Prospect Theory in Decision Making Process

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ABSTRACT

This thesis purposes the investigation of the prospect theory in decision making process. The theoretical aspects and basic concepts of the prospect theory for the decision making under uncertainty and risk are analyzed. The editing and evaluation phases of prospect theory are discussed. The expected utility, value and weighting functions are calculated. The utility functions for rational decision making by evaluating gains and losses are considered.

Keywords: Prospect theory, Decision making, Expected utility, Value and weighting functions, Utility functions

ÖZ

Bu tezin amacı karar verme sürecinde beklenti teorisini araştırmaktır. Kararsızlık ve risk durumlarında karar verme için beklenti teorisinin teorik yönleri ve temel kavramları incelenir. Bekleme teorisinin düzenleme ve değerlendirme aşamaları ele alınır. Beklenen fayda, değer ve ağırlık verme fonksiyonları hesaplanır. Rasyonel karar verme için kazanç ve zarar değerlendirilerek fayda fonksiyonları dikkate alınır.

Anahtar Kelimeler: Beklenti teorisi, Karar verme, Beklenen fayda, Değer ve ağırlık verme fonksiyonları, Fayda fonksiyonları

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Chapter 1

INTRODUCTION

We like it or not, we expect to face risk on a daily basis of our lives. To choose a route from work to home, in choosing a mate, we tend to seldom grasp before and with certainty what the end results of our selections are. Thus, we consider the possible outcomes whether they are attractive or unattractive, and the probability of incidence of these outcomes.

The main conception of "risk" is related with hazards, and is poorly understood by people [1]. People consider a risk as a case that should be overcome, and come to the conclusion that a risk is getting higher with the magnitude of potential losses [2]. The scientists in decision theory, in distinction, consider risk as increasing with variance within the chance distribution of attainable outcomes, in spite of whether or not a possible loss is concerned. As an example, a possibility that provides a case with 50-50 chance of paying $100 or nothing is more risky than a case offering $50 as expected. Since Knight, economists decided that it is necessary to distinguish choices to be made under risk and under uncertainty [3]. While making choices under risk, the decision maker knows a preciseness of the chance distribution of the possible outcomes.

Prospect theory was firstly proposed by Markowitz [4]. Afterwards two other scientists Kahneman and Tversky in 1979 described a decision making in another form by comparing it with the expected utility theory.

Expected utility theory is simple, easy for modeling, and is used for decision making under uncertainty, and is mostly applied in situations when there is an incompleteness of information.

Kahneman and Tversky found by trial that individuals underweight outcomes that are simply probable as compared with outcomes that are obtained with certainty; additionally individuals typically discard parts that are shared by all prospects into account. Under prospect theory, price is allotted to gains and losses instead of to final assets; additionally possibilities are replaced by decision weights. The worth operate is outlined on deviations from a reference and is often recessed for gains (implying risk aversion), normally convex for losses (risk loving) and is usually vessel for losses than for gains (loss aversion). Decision weights are typically under the corresponding possibilities, except in the range of low possibilities. Prospect theory has managed to clarify some major violations of expected utility theory with relation to a tiny low range of outcomes, and Kahneman and Tversky obtained that because of these systematic violations of expected utility theory people basically prefer optimal decision by maximizing expected utility instead of real life chooses characteristic to prospect theory.

Prospect theory describes human behavior in classification of expected prospects as positive and negative. Prospect theory shows that in a real life people differently react to risky situations according to the outcomes which can be expected as gains or losses. People may change their behavior and desire if they expect changing prospect from sure gain to a probable gain.

Kahneman and Tversky verified in some experiments that the day-after-day reality of decision makers varies from the assumptions command by economists.

In comparison with a classical expected utility theory, a prospect theory is more improved decision making technique. Many disadvantages of the expected utility theory can be easily determined by using prospect theory [5].

Prospect theory is the best description to be offered as a choice method. It summarizes many centuries value of findings and insights regarding behavior of human decision. Moreover, the latest insights and predictions are taken into account.

While making a decision under prospect theory, people try to follow two stages: firstly according to some heuristics they consider the possible outcomes of the events, order them in order of preference, assign reference points, evaluate possible minimum losses and maximum gains; and secondly after assigning some utilities the final decision is taken.

Prospect theory has been playing an important role in many areas such as economics, computer science, marketing, human behavior, psychology, medicine, engineering, political sciences etc.

Prospect theory brought science into the center of economic analysis in comparison to other approaches. Prospect theory has gained a big popularity in recent years, and currently actually occupies second place on the analysis agenda for economists.

Chapter 2

REVIEW OF EXISTING LITERATURE ON PROSPECT THEORY

[6] is about the prospect theory under risk, and the comparison between the outcomes under certainty and the one that is merely probable is discussed. An expected utility theory (EUT) as a descriptive model is criticized in this paper while taking appropriate decision under risk, and developed an alternate model, referred to as prospect theory. The descriptive foundation of prospect theory containing certainty, isolation, and reflection effects and prospect theory functions such as value function and weighted function are explained. Another theory of selection is proposed, and in this theory instead of final assets it is recommended that value is assigned to gains and losses.

Kahenman's and Tversky's prospect theory in the commercial and banking industry is examined in [7]. The paper studies the risk-taking behavior and how the prospect predicts the growth by taking into consideration the target outcome. Cross-sectional medians of return on assets, on equity and primary capital ratio are used as target.

In [8] the portfolio choice problem is considered. Some of the applications of prospect theory as well as the study show how the prospect theory will be re-designed if one is willing to apply it on a portfolio selection. The study is based on the results of Kahneman & Tversky's theory in 1979 and cumulative prospect theory of them in 1992.

In [9] the momentum strategy probability is generated by the variable proxy for aggregate unrealized capital gains. The paper shows the solutions of some cases when investors hold their losing stocks that are driven by mental accounting.

The consistency principle is proposed in [10] for the elicited probability of midpoints that require a consistent treatment of gains and losses. When all other standard preference conditions are present, the consistency principle implies prospects.

In [11] the behaviors of decision makers under risky chance are discussed. It is defined that the framing of question provides different information about the interpretation of a decision maker's action. The study shows that a rational need to avoid wanting unskilled could facilitate many anomalies related to the prospect theory, as well as probability weighting, loss aversion, and framing.

In [12] the liquidation problem is solved for an agent, depending on break-even point and the relatively sharp ratio.

The best fitting parameters for two graphs in cumulative prospect theory is obtained by using finite mixture models [13].

[14] provides a preference foundation of a prospect theory for complicated probabilities, and tendency to consider an obtained preference foundations for special cases of prospect theory.

Equilibrium trading strategies and market economy are discussed in [15], and the speculators use preferences according to prospect theory. The loss aversion and caution have nontrivial and state-dependent effects on equilibrium liquidity, market efficiency, and trading volume.

[16] shows the importance of the theory of probability in wealth management and describes how the investor understands the risks and makes the rational decision-making. The use of probability theory in the consultative process for customers is described.

In [17] the fourfold pattern (FFP) of risk attitudes is presented. Two cases are considered: 1) risk seeking (low-probability for gains and high-probability for losses); 2) risk-aversion (low probability for losses and high-probability for gains). Using real and simple gamble also provides a direct test of the fourfold pattern.

The paper [18] studies the importance of the prospect theory for jurists and the approach how to use the theory of decision-making in law. Probability theory contains many proposals and analysis developed by Daniel Kahneman and Amos Tversky's. The paper mainly focuses on how individuals usually use their choices in risk-aversion selections when selecting between “gains” and risk-seeking selections when choosing between “losses”. It is underlined that the prospect theory is a valuable tool for legal scholars and policy makers.

In [19] the importance of the prospect theory in the social sciences, especially in the field of behavioral economics is considered. The theoretical problems for the explanation of the political decisions are investigated.

[20] presents how the patient selects the best treatment option according to his/her preference point.

In [21] it is considered that the prospect theory is not better than cumulative theory because the cumulative theory can give different and new predictions, and the cumulative theory is not an extension for prospect theory.

In [22] an axiomatization for the decision under risk is concerned. The resulting axiomatization under risk is simpler than that for uncertainty.

The risk aversion in cumulative theory is described in [23]. The investigation is also about the possible relationship between risk and loss aversions. The convex weighting functions for gains and losses are implied by the risk aversion.

Some formats are used in [24] to eliminate or reduce the violation of cumulative prospect theory (cancellation and combination). Some data are used to discuss the patterns of evidence that violate cumulative prospect theory.

In [25] a new prospect theory called cumulative prospect theory is discussed. The study extends the theory in several prospects. The author confirms a distinctive fourfold pattern (FFP) of risk attitudes and also two principles that are used to justify the characteristic curvature of the value function and the weighting functions.

The risk aversion analysis for both gains and losses is discussed in [26]. The gains are transformed to losses by reflecting them around zero. The frequency of reflection (about 10 percent) isn't larger than the frequency of reverse reflection (risk loving for gains and risk aversion for losses), and the risk aversion is less common in a loss domain.

In [27] it is underlined that the existing functional forms are inappropriate for estimations of behavioral patterns with a single set of parameter. It is shown that none of this parameterization can account simultaneously for gambling on unlike gain, and the proposed method is very useful to get a reasonable risk premia.

The cumulative prospect theory is planned and proposed as an alternative approach to expected utility theory to clarify irregular behavior by economic agents [28].

A cumulative prospect theory portfolio choice model is developed in [29]. Three key elements of cumulative prospect theory (CPT) are studied: S-shaped utility, reference point, and probability weighting. A new measure of a loss aversion called large-loss aversion degree (LLAD) is introduced.

Majority of studies on the prospect theory shows that the prospect theory is based on the status quo and not on the final wealth as in expected utility theory. [30] is about the value function depending on the final wealth, and the cumulative prospect theory is developed to put condition for a preference between dependence on the status quo and the dependence on final wealth.

Chapter 3

THEORETICAL ASPECTS AND BASIC CONCEPTS OF PROSPECT THEORY

3.1 Prospect Theory and Decision Making

In a presence of uncertainty it is very difficult to predict or expect the results of the events in a clear way. The decision contains inner struggle over value of tradeoff. Prospect theory is basically dealing with the problem of framing and evaluation of the choices in the process of decision making [31].

In 1979, Kahneman and Tversky gave a definition for the prospect (p_i, x_i) to be a convention that yields the result (outcome) x_i related with the probability (p_i), where the summation of p_i is 1.

In prospect theory people are making their decisions on options, and all are shown in terms of prospect. In a prospect theory, the value of a prospect, $V(x, p)$, is described by the formula given below:

$$V(x, p) = w(p_1)v(x_1) + w(p_2)v(x_2) + \cdots + w(p_i)v(x_i) + \cdots \ (1)$$

Where v measures the subjective value of the consequence, x and w measure the impact of probability p on the attractiveness of the prospect. There is a similarity between value function and utility function in expected utility theory, and a value function is denoted by v, and it assigns a number $v(x_i)$ to each outcome x_i. The worth of the indicator (reference point) should be neutral, and we assign $v(0) = 0$. Additionally, $v(x_i)$ is assumed to be a nonstop, strictly increasing perform. The prospects supported by their values are evaluated [32].

In other words, prospect theory gives prevention that people are risk averse in case of gains or when everything runs well, and comparatively risk searching in case of losses. The prospect theory is descriptive and empirical in nature, and is intended to clarify a mutual pattern of a choice. There are two phases in a prospect theory: editing phase and evaluation phase. The editing phase includes framing effects, and the evaluation phase includes decision making by choosing among many available options. This decision is affected by two procedures, and the first one is related to subjective value, and another one is related to perceptual likelihood.

3.2 Descriptive Foundation of Prospect Theory

In 1979, the experiments conducted by Kahneman and Tversky involved some hypothetic decisions, and it is helpful to summarize foremost vital findings. Most of their examples consult with risky selection concerning financial outcomes; however, several of their findings are generalized to alternative types of risky selection.

(i) Individuals suppose in terms of gains and losses instead of in terms of their web assets, and so encrypt selections in terms of deviations from a reference point.

(ii) Individuals differently treat gains and losses because of two reasons. First, people are risk-averse with regard to gains and risk-loving with regard to losses. In very typical experiments conducted by Kahneman and Tversky in 1979, most of the people would select an exact gain of $30 over a simple fraction probability of gaining $70. However, they might select a simple fraction probability of losing $70 (and simple fraction probability of losing nothing) over an exact loss of $30. The outcomes truly involve completely different domains (gain versus loss), that is, they disagree in sign (+$30 versus -$30). The conducted experiments show that individual utility functions are concave and convex within the domain of gains and losses, respectively.

Reflection effect involves gambles whose outcomes are opposite in sign, though they have a similar magnitude. Reflection effect is predicted in prospect theory by the S shape of the value function: the concave for gains refers to risk aversion and the convex for losses is pointing risk loving.

(iii) Gains also are treated in different ways than losses. As Jimmy Conner exclaimed, “I hate to lose more than I like to win”. The development of a loss aversion leads to the fact that individuals like the established order over a 50-50 likelihood for positive and negative alternatives with a similar definite quantity. It also shows that individuals differently value available and unavailable things. People give more value to things they own compare to things they do not own, and it is known as endowment effect [33].

The loss aversion and also the endowment impact imply that commercialism costs ought to be over shopping for prices: the marginal compensation individuals demand to relinquish up an honest is usually many times larger than the utmost quantity they are willing to obtain a conterminous title. The development of a loss aversion is aggravated by different psychological factors. First, the variations between choices will appear more necessary if they are framed in terms of losses instead of if they are framed in terms of gains. Second, any addition of a loss to a specific selection can hurt it, but any addition of an advantage will facilitate it.

In a loss aversion people give more weight to losses than to gains - they are loss averse. So, if you gain $200 and lose $190, it's going to be thought-about a net loss in terms of satisfaction, even if your profit is $10, as a result you will specialize in what quantity you lost, not on what quantity you gained.

The endowment impact (also referred to as divestiture aversion) is that the hypothesis that one is willing to accept compensation a good is larger than their willing to pay for it once their characteristic to that has been established. Individuals pay more to retain some things they own than to get something closely-held by somebody else - even once there is no cause for attachment, or maybe if the item was solitary obtained minutes ago. This happens often because of the actual fact that after one owns the item, foregoing it looks like a loss, and people are loss-averse. The money spent on goods is not considered by people as a loss, and items purchased for ultimate sale do not lead for a generation of an endowment impact.

(iv) Studies have shown that people overweight outcomes that are certain relative to outcomes that are simply probable - the certainty effect. They furthermore overweight tiny chances and underweight moderate and high chances, and also the latter result is additional pronounced than the previous. Very seemingly unsure outcomes are typically treated as if they were sure, and it is invoked the pseudocertainty impact by Kahneman and Tversky [34]. The term pseudocertainty effect comes from the prospect theory, and individuals consider uncertain outcomes as certain outcomes but in reality it is uncertain [34]. Consequently, changes in chances close to zero or one have more effect compare to changes within the middle of the likelihood vary, that ends up in the attribution distinction principle or sub proportionality: the impact of any fastened positive distinction between two amounts will increase with their attribution. The differential analysis of the whole elimination as hostile to the risk decrement is realized with actual fact that individuals are ready to spend more money for decreasing the risk of a ruinous loss from 0.1 to zero than from 0.2 to 0.1, even the modification in expected utility is that the same.

(v) A proof used for the statement that for changing the selection between alternatives people usually disregard parts which are to be same to every different choice, and focus on parts that are totally different. This isolation effect causes totally distinct preferences as a result of there could also be quite a method to decompose prospects into shared and distinctive parts [6].

3.3 Editing Phase of Prospect Theory

The editing phase is considered as a first phase in prospect theory. Using this phase enables to represent actions, results or outcomes, and probabilities to be convenient with accurate best problem. In the first stage of analysis the editing phase consisting of the offered prospects frequently yields an easier illustration of those prospects. Within the second phase (evaluation phase), the evaluation process of the edited prospects are performed and so the prospect with highest value is selected. We tend to next to define the editing phase, and to develop an appropriate model for the second phase. The editing phase arranges and develops the choices to modify the evaluation to be considered as a subsequent one and selection. The editing phase also includes the appliance of many operations that remodel the outcomes and chances related to the offered prospects. The main operations used in editing phase are given below:

Combination: Prospects will generally be simplified by combining the chances related to identical outcomes. For instance, the prospect (400, 0.25; 400, 0.25) is reduced to (400, 0.50), and evaluation is performed according to this form**.**

Segregation: Some prospects consist of a risk-less part which is sequestered from the part to be risky in editing phase. As an example, the prospect (400, 0.70; 300, 0.30) is naturally decomposed into a certain gain of three hundred and therefore the risky prospect is (100, 0.70). Similarly, the prospect (-500, 0.30; -100, 0.70) is naturally decomposed into a certain loss of one hundred and the prospect (-400, 0.30). The combination operation is applied to each prospect separately. The preceding operations (cancellation, simplification, transparent dominance) can be used for being applied to two or more prospects.

Cancellation: This operation is used for the cancelation of the shared components of choices to be offered together - for example, an alternative between ($20, 0.1; $60, 0.1) or ($20, 0.1; $30, 0.2) would be naturally portrayed as an alternative between a ($60, 0.1) or ($30, 0.2).

Simplification: This operation is used to alter and round prospects for very unlikely outcomes: for example, ($99, 0.51; $5, 0.0001) may well be delineated as ($100, 0.5).

Transparent dominance: The choices are rejected by decision makers if these choices are clearly dominated by alternative choices - for example, given a selection between ($19, 0.1; $20, 0.1; $30, 0.1) or ($30, 0.3), most people would obviously reject the primary choice because of being dominated by the second choice.

3.4 Evaluation Phase of Prospect Theory

In the evaluation part of prospect theory, a person examines all the altered prospects and chooses the one with the best price. The price of an opening, denoted by V, is expressed in terms of two scales: π and v.

The scale π associates with every probability p and is indicated as $\pi\ (p)$ that reflects the impact of p in all the prospects. π shows that $\pi\ (p)\ +\ \pi\ (l\ -\ p)$ is often less than unity. v is considered as a second scale, and assigns every outcome x to variety $v\ (x)$ that conveys the subjective worth of the given outcome. These outcomes are outlined to a point of reference relatively, that is the zero of the worth scale. The worth (i.e. losses and gains) of deviations from that indicator is measured by v.

The given formulation cares with easy prospects of the shape $(x, p;\ y, q)$, that have at most two outcomes that are not equal to zero. In this probability, x is received with likelihood p, and y is received with likelihood q, and the likelihood 1– p–q is related with no prospect, wherever $p\ +\ q\ \leq\ 1$. The prospect which is offered is mentioned as strictly positive if all the outcomes of the prospect are positive, which means $x, y\ >\ 0$ and $p\ +\ q\ = 1$. An offered prospect can be mentioned as strictly negative if all the outcomes of the prospect are negative. A prospect which is not strictly positive and also not strictly negative is called a regular prospect.

The considered idea describes a style during which $\pi\ and\ v$ should be combined to work out the overall worth of normal prospects.

If $(x, p;\ y, q)$ is a regular prospect

$$(i.e., either\ p\ +\ q\ <\ 1, or\ x \leq 0 \leq\ y, or\ x \geq\ 0 \geq y)\ then$$

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y) \cdots\ (2)$$

Where $v(0) = 0, \pi(0) = 0,$ and $\pi(1) = 1$. In case of availability of sure prospects, two scales come together and coincide, where $V(x, 1.0) = V(x) = v(x)$ [6].

Equation (2) is a generalized form of an expected utility theory by restful the expectation principle.

While analyzing strictly positive and strictly negative prospects it must be mentioned that the segregation of the prospects can be realized with two components**:**

i) The safe element, i.e., the minimum value of either gain or loss should certainly be determined or paid;

ii) The risky element, i.e., the extra gain or loss that is truly open for changing.

Such prospects are described with the following equation [6]:

If $p + q = 1$, and either $x > y > 0$ or $x < y < 0$, then

$$V(x, p; y, q) = v(y) + \pi(p)[v(x) - v(y)] \cdots (3)$$

The value of either strictly positive or strictly negative prospect is equal to the value of risk-free element and the value which is calculated as a difference between the outcomes, increased by the burden related to the additional extreme outcome.

Example:

$$V(300,0.35;\ 200,0.65) = v(200) + \pi(0.35)[v(300) - v(200)]$$

3.5 Fourfold Pattern of Risk Attitudes

Together, the value and weight functions cause a fourfold pattern of risk attitudes. There are two different stimulation procedures in fourfold pattern of risk attitudes. The first one is risk-averse behavior when gains have high probabilities and losses have small probabilities; and the second one is risk-loving behavior when losses have high probabilities compare to small probabilities of gains.

The example related with the probabilities of gain and loss for risk aversion and risk loving is described in the table 1.

Table 1: The probabilities of gain and loss for risk aversion and risk loving cases

Example	Gains	Losses
High Probability	90% probability of winning $50000 (RISK AVERSE PROSPECT)	90% probability of losing $50000 (RISK LOVING PROSPECT)
Low Probability	10% probability of winning $50000 (RISK LOVING PROSPECT)	10% probability of losing $50000 (RISK AVERSE PROSPECT)

The table 1 reveals a fourfold pattern of risk attitudes: risk loving and risk aversion. In risk loving case gains are measured with low probabilities, and losses are measured with high probabilities whereas in risk aversion case the probabilities for gains and losses are high and low, respectively. The choices in line with this fourfold pattern are discovered in many studies [6], [35]-[38].

3.6 Value Function

There are two parts of the evaluation phase in the prospect theory: 1) value function; 2) weighting function. The proposed value function with three characteristics is illustrated in figure 1:

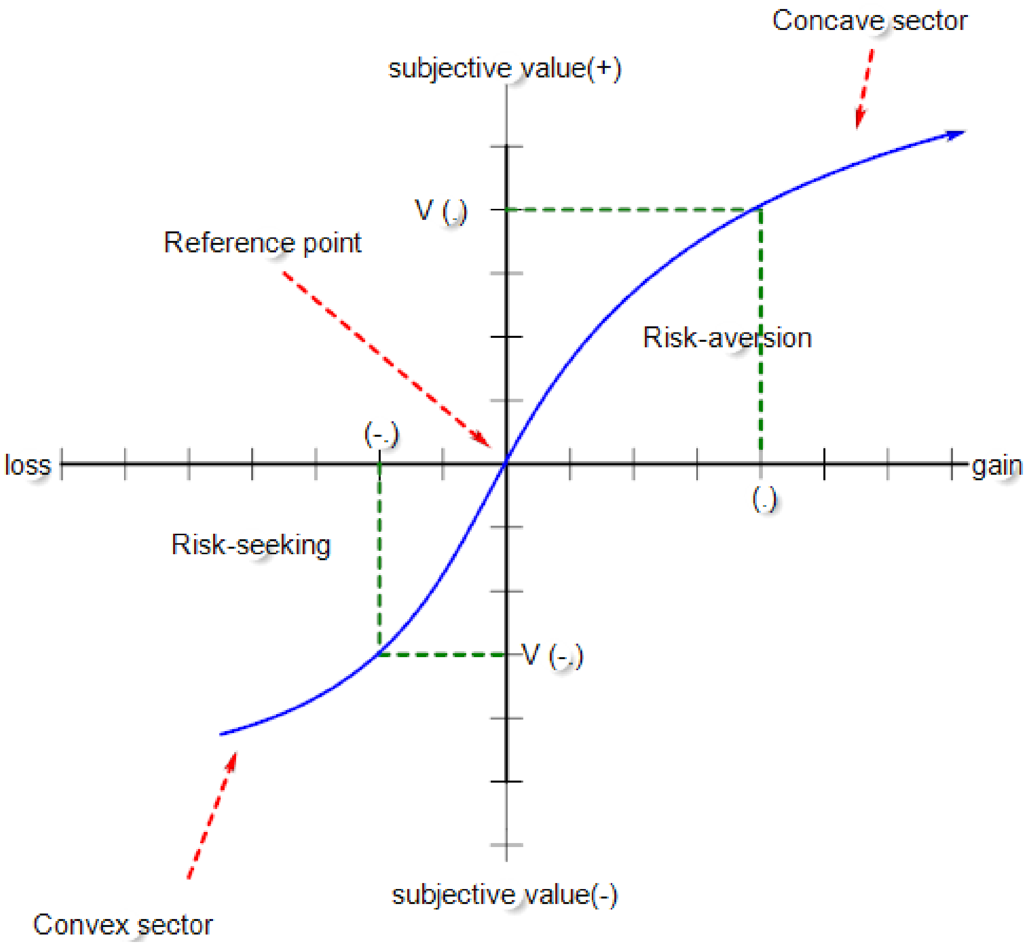

Figure 1: Illustration of three characteristics of a value function

There are three characteristics of the value function. The primary one is that it is outlined in terms of gains and losses relative to the reference. This kind is completely different from the expected utility theory, and assumes that the final quality position is definitive in hard subjective utility and predicting selection. The emphasis on modification from the point of reference in prospect theory is to keep with basic human sensory activity processes that tend to note shifts over resting states. While applying a prospect theory, the value can be considered as a function of this modification, in an exceedingly positive or negative. The importance of the point of start is underlined.

The second necessary thing is S-shaped value curve; and according to the reference point the convex is below the indicator and the concave is mapped to be above it. In sensible terms, usually the established order is the operative reference purpose. The right - hand aspect of the graph is related to the domain of gains; the left - hand aspect is related to the domain of losses. For any given modification, there is a lot of impact nearer to the point of start than farther away from starting point. It can be confirmed by the observation that the distinction in value between the gains of 100 and 200 seems to be larger than the distinction between the gains of 1100 and 1200. Same as, the distinction between the losses of 100 and 200 seems larger than the distinction between the losses of 1100 and 1200, except if the bigger loss is intolerable. Thus, we have a tendency to anticipate that the value function for changes of wealth is that concave is greater than the reference point ($v''(x) < 0, for\ x > 0$), and sometimes the reference point is greater than the concave

$(v''(x) > 0, for\ x < 0)$[6]. It is to conclude that the marginal value of each gain and loss decreases with its magnitude.

The third side of the function of value is an uneven nature of the curve of value; its steeper within the loss domain than in gains domain. It is possible to say that losing causes more hurts than gain pleases. As an example, losing $10 hurts over finding $10 gratifies. The general public notice symmetrical bets of the shape $(x, 0.5; -x, 0.5)$ clearly unattractive. Moreover, the prevarication of symmetrical truthful bets usually will increase with the scale of the stake.

That is, if $x > y \geq 0$, then $(y, 0.5; -y, 0.5)$ is preferred to $(x, 0.5; -x, 0.5)$. According to equation (2), we have

$v(y) + v(-y) > v(x) + v(-x)$, and $v(-y) - v(-x) > v(x) - v(y)$.

Setting $y = 0$ yields $v(x) < -v(-x)$, and if y approaches x, then $v'(x) < v'(-x)$, provided v', which is a derivative of v, exists. So, the function of value for losses is sheerer than the function value used for gains [6].

3.7 Weighting Function

The probability (likelihood) - weighting function is used to measure the impact of the probability of the evidence on a prospect desirability. The weights of decision are not considered as probabilities.

Factors other than probabilities affect the decision weights, and it includes "ambiguity" about the level of uncertainty or risk [6], [39]-[40]. A typical probability - weighting function, evoked from experimental proof, is illustrated in figure 2 [41].

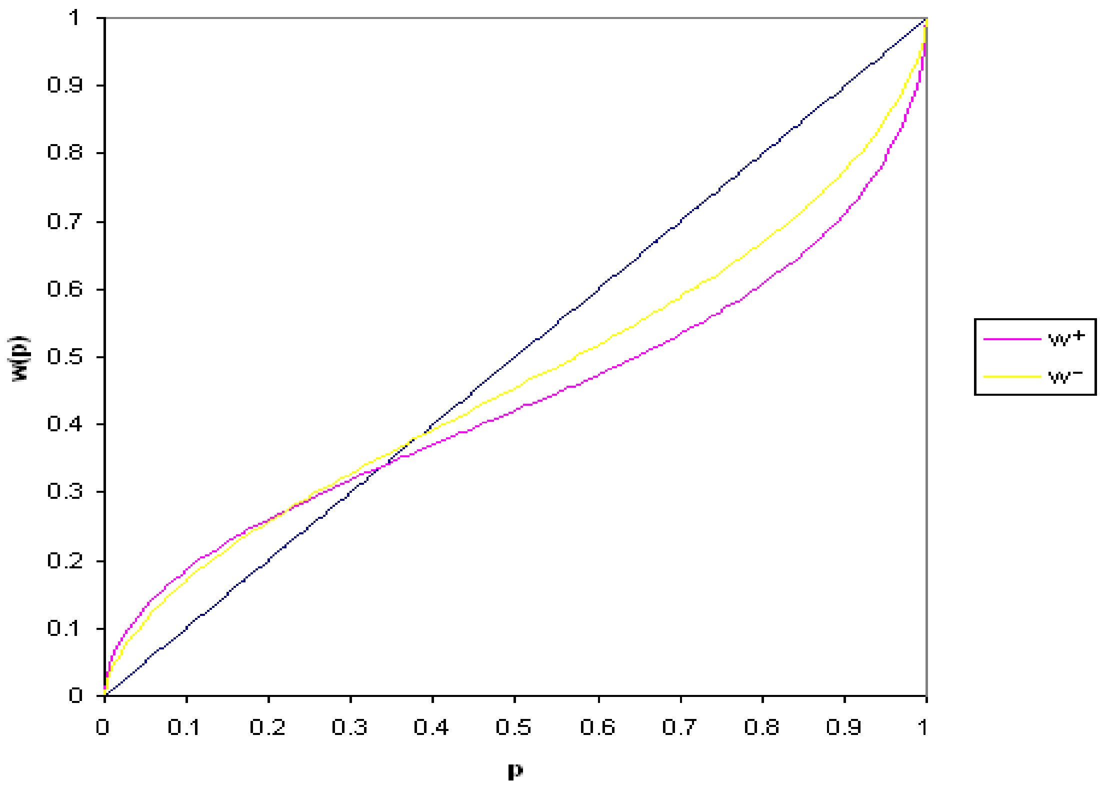

Figure 2: Weighting functions for gains (w+) and losses (w-)

The weighting function has many characteristics. First, the weight function is not well-behaved close to its endpoints. The unpredictability of behavior conditions of very tiny or very giant probability is used. In alternative words, the variance within the weighting function of probability is a giant within the region close to 0 or 1.

Kahneman and Tversky (1979) acknowledged this unpredictability, and argued that because individuals are restricted in their ability to grasp and judge extreme possibilities, extremely unlikely events are either unnoticed or overweighted.

The fact that the function of weighting isn't well-behaved close to its endpoints, and that is by definition $w\ (0) = 0$ and $w\ (1) = 1$: there is a point (through somewhat indeterminate) increase within the function of weighting in these regions.

The changes in possibilities close to 0 or 1 have disproportionately massive effects on the prospects evaluation. A third characteristic of the function of weight is that its slope is a smaller amount than one across its entire varies, apart from the tiny region close to its endpoints.

Fourth, aside from the indefiniteness of behavior for terribly little possibilities (probabilities), little possibilities are overweight whereas larger are underweight. It is not fully proven that the particular point at those overweighting shifts to underweighting, or whether or not this particular point varies considerably across individual, and as it seems from [42], this point is in the range 0.1-0.15.

The fifth characteristic of the weighting function is: for all $0 < p < 1$, $w\ (p) + w\ (1 - p) < 1$ [6]. It means that the decision weights do not sum to one for decisions between two choices.

Chapter 4

EXPECTED UTILITY FUNCTION

4.1 Expected Utility Theory

Expected utility theory (EUT) was initially projected by Bernoulli in 1738 [43], and developed by John Von Neumann and Oskar Morgenstern in 1944 [44]. EUT has been used as a regard to find the best solution in several areas of social science [44]. This was a tool aimed to assist making choices among variety of alternatives. This is also a theory about how to balance risk versus reward employing a formal, mathematical relation. When one experiences different options and alternatives, expected utility theory recommends that one simply calculates the expected utility of every selection and so opt for the one with highest expected.

4.2 Utility

Utility is somebody's preference among various alternatives. From these preferences "if they are rational!" we will deduce a utility function that represents preferences by order relations between numbers. This solely works if somebody's preferences are during a sure sense, rational. For any two baskets,

$X \text{ and } Y, U(X) > U(Y),$ iff this person prefers (strictly) X to Y

$U(X) = U(Y),$ iff this person is indifferent between X and Y

$U(X) < U(Y)$, iff this person prefers (strictly) Y to X.

U(.) is a function that assigns number to things (represented as variables (A,B,C,…,X,Y,Z) [45].

If someone prefers X to Y, and he/she prefers Z to X, we say that he/she prefers Z to Y. We additionally need that, for any two things, an individual prefers one to the other, or is indifferent between these things.

The numbers appointed by $u(.)$ should match how much is preferably one of the things to the other. If somebody assigns $u(X\ wins) = 100$ and $u(Y\ wins) = 1$, then it is favor to see that $X\ wins$ is a hundred times more than $Y\ wins$.

Suppose we wish to form a utility function for a friend of the “Planet of the mice” movies. There have been 5 movies within the series: 1 - Planet of the mice, 2 - Beneath the earth of the mice, 3 - Escape From the earth of the mice, 4 - Conquest of the earth of the mice, and 5 - Battle for the earth of the mice.

Our fan likes “Planet” the most effective out of the 5, prefers “Planet” to “Escape”, is indifferent between “Escape” and “Conquest”, prefers “Escape” to “Beneath”, and prefers “Beneath” to “Battle”. Here could be a utility perform that would represent the fan's preferences:

$u(Planet) = 15$

$u(Escape) = u(Conquest) = 10$

$u(Beneath) = 8$

$u(Battle) = 2$

The utility means in terms of what quantity one should get hold of every of those things or what quantity this stuff is valuable to you.

4.3 Expected Utility Calculation

Appealing an exact selection depends not solely on the payoffs of that selection, but how possible these payoffs are. A big payoff of a lottery is actually appealing, however it is therefore unlikely that purchasing a lottery price ticket is just about a waste of cash. One thinks of act like shopping for a lottery ticket as having variety of winning or losing outcomes. Given utility function and its degrees of belief in each of the possible outcomes, one will calculate the expected utility of any act. This can be done as following.

Let’s assume that the act is tagged $X.$ Let $o_1, o_2, ...$ be the varied double outcomes of X (there has to be a minimum of one outcome, however there may will be many). To every outcome o_i there is an associated likelihood $Pr(o_i)$ that measures how seemingly that outcome is, and a utility $u(o_i)$ that measures the spot of that outcome during the preference relation is chosen by this person.

Calculation of the expected utility of X is done as following [45]:

$$E(X) = \mathrm{u}(\mathrm{o}_1) \cdot \mathrm{Pr}(\mathrm{o}_1) + \mathrm{u}(\mathrm{o}_2) \cdot \mathrm{Pr}(\mathrm{o}_2) + \cdots + \mathrm{u}(\mathrm{o}_n) \cdot \mathrm{Pr}(\mathrm{o}_n)$$

When there are many choices among multiple actions $x_1, x_2, \ldots x_n$, the expected utility theory is used to choose acting in terms of the expected higher interest. This is done by calculating $E(x_1), E(x_2), \cdots, E(x_n)$, and then by choosing the act that has the highest expected benefit associated with it.

Let's consider another example. Suppose you have three options to spend tonight. There is a birthday celebration in the apartment complex you can attend, or there is a football game of your favorite team you can go to, or stay at home and choose one of the best available DVDs, and watch it. If you stay home and watch the DVD, you recognize evidently you may receive a utility of sixty. You assign a utility of two hundred and fifty to the win of your favorite team; however you furthermore might recognize that they solely have a thirty likelihood of winning tonight. A loss of a favorite team has a utility of fifteen. If you attend the birthday celebration, the likelihood is ninety that it will be lame, and also the utility of a lame birthday celebration for you is barely thirty. There is 8% likelihood that the birthday celebration can have good music, an outcome that you just assign utility sixty to. Finally, there is 2% likelihood that you simply can indiscriminately meet the person of your dreams at the birthday celebration, which might yield utility of two thousand.

What must you do? The calculation of the expected utility for each of the three acts should be made:

$$E(watch\ DVD) = u(watch\ DVD) \cdot Pr(watch\ DVD) = 60 \times 1.00 = 60$$

$$E(Favorite\ team\ game)$$
$$= u(Favorite\ team\ wins) \cdot Pr(Favorite\ team\ wins)$$
$$+ u(Favorite\ team\ loses) \cdot Pr(Favorite\ team\ loses)$$

$$E(Favorite\ team\ game) = 250 \times 0.30 + 15 \times 0.70 = 85.5$$

$$E(Birthday\ celebration)$$
$$= \mathrm{u(lame)\ Pr(lame) + u(good\ music)\ Pr(good\ music)}$$
$$+ \mathrm{u(dreams)Pr\ (dreams)}$$

$$E(Birthday\ celebration) = 0.90 \times 30 + 0.08 \times 60 + 0.02 \times 2000 = 71.8$$

The best alternative is the second one, and so the decision is to go to the favorite team's game.

4.4 Ordinal Utility

This is one of the main kinds of measure of utility implemented by economists. An ordinal utility could be a rule that assigns variety (known as utility) to every existent consumption plan, wherever the upper the number assigned, the more preferred the arrange is [46]. The only property of a utility assignment is its vitality to order the bundles of products. Because of this stress on ordering bundles of goods, this sort of utility is referred to as ordinal utility. In the table 2 the preference between different bundles ($X, Y,$ and Z) is shown.

Table 2: The preference between different bundles (X, Y, Z)

	U1	U2	U3	U4
Bundle X	5	16	4	-2
Bundle Y	3	7	3	-3
Bundle Z	2	1	1	-4
	X=5+16+4-2=23, Y=3+7+3-3=10, Z=2+1+1-4=0 X is preferred to Y, and Y is preferred to Z.			

Ordinal utility holds that utility can't be measured but is often ordered in keeping with consumer's preferences between pairs of alternative bundles. The different pairs of alternative bundles (combinations) of goods show the same utility and these bundles of the same utility contain the indifferent curve.

If the utility function of the bundle x is 15, and the utility function of the bundle y is 3, then it is to say that the preference of the bundle x is strictly more than bundle y, but it does not mean that the bundle x is exactly preferred "five times more" than the bundle y.

Let's consider the utility function for two goods $U(x, y) = xy$, and suppose there are two utility levels 8 and 12. Then the corresponding reference points to the first utility level are (1,8), (2,4), (4,2), (8,1), and (1,12), (2,6), (6,2), and (12,1). The indifference curves for the above bundles are represented in Figure 3.

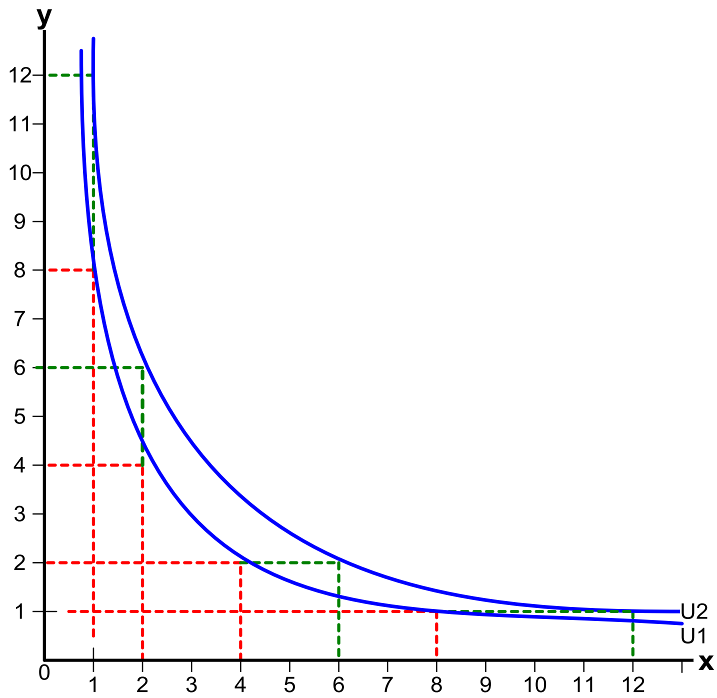

Figure 3: Indifference curves

Indifference curve is a locus of points representing completely different bundles of goods, and all these bundles of goods yield the same level of total utility [47] as shown in figure 4.

Suppose there are two sorts of cakes: pineapple cake, and strawberry cake, and the consumer may choose three items of them. As it can be seen from the figure 4, the bundles (1,2) and (2,1) belong to the same indifference curve; it means that the consumer may be indifferent in choosing of two pineapple cakes and one strawberry cake, or one pineapple cake and two strawberry cakes.

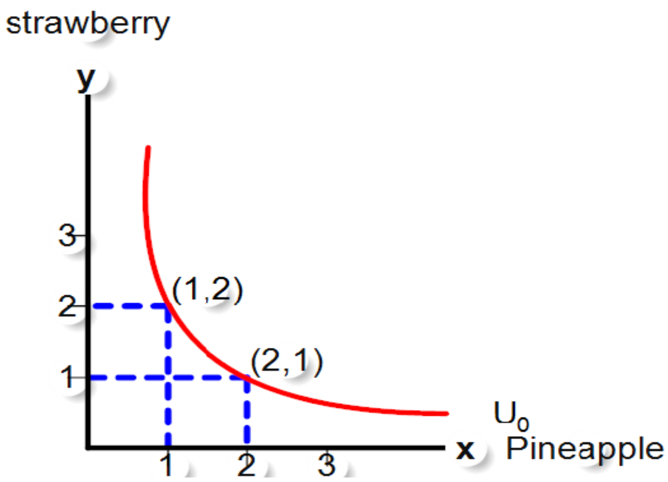

Figure 4: Representation of bundles belonging to the same indifference curve

If a consumer is indifferent in choosing one of the given two bundles, one of the effective approaches to decide is the mixing of these goods by calculating the average value of them, and the average bundle is more preferred than both original bundles. For example, the bundle (X,Y) with two goods X and Y, where X represents banana, and Y represents apple, and a consumer is indifferent in choosing one of the bundles (5,7) and (3,9), then the average of these bundles is ((5+3)/2, (7+9)/2)) = (4,8), then it can be claimed that the bundle (4,8) > (5,7), and (4,8) > (3,9).

The preferences between some bundles can be also transitive. In other words, if there are three bundles compared, and if the first bundle is preferred to the second bundle and the second bundle is preferred to the third bundle, then the first bundle is preferred to the third bundle.

4.5 Uncertainty and Expected Value

Suppose there are measures of two states of nature (good day, dangerous day), and a person's wealth W, depends on these states. The chance of every state is given by

p_i $(i = 1,2)$ and also the wealth in each state is w_i $(i = 1,2)$. Together, the two chances and also the two values of wealth are noted as a Risky Prospect.

The expected value (EV) of a risky prospect is described by the following formula [48]:

$$EV = p_1 w_1 + p_2 w_2$$

Since $p_1 + p_2 = 1$, we see $p_1 = (1 - p_2)$. We can substitute to eliminate p_1:

$$EV = (1 - p_2)w_1 + p_2 w_2$$

$$EV = (1 - p)w_1 + p w_2.$$

Let's consider an example to understand it.

Assume that you have a bingo cage containing 100 balls, and 70 of them are red balls, and 30 are yellow balls. If you pull a red ball, you will gain $4, but if you pull the yellow ball, you will loss $1. What's the probability of every event? What is the expected value?

$$Events = (yellow\ and\ red)$$

$$P(yellow) = 30/100 = 0.3$$

$P(red) = 70/100 = 0.7$

$EV = (0.7 \times \$4) + ((0.3 \times \ \$(-1)) = \$2.5$

Now suppose that tier of wealth offers someone a particular level of utility such $U = U(W)$, and the utility function is $U = \sqrt{w}$ (risk averse). Expected utility (EU) is given by [48]:

$EU = (1 - p)U(w_1) + (p)U(w_2)$

It's the same way to find expected value.

Suppose we have to select between the subsequent two gambles given in the table 3:

Table 3: Selection between two gambles X and Y

Gamble X	Win $200	With probability = 1
Gamble Y	Win $320 Win $100	0.5 0.5

Let's show that in order to maximize expected value, the Gamble Y should be chosen, and in order to maximize expected utility with $U(w) = \sqrt{w}$, we should choose Gamble X.

$EV_{gamble\ X} = \$200$, $EU_{gamble\ X} = \sqrt{200} = \$14.14$

$$EV_{Gamble\ Y} = 0.5(\$320) + 0.5(\$100) = \$210$$

$$EV_{Gamble\ Y} > EV_{gamble\ X}$$

$$EU_{gamble\ y} = 0.5(\sqrt{320}) + 0.5(\sqrt{100}) = 0.5(\$17.88) + 0.5(\$10)$$

$$= \$13.94$$

$$EU_{gamble\ x} > EU_{gamble\ y}.$$

It is concluded that there is no need to take risk, and this situation is called risk aversion, and graphically is represented in figure 5.

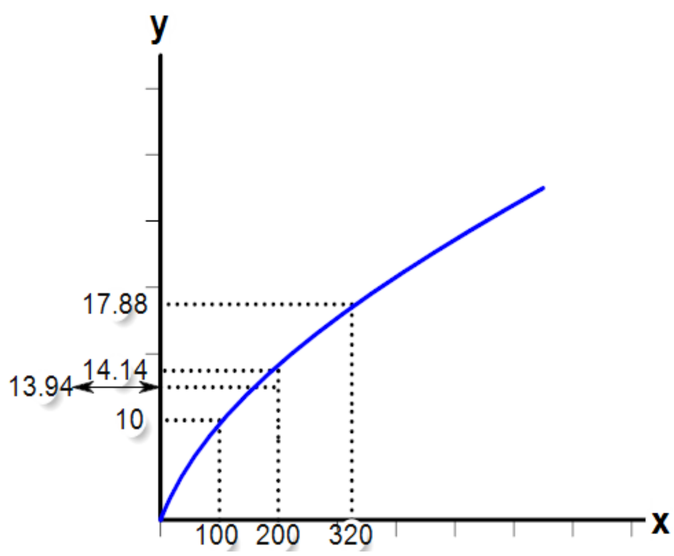

Figure 5: Graphical representation of risk-aversion

4.6 Attitudes toward Risk (Risk-Averse, Risk-Neutral, and Risk-Loving Utility Functions)

In risk aversion the person prefers to accept a deal with certain profits rather than a deal with uncertain profit even if the profit is higher. For instance, a risk-averse person would possibly opt to place his or her cash into a checking account however secure rate, instead of into a stock that will have high expected returns, however additionally involves an opportunity of losing worth.

A person who is a risk neutral is a rational person, and does not have any preference. Every possibility offers equal expected utility. However, not many of people are risk neutral.

In risk-loving, individuals prefer high risk.

In risk-averse $E(U(W)) < U(E(W))$, in risk-neutral $E(U(W)) = U(E(W))$, and in risk-loving $E(U(W)) > U(E(W))$[49]. The graphical representations of expected utilities of wealth for risk-averse, risk-neutral, and risk-loving individuals are shown in figures 6, 7, and 8, respectively [50].

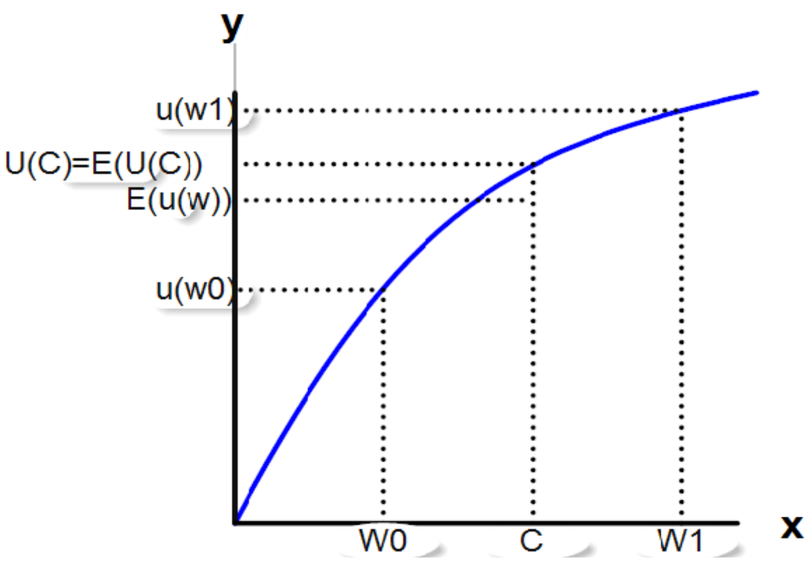

Figure 6: Expected utility of wealth for risk-averse individual

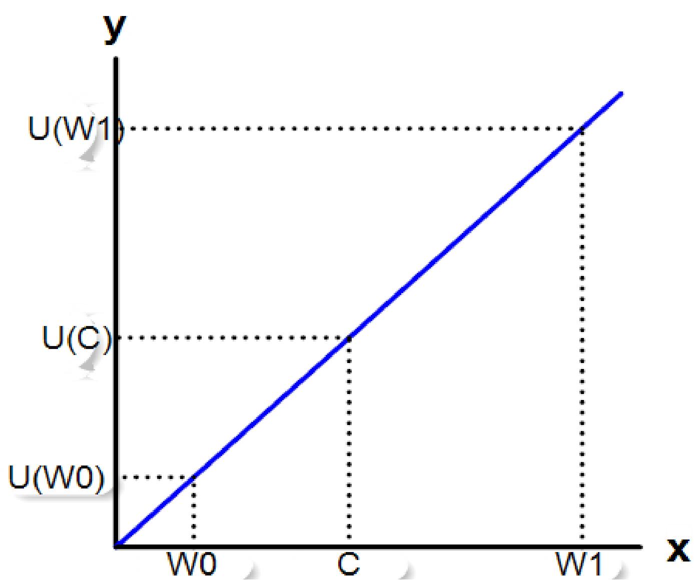

Figure 7: Expected utility of wealth for risk-neutral individual

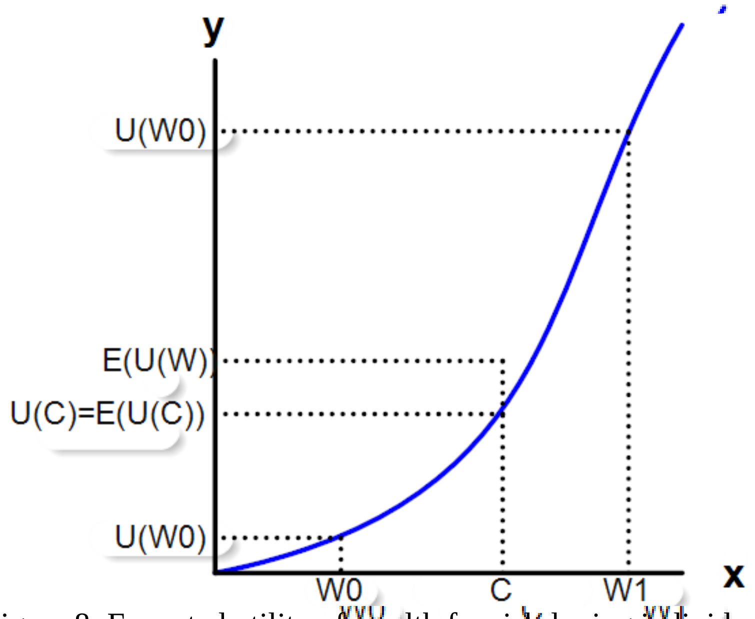

Figure 8: Expected utility of wealth for risk-loving individual

Such that

C - Certain value;

$E(U(W))$ - Expected value of the utility (expected utility) of the uncertain payment;

$U(C)$ - Utility of the certain value;

$U(w_0)$ - Utility of the minimal payment;

$U(w_1)$ - Utility of the maximal payment;

w_0 - Minimal payment;

w_1 - Maximal payment;

$E(U(W)) = U(W)$ (For a certain wealth) [50].

Let us illustrate it in the following example. Suppose the representative wants to choose between two options where one of them is useful guaranteed $50 and the other has the benefit of unsecured, is a reflection of the same coin to decide whether a person receives $100 or receives nothing.

The expected payoff for each contract is $50 meaning that a person who was insensitive to risk would not care whether or not they took the secured payment or the gamble. However, people might have totally different risk attitudes.

Risk-averse utility function: if a person prefers particular payment even if it is less than $50 (for example, $45), instead of taking the gamble and receiving nothing.

Risk-neutral utility function: if a person is indifferent between the bet and a particular $50 payment. Every possibility offers equal expected utility. It should be mentioned that not many people are perfectly unbiased.

Risk-loving utility function: if a person prefers the bet even when the guaranteed payment is over $50 (for example, $55).

Suppose there is someone who has a wealth of $400. He/she plays gambling by flipping the coin. If coin comes up head, he/she will win $500 with the possibility of 50%, and if coin comes up tail, he/she will loss (-$400).

Suppose a person has different utility functions:

$U1 = \sqrt{W}$, $U2 = W - 380$, $U3 = W^2/8000$

$W = 400$

In case without gamble:

$U1(W) = \sqrt{400} = 20$

$U2(W) = 400 - 380 = 20$

$U3(W) = (400)^2/8000 = 20.$

$E\big(U(W)\big) = U(W)(for\ certain)$

Second case with gamble:

Their expected value is:

$EV = p(win).v(win) + p(loss).v(loss)$

$= 0.5\ (500 + 400) + 0.5\ (400 - 400) = 450.$

$U(E(W))$ is:

$(\sqrt{450}) = 21.213$

$(450 - 380) = 70$

$(450)^2/8000 = 25.31$

Their expected utility is:

$U1 = \sqrt{W}$

$EU = \ p(win).u(win) + \ p(loss).u(loss)$

$\quad = 0.5(\sqrt{900}) + 0.5(\sqrt{0}) \ = 15$

$E\big(U(w)\big) < U\big(E(w)\big), 15 < 21.213\ (risk - averse)$ (Figure 9).

$U2 = W - 380$

$EU = p(win).U(win) + \ p(loss).(loss)$

$\quad = 0.5(400 + 500 - 380) + 0.5(400 - 400 - 380)$

$\quad = 0.5(900 - 380) + 0.5(-380)$

$\quad = 260 - 190 = 70 = 70$

$E(U(w)) = U(E(W))$ $(risk - neutral)$ (Figure 10).

$U3 = W^2/8000$

$EU = p(win).U(win) + \ p(loss).U(loss)$

$= 0.5((900)^2/8000) + 0.5((0)^2/8000))$

$= 0.5(101.25) + 0$

$= 50.62 \ > \ 25.31$ $(risk - loving)$ (Figure11).

For risk-averse case in Figure 9, $U(E(W)), U(450\$)$ is larger than $E\big(U(W)\big) = 0.5U(900) + 0.5U(0)$. For risk-neutral case in Figure 10, the $U(E(W)), U(450\$)$ is equal to $\big(U(W)\big) = \ 0.5U(900) + 0.5U(0)$. For risk-loving case in Figure 11, the $U(E(W)), U(450\$)$ is less than $E\big(U(W)\big) = 0.5U(900) + 0.5U(0)$.

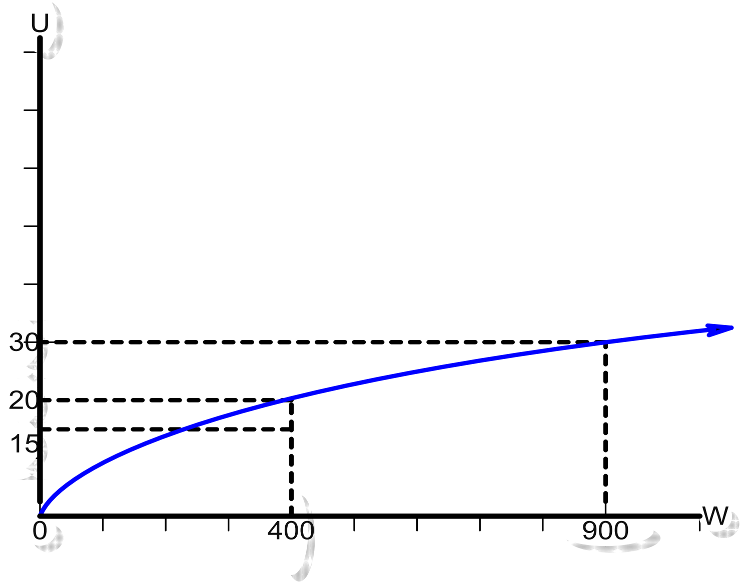

Figure 9: The utility function and risk-averse

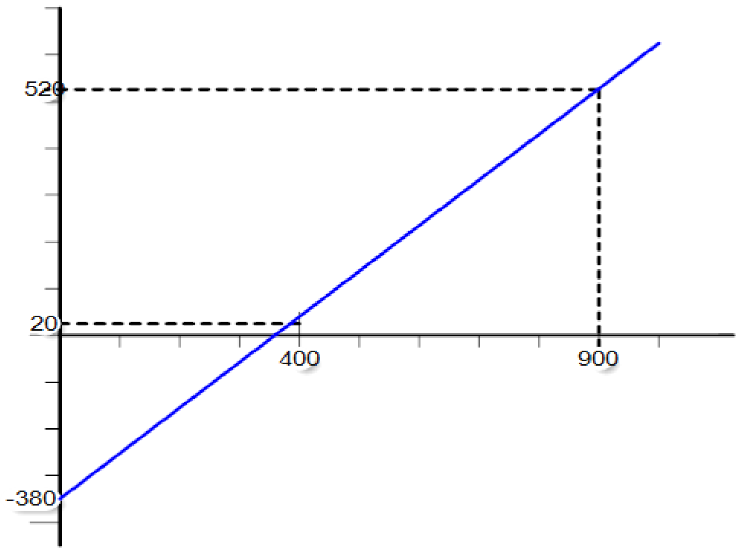

Figure 10: The utility function and risk-neutral

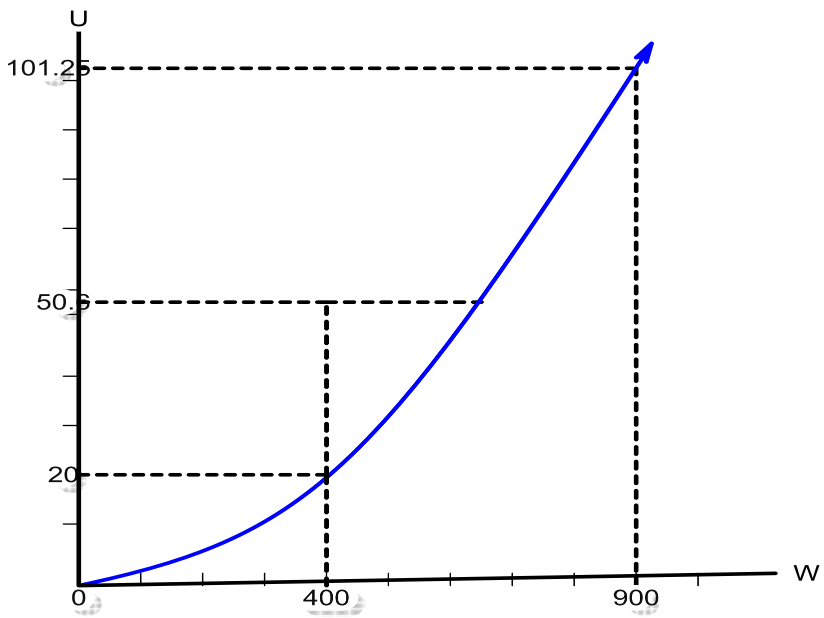

Figure 11: The utility function and risk-loving

In square root utility function the first derivative should be more than zero and the second derivative should be less than 0 to claim that a utility function used is a legitimate utility function [51]:

$$\mathrm{U(W)} = \sqrt{W} = W^{0.5}$$

Note that:

$$U' = 0.5W^{-o.5} > 0$$

$$U''(W) = -0.25W^{-1.5} < O$$

So the square root utility function is a legitimate utility function.

In risk-averse, risk-neutral, and risk-loving cases a person will differently decide about the acceptance of a fair gamble. A fair gamble means that if an initial wealth to be bet is equal to the expected return from the gamble. For example, if an initial wealth is $50, and there is 60% chance to win $30 and 40% chance to win $80, then expected utility becomes $EU = 0.6 * 30 + 0.4 * 80 = 50$ which is as much as an initial wealth.

It can be concluded that a person rejects a gamble in case expected utility is less than an initial wealth to be bet (risk-aversion), a person is indifferent in decision whether to accept or reject gamble if expected utility is equal to the initial wealth to be bet (risk-neutral), and a person accepts a gamble if expected utility is more than initial wealth to be bet (risk-loving).

How much loss is expected if the risk-averse person avoids taking risk? In other words, what should be a loss if a person gives up risky income instead of riskless income? In order to calculate the risk premium, the following steps should be followed for the problem given below:

Suppose a person has two options:

1) It is possible to have an income of $16 with probability 0.5, and income of $36 with probability 0.5;

2) It is possible to have an income of $25 with certain.

Suppose the utility function of the person to be used for the income is $u(I) = \sqrt{I}$.

The expected payoff of the first option is

$$E(I) = 0.5 * 16 + 0.5 * 36 = 26$$

The expected payoff of the second option is

$$E(I) = 25$$

The expected utility of the first option is

$$E\big(u(I)\big) = 0.5 * u(16) + 0.5 * u(36) = 0.5 * \sqrt{16} + 0.5 * \sqrt{36} = 0.5 * 4 + 0.5 * 6$$
$$= 5$$

The expected utility of the second option is:

$$E\big(u(I)\big) = u(25) = \sqrt{25} = 5$$

The expected payoff of the first option is more than the expected payoff of the second option ($26>$25), and a risk-premium is $26 - $25 = $1, but we can also see that a person is indifferent between choosing the options 1 and 2, since their expected utilities are same.

Chapter 5

CONCLUSION

Decision making process under risk has been actual for many years. While evaluating gains and losses a dominated idea was a choice according to maximization of expected utility, but the alternative theory called prospect theory proposed by Kahneman and Tversky has shown that people are more emotianally sensitive to losses compare to gains if even they have equivalent final economic outcomes, i.e. rational decision to choose a best option among some risky or uncertain prospects is not up to the maximum utility value, but up to human behaviour. In other words, there is asymmetry between gains and losses to be evaluated in case of dealing with risky situations.

This thesis considers the prospect theory for decision making process in which people make a reasonable decision among some gambles. The theoretical aspects and basic concepts of prospect theory are presented to define its best features such as editing and evaluation phases, value and weighting functions.

In comparison with the prospect theory, expected utility theory is used to calculate the expected value of the utility of the preferences. In expected utility theory the utility and probability of each outcome are known, and the maximization of the expected utility is leading to the determination of best solution between risky options.

This thesis investigates the expected utility theory and computations of expected utility. The ordinal utility is considered to analyze the indifference curves.

A utility function is used to prefer outcomes with higher utilities to outcomes with lower probabilities. The risk-averse, risk-neutral, and risk-loving utility functions are also presented in this thesis.

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