Optimization of Selected 2-Dimensional Steel Truss Shapes Using a New Mathematical Formulation

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ABSTRACT

The purpose of this study is to carry out an investigation on the existing truss systems in order to introduce a mathematical formulation relating to the geometrical shape of the truss so that the mid-span deflection of the truss can be optimized. Every time there is a need to use a truss structure it is difficult to decide which truss type, bay width and height would produce the optimum truss shape with minimum mid-span deflections and its corresponding minimum bottom chord stress for a specific span length. The results of this study is expected to produce a set of guidelines to help researchers, designers and practicing engineers to determine the most appropriate and efficient truss system for their specific usages. In order to achieve this aim, a total of two sets of two-dimensional trusses with eleven different shapes of common symmetry, made of steel with pinned and roller supports were studied to identify which truss shapes and sizes are efficient for the purpose of this study. The design loads are applied to the joints so that there is no moment to be resisted by the members. Initially, the virtual work method was applied on selected truss shapes in order to obtain the amount of deflection at mid-span of the trusses. Afterwards, hand calculation was carried out followed by computer analysis using MAPLE 12 and then the use of TABLE CURVE 2D v5.01 for mathematical approach to derive the deflection formula. Finally, STAAD Pro was used to analyze and design the truss structures. The analysis of all sets of trusses enabled the comparison among the various spans, height and bay width of trusses. Thus, the changes in mathematical deflection formula, due to the number of bays and shapes of trusses were lead to obtain the specific optimum height and minimum deflection for each truss system. In

other words, the occurrence of minimum deflection along the truss span and optimum height presents the optimum truss.

Besides the above mentioned outcomes a significant advantage was achieved due to mathematical formulation. The formula derived demonstrates an easy, fast and accurate way of calculating the deflection value at mid-span of trusses. Currently, virtual work method is the most efficient and accurate way of determining the deflection. Although this method is the most common one it is dependent on long and complicated procedure. The formula derived in this study has introduced a new approach to determine the mid-span deflection of trusses in an extremely short and easy way.

ÖZ

Bu araştırmanın amacı mevcut makas sistemlerini inceleyerek geometrik şekillerine ilişkin bir matematiksel formül üretmek ve böylece makas orta noktasında oluşacak sehimi olabilecek en az seviyeye çekmektir. Makas kullanımına ihtiyaç duyulan her durumda, hangi makas şekli, dikey eleman açıklıkları, yükseklikleri ve makas uzunluğunun kullanımı ile optimum makas şekli ve buna bağlı olarak makas orta noktasında en az sehim ve makas altı gerilme elemanlarında en az çekme basıncının oluşacağına karar vermek çok zordur. Bu çalışmanın sonuçlarının, araştırmacı, tasarım yapan ve pratikte çalışan mühendislerin, kullanım ihtiyaçları doğrultusunda en uygun ve etkin makas sistemini bulmaları için yol gösterici olması beklenmektedir.

Bu amaca ulaşmak için çelikten yapılmış, destekleri basit ve yatay yönde hareketli, iki gurup, iki düzlemli ve ortak sistemi olan 11 adet makas seçilmiş ve hangilerinin bu amaca uygun oldukları incelenmiştir. Tasarım için kullanılan yükler taşıyıcı elemanlarda herhangi bir momente neden olmaması için yatay ve dikey elemanların birleştiği düğüm noktalarına yüklenmiştir. Önce sanal çalışma yöntemi kullanılarak seçilmiş makaslara yükleme yapılmış ve makas uzunluğunun orta noktasında oluşan sehimler elde edilmiştir. Bunu takiben, önce elde ve sonrasında bigisayar kullanarak (MAPLE 12 ve TABLE CURVE 2D v5.01) matematiksel bir sehim formülü üretmek için analizler yapılmıştır. En sonunda STAAD Pro kullanılarak makas yapıları analiz ve tasarımı yapılmıştır. Tüm makasların analiz sonuçları kendi içerisinde açıklıkları, yükseklikleri ve de dikey eleman açıklıkları açısından karşılaştırılmasına olanak sağlamıştır.

Böylece, sehim formülünde makas şekli ve dikey aralıklardan dolayı oluşan değişimler her makas yapısı için optimum yükseklik ve en az sehimin elde edilmesine yardımcı olmuştur. Diğer bir değişle, makas boyunda elde edilen en az sehim ve en az yükseklik o makasın optimum bir makas şekli olduğunu göstermektedir.

Matematiksel formülasyon yukarıda belirtilen sonuçlara ilaveten önemli bir avantaj elde edilmesine neden olmuştur. Oluşturulan yeni formül kullanılarak makas orta noktalarında kolay, hızlı ve doğru bir şekilde sehim hesaplaması yapılabilmektedir. Şu anda, sanal çalışma yöntemi sehim hesaplamaları için en doğru ve etkin sonuç veren yaklaşımdır. Bu yöntem hernekadar da yaygın bir şekilde kullanılıyor olsa da uzun ve karmaşık prosedür gerektiren bir yaklaşımdır. Bu araştırma sonucunda elde edilen formül, makas orta noktalarında oluşan sehimi çok kısa sürede ve kolayca hesaplayabilecek yeni bir yaklaşımdır.

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LIST OF SYMBOLES

Δ= joint displacement caused by the real loads on the truss

N= internal force in a truss member caused by the real loads

n= internal virtual force in a truss member caused by the external virtual unit load

w= multiple loads on the structure that are applied at joints

L= length of the member

A= cross-sectional area of member

E= modulus of elasticity of a member

k= number of bays in half length of the truss

a= distance between the joints of truss members (bay width)

b= height of the truss

$\sum F_x$= total force at direction – x

$\sum F_y$= total force at direction – y

$\sum M$= total moment

DL = dead load

LL = live load

b = unknown bar force

r = reaction

D = degree of indeterminacy

σ = stress

Chapter 1

INTRODUCTION

1.1 Introduction

"A truss is a structural element composed of a stable arrangement of slender interconnected bars. The pattern of bars, which often subdivides the truss into triangular areas, is selected to produce an efficient, lightweight, load-bearing member" [1]. Since the members are connected at joints by frictionless pins, no moment can be transferred through this joints. Truss members are assumed to carry only axial forces, either tension or compression. Because of the fact that stress is produced through the length of truss members, they carry load efficiently and often have relatively small cross section [1].

Basically, in truss design, compressive and tensile forces act seperately on each member, causing less consumption of material and increase in the economic revenue. In fact, the structural beheviour of many trusses is similar to that of a beam. The chords of a truss correspond to the flanges of beam. The forces that developed in these members make up the internal couple that carries the moment produced by the applied loads. The webs give stability to the truss system. Therefore, they transfer vertical force (shear) to the supports at the ends of the truss [1].

Before steel became an economically useful material, trusses were made of wood or iron. Nowadays, most of the trusses are made of wood, steel, wood and steel, or aluminium. Concrete trusses are not common but do exist, usually in very large structure.

The members of the most modern trusses are arrenged in triangular patterns because even when the joints are pinned, the triangular form is geometrically stable and will not collaps under load. In contrast, a pin-connected rectangular element, which acts like an unstable connection, will collapse under load. On the grounds that the triangel configuration gives them high strength- to-weight ratios, which permit longer spans than conventional framing, and offers greater flexibility in floor plan layouts. Long spans without intermediate supports create large open spaces that architects and designers can use with complete freedom [1].

The design of truss structures has to be carried out according to the two important requirements; the best geometrical layout and the most adequate cross-sections. In general, the structural shape depends on the design standards and partially on economical, aesthetical, construction techniques, application and environmental aspects. Moreover, for any truss design there must be an optimum shape and a cross-section that is adapted for external loads [2].

In the past decades, the subject of optimization has made important progress in most of the scientific fields. In recent years, the development in computational abilities made an impressive improvement in design optimization schemes for majority of the engineering issues, including those issues relating to structural engineering. The development of structural optimization algorithms has obtained an adequate horizon for engineers to find the most suitable structural shape for a particular loading system.

1.2 Objectives of Research

This research is aimed to carry out an investigation on the existing truss systems in order to introduce a mathematical optimization approach. This approach is expected to produce a set of guidelines to help researchers and practicing engineers in order to select a suitable truss system for their specific usages. Therefore, objective of the optimization was to minimize deflection at mid-span with constraints; loading, spans and truss chord member spacing. Finally, the research has fulfilled the following objectives:

- To identify the efficient truss shapes in terms of deflection among the eleven selected geometry of truss shapes in proportion of height and distance between joints (Bay) using mathematical formulation.
- To develop the general deflection formulas using existing virtual work method to demonstrate an easy, fast and accurate way of calculating the deflection value.
- To compare the deflection of the selected and optimized truss shape with the same ones in the construction industry.
- To determine which optimum shape of truss can be applied to a given span, under height and bay circumstances.

1.3 Outline of Research

Chapter 2 provides a discussion of characteristic of truss systems, structural optimization and background with regard to behavioural construction, mathematical formulation for optimal and effective solution procedures to introduce the different techniques to obtain the optimal truss structure.

In chapter 3, the eleven shapes of common symmetry trusses in 2-D position are categorized to find the most appropriate type for the design of the selected truss structure. Therefore, a mathematical assessment carried out to introduce a common formula to guide engineers, designers and decision makers in choosing the most optimal deflected truss type for given spans.

Chapter 4 reveals the methods in detail based on the objective of the research. Initially, the virtual work method (force method) applied on selected truss structure in order to obtain the amount of deflection. Then hand calculation was carried out followed by a computer application analysis using MAPLE and then the use of Table Curve 2D for mathematical approach to create the deflection formula. Finally STAAD Pro computer software was used to analyze and design the truss structure. As a result the deflection formula was derived and applied to the selected models in order to determine the optimum truss. Therefore, the output of simulated models for different span lengths and bays would demonstrate the least deflection and minimum stress simultaneously.

Consequently, Chapter 5 provides the advantages of introduced method for engineers, designers and decision makers to make the most efficient and accurate truss design.

Chapter 2

LITERATURE REVIEW

2.1 Trusses

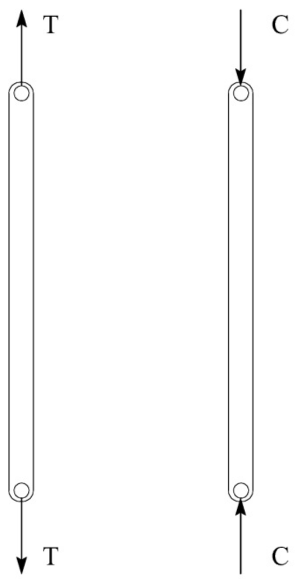
2.1.1 Characteristics of Triangulation, Joints and Member Forces of Trusses

A truss is an assembly of long, slender structural elements that are arranged in a triangle or series of triangles to form a rigid framework. Since a basic triangle of members is a stable form, it follows that any structure made of an assembly of triangulated members is also a rigid and stable structure. This idea is the fundamental principle of the viability and usefulness of trusses in buildings as a light and un-yielding structure. The most usages of trusses are in single story industrial buildings, large span and multi-storey building roofs carrying gravity loads. Also it is used for the walls and horizontal planes of industrial buildings to resist lateral loads and provide lateral stability [1].

The joints of a truss are usually rigid and the members being either welded to each other or welded or bolted to a gusset plate. The behaviour of a braced framed is essentially the same as pin joints. As a result joints could be considered as pinned in any sort of construction mode. In addition, the procedure of analysis is greatly simplified when considering the implementation of joints.

All truss members are acting as a two force member and as a result, the forces at the ends of member must be delivered to the axis of member length. If the force has a tendency to elongate the member, then it is a tensile force (T), Fig 1a; otherwise it is a compressive force (C) and would try to shorten the member, Fig 1b. In terms of truss design, it is important to state the nature of the force at first (tensile or compressive).

> Often, compression members must be heavier and/or stronger than tension members because of the buckling or column effect that occurs when a member is in compression. [3]



(a) Tension (b) Compression

Figure 1: Truss Force Members

2.1.2 Determinacy and Stability of Trusses

Before deciding on the determinacy or indeterminacy of a structure the stability of structural system should be assessed. “Stability is the ability of a component or structure to remain stationary or in a steady state” [4]. Therefore, stability is an inherent quality generally having to do with the nature of arrangement of members and joints or with the support conditions.

> Determinacy is the ability to compute support reactions using statics. That is, if a structure is determinate, the equations of equilibrium are sufficient to find all the forces. If it is indeterminate, there are too many reactions to solve for. This is the classic problem of having more unknowns than

independent equations to solve for the unknowns. If there are too few reactions, then the structure is unstable. [4]

A large percentages of the trusses used in buildings have regular forms with limited number of ordinary situations. The basic device of trussing that may be used in order to produce a range of possible structures is triangulation framework. When truss forms are complex or unusual, a basic determination that must be made early in the design phase is the condition of the particular truss configuration with regard to its stability and determinacy.

In general, all of the joints and members of a truss are in equilibrium if the loaded truss is in the equilibrium. If the load is only applicable in the joints and all truss members are supposed to bear only axial load, then the forces acting on free-body diagram of a joint will constitute a simultaneous force system. In other words, a stable truss system is dependent on equilibrium of the below given equations:

$$\sum F_{x=0} \qquad (1)$$

$$\sum F_{y=0} \qquad (2)$$

There are two equilibrium for each joint in a truss, therefore in order to determine the unknown bar forces *(b)* and reactions ***(r)*** there would be totally *2n* number of equilibrium equations which is given below:

Where n is equal to the total number of joints:

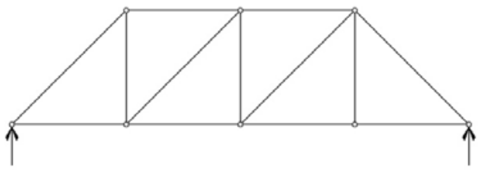
(1) If $b + r = 2nb + r = 2n$ Truss is stable and determinate.

(2) If $b + r > 2nb + r > 2n$ Truss is stable and indeterminate.

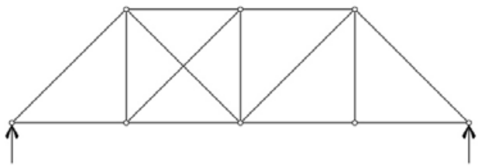
The degree of indeterminacy D equals $D = r + b - 2n$

(3) If $b + r < 2nb + r < 2n$ Truss is unstable.

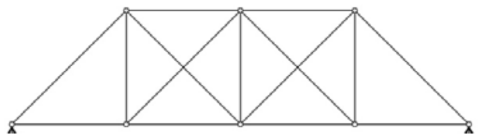
The figure 2 is demonstrated the conditions for stability and determinacy of a truss system to find out stability and instability of that system [1].



Unstable Truss
b + r =15 < 2n =16



Stable Truss (Stabilized by completion of triangulation pattern).
b + r =16 =2n =16



Stable and Indeterminate Truss
b + r =17 > 2n =16
D= r + b – 2n =1

Figure 2: Classifying Trusses

It is noticed that the statical determinacy of a truss structure does not depend on to the applied load system. It only depends on the geometry of the framework.

2.2 Structural Optimization

After four decades the structural optimization is still a new and developing field for research and study. In recent years, the approaches in structural optimization had enough reason to make it a helpful device for designers and engineers. Despite the 40 years of investigation on structural optimization it has not been frequently used as an engineering device for design until high performance computing systems become widely available. Structures are becoming lighter, stronger and cheaper as industry adopts higher forms of optimization. Therefore, the main objective of the current engineering industry should be to find a solution and improvement for the above mentioned issues.

According to the article of *In Structural Optimization,* by N.Olhoff and J.E.Taylor (1983) in their paper entitled *On Structural Optimization*, in optimization of structures, experience has shown that particular attention must be paid to the following five principle points so that an efficient and practical design may be obtioned:

(1) The objective or cost function must be taken as realistic as possible;

(2) The largest possible number of design variables for different types of trusses must be selected;

(3) As much as possible, many realistic design requirements (behavioral constructions) must be considered;

(4) The mathematical formulation must accomodate for unexpected properties of the optimal solution; and

(5) Effective solution procedures are necessery.

2.2.1 Optimization Problem

Optimization problems are categorized according to design variables by considering the type of equations.

In other words a design is optimum if a certain objective function is minimum (or maximum) while it meets its design requirments.

Optimization techniques, which are based on an optimality criteria approach, mathematical programming and genetic algorithms are widely employed (Kuntjoro and Mahmud 2005).

> In the mathematical optimization if the objective function and the constraints involving the design variable are linear then the optimization is termed as linear optimization problem. If even one of them is nonlinear it is classified as the non-linear optimization problem [5].

This research deals with structural optimization based on mathematical programming. The deflection of the structure is to be minimized and it is formulated as the objective function. The design variables are structural parameters, the values that are going to be varied during the optimization process. The design requirments, such as height and width, are formulated as the design constraints. The flow chart of the design optimization which is obtained by using a mathemathical programming is shown in Figure 3.

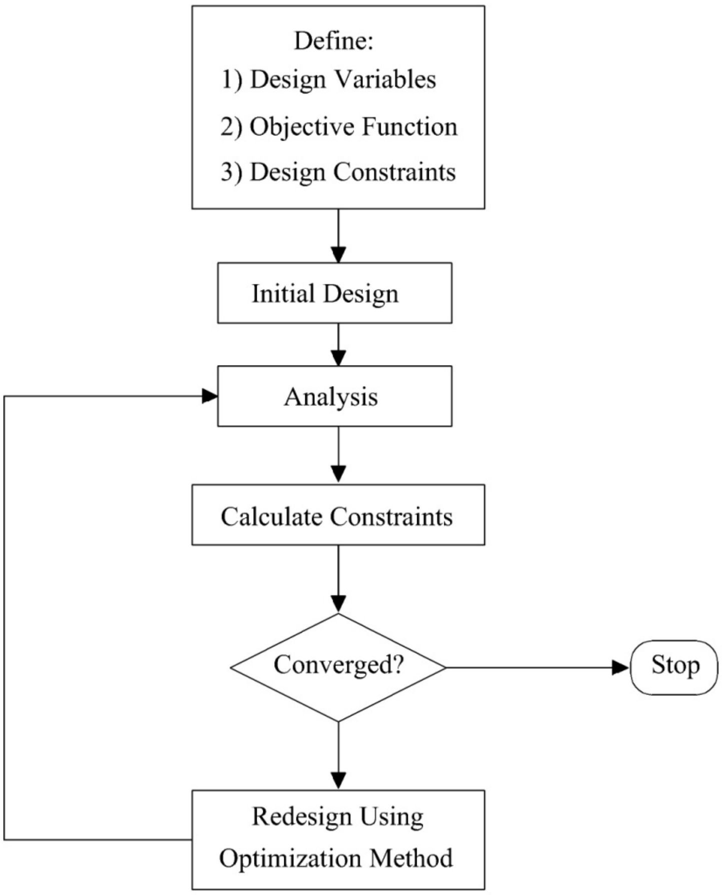


Figure 3: Flow Chart of Optimization Procedure

2.2.2 Structural Optimization Problem Statements

In structural optimization problems for design variables, objective function and constraints are summarized in the following formulation of the optimization problem.

Minimize $f(x)$

Such that $g_i(x) \geq 0 \qquad j = 1, \ldots, n_g$

$$h_k(x) = 0 \qquad k = 1, \ldots, n_e \qquad (3)$$

Where x is denoted as a vector of design variables with components; $x_i, i = 1, \ldots, n$ x_i and $i = 1, \ldots, n$. The equality constraints $h_i(x)$ and the inequality constraints $g_i(x)$ are assumed to be transformed into the form (3). The optimization problem is assumed to be the minimization rather than a maximization problem. Therefore, it is not restrictive since, instead of maximizing a function it is always possible to minimize its negative value. Similarly, if we have an inequality of opposite type, that is

$$g_i(x) \leq 0 \qquad (4)$$

It can be transformed into a greater – than –zero type by multiplying Eq. (4).

An optimization problem is said to be linear when both the objective function and the constraints are linear functions of the design variables x_i, x_i, that is to say they can be expressed in the form of:

$$f(x) = c_1x_1 + c_2x_2 + \ldots c_nx_n = c^Tx. \qquad (5)$$

Linear optimization problems are solved by a branch of mathematical programming called *linear programming*.

The linear programming problem was interpreted as maximizing or minimizing a linear function which is subjected to linear constraints. The constraints were stated as either equalities or inequalities. “In fact, linear programming is the process of taking various linear inequalities relating to a specific situation and finding the "best" value obtainable under those conditions” [6]. In this research, linear programming has been used for the mathematical approach of the deflection formula.

2.3 Previous Researches in Truss Optimization

In recent decades, optimization of truss design has become a significant term in structural optimization. Classical optimization problems are divided into three types: size, geometry/shape and topology. In fact in comparison to other types of structures, the design and analysis of trusses are quite simple process which could be easily written in a mathematical form. As a result, to obtain the optimal truss structure due to classical optimization methods, different investigation has been developed in research papers. Early works were based on the deterministic methods such as mixed integer programming [7], branch and bound techniques [8], dual formulation [9], penalty approach [10], segmental approach with Linear programming [11], and so forth.

Another category of methods that belongs to the nondeterministic methods is simulated annealing [12], genetic algorithm [13] and other methods have been used successfully to solve optimal design problem with discrete variable. "Structural optimization with discrete design is usually very much complicated" [13]. *Yates et al. (1982)* have mathematically proven that discrete optimization problems are NP-complete and consequently they are unsolvable by polynomial algorithms. Also genetic algorithm is one of the efficient subset of discrete variable optimization method [14]. Genetic algorithms are based on the concepts of natural selection and natural genetics (Holland 1975; Goldberg 1989). Although these algorithm are randomized, genetic algorithm are not a simple random walk in the space of solution [15]. *Rajeev and krishnamoorthy (1997)* presented a genetic algorithm(GA)-based methodologies for obtaining optimal design solution simultaneously considering topology, configuration, and cross sectional parameters in unified manner [16] . They have already presented a genetic algorithms-based method for discrete optimization of trusses [13]. Two improved methods are presented in this paper; the first one, simple genetic algorithm (SGA), is adopted to solve size and configuration optimization problem and "second method based on a variable string length genetic algorithm (VGA), addresses the topology optimization problem, taking into account a number of practical issues" [13]. The classical 10 and 18-bay truss problems are solved to illustrated working of the methods and then the values of design variables compared with the previous researches. Comparison of results with those of the report, genetic algorithms-based optimal design methodologies are simple and less mathematically complex and better solutions are obtained using the proposed methodologies than those obtained from the classical optimization methods based on mathematical programming techniques. *Komousis et al. (1994)* have solved the sizing optimization problem of steel roof truss with a genetic algorithm. They have proved that traditional optimization methods based on mathematical programming are not effective in discrete optimization problem and robust algorithm can satisfy the design purposes [17]. It is indicated in *Numerical method in engineering* (*Kaveh and kalatjari, 2003)* the optimization of trusses due to their size and topology by using a genetic algorithm (GA), the force method concept and some perception of graph theory.

> Whereas the optimization with genetic algorithm has a difficulty in the cognition of parameters, existence the application of some concepts of the force method, together with theory of graphs and genetic algorithm make the generation of a suitable initial population well-matched with critical paths for the transformation of internal forces feasible. [18]

The examples studied in this research show that the optimal form of structure depends on the number of nodes considered for the ground structure. Indeed the application of this concept can easily be extended to rigid jointed structures, since more efficient combinational approaches are available for the analysis by the force method.

Until now most of the discussed papers related to this subject dealt with optimal design under static displacement and stresses constraints. On the other side, a little effort has been made due to optimal design based on structural dynamic aspect. *Tong and Liu* suggested:

> Two-step optimization procedure for the optimal design of truss structures with discrete design variables under dynamic constraints. At first, a global normalized constraint function (GNCF) has been defined. At the second step, the discrete values of the design variable are determined by analysing differences quotient at the feasible basic point and by converting the structural dynamic optimization process into a linear zero-one programming. [19]

Since, the above mentioned optimization procedure for optimal design has successfully been applied to some of the truss structures; the result demonstrated that the method is practical and efficient. Also, it is noted that the optimal design deal with constraints of stress and displacement, simultaneously with natural frequency and frequency response.

As has been perceived in the previous paragraphs, a considerable amount of work has been carried out relating to optimization with genetic algorithm method while the other methods of optimization has been investigated far less due to their complexity. Therefore, some methods developed using size; geometry and topology for optimization are presented. *Rahami et al. (2008), in Journal of Engineering Structure* "have used a combination of energy and force method for minimizing the weight of truss structures. The main idea proposed in this research is the manner in which the input variables are reduced" [20]. As a result, the formulation based on energy concepts permits an efficient use of GA in optimization. In fact a mixed formulation is presented for the optimization of structures using a genetic algorithm. "The method employs basic idea from the force method and the complementary energy approach, and uses a simple genetic algorithm as a powerful optimization technique" [14]. Moreover *Farshi and Alinia-ziazi (2010)* have described a force method based on the method of centre points as a new approach to optimum weight design of truss structures. It is indicated in their research that:

> Design variables are the member cross-sectional areas and the redundant forces evaluated for each independent loading condition acting on structure. Forces in each member are consisted to have two parts; the first part corresponds to the response of the determinate structure as defined from the whole structure, and the second part takes care of the effect of forces in the redundant members. [21]

The comparison of the results of this research with the examples selected from similar works has illustrated that:

> The analysis step is embedded within the optimization stage using the force formulation; avoiding tedious separate analyses. Also it should be noted that in cases of low degrees of redundancy effectiveness of the proposed method will be more prominent, since few additional variables (i.e. redundant forces) should be added to the design variables (cross-sectional areas), requiring less computational efforts. [21]

One of the other approaches in optimal truss design that has been widely investigated is truss optimization under stress, displacement, and local buckling constraints with minimum weight. It was introduced in journal of structural and multidisciplinary optimization, *Gou et al.* made a new appeal to "the solution of singular optimal of truss topology optimization problems caused by stress and local buckling constraints. First, a second-order smooth-extended technique is used to make a disjoint feasible regions connect, and then the so-called $\varepsilon\varepsilon$-relaxed method is applied to eliminate the singular optima from the problem formulation" [22]. As a result the given numerical examples in this study indicated an efficient approach to optimization of truss topology problems which are subjected to local bukling and stress constraints. In addition, it was concluded that the traditional stress formulation method is not appropriate especialy in case of local bukling constraints. Therefore, the proposed εε-relaxed approach is recommended in order to truss topology optimization with local buckling constraints [22] . *Bojczuk and Mroz (1999)* in the journal structural optimization *were* presented “a heuristic algorithm for optimal design of trusses with account for stress and buckling constraints. The design variables are constituted by cross-sectional areas, configuration of nodes and the number of nodes and bars” [23]. The main idea of this study was associated with “the assumption that topology variation occurs at a discrete set of states when the optimal design evolves with the selected size parameter” [23]. In fact this research was introduced three virtual topology variation modes with their applicability by solving particular examples;

> (1) A new node at the centre of the existing bar that connected to the closest existing node.
> (2) The separate existing node and a new bar that connected to two nodes.
> (3) Two nodes at the centre of a compressed bar that separated by a connecting bar. As a result, the examples demonstrate that topology variations coupled with configuration optimization can provide very effective designs. [23]

In the geometry and topology optimization subject, *W. Achtziger (2007)* has introduced the classical problem of optimal truss design where cross-sectional areas and the position of joints are simultaneously optimized. In fact, he focused on the difference between simultaneous and alternating optimization of geometry and topology and recalled a rigorously mathematical approach based on implicit programming technique which considers the classical single load minimum compliance problem subject to a volume constraint. Two numerical examples are presented to illustrate that simultaneous optimization of geometry and topology may result in very reasonable structures even for small problem size and very sparse ground structures [24]. Then after, *Martinez et al. (2007)* in journal of journal of structural and multidisciplinary optimization studied about “a novel growth method for the optimal design in a sequential manner of size, geometry, and topology of plane trusses without the need of ground structure. Actually, the most used method for truss topology design by computational methods is the ground structure approach” [25]. This method was associated with the design of optimal plane trusses which are subjected to the stress constraints.

> The growth method begins with a simplest structure and would continually modify it by adding iteratively, joints and members optimizing the variable of size, geometry and topology at each step. The characteristic of method and the result of the three examples illustrated that this method requires a minimal amount of initial data and allows the optimal structure to be obtained with a given number of joints. [25]

Also the research was clarified that this method “is very flexible and permits the fulfilment of different design conditions. Moreover, the computational cost is lower than the procedures based on the Ground Structure approach” [25].

A few attempts have also been reported on configuration optimization, in which both size and configuration variables. *Wang et al. (2002)* were proposed “an evolutionary optimization method to optimize the shape and size of a truss structure for its weight minimization. The stress, local buckling and displacement constraints in one load case are imposed on the structure” [26]. The research was argued that “the FSD algorithm is an intuitive and efficient optimality criterion for size optimization of structural members. In fact the elements designed with FSD are fully stressed only for statically determinate structures” [26]. The concluded of the study stated, this “approach needs further study to be extended to more general situation with constraints of stress, local stability and multiple nodal displacements under multiple load cases” [26] . *Gil and Andreu (2001)* were presented a new approach for the identification of the optimal shape and cross-sections of a plane truss structure under stress and geometrical constraints. The optimization algorithm includes the treatments of constraints using penalty functions, optimization of cross section and optimization of nodal coordinates. In the study, the cross-section optimization is achieved by the fully stress design (FSD) strategy and the coordinates optimization is driven by the conjugate-gradients strategy. In fact the strategy outlined in this paper has demonstrated to be highly stable, even when starting from initial structures which are very far from optimum. So the optimized structure is observed to the applied load or related shapes to the bending stress [2]. *Nishino and Duggal (1990)* have carried out a shape optimum design of trusses under multiple loading conditions. The weight of a truss is minimized subject to nodal equilibrium and permissible stress constraints, and constraints to ensure uniqueness of the stress-free length of each member. The optimization procedure includes selection of topology, geometry and sectional properties [27].

Chapter 3

METHODOLOGY

3.1 Introduction

In architecture and structural engineering, truss is a structure that is constructed out of one or more triangular units with straight members which ends are connected at joints referred to as nodes.

So far due to the literature studies, structural optimisation is dealing with; largest possible number of design variables, behavioural construction, mathematical formulation for optimal solution and effective solution procedures. As a result truss systems also turned to be a remarkable issue in structural optimization. The simple characteristics of truss systems in design and analysis made an easy mathematical model opportunity for classical truss optimization when compared to other types of structures. Therefore, different techniques are introduced to obtain the optimal truss structures. These developed methods are listed as below:

A) Deterministic methods

Mixed integer programming

Branch and bound techniques Dual formulation

Penalty approach

Segmental approach with LP

B) Nondeterministic methods

Simulated annealing

Genetic algorithm

This research is aimed to carry out an investigation on the existing truss systems in order to introduce a mathematical optimization approach. This approach is expected to lead to an efficient method for designers and decision makers so that they can find the most appropriate truss structure (listed in this research) for their design purposes. The suggested method is clarified in detail based on the below given critical questions to identify which of the selected trusses (in this research) could be suitable for the chosen span based on:

- How the optimal truss is identified among different changes in proportion of height and distance between joints (Bay)?
- What would be the amount of deflection of optimized truss?

To achieve these some of the common types of trusses made of steel are studied to identify their efficient sizes and shapes. Therefore, it is decided to produce a mathematical deflection formula by considering loading and truss spaces as our constraints and defined variables as; shape, span and height. Also deflection of the structure is minimized and formulated as an objective function.

Initially the force method is applied on truss structure in order to obtain the amount of deflection. Then hand calculation was carried out followed by computer application analysis using MAPLE and then the use of Table Curve 2D for mathematical approach to create the deflection formula. Finally STAAD Pro structural design computer software has been used to analyse and design the truss structure.

As a result, the changes in mathematical deflection formula, due to the number of frames and shapes of trusses, are lead to obtain the specific optimum height and minimum deflection for each truss system. In other words, the occurrence of minimum deflection along the truss span and optimum height presents the optimum truss.

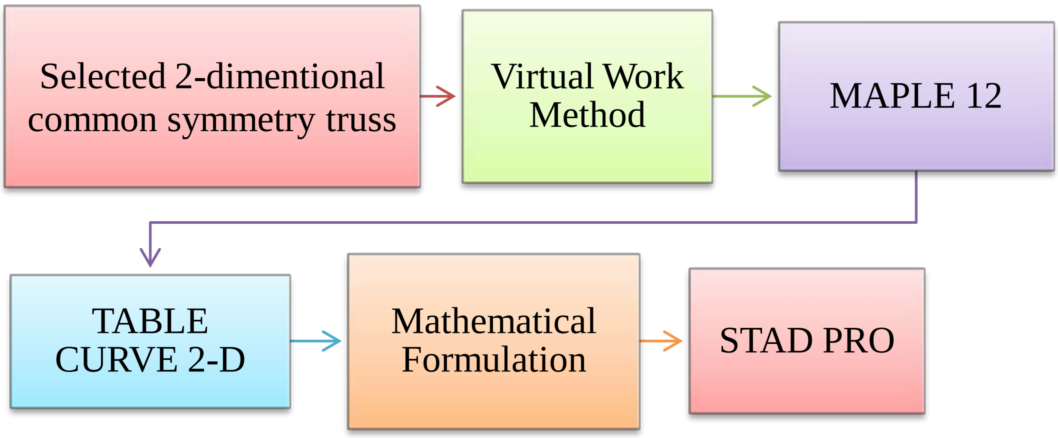


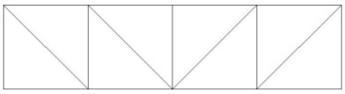
Figure 4: Scheme of methodology stages

3.2 Truss Shapes

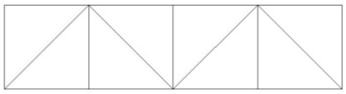
Basically 11 shapes of common symmetry trusses in 2-D position are categorized into 2 groups as shown below:

a) Trusses with horizontal top chords

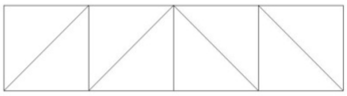
b) Trusses with a constant slope top chords



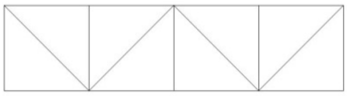
Truss 1



Truss 2



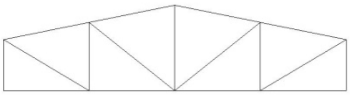
Truss 3



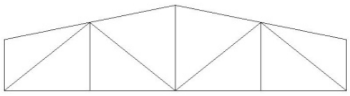
Truss 4

Truss 5

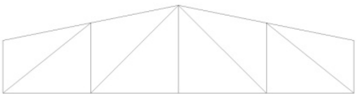
Figure 5: Trusses with Horizontal Top Chords



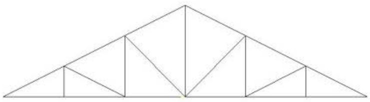
Truss 6



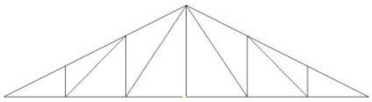
Truss 7



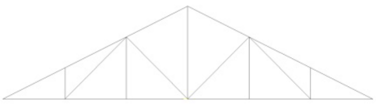
Truss 8



Truss 9



Truss 10



Truss 11

Figure 6: Trusses with Constant Slope

Figures 5 and 6 show the 11 truss models; flat, warren and triangular with 5, 3 and 3 types of each one are used respectively. Different span lengths 10, 20, 30 and 40 meters were applied for all types of the trusses to find the least deflection mode. It should be noted that the variety of trusses selected are not randomly assumed. These are the most frequently used trusses in real life. When flat trusses are considered the five types used in this research are generally the ones used in real life. However, for warren and triangular type trusses a sample of the most common types were considered. In order to reduce the wide range of analysis and to achieve more accurate outputs from the analysis only 3 types from each of warren and triangular trusses were studied.

3.3 Assumptions Used in this Research

This research is aimed to present a mathematical method for the optimum deflection of the plane truss structures subject to multiple loads and stresses. To achieve a mathematical statement with constraints and variables, the proportion between the height of the truss and the horizontal distances between the joints are investigated in advance. As a result, the cross sectional areas of members, distance between joints of chords and heights of trusses are assumed to be as variables of design. Therefore, objective of the optimization is specified as the minimization of deflection at mid-span with constraints on loading, spans and truss chord member spacing.

The structural analysis is further base on the following assumptions:

a) The mathematical model for the plane truss consists of a set of joints which are connected by straight members and carry only axial load. Also the deadweight of the members is neglected.

b) All members are connected to joints by frictionless pins. That is to say, no moment can be transferred between the ends of a member and the joints to which they are connected.

c) The selected trusses are loaded in a similar manner and only dead and live loads are considered, 1.25kN/m^2 and 0.75kN/m^2, respectively. Also, cladding system, insulation, self-weight of truss members and purlins are considered as dead load.

d) All loads on the structure are applied only at joints. Purlins are arranged in such a way that the loads are applied on the purlins that are placed directly where the vertical truss member joins the top chord. These are considered to be nodes of the truss. Hence, all members of truss are assumed to be subjected to pure axial loads. Moments acting on the joints or intermediate loads acting directly on the members is not permitted. No shear force or bending moment exists in the members.

e) Only translation restraints may exist at the support joints. Therefore, only pinned or roller supports which translate in the plane of the structure are permitted.

f) Each shape of truss (in terms of geometrical arrangement of members) is constant while the distance between vertical members (bays) and height of the truss are changed.

Chapter 4

MATHEMATICAL FORMULATION AND RESULTS

4.1 Mathematical Formulation

4.1.1 Hand Calculation

4.1.1.1 Virtual Force Method

When a structure is loaded, deformation on stressed elements will take placed. As a result of the changes on the structural shape, the nodes of the structure will be displaced. In a well-designed structure, these displacements are substantially small. For instance, Figure 6 shows that the changes occurred on the structural elements will have some effect on the displacement point of the given truss. The applied load P produced the axial forces F_1, F_2 and F_3 in the members. It is obvious (Fig. 7) that the members are deformed axially (dashed lines) and joint B of the truss is displaced diagonally toB'.

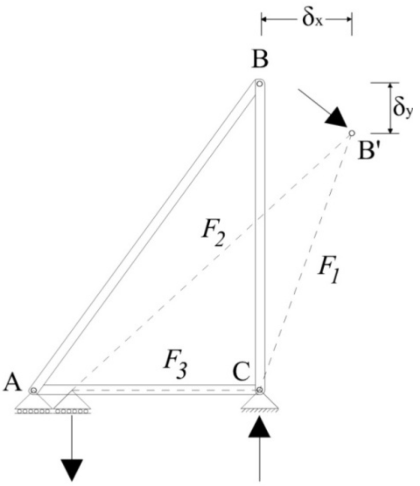


Figure 7: Deformations of Truss after Load is Applied

The Virtual force method is applied to determine the deflection of trusses. The virtual work principle is defined as such that the deflection can be calculated by the following equation:

$$1(\Delta) = \sum n(\delta) \qquad (6)$$

Where n is equal to the virtual force in the member and δ equal to the change in length of the member.

Therefore, the deflection that occurred due to the changes in length of the truss members can be calculated. These changes in length are caused by; the effect of applied loads on the behaviour of each truss member, changes in temperature and fabrication errors.

In order to determine the member forces in a truss one can use either the method of joints or the method of sections [3]. Once the member forces are known then the axial deformation of each member can be determined by using the below given equation:

$$\delta = \frac{NL}{AE} \qquad (7)$$

The deflection formula can be modified by the substitution $1.(\Delta)$, from equation (6) instead of δ in equation (7).

$$1.(\Delta) = \sum \frac{nNL}{AE} \qquad (8)$$

Here:

1= external virtual unit load acting on the truss joint in the stated direction of Δ

Δ= joint displacement caused by the real loads on the truss

n= internal virtual force in a truss member caused by the external virtual unit load

N= internal force in a truss member caused by the real loads

L= length of the member

A= cross-sectional area of member

E= modulus of elasticity of a member

The external virtual unit load creates internal virtual “n” forces in each of truss members. When the real loads are applied to the truss, then the truss joint will displaced Δ in the same direction as the virtual unit load, and each member undergoes a displacement NL/AE, in the same direction as its respective n force. Consequently, the external virtual $1.\Delta$ is equals to the internal virtual work or the internal (virtual) strain energy stored in all the truss members, i.e., Equation 6.

4.1.2 Problem Statements

As it has been explained in the previous chapters, this research is aimed to provide an optimum truss shape which is subjected to minimum deflection by using the virtual force method. Hence, this method is applied in order to create a general deflection formula to achieve a specific approach in deflection minimization.

In each type of trusses that is categorized at the beginning of this chapter, deflection of trusses are calculated to create a general formula based on an assumed interval for k (k=1 till k=10), whereas n is the number of bays in one side of a symmetrical truss. In this way some mathematic software like “MAPLE 12” and “TABLE CURVE 2D” are used for mathematical approach of deflection formula.

First approach is investigated based on a 2-D symmetrical warren type flat truss (Fig. 8) with a multiple load (W) that are applied on the joints to determine the vertical deflection at joint E (the mid-span of truss).

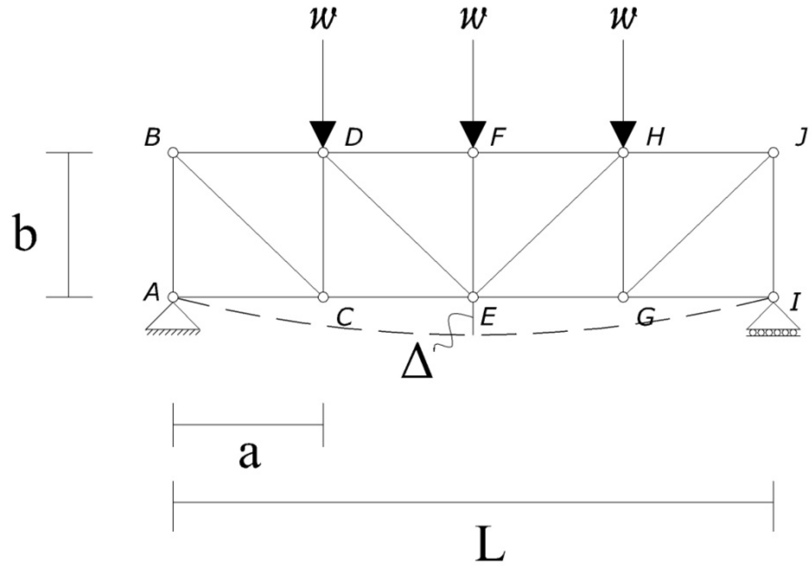


Figure 8: Warren Type Flat Truss with Multiple Loads

4.1.3 Analysis Procedure

The following procedure provides a method that may be used to determine the displacement of any joint on a truss, by applying the virtual force method. The internal force on each element, are determined in two sections. Once it is caculated based on real forces (N) then the virtual force is applied. Based on outputs of hand calculation an individual deflection formula (Δ) for each n (1< k <10) is generated. As a result, the investigation on the deflection formulas and by using the mathematical software helped to lead us to create a general deflection formula (Δn) for each of the 11 trusses. Finally, the formula was entered into the MAPLE program under a paticular mathematical circumastences; deflection, virtual force method formula is generated. The following sections of this chapter discusses the derivation of the formula in mor detail.

STEP 1: Calculate the Internal Forces, N

Initially the internal forces in each member should be determined. These forces are resulted solely from the real behaviour of truss under the applied external loads.

It is assumed that the tensile forces are positive and the compressive forces are negative.

a) Calculate the Support Reactions, due to the Applied Real Loads

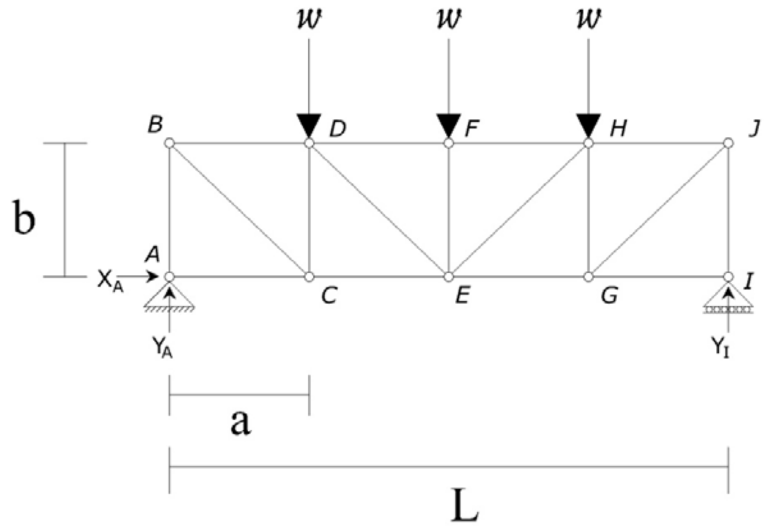


Figure 9: Frame Structure with Applied Real Forces

Calculate the support reactions (caused by the applied loads in Figure 9) through summation of the moments at A and E:

$$\sum M_A = 0 \;\Rightarrow\; Y_I \times 4a - w \times a - w \times 2a - w \times 3a = 0 \Rightarrow Y_I = \frac{3}{2}W$$

Since, the truss is symmetrical then:

$$Y_A = Y_I = \frac{3}{2}W$$

$$\sum F_X = 0 \;\Rightarrow\; X_A = 0$$

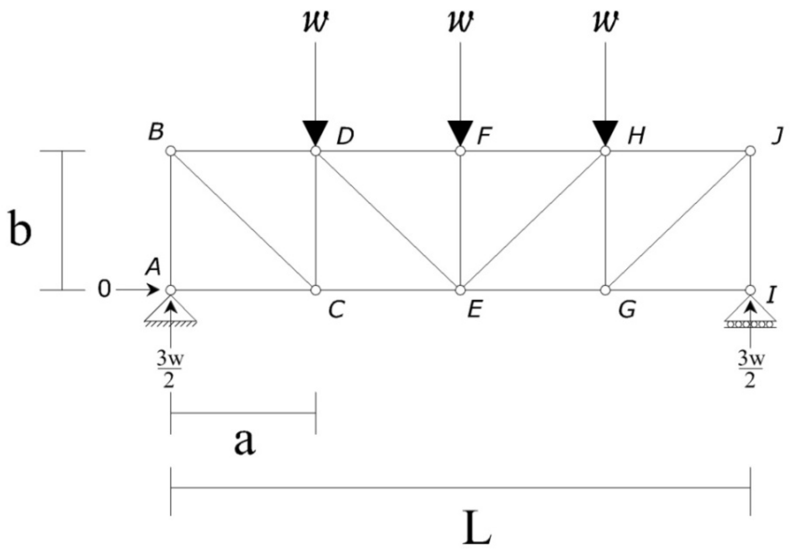


Figure 10: Support Reactions due to Applied Real Loads

b) Use the Method of Joints to Determine The Internals Force in Each Member, due to the Applied Real Loads

For equilibrium at joint A;

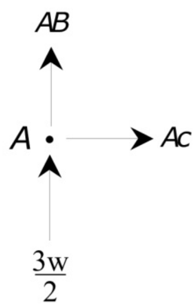


Figure 11: Joint Equilibrium at joint A

Summation of vertical and horizental forces to determine the forces in each member

$$\sum F_Y = 0 \;\Rightarrow\; F_{AB} + \frac{3w}{2} = 0 \;\Rightarrow F_{AB} = -\frac{3w}{2}$$

$$\sum F_X = 0 \;\Rightarrow\; F_{AC} = 0$$

Hence, by applying this method for each joint the internal forces on each member is caculated as shown in Figure 12.

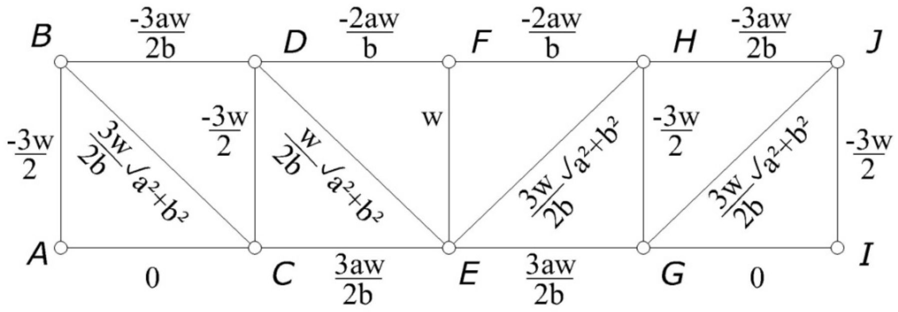


Figure 12: Truss Diagram with Internal Forces due to Applied Real Loads

STEP 2:

a) Apply Virtual Force, n

Place the virtual unit load on the truss at the joint where the desired displacement is to be determined. The load should be directed along the line of action of the displacement. With the unit load so placed and all the real loads removed from the truss, the internal *n* force in each truss member is calculated. Agian, it was assumed that the tensile forces are positive and the compressive forces are negative. The unit load was applied at point E with the intention of determining the deflection at that point (Fig. 13) which is in the center of the assumed symetrical truss system.

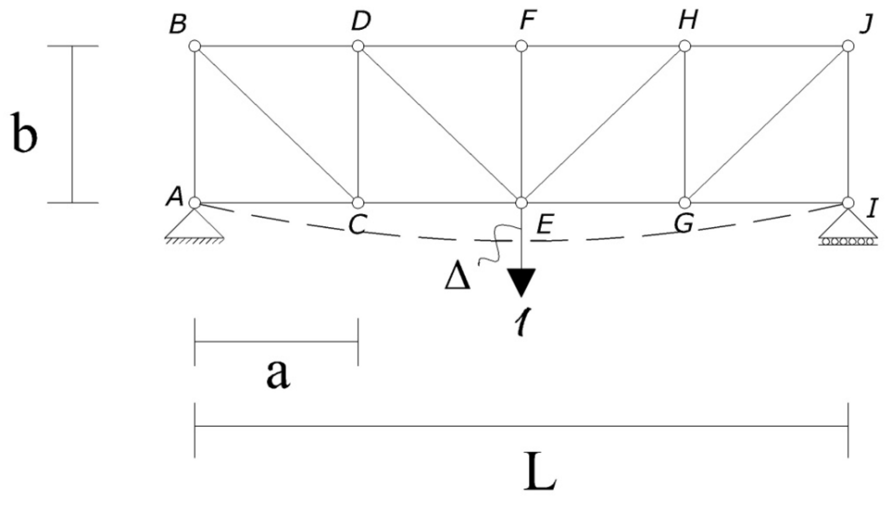


Figure 13: Truss with Virtual Unit Force Applied

b) Solve For the Support Reactions due to The Virtual Force

The aformentiond procedure is applied to caculate the reaction at each support which is resulted by the virtual forces (Fig. 14).

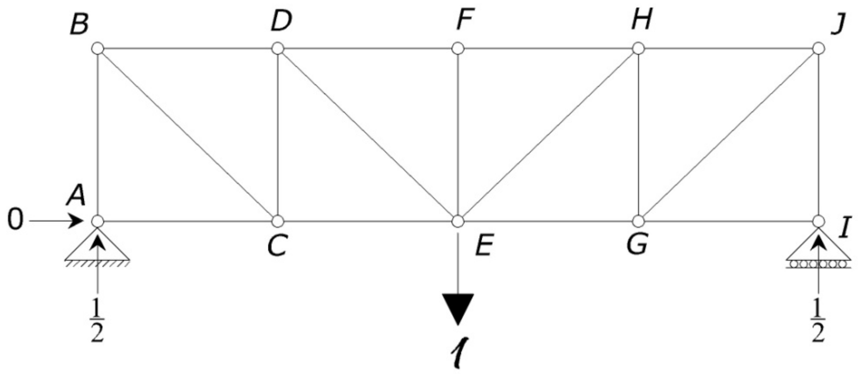


Figure 14: Support Reactions due to Applied Virtual Forces

c) Use Method of Joints to Determine the Virtual Force in Each Member

The virtual forces on each member are calculated by applying the method of joints that is illustrated in the applied real load (Fig. 15).

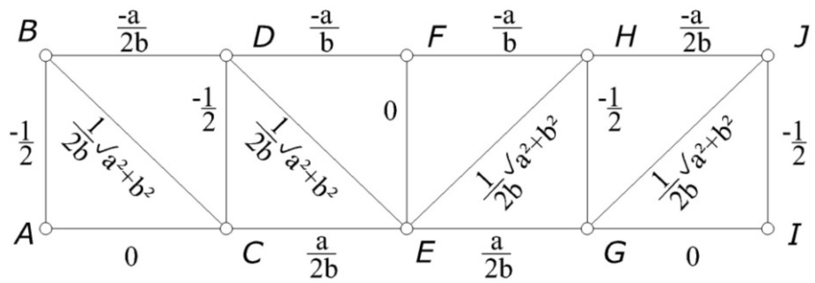


Figure 15: Truss Diagram with Internal Forces due to Virtual Force

STEP 3:

a) Calculate the Deflection

The deflection of the truss can now be determined by computing the equation 3:

$$1.(\Delta) = \sum \frac{nNL}{AE} \quad (8)$$

Table 1: Calculate the mid-span Deflection at Joint E

Member	**n**	**N**	**L**	**nNL**
AB	$-1/2$	$-3W/2$	b	$3W/4$
AC	0	0	a	0
BC	$\frac{\sqrt{a^2+b^2}}{2b}$	$\frac{3W\sqrt{a^2+b^2}}{2b}$	$\sqrt{a^2+b^2}$	$\frac{3w}{b^2}(a^2+b^2)\sqrt{a^2+b^2}$
BD	$-a/2b$	$-3aw/2b$	a	$3a^3w/4b^2$
CD	$-1/2$	$-3W/2$	b	$3W/4$
CE	$a/2b$	$3aw/2b$	a	$3a^3w/4b^2$
ED	$\frac{\sqrt{a^2+b^2}}{2b}$	$\frac{W\sqrt{a^2+b^2}}{2b}$	$\sqrt{a^2+b^2}$	$\frac{w}{4b^2}(a^2+b^2)\sqrt{a^2+b^2}$
DF	$-a/b$	$-2aw/b$	a	$2a^3w/b^2$
EF	0	w	b	0
FH	$-a/b$	$-2aw/b$	a	$2a^3w/b^2$
EH	$\frac{\sqrt{a^2+b^2}}{2b}$	$\frac{W\sqrt{a^2+b^2}}{2b}$	$\sqrt{a^2+b^2}$	$\frac{w}{4b^2}(a^2+b^2)\sqrt{a^2+b^2}$
EG	$a/2b$	$3aw/2b$	a	$3a^3w/4b^2$
GH	$-1/2$	$-3W/2$	b	$3W/4$
HJ	$-a/2b$	$-3aw/2b$	a	$3a^3w/4b^2$
GJ	$\frac{\sqrt{a^2+b^2}}{2b}$	$\frac{3W\sqrt{a^2+b^2}}{2b}$	$\sqrt{a^2+b^2}$	$\frac{3w}{b^2}(a^2+b^2)\sqrt{a^2+b^2}$
GI	0	0	a	0
IJ	$-1/2$	$-3W/2$	b	$3W/4$

The total deflection (for selected case) at point E is:

$$\Delta_k = \frac{7w}{b^2}a^3 + 3wb + \frac{2w}{b^2}(a^2+b^2)^{\frac{3}{2}} /AE \tag{9}$$

In this case the number of the frame on one half of the structure is equal to two then :

$$\Delta_k = \Delta_2 = \frac{7w}{b^2}a^3 + 3wb + \frac{2w}{b^2}(a^2+b^2)^{\frac{3}{2}}/AE \tag{10}$$

Following the same procedure used previously, calculate the deflections (k=1 until k=10) to find the general deflection formula.

Table 2: Determine the deflection formulas due to the number of bays on one half of the symmetrical flat truss

k	**Deflection Formula (Δ_k)**
1	$\Delta_1 = \frac{1}{2} * \frac{w}{b^2}\, a^3 + \frac{1}{2} * wb + \frac{1}{2} * \frac{w}{b^2}\, (a^2+b^2)^{\frac{3}{2}} \,/\, AE$
2	$\Delta_2 = 7 * \frac{w}{b^2}\, a^3 + 3 * wb + 2 * \frac{w}{b^2}\, (a^2+b^2)^{\frac{3}{2}} \,/\, AE$
3	$\Delta_3 = \frac{69}{2} * \frac{w}{b^2}\, a^3 + \frac{13}{2} * wb + \frac{9}{2} * \frac{w}{b^2}\, (a^2+b^2)^{\frac{3}{2}} / AE$
4	$\Delta_4 = 108 * \frac{w}{b^2}\, a^3 + 11 * wb + 8 * \frac{w}{b^2}\, (a^2+b^2)^{\frac{3}{2}} / AE$
5	$\Delta_5 = \frac{525}{2} * \frac{w}{b^2}\, a^3 + \frac{33}{2} * wb + \frac{25}{2} * \frac{w}{b^2}\, (a^2+b^2)^{\frac{3}{2}} \,/\, AE$
6	$\Delta_6 = 543 * \frac{w}{b^2}\, a^3 + 23 * wb + 18 * \frac{w}{b^2}\, (a^2+b^2)^{\frac{3}{2}} / AE$
7	$\Delta_7 = \frac{2009}{2} * \frac{w}{b^2}\, a^3 + \frac{61}{2} * wb + \frac{49}{2} * \frac{w}{b^2}\, (a^2+b^2)^{\frac{3}{2}} \,/\, AE$
8	$\Delta_8 = 1712 * \frac{w}{b^2} a^3 + 39 * wb + 32 * \frac{w}{b^2}\, (a^2+b^2)^{\frac{3}{2}} / AE$
9	$\Delta_9 = \frac{5481}{2} * \frac{w}{b^2}\, a^3 + \frac{97}{2} * wb + \frac{81}{2} * \frac{w}{b^2}\, (a^2+b^2)^{\frac{3}{2}} / AE$
10	$\Delta_{10} = 4175 * \frac{w}{b^2}\, a^3 + 59 * wb + 50 * \frac{w}{b^2}\, (a^2+b^2)^{\frac{3}{2}} / AE$

4.1.4 Calculate the General Formula Using Maple 12

MAPLE is a powerful mathematical software package. It can be used to obtain symbolic and numerical solutions of problems in arithmetic, algebra, and calculus and to generate plots of the solutions it generates [28].

In this section, constructing a deflection formula by MAPLE to obtain a different mathematical approach for the calculation of the deflection of trusses will be discussed.

4.1.5 Equation of the Coefficients

The first step is to specify the data as a collection of points, or as separate collection of independent and dependent values. Table 3 shows the coefficients of the previous deflection formulas as a collection of points

Table 3: The Coefficients of Deflection Formula

k	**Coefficients**		
	a^3	b	$(a^2+b^2)^{\frac{3}{2}}$
1	1	1	1
2	14	6	4
3	69	13	9
4	216	22	16
5	525	33	25
6	1086	46	36
7	2009	61	49
8	3424	78	64
9	5481	97	81
10	8350	118	100

The second step is to provide a mathematical formula for the specific datas by using the CurveFitting [Interactive] command in MAPLE (Fig. 16).

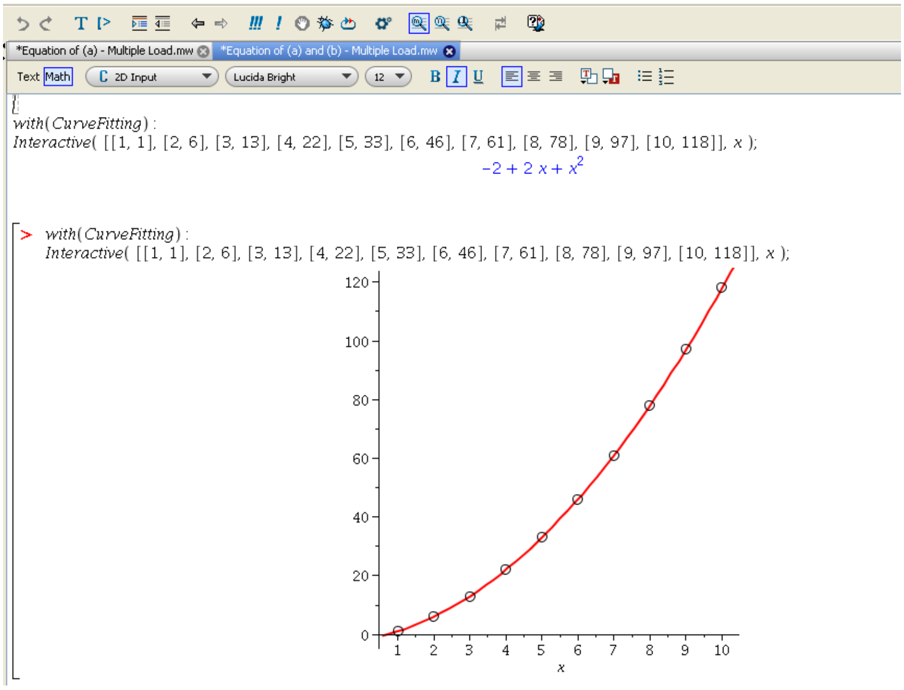


Figure 16: Determiniation of coeficient equation in MAPLE

Finally, the general deflection formula (Δ_k) has been calculated :

$$\Delta_k = \frac{1}{2}\left(\frac{1}{6}k^2 + \frac{5}{6}k^4\right)\frac{w}{b^2}a^3 + \frac{1}{2}(k^2 + 2k - 2)wb + \frac{1}{2}k^2\frac{w}{b^2}(a^2 + b^2)^{3/2}/AE \qquad (11)$$

Here:

Δ= joint displacement caused by the real loads on the truss

k= number of bays on one half of the symmetrical truss

w= multiple loads on the structure that are applied at joints

a= distance between the joints of truss members (bay width)

b= height of the truss

A= cross-sectional area of members

E= modulus of elasticity of a members

4.1.6 Ratio of Height

The main aim of this section is to find a relative optimum height of truss to reach the minimum optimum deflection in each truss case. For this purpose, the deflection formula obtained in the previous section is converted into a mathematical function *(f(k))*. Since *a* and *b* are the two parametric values which are representing the distance between the horizontal truss joints (bay width) and the height of the truss system respectively, then it is assumed that the ratio of ***b*** to ***a*** can be equal to one single parameter, ***x***. As a result, instead of getting the derivative of *a* and *b* in *(f (k))*, based on one single parameter of ***x*,** the calculation and results will become less complicated and more accurate.

$$f(k) = \frac{1}{2}\left(\frac{1}{6}n^2 + \frac{5}{6}n^4\right)\frac{w}{b^2}a^3 + \frac{1}{2}(n^2 + 2n - 2)wb + \frac{1}{2}n^2\frac{w}{b^2}(a^2 + b^2)^{3/2} \qquad (12)$$

Assume:

$$\frac{b}{a} = x \Rightarrow b = ax \qquad (13)$$

then by substituting equation (13) in equation (12) the following is obtained:

$$f(x) = \frac{1}{12}k^2wa\ \frac{1 + 5k^2 + 6x^3\left(1 + \frac{2}{k} - \frac{2}{k^2}\right) + 6\sqrt{1 + x^2} + 6\sqrt{1 + x^2}x^2}{x^2}$$

$$= \frac{1}{2}k^2wa\ g(x) \qquad (14)$$

in which:

$$g(x) = \frac{1 + 5k^2 + 6x^3\left(1 + \frac{2}{k} - \frac{2}{k^2}\right) + 6\sqrt{1 + x^2} + 6\sqrt{1 + x^2}x^2}{x^2} \qquad (15)$$

The derivative of equation read:

$$\frac{dg}{dx} =$$

$$\frac{6k^2\sqrt{1 + x^2}x^3 + 12x^3\sqrt{1 + x^2}k - 12x^3\sqrt{1 + x^2} - 2k^2\sqrt{1 + x^2} - 10\sqrt{1 + x^2}k^4}{k^2\sqrt{1 + x^2}x^3}$$

$$+ \frac{6x^4k^2 - 6x^2k^2 - 12k^2}{k^2\sqrt{1 + x^2}x^3} \qquad (16)$$

Set $\frac{dg}{dx} = 0$ which implies;

$$2(3k^2x^3 + 6kx^3 - 6x^3 - k^2 - 5k^4)\sqrt{1 + x^2} + 6k^2(x^4 - x^2 - 2) = 0 \qquad (17)$$

To find x, plot the later function in terms of x for different values of k. This is illustrated in Figure 17 which in captured from MAPLE application. The ratio of height to joint distance is obtained via the assumed parameter of $\boldsymbol{x}$ and the next step is to determine the amount of deflection corresponding to this ratio.

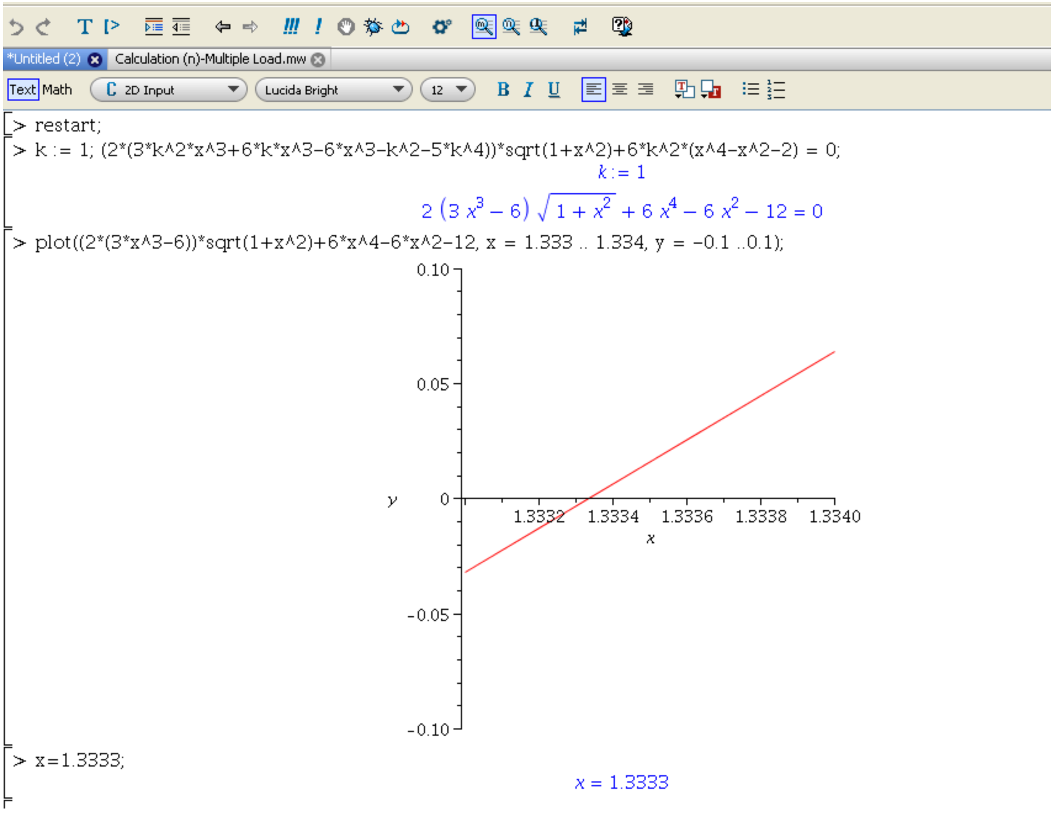


Figure 17: Calculation of x according to the graph drawn in MAPLE application

Following the same procedure would lead to the calculation of the ratio ***(b/a)*** for different values of *k* (k=1 ...k=10) for each assumed truss model. It is important to identified that the interval assumed for x = b/a, and the selection of values of k between 1 to 10 is purely intended to get more accurate and adequate results for x (Table 4).

Then the calculated ratios (*x*= *b/a*) are brought togather in order to create an equation out of the determined series of x. To achieve the expected equation, the TABLE CURVE application is used.

Table 4: Different ratios of b/a based on the interval assumed for n

k	b/a
1	1.33330000
2	1.58870000
3	1.97493600
4	2.35418000
5	2.71584870
6	3.06026000
7	3.38936800
8	3.70514060
9	4.00928040
10	4.30319927

4.1.7 Calculation of Ratio by using Table Curve 2D v5.01

Table Curve 2D is a linear and non-linear Curve fitting software package for engineers and scientists that automates the curve fitting process and in a single processing step instantly fits and ranks 3,600+ built-in frequently encountered equations enabling users to easily find the ideal model for their 2D data within seconds [29].

The expected final equation is done by TABLE CURVE computer application instead of MAPLE application. TABLE CURVE is used for the formation of the equation since it has extensive variety of equations (2600 equation only for each curve fitting) and variety of equation formats (e.g. linear and non-linear equation at the same time with wider interval). In addition, equations with insignificant terms have been removed from the equation list at the end of the curve fitting. Some other equations that may be absent from the list are due to not being fitted. For example, there is no point in fitting an equation with an *ln(x)* term if there are negative x values in the data set.

Therefore, among the possible equations one of the best studied equations is selected for further approaches (Fig. 18).

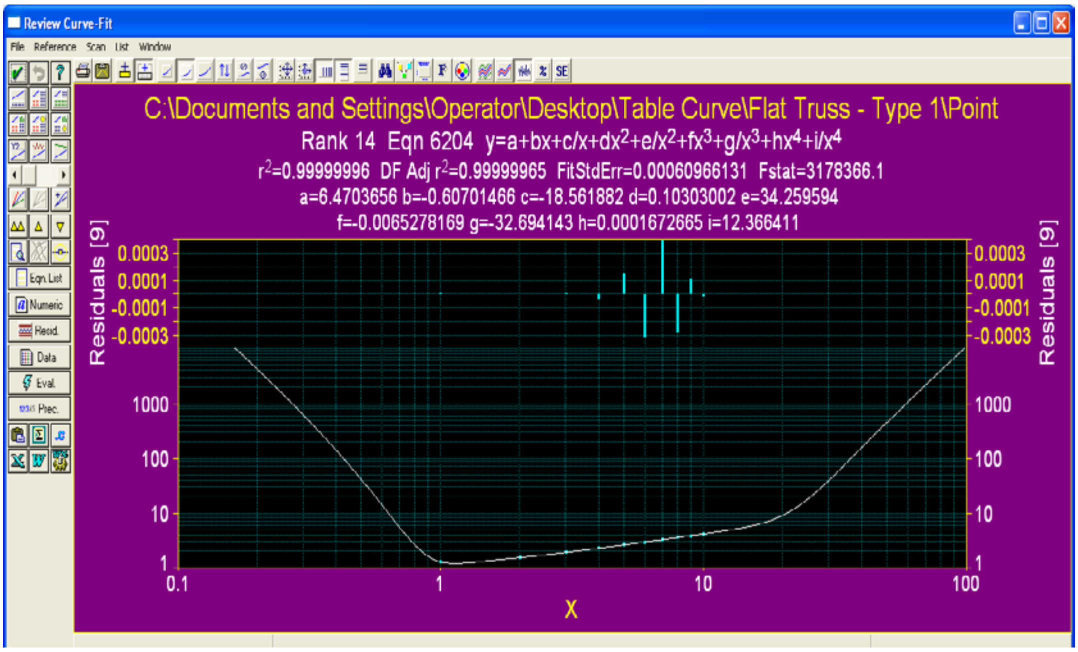


Figure 18: Determining Equation Ratio by applying TABLE CURVE

The equation was constructed based on 10 different ***b/a*** ratio's from the best fitted to the input set as below:

$$f(x) = 6.470 - 0.6075x - \frac{18.5754}{x} + 0.1037x^2 + \frac{34.2931}{x^2} - 0.00653x^3 - \frac{32.7393}{x^3} + 0.0001673x^4 + \frac{12.392}{x^4} \quad (18)$$

The resulted equation for the TABLE CURVE application is imported in MAPLE so that the minimum amount of *k* can be achieved.

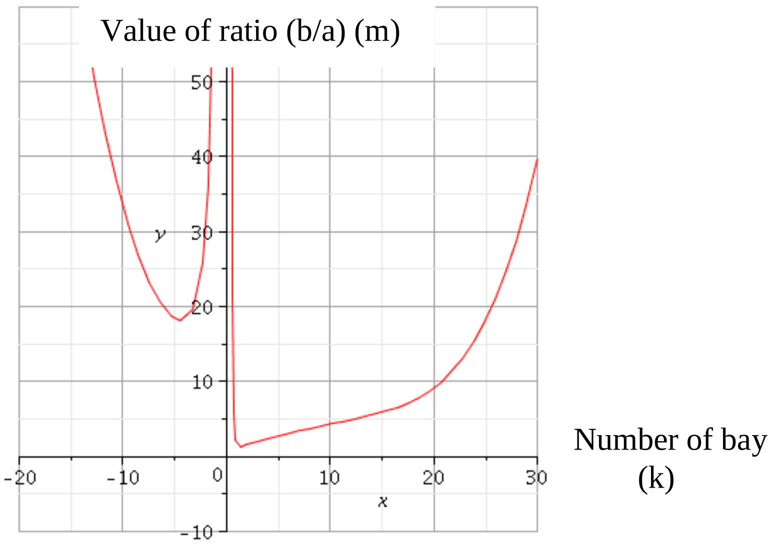


Figure 19: Curve plotted in MAPLE selected out of the imported TABLE CURVE equations

Therefore, the minimum value of *k* is calculated based on the plotted graphs in MAPLE (Fig. 19). In other words the minimum value of *k* is introduced for the truss model that promised to deliver the minimum deflection among all the 10 selected models. Briefly, the truss with the minimum value of *k* demonstrated the minimum mid-span deflection for the truss.

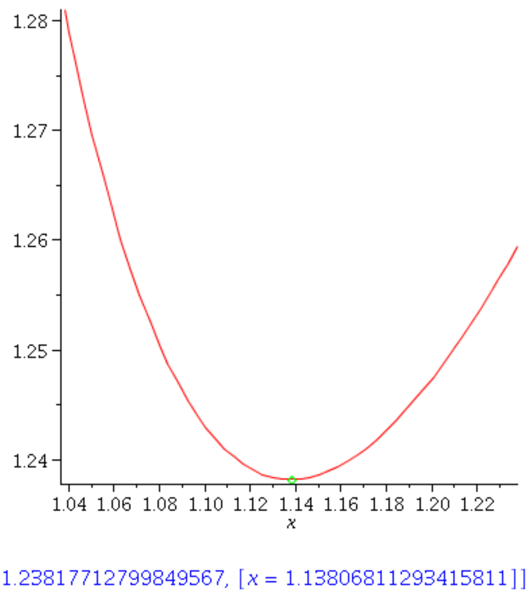


Figure 20: Determining the minimum value of n from the TABLE CURVE

For instance, for the selected truss model as shown in Figure 20 the minimum value in the X coordinate (1.13807) is presented the minimum value for k to achieve the minimum deflection in the truss span. Furthermore, the minimum value of k on X coordinate (1.13807) is intersected with Y coordinate at point 1.23 only, which is named as height ratio (b/a). Therefore, the selected symmetrical truss model is delivered the minimum deflection amount (among the ten defined possibilities for n) if and only if the frame carried maximum of 2 frames on each side. In other words when, $1 < minimum\ point\ < 2$ we are allowed to assign a minimum of one and a maximum of two frames on each half of the selected symmetrical truss model.

4.1.8 Loading

It is discussed earlier in this chapter that the selected trusses were loaded in a similar manner and only dead and live loads were considered with $1.25kN/m^2$ and $0.75kN/m^2$ load factors respectively (wind load was not considered). Also, it was assumed that the weight of the cladding system, isolation and self-weight of the truss and the purlins were considered as dead load. Therefore, the load of flat truss is calculated as a sample to illustrate the whole procedure followed to achieve the total load acting on the nodes for each type of truss models.

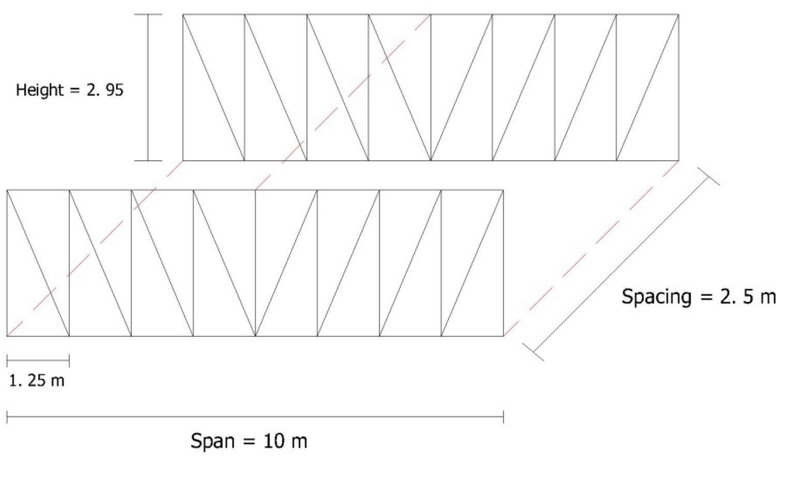


Figure 21: Typical Layout of Trusses with Labels

Data:

Spacing of Truss = 2.5 m

Height of Truss = 2.95 m

Dead Load (on Plan) = 1.25 kN/m^2

Live Load (on Plan) = 0.75 kN/m^2

Calculation of point load on nodes:

Dead load (on slope) = 1.25 kN/m^2

Total dead load = $1.25 \times 0.625 = 0.78125$ kN/m $\times 2 = 1.5625$ kN/m

= $1.5625 \times 2.5 = 3.90$ kN

Live load (on slope) = 0.75 kN/m^2

Total dead load = $0.75 \times 0.625 = 0.46875$ kN/m $\times 2 = 0.9375$ m

= $1.5625 \times 2.5 = 2.34$ kN

Total Point load, p = 1.4 DL +1.6 LL

= 1.4 (3.90 kN) +1.6 (2.34 kN)

= 9.2 kN

The point loads determined were applied on each node (Fig. 22) in order to analyses the truss model in STAAD Pro. Similarly, the load on each joint is obtained for all assumed truss models as is shown in Table 5.

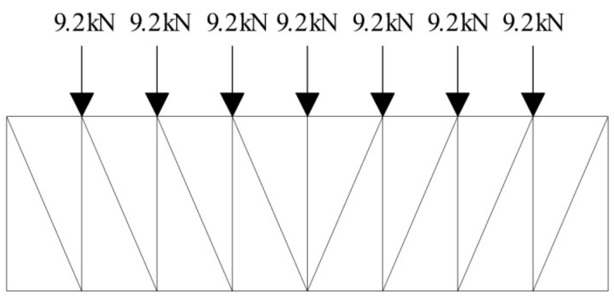


Figure 22: Point Loads Acting on Nodes

Table 5: The Calculated loads associated with the selected spans and bays

Span (m)		Truss Spaces (m)	DL (kN/m^2)	LL (kN/m^2)	Total DL (kN)	Total LL (kN)	Total PL (kN)
k=4	10	2.5	1.25	0.75	3.90	2.34	9.20
	20	5	1.25	0.75	15.60	9.40	36.88
	30	5	1.25	0.75	23.44	14.10	55.32
	40	5	1.25	0.75	31.25	18.75	73.75
k=5	10	2.5	1.25	0.75	3.12	2.00	7.57
	20	5	1.25	0.75	12.50	7.50	29.50
	30	5	1.25	0.75	18.75	11.25	44.25
	40	5	1.25	0.75	25.00	15.00	59.00
k=8	10						
	20	5	1.25	0.75	7.80	4.70	18.44
	30	5	1.25	0.75	11.72	7.00	27.10
	40	5	1.25	0.75	15.62	9.40	36.91
k=10	10						
	20						
	30	5	1.25	0.75	9.40	5.60	22.12
	40	5	1.25	0.75	12.5	7.50	29.50

4.1.9 Analysis by using STAAD Pro

It is indicated at the beginning of this chapter that the methodology of optimal truss determination mentioned in this research is by hand calculation, then creating the general formula in Maple, followed by the ratio calculated from Table Curve 2D and finally identify the member section properties required as a result of the analysis by STAAD Pro (Fig. 23). To achieve the deflection for each of the 11 Analysis procedures selected trusses, section properties from the analysis of all the truss members were substituted in the deflection formula (Δ) created in Maple. Therefore, the numerical amount of deflection for each selected truss is calculated and then compared to each other as detailed in the following chapter (Discussion and Conclusion).

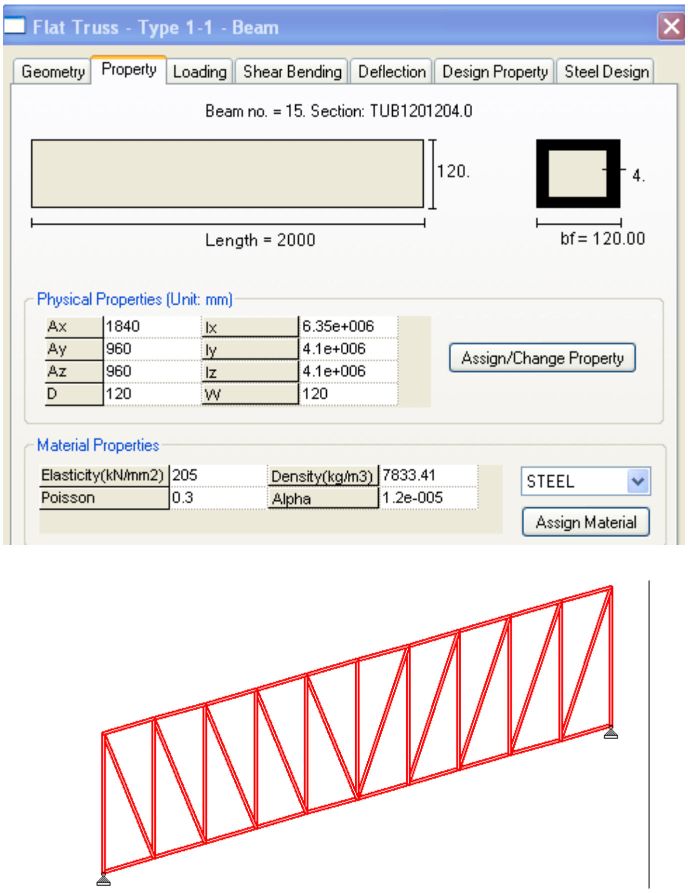


Figure 23: Section Property Resulted from STAAD Pro Analysis

4.2 Analysis Results

The deflection formula is derived and applied to the selected models in order to predict the optimal truss. The results are analysed based on the ratio of b/a for each truss model to highlight the least deflection value. The determination of the optimum truss is investigated by considering the characteristics of minimum deflection and stress as discussed below.

4.2.1 Determination of Optimal Truss

The deflections resulted from the analysis of different span lengths for all 11 types of flat, triangular and warren trusses were recorded. Hence, the least deflection and minimum stress was identified for each truss due to its model, type and span length.

The following tables and graphs are prepared to demonstrate the least deflection and minimum stress for the number of bays in each type and model of truss systems.

4.2.2 Deflection outputs

The deflection calculated for each model (flat, triangular and warren) are grouped in 3 individual tables (Tables 6, 7 and 8). Also the deflections for each stated circumstance of truss model, type and number of bays are presented for further discussions.

- **Flat Truss:** The mid-span deflections for the 5 different types of flat trusses are calculated as given below (Table 6).

Table 6: Deflections for various spans of Flat Trusses

Span(m) \ Type		Deflection (Δ) (mm)				
		1	**2**	**3**	**4**	**5**
k=4	**10**	2.12	**0.85**	1.12	1.07	1.30
	20	4.33	**1.72**	2.70	2.55	3.80
	30	5.83	**2.66**	3.94	3.74	4.52
	40	6.88	**3.14**	5.00	4.73	5.73
k=5	**10**	2.53	1.81	1.50	**1.02**	1.92
	20	4.83	3.45	3.86	**2.27**	4.33
	30	7.00	4.12	4.61	**3.00**	5.84
	40	7.30	5.22	5.83	**4.24**	6.60
k=8	**10**					
	20	7.60	**3.70**	5.40	4.25	7.03
	30	9.00	**4.40**	7.70	5.04	8.31
	40	11.88	**5.60**	10.22	6.43	11.04
k=10	**10**					
	20					
	30	9.50	**5.70**	8.41	6.40	8.95
	40	10.7	**6.40**	9.47	7.20	10.1

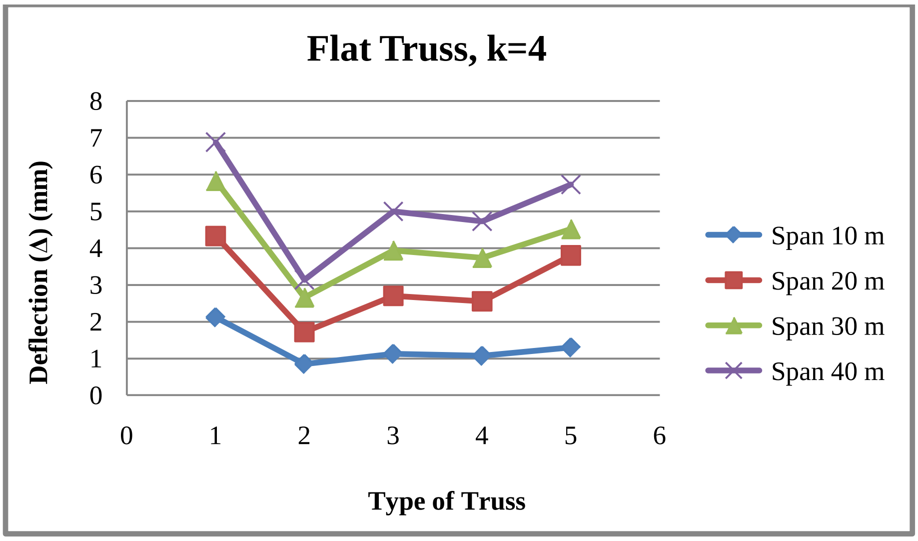


Figure 24: Graphical comparison of deflections obtained for Flat Trusses with k=4

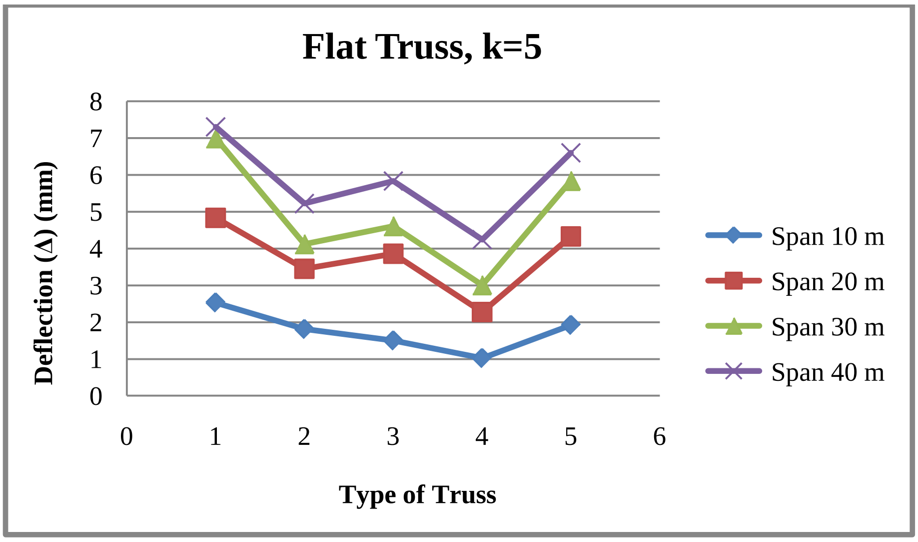


Figure 25: Graphical comparison of deflections obtained for Flat Trusses with k=5

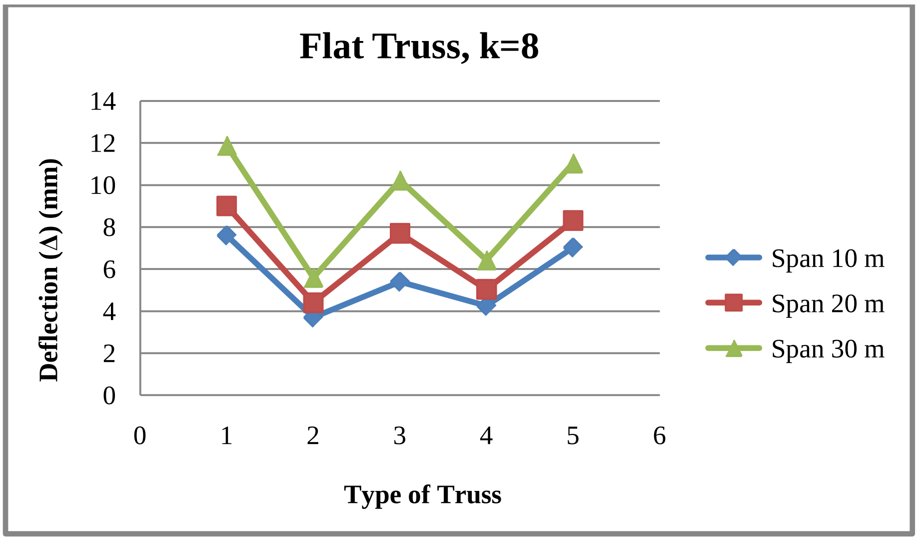


Figure 26: Graphical comparison of deflections obtained for Flat Trusses with k=8

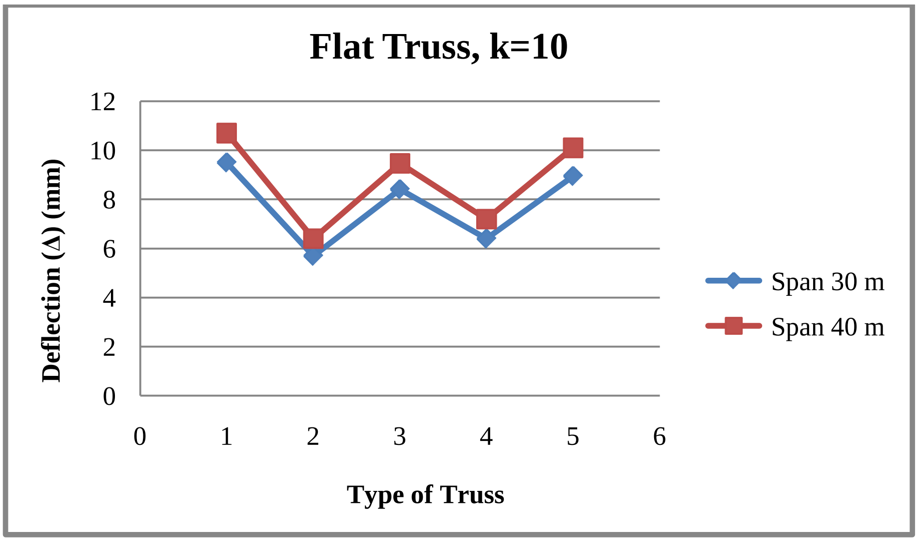


Figure 27: Graphical comparison of deflections for the Flat Trusses with k=10

- **Warren Truss**: Three different types of warren trusses with three different slopes of 10%, 15% and 20% were considered and the deflections for each type and top chord slope were calculated in Table 7.

Table 7: Deflections of Warren trusses with different top chord slopes

Type / Span (m)		Deflection (Δ) (mm)								
		1			2			3		
		10%	**15%**	**20%**	**10%**	**15%**	**20%**	**10%**	**15%**	**20%**
k=4	**10**	1.80	1.67	**1.55**	0.64	0.61	**0.58**	0.99	0.93	**0.87**
	20	4.46	4.13	**3.78**	1.53	1.45	**1.40**	2.40	2.30	**2.10**
	30	5.30	4.92	**4.58**	2.40	2.27	**2.16**	3.50	3.30	**3.10**
	40	7.00	6.50	**6.00**	2.80	2.67	**2.54**	4.40	4.15	**3.90**
k=5	**10**	2.15	1.98	**1.85**	1.10	1.00	**0.96**	1.48	1.37	**1.28**
	20	5.50	5.10	**4.73**	3.00	2.80	**2.66**	3.34	3.11	**2.90**
	30	6.36	5.87	**5.46**	4.04	3.80	**3.60**	4.51	4.20	**3.90**
	40	8.70	8.00	**7.50**	4.80	4.50	**4.24**	5.33	5.00	**4.60**
k=8	**10**									
	20	6.92	6.30	**5.75**	3.10	2.97	**2.80**	6.00	5.50	**5.03**
	30	7.88	7.16	**6.53**	4.30	3.98	**3.74**	6.90	6.30	**5.80**
	40	10.86	9.80	**9.00**	5.10	4.75	**5.92**	9.52	8.70	**7.94**
k=10	**10**									
	20									
	30	10.38	9.36	**8.48**	5.20	4.80	**4.50**	7.92	7.20	**6.50**
	40	12.13	10.94	**9.91**	6.86	6.35	**5.91**	10.88	9.84	**8.90**

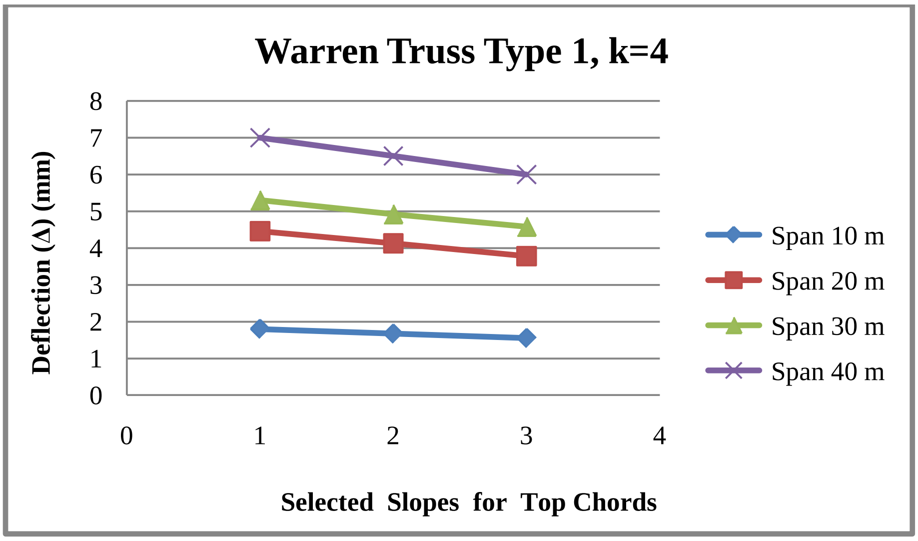


Figure 28: Graphical comparison of deflections of Warren Trusses Type 1 with k=4 and for the three different top chord slopes

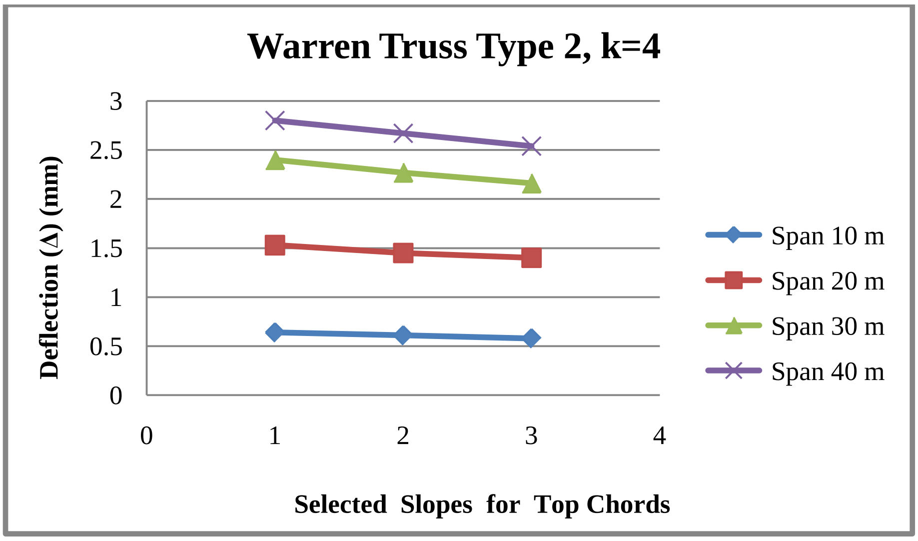


Figure 29: Graphical comparison of deflections of Warren Trusses Type 2 with k=4 and for the three different top chord slopes

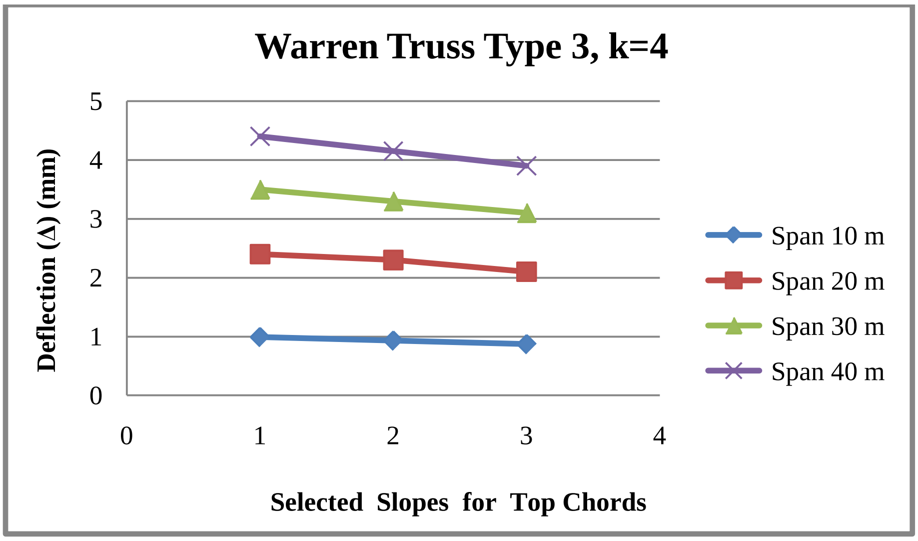


Figure 30: Graphical comparison of deflection of Warren Trusses Type 3 with k=4 and for the three different top chord slopes

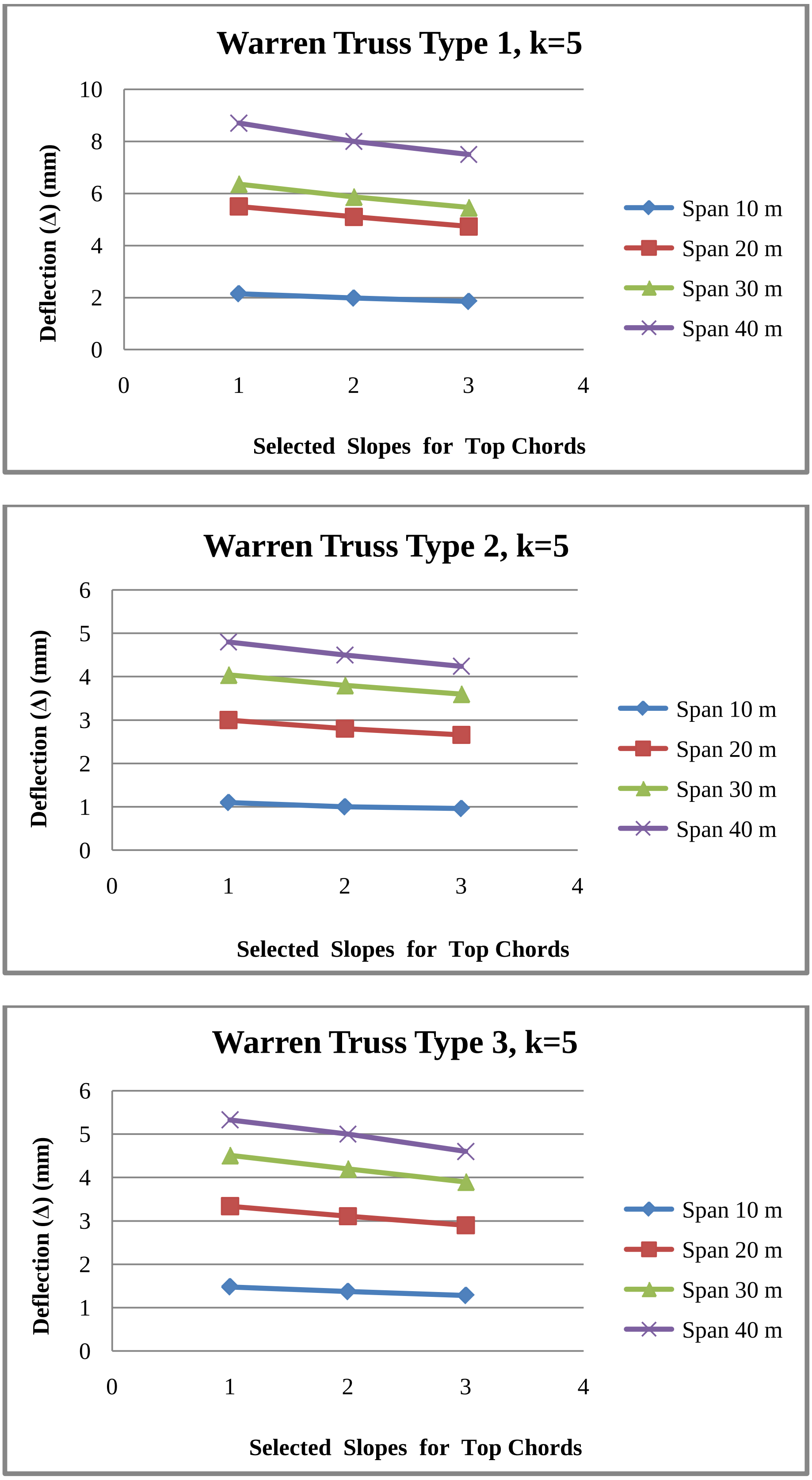
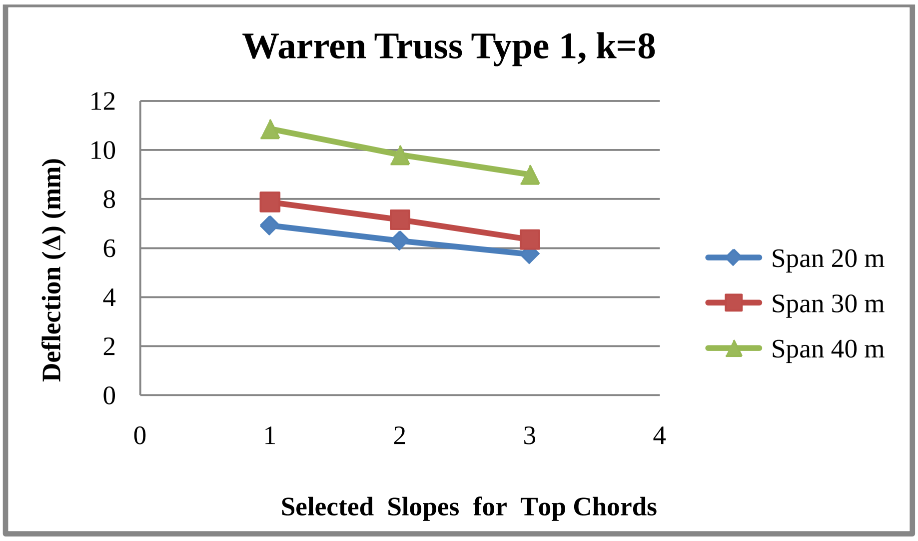
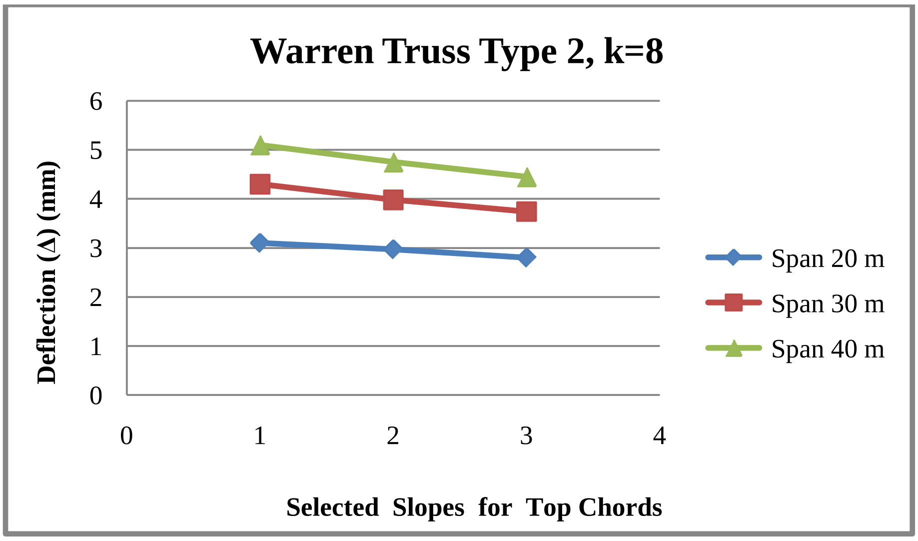


Figure 31: Graphical comparison of deflection of Warren Trusses with n=k and for the three different top chord slopes





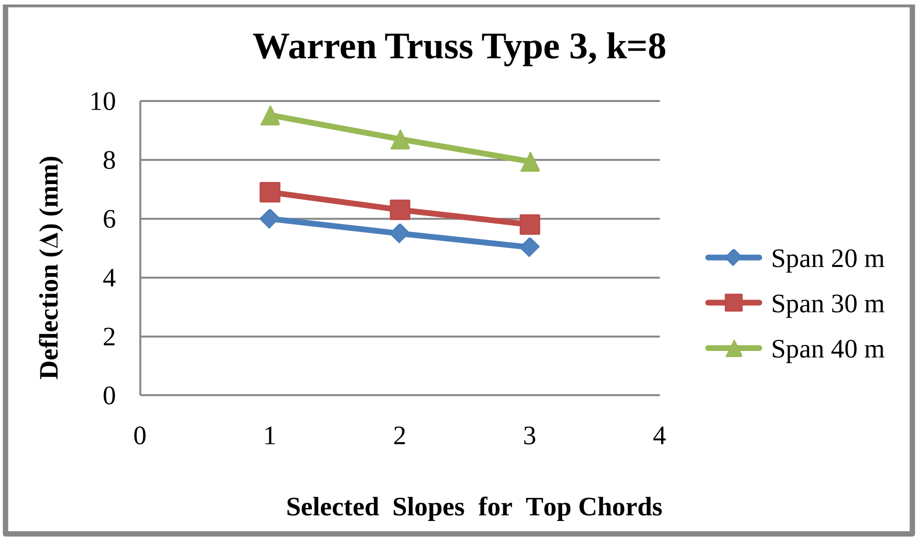
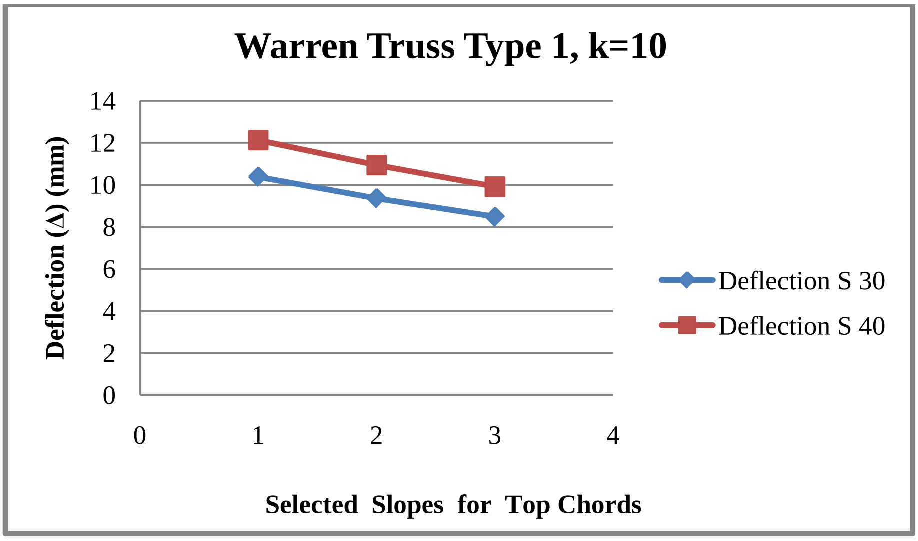
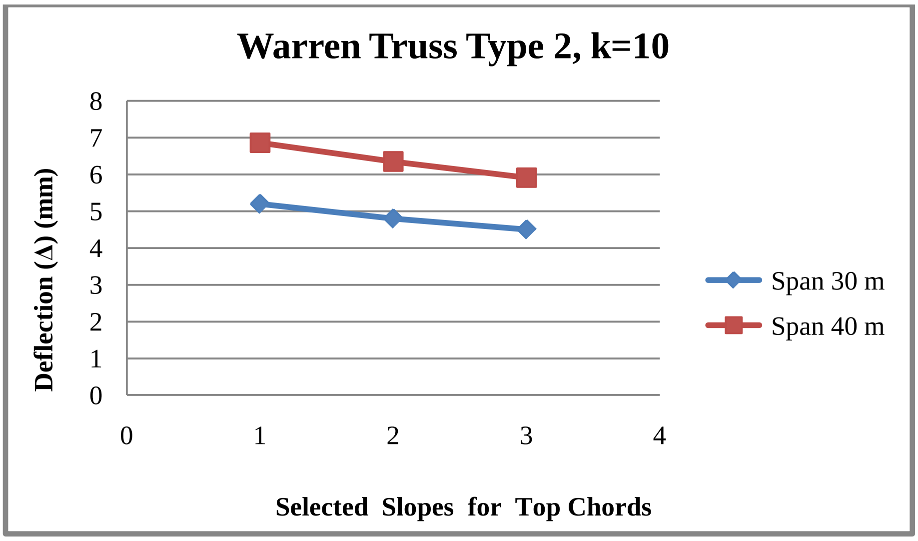


Figure 32: Graphical comparison of deflection of Warren Trusses with k=8 and for the three different top chord slopes





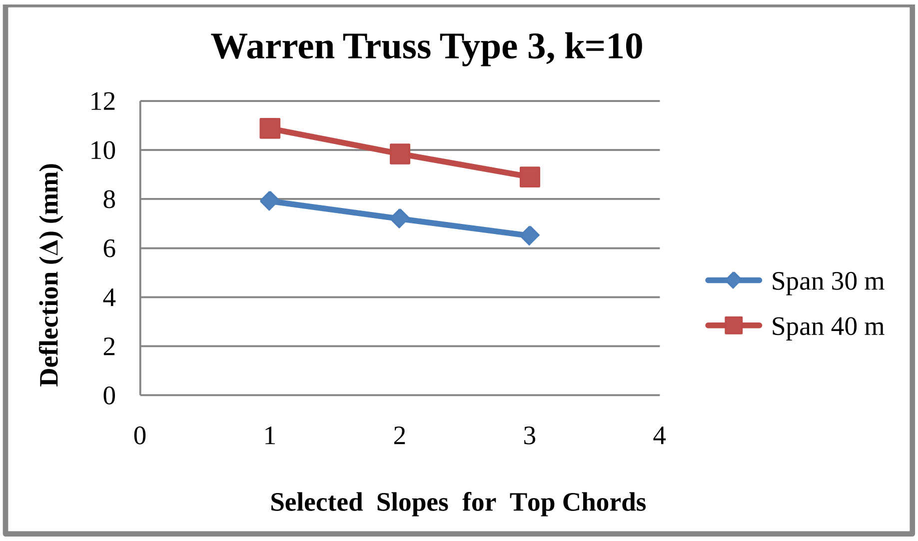


Figure 33: Graphical comparison of deflection of Warren Trusses with k=10 and for the three different top chord slopes

The comparison of the deflection values of warren trusses for three different slopes resulted in identifying the degree of slope that contributes to the most optimum deflection. The results are given in the following graphs in Figures 34 to 37 for n values of 4, 5, 8 and 10 respectively.

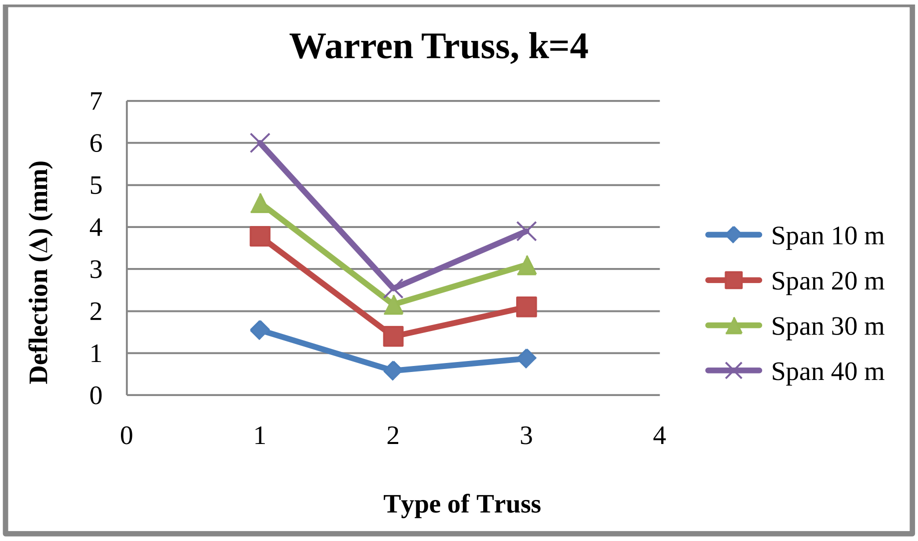


Figure 34: Graphical comparison of deflection occurred due to optimal slope for Warren Truss with k=4

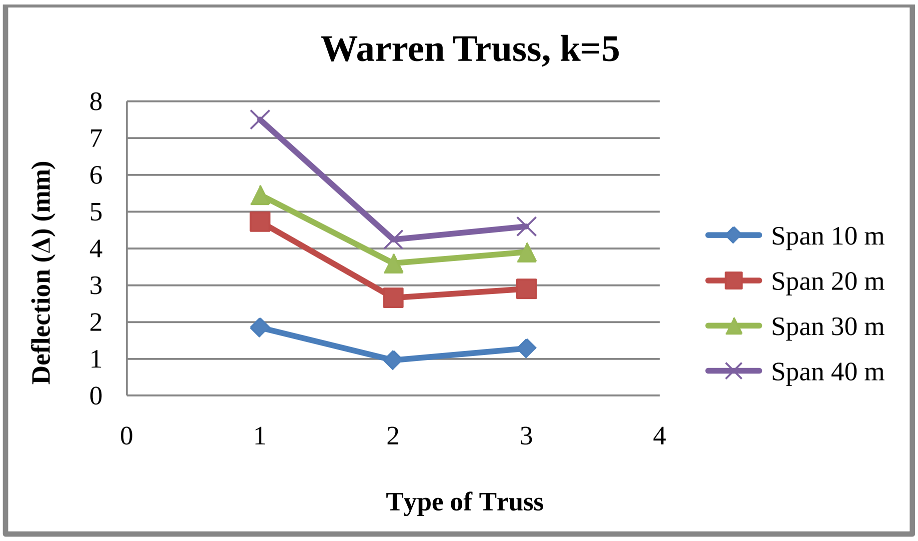


Figure 35: Graphical comparison of deflection occurred due to optimal slope for Warren Truss with k=5

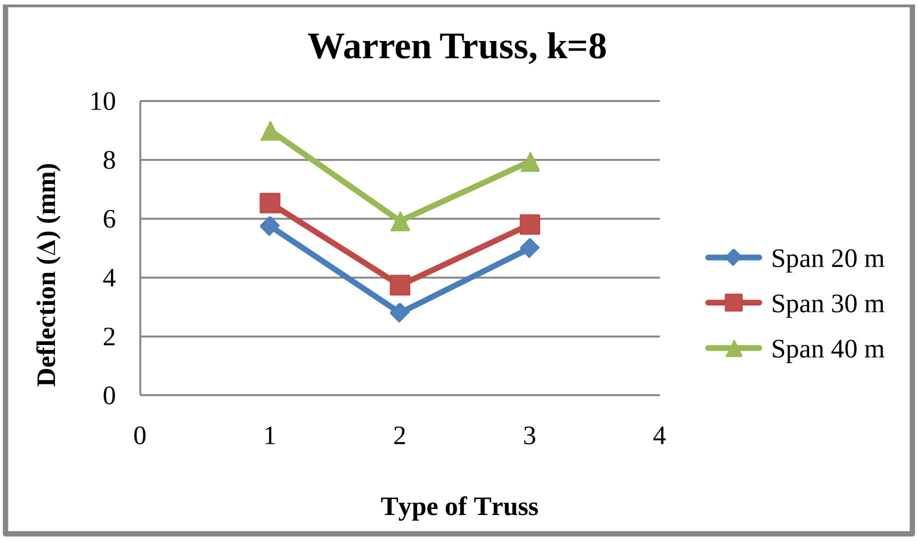


Figure 36: Graphical comparison of deflection occurred due to optimal slope for Warren Truss with n=8

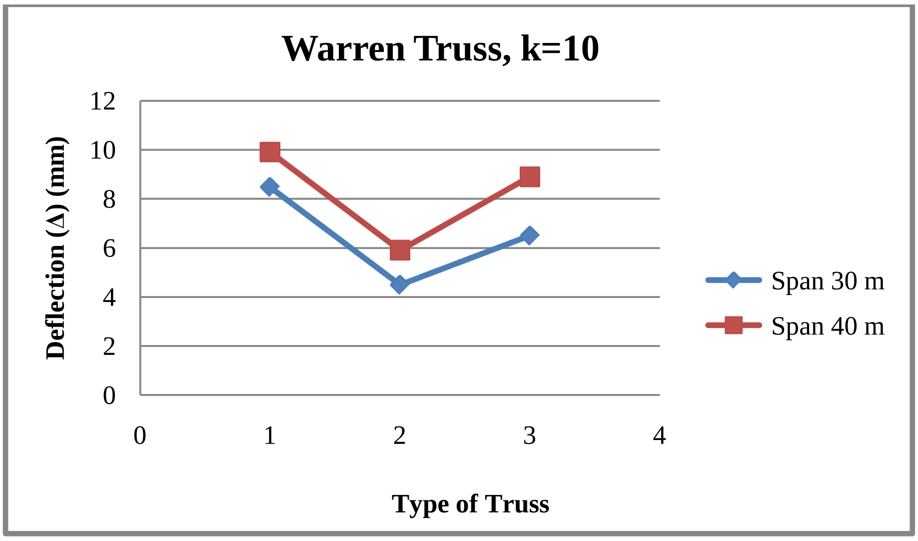


Figure 37: Graphical comparison of deflection occurred due to optimal slope for Warren Truss with k=10

- **Triangular Truss**: Three different types of triangular trusses were investigated, deflections calculated and presented in Table 8 and Figures 38, 39 and 40.

Table 8: Deflection obtained from the analysis of Triangular Trusses

Span(m) \ Type		Deflection (Δ) (mm) 1	2	3
k=4	10	0.80	**0.58**	0.82
	20	1.93	**1.39**	1.96
	30	2.73	**1.98**	2.78
	40	2.16	**1.58**	2.21
k=5	10	0.66	**0.46**	0.677
	20	1.75	**1.24**	1.80
	30	2.81	**1.98**	2.90
	40	2.23	**1.60**	2.30
k=8	10			
	20	1.83	**1.24**	1.86
	30	2.00	**1.37**	2.05
	40	2.12	**1.44**	2.15

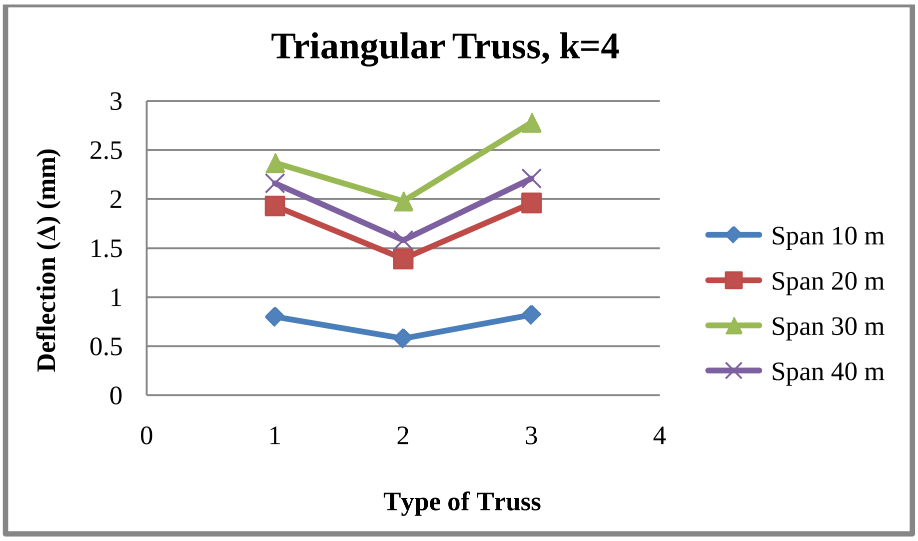


Figure 38: Graphical comparison of deflections obtained from the analysis of Triangular Trusses with k=4

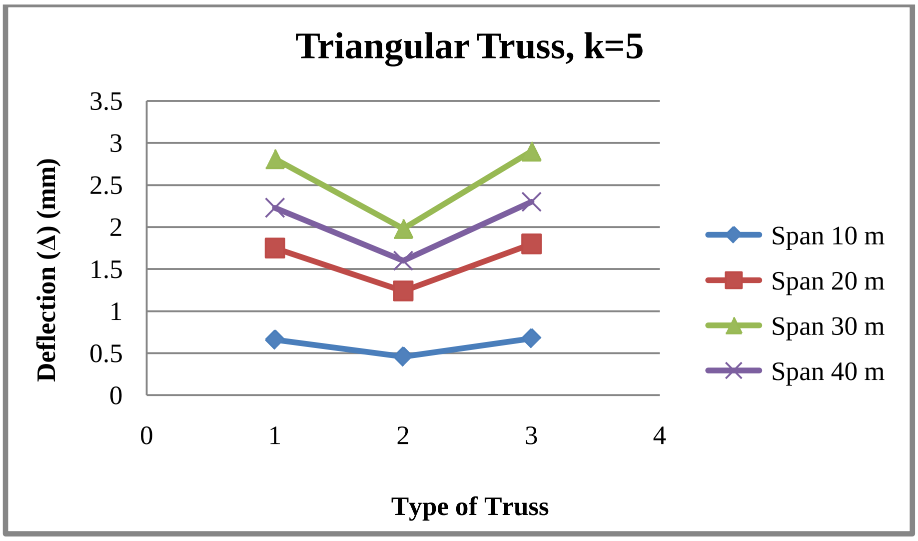


Figure 39: Graphical comparison of deflections obtained from the analysis of Triangular Trusses with k=5

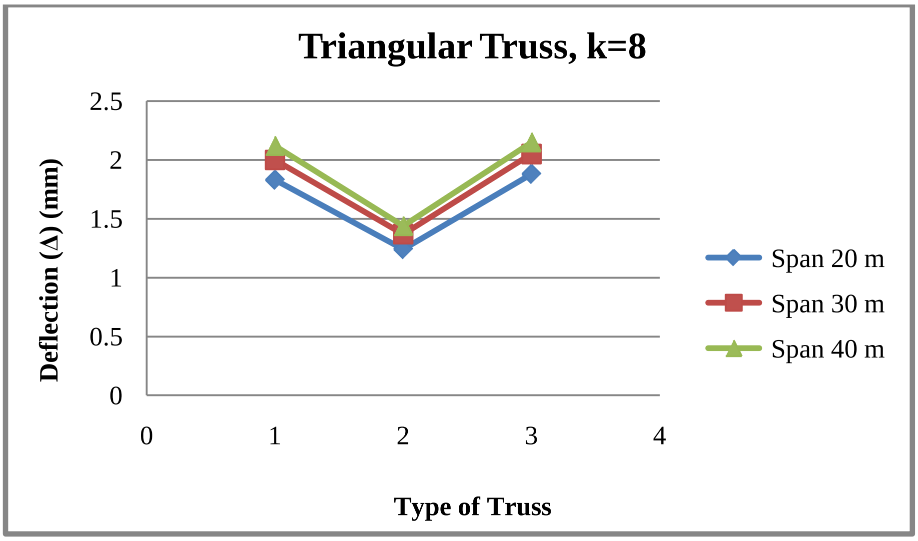


Figure 40: Graphical comparison of deflections obtained from the analysis of Triangular Trusses with k=8

4.2.3 Truss Members' Axial Stresses

The minimum stress obtained as a result of the analysis of each model (flat, triangular and warren) is grouped in similar manner as the deflection for each model in 3 single tables (Tables 9, 10 and 11). Also each stress obtained from different truss models, truss types and numbers of bays are plotted individually in Figures 41 to 55.

- **Flat Truss:** The trusses selected are flat top chord with 5 different types. The stresses were calculated for each type.

Table 9: Stress values obtained for Flat Trusses

Span(m) \ Type		Stress (σ) (kN/mm²) 1	2	3	4	5
k=4	**10**	0.0434	**0.0322**	0.0400	0.0380	0.0430
	20	0.0510	**0.0325**	0.0477	0.0464	0.0514
	30	0.0507	**0.0340**	0.0470	0.0455	0.0500
	40	0.0480	**0.0300**	0.0440	0.0432	0.0480
k=5	**10**	0.0500	**0.0400**	0.0466	**0.0380**	0.0588
	20	0.0740	**0.0610**	0.0650	**0.0430**	0.0697
	30	0.0680	**0.0468**	0.0520	**0.0390**	0.0626
	40	0.0670	**0.0450**	0.0500	**0.0420**	0.0530
k=8	**10**					
	20	0.1000	**0.0660**	0.0800	0.0710	0.1000
	30	0.0812	**0.0520**	0.0770	0.0560	0.0800
	40	0.0800	**0.0500**	0.0760	0.0540	0.0790
k=10	**10**					
	20					
	30	0.0830	**0.0640**	0.0785	0.0684	0.0810
	40	0.0700	**0.0530**	0.0660	0.0570	0.0670

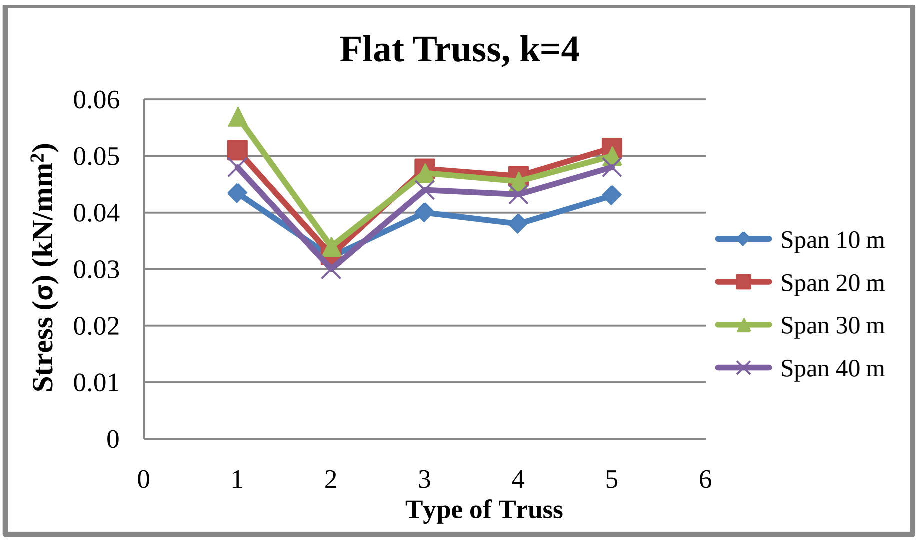
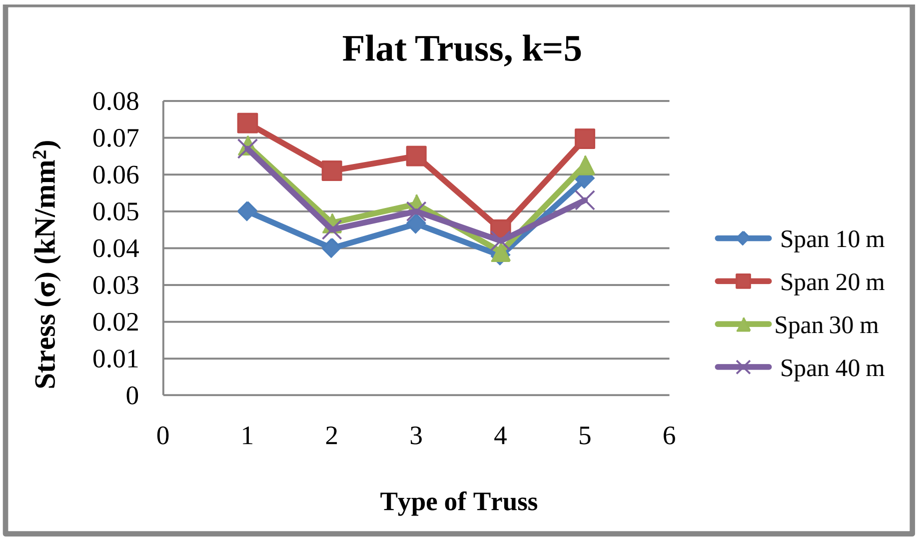
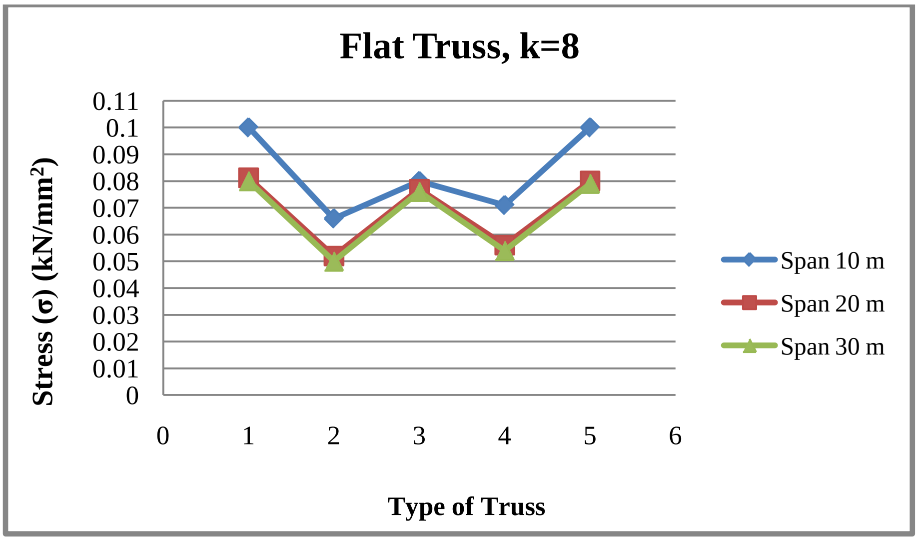


Figure 41: The comparison of stresses for Flat Trusses with k=4





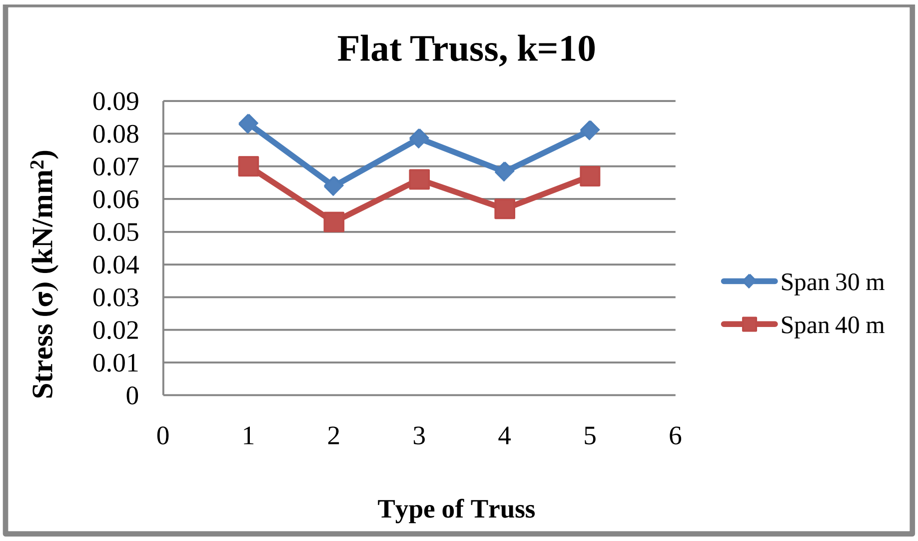


Figure 42: The comparison of stresses for Flat Trusses with k=5, 8, and 10

- **Warren Truss:** Since the truss assumed to be Warren with 3 different types, therefore related deflection are calculated and compared first based on assumed slopes and then through the stated characteristics of stress.

Table 10: The obtained amount for stress with 3 assumed slope in Warren Truss

Span (m) \ Type		Stress (σ) (kN/mm^2)								
		1			2			3		
		10%	15%	20%	10%	15%	20%	10%	15%	20%
k=4	10	0.062	0.061	**0.060**	0.028	0.027	**0.026**	0.037	0.036	**0.035**
	20	0.077	0.075	**0.074**	0.033	0.032	**0.031**	0.044	0.043	**0.042**
	30	0.065	0.064	**0.063**	0.034	0.033	**0.032**	0.043	0.042	**0.041**
	40	0.060	0.059	**0.058**	0.030	0.029	**0.028**	0.041	0.040	**0.039**
k=5	10	0.070	0.068	**0.066**	0.041	0.040	**0.039**	0.053	0.052	**0.050**
	20	0.097	0.095	**0.092**	0.058	0.057	**0.056**	0.063	0.061	**0.059**
	30	0.071	0.070	**0.068**	0.052	0.051	**0.050**	0.056	0.054	**0.052**
	40	0.073	0.072	**0.070**	0.046	0.045	**0.044**	0.049	0.048	**0.046**
k=8	10									
	20	0.110	0.106	**0.103**	0.063	0.061	**0.058**	0.100	0.097	**0.094**
	30	0.083	0.081	**0.078**	0.056	0.055	**0.053**	0.077	0.075	**0.072**
	40	0.086	0.083	**0.079**	0.050	0.049	**0.047**	0.080	0.077	**0.074**
k=10	10									
	20									
	30	0.106	0.103	**0.097**	0.065	0.063	**0.061**	0.084	0.081	**0.077**
	40	0.092	0.089	**0.085**	0.064	0.062	**0.060**	0.087	0.083	**0.079**

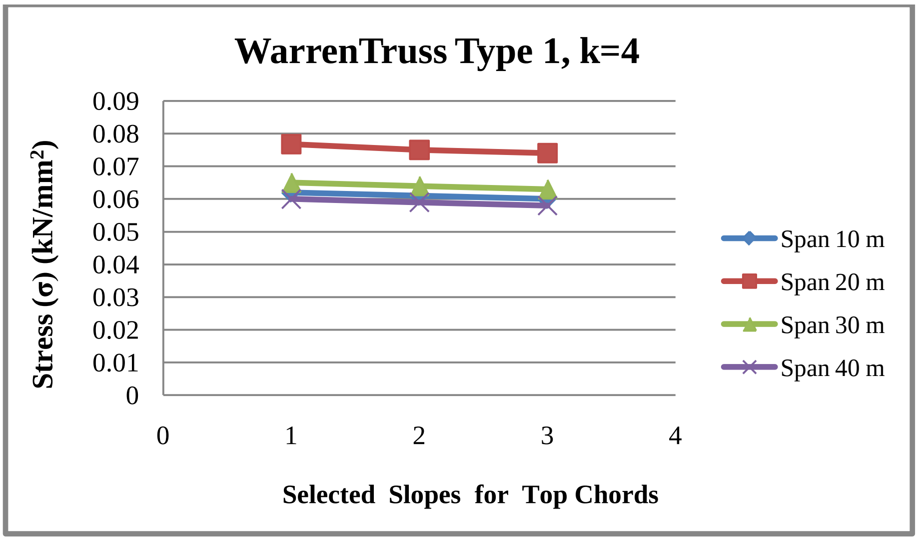


Figure 43: Graphical comparison of stress of Warren Truss, Type 1, with k=4 and for the three different top chord slopes

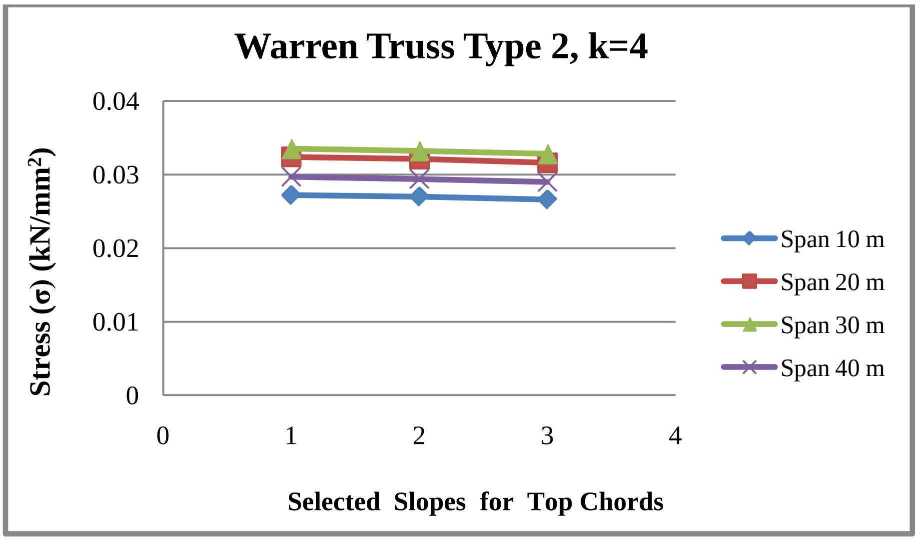


Figure 44: Graphical comparison of stress of Warren Truss, Type 2, with k=4 and for the three different top chord slopes

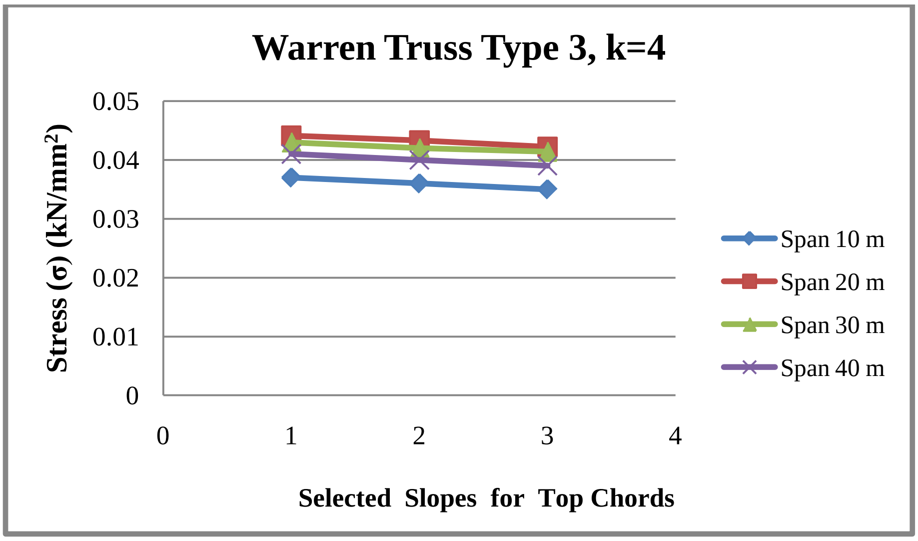


Figure 45: Graphical comparison of stress of Warren Truss, Type 3, with k=4 and for the three different top chord slopes

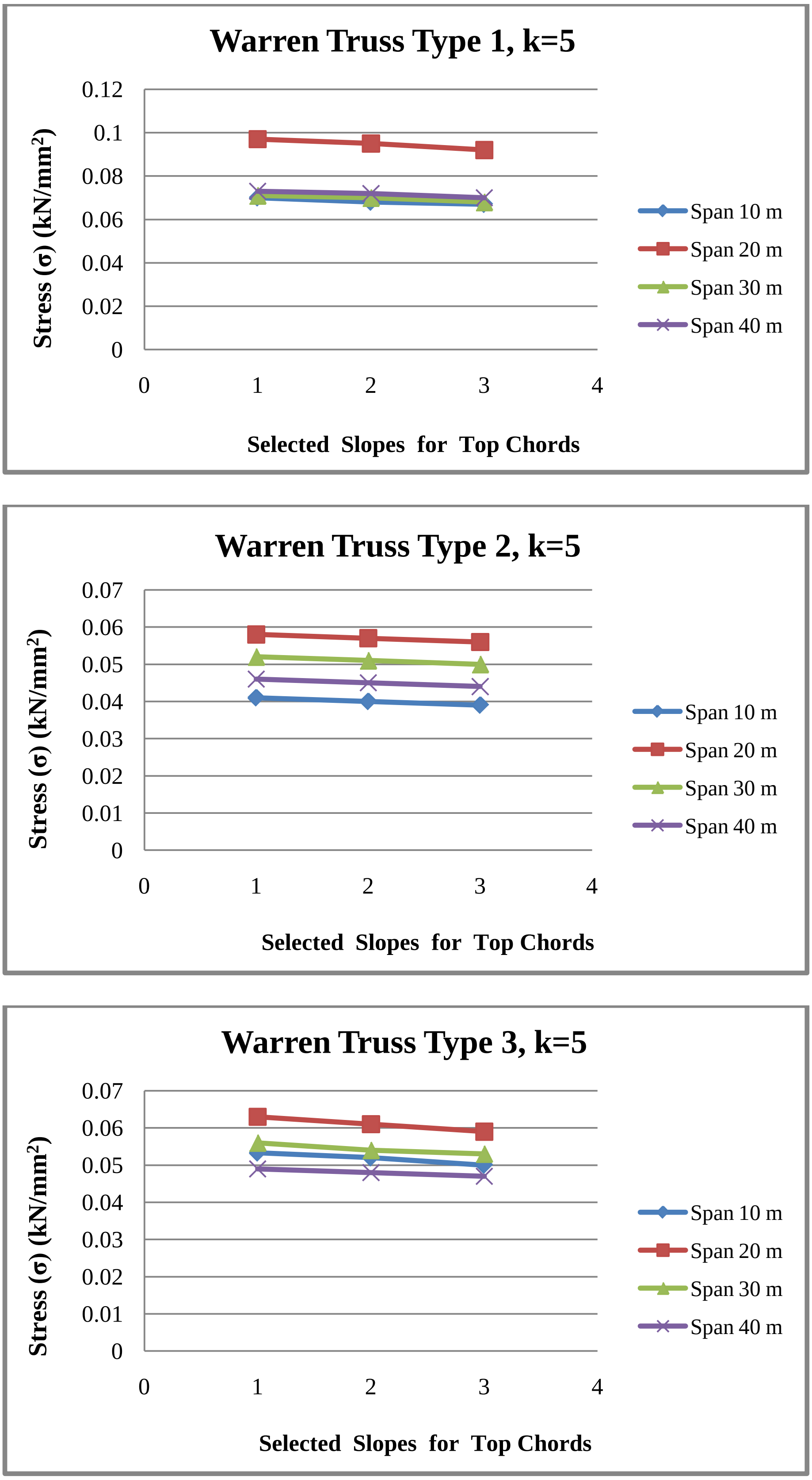


Figure 46: Graphical comparison of stress of Warren Trusses with k=5 and for the three different top chord slopes

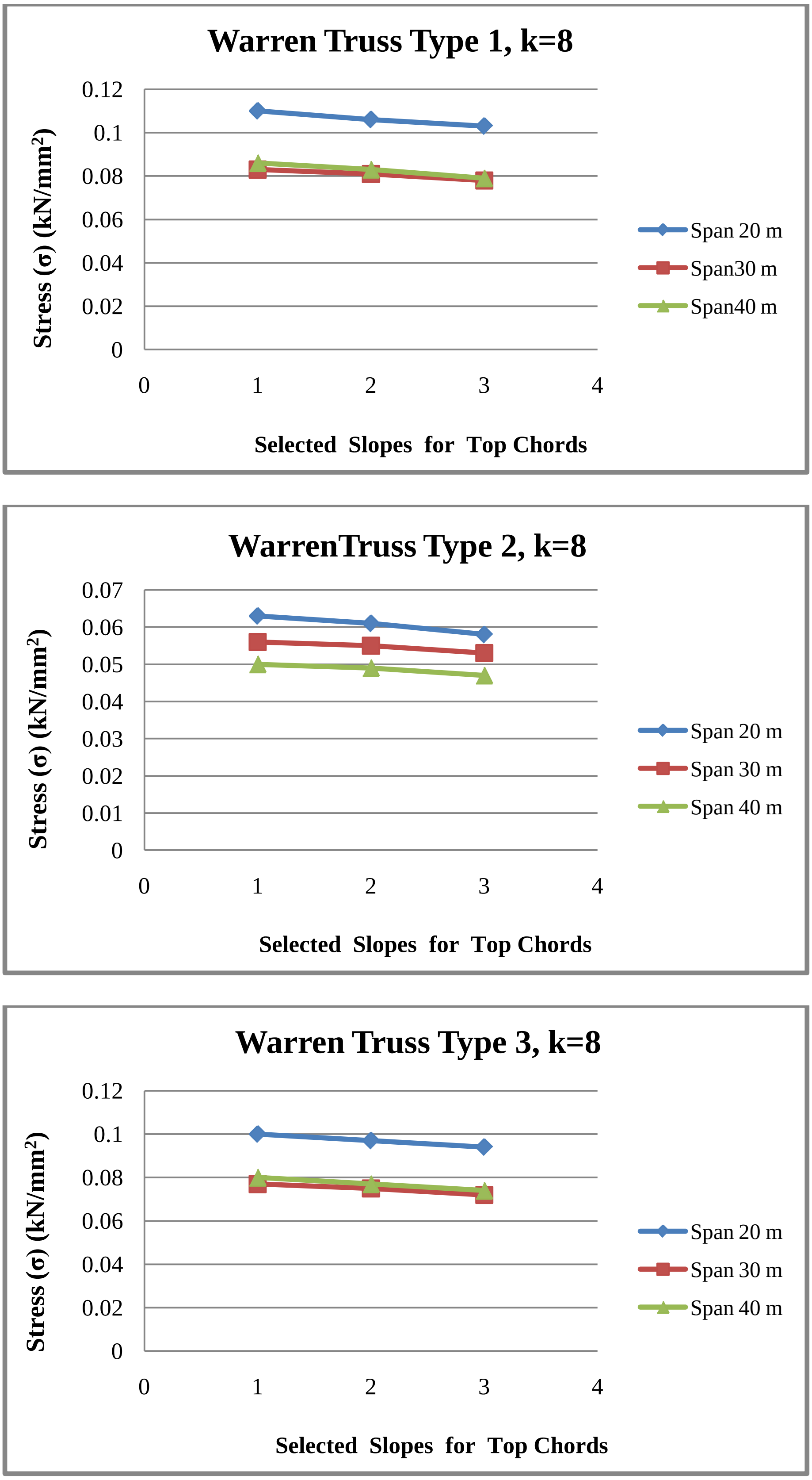


Figure 47: Graphical comparison of stress of Warren Trusses with k=8 and for the three different top chord slopes

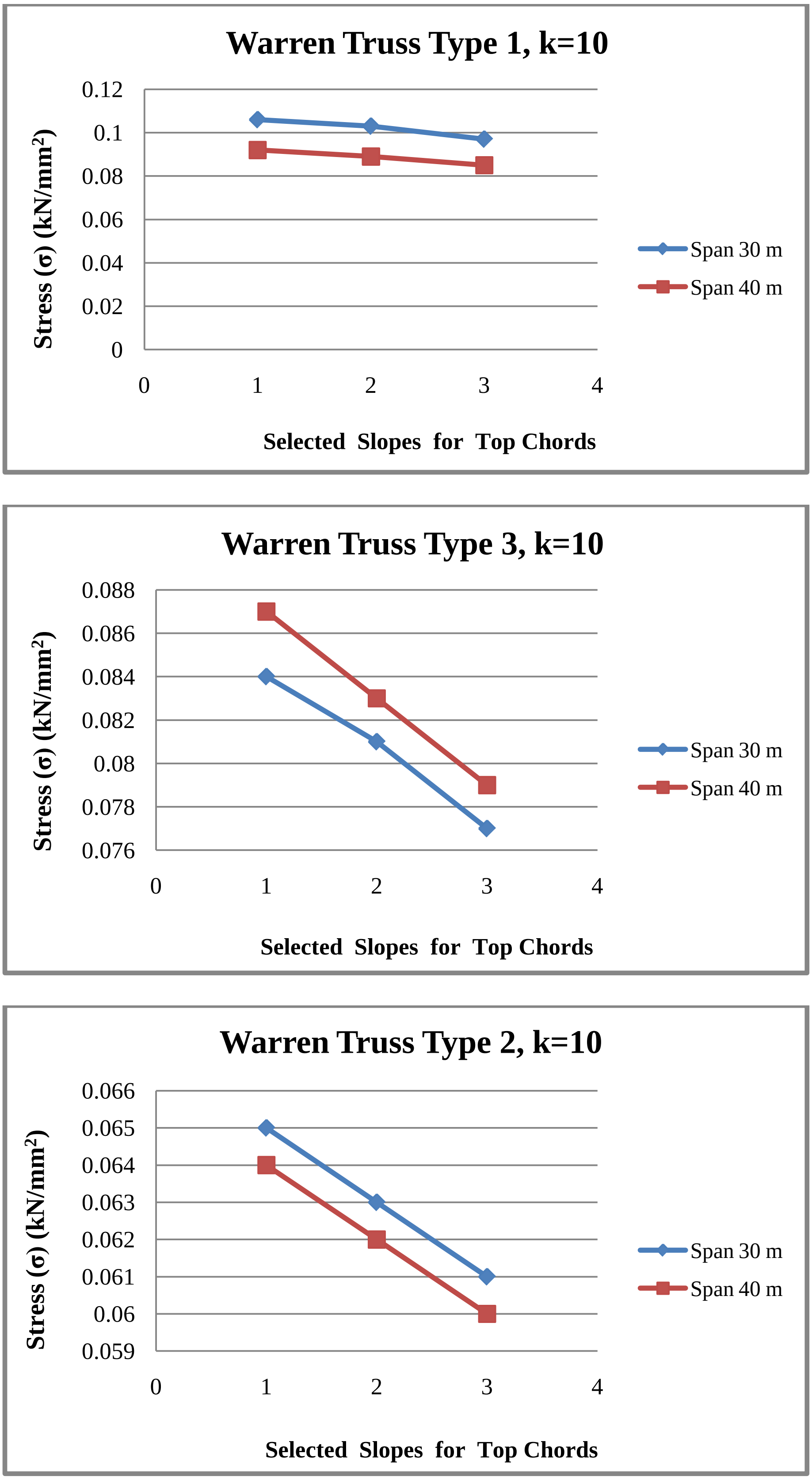


Figure 48: Graphical comparison of stress of Warren Trusses with k=10 and for the three different top chord slopes

The comparison of warren truss stresses were carried out for the three different slopes first and then it was done for the optimum slope selected. The following graphs in Figures 49 to 52 gives the stress values for the truss types with the optimum slope values.

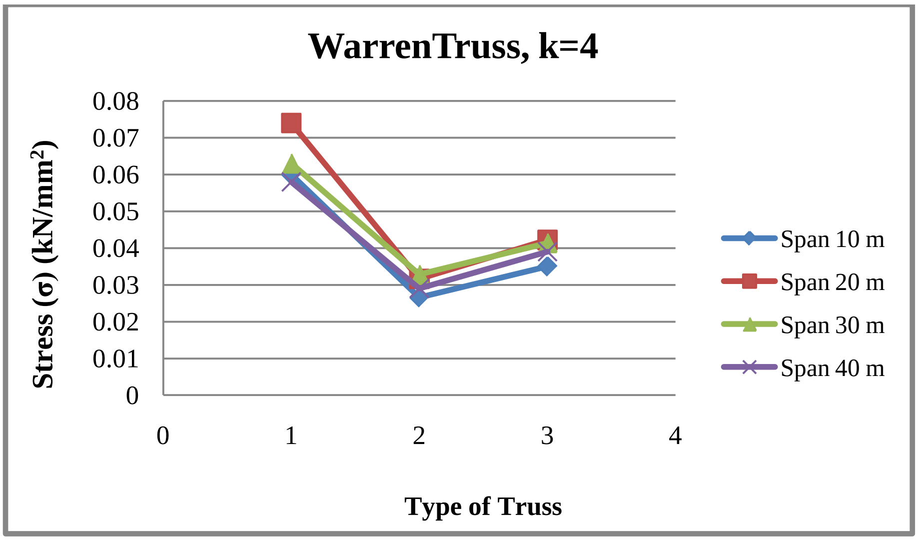


Figure 49: The comparison of stresses due to optimal slope for Warren Trusses with k=4

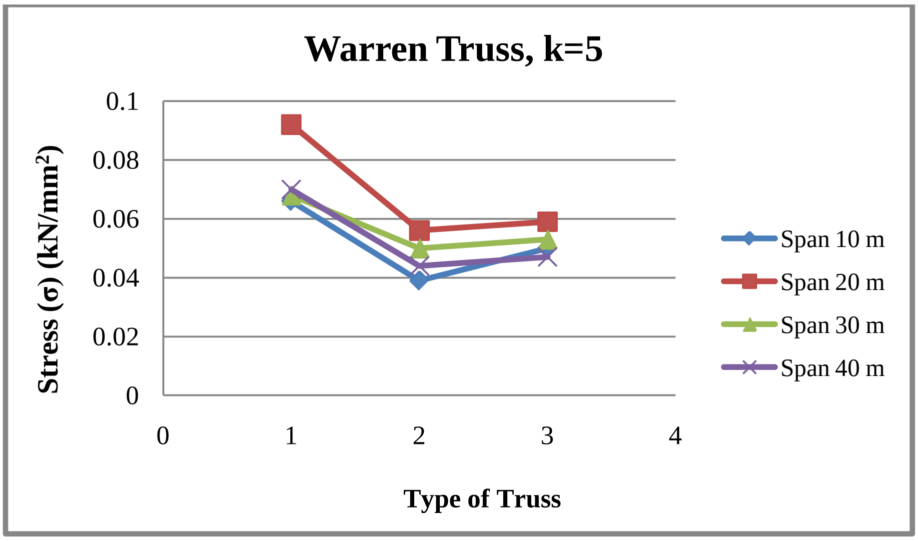


Figure 50: The comparison of stresses due to optimal slope for Warren Trusses with k=5

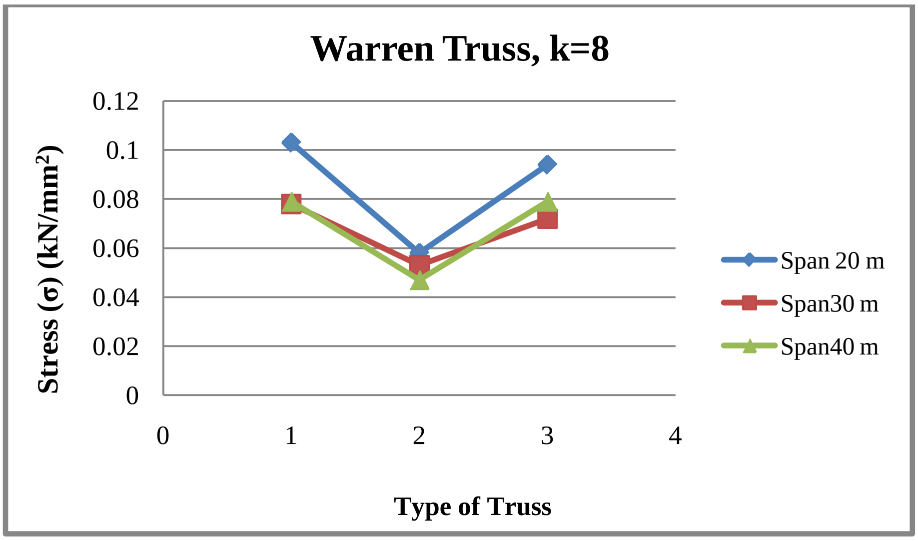


Figure 51: The comparison of stresses due to optimal slope for Warren Trusses with k=8

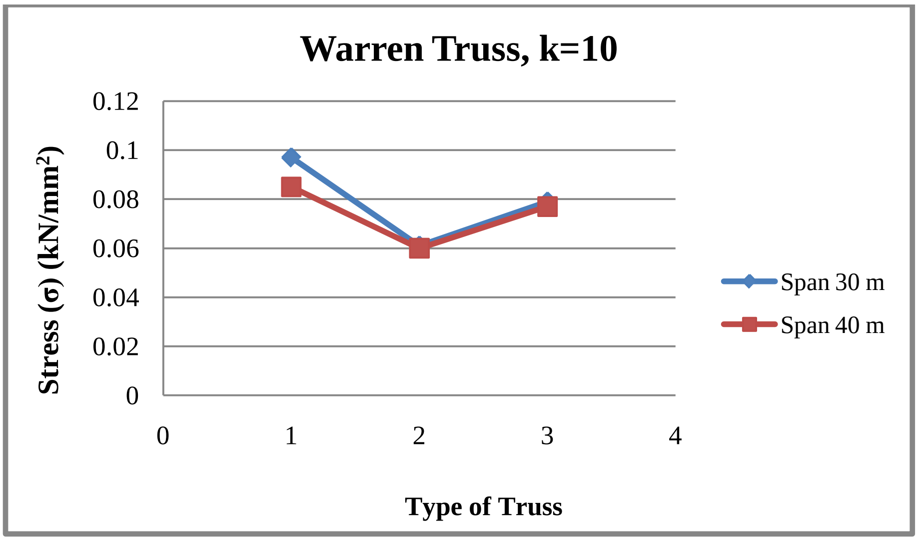


Figure 52: The comparison of stresses due to optimal slope for Warren Trusses with k=10

- **Triangular Truss**: Three different types of triangular trusses are analyzed and the stresses are presented in (Table 11).

Table 11: Triangular Truss stresses obtained from the analysis

Span(m) \ Type		Stress (σ) (kN/mm^2)		
		1	**2**	**3**
k=4	**10**	0.0186	**0.0134**	0.0188
	20	0.0220	**0.0160**	0.0225
	30	0.0212	**0.0150**	0.0213
	40	0.0126	**0.0090**	0.0127
k=5	**10**	0.0140	**0.0103**	0.0142
	20	0.0193	**0.0143**	0.0196
	30	0.0206	**0.0153**	0.0209
	40	0.0123	**0.0091**	0.0125
k=8	**10**			
	20	0.0184	**0.0145**	0.0185
	30	0.0136	**0.0107**	0.0137
	40	0.0102	**0.0080**	0.0103

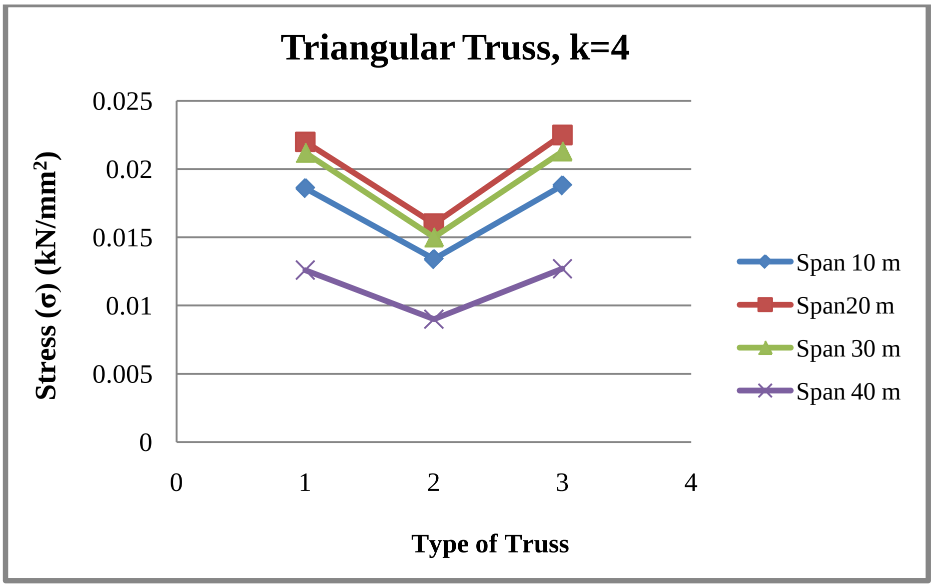


Figure 53: The comparison of stress for Triangular Trusses with k=4

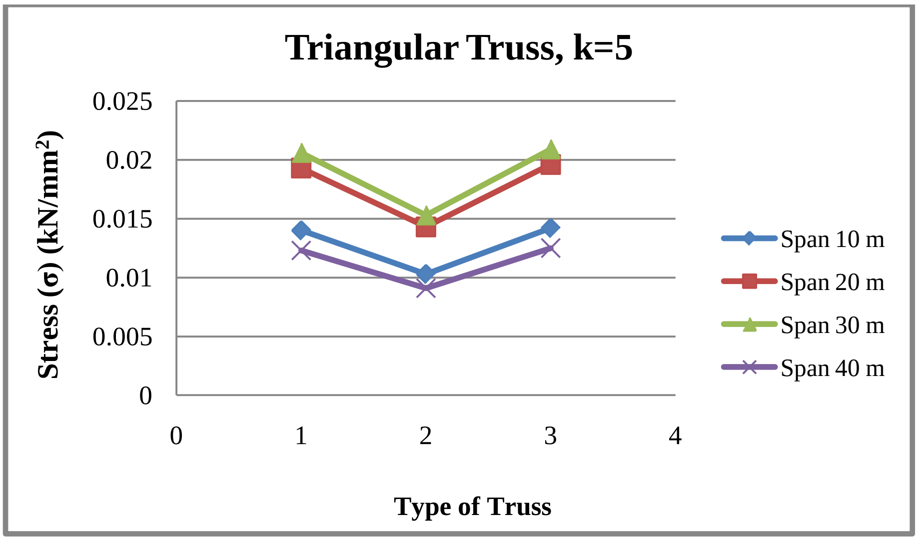


Figure 54: The comparison of stress for Triangular Trusses with k=5

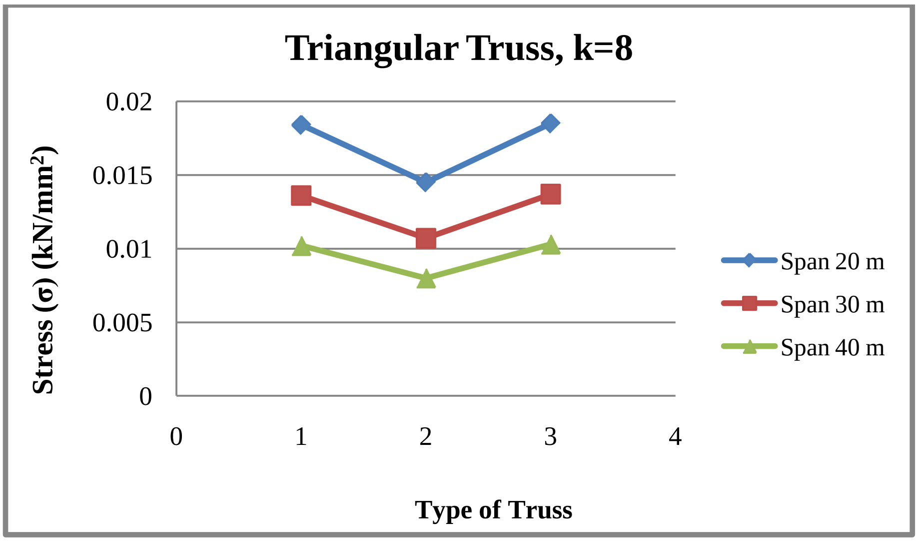


Figure 55: The comparison of stress for Triangular Trusses with k=8

So far, the deflections and stresses obtained from the analysis of trusses are presented numerically and graphically. The main aim is to determine the truss type with the least deflection and minimum stress value. This will lead to the optimum truss type. Chapter 5 (Discussions and Conclusion) will discuss in details about the study of finding the optimum truss type via investigating all the results from the truss types considered. In addition, a brief discussion of results is included about each optimum truss identified.

Chapter 5

DISCUSSION AND CONCLUSION

5.1 Discussion

The truss models selected for this research and the details given in chapter 3 (methodology) were studied for different span lengths (chapter 4 Mathematical Formulation and Analysis). The investigation was carried out by applying multiple point loads at truss nodal points by considering height and bay as parametric terms. The purpose was to find the optimum truss with sufficient height and bay for the various selected spans. Meanwhile this approach was emerged solely by obtaining the minimum stress and the minimum deflections at mid span of trusses (chapter 4). As a result Tables 12 and 13 give the types of trusses that satisfy the minimum stress and deflection values. The optimum trusses among the 11 selected models with various span lengths are highlighted. Moreover these results are compared with the existing cases. Hence it was found out that there is a substantial difference between the deflection values of the two approaches.

It is stated in previous chapter that the truss span lengths were applied in a different manner to various types of the truss models. Therefore, in case of trusses with 4 and 5 bays spans of 10, 20, 30 and 40 meters were considered. But in the case of 8 and 10 bays the span lengths used were 20, 30 and 40 meters and 30 and 40 meters, respectively. In other words, if the length of span is increasing the number of bays (***k***) should increase correspondingly. Otherwise the proportion for each bay length to the height of the related truss member would be a non-practical geometrical shape with inefficient long length in each bay. In addition, the optimum truss types were satisfied in all the selected span lengths but because of the above mentioned reasons only the most suitable ones were considered for further comparison with the existing method.

Thus the results obtained are only focused on specific span lengths which produced the optimum trusses. It should be noted that the following given table is generated by using different span lengths for the given number of bays. Moreover, the optimum truss types were changed due to the use of odd and even numerical values for the bays.

In order to get a better perception of the optimum truss, clarification of all the above discussions were investigated in each truss model numerically and graphically.

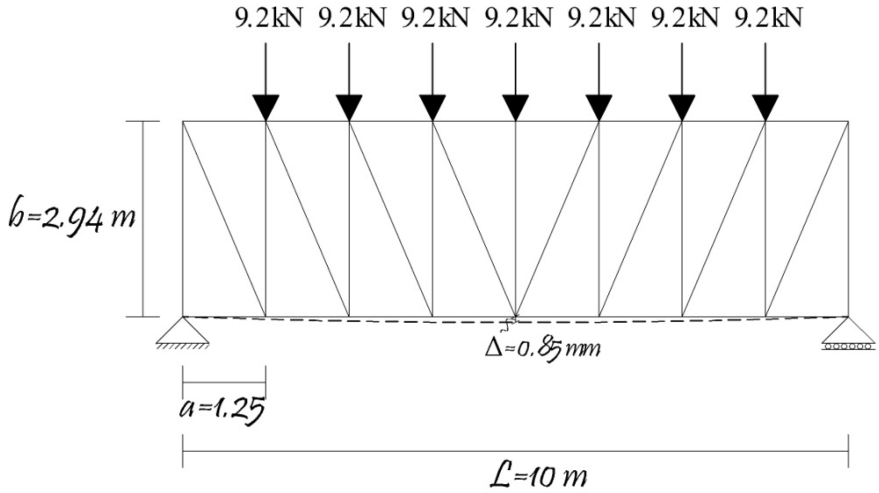
5.1.1 Deflection and Stress Approaches

Optimum truss for group one (flat truss) is obtained for the selected span lengths for types 2 and 4 in case of even number of bays and odd number of bays respectively. It is obvious that, for all the selected span lengths with k=4 the optimum flat truss is obtained for type two truss having the least mid-span deflection equal to 0.85 mm. As it is given in Table 12, for the same truss type, the minimum deflection is obtained as 1.83 mm by using the current available method of deflection calculation, and this value is greater than the 0.85 mm deflection obtained from the optimized deflection formula. Similarly, for 30 meters of span length with k= 8 and for 40 meters of span length with k=10 the optimum trusses are from type two with considerable differences when compared to the current available method of deflection calculation. Only in case of truss span with 20 meter length and k=5 the optimum truss was obtained from type four. Although the optimum truss is type four in this case, there is still substantial difference between the values of optimized minimum deflection and the one obtained from the current method.

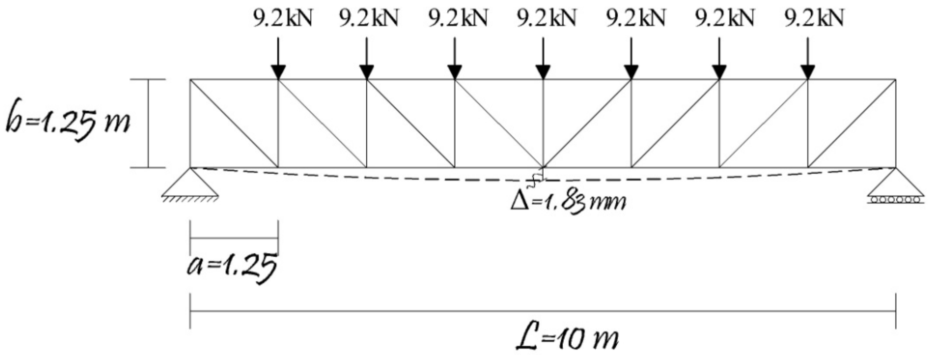
Table 12: Comparison of optimum truss deflections for three groups of truss models

Span = 10 m , k= 4			
	Optimal Truss Deflection Formula	**Existing Deflection Calculation Method**	**Decrease in Deflection (%)**
Group 1	Truss 2 (0.85 mm)	Truss 2 (1.83 mm)	53.55
Group 2	Truss 7 (0.58 mm)	Truss 7 (1.07 mm)	45.80
Group 3	Truss 10 (0.58 mm)	Truss 10 (2.58 mm)	77.52
Span = 20 m , k= 5			
Group 1	Truss 4 (2.27 mm)	Truss 4 (6.47 mm)	65.00
Group 2	Truss 7 (2.66 mm)	Truss 7 (3.82 mm)	30.40
Group 3	Truss 10 (1.24 mm)	Truss 10 (9.30 mm)	86.70
Span = 30 m , k= 8			
Group 1	Truss 2 (4.40 mm)	Truss 2 (11.15 mm)	60.54
Group 2	Truss 7 (3.74 mm)	Truss 7 (5.36 mm)	30.22
Group 3	Truss 10 (1.37 mm)	Truss 10 (12.93 mm)	89.40
Span = 40 m , k= 10			
Group 1	Truss 2 (6.40 mm)	Truss 2 (14.60 mm)	56.20
Group 2	Truss 7 (5.91 mm)	Truss 7 (8.00 mm)	26.12
Group 3			

Figures 56 show the comparison of the amount of deflection obtained from the optimized deflection formula and the deflection of the same truss type by using the current available method of deflection calculation.



Optimized Truss Deflection



Existing Truss Deflection

Figure 56: Comparison of optimum flat truss and existing flat truss system in case k=4 and S= 10 m

The following graphs in Figures 57 to 60 gives the comparison of deflection values between the optimum and existing flat truss type with the different span lengths.

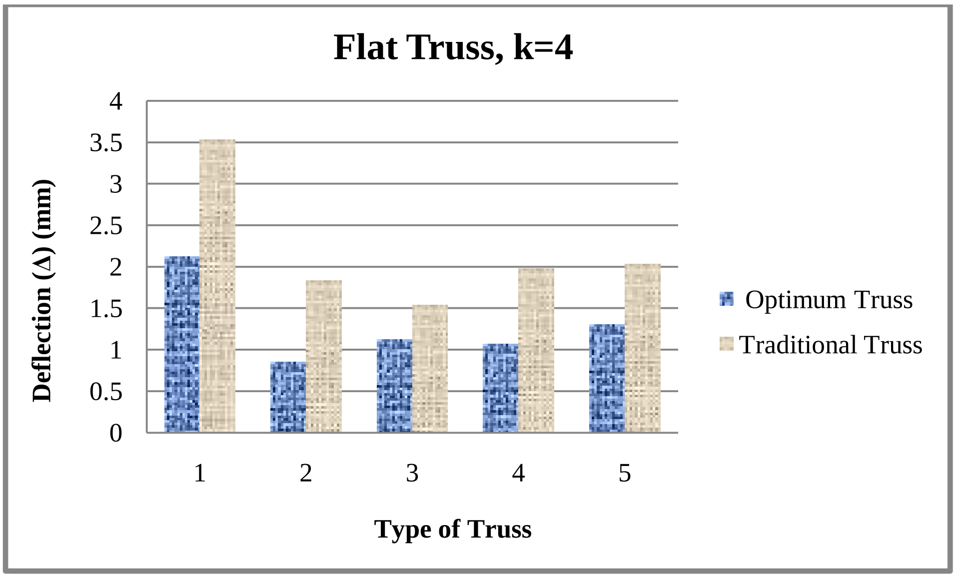


Figure 57: The graphical comparison of deflection values between the optimum truss and the traditional truss system in case of k=4 and S=10 m

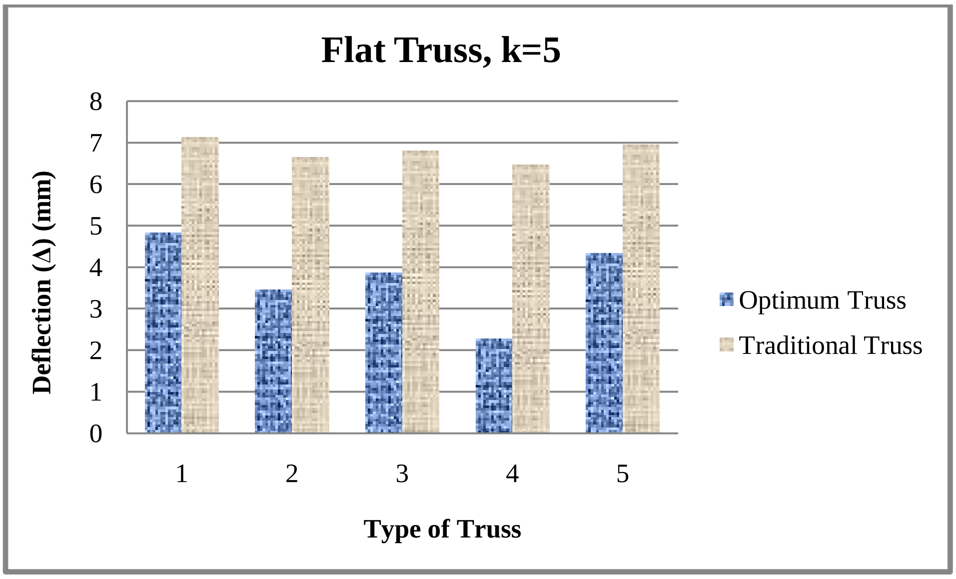


Figure 58: The graphical comparison of deflection values between the optimum truss and the traditional truss system in case of k=5 and S=20 m

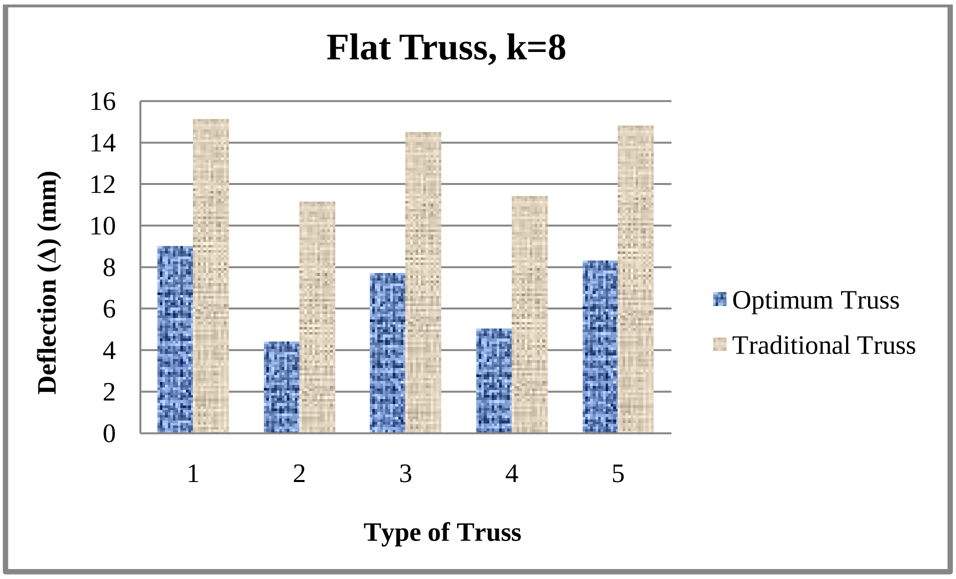


Figure 59: The graphical comparison of deflection values between the optimum truss and the traditional truss system in case of k=8 and S=30 m

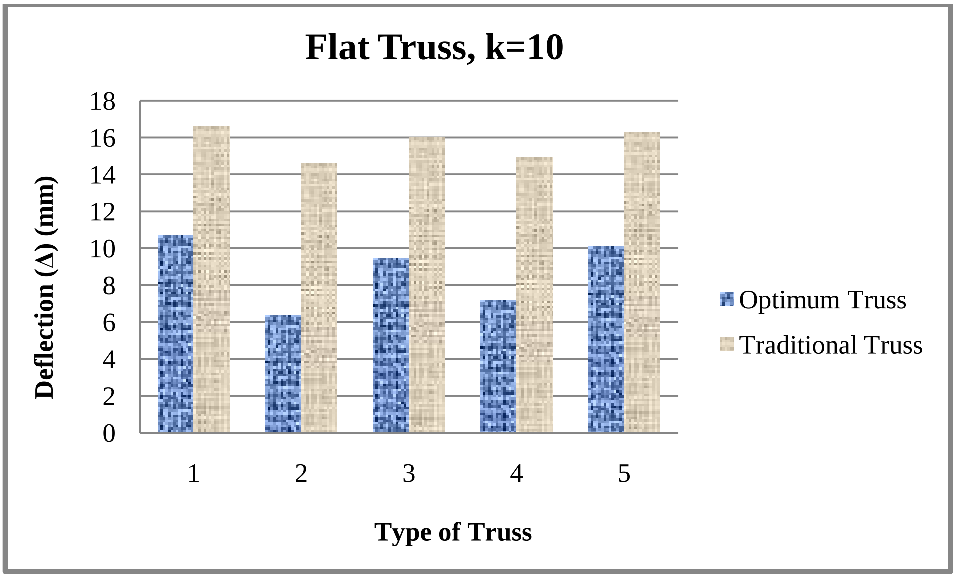


Figure 60: The graphical comparison of deflection values between the optimum truss and the traditional truss system in case of k=10 and S=40 m

The results of the stress output also demonstrated that the optimum truss for deflection is also the optimum truss for the stress. Thus, the optimality is once again for the second type for even and forth type for odd number of bays respectively. Like the results of deflection there are noticeable differences between the stresses obtained for optimum truss and the ones from STAAD Pro analysis in Table 13. Since the least deflection with minimum stress is existed for the above identified types of trusses then there would be no other optimum truss for the specified model.

Table 13: Comparison of optimum truss stresses for three groups of truss models

Span = 10 m , k= 4			
	Optimal Truss kN/mm^2	**Existing Truss kN/mm^2**	**Decrease in Stress (%)**
Group 1	Truss 2 (0.032)	Truss 2 (0.100)	68.00
Group 2	Truss 7 (0.027)	Truss 7 (0.054)	50.00
Group 3	Truss 10 (0.013)	Truss 10 (0.050)	74.00
Span = 20 m , k= 5			
Group 1	Truss 4 (0.043)	Truss 4 (0.083)	48.20
Group 2	Truss 7 (0.056)	Truss 7 (0.093)	40.00
Group 3	Truss 10 (0.014)	Truss 10 (0.085)	83.53
Span = 30 m , k= 8			
Group 1	Truss 2 (0.052)	Truss 2 (0.134)	61.20
Group 2	Truss 7 (0.053)	Truss 7 (0.086)	38.40
Group 3	Truss 10 (0.011)	Truss 10 (0.066)	83.40
Span = 40 m , k= 10			
Group 1	Truss 2 (0.053)	Truss 2 (0.133)	60.15
Group 2	Truss 7 (0.060)	Truss 7 (0.093)	35.48
Group 3			

The following graphs in Figures 61 to 64 gives the comparison of stress values between the optimum and existing flat truss type with the different span lengths.

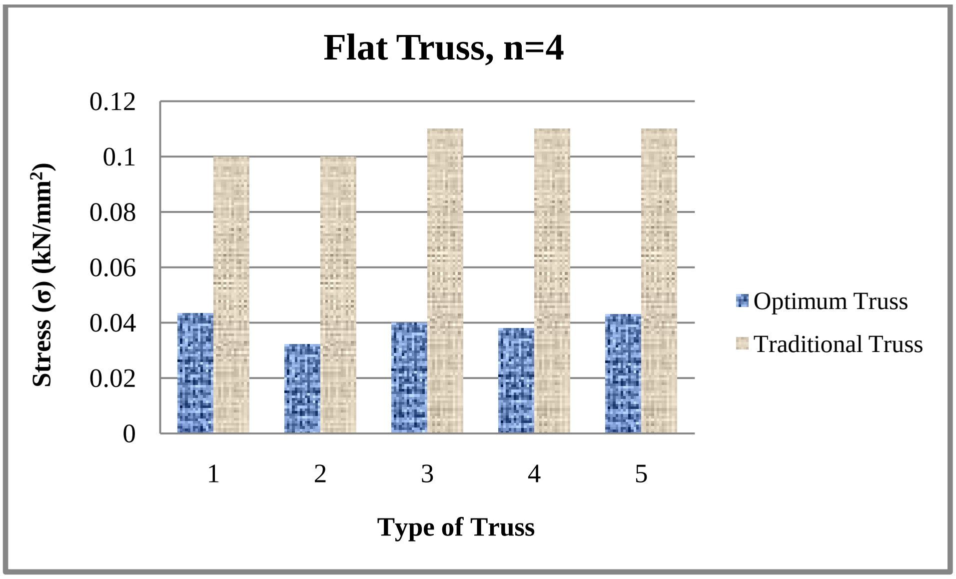


Figure 61: The graphical comparison of stress values between the optimum truss and the traditional truss system in case k=4 and S=10 m

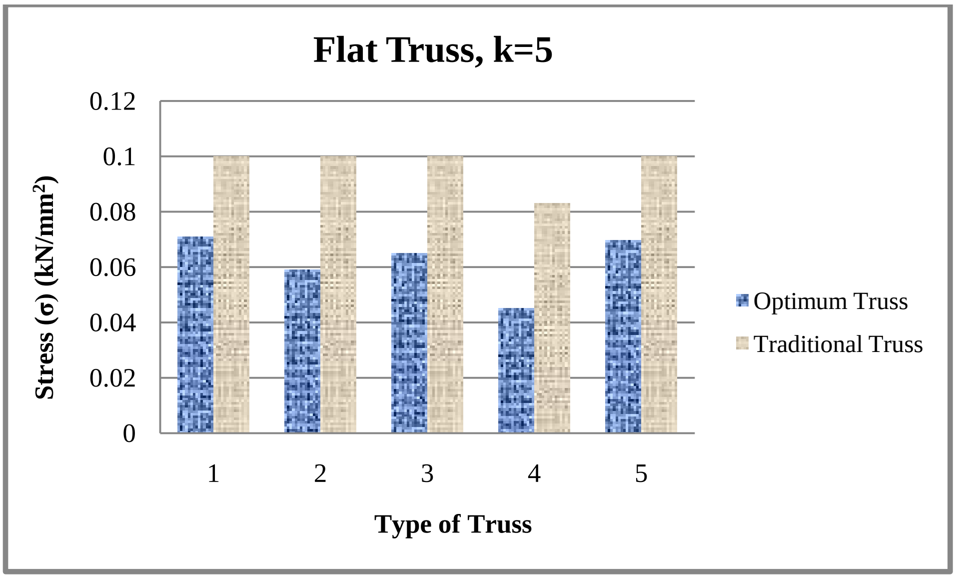


Figure 62: The graphical comparison of stress values between the optimum truss and the traditional truss system in case of k=5 and S=20 m

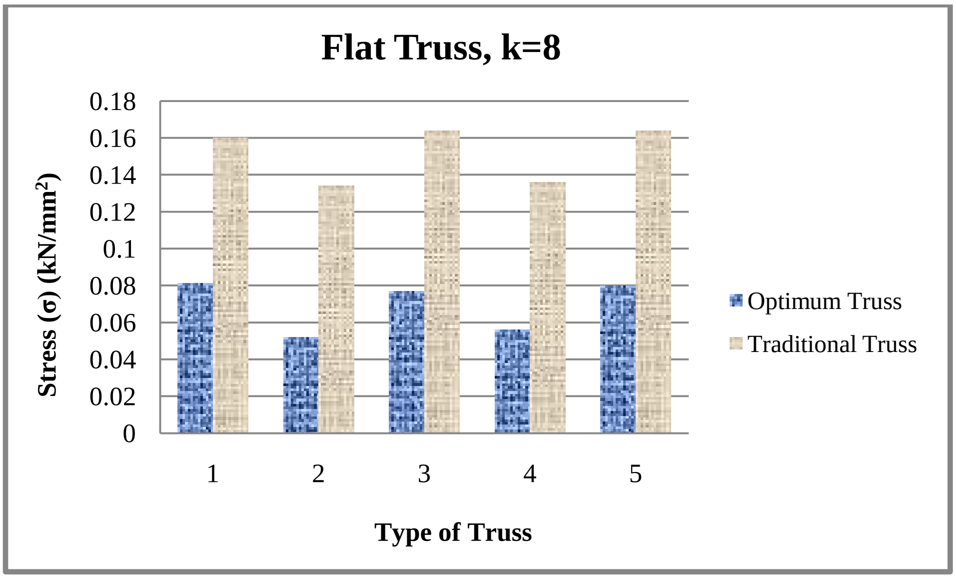


Figure 63: The graphical comparison of stress values between the optimum truss and the traditional truss system in case of k=8 and S=30 m

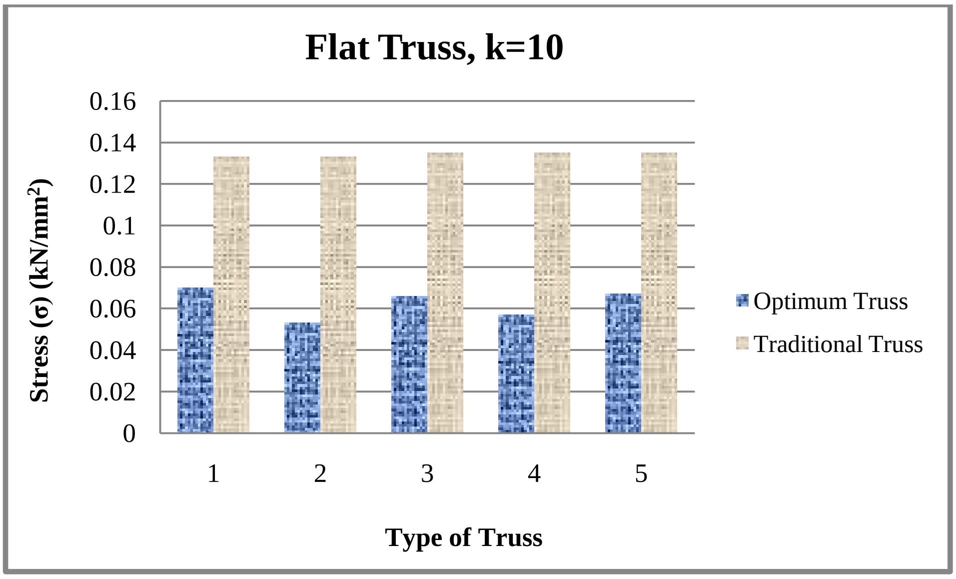


Figure 64: The graphical comparison of stress values between the optimum truss and the traditional truss system in case of k=10 and S=40 m

The optimum truss type for the second group of truss model or warren truss system is located in type 2. Before any further investigation is carried out on warren trusses it is important to mention that the calculation of optimum warren truss is based on 3 selected roof slopes (10%, 15% & 20%). It was observed that the amount of deflection is decreased by increasing the degree of truss top chord slope. As demonstrated in Table 12 the minimum deflection was obtained in truss type two with 20% slope. Likewise for the flat truss the least deflection obtained was much lower than the value obtained by using the current truss deflection calculation method. Therefore, the optimum truss for 10 meter span with k=4, 20 meter span with k=5, 30 meter span with k=8 and 40 meter span with k=10 were found to have 0.58, 2.66, 3.74 and 5.91 millimetres mid-span deflections respectively. It was observed that there is significant difference between the deflections of optimised truss and those that are obtained by using the existing truss deflection calculation.

Similarly, the outputs for the minimum stress values (Table 13) in warren truss are following the same trend as the deflection values. Therefore, if and only if, the minimum stress and the deflection are observed at the same time then the truss type is an optimal warren truss.

The third group of truss model (Triangular model) was studied almost in a similar manner as the flat and warren truss models. Also the 10m truss span cannot be used for this study due to its non-practical geometrical shape and irregular distance between the joints. As a result the occurred deflections obtained for 10, 20 and 30 meter span are 0.58, 1.24 and 1.37 mm respectively. A comparison between the optimized truss systems with traditional approach of truss design proved to be considerably different from each other.

The stresses for the truss members (Table 13) followed the same trend those for the deflection in triangular trusses. Consequently, the optimum triangular truss was achievable due to the least deflection and minimum stress.

5.1.2 Revised Deflection Calculation

Besides the above mentioned outcomes a significant advantage is achieved due to mathematical formulation. The mathematical formula created an easy, fast and accurate way to calculate the deflections of trusses considered in this research. Currently, the most efficient and accurate way for determining the deflection value is virtual work method. Although the current method is the most widely used method for this purpose it is dependents on long and complicated procedure. The suggested formula is introduced a new approach to determine the deflections for the trusses in an extremely short and easy procedure. Essentially, the proposed deflection calculation method is specified an individual formula for each type of the truss models (Table 2) studied in this thesis. Thus, by deciding on the values of the unknowns (n, a, b, w, A and E) and substituting them into the related formula the deflection can be calculated for the first type of flat truss (Eq. 11).

Since the above given unknowns are available in each truss type simply by substituting them in the suggested formula for the specific truss type the deflection could be obtained in a very short time with minimum mathematical calculation and less possibility of making a mistake due to long mathematical calculations.

Although the new approach given is opened a considerable perception in design of truss system, still there are further studies out of scope of this research like; non-linear optimization of the weight, cost, stress, height and deflection of trusses at once.

5.2 Conclusion

A unique set of design methods or guides for truss systems with different span lengths is not yet established. Although there are some reliable experimental estimation methods and considerable mathematical optimization formula for design of trusses still there is no standards for the design of three truss models; flat, warren and triangular truss.

Therefore, a mathematical assessment is carried out to introduce a common formula to guide engineers, designers and decision makers in choosing the truss type with the optimum deflection for the given spans. The created mathematical formula is based on virtual work method to achieve the truss type with the least deflection and minimum stress in order not to unnecessarily over design and also to reduce the secondary effects. Simultaneously, effort was made to derive a formula that can be applicable for most common types of the flat, warren and triangular truss models.

The information provided in the previous chapters and the discussions in this chapter makes it clear of how to choose the most efficient truss type and geometrical shape for different span lengths.

The approach introduced as a result of this study would lead the engineers, designers and decision makers to be able to carry out the most efficient and accurate truss design. As a result, the matters relating to over design, inaccurate design estimation and time consumption due to lengthy calculations can be a matter of history with this approach. Also a large number of possible design variables with mathematical formulation for optimal solution and effective solution procedures are achieved.

To conclude it is suggested to undertake the investigated method to design optimized truss systems instead of using existing methods.

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