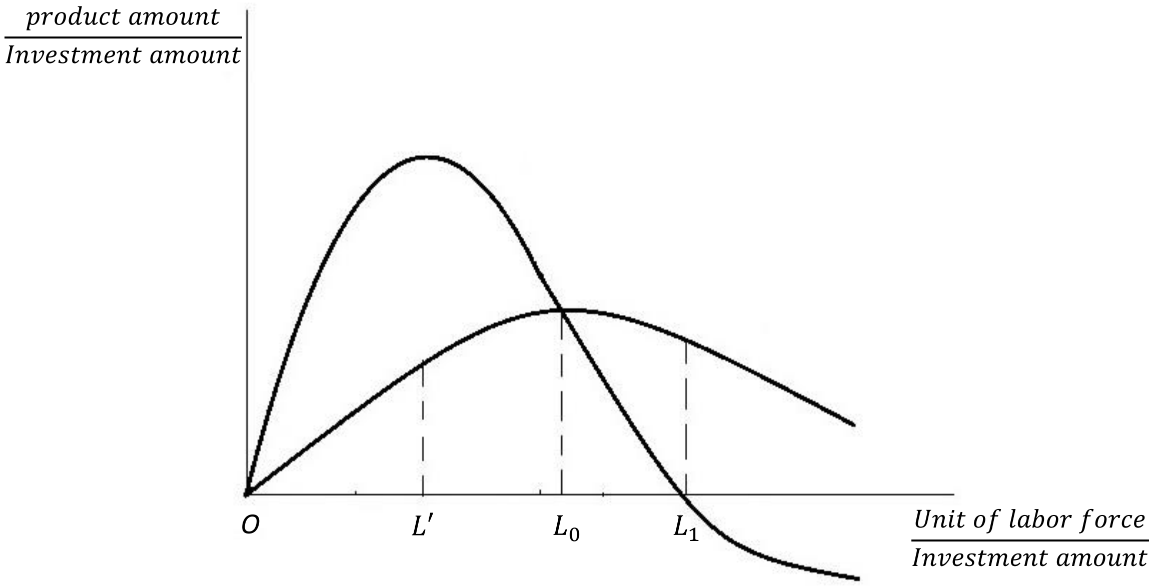

Figure 2.4. Production Curve

2.3.2 Average Product Function (AP)

Average production curve is the result of the following equation.

$$AP = \frac{y}{x_1} = \frac{f(x_1|x_2=20)}{x_1} \tag{2.4}$$

Average production curve of labor could be easily extracted from the average production curve (fig2.3). Average production curve is equal to the slope of the vector, which links center of coordinates to the total product curve. Since Average production is equal to total product divided by number of labors, Average production for each individual labor force would be equal to:

$$\frac{L' A'}{OL'} = Slope\ of\ OA' \tag{2.5}$$

While number of labors increases from zero to L_0, vectors with similar slopes to $0A'$ and average production of labor, also increases. While applying L_0 labor, slope of OA_0 vector is greater than the corner coefficient of OA vector. Hence, average production of labor in this point L_0 is at the possible maximum of it. If the applied number of labors is more than L_0, average production of labor will decreases and if again this number is greater than L_0 the average production will stay a positive number. In figure 2.4 average production curve is shown by "AP".

2.3.3 Marginal Product Curve (MP)

Marginal product curve is being defined as the extra produced output which is the result of adding one more unit of input to the procedure by holding all other inputs unchanged. The mathematical equation of marginal product of resource i is as follow:

$$MP_i = \frac{\partial Y}{\partial X_i} = \frac{\partial F(X_i\,|fix\ inputs)}{\partial X_i} \tag{2.6}$$

Slope of marginal product curve and total product curve is the same (Douglas 1372). Since marginal product of labor is the increase in the marginal product with regard to the increase in all labor force within a unit, Slope of marginal product for a specific number of labor force is equal to the marginal product of labor. Marginal product in B, which is the turning point of the curve, will be maximum. When number of used labors reachesL_1, marginal product will be maximum. Hence, marginal product in this point would be equal to zero. Using more labor force greater than L_1 could cause the total product to decrease and marginal product to become negative.

The relationship between marginal product curve and average product curve could be used to realize the location and slope of the marginal product curve. While average product is increasing, marginal product would be greater than average product. When average product is maximum, it would be equal to marginal product and when it is decreasing, marginal product would be smaller than average product (Leftwitch 1974).

2.4 Return to Scale

Return to scale is defined as the ratio of outputs to inputs in long run. This ratio could be constant, increasing or decreasing. It is constant when input is increasing, output increases by exact amount. When input is increasing and the output increases more rapid than input the ratio increases. If output increases slower than the increase in inputs the ratio will be decreasing. Returns to scale for production function as a mathematical equation is shown in the following table 2.1:

Table 2.1. Mathematical Relationship between Returns to Scale

Returns to scale	defined
Constant	$f(ax_1, ax_2 \dots) = af(x_1, x_2 \dots)$
increasing	$f(ax_1, ax_2 \dots) > af(x_1, x_2 \dots)$
decreasing	$f(ax_1, ax_2 \dots) < af(x_1, x_2 \dots)$

Suppose that in the function of $f(x_1, x_2 \dots)$ All factors of production are multiply on constant value like k, it means all inputs increase k times. production function shown in the following:

$$hy = f(kx_1, kx_2, kx_3)$$

In relation to the above:

If $h>k$, the production function has increase returns to scale.

If $h<k$, the production function has decrease returns to scale.

If $h=k$, the production function has constant returns to scale.

This relationship expresses the relationship on the Table 2.1.

2.5 Efficiency

Efficiency is the proper use of all resources such as time, cost, labor etc, to intend a task. Efficiency is calculated from the following ratio:

$$\text{Efficiency} = \frac{\frac{Real\ output}{Real\ input}}{\frac{respect\ output}{respect\ input}} = \frac{\text{real output}}{\text{real input}} \quad (2.7)$$

For example if the efficiency of a labor is 120 parts in one hour and the standard product number in an hour is 180, labor efficiency is equal to 120/180=0.66.

2.6 Ratio in Measuring Efficiency

As it has been mentioned earlier, ratios are a tool to measure the efficiency. Efficiency is the ratio of inputs to outputs. This ratio is easy to calculate for those units which use only one input and one output. Generally units use a various number of inputs and outputs in real world. The use of ratios will be explained in upcoming sections.

2.6.1 One Input and Two Outputs

Usually in reality, units use a various number of inputs and outputs. For instance imagine two outputs, 1- number of interactions on personal accounts, 2- number of interactions on trading account and an input which is the number of staff.

The information on this example is given in table 2.2 For example the second branch has used 16 staff that has done 44000 interactions on personal accounts and 20000 interactions on trading accounts during a year. Now here is the question, how the comparison between these branches on their efficiency should be done?

Table 2.2. Input and Output Branches

number of staff	number of interactions on trading account	Interactions on personal accounts	branch
18	50	125	1
16	20	44	2
17	55	80	3
11	12	23	4

Here the ratios could be used again. In this case, input, which is the number of staff, is divided over the output, which is the number of interactions on both personal and trading accounts. The result of the division is given in table 2.3.

Table 2.3. Ratio of Output to Input

number of interactions on trading account/ number of staff	Interactions on personal accounts/ number of staff	Branch
2.78	6.94	1
1.25	2.75	2
3.24	4.71	3
1.09	2.09	4

As it is shown the first branch has the highest ratio of the number of interaction on personal accounts and number of staff. The third branch has the highest ratio of the number of interaction on trading accounts and number of staff. One of the main problems caused by comparing different ratios is that different ratios have different purposes. So it is difficult to combine the final results of each ratio and analyze the final data. Now, imagine the 2nd and fourth branch, 2nd branch is more efficient than the 4th branch by 1.32 times ($\frac{2.75}{2.09} = 1.32$) on the completed interactions on personal accounts. It is 1.15 times more efficient than the 4th branch in completed interactions on trading accounts. How is it possible to combine these results to get the best true result while each of them represents a different criterion? This issue is observable when the numbers of inputs and out puts are increasing.

2.7 Diagram Analysis

One of the common ways to analyze these ratios is diagram analysis. This model is applicable for those units with 2 outputs and one input. Imagine the ratios for each branch in table 2.2 are illustrated in the following diagram.

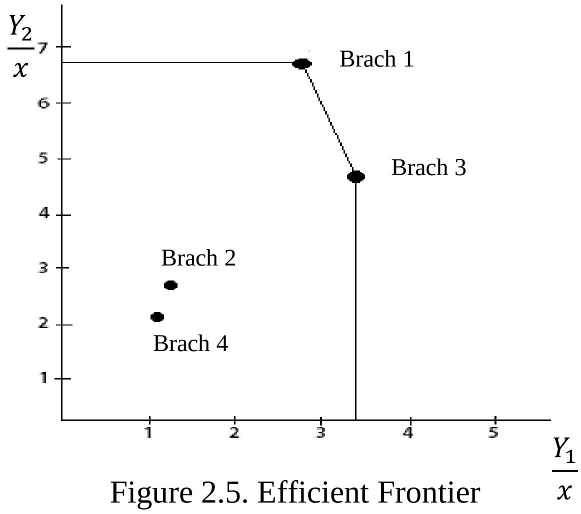

Figure 2.5. Efficient Frontier

Branch 1 and branch 3 on the curve, represent the level of efficiency which is better than branch 2 and 4. The horizontal line, which links axis Y_2/x to nod of branch 1 and from nod of branch 1 to branch 3 and from this point to Y_1/x axis, is called efficiency frontier. Efficiency frontier, which illustrates the maximum efficiency of each branch according to the sample data, is considered a standard guide for those branches that are below the frontier to try harder and reach it.

2.7.1 Efficiency Calculation of Inefficient Units

Branches number 2 and 4 in fig 2.5 have lower ratios rather than the first and third branch. Simply, it could be said that these 2 branches have lower efficiency than 100% but the important question here is, how much of a percentage/number are they behind the proper efficiency level?

Now imagine the 4th branch. The data related to this branch is:

1) Number of employees 11 people
2) Number of interactions on personal accounts 23(thousands)
3) Ratio of number of interactions on personal accounts over number of employees ($\frac{23}{11} = 2.09$)
4) Number of interactions on trading accounts 12 (thousands)
5) Ratio of number of interactions on trading accounts over number of employees ($\frac{12}{11} = 1.09$)

For this branch, the ratio of number of interactions on personal accounts over number of interactions on trading accounts is equal to $\frac{23}{12} = 1.92$. This number shows that for each interaction on trading account, 1.92 interaction are done on personal accounts. In other words, 1.92 shows the ratio of number of interactions on personal accounts over Number of employees and the ratio of Number of interactions on trading accounts Number of employees ($\frac{2.9}{1.09} = 1.92$). By looking at the following diagram, it is shown that those branches with Ratio of number of interactions on personal accounts and number of employees over Ratio of number of interactions on trading accounts and number of employees equal to 1.92, would be placed on a direct line which links the center of coordinates to the 4th branch.

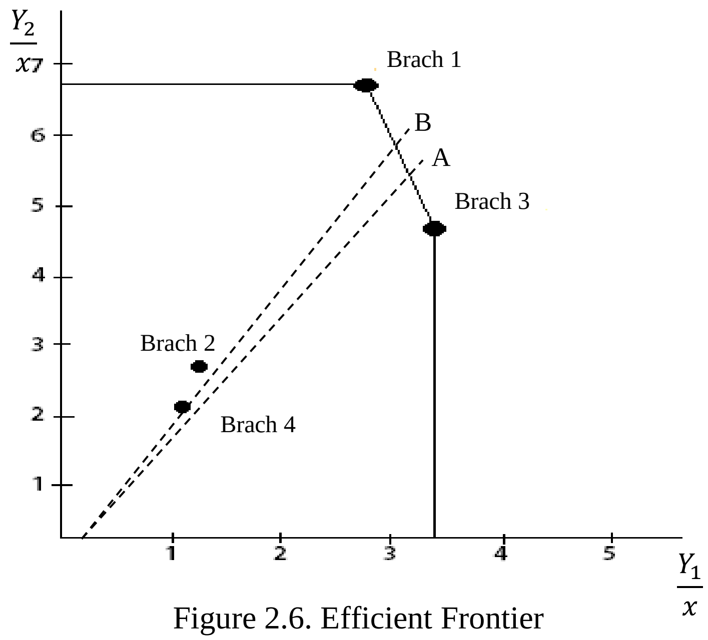

Figure 2.6. Efficient Frontier

Hence if the fourth branch decides on continuing its business strategy (for each trading interaction it should complete 1.92 personal interactions) but make changes in number of employees, in that case in fig 2.6, it will be located on a line which links the center of coordinates to the fourth branch. It could be concluded that the fourth branch is expected to have its maximum efficiency on location A. This location is the result of contact between the centers of coordinates to the fourth branch and the forth branch to the efficiency frontier. Since this location is placed on the frontier line, it is called an efficient location, which is considered to be the ultimate target point of each efficient unit. Accordingly, location B shows an efficient target for the second inefficient branch. It could be concluded that the relative efficiency of location B could be calculated from the following equation:

$$\frac{\text{Length of the line form the center of coordinates to the fourth branch}}{\text{length the center of coordinates to the fourth branch and the forth branch to the efficiency frontier}}$$

To show the result as percentage, it has to be multiplied by 100. According to the mentioned formula, efficiency of the forth branch is estimated to be 36%. The reasoning behind all this is to compare the current efficiency of the forth branch with the possible maximum efficiency of it.

2.9 Reaching the Efficiency Frontier

Imagine the location on efficiency frontier, which expresses the possible maximum efficiency for the fourth branch (A). This location is expected for the fourth branch to be at. There are several approaches for the fourth branch to reach this point.

1) To decrease the input (number of employees) while, the output is kept constant.
2) To increase the output while the ratio of number of personal interaction to trading interactions is equal to 1.92 and input being kept constant.
3) Combination of the two previous approaches.

Chapter 3

MODELING AND METHODOLOGY

3.1 Data Envelopment Analysis

Measuring the efficiency plays an important role in a firm that is why it has always been center of attention to the researches. (Farrell 1975) measured the efficiency by developing a new approach of measuring the efficiency in engineering fields. He used his model to estimate the efficiency in the agriculture industry of United States and then he compared the results with other countries. Although he was not successful on developing his model to capture the accurate efficiency when there are several inputs and outputs.

3.2 CCR Model

Charnes, Cooper and Rhodes (1976) developed Farrell's model and enhanced it to be applicable for those systems with several inputs and outputs. The first model on Data Envelopment Analysis is named after its creators' initials, CCR. This model tries to measure the efficiency of organizations such as factories, hospitals and banks with several similar inputs and outputs. Furthermore the model runs a comparison between the efficiency of the mentioned organizations. Different models on CCR will be introduced and discussed shortly. One of the most outstanding features of Data Envelopment Analysis is the return to scale structure of it. Return to scale could be either variant or invariant. It means that increase in inputs is supposed to increase the outputs with the same amount which keeps the return to scale constant. Invariant return to scale shows that increase in outputs could be more or less than inputs. CCR is listed as constant return to scale models.

3.2.1 CCR Ratio Model

(Farrell 1978) used the following formula to measure the ratio model of units.

$$\text{Efficiency} = \frac{\text{sum of (wights* outputs)}}{\text{sum of (wights* inputs)}} \qquad (3.1)$$

The following formula is used to measure the efficiency of n units with m inputs and s outputs for each unit, efficiency of unit j ($j = 1,2,3 \dots \dots , n$):

$$\text{Efficiency of unit j} = \frac{\Sigma_{r=1}^{ms} u_r y_{rj}}{\Sigma_{i=1}^{m} v_i x_{ij}} \qquad (3.2)$$

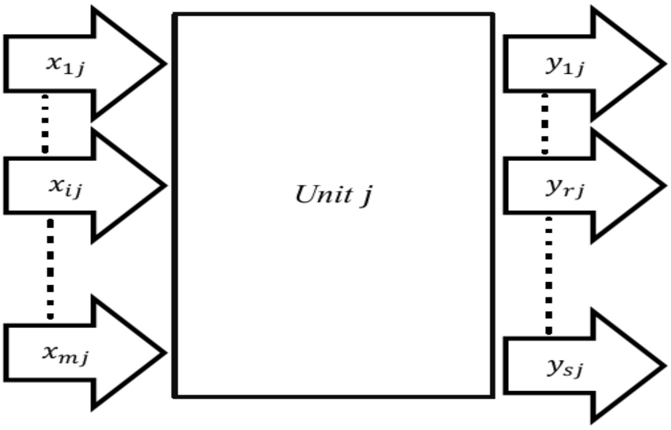

Figure 3.1. CCR Model

In Fig 3.1:

x_{ij} : Amount of inputs of *i* for unit *j* $\quad i = (1,2,3 \dots \dots , m)$

y_{rj} : Amount of outputs of *r* for unit *j* $\quad r = (1,2,3 \dots \dots , s)$

u_r : Weight of output *r* (cost of output r)

v_i : Weight of input *i* (cost of input i)

Allocating weights for all units is one of the most important tasks to be done for the model in order to measure more accurate. The decision maker units (DMU) are those units which by using a certain amount of inputs provide amount of outputs. These units are responsible on how to use and process the inputs.

When applying the mentioned formula (3.2), two important factors must be considered.

1) Inputs and outputs could have different values which makes it difficult to allocate the proper value.
2) Outputs having different values could be a possible result of different acts in different units. Hence there should be different weights defined.

If the allocated weights to outputs are shown by $u_1, u_2, u_3, \dots\dots, u_m$, and allocated weights to inputs are shown by $v_1, v_2, v_3, \dots\dots, v_m$ to calculate the maximum efficiency the following division should be maximum.

$$\frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{i=1}^{m} v_i x_{i_0}} \tag{3.3}$$

This formula should be applied for other units too.Variables in the previous formula are weights and the result of the formula finds the best answer for the weight of those units at level zero. The following equation describes it better:

$$MAX\ Z_0 = \frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{i=1}^{m} v_i x_{i_0}} \tag{3.4}$$

St: $\frac{\sum_{r=1}^{ms} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \qquad (j = 0,1,2,3 \dots\dots, n)$

$u_r v_i \geq 0$

In the previous formula if u_r are massive and v_i are petit the result of the ratio will be either infinite or unlimited. To overcome the issue, all the ratios (efficiency of units) are assumed to be less or equal to one. It is considered to be a limitation and then being entered to the formula. It is worthy to mention that in limitations, instead of one, every other possible positive number such as K could be used. In this situation the efficiency of units will be assessed toward K. It also should be mentioned that number of limitations in CCR ratio model is equal to the number of units and variables which itself is equal to sum of all inputs and outputs.

3.3 Linear-fractional Programing

The formula of fractional programing model is the result of division of two first degree equations. The limitations in this model are linear. Hence, the CCR ratio model of it, will be a fractional programing model.

To change the model and have a brand new fractional linear programing model, changes in variables will be needed, twice. So, let's have another look at CCR model:

$$MAX\ Z_0 = \frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{i=1}^{m} v_i x_{i_0}} \tag{3.5}$$

St: $\frac{\sum_{r=1}^{ms} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \quad (j = 0,1,2,3 \dots \dots, n)$

$u_r v_i \geq 0$

First change of variables should be done, accordingly:

$$\frac{1}{\sum_{i=1}^{m} v_i x_{ij_0}} = t \tag{3.6}$$

Limitations in model would be as following:

$$MAX\ Z_0 = \sum_{r=1}^{s} u_r\, y_{rj_0} t \qquad (3.7)$$

1) $\Sigma_{r=1}^{s} u_r y_{rj} - \Sigma_{i=1}^{m} v_i x_{ij} \leq 0$

2) $\Sigma_{i=1}^{m} v_i x_{ij_0} t = 1$

$u_r, v_i, t \geq 0$

Equation 3.7 actually represents changes in variables. Now equation 3.4 will be multiplied by t and for the second time changes in variables should be done within it.

$tu_r = \mu_r, tv_i = \omega_r$

Hence the mentioned formula would be as following:

$$MAX\ Z_0 = \sum_{r=1}^{s} u_r\, y_{rj_0} t \qquad (3.8)$$

$$st: \sum_{r=1}^{s} y_{rj}\mu_r - \sum_{i=1}^{m} x_{ij}\omega_i \leq 0$$

$$\sum_{i=1}^{m} x_{ij_0}\omega_i = 1$$

$\omega_i\,, \mu_r \geq 0$

3.4 CCR Linear Programing

To convert CCR ratio model to a CCR linear programing model, the main focus is on the solution that Charnes, Cooper and Rhodes (1978) have developed. This solution suggests that to maximize a fraction there are two ways:

1) Holding the denominator constant and maximizing the nominator
2) Holding the nominator constant and minimizing the denominator

The result could be shown in two different approaches which depend on the mentioned solutions. Input oriented and output oriented model.

3.4.1 CCR Linear Programing Based on Input Oriental Model

Generally, input oriental models are divided in to two different models:

1) Envelopment model
2) Multiplier model

3.4.1.1 Multiplier Model

Here to convert CCR linear programing to a Multiplier input oriented model, the fraction should be equal to 1, as the result the nominator will be maximum. The equation will be as following:

The objective function:

$$MAX\ Z_0 = \frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{i=1}^{m} v_i x_{i_0}} \tag{3.9}$$

The objective function of CCR input oriental:

$$MAX\ Z_0 = \sum_{r=1}^{s} u_r\, y_{r_0} \tag{3.10}$$

$$\frac{1}{\sum_{i=1}^{m} v_i x_j} = 1$$

And,

$$MAX\ Z_0 = \sum_{r=1}^{s} u_r\, y_{r_0} \tag{3.11}$$

$$\sum_{i=1}^{m} v_i x_j = 1$$

The new model is called CCR Multiplier input oriented model.

$$MAX\ Z_0 = \sum_{r=1}^{s} u_r\, y_{r_0} \tag{3.12}$$

$$st\colon \sum_{i=1}^{m} x_{i_0} v_i = 1$$

$$st: \sum_{r=1}^{s} y_{rj}u_r - \sum_{i=1}^{m} x_{ij}v_i \leq 0 \qquad (j = 1,2,\dots,n)$$

$u_r, v_i \geq 0$

3.4.1.2 CCR Envelopment Input Oriented Model

During developing a new data envelopment analysis model Charnes, Cooper and Rhodes (1987) found a new practical model between the units being assessed and number of inputs and out puts. The concept of the model is as follow:

Units being assessed ≥ 3(number of inputs+ number of outputs)

Being on the verge of efficiency border for many units could be the result of not considering the mentioned concept. In this case those units which are not the most efficient ones will be considered as most efficient. But by using the model the different between efficient and most efficient units will be appeared. A reliable model is the one with a great power of separation. A model which calculates the efficiency of most or all units as one is not able to determine the proper and real efficiency of units. For those models which have more limitations than variables and since solving by simplex is more dependent on limitations rather than variables, the question will be solved in a dual equation which requires less calculations.

If the allocated variable to the limitation $\Sigma_{i=1}^{m} v_i x_{ij_0} t = 1$ is entered in secondary equation as θ and if the allocated variable to the limitation $\Sigma_{r=1}^{s} u_r y_{rj} - \Sigma_{i=1}^{m} v_i x_{ij} \leq 0$ is entered in secondary equation as λ_j the dual model will be as following:

$$Min\ Y_0 = \theta \qquad (3.13)$$

$$st: \sum_{j=1}^{n} \lambda_j\, y_{rj} \geq y_{r_0} \qquad (r = 1,2,\dots,s)$$

$$\theta_{xi_0} - \sum_{j=1}^{n} \lambda_j \, x_{ij} \geq 0 \qquad (i = 1,2, \ldots., m)$$

$$\lambda_j \geq 0 \qquad \theta \, free \qquad (j = 1,2, \ldots., n)$$

By making some changes in the previous formula will be as following. The new model is called envelopment form.

$$Min \, Y_0 = \theta \tag{3.14}$$

$$st: \sum_{j=1}^{n} \lambda_j \, y_{rj} \geq y_{r_0} \qquad (r = 1,2, \ldots., s)$$

$$\sum_{j=1}^{n} \lambda_j \, x_{ij} \leq \theta_{xi_0} \qquad (i = 1,2, \ldots., m)$$

$$\lambda_j \geq 0 \qquad \theta \, free \qquad (j = 1,2, \ldots., n)$$

In the equation, *m* represents the inputs, *s* outputs and *n* represents number of units. According to it, the secondary equation will have (n+1) variables which results in fewer limitations with respect to the main equation. The main goal of the model is to decrease the level of inputs withθ. The above secondary model is envelopment model.

3.4.1.3 Modified CCR Input Oriented Model

In CCR multiplier model, u_r and v_i are non-negative variables (≥0) and there is always this possibility that a variable becomes zero. For instance if the result of modified CCR model is u_1^*=2 and v_2^* =3/2 and v1=0, v_1^*=0 causes the first input to be ignored during the efficiency measurement. To overcome the solution in 1979 it was suggested that the variables values of the model (u_r,v_i) should be greater than a small amount of ε. Hence, the final model is as following:

$$MAX \, Z_0 = \sum_{r=1}^{s} u_r \, y_{r_0} \tag{3.15}$$

$$st: \sum_{i=1}^{m} v_i \, x_{i_0} = 1$$

$$\sum_{r=1}^{s} y_{rj} u_r - \sum_{i=1}^{m} x_{ij} v_i \leq 0 \quad (j = 1,2, \dots, n)$$

$$u_r, v_i \geq \varepsilon$$

The secondary equation of the model is as follow:

$$Min\, Y_0 = \theta - \varepsilon(\sum_{r=1}^{s} s_r^+ + \sum_{r=1}^{s} s_r^-) \tag{3.16}$$

$$st: \sum_{j=1}^{n} \lambda_j \, y_{rj} - s_r^+ \geq y_{r_0} \quad (r = 1,2, \dots, s)$$

$$\theta_{xi_0} - \sum_{j=1}^{n} \lambda_j \, x_{ij} + s_r^- \geq \theta_{xi} \quad (i = 1,2, \dots, m)$$

$$\lambda_j, s_r^-, s_r^+ \geq 0 \quad \theta\, free \quad (j = 1,2, \dots, n)$$

Auxiliary variable s_r^+ shows the lack of production for the certain output of r and s_r^- is another auxiliary variable which represents the used amount of input i.

1) $\theta^* = 1$
2) $s_r^- = s_r^+ = 0$

3.4.2 CCR Output Oriented Model:

As it has been mentioned earlier, efficiency could be assessed from two perspectives. Input and output oriented. Charnes, Cooper and Rhodes (1981) defined efficiency according to these two perspectives.

1) In an input oriented model, a DMU is considered to be inefficient if the possibility of decreasing in each input without any increase in other inputs or decreases in any of outputs exists.

2) In an output oriented model, a unit is considered to be inefficient if the possibility of decreasing in each output without any increase in other inputs or decreases in any of outputs exists.

If there is no chance of the above incidents to happen, the unit will be efficient. Efficiency less than 1 states that linear combination of other units could make the same outputs by using fewer inputs.

CCR multiplier and envelopment models are as following:

$$Min\ Z_0 = \sum_{i=1}^{m} v_i x_{i_0} \tag{3.17}$$

$$st{:} \sum_{r=1}^{s} u_r y_{r_0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad (j = 1,2,\dots,n)$$

$$u_r, v_i \ge 0$$

By assuming θ and λ_j in the secondary model as following, the envelopment model would be:

$$Min\ Y_0 = \theta \tag{3.18}$$

$$st{:} \sum_{j=1}^{n} \lambda_j\, y_{rj} \ge \theta y_{r_0} \qquad (r = 1,2,\dots,s)$$

$$\sum_{j=1}^{n} \lambda_j\, x_{ij} \le x_{j_0} \qquad (i = 1,2,\dots,m)$$

$$\lambda_j \ge 0 \qquad \theta\ free \qquad (j = 1,2,\dots,n)$$

The main focus here is to reach the maximum output. In this model $\theta^* \ge 1$ and λ_{θ^*} represents the efficiency. The modified model of the previous two models is as following:

$$Min\ Z_0 = \sum_{i=1}^{m} v_i x_{i_0} \tag{3.19}$$

$$st: \sum_{r=1}^{s} u_r y_{r_0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} x_{ij} \leq 0 \qquad (j = 1,2, \ldots., n)$$

$$u_r \geq \varepsilon, v_i \geq \varepsilon$$

The secondary equation of the model is as follow:

$$Min\ Y_0 = \theta - \varepsilon \left(\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{s} s_i^- \right) \tag{3.20}$$

$$st: \sum_{j=1}^{n} \lambda_j \, y_{rj} - s_r^+ = y_{r_0} \qquad (r = 1,2, \ldots., s)$$

$$\sum_{j=1}^{n} \lambda_j \, x_{ij} + s_i^- = x_{i_0} \qquad (i = 1,2, \ldots., m)$$

$$\lambda_j, s_r^-, s_r^+ \geq 0 \qquad \theta\ free \qquad (j = 1,2, \ldots., n)$$

3.5 Theorem

Let Z^* be an optimal solution for the input oriented model in (3.15). Then $(\frac{1}{Z^*}) = Z'^*$ is optimal for the corresponding output oriented model. Similarly if (Z'^*) is optimal for the output oriented model then $(\frac{1}{Z'^*}) = Z^*$ is optimal for the input oriented model (William W. Cooper, Lawrence M. Seiford and Joe Zhu).

3.6 Modify and Facet Analysis

The CCR- impressive DMUs, in which the optimum avail of over problem is nonzero, are those that can be situated on the connection of the impressive boundary and the feeble impressive boundary hyper planes (Figure 3.2).

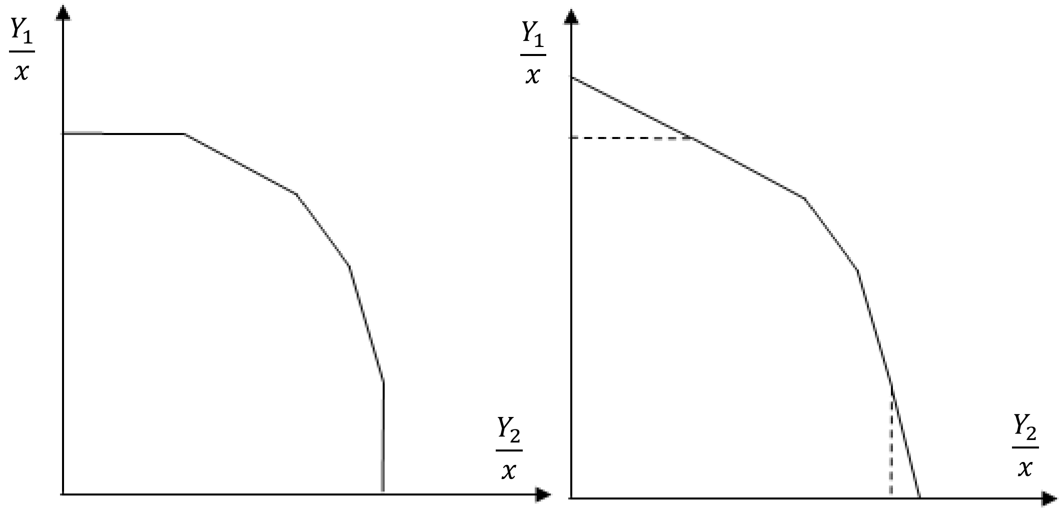

Figure 3.2. Weak Efficient Frontier

Figure 3.2 show weak efficient frontier.

$$Max\ v_i \qquad (3.21)$$

$$st. \sum_{i=1}^{m} v_i x_{i0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad j = 1,2 \ldots n$$

$$v_i \geq 0 \quad u_i \geq 0$$

And

$$Max\ u_i \qquad (3.22)$$

$$st. \sum_{i=1}^{m} v_i x_{i0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \; j = 1,2 \dots n$$

$$v_i \geq 0 \;\; u_i \geq 0$$

Imagine that the optimum avail for (3.21) and (3.22) are it has been shown by v_i^{+} and u_r^{+}severally. To decrease the number of problems, it is advised that the problems (3.21) and (3.22) just to be Resolved for v_is and v_is with identical indices when $S_i^{-} > 0$ and $S_r^{+} > 0$, in optimum solution of problem. Even so for each r=(1,…,s) and i=(1,…,m) imagine that:

$$\epsilon_r = min\{u_r^{+} | DMU \; \in B\} \; \forall r = 1,2 \dots s \quad \text{(I)}$$

$$\epsilon_i = min\{v_i^{+} | DMU \; \in B\} \; \forall i = 1,2 \dots m \quad \text{(II)}$$

Now according to (I) and (II) the CCR method, modified as below:

$$Max \; u_r \qquad (3.23)$$

$$st. \sum_{i=1}^{m} v_i x_{i0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \; j = 1,2 \dots n$$

$$v_i \geq \varepsilon_i \;\; u_r \geq \varepsilon_r$$

With acknowledgment to explanation of ϵ, r = 1,…, s and ϵ, i = 1,…, s, using them as a lower bound of each factor weights in CCR model, produces passable hyper planes. These hyper planes are replaced by hyper planes of weak boundary. This substitution preserves the possibility of modified CCR model in multiple side and unbounded, in envelopment side.

3.7 Choice of Software

There has been different software introduced for the mean of linear programing. Each one of them has a specific feature though. According to the needs of the very study, the author has used LINGO 11 released by LINDO Company in 2011. The program provides a user friendly layout where basic coding could be written. The procedure of entering and analyzing the data is more described in the following chapters.

3.8 Data

The current study uses secondary data. Data has been obtained from previous major studies published in outstanding journals (APIEMS 2007). Previous studies have used CCR model to calculate the efficiency. This study uses the results and the data, resolve the equations according to the methodology represented in this study and compare the results to those from previous ones.

Chapter 4

EMPIRICAL ANALYSIS AND RESULTS

This chapter tries to estimate the efficiency of a production line of an organization by using data envelopment analysis. The procedure is defined in to three different levels. At first the sample is solved by CCR, after that Modified CCR is applied to it and in the end, Facet analysis is used to solve it. The aim behind this chapter is to find the most efficient production line and also to estimate the other production lines efficiency.

Production and profit are two definitions which are closely related. The importance of this relation becomes clearer when most managers in different organizations try to achieve the maximum profit out of their products. It could be said that all the future decisions and strategies are mostly dependent on the predicted capacity of a product.

On the other hand, a firm tries to find those right spots in a market which could achieve the competitive advantage for the firm with respect to its competitors. This competitive advantage is (if it is a product) expected to capture the most benefit of the market for the firm. An index in a market is defined as a set of customers with certain demands which are expected to be satisfied by suppliers. The utopia for each firm is to capture a massive share of a market for itself. When the firm reaches to the point where all the possible demands are being satisfied and supplied for, the stability would be at its maximum.

So far, maximizing both the profit and market coverage has been explained as two important principles for a manufacturer. On the other hand, for a new product which has the mentioned specific features, an initial investment would in need. Maximizing the profit is the focus of each firm but on the other hand minimizing the costs and specifically the initial investment for a new product should also be considered. To do so, firms are supposed to use the raw material similar to those material used for previous products. This procedure is called, product combinability index or PCI. The following table is for a manufacturer which combines 5 different inputs and generates 15 different outputs.

Table 4.1. Input and Output Data

Product Mix	PCI (%)	Profit ($)	Market Coverage (%)
1 2 3 4 5	36.5	$45,543,018	80
1 2 3 4	40.7	$36,280,518	70
1 3 4 5	40.7	$26,389,514	60
1 5 4 2	43.1	$48,793,824	80
2 1 3 4	43.1	$39,817,768	80
2 1 3 5	59.2	$17,127,014	50
2 1 4 5	42.9	$30,555,268	70
2 3 4 5	42.9	$20,664,264	60
3 1 2	42.9	$39,531,324	70
3 1 4	42.9	$29,640,320	0.6
3 1 5	63.3	$28,416,004	0.3
3 2 4	51.7	$65,997,295	97
3 2 5	51.3	$65,486,678	83
3 4 5	29.4	$33,283,961	88
4 1	30.5	$31,479,280	91

PCI is considered as input for each product and profit and market coverage are considered as outputs. This chapter tried to calculate the maximum profit and market coverage by solving the sample. In other words, determining efficient and inefficient products both in profitability and market coverage by considering PCI constant was the aim of this chapter.

4.1 Normalizing Data

One of the best ways of making sure there is not much imbalance in the data sets is to have them at the same or similar magnitude. A way of making sure the data is of the same or similar magnitude across and within data sets is to mean normalize the data. The process to mean normalize is taken in two simple steps. First step is to find the mean of the data set for each input and output. The second step is to divide each input or output by the mean for that specific factor (Table 4.2).

Table 4.2. Normalizing Data

DMU	PCI	Profit	Market Coverage
1	1	0.96985	0.7825
2	1	0.692875	0.613929
3	1	0.503979	0.076429
4	1	0.879961	0.662857
5	1	0.718084	0.662857
6	1	0.224872	0.301429
7	1	0.553611	0.5825
8	1	0.374402	0.499286
9	1	0.716242	0.5825
10	1	0.537033	0.004643
11	1	0.348927	0.001429
12	1	1	0.478571
13	1	1	0.371429
14	1	0.802233	1
15	1	0.724506	1

4.2 Diagram Analysis

This model is applicable for those units with two outputs and one input. Imagine the ratios for each DMU in table 4.2 are illustrated in the following diagram.

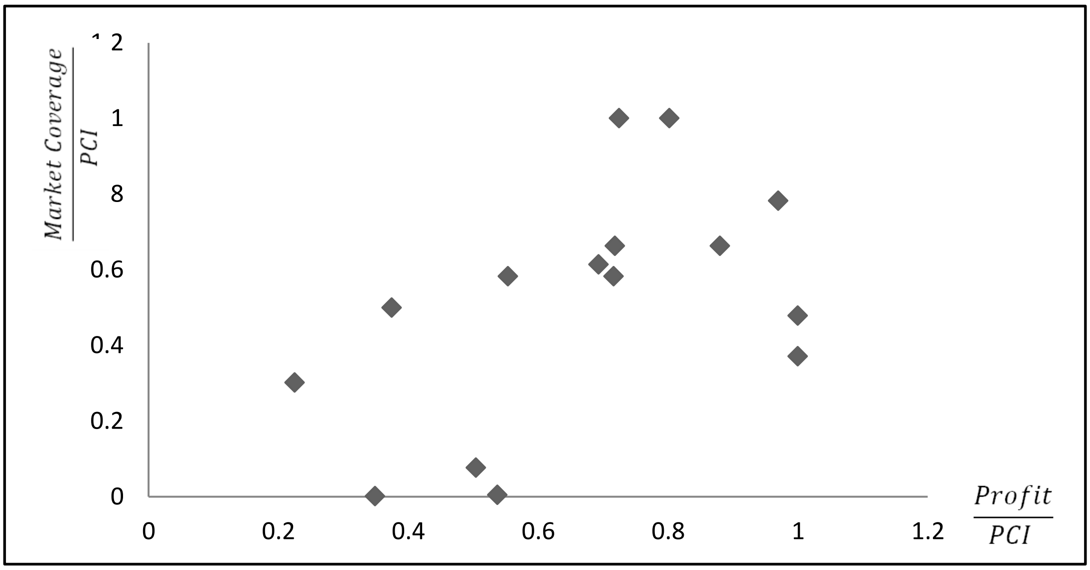

Figure 4.1. Diagram Analysis

The aim is to apply DEA model to help the firm in choosing the most efficient and accurate product (among the other products with the same features). The case study which is represented in this study includes 15 different mixture and combinations of the mentioned products. This study has also used the values of product line commonality index (PCI) and other information such as profit and market coverage. All the relevant data is represented in table 4.1. Kota et all. (2000) introduced the concept of PCI for the first time. PCI aims to measure the commonality of product line with in different aspects. The amount of PCI usually fluctuates between “0 to 100”. The greater the number, the more commonality exists between the products. The other two measures market coverage and profit, shows whether the product was successful from the manufacturers’ point of view. If the product is successful to cover a wide range of market and brings profit to the firm, the manufacturers are likely to produce a product family with high commonality of product. One of the advantages of having high commonality is that the complications of manufacturing process will be reduced and as the result the cost of producing a product will decrease.

From a manufacturing point of view, along with the high profit and wide market coverage, the company might prefer to produce a product family with high product commonality. High commonality in products would reduce the complexity in manufacturing processes, and also reduce the production costs.

4.3 Results

4.3.1 CCR Model

We have CCR model :

$$MAX\ Z_0 = \sum_{r=1}^{s} u_r\, y_{r_0} \tag{4.1}$$

$$st{:} \sum_{i=1}^{m} x_{i_0} v_i = 1$$

$$st{:} \sum_{r=1}^{s} y_{rj} u_r - \sum_{i=1}^{m} x_{ij} v_i \leq 0 \qquad (j = 1,2, \dots, n)$$

$$u_r, v_i \geq 0$$

We use this formula for assessment DMU_{15}:

$$MAX\ Z_1 = 0.724506 u_1 + u_2$$

$$st{:}\ v_1 = 1$$

$$0.96985 u_1 + 0.7825 u_2 - v_1 \leq 0$$

$$0.692875 u_1 + 0.613929 u_2 - v_1 \leq 0$$

$$0.503979 u_1 + 0.076429 u_2 - v_1 \leq 0$$

$0.879961u_1$+$0.662857u_2$-$v_1 \leq 0$

$0.718084u_1$+$0.662857u_2$-$v_1 \leq 0$

$0.224872u_1$+$0.301429u_2$-$v_1 \leq 0$

$0.553611u_1$+$0.5825u_2$-$v_1 \leq 0$

$0.374402u_1$+$0.499286u_2$-$v_1 \leq 0$

$0.716242u_1$+$0.5825u_2$-$v_1 \leq 0$

$0.537033u_1$+$0.004643u_2$-$v_1 \leq 0$

$0.348927u_1$+$0.001429u_2$-$v_1 \leq 0$

$1u_1$+$0.478571u_2$-$v_1 \leq 0$

$1u_1$+$0.371429u_2$-$v_1 \leq 0$

$0.802233u_1$+$1u_2$-$v_1 \leq 0$

$0.724506u_1$+$1u_2$-$v_1 \leq 0$

u_1,$u_2, v_1 \geq 0$

From this equation maximum of Z_1 is equal to 1.

4.3.2 Dual-CCR Model

However for these models which have more limitations than variables, it's better to use dual equation which requires less calculations, dual model will be as following:

$$Min\ Y_0 = \theta \qquad (4.2)$$

$$st\colon \sum_{j=1}^{n} \lambda_j\, y_{rj} \geq y_{r_0} \qquad (r = 1,2, \ldots, s)$$

$$\theta_{xi_0} - \sum_{j=1}^{n} \lambda_j\, x_{ij} \geq 0 \qquad (i = 1,2, \ldots, m)$$

$$\lambda_j \geq 0 \qquad \theta\ free \qquad (j = 1,2, \ldots, n)$$

Then the dual linear program for DMU_{15} is defined by:

$Min\, Y_0 = \theta$

St:

0.96985λ_1+0.692875λ_2+0.503979λ_3+0.879961λ_4+0.718084λ_5+0.224872λ_6+0.5536 11λ_7+0.374402λ_8+0.716242λ_9+0.537033λ_{10}+0.348927λ_{11}+1λ_{12}+1λ_{13}+0.802233 λ_{14}+0.724506λ_{15} ≥0.724506

0.7825λ_1+0.613929λ_2+0.076429λ_3+0.662857λ_4+0.662857λ_5+0.301429λ_6+0.5825 λ_7+0.499286λ_8+0.5825λ_9+0.004643λ_{10}+0.001429λ_{11}+0.478571λ_{12}+0.371429λ_{13}+ 1λ_{14}+1λ_{15} ≥1

θ-λ_1- λ_2-λ_3-λ_4-λ_5-λ_6-λ_7-λ_8-λ_9-λ_{10}-λ_{11}-λ_{12}-λ_{13}-λ_{14}-λ_{15} ≥0

$\lambda_j \geq 0 \qquad \theta\, free \qquad\qquad (j = 1,2, \ldots, n)$

In this equation maximum of Z_1 is equal to 1 and it means DMU_{15} is efficient for this company with this amount:

For maximum of Z_1 global optimal solution found.

Objective value: 1.000000

Infeasibilities: 0.000000

Total solver iterations: 2

Variable	Value
U_1	0.000000
U_2	1.000000
v_1	1.000000

4.3.3 Modified-CCR Model

Value of u_1 in this case is equal to zero, it means in this model one of the output is not operational, therefor suggested that the variables values of the model (u_r,v_i) should be greater than a petit amount of ε. Hence, the Modified CCR model is as following equation:

$$MAX\ Z_0 = \sum_{r=1}^{s} u_r\, y_{r_0} \tag{4.3}$$

$$st: \sum_{i=1}^{m} v_i\, x_{i_0} = 1$$

$$\sum_{r=1}^{s} y_{rj} u_r - \sum_{i=1}^{m} x_{ij} v_i \leq 0 \quad (j = 1,2, \ldots, n)$$

$$u_r, v_i \geq \varepsilon$$

Hence the mentioned formula for DMU_{15} would be as following:

$MAX\ Z_1 = 0.724506u_1 + u_2$

$st: v_1 = 1$

$0.96985u_1 + 0.7825u_2 - v_1 \leq 0$

$0.692875u_1 + 0.613929u_2 - v_1 \leq 0$

$0.503979u_1 + 0.076429u_2 - v_1 \leq 0$

$0.879961u_1 + 0.662857u_2 - v_1 \leq 0$

$0.718084u_1 + 0.662857u_2 - v_1 \leq 0$

$0.224872u_1 + 0.301429u_2 - v_1 \leq 0$

$0.553611u_1 + 0.5825u_2 - v_1 \leq 0$

$0.374402u_1 + 0.499286u_2 - v_1 \leq 0$

$0.716242u_1 + 0.5825u_2 - v_1 \leq 0$

$0.537033u_1 + 0.004643u_2 - v_1 \leq 0$

$0.348927u_1$+0.001429u_2-$v_1 \leq 0$

$1u_1$+0.478571u_2-$v_1 \leq 0$

$1u_1$+0.371429u_2-$v_1 \leq 0$

0.802233u_1+$1u_2$-$v_1 \leq 0$

0.724506u_1+$1u_2$-$v_1 \leq 0$

$u_1 \geq \varepsilon$

$u_2 \geq \varepsilon$

$v_1 \geq \varepsilon$

To solve this problem we consider $\varepsilon = 0.001$, so we have:

Global optimal solution found.

Objective value:	0.9999223
Infeasibilities:	0.000000
Total solver iterations:	2

Variable	Value
U_1	0.1000000E-02
U_2	0.9991978
v_1	1.000000

By assuming $\varepsilon = 0.001$ amount of u_1 is equal to nonzero although DMU_{15} is not efficient for this company anymore and efficiently for this DMU is equal to 0.9999223.

4.3.3 Facet Analysis for Modified-CCR Model

Consider amount of ε can show us weakly efficiency DMU although it cannot show exact amount of efficiency for each DMU in weak frontier, As noted, by using the following equation to determine the amount of ε to be:

$$Max\ v_i \tag{4.4}$$

$$st. \sum_{i=1}^{m} v_i x_{i0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \; j = 1,2 \dots n$$

$$v_i \geq 0 \;\; u_i \geq 0$$

And

$$Max\ u_i$$

$$st. \sum_{i=1}^{m} v_i x_{i0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \; j = 1,2 \dots n$$

$$v_i \geq 0 \;\; u_i \geq 0$$

By applying DEA and Modified DEA method for set of data and evaluating the amount of Ɛ it is possible to compare the result.

Ɛi 1= 0.07244 Ɛi 2= 0.03714 Ɛr =0.0238

With consider this amount of Ɛ modify CCR model for DMU_{15} can be:

$MAX\ Z_1 = 0.724506u_1 + u_2$

$st: v_1 = 1$

0.96985u_1+0.7825u_2-v_1 ≤ 0

0.692875u_1+0.613929u_2-v_1 ≤ 0

0.503979u_1+0.076429u_2-v_1 ≤ 0

0.879961u_1+0.662857u_2-v_1 ≤ 0

0.718084u_1+0.662857u_2-v_1 ≤ 0

0.224872u_1+0.301429u_2-v_1 ≤ 0

$0.553611u_1$+0.5825u_2-$v_1 \leq 0$

$0.374402u_1$+0.499286u_2-$v_1 \leq 0$

$0.716242u_1$+0.5825u_2-$v_1 \leq 0$

$0.537033u_1$+0.004643u_2-$v_1 \leq 0$

$0.348927u_1$+0.001429u_2-$v_1 \leq 0$

$1u_1$+0.478571u_2-$v_1 \leq 0$

$1u_1$+0.371429u_2-$v_1 \leq 0$

$0.802233u_1$+1u_2-$v_1 \leq 0$

$0.724506u_1$+1u_2-$v_1 \leq 0$

$u_1 \geq 0.07244$

$u_2 \geq 0.03714$

$v_1 \geq 0.0238$

And we have:

Global optimal solution found.

Objective value: 0.9943695

Infeasibilities: 0.000000

Total solver iterations: 2

Variable	Value
U_1	0.7244000E-01
U_2	0.9418862
v_1	1.000000

Efficiently for this DMU in this model is equal to 0.9943695.

Table 4.3 show efficiently for all of DMU by use classic CCR model, CCR/$\varepsilon =$ 0.001 and modified CCR model:

Table 4.3. Input Oriental Models

DMU	θ^*	θ^* (CCR/ε)	θ^* (modified)
1	1	1	1
2	0.7419	0.7413133	0.7412
3	0.5039	0.5038142	0.5009
4	0.9028	0.9028	0.9028
5	0.7813	0.7813	0.7813
6	0.3015	0.3014121	0.3010
7	0.6373	0.6373	0.6373
8	0.4992	0.4992	0.4983
9	0.7407	0.7407	0.7407
10	0.5370	0.5367806	0.5320
11	0.3489	0.3487614	0.3467
12	1	1	1
13	1	0.9998929	0.9960
14	1	1	1
15	1	0.9999223	0.9924

According to Theorem 3.1, since the aim of this model is to maximize the constant output regardless of the input (output-oriented model) we will have the following table:

Table 4.4. Output Oriental Models

DMU	θ^*	θ^* (CCR/ε)	θ^* (modified)
1	1	1	1
2	1.3478	1.3489	1.3491
3	1.9845	1.9849	1.9964
4	1.1076	1.1076	1.1076
5	1.2799	1.2799	1.2799
6	3.3178	3.3178	3.3222
7	1.5691	1.5691	1.5691
8	2.0032	2.0032	2.0068
9	1.35	1.35	1.35
10	1.8621	1.8632	1.8796
11	2.8661	2.8677	2.8843
12	1	1	1
13	1	1.0002	1.0040
14	1	1	1
15	1	1.0001	1.0076

According to Table 4.4 and Figure 4.1 and 4.2 DMU 15 is proved to be a weak efficient DMU and DMU 6 is the one that is compared to the frontier with weak efficiency. Classical CCR model proved that DMU 15 is efficient since weak efficiency frontier its location.in CCR\ ε model, the mentioned DMU is compared by a hyper plane that produced related on amount of ε (ε=0.001). The amount of efficiency in this DMU is decreased to 0.999. Now when modified CCR is applied the efficiency amount would be equal to 0.992 because in this case DMU A is compared by an admissible hyper plane.

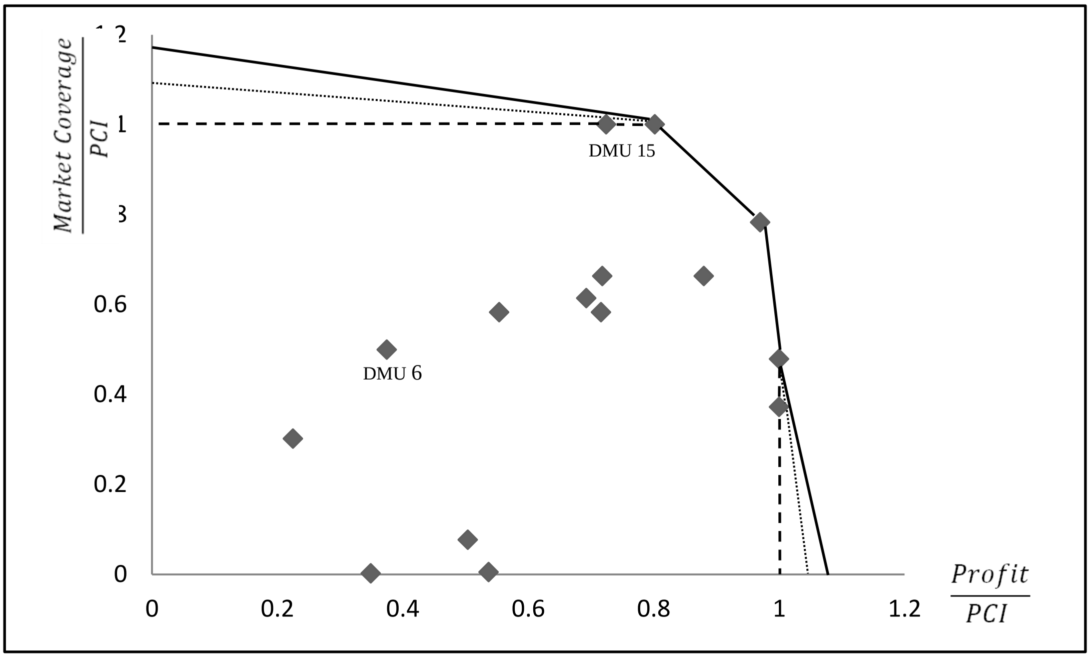

Figure 4.2. Weak Frontier

Imagine the comparison of DMU 6 to weak frontier in normal CCR model. The efficiency in this model is calculated to be 0.3015. Now if it is compared to a hyper plan in CCRR\0.05 model, its efficiency will decrease and would be equal to 0.3014. Figure 4.2 illustrates that when DMU 6 is compared to a hyper plan its efficiency would be equal to 0.3010. This procedure is also true for other DMUs.

Chapter 5

CONCLUSIONS

Choosing a substitute of potential product variants which would be capable of both proliferation of product and the coverage of market is known as Product line selection. To choose the product which could considered as the most efficient is a complicated task which needs to be done through different filters. Now to make the choice easier this thesis proposes a method based on Data Envelopment Analysis (DEA) for product line selection. DEA is a technique based on simple linear programming. This method is often used to measure the performance of Decision Making Units (DMUs) and to choose the most efficient ones which could generate multiple outputs via multiple inputs. DEA has been used in different sciences from mechanical and industrial engineering to economics and finance and results show that the accuracy of method is substantial. it should be mentioned that this method is not used before.. Hence this study tries to investigate the matter on product line selection problem by testing the DEA methodology. To generate the evidence in a quantitative manner, a real life sample is discussed and the results are argued. Results of this thesis could be used by managers to make the correct decisions in order to achieve both maximum consumers' satisfaction and maximum profitability.

This study applied a decision making method (DEA) on product line selection to overcome the possible problems with product line selection. According to Table 4.4 and Figure 4.1 DMU 15 and 13 are proved to be a weak efficient DMU and DMU

2,3,6,8,10,11 that are compared to the frontier with weak efficiency. Classical CCR model proved that DMU 15 and 13 are efficient since weak efficiency frontier its location.in CCR\ ε model, the mentioned DMU is compared by a hyper plane that produced related on amount of ε (ε=0.001).Now when modified CCR is applied the efficiency amount would be change because in this case DMU A is compared by an admissible hyper plane.

By following the results, it is proved that the method that this study is used could be helpful for different industrial units. This method could be used as an application for those industries which are likely to face product line selection problems.

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APPENDIX

Table of Calculations with Lingo Software:

```
DMU1:

CCR Model:
Global optimal solution found.
  Objective value:                              1.000000
  Infeasibilities:                              0.000000
  Total solver iterations:                             3


                       Variable           Value        Reduced Cost
                            U_1       0.6357740            0.000000
                            U_2       0.4899611            0.000000
                            V_1        1.000000            0.000000

CCR/ε Model:
Global optimal solution found.
  Objective value:                              1.000000
  Infeasibilities:                              0.000000
  Total solver iterations:                             3


                       Variable           Value        Reduced Cost
                            U_1       0.6357740            0.000000
                            U_2       0.4899611            0.000000
                            V_1        1.000000            0.000000
Modified CCR Model:
Global optimal solution found.
  Objective value:                              1.000000
  Infeasibilities:                              0.000000
  Total solver iterations:                             3


                       Variable           Value        Reduced Cost
                            U_1       0.6357740            0.000000
                            U_2       0.4899611            0.000000
                            V_1        1.000000            0.000000
```

```
DMU2:

CCR Model:
Global optimal solution found.
  Objective value:                              0.7419133
  Infeasibilities:                               0.000000
  Total solver iterations:                              3


                       Variable           Value        Reduced Cost
                            U_1       0.6357740            0.000000
                            U_2       0.4899611            0.000000
                            V_1        1.000000            0.000000

CCR/ε Model:
Global optimal solution found.
  Objective value:                              0.7413133
  Infeasibilities:                               0.000000
  Total solver iterations:                              3


                       Variable           Value        Reduced Cost
                            U_1       0.6357740            0.000000
                            U_2       0.4899611            0.000000
                            V_1        1.000000            0.000000
Modified CCR Model:
Global optimal solution found.
  Objective value:                              0.7412133
  Infeasibilities:                               0.000000
  Total solver iterations:                              3


                       Variable           Value        Reduced Cost
                            U_1       0.6357740            0.000000
                            U_2       0.4899611            0.000000
                            V_1        1.000000            0.000000
```

```
DMU3:

CCR Model:
Global optimal solution found.
  Objective value:                              0.5039790
  Infeasibilities:                               0.000000
  Total solver iterations:                              1


                       Variable           Value        Reduced Cost
                            U_1        1.000000            0.000000
                            U_2        0.000000           0.1647607
                            V_1        1.000000            0.000000

CCR/ε Model:
Global optimal solution found.
  Objective value:                              0.5038142
  Infeasibilities:                               0.000000
  Total solver iterations:                              1


                       Variable           Value        Reduced Cost
                            U_1       0.9822259            0.000000
                            U_2       0.3714000E-01        0.000000
                            V_1        1.000000            0.000000
Modified CCR Model:
Global optimal solution found.
  Objective value:                              0.5009134
  Infeasibilities:                               0.000000
  Total solver iterations:                              1


                       Variable           Value        Reduced Cost
                            U_1       0.9995214            0.000000
                            U_2       0.1000000E-02        0.000000
                            V_1        1.000000            0.000000
```

```
DMU4:

CCR Model:
Global optimal solution found.
  Objective value:                              0.9028543
  Infeasibilities:                               0.000000
  Total solver iterations:                              3


                       Variable           Value        Reduced Cost
                            U_1       0.9546771            0.000000
                            U_2       0.9470473E-01        0.000000
                            V_1        1.000000            0.000000
CCR/ε Model:
Global optimal solution found.
  Objective value:                              0.9028543
  Infeasibilities:                               0.000000
  Total solver iterations:                              3


                       Variable           Value        Reduced Cost
                            U_1       0.9546771            0.000000
                            U_2       0.9470473E-01        0.000000
                            V_1        1.000000            0.000000
Modified CCR Model:
Global optimal solution found.
  Objective value:                              0.9028543
  Infeasibilities:                               0.000000
  Total solver iterations:                              3


                       Variable           Value        Reduced Cost
                            U_1       0.9546771            0.000000
                            U_2       0.9470473E-01        0.000000
                            V_1        1.000000            0.000000
```

```
DMU5:

CCR Model:
Global optimal solution found.
  Objective value:                              0.7813133
  Infeasibilities:                               0.000000
  Total solver iterations:                              3


                       Variable           Value        Reduced Cost
                            U_1       0.6357740            0.000000
                            U_2       0.4899611            0.000000
                            V_1        1.000000            0.000000
```

CCR/ ε Model:

```
Global optimal solution found.
  Objective value:                              0.7813133
  Infeasibilities:                               0.000000
  Total solver iterations:                              3


                       Variable           Value        Reduced Cost
                            U_1       0.6357740            0.000000
                            U_2       0.4899611            0.000000
                            V_1        1.000000            0.000000
Modified CCR Model:
Global optimal solution found.
  Objective value:                              0.7813133
  Infeasibilities:                               0.000000
  Total solver iterations:                              3


                       Variable           Value        Reduced Cost
                            U_1       0.6357740            0.000000
                            U_2       0.4899611            0.000000
                            V_1        1.000000            0.000000
```

```
DMU6:

CCR Model:
Global optimal solution found.
  Objective value:                              0.3015290
  Infeasibilities:                               0.000000
  Total solver iterations:                              2


                       Variable           Value        Reduced Cost
                            U_1        0.000000        0.1694429E-01
                            U_2        1.000000          0.000000
                            V_1        1.000000          0.000000
CCR/ε Model:
Global optimal solution found.
  Objective value:                              0.3014016
  Infeasibilities:                               0.000000
  Total solver iterations:                              2


                       Variable           Value        Reduced Cost
                            U_1       0.7244000E-01        0.000000
                            U_2       0.9418862            0.000000
                            V_1        1.000000            0.000000
Modified CCR Model:
Global optimal solution found.
  Objective value:                              0.3010121
  Infeasibilities:                               0.000000
  Total solver iterations:                              2


                       Variable           Value        Reduced Cost
                            U_1       0.1000000E-02        0.000000
                            U_2       0.9991978            0.000000
                            V_1        1.000000            0.000000
```

```
DMU7:

CCR Model:
Global optimal solution found.
  Objective value:                              0.6373738
  Infeasibilities:                               0.000000
  Total solver iterations:                              3


                       Variable           Value        Reduced Cost
                            U_1       0.6357740            0.000000
                            U_2       0.4899611            0.000000
                            V_1        1.000000            0.000000
CCR/ε Model:
Global optimal solution found.
  Objective value:                              0.6373738
  Infeasibilities:                               0.000000
  Total solver iterations:                              3


                       Variable           Value        Reduced Cost
                            U_1       0.6357740            0.000000
                            U_2       0.4899611            0.000000
                            V_1        1.000000            0.000000
Modified CCR Model:
Global optimal solution found.
  Objective value:                              0.6373738
  Infeasibilities:                               0.000000
  Total solver iterations:                              3


                       Variable           Value        Reduced Cost
                            U_1       0.6357740            0.000000
                            U_2       0.4899611            0.000000
                            V_1        1.000000            0.000000
```

```
DMU8:

CCR Model:
Global optimal solution found.
  Objective value:                              0.4992860
  Infeasibilities:                               0.000000
  Total solver iterations:                              2


                       Variable           Value        Reduced Cost
                            U_1        0.000000        0.2614171E-01
                            U_2        1.000000          0.000000
                            V_1        1.000000          0.000000

CCR/ε Model:
Global optimal solution found.
  Objective value:                              0.4992860
  Infeasibilities:                               0.000000
  Total solver iterations:                              2


                       Variable           Value        Reduced Cost
                            U_1       0.7244000E-01        0.000000
                            U_2       0.9418862            0.000000
                            V_1        1.000000            0.000000
Modified CCR Model:
Global optimal solution found.
  Objective value:                              0.4992860
  Infeasibilities:                               0.000000
  Total solver iterations:                              2


                       Variable           Value        Reduced Cost
                            U_1       0.1000000E-02        0.000000
                            U_2       0.9991978            0.000000
                            V_1        1.000000            0.000000
```

```
DMU9:

CCR Model:
Global optimal solution found.
  Objective value:                              0.7407704
  Infeasibilities:                               0.000000
  Total solver iterations:                              3


                       Variable           Value        Reduced Cost
                            U_1       0.6357740            0.000000
                            U_2       0.4899611            0.000000
                            V_1        1.000000            0.000000
CCR/ε Model:
Global optimal solution found.
  Objective value:                              0.7407704
  Infeasibilities:                               0.000000
  Total solver iterations:                              3


                       Variable           Value        Reduced Cost
                            U_1       0.6357740            0.000000
                            U_2       0.4899611            0.000000
                            V_1        1.000000            0.000000

Modified CCR Model:
Global optimal solution found.
  Objective value:                              0.7407704
  Infeasibilities:                               0.000000
  Total solver iterations:                              3


                       Variable           Value        Reduced Cost
                            U_1       0.6357740            0.000000
                            U_2       0.4899611            0.000000
                            V_1        1.000000            0.000000
```

```
DMU10:

CCR Model:
Global optimal solution found.
  Objective value:                              0.5370330
  Infeasibilities:                               0.000000
  Total solver iterations:                              1


                       Variable           Value        Reduced Cost
                            U_1        1.000000            0.000000
                            U_2        0.000000           0.2523654
                            V_1        1.000000            0.000000
CCR/ε Model:
Global optimal solution found.
  Objective value:                              0.5367660
  Infeasibilities:                               0.000000
  Total solver iterations:                              1


                       Variable           Value        Reduced Cost
                            U_1       0.9822259            0.000000
                            U_2       0.3714000E-01        0.000000
                            V_1        1.000000            0.000000
Modified CCR Model:
Global optimal solution found.
  Objective value:                              0.5320806
  Infeasibilities:                               0.000000
  Total solver iterations:                              1


                       Variable           Value        Reduced Cost
                            U_1       0.9995214            0.000000
                            U_2       0.1000000E-02        0.000000
                            V_1        1.000000            0.000000
```

```
DMU11:

CCR Model:
Global optimal solution found.
  Objective value:                              0.3489270
  Infeasibilities:                               0.000000
  Total solver iterations:                              1


                       Variable           Value        Reduced Cost
                            U_1        1.000000            0.000000
                            U_2        0.000000           0.1655573
                            V_1        1.000000            0.000000

CCR/ε Model:
Global optimal solution found.
  Objective value:                              0.3487614
  Infeasibilities:                               0.000000
  Total solver iterations:                              1


                       Variable           Value        Reduced Cost
                            U_1       0.9822259            0.000000
                            U_2       0.3714000E-01        0.000000
                            V_1        1.000000            0.000000

Modified CCR Model:
Global optimal solution found.
  Objective value:                              0.3467614
  Infeasibilities:                               0.000000
  Total solver iterations:                              1

                       Variable           Value        Reduced Cost
                            U_1       0.9995214            0.000000
                            U_2       0.1000000E-02        0.000000
                            V_1        1.000000            0.000000
```

```
DMU 12:

CCR Model:
Global optimal solution found.
  Objective value:                              1.000000
  Infeasibilities:                              0.000000
  Total solver iterations:                             3


                       Variable           Value        Reduced Cost
                            U_1       0.9546771            0.000000
                            U_2       0.9470473E-01        0.000000
                            V_1        1.000000            0.000000

CCR/ε Model:
Global optimal solution found.
  Objective value:                              1.000000
  Infeasibilities:                              0.000000
  Total solver iterations:                             3


                       Variable           Value        Reduced Cost
                            U_1       0.9546771            0.000000
                            U_2       0.9470473E-01        0.000000
                            V_1        1.000000            0.000000
Modified CCR Model:
Global optimal solution found.
  Objective value:                              1.000000
  Infeasibilities:                              0.000000
  Total solver iterations:                             3


                       Variable           Value        Reduced Cost
                            U_1       0.9546771            0.000000
                            U_2       0.9470473E-01        0.000000
                            V_1        1.000000            0.000000
```

```
DMU13:

CCR Model:
CCR Model:
Global optimal solution found.
  Objective value:                              1.000000
  Infeasibilities:                              0.000000
  Total solver iterations:                             3


                       Variable           Value        Reduced Cost
                            U_1       0.6357740            0.000000
                            U_2       0.4899611            0.000000
                            V_1        1.000000            0.000000
CCR/ε Model:
Global optimal solution found.
  Objective value:                              0.9998929
  Infeasibilities:                              0.000000
  Total solver iterations:                             3


                       Variable           Value        Reduced Cost
                            U_1       0.6357740            0.000000
                            U_2       0.4899611            0.000000
                            V_1        1.000000            0.000000
Modified CCR Model:
Global optimal solution found.
  Objective value:                              1.996001
  Infeasibilities:                              0.000000
  Total solver iterations:                             3


                       Variable           Value        Reduced Cost
                            U_1       0.6357740            0.000000
                            U_2       0.4899611            0.000000
                            V_1        1.000000
```

```
DMU14:

CCR Model:
Global optimal solution found.
  Objective value:                              1.000000
  Infeasibilities:                              0.000000
  Total solver iterations:                             2


                       Variable           Value        Reduced Cost
                            U_1        0.000000            0.000000
                            U_2        1.000000            0.000000
                            V_1        1.000000            0.000000
CCR/ε Model:
Global optimal solution found.
  Objective value:                              1.000000
  Infeasibilities:                              0.000000
  Total solver iterations:                             2


                       Variable           Value        Reduced Cost
                            U_1       0.1000000E-02        0.000000
                            U_2       0.9991978            0.000000
                            V_1        1.000000            0.000000
Modified CCR Model:
Global optimal solution found.
  Objective value:                              1.000000
  Infeasibilities:                              0.000000
  Total solver iterations:                             2


                       Variable           Value        Reduced Cost
                            U_1       0.7244000E-01        0.000000
                            U_2       0.9418862            0.000000
                            V_1        1.000000            0.000000
```

```
DMU15:

CCR Model:
Global optimal solution found.
  Objective value:                              1.000000
  Infeasibilities:                              0.000000
  Total solver iterations:                             2


                       Variable           Value        Reduced Cost
                            U_1        0.000000        0.7772700E-01
                            U_2        1.000000          0.000000
                            V_1        1.000000          0.000000
CCR/ε Model:
Global optimal solution found.
  Objective value:                             0.9999223
  Infeasibilities:                              0.000000
  Total solver iterations:                             2


                       Variable           Value        Reduced Cost
                            U_1       0.1000000E-02        0.000000
                            U_2       0.9991978            0.000000
                            V_1        1.000000            0.000000


Modified CCR Model:
Global optimal solution found.
  Objective value:                             0.9924369
  Infeasibilities:                              0.000000
  Total solver iterations:                             2

                       Variable           Value        Reduced Cost
                            U_1       0.7244000E-01        0.000000
                            U_2       0.9418862            0.000000
                            V_1        1.000000            0.000000
```