Investigation of Space Truss Using the Integrated Force Method

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ABSTRACT

This thesis investigates the usage of integrated force method in the analysis of statically indeterminate space truss. Computer codes are written to generate the equilibrium equations and then calculate the unknowns as: internal forces, nodal displacements, deformations and support reactions.

The first two programs of space truss analysis find the member forces in statically indeterminate cases. These two programs are based on integrated force method (IFM) which recently developed as solution approach. The main step of integrated force method is obtaining of compatibility condition and the difference between these programs is related to the calculation of compatibility condition. In the first program null space of equilibrium equation is used to find the compatibility condition and in the second program singular value decomposition is used.

The third program is based on displacement method termed dual integrated force method. In this method the main step is generation of global stiffness method which is assembled by the matrix multiplication of the equilibrium equation, its transpose and the diagonal matrix of the inverse of the flexibilities of members. In this method displacements are primary unknowns and internal forces can be back-calculated.

Keywords: equilibrium equation, integrated force method, dual integrated force method.

ÖZ

Bu tezde Bileşik Kuvvet Metodunu kullanarak statik belirsiz uzay kafes kirişler incelenmektedir. Geliştirilmiş analiz paketleri denge denklemlerini elde edip daha sonra eleman uç noktalarındaki kuvvetler, düğüm noktalarındaki deplasmanlar, eleman deformasyonları ve mesnet reaksiyonlarını hesaplar.

İlk iki program statik belirsiz uzay kafes kirişlerin eleman kuvvetlerini bulur. Bu iki program son yıllarda geliştirilmiş Bileşik Kuvvet Metodunu (IFM) kullanmaktadır. Bileşik Kuvvet Metodunda en önemli işlem uygunluk şartlarının elde edilmesidir ve yazılmış olan programların bibirinden farkı da uygunluk şartlarının hesaplanmasıdır. Bu iki analiz paketi sırası ile Null Space ve Singular Value Decomposition yöntemlerini kullanarak uygunluk matrislerini elde eder.

Bir ilave analiz paketi de deplasman yöntemini kullanan Çift Bileşik Kuvvet Metodudur (IFMD). Bu metodda global rijitlik matrisi denge denklemleri, diyagonal fleksibilite matrisinin tersi ve denge denklemleri transpozunun matris çarpımı kullanarak elde edilir. Çift Bileşik Kuvvet Metodudunda ana bilinmeyenler düğüm noktalarındaki deplasmanlardır ve eleman uç noktalarındaki kuvvetler deplasmanlar kullanılarak daha sonra hesap edilir

Anahtar kelimeler: Denge Denklemleri, Bileşik Kuvvet Metodu, Çift Bileşik Kuvvet Metodu.

TO MY FAMILY

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I would like to express my deepest gratitude and appreciation to my supervisor, Dr. Erdinç Soyer for his patient guidance and encouragement throughout this study. His experience and knowledge have been an important help for my work.

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LIST OF SYMBOLS

MATRIX QUANTITIES

[A]	Equilibrium Equation
$\left([A]^T\right)^{pinv}$	Moore-Penrose Pseudo Inverse of $[A]^T$
[C]	Compatibility Condition
[D]	Direction Cosine
[G]	Unconnected Flexibility Matrix
[I]	Identity Matrix
[J]	Transpose Matrix of $[S]^{-1}$
$[K]_{ifmd}$	Pseudo Stiffness Matrix
[M]	Singular Value Decomposition Matrix
$[M_u]$	Orthogonal Matrix
$[M_v]$	Orthogonal Matrix
$[M_\sigma]$	Diagonal Matrix
[NS]	Null Space Matrix of [A]
[S]	Matrix of Coupling [A] with [C]

NON MATRIX QUANTITIES

A	Cross-Sectional Area
E	Modulus of Elasticity
F	Internal Force

f_s	Flexibility of Element
{F}	Internal Forces Vector
L	Length of Element
l_{ij}	Direction Cosine Along X Axis
m	Number of Elements
m_{ij}	Direction Cosine Along Y Axis
n	Number of Nodes
n_{ij}	Direction Cosine Along Z Axis
{P}	Applied Load Vector
R	Support Reactions
{X}	Displacement Vector
$\{\beta\}$	Deformations Vector
$\{\Delta R\}$	Initial Deformation Vector

LIST OF ABBREVIATIONS

CC	Compatibility Condition
DOF	Degree of Freedom
DDR	Deformation Displacement Relations
EE	Equilibrium Equations
IE	Internal Energy
IFM	Integrated Force Method
IFMD	Dual Integrated Force Method
SVD	Singular Value Decomposition
W	Work Done

Chapter 1

INTRODUCTION

1.1 Introduction

While a building is constructed, it is necessary to be in equilibrium and stable. Otherwise the built structure will fail. For this aim, according to the concept of Newton's Law, internal forces should be equal to the sum of exterior forces and imported loads. Therefore, to design the structure and evaluate the member section and supports conditions and other properties of building the equilibrium equation of structure should be assembled. (S. N. Patnaik, D. A. Hapkins, and G. R. Halford, 2004), (West, 1993)

For the constructions and buildings with large scales, attempting to write equilibrium equations and calculating unknowns will be very hard and time consuming. Therefore, utilizing computer programs is unavoidable to solve the problems.

In this thesis, structures which are studied are space trusses. Automatic generation of the equilibrium equations are carried out and later used to obtain unknown member forces by adopting the Force Method.

In determinate truss, numbers of unknowns are same as number of equation, then only with generation of equilibrium equation and straightforward solution will be enough to determine the unknown forces of the structure. Whereas for indeterminate truss, only with generation of equilibrium equation it is not possible to solve the unknowns and needs additional equation set to obtain the unknowns. (S. N. Patnaik, D. A. Hapkins, and G. R. Halford, 2004), (Saouma, 1999)

This study to solve the indeterminate structures used a method which is known as Integrated Force Method (IFM). To give whole concept of this method (IFM) and make the readers familiar, a brief history of Integrated Force Method is presented.

Navier (1785-1836) tried to calculate the four reactions of four-leg table and he wrote the equilibrium equation, but there were three equations with four unknowns. Then Navier found that structure is indeterminate and he could not solve the problem. The main and important point to solve this problem was a need of an additional equation to make the Navier's (3x4) rectangular equation matrix to square. This additional equation was called compatibility condition (1x4) matrix which was identified by Patnaik and his research group. (S. N. Patnaik, D. A. Hapkins, and G. R. Halford, 2004)

Coupling the Navier's (3x4) rectangular equilibrium equation matrix with Patnaik's (1x4) compatibility condition matrix created a new analysis method for indeterminate structures with the name of Integrated Force Method which is used in this thesis. In this method (IFM) after generation of equilibrium equation matrix, two methods that have been used to obtain the compatibility condition are algebraic methods of: (S. N. Patnaik, D. A. Hapkins, and G. R. Halford, 2004)

- Null Space

- Singular Value Decomposition

In the usage of Integrated Force Method the general steps are:

  - Generation of equilibrium equation matrix,
  - Generation of compatibility condition matrix,
  - Coupling the equilibrium equation with compatibility condition to assemble the [S] matrix,
  - Use [S] matrix to obtain internal forces.

1.2 Purpose of This Study

The main purpose of this study is the analysis of space trusses to evaluate the internal forces directly by using of integrated force method with generating equilibrium equations and compatibility condition with computer codes.

- The principal motivation of this study is developing the force method analysis, because in the consideration of the state of art it is discovered that there are many computer codes which are based on stiffness method but a few computer codes are available that are based on integrated force method.
- Other motivation of this study is the process of the analysis of integrated force method. Methods like stiffness and displacement, the primary unknowns are the displacement, and then internal forces are back calculated whereas in integrated force method internal forces can be obtained directly.
- In majority universities in the related courses to the structural analysis, only two dimensional structures are discussed, because analyzing of three dimensional structures are very hard to evaluate manually and consumes a lot of time. Whereas larger structures it is impossible to analyze without any computer

program. Then this study can be helpful to analyze the space trusses more quickly and easily in the basic structural analysis courses.

- Other purpose is making students and readers familiar with integrated force method, because a few books and documents are available which are written in details about this method and the using of equilibrium equation.
- As another purpose of this research it can be expressed that a few computer codes exist which are used to generate the equilibrium equation automatically and solve the assembled equilibrium equation by using of null space and singular value decomposition methods.
- In this study the computer software used for writing the codes and programs to analyze the structure in Mathematica. Then it also emphasis that Mathematica is not related only to mathematic courses but it can be used in the structure analysis courses and other relevant engineering fields.

1.3 Research Problems

The questions which this study attempt to answer, are:

- In which way computer codes must be written to collect and generate the equilibrium equation,
- How equilibrium equation can be utilized to analyze the space truss,
- Which properties and relations of matrix course can be used to generate equilibrium equations matrix and obtain compatibility condition matrix,
- What are the main differences between the integrated force method and other analysis methods like stiffness.

1.4 Objectives of Research

The study intentions summary is as following:

- To introduce a new method which can be helpful in structural analysis of space truss
- To analyze different space trusses with written computer codes that generates equilibrium equation and puts in matrix format.
- To help students in space truss analysis to avoid a lot of time consumption to analyze.
- To make the examples practical and establish closer relationship between theoretical truss and the actual one.
- To analyze the large trusses quickly which need to generate the big matrix of equilibrium equations and solving it.

1.5 Summary of Thesis

This thesis consists of 7 chapters:

Chapter 1 is an introduction giving:

- Brief introduction of integrated force method
- Purpose of this study
- Research problems
- Objectives to researches.

Chapter 2 gives basic and necessary information about generation of equilibrium equation. For this aim, a determinate space truss is solved and all of the relations and formulas are presented step by step.

Chapter 3 explains the three methods which are used in this thesis for writing the computer codes as:

- Integrated Force Method via Null Space
- Integrated Force Method via Singular Value Decomposition
- Dual Integrated Force Method

In chapter 4, how the equilibrium equations can be generated automatically by written computer codes, is explained. In this chapter an example is solved to better illustrate the computer codes and at each step the related relations and equations are presented.

In chapter 5, the written computer codes to solve generated equilibrium equation in chapter 4 are explained by using an example of indeterminate space truss. Also in this chapter three algorithms for each of the three methods are expressed. In these algorithms needed formulation for each step has been placed.

In chapter 6, there are six illustrative examples which are indeterminate cases with different number of nodes and members and also with different degree of indeterminacy and support conditions. Two first examples are solved by IFM via null space, the examples 3 and 4 are solved by IFM via singular value decomposition and last two examples are analyzed by using of dual integrated force method (IFM). At the end of solution of each example to prove and compare the obtained unknowns, the result of Mastan software for that truss is presented.

In chapter 7, summary of research and conclusions are expressed, then some directions for possible future researches are recommended.

Finally the used sources and publications during this study are provided at References section.

Chapter 2

BACKGROUND INFORMATION

2.1 Introduction

In this chapter some data are presented about the previous works done on integrated force method. Also some primary and basic information are expressed about space truss structures and their general characteristics. Then it is intended to explain how the equilibrium equation must be written in truss analysis and how generated equations can be solved.

2.2 Previous Works Done by Integrated Force Method

After Navier, who wrote the equilibrium equation for four-leg table but because of the indeterminacy nature of structure he could not solve it. Patnaik attempted to find a method to solve this problem with force method by using equilibrium equation. This issue led to development of integrated force method. Patnaik was one of the few researchers that worked and toiled on force method especially on integrated force method. He has collected his approaches and made them like a book which helped to write this thesis and is the main reference of this study. But in this collection of Patnaik's book there a few examples on plane truss where space truss was not discussed and explained in detail. (S. N. Patnaik, D. A. Hapkins, and G. R. Halford, 2004)

Patnaik continued to study on integrated force method and tried to constitute this method with other subjects for example he has considered behavior of initial deformation in integrated force method. Later Patnaik and his research group developed structural analyzing of finite elements by using integrated force method for two dimensional structures in which space framed structures have not been discussed. Nonlinear analyzing of structures by this method (IFM) was another subject that has been studied by other researchers. (S. N. Patnaik, D. A. Hapkins, and G. R. Halford, 2004), (N. R. B. Krishnam Raju, and J. Nagabhushanam, 2000)

Other recent researches by integrated force method are related to Eastern Mediterranean University's Civil Engineering department. One of the students studied on analyzing of two dimensional truss structures and later other student has worked on two dimensional analyses of frame structures. (S.Khosravi, 2005), (S.Kamkar, 2010)

2.3 Description of Space Truss

Space truss is a kind of structures that often are analyzed base on equilibrium. This type of structure (space truss) consists of four members at least and all of the joints are pins which are not capable to transmit moment. Generally, stability of space truss is realized by forming of joined four-face units.

2.3.1 Assumptions in Space Truss

To analyze the space truss in this study some assumptions are intended:

- The members are connected together with pin joints without friction.
- All of the loads and reactions subjected to space truss only and only applied on nodes.

- The central axis of each member is straight and it is coincident with connecting line between the end nodes of member.
- In truss the subjected loads are concentrated type (not distributed load)

2.3.2 Stability and Determinacy of Space Truss

Each nodes of a space truss consists of intersecting forces in which three moment equations are satisfied automatically. Therefore, only three independent force equilibrium equations must be written for each node. The condition that is essential but not enough to be determinate truss is written as:

$$b + r = 3j \tag{2.1}$$

Where

b: is the number of members which is equal to number of unknown forces

r: is the number of support's reactions

j: is the number of joints or nodes

If $b + r < 3j$, truss is unstable

If $b + r = 3j$, truss is determinate (2.2)

If $b + r > 3j$, truss is indeterminate

2.4 Generation of Equilibrium Equation for Space Truss

As it was explained above, for each node of space truss three equilibrium equation must be written. To analyze a member of space truss which consist of two nodes, six equilibrium equations are written. To explain the method of assembling the equilibrium equation, a part of space truss shown in Figure 1 is used and member i-j is separated from structure as shown in Figure2.

According to Figure1, node i is a support indicator in which reaction Ri has been resolved to its three components. Node j is a joint indicator in the structure in which subjected load Pj has been resolved to its components. The axial force Fij is assumed to be tension force that affects in nodes I and j as a member force. All the forces are shown in positive direction.

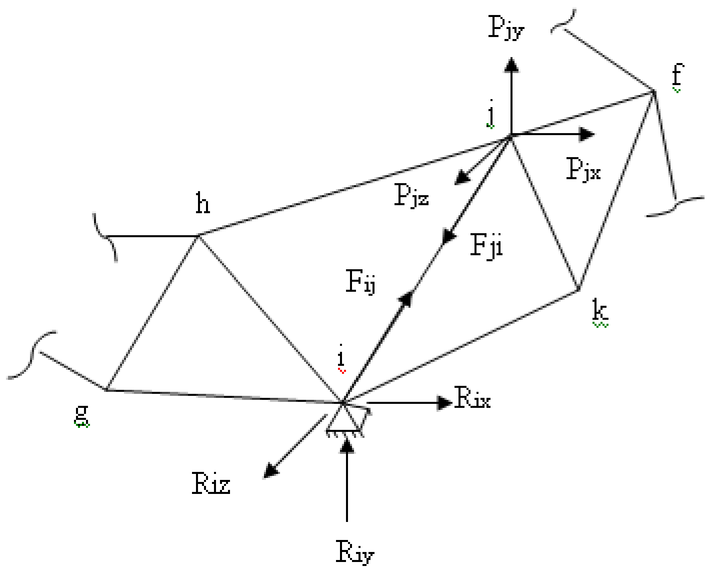

Figure 1. Separated Part of Space Truss

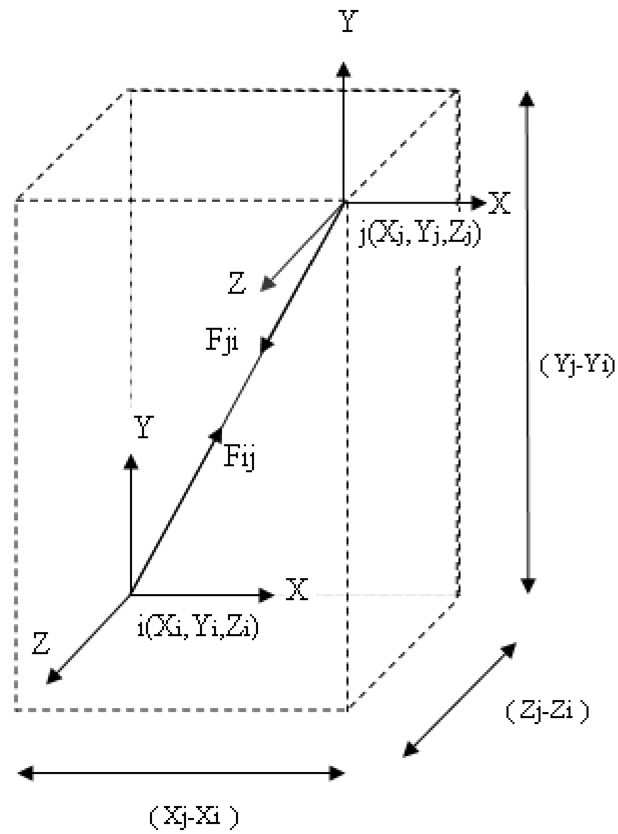

Figure 2. Member i-j of Truss

Figure 2 shows the separated member i-j with its end nodes coordinates in global coordinate system. At the end member i, force Fij is resolved to its parameters X_{ij}, Y_{ij} and Z_{ij} in the direction of X, Y and Z respectively. These parameters can be written as:

$$X_{ij} = F_{ij}\left(\frac{X_j - X_i}{L_{ij}}\right) = F_{ij}\, l_{ij}$$

$$Y_{ij} = F_{ij}\left(\frac{Y_j - Y_i}{L_{ij}}\right) = F_{ij}\, m_{ij} \qquad 2.3$$

$$Z_{ij} = F_{ij}\left(\frac{Z_j - Z_i}{L_{ij}}\right) = F_{ij}\, n_{ij}$$

In this terms quantities l_{ij}, m_{ij} and n_{ij} are the direction cosines which indicate the angles between member i-j and axes x, y and z respectively. Also at the j end, the force F_{ji} in way shown is resolved to its components X_{ij}, Y_{ij} and Z_{ij} as written below:

$$Xji = F_{ji}\left(\frac{X_i - X_j}{L_{ij}}\right) = Fji\ lji$$

$$Yji = F_{ji}\left(\frac{Y_i - Y_j}{L_{ij}}\right) = Fji\ mji \qquad 2.4$$

$$Zji = F_{ji}\left(\frac{Z_i - Z_j}{L_{ij}}\right) = Fji\ nji$$

Where: l_{ji}, m_{ji} and n_{ji} are the direction cosines of member. By checking the equations 2.3 and 2.4 it is observed that :

$l_{ji} = -l_{ij}$ $\qquad$ $m_{ji} = -m_{ij}$ $\qquad$ $n_{ji} = -n_{ij}$

Also since the member is under tension it can be written: Fji =Fij then the relations below can be concluded:

$$X_{ji} = F_{ij}\ (-l_{ij})$$

$$Y_{ji} = F_{ij}\ (-m_{ij}) \qquad 2.5$$

$$Z_{ji} = F_{ij}\ (-n_{ij})$$

The length of the member Lij can be calculated by following formulation:

$$Lij = \sqrt{\left(\left(X_j - X_i\right)^2 + \left(Y_j - Y_i\right)^2 + \left(Z_j - Z_i\right)^2\right)} \qquad 2.6$$

$$l_{ij} = \frac{X_j - X_i}{L_{ij}}$$

$$m_{ij} = \frac{Y_j - Y_i}{L_{ij}} \qquad 2.7$$

$$n_{ij} = \frac{Z_j - Z_i}{L_{ij}}$$

Now, according to structure in Fig.1 and stated relations the equilibrium equation for member i-j can be generated:

$$X_{ig} + X_{ih} + X_{ij} + X_{ik} + R_{ix} = 0$$

$$Y_{ig} + Y_{ih} + Y_{ij} + Y_{ik} + R_{iy} = 0$$

$$Z_{ig} + Z_{ih} + Z_{ij} + Z_{ik} + R_{iz} = 0 \qquad 2.8$$

$$X_{jh} + X_{ji} + X_{jk} + X_{jf} + P_{jx} = 0$$

$$Y_{jh} + Y_{ji} + Y_{jk} + Y_{jf} + P_{jy} = 0$$

$$Z_{jh} + Z_{ji} + Z_{jk} + Z_{jf} + P_{jz} = 0$$

According to equations 2.7, a general formulation for a truss that consists of n nodes can be determined as :

$$\begin{pmatrix}
\cdots & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\cdot & \cdot & \cdot & \cdot & \cdot\cdot \\
\cdots & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\cdot & \cdot & \cdot & \cdot & \cdot\cdot \\
\cdots & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\cdot & \cdot & \cdot & \cdot & \cdot\cdot \\
\cdots & -l_{ig} & -l_{ih} & -l_{ik} & -l_{ij} & \cdot & \cdot & \cdot & \cdot\cdot & -1 & \cdot & \cdot & \cdot\cdot \\
\cdots & -m_{ig} & -m_{ih} & -m_{ik} & -m_{ij} & \cdot & \cdot & \cdot & \cdot\cdot & \cdot & -1 & \cdot & \cdot\cdot \\
\cdots & -n_{ig} & -n_{ih} & -n_{ik} & -n_{ij} & \cdot & \cdot & \cdot & \cdot\cdot & \cdot & \cdot & -1 & \cdot\cdot \\
\cdots & \cdot & \cdot & \cdot & l_{ij} & -l_{jh} & -l_{jk} & -l_{jf} & \cdot\cdot & \cdot & \cdot & \cdot & \cdot\cdot \\
\cdots & \cdot & \cdot & \cdot & m_{ij} & -m_{jh} & -m_{jk} & -m_{jf} & \cdot\cdot & \cdot & \cdot & \cdot & \cdot\cdot \\
\cdots & \cdot & \cdot & \cdot & n_{ij} & -n_{jh} & -n_{jk} & -n_{jh} & \cdot\cdot & \cdot & \cdot & \cdot & \cdot\cdot \\
\cdots & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\cdot & \cdot & \cdot & \cdot & \cdot\cdot \\
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\cdots & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\cdot & \cdot & \cdot & \cdot & \cdot\cdot \\
\cdot\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\cdot & \cdot & \cdot & \cdot & \cdot\cdot
\end{pmatrix}
\begin{pmatrix}
\cdot \\ \cdot \\ \cdot \\ F_{ig} \\ F_{ih} \\ F_{ik} \\ F_{ij} \\ F_{jh} \\ F_{jk} \\ F_{jf} \\ \cdot \\ \cdot \\ R_{ix} \\ R_{iy} \\ R_{iz} \\ \cdot \\ \cdot
\end{pmatrix}
=
\begin{pmatrix}
\cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ P_{jx} \\ P_{jy} \\ P_{jz} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot
\end{pmatrix} \qquad 2.9$$

Also the equation 2.8 can be written in abbreviated form as:

$$[D]\begin{Bmatrix} \{F\} \\ \{R\} \end{Bmatrix} = \{P\} \qquad 2.10$$

In which: [D] is the matrix of direction cosines or universal statics matrix, {F} is member forces vector,

{R} is support reactions vector

{P} is load vector that presents subjected external loads.

In equation 2.9 support reaction vector {R} can be eliminated temporarily from the equation, after obtaining of member forces they can be back calculated if they are needed.

2.5 Evaluation of Unknown Forces in Space Truss

While assembling of equilibrium equations of space truss two cases may exist:

- If the equilibrium equation matrix is square then the structure is determined.
- If the equilibrium equation matrix is rectangle then the structure is indeterminate.

2.5.1 Calculation of Forces for Determinate Space Truss by Using the Generated Equilibrium Equations

By using of equilibrium equation the determinate space truss's member forces can be obtained easily only with linear solution of following relation:

$$[D]\{F\} = \{P\} \qquad 2.11$$

This relation can be computed manually but there is need to know related solving methods of matrix and be familiar attributes of some special matrices. It also can be solved by some computer software like Mathematica intended software to this study.

In this software first two factors of matrix [D] and vector {P} must be entered, then by inserting of command of "LinearSolve" the forces will be calculated.

To realize better for this method an example is solved. In this example as shown in Figure 3 internal forces and support reactions are to be obtained obtain.

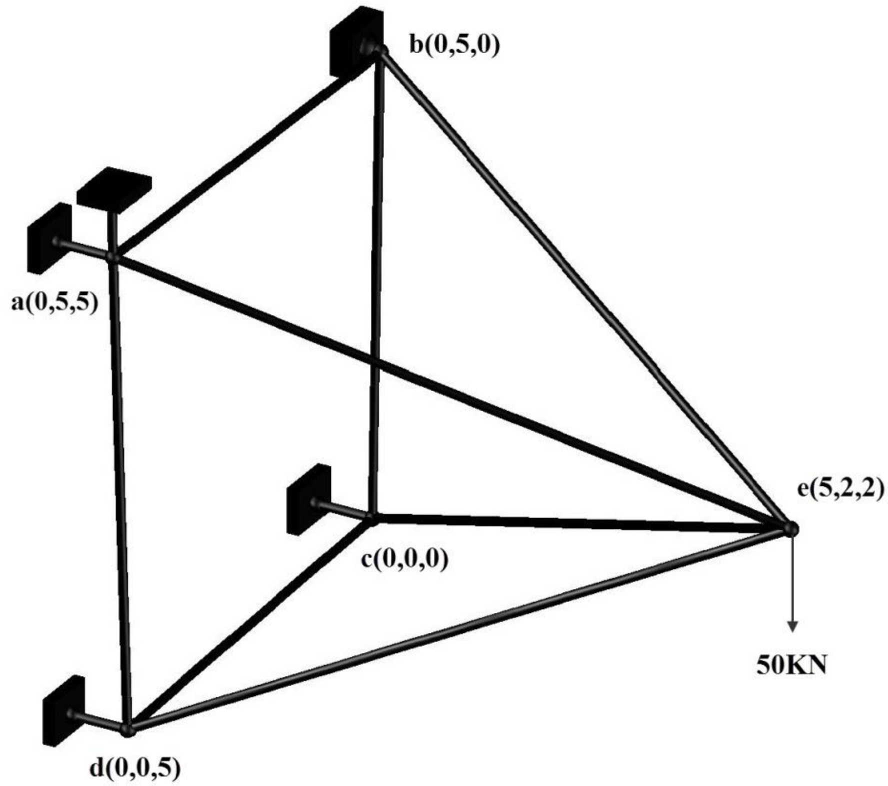


Figure 3. Eight-Member Determinate Space Truss

According to the Figure3 and relation 2.1 and 2.2 this truss is determinated because:

3 (5) = 8 + 7;

nodes = 5, member = 8, reastions = 7.

Then for truss of Figure 3 equations 2.3 – 2.6 are produced as Table 1.

Table 1. Direction Cosines of Example Truss

Member Ij	$(x_j - x_i)$ M	$(y_j - y_i)$ m	$(z_j - z_i)$ m	L_{ij} m	l_{ij}	m_{ij}	n_{ij}
Ae	5.0	-3.0	-3.0	6.56	0.762	-0.457	-0.457
Ea	-5.0	3.0	3.0	6.56	-0.762	0.457	0.457
Be	5.0	-3.0	2.0	6.16	0.812	-0.487	0.325
Eb	-5.0	3.0	-2.0	6.16	-0.812	0.487	-0.325
Ce	5.0	2.0	2.0	5.74	0.871	0.348	0.348
Ec	-5.0	-2.0	-2.0	5.74	-0.871	-0.348	-0.348
De	5.0	2.0	-3.0	6.16	0.812	0.325	-0.487
Ed	-5.0	-2.0	3.0	6.16	-0.812	-0.325	0.487
Ab	0	0	-5.0	5.0	0	0	-1.0
Ba	0	0	5.0	5.0	0	0	1.0
Bc	0	-5.0	0	5.0	0	-1.0	0
Cb	0	5.0	0	5.0	0	1.0	0
Cd	0	0	5.0	5.0	0	0	1.0
Dc	0	0	-5.0	5.0	0	0	-1.0
Da	0	5.0	0	5.0	0	1.0	0
Ad	0	-5.0	0	5.0	0	-1.0	0

After substitution of quantities of Table1 and loads in relation 2.8 the equilibrium equations in matrix form is produced as:

$$\begin{pmatrix} 0 & -0.762 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.457 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0.457 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -0.812 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0.487 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.325 & -0.871 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & -0.348 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.348 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.812 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & -0.325 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.487 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.762 & 0 & 0 & 0.812 & 0.871 & 0 & 0.812 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.457 & 0 & 0 & -0.487 & 0.348 & 0 & 0.325 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.457 & 0 & 0 & 0.325 & 0.348 & 0 & -0.487 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} F_{ab} \\ F_{ae} \\ F_{ad} \\ F_{bc} \\ F_{be} \\ F_{ce} \\ F_{cd} \\ F_{de} \\ R_{ax} \\ R_{ay} \\ R_{bx} \\ R_{by} \\ R_{bz} \\ R_{cx} \\ R_{dx} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -50 \\ 0 \end{pmatrix}$$

After linear solve for generated matrix the unknown forces and support reactions can be obtained as:

$$\begin{Bmatrix} \{F\} \\ \{R\} \end{Bmatrix} = [D]^{-1}\{P\} = \begin{Bmatrix} F_{ab} \\ F_{ae} \\ F_{ad} \\ F_{bc} \\ F_{be} \\ F_{ce} \\ F_{cd} \\ F_{de} \\ R_{ax} \\ R_{ay} \\ R_{bx} \\ R_{by} \\ R_{bz} \\ R_{cx} \\ R_{dz} \end{Bmatrix} = \begin{Bmatrix} -12.0 \\ 26.24 \\ 8.0 \\ 12.0 \\ 36.96 \\ -34.44 \\ 12.0 \\ -24.64 \\ -20.0 \\ 20.0 \\ -30.0 \\ 30.0 \\ 0.0 \\ 30.0 \\ 20.0 \end{Bmatrix} kN$$

2.5.2 Calculation of Forces for Indeterminate Space Truss by Using the Generated Equilibrium Equations

When generated equations matrix of equilibrium is rectangular, it means that truss which is analyzed is indeterminate and it cannot be solved by using of equilibrium equations only.

To obtain the forces by written equations in indeterminate truss, there are some methods as following:

- Force Method
- Displacement Method

In displacement method presented by Navier (S. N. Patnaik, D. A. Hapkins, and G. R. Halford, 2004), Primary unknowns are displacements of nodes and the member forces are back calculated. Two approaches in displacement method are:

- Direct Stiffness Method

- Dual Integrated Force Method

Direct Stiffness method is not discussed in this thesis, because there are a lot of sources and computer codes based on this method.

Dual Integrated Force Method is one of the intended analyzing method in this research and is explained in detail in chapter three.

In the force method, member forces are taken as primary unknowns. There are two principal approaches in this method that are:

- Classical Force Method
- Integrated Force Method

In classical force method when structure is indeterminate, some members can be "cut" to make a stable and determinate structure. Then by using of related equations and relations member forces are obtained.

The main approach of force method that is used in this thesis is Integrated Force Method. In chapter three complete explanation of this method is presented but here a brief review is inscribed.

After generation of equilibrium equation in indeterminate space truss, since the matrix is rectangular an additional equation should be produced. This additional equation termed compatibility condition and it makes the equilibrium equation matrix square. In this research the additional compatibility condition equation is achieved by using two methods:

- Null Space
- Singular Value Decomposition

Then the steps to use of integrated force method are:

- Assembling of equilibrium equation
- Writing of compatibility condition
- Coupling the equilibrium equation with compatibility condition to obtain [S] matrix given by:

$$[S] = \begin{bmatrix} [A] \\ [C] \quad [G] \end{bmatrix} \tag{2.12}$$

- Evaluation of forces by using of:

$$\begin{bmatrix} [A] \\ [C] \quad [G] \end{bmatrix} \{F\} = \begin{Bmatrix} P \\ \Delta R \end{Bmatrix} \tag{2.13}$$

2.6 Transformation Matrix in Space Truss

When elements of a space truss have been oriented in different direction, there is a necessary need to transform the member relations from their local coordinate system to global coordinate system. Transformation matrices in space truss structures are obtained by:

$$T = \begin{bmatrix} Cos\theta_X & Cos\theta_Y & Cos\theta_Z & 0 & 0 & 0 \\ 0 & 0 & 0 & Cos\theta_X & Cos\theta_Y & Cos\theta_Z \end{bmatrix} \tag{2.14}$$

Where:

$$Cos\theta_X = \frac{X_2 - X_1}{L}$$
$$Cos\theta_Y = \frac{Y_2 - Y_1}{L} \tag{2.15}$$
$$Cos\theta_Z = \frac{Z_2 - Z_1}{L}$$

$$L = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2} \tag{2.16}$$

Chapter 3

METHODOLOGY

3.1 Introduction

In structure to analyze the relation of [S]{F}= {P} is used. In determinate structure matrix [S] is square (m × m) and it can be solved directly. Then internal forces are obtained and displacements can be back-calculated if needed.

In indeterminate structures the generated equilibrium matrix of structure is rectangular with dimension of (m × n), where m is number of equilibrium equations (EE) and n is the number of unknown forces. In this condition that [S] matrix is not a square an additional equation termed Compatibility Condition must be generated. The methods this study utilizes to solve the indeterminate space truss structures by using of equilibrium equation are:

- Integrated Force Method
    - Null Space
    - Singular Value Decomposition
- Displacement Method
    - Dual Integrated Force Method

3.2 Integrated Force Method (IFM)

Base of the new force method which has been expanded is essentially equation below:

$$\left[\frac{\text{Equilibrium Equation}}{\text{Compatibility condition}}\right]\{\text{Forces}\}=\left\{\frac{\text{Mechanical Load}}{\text{Initial Deformation}}\right\} \qquad 3.1$$

Where, equilibrium equation (EE) and compatibility condition (CC) are coupled together. This method is termed as Integrated Force Method (IFM). Indeterminate space truss analysis, objective of this study, has need of the same EE and CC. Since integrated force method uses both equilibrium equation and compatibility condition, this method can be expanded systematically and can construct dependable solutions even for large structures with complicated topology. (S. N. Patnaik, and D. A. Hopkins, 1998), (S. N. Patnaik, D. A. Hapkins, and G. R. Halford, 2004)

The equilibrium equations are generated base on forces, but compatibility condition is written in term of deformations and displacements. Then there is need to write the compatibility condition in terms of forces because it must be coupled to equilibrium equation which is base on forces.

Therefore the governing formulation of integrated force method is:

$$\begin{bmatrix} [A] \\ [C]\quad [G] \end{bmatrix}\{F\}=\begin{Bmatrix} P \\ \Delta R \end{Bmatrix} \qquad 3.2$$

Where:

Matrix [A] is equilibrium equation,

Matrix [C] is compatibility condition,

Matrix [G] is unconnected flexibility,

Vector {F} is internal forces,

Vector {P} is external loads and

Vector {δR} is initial deformations.

Also the equation 3.2 can be written as:

$$[S]\{F\}=\{P^*\} \qquad 3.3$$

In equation 3.3 matrix [S] is square and is produced by coupling of equilibrium equation, compatibility condition and flexibility matrix. In vector {P*} number of rows is equal to number of rows of vector of external loads. Therefore, in the cases that there is not any initial deformation, to balance the equation, zero should be placed.

In integrated force method in which primary unknowns are member forces, displacements if are needed can be calculated by: (S. N. Patnaik, D. A. Hapkins, and G. R. Halford, 2004)

$$\{X\}=[J][G][F] \qquad 3.4$$

Where:

Vector {X} is nodal displacements,

Matrix [J] is transpose matrix of inversed [S]

$$J=\left[[S]^{-1}\right]^{T} \qquad 3.5$$

Matrix [G] is unconnected flexibility matrix and

[F] is member forces which have been calculated.

3.2.1 Assembling of compatibility condition in IFM

In indeterminate structures, to solve the equation 3.2 compatibility condition should be generated and then couple with equilibrium equation. In integrated force method, as

Patnaik explained and utilized, compatibility condition should be written as following steps: (Patnaik S. , 1999)

- The relations of deformation-displacement (DDR) must be derived
- The displacements must be eliminated from deformation displacement relation

To write deformation-displacement relation, energy theory in structures is used as:

$$IE = \frac{1}{2}\{F\}^T\{\beta\} \quad 3.6$$

In truss structures deformations $(\beta_1, \beta_2, ..., \beta_m)$ are corresponding to internal forces $(F_1, F_2, ..., F_m)$. External loads lead to be done work in structure:

$$W = \frac{1}{2}\{P\}^T\{X\} \quad 3.7$$

Here the nodal displacements are corresponding to the external loads. Then, according to conservation of work-energy:

$$IE = W \quad 3.8$$

The equations 3.6 and 3.8 can be written as:

$$\frac{1}{2}\{F\}^T\{\beta\} = \frac{1}{2}\{P\}^T\{X\} \quad 3.9$$

By substituting equilibrium equation (EE) 3.2 into equation 3.9 it can be expressed as:

$$\{F\}^T\left(\left\{\beta - [A]^T\{X\}\right\}\right) = 0 \quad 3.10$$

And also equation 3.10 can be expressed as:

$$\{\beta\} = [A]^T\{X\} \quad 3.11$$

Because internal forces {F} are not null vector. Equation 3.11 indicates m deformation in terms of n displacement. Now, according to step 2 of writing of compatibility condition (CC), displacements must be eliminated from equation to obtain ρ= m – n compatibility condition:

$$[C]\{\beta\}=\{0\} \qquad 3.12$$

The CC has (m – n) rows and columns.

3.2.2 Null Property of Equations

By using of equations 3.11 and 3.2 null property of equilibrium equation can be proved and subsequently compatibility condition can be obtained. According reference (Patnaik S. , 1999), (S. N. Patnaik and K. T. Joseph, 1986) , if deformations are removed between equations 3.11 and 3.12, compatibility condition can be generated as:

$$[C][A]^T\{X\}=\{0\} \qquad 3.13$$

In equation above, since displacements are subjective and not null vector, its coefficient can be withdrawn, then:

$$[C][A]^T=\{0\} \qquad 3.14$$

Or $$[A][C]^T=\{0\} \qquad 3.15$$

Thus, when equilibrium equation is generated and compatibility condition is written, the null property of them must be examined by using of equation 3.14 or 3.15.

3.2.3 Assembling of Compatibility Condition by Using of Null Space

According to references (S. N. Patnaik and K. T. Joseph, 1986), compatibility condition is obtained by null space of equilibrium equation (EE) and then this compatibility condition (CC) and EE are coupled. As it was mentioned before the used software this study is Mathematica and finding of null space of matrix is one of the several built in commands in this software. Then to find null space of equilibrium equation, it should be written as: NullSpace [A] (EE matrix).

3.2.4 Assembling of Compatibility Condition by Using of Singular Value Decomposition (SVD)

Another alternative to calculate the compatibility condition is utilizing of singular value decomposition termed as [M] matrix which is (S. N. Patnaik and K. T. Joseph, 1986), (Patnaik S. , 1999):

$$[M]=\left\lfloor [I]-[A]^T\left([A]^T\right)^{pinv} \right\rfloor \quad 3.16$$

In which:

[I] is identity matrix and number of its columns and rows are equal to number of members,

$[A]^T$ is transpose of equilibrium equation,

$\left([A]^T\right)^{pinv}$ is the Moore-Penrose pseudo inverse of $[A]^T$ which is obtained by:

$$\left([A]^T\right)^{pinv}=\left([A][A]^T\right)^{-1}[A] \quad 3.17$$

Then singular value decomposition (SVD) is applied to matrix [M] to obtain:

$$[M]=[M_u][M_\delta][M_v]^T \quad 3.18$$

Where: $[M_u]$ and $[M_v]$ are orthogonal matrix and number of their columns and rows are equal to number of element,

$[M_\delta]$ is obtained as:

$$[M_\delta]=\begin{pmatrix} \Lambda & 0 \\ 0 & 0 \end{pmatrix} \quad 3.19$$

In which, $\left[M_{\delta}\right]$ is square matrix,

$$\Lambda = diag\ (\Lambda_1, \Lambda_2, ..., \Lambda\rho)$$

And ρ is degree of indeterminacy and

$$\Lambda_1 \geq \Lambda_2 \geq ..... \geq \Lambda\rho \geq 0$$

According to references (Patnaik S. , 1999), (S. N. Patnaik and K. T. Joseph, 1986), it can be written as:

$$[M] = [M_u]\begin{bmatrix}[NS] \\ [0]\end{bmatrix} \qquad 3.20$$

$$[C] = [NS][G] \qquad 3.21$$

Where: [C] is compatibility condition matrix and will be calculated by equations 3.20and 3.21 and [NS] is null space matrix of equilibrium equation.

3.3 Dual Integrated Force Method (IFMD)

According to references (S. N. Patnaik, and D. A. Hopkins, 1998), (S. N. Patnaik, D. A. Hapkins, and G. R. Halford, 2004), Patnaik formulated and expanded the dual integrated force method. In this method termed (IFMD), main equation is:

$$[K]_{ifmd}\{X\} = \{P\}_{ifmd} \qquad 3.22$$

Where: [K] is Pseudo Stiffness matrix and is generated by:

$$[K]_{ifmd} = [A][G]^{-1}[A]^T \qquad 3.23$$

In which:

[A] is equilibrium equation matrix,

$[G]^{-1}$ is inverse of flexibility matrix which is a block diagonal matrix and each block is unconnected flexibility matrix for each member,

[X] is vector of displacements and

[P] is external loads or applied loads.

In truss structure flexibility matrix will be obtained as:

$$f_s = \frac{L_i}{E_i A_i} \quad 3.24$$

$$[G] = \begin{bmatrix} f_1 & & & 0 \\ & f_2 & & \\ & & \ddots & \\ 0 & & & f_s \end{bmatrix} \quad 3.25$$

The governing formulation for load vector is:

$$\{P\}_{ifmd} = \left\{\{P\} + \left([A][G]^{-1}\{\beta^0\}\right)\right\} \quad 3.26$$

Where, vector $\{\beta^0\}$ is initial deformation and its rows and columns are equal to number of total degree of freedom, but in this thesis initial deformation of supports are not considered and the vector $\{\beta^0\}$ will be zero. Then equation 3.25 in this thesis can be written as:

$$\{P\}_{ifmd} = \{P\} \quad 3.27$$

After assembling of $\{K\}_{ifmd}$ matrix and finding displacements with equation 3.22, the internal forces can be calculated by:

$$\{F\} = [G]^{-1}[A]^T\{X\} \quad 3.28$$

The similarity between integrated force method and dual integrated force method is using of equilibrium equation matrix [A], but difference between these two methods is that primary unknowns in integrated force method is internal forces however in dual integrated force method primary unknowns are displacements.

3.4 Overview of Solution Approach

As it was explained, the governing equation to analyze structure is:

$$[A]\{F\} = \{P\} \quad 3.29$$

This section discusses how equilibrium equation [A] is utilized to fine the internal forces. It also presents an overview of computer programming process with algorithms.

3.4.1 Overview of Usage of Equilibrium Equation

The truss structures are either determinate or indeterminate, equilibrium equation is used in both cases.

In determinate truss structures, writing of equilibrium equation is enough to solve the unknowns which are member forces, because number of equilibrium equations is equal to the number of unknowns. After obtaining internal forces the deformation and the displacements can be calculated. Therefore in determinate truss structures finding of deformations and displacements are straightforward after finding internal forces.

In indeterminate truss structures number of equilibrium equations is not equal to the number of unknowns, because number of members (unknowns) is more than the number of unrestrained degree of freedom. For overcoming this problem some additional relations must be supplied.

In structural analysis, displacement method and force method are two main and important methods which are used to find displacements and forces, respectively. In displacement method primary unknowns are nodal displacements and in force methods primary unknowns are member forces (internal forces).

The methods which are used in this thesis are shown in Figure 4. As shown in Figure dual integrated force method which is one of displacements method is used to find displacements and integrated force method one of force methods is used to calculate internal forces as primary unknowns.

As it was explained, compatibility condition is need in integrated force method and it is generated by null space or singular value decomposition.

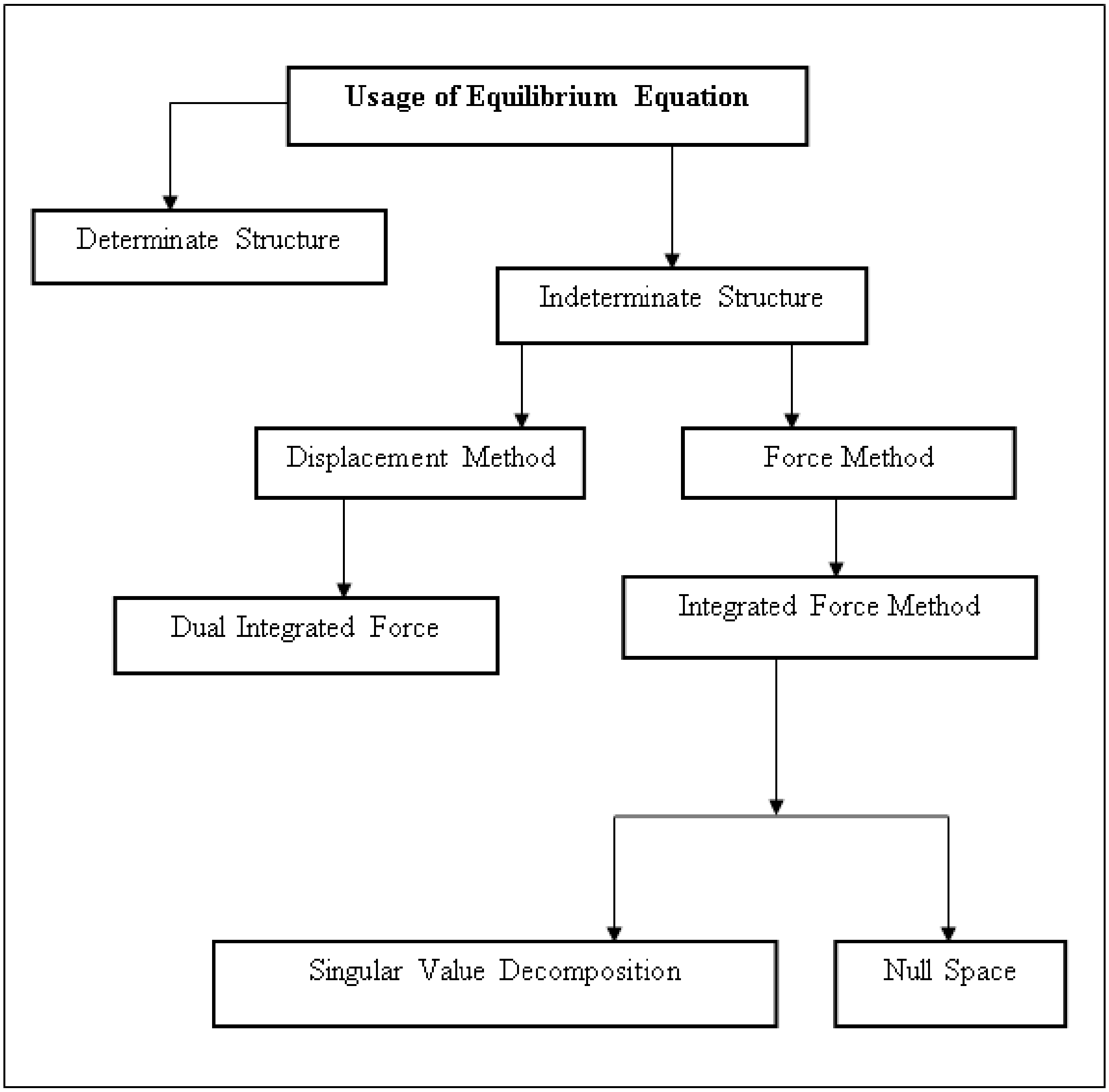


Figure 4.Classification of Usage of Equilibrium Equations

3.4.2 Overview of Computer Programming with Algorithms

When the space truss structures are in large scale, hand calculation are not practical and it consumes much time and at the end of calculation, results may be obtained that are not correct or exact. Therefore, there is a need to automate the solving procedure shown in Figure 4. In this section shown methods in Figure 4 is discussed in algorithm models.

3.4.2.1 Integrated Force Method

For integrated force method the main steps are: generation equilibrium equation, finding compatibility condition, coupling compatibility condition and equilibrium equation to find internal forces.

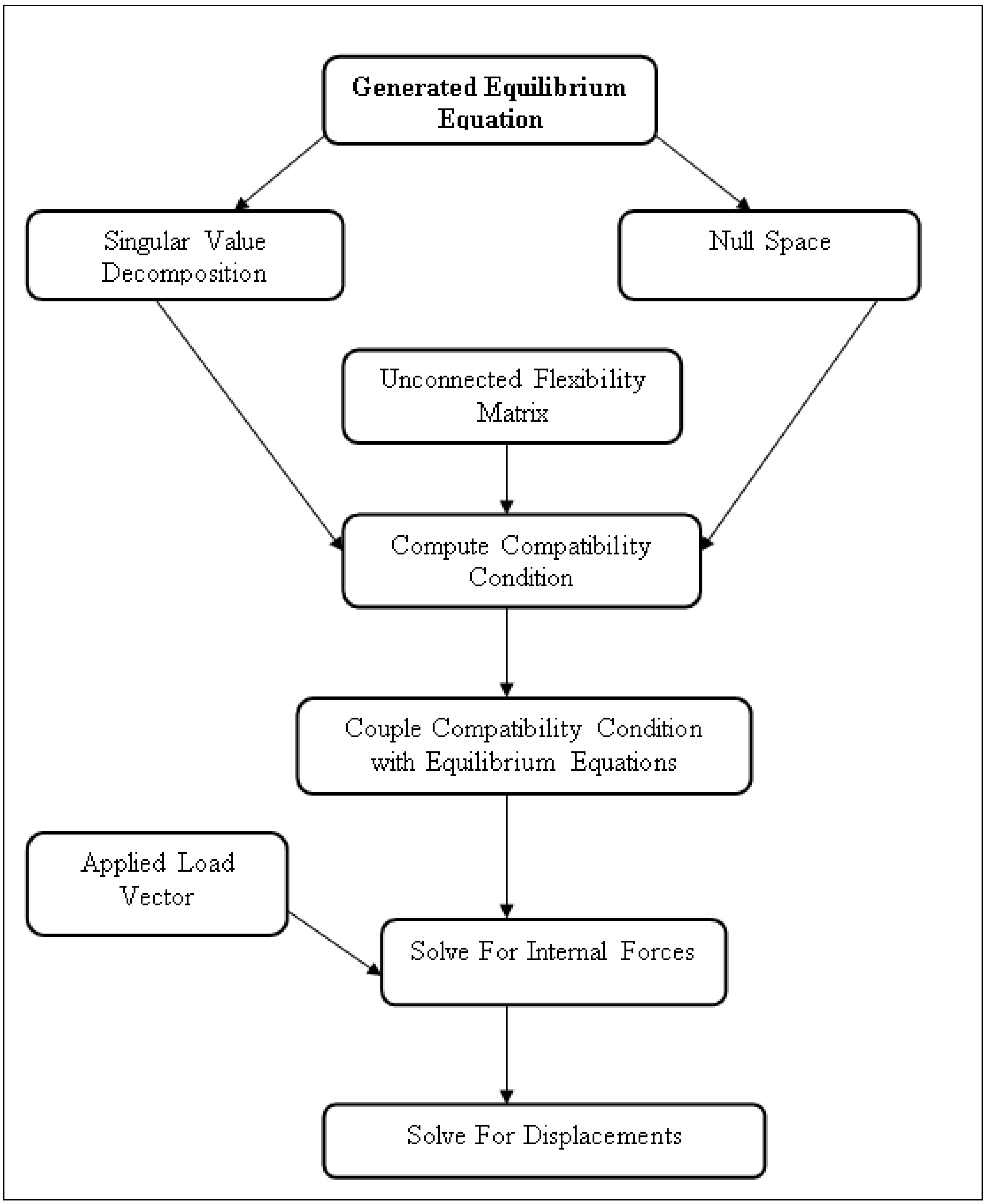


Figure 5. Overview of Integrated Force Method Programming

3.4.2.2 Dual Integrated Force Method

For dual integrated force method the main steps are: generation of equilibrium equation, generation global $[K]_{ifmd}$ matrix and then solve for displacements.

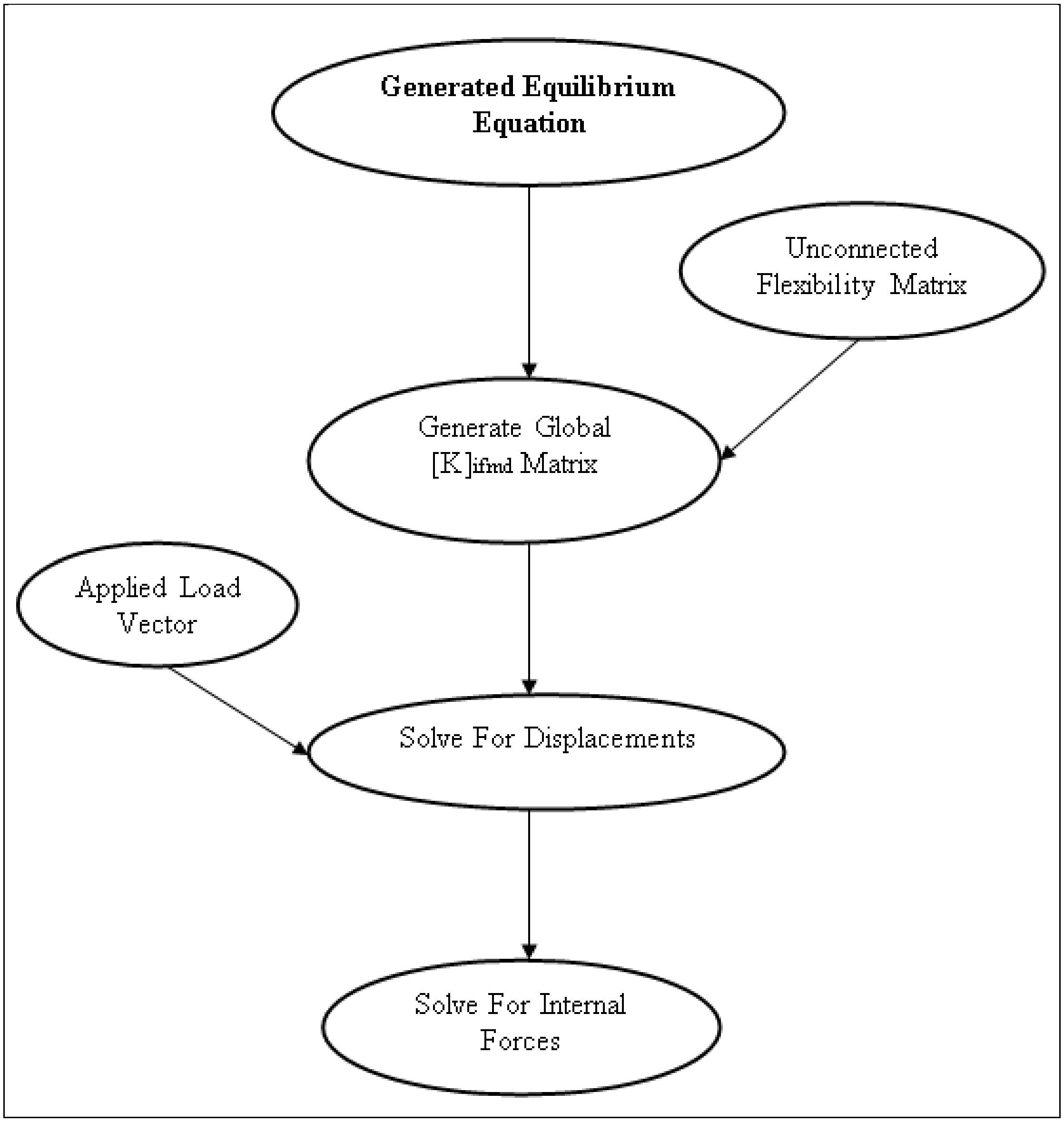


Figure 6. Overview of Dual Integrated Force Method Programming

The presented procedures above are for indeterminate space truss and in determinate cases calculation of internal forces are straightforward as shown in Figure 7.

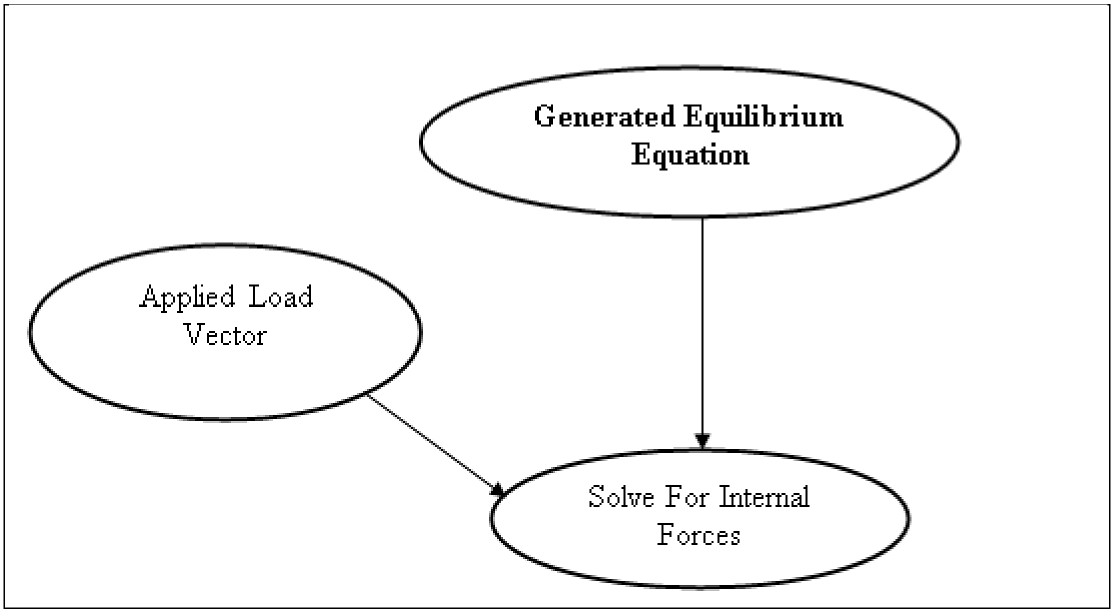


Figure 7. Overview of Determinate Structure Analysis

3.5 Programming

Actual programming for the expressed methods consists of:

- Matrix Operations
- Matrix Decomposition
- Solution of Linear System of Equation
- Making Scatter Plot of Matrix
- Import and Export of Data
- Using Symbolic and Numerical Mathematics

Therefore one of the ways to overcome these equations is using of computer algebra system like Mathematica 8 which is used in this thesis.

Mathematica software is numeric and symbolic computational engine, programming system, documentation system, graphics system, programming language and strong connectivity to other applications.

Other attribute of this software is being easy to use. Usually Mathematica is worked on its notebook interface and also results are visible in notebook interface. Mathematica has capability to present programs and its result in slides. For this purpose, it has toolbar with buttons to navigate between slides. This software has import and export filters for over 40 popular formats including DOC and JPEG. It is possible to open a file to read data form, and return an input stream object.

Additional packages for specialized analysis include:

- Linear Algebra 'Matrix Manipulation'
- Statics

3.5.1 Desired Features of the Programs for Integrated Force Method

In the survey of the state of the art, it was discovered that Patnaik used the following steps to obtain the compatibility condition:

- Writing of deformation displacement relations
- Eliminating the displacement
- Obtain the compatibility condition

In this study, two alternative algebraic methods are utilized to obtain the compatibility condition of space truss. After generation of equilibrium equation the following techniques will be used:

- Null space property of equilibrium equation matrix and flexibility matrix are combined
- Singular value decomposition of equilibrium equation matrix and flexibility condition are combined

3.5.2 Desired Features of the Programs for Dual Integrated Force Method

In the documentation and computer codes which were written previously, the main accent is generation of global stiffness matrix, however in the computer codes supplied by this thesis, the global stiffness matrix in dual integrated force method is written easily by using of generated equilibrium equation and manipulation capabilities of the algebra system of Mathematica 8 software. In the written programs in this study the global stiffness matrix is generated in only one command line. Therefore, by using of Mathematica 8 some programming advantages can be helpful.

3.5.3 Other Attributes of the Prepared Analysis Packages for IFM and IFMD

- Easy to use: there is no need to read or learn any documentation for the first time users, because the programs are easy to operate and use.
- Simple: in the comparison with the existing computer codes packages of structural analysis the running and understanding of analyzing procedure of prepared package for this study is simple.
- Transparent Theory: the theory of methods and used relations and equations are displayed in each of level of analyzing.
- Results: the analyzing results of each section are presented separately, then if user is suspicious of the result, a quick review of procedure is possible to find the mistake.

- Educational: the programs are like tutorial, because all of the theories and formulation at each section of procedure for IFM via null space, IFM via singular value decomposition and also IFMD are introduced step by step to users.

Chapter 4

AUTOMATIC ASSEMBLY OF EQUILIBRIUM EQUATIONS (EE) AND SOLUTION ALGORITHMS

4.1 Introduction

In this chapter the technique which is used to generate equilibrium equation is discussed. Also an example will be solved manually to show how this technique can be used in space truss structures. Then the process of solving of space truss is described step by step in algorithms and required relations and formulations are placed in these algorithms.

4.2 Formulation of Equilibrium Equations

In chapter two, basic concept of equilibrium equation and how it can be written, were explained. In this section a systematic method of assembling the equilibrium equations is established. This issue will be helpful to supply a computer code to write the equilibrium equations automatically.

To illustrate technique, consider space truss with four members as shown in Figure 8. Also the data of truss has been expressed in table 2.

According to section 2.3 and relation 2.1 the number of degree of indeterminacy is:

$$(4+12) - 5(3) = 1$$

And the only free degree of freedom (dof) exists at node number five.

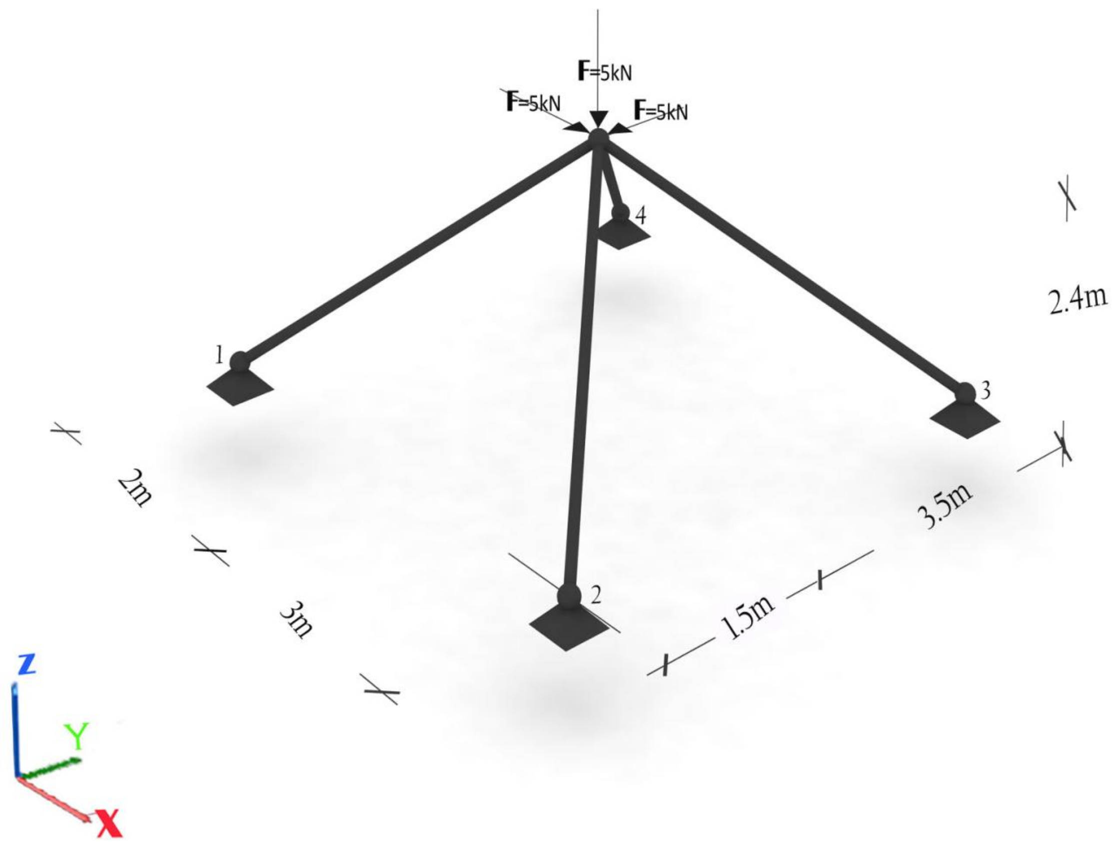


Figure 8. Space Truss with Four Members

Table 2. Nodal Data of Space Truss

Node Number	Coordinate (m)			Applied Load (kN)			Restraints		
	x	y	z	x	y	z	x	y	z
1	0	0	0	0	0	0	Fixed	Fixed	Fixed
2	5	0	0	0	0	0	Fixed	Fixed	Fixed
3	5	5	0	0	0	0	Fixed	Fixed	Fixed
4	0	5	0	0	0	0	Fixed	Fixed	Fixed
5	2	1.5	2.4	5	-5	-5	Free	Free	Free

Then equilibrium equation is written attention to free body diagram of truss. Therefore, according to relations 2.14 and 2.15 the length and the direction cosines of members are:

For element 1:

$$L_1 = \sqrt{2^2 + 1.5^2 + 2.4^2} = 3.465$$

$$Cos\theta_X = \frac{2}{3.465} = 0.577$$

$$Cos\theta_Y = \frac{1.5}{3.465} = 0.433$$

$$Cos\theta_Z = \frac{2.4}{3.465} = 0.700$$

For element 2:

$$L_2 = \sqrt{(-3)^2 + (1.5)^2 + 2.4^2} = 4.124$$

$$Cos\theta_X = \frac{-3}{4.124} = -0.727$$

$$Cos\theta_Y = \frac{1.5}{4.124} = 0.363$$

$$Cos\theta_Z = \frac{2.4}{4.124} = 0.582$$

For element 3:

$$L_3 = \sqrt{(-3)^2 + (-3.5)^2 + 2.4^2} = 5.197$$

$$Cos\theta_X = \frac{-3}{5.197} = -0.577$$

$$Cos\theta_Y = \frac{-3.5}{5.197} = -0.673$$

$$Cos\theta_Z = \frac{2.4}{5.197} = 0.461$$

For element 4:

$$L_4 = \sqrt{(2)^2 + (-3.5)^2 + 2.4^2} = 4.700$$

$$Cos\theta_X = \frac{2}{4.7} = 0.425$$

$$Cos\theta_Y = \frac{-3.5}{4.7} = -0.745$$

$$Cos\theta_Z = \frac{2.4}{4.7} = 0.510$$

Then, according relations above member's data can be collected as shown in Table 3:

Table 3. Member Data of Space Truss

Element Number	**connectivity**		**Length**	**Cosine Direction**		
	Start Node	End Node		l_{ij}	m_{ij}	n_{ij}
1	1	5	3.465	0.577	0.433	0.70
2	2	5	4.124	-0.727	0.363	0.582
3	3	5	5.197	-0.577	-0.673	0.461
4	4	5	4.700	0.425	-0.745	0.510

Where: $Cos\theta_X$, $Cos\theta_Y$ and $Cos\theta_Z$ are replaced by the terms of l_{ij}, m_{ij} and nij respectively. These terms are the same direction cosines which are expressed at chapter 2 and equation 2.7.

According to section 2.6, to transform the coordinate of internal forces from local system to global system in each member, a transformation matrix λ^i is used:

$$\lambda^i = \begin{bmatrix} l_{ij} & m_{ij} & n_{ij} & 0 & 0 & 0 \\ 0 & 0 & 0 & l_{ij} & m_{ij} & n_{ij} \end{bmatrix} \qquad 4.1$$

And also to convert basic truss force to elemental equilibrium equations in local system, matrix β^i is used:

$$B^i = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \qquad 4.2$$

In addition to form element equilibrium equations in global system, $\overline{B^i}$ is needed which is obtained by:

$$\overline{B^i} = \left(\lambda^i\right)^T B^i \qquad 4.3$$

Therefore, for each element it can be written:

$$\overline{B^1} = \left(\lambda^1\right)^T B^1 = \begin{bmatrix} 0.577 & 0 \\ 0.433 & 0 \\ 0.70 & 0 \\ 0 & 0.577 \\ 0 & 0.433 \\ 0 & 0.70 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{matrix} 1 \\ 2 \\ 3 \\ 13 \\ 14 \\ 15 \end{matrix} \begin{bmatrix} -0.577 \\ -0.433 \\ -0.70 \\ 0.577 \\ 0.433 \\ 0.70 \end{bmatrix}$$

$$\overline{B^2} = \left(\lambda^2\right)^T B^2 = \begin{bmatrix} -0.727 & 0 \\ 0.363 & 0 \\ 0.582 & 0 \\ 0 & -0.727 \\ 0 & 0.363 \\ 0 & 0.582 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{matrix} 4 \\ 5 \\ 6 \\ 13 \\ 14 \\ 15 \end{matrix} \begin{bmatrix} 0.727 \\ -0.363 \\ -0.582 \\ -0.727 \\ 0.363 \\ 0.582 \end{bmatrix}$$

$$\overline{B^3} = \left(\lambda^3\right)^T B^3 = \begin{bmatrix} -0.577 & 0 \\ -0.673 & 0 \\ 0.461 & 0 \\ 0 & -0.577 \\ 0 & -0.673 \\ 0 & 0.461 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{matrix} 7 \\ 8 \\ 9 \\ 13 \\ 14 \\ 15 \end{matrix} \begin{bmatrix} 0.577 \\ 0.673 \\ -0.461 \\ -0.577 \\ -0.673 \\ 0.461 \end{bmatrix}$$

$$\overline{B^4} = \left(\lambda^4\right)^T B^4 = \begin{bmatrix} 0.425 & 0 \\ -0.745 & 0 \\ 0.510 & 0 \\ 0 & 0.425 \\ 0 & -0.745 \\ 0 & 0.510 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{matrix} 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \end{matrix} \begin{bmatrix} -0.425 \\ 0.745 \\ -0.510 \\ 0.425 \\ -0.745 \\ 0.510 \end{bmatrix}$$

The superscript T indicates transpose of matrix. Note that the numbers from1 to 15 used in the row numbers of matrix $\left(\lambda^i\right)^T B^i$ indicate the global numbering of degree of freedom for each node as:

Node 1: 1, 2, 3

Node 2: 4, 5, 6

Node 3: 7, 8, 9

Node 4: 10, 11, 12

Node 5: 13, 14, 15

Therefore, according to equations 2.4 to 2.9 can be written and then the equilibrium equations for space truss can be assembled directly by transferring each entry from $\left(\lambda^{i}\right)^{T} B^{i}$ to overall equilibrium equations [A].this is carried out according global degree of freedom:

$$
\begin{bmatrix}
-0.577 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.433 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.70 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.727 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.363 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.582 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.577 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.673 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
00 & 0 & -0.461 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.425 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0.745 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & -0.510 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0.577 & -0.727 & -0.577 & 0.425 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.433 & 0.363 & -0.673 & -0.745 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.70 & 0.582 & 0.461 & 0.510 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \\ R_7 \\ R_8 \\ R_9 \\ R_{10} \\ R_{11} \\ R_{12} \end{bmatrix}
=
\begin{bmatrix} 5 \\ -5 \\ -5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

But according to section 2.4, the equilibrium equations can also be written only for free degree of freedom, which means that reactions can be eliminated. Therefore to three degree of freedom and four elements, the equilibrium equation matrix above is reduced to:

$$\begin{bmatrix} 0.577 & -0.727 & -0.577 & 0.425 \\ 0.433 & 0.363 & -0.673 & -0.745 \\ 0.70 & 0.582 & 0.461 & 0.510 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \\ -5 \end{bmatrix}$$

Equilibrium Equation [A] | Internal Forces {F} | Applied Loads {P}

Now the matrix of equilibrium equation is generated and this matrix [A] will be used to couple with some additional equations to generate the final square equilibrium equation, matrix [S] and then by using of relation 3.3, internal forces will be calculated.

4.3 Algorithm for Automatic Assembly of Equilibrium Equations

In this section the method by which computer codes can be written to generate the reduced equilibrium equations automatically, is explained. Following steps are used to generate the equilibrium equations matrix:

- **Step 1:** Writing of X, Y and Z coordinates of element at the each end according to the element connectivity,
- **Step 2:** Use equation 2.6 and 2.7 to calculate the length and direction cosine of each member.
- **Step 3:** Use relations 4.1 and 4.2 to obtain the matrixes λ^i and B^i.
- **Step 4:** Use equation 4.3 to obtain $\overline{B^i}$ matrix.
- **Step 5:** Establish the 3j×m zero matrix(j is number of joints and m is number of elements)

Number of members

Degree of freedom

$$\begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \qquad 4.4$$

- **Step 6:** Place each $\overline{B^i}$ into the columns of the zero matrix:

Number of members

Degree of freedom

$$\begin{bmatrix} \overset{\overline{B^1}}{0} & \overset{\overline{B^2}}{0} & \cdots & \overset{\overline{B^m}}{0} \\ \vdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \qquad 4.5$$

If a degree of freedoms corresponds to a restrained degree of freedoms then no entry is made into that row.

- **Step 7:** Rows which are containing complete zero entries will be deleted,
- **Step 8:** The resulting matrix has a size of:

  (3j – number of restraints) × m 4.6

These following steps are used for the truss shown in Figure 8.

Step 1: The nodal data has been written in Table 2.

Step 2: The member data has been written in Table 3.

Step 3: λ^i and B^i are:

$$\lambda^1 = \begin{bmatrix} 0.577 & 0.433 & 0.70 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.577 & 0.433 & 0.70 \end{bmatrix}$$

$$\lambda^2 = \begin{bmatrix} -0.727 & 0.363 & 0.582 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.727 & 0.363 & 0.582 \end{bmatrix}$$

$$\lambda^3 = \begin{bmatrix} -0.577 & -0.673 & 0.461 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.577 & -0.673 & 0.461 \end{bmatrix}$$

$$\lambda^4 = \begin{bmatrix} 0.425 & -0.745 & 0.510 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.425 & -0.745 & 0.510 \end{bmatrix}$$

$$B^1 = B^2 = B^3 = B^4 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Step 4: The $\overline{B^i}$ matrixes are:

$$\overline{B^1} = \left(\lambda^1\right)^T B^1 = \begin{bmatrix} 0.577 & 0 \\ 0.433 & 0 \\ 0.70 & 0 \\ 0 & 0.577 \\ 0 & 0.433 \\ 0 & 0.70 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{matrix} 1 \\ 2 \\ 3 \\ 13 \\ 14 \\ 15 \end{matrix} \begin{bmatrix} -0.577 \\ -0.433 \\ -0.70 \\ 0.577 \\ 0.433 \\ 0.70 \end{bmatrix}$$

$$\overline{B^2} = \left(\lambda^2\right)^T B^2 = \begin{bmatrix} -0.727 & 0 \\ 0.363 & 0 \\ 0.582 & 0 \\ 0 & -0.727 \\ 0 & 0.363 \\ 0 & 0.582 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{matrix} 4 \\ 5 \\ 6 \\ 13 \\ 14 \\ 15 \end{matrix} \begin{bmatrix} 0.727 \\ -0.363 \\ -0.582 \\ -0.727 \\ 0.363 \\ 0.582 \end{bmatrix}$$

$$\overline{B^3} = \left(\lambda^3\right)^T B^3 = \begin{bmatrix} -0.577 & 0 \\ -0.673 & 0 \\ 0.461 & 0 \\ 0 & -0.577 \\ 0 & -0.673 \\ 0 & 0.461 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{matrix} 7 \\ 8 \\ 9 \\ 13 \\ 14 \\ 15 \end{matrix} \begin{bmatrix} 0.577 \\ 0.673 \\ -0.461 \\ -0.577 \\ -0.673 \\ 0.461 \end{bmatrix}$$

$$\overline{B^4} = \left(\lambda^4\right)^T B^4 = \begin{bmatrix} 0.425 & 0 \\ -0.745 & 0 \\ 0.510 & 0 \\ 0 & 0.425 \\ 0 & -0.745 \\ 0 & 0.510 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{matrix} 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \end{matrix} \begin{bmatrix} -0.425 \\ 0.745 \\ -0.510 \\ 0.425 \\ -0.745 \\ 0.510 \end{bmatrix}$$

Step 5: Generate zero matrix:

$$\begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

Step 6: Put $\overline{B^i}$ in to zero matrix as explained in section 4.3.

$$
\begin{array}{r}
\\ 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15
\end{array}
\begin{array}{cccc}
\overline{B^1} & \overline{B^2} & \overline{B^3} & \overline{B^4} \\
\end{array}
$$

$$
\begin{array}{r|cccc|}
 & \overline{B^1} & \overline{B^2} & \overline{B^3} & \overline{B^4} \\
1 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & 0 \\
5 & 0 & 0 & 0 & 0 \\
6 & 0 & 0 & 0 & 0 \\
7 & 0 & 0 & 0 & 0 \\
8 & 0 & 0 & 0 & 0 \\
9 & 0 & 0 & 0 & 0 \\
10 & 0 & 0 & 0 & 0 \\
11 & 0 & 0 & 0 & 0 \\
12 & 0 & 0 & 0 & 0 \\
13 & 0.577 & -0.727 & -0.577 & 0.425 \\
14 & 0.433 & 0.363 & -0.673 & -0.745 \\
15 & 0.70 & 0.582 & 0.461 & 0.510
\end{array}
$$

Step 7: Delete the rows with all zero entries.

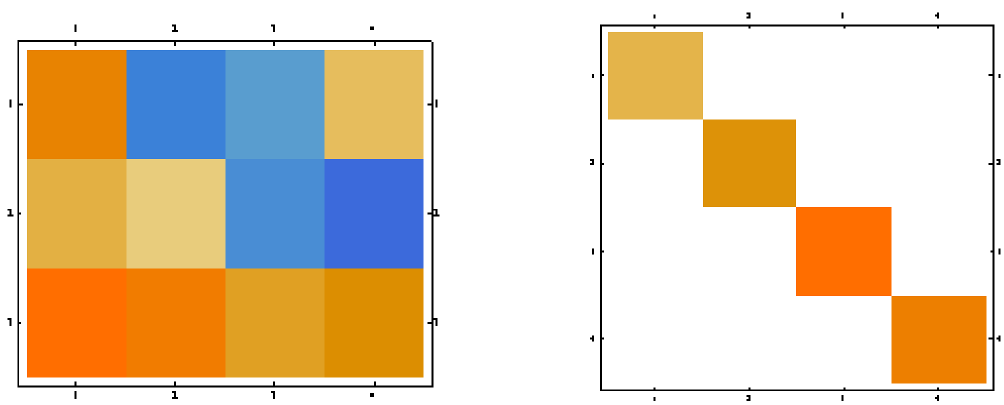
$$
\begin{array}{r|cccc|}
 & \overline{B^1} & \overline{B^2} & \overline{B^3} & \overline{B^4} \\
13 & 0.577 & -0.727 & -0.577 & 0.425 \\
14 & 0.433 & 0.363 & -0.673 & -0.745 \\
15 & 0.70 & 0.582 & 0.461 & 0.510
\end{array}
$$

4.4 Solution Algorithms

As it was expressed in chapter 2 and 3, this study uses the integrated force method and dual integrated force method to analyze the space truss structures. In the both of methods the beginning step of analysis procedure is generation of equilibrium equation and it is same at both of them. The difference between these methods starts after writing of equilibrium equation matrix. This section presents the algorithms of methods which are used in this thesis and intended computer codes are based on these algorithms.

4.4.1 Integrated Force Method via Null Space

In integrated force method, null space property of the equilibrium equation is used to obtain the compatibility condition. Then obtained compatibility condition and equilibrium equation are couples together to generate the square matrix [S]. Finally this square matrix is utilized to calculate the unknowns by using of equation 3.2. The procedure of integrated force method via null space is shown in Figure 11.Applying this method the equilibrium equation and unconnected flexibility matrix for space truss illustrated in Figure 8 are obtained as shown in Figure 9 in scatter plots form.



(a) (b)

Figure 9. Figure (a); Matrix Plot of EE Figure (b); Matrix Plot of Flexibility

Also the scatter plot of coupled equilibrium equation with compatibility condition matrix is shown in Figure 10.

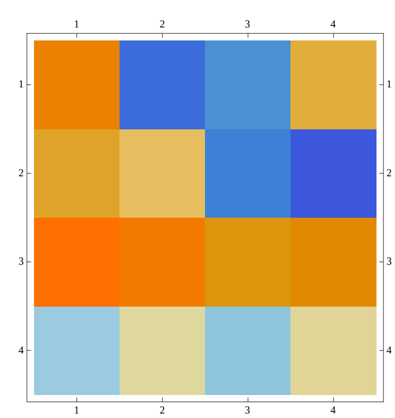


Figure 10. Matrix Plot of Coupled EE and CC via Null Space

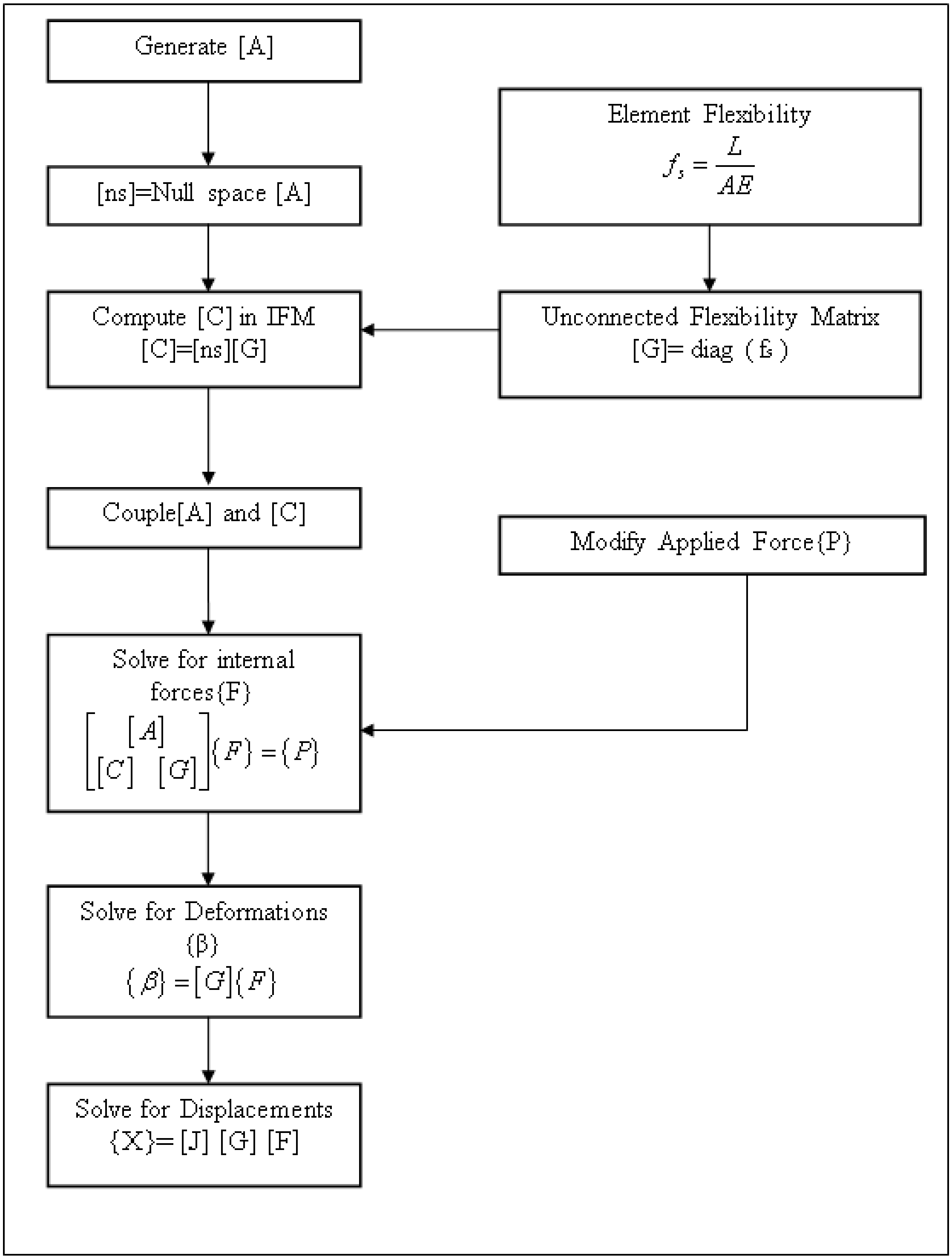


Figure 11. Algorithm of Integrated Force Method via Null Space

4.4.2 Integrated Force Method via Singular Value Decomposition

Another method to find the compatibility condition in the integrated force method is using of singular value decomposition outlined in section 3.2.4. When equilibrium equation is generated, matrix $\left([A]^T\right)^{pinv}$, is obtained with equation 3.17. Then matrix [M] is calculated by using of equation 3.18. Then the singular value decomposition (SVD) of the matrix [M] is carried out to obtain matrices $[M_u],[M_v]$ and $[M_\delta]$. Then compatibility condition can be calculated by equations 3.20 and 3.21. This procedure of integrated force method is shown in Figure 13.

Applying this method to the truss shown in Figure 8, the equilibrium equation and flexibility matrix will remain same. Then scatter plot of them will be same as null space method. Also the scatter plot of couples equilibrium equation and compatibility condition is shown in Figure 12.

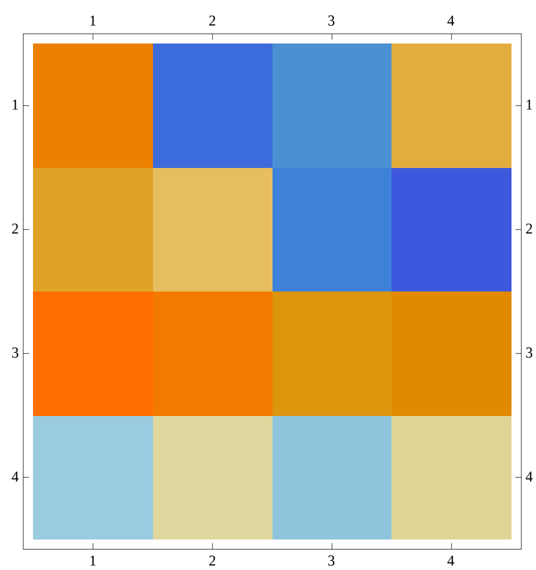


Figure 12. Matrix Plot of Coupled EE and CC via SVD

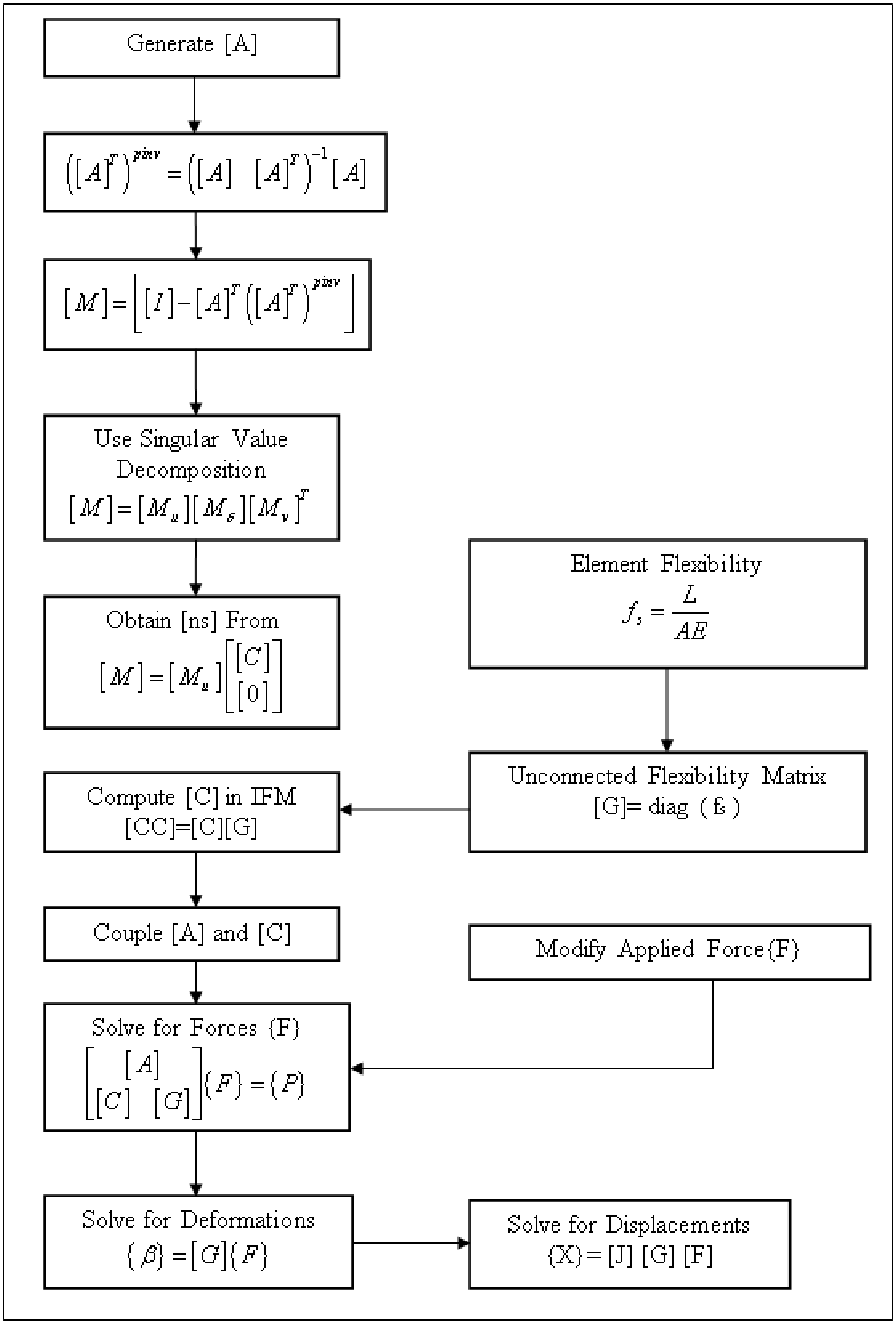


Figure 13. Algorithm for Integrated Force Method via SVD

4.4.3 Dual Integrated Force Method

The procedure and equations for the dual integrated force method is shown in Figure 15. After generation of equilibrium equation in dual integrated force method by using of equation 3.23 the matrix $[K]_{ifmd}$ is assembled. To obtain the $[K]_{ifmd}$ inverse of flexibility matrix is used. Then the nodal displacements are obtained by equation 3.22. Finally the internal forces can be calculated by equation 3.28.

In this method the scatter plot of equilibrium equation remains same because the element and nodal numbering system are same and the scatter plots of flexibility matrix and $[K]_{ifmd}$ matrix is shown in Figure 14.

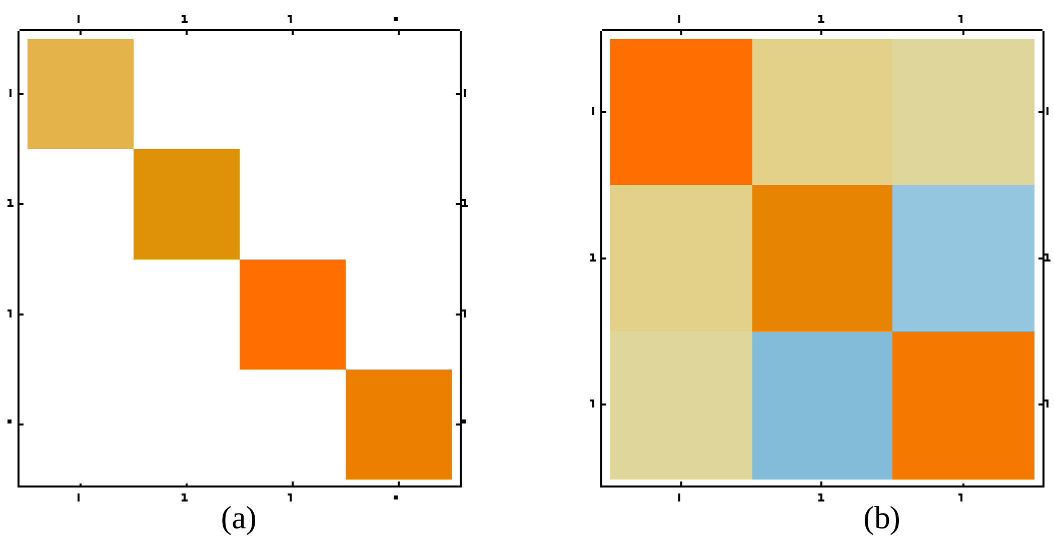


Figure 14. Figure (a); Matrix Plot of Flexibility and Figure (b); Plot of Pseudostiffness Matrix

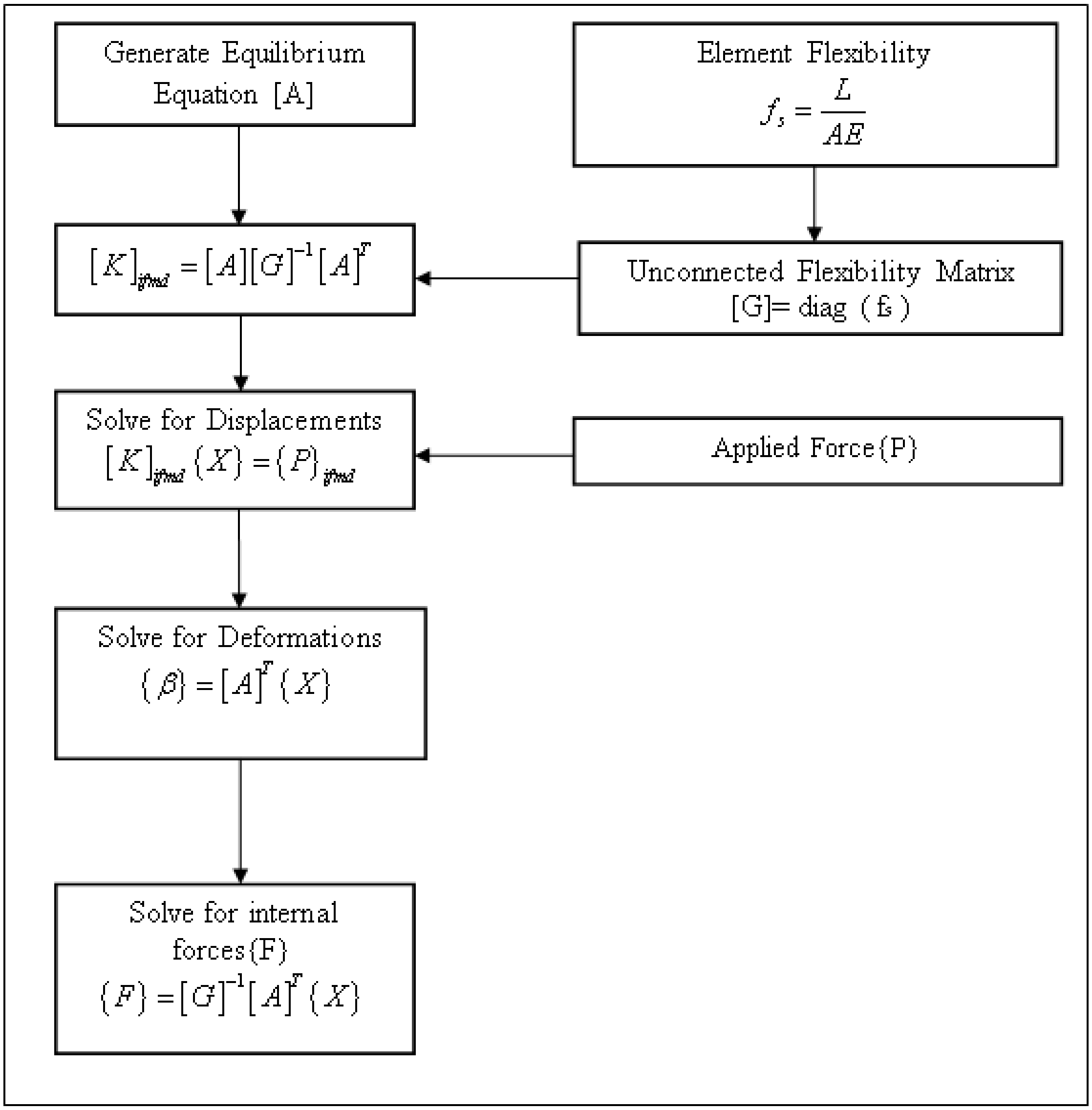


Figure 15. Algorithm for Dual Integrated Force Method

Chapter 5

SPACE TRUSS ANALYSIS PACKAGE

5.1 Introduction

The main and principal purpose of this study is to establish a computer code to analyze the indeterminate space truss structures base on integrated force method, because majority of existing computer codes and analysis software are based on stiffness method and during the survey of the state of the art for this thesis, the computer codes for space truss analysis with integrated force method were not found.

Three packages of computer codes are written for this study to analyze the indeterminate space truss structures. Each package uses different theories which were explained in chapter 3. Then the space truss analysis packages are:

- Package 1: Integrated Force Method via Null Space
- Package 2: Integrated Force Method via Singular Value Decomposition
- Package 3: Dual Integrated Force Method.

In this chapter, to introduce the all of three programs an indeterminate space truss as Figure 16 is discussed as a simple illustrative example. Also how these programs can be used and how the data of structures should be entered are illustrated in this chapter.

After that all sections and steps of analysis procedure in these packages are shown and explained.

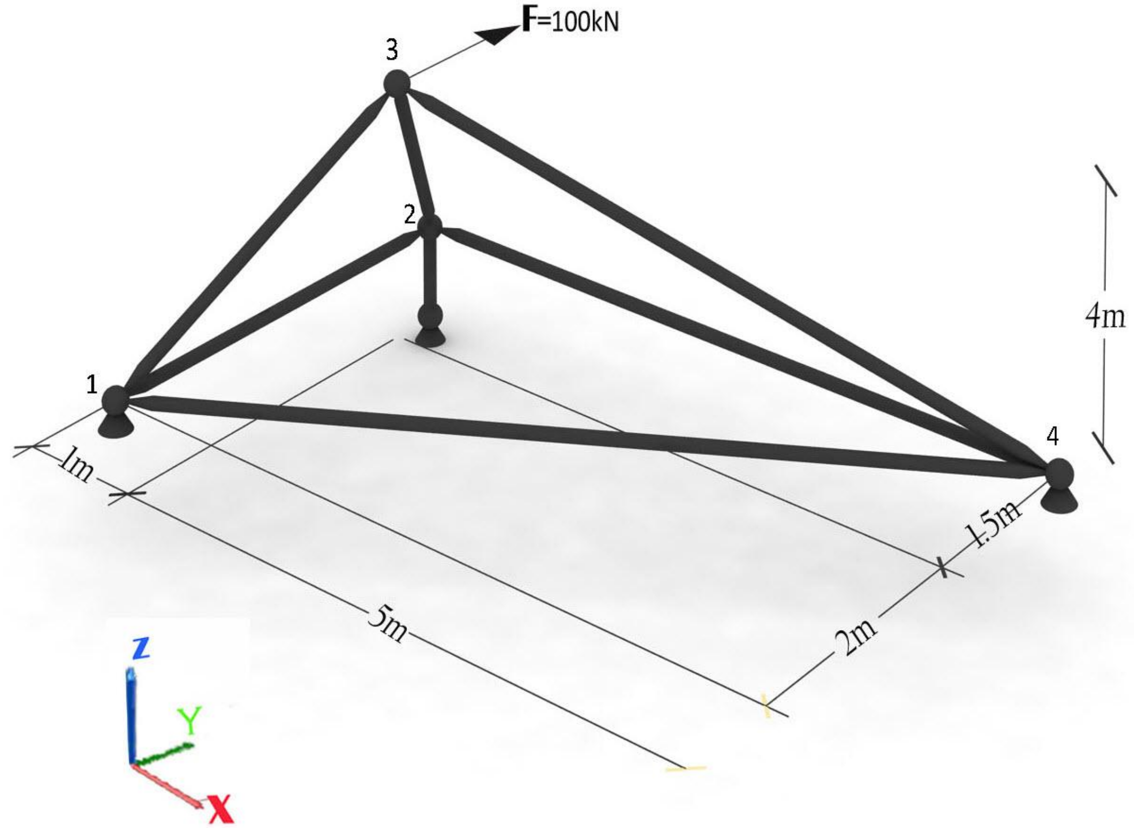


Figure 16. Space Truss with Six Members and Four Nodes

The package consists of three main common phases as:

- Data Input Phase
- Calculation and Reporting Phase

The space truss of Figure 16 has 6 members with 4 nodes and the coordinates of nodes and member connectivity are shown in Table 4 and 5 receptively. The area for each member is A= 0.0025 m² and modulus of elasticity is $E = 2 \times 10^8$ kN/m². Only one external load which enters in node 3 is applied to truss.

The structures of the matrices assembled for this truss will be illustrated by using of their matrix plot. Each matrix plot displays the nonzero entries in color and according to value of the nonzero entries the matrix plot colors change

Table 4. Nodal Data of Truss Structure of Figure 16

Node Number	Coordinates			Applied Loads (kN)			Restraints		
	x	y	z	x	y	z	x	y	z
1	0	0	0	0	0	0	Fixed	Fixed	Fixed
2	0	3.5	0	0	0	0	Free	Free	Fixed
3	1	2	4	0	100	0	Free	Free	Free
4	5	3.5	0	0	0	0	Fixed	Fixed	Fixed

Table 5. Elemental Data of Truss Structure of Figure 16

Element Number	Connectivity	
	Start Node	End Node
1	1	2
2	2	3
3	1	3
4	3	4
5	2	3
6	1	4

5.2 Data Input Phase

This phase describes the problems. Data input phase is depicted to assist in demonstration of general input of truss (number of nodes and elements), geometry of truss (member connectivity, coordinates of nodes and restraints of joints), properties of elements and materials (member area and modulus of elasticity) and loading case (joint loads). A diagram of skeleton of input phase is shown in Figure 17.

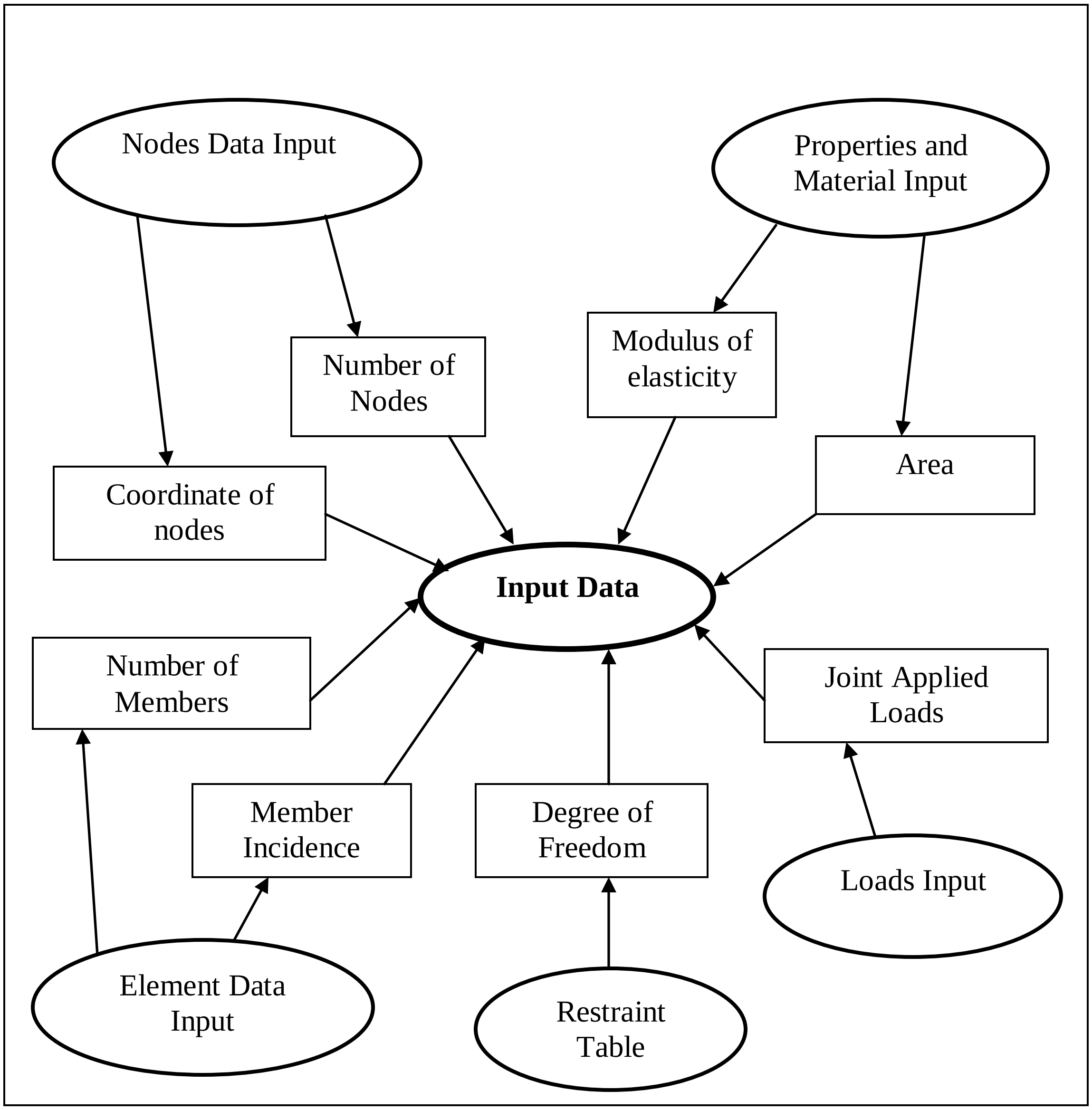


Figure 17. Input Phase skeleton Diagram

5.2.1 Interface of Data Input Phase

The user interface of data input phase consists of following sections:

- Nodes Data Input
- Element Data Input
- Restraint Data Input

- Loads Data Input
- Properties and Material Input

When these five sections are completed by user, data input phase is completed and program can be run.

5.2.1.1 Nodes Data Input

In this part the user should give number of nodes and coordinates as shown in Figure 18. This part includes n × 4 matrix. The first column of this matrix shows the number of each node. Second, third and fourth columns of this matrix show X, Y and Z coordinates of node respectively.

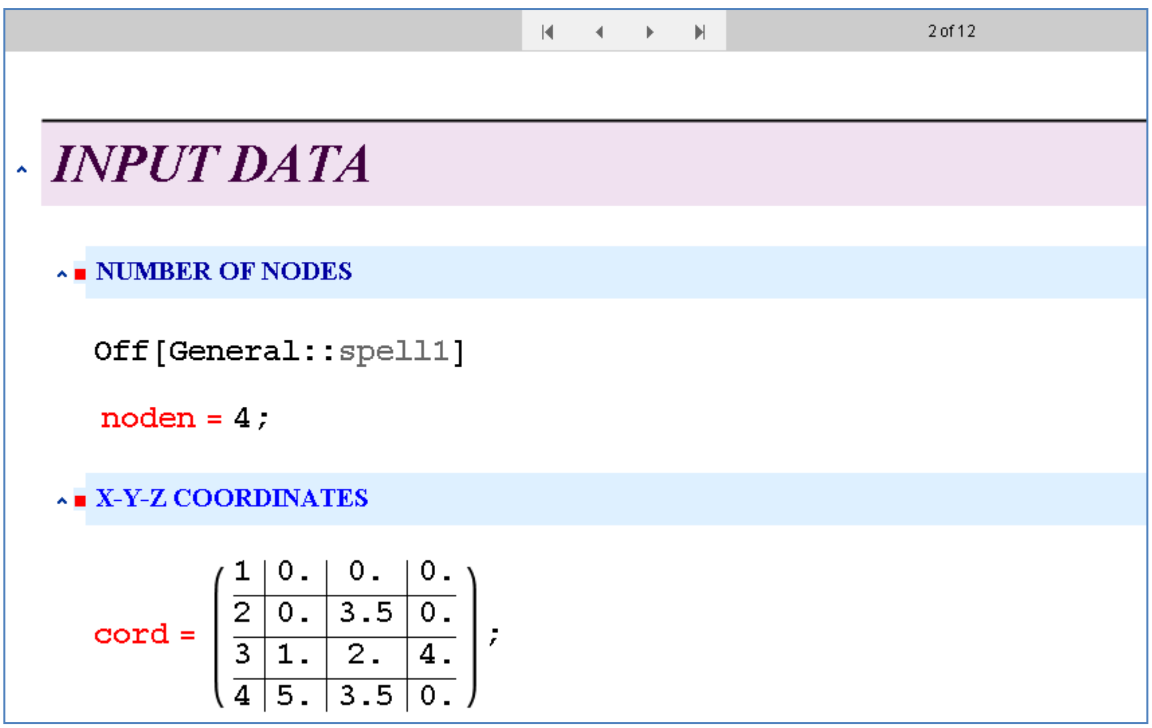


Figure 18. Nodes Numbers and Coordinates

5.2.1.2 Element Data Input

In this step of procedure user enters the element information like number of elements and connectivity to indicate geometry of truss. This part has m × 3 matrix in which m is equal to number of members and the first column shows number of element, second column shows number of starting node and third column shows the number of end node of each element. (Figure 19)

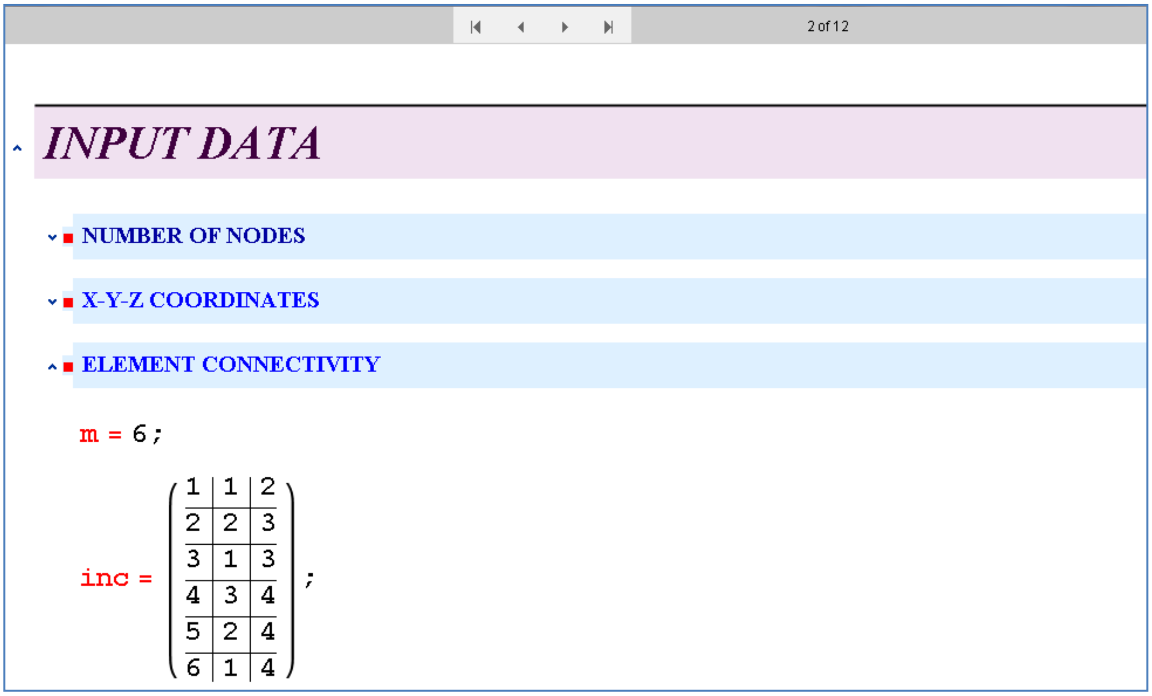


Figure 19. Element Number and Connectivity

5.2.1.3 Restraint Data Input

This part is indicates freedom condition of joints which includes n × 4 matrix. Where first column shows the number of joints, second, third and fourth columns show condition of supports and degree of freedom of in x, y and z axes directions to be free or restrained in each node. Each nodes of space truss has three degree of freedom along three axes of x, y and z. Then to specify the degree of freedom 0 will be replaced at the related direction of axes to show that there is not any restraint and at the supports and nodes in which there is restraint condition 1 will be replaced in the related direction (Figure 20).

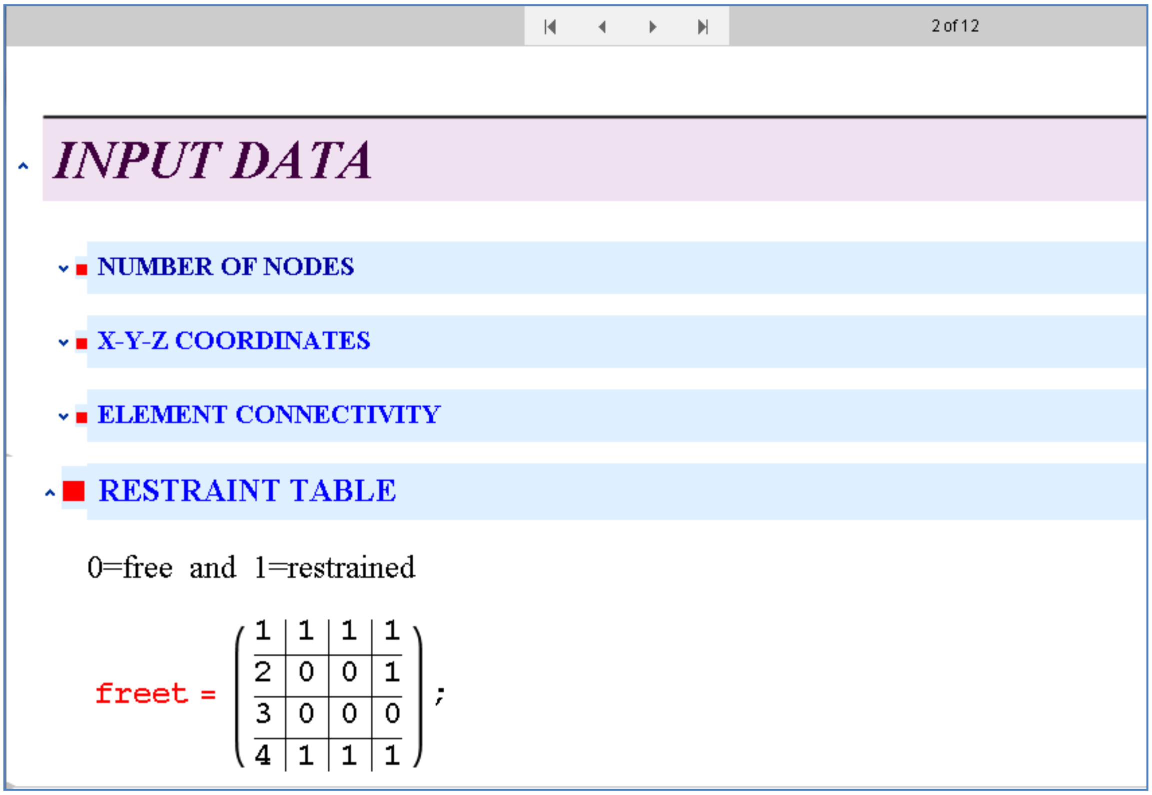


Figure 20. Freedom Conditions of Joints

5.2.1.4 Loads Data Input

In this part user should define the point loads (concentrated load). In truss structures the external loads are applied at the joints. Then in this step there is n × 4 matrix in which first column shows number of joint, second, third and fourth shows x, y and z axes direction. Then if there is any load applied at joint, in front of number of that node value of load should be entered in the related direction of axes and if there is no applied point load at a node or some axes direction of node, then the value of zero should be entered. (Figure 21).

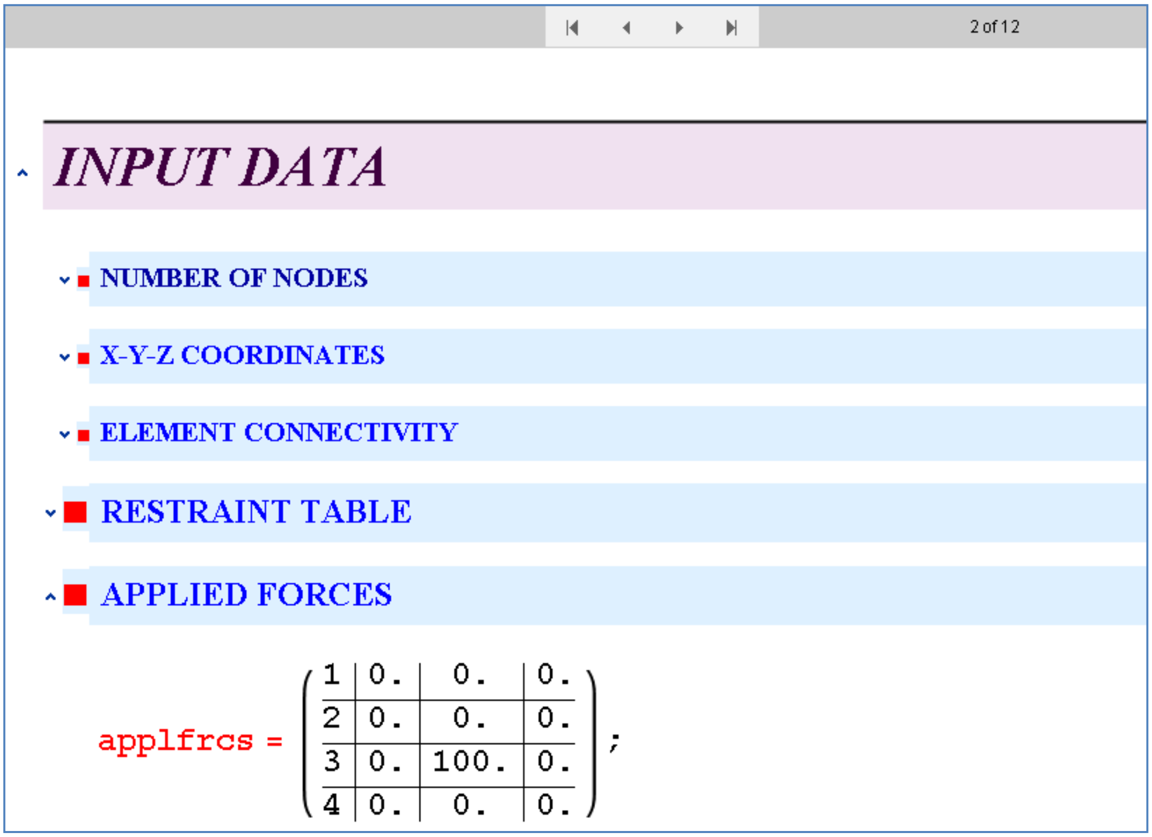


Figure 21. Applied Loads at Joints

5.2.1.5 Properties and Material Input

In this part the value of modulus elasticity are entered. The value that used in this study is $E = 2 \times 10^8$ and its unit is kN/m². Also this part contains the values of member section area. The used sections area for all members may change. This package has flexibility in that structure problems consist of different member area (Figure 22).

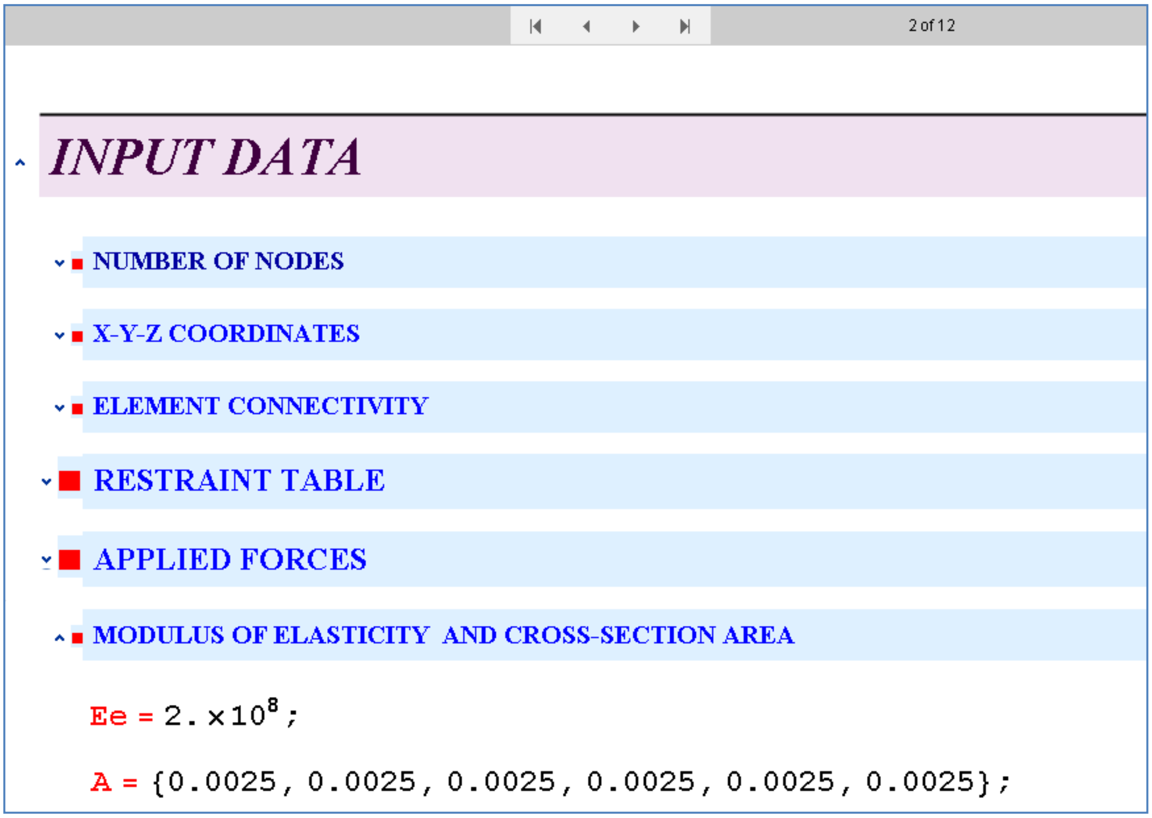


Figure 22. Material and Section Property of Members

5.3 Calculation and Reporting Phase

This phase tries to calculate the structures similar to hand calculation which were explained in chapter 4. Therefore, the user can see each step of calculation like the way students do in exam papers. Also user can see related computer codes to the solving and analyzing steps and formulations at each level.

In the calculation phase the computer codes for generation of equilibrium equations is same for all of three methods intended for this study. The formulas and algorithm of writing of generation was explained in chapter 4 and 5.

After generation of equilibrium equation matrix the computer codes will vary for the three methods. Then the computer codes for calculation phase, after assembling of equilibrium equation consists of following parts:

- IFM via Null Space
- IFM via Singular Value Decomposition
- Dual Integrated Force Method

5.3.1 Computer Codes for Assembling of Equilibrium Equation Matrix

The computer codes which are written to generate of equilibrium equation matrix can be divided into three parts generally:

At the first part the computer codes are written to read the input data of truss like node coordinates and freedom condition and element section area and connectivity to obtain the member length and direction cosines as shown in Figure 23 by using of 2.6 and 2.7:

$$L = \sqrt{\left(\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2 + \left(z_j - z_i\right)^2\right)}$$

$$l_{ij} = \frac{x_j - x_i}{L_{ij}}, \quad m_{ij} = \frac{y_j - y_i}{L_{ij}}, \quad n_{ij} = \frac{z_j - z_i}{L_{ij}}$$

The second part of this section is related computer codes to calculate and write the elemental transformation matrix and then elemental equilibrium equations (Figure 24). The used relations for this purpose are:

$$\lambda^i = \begin{bmatrix} l_{ij} & m_{ij} & n_{ij} & 0 & 0 & 0 \\ 0 & 0 & 0 & l_{ij} & m_{ij} & n_{ij} \end{bmatrix}$$

$$B^i = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\overline{B^i} = \left(\lambda^i\right)^T B^i$$

EQUILIBRIUM EQUATIONS

THE COMPUTER CODE

```
Off[General::spell]

kmem = {};
Lmem = {};
bgmem = {};
bmem = {};
tmem = {};

Off[General::spell]

Do[

  mincb = inc[[i, 2]];
  mince = inc[[i, 3]];
  dtabl = {3 * mincb - 2, 3 * mincb - 1, 3 * mincb,
           3 * mince - 2, 3 * mince - 1, 3 * mince};
  xd = cord[[mince, 2]] - cord[[mincb, 2]];
  yd = cord[[mince, 3]] - cord[[mincb, 3]];
  zd = cord[[mince, 4]] - cord[[mincb, 4]];
  If[freet[[mincb, 2]] == 1, ReplacePart[dtabl, 0, 1]];
  If[freet[[mincb, 3]] == 1, ReplacePart[dtabl, 0, 2]];
  If[freet[[mincb, 4]] == 1, ReplacePart[dtabl, 0, 3]];
  If[freet[[mince, 2]] == 1, ReplacePart[dtabl, 0, 4]];
  If[freet[[mince, 3]] == 1, ReplacePart[dtabl, 0, 5]];
  If[freet[[mince, 4]] == 1, ReplacePart[dtabl, 0, 6]];
  Lm = Sqrt[xd^2 + yd^2 + zd^2];
  ldc = xd/Lm;
  mdc = yd/Lm;
  ndc = zd/Lm;
```

Figure 23. Computer Codes for Find the Length and Direction Cosines of members

```
b = ( -1. / 1. );
t = ( ldc/z | mdc/z | ndc/z | z/ldc | z/mdc | z/ndc );
bg = Transpose[t].b;
AppendTo[bmem, b];
AppendTo[Lmem, Lm];
AppendTo[tmem, t];
AppendTo[bgmem, bg];
Print["b  =", i, "   ", MatrixForm[b]];
Print["i  =", i, "   ", ldc, "   ", mdc, "   ",
 ndc, "   "];
Print["bg  =", i, "   ", MatrixForm[bg]];
Print[mincb, "---", mince],
{i, 1, m}];


Lmem

{3.5, 4.38748, 4.58258, 5.85235, 5., 6.10328}


rest = 0;
Do[If[freet[[i, 2]] == 1, rest = rest + 1];

 If[freet[[i, 3]] == 1, rest = rest + 1];
 If[freet[[i, 4]] == 1, rest = rest + 1],

 {i, 1, noden}]

rest

7

dof = Table[jj, {jj, 1, 3 * noden}]

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
```

```
Clear[kk]
kk = 0;
Do[
 cj1 = 3 * i - 2;
 cj2 = 3 * i - 1;
 cj3 = 3 * i;
 Print[i, "    ", cj1, "    ", cj2, "    ", cj3];
 If[freet[[i, 2]] == 1, dof[[3 * i - 2]] = 0];
 If[freet[[i, 2]] == 0, kk = kk + 1];
 If[freet[[i, 2]] == 0, dof[[3 * i - 2]] = kk];
 If[freet[[i, 3]] == 1, dof[[3 * i - 1]] = 0];
 If[freet[[i, 3]] == 0, kk = kk + 1];
 If[freet[[i, 3]] == 0, dof[[3 * i - 1]] = kk];
 If[freet[[i, 4]] == 1, dof[[3 * i]] = 0];
 If[freet[[i, 4]] == 0, kk = kk + 1];
 If[freet[[i, 4]] == 0, dof[[3 * i]] = kk],
 {i, 1, noden}]
1    1    2    3
2    4    5    6
3    7    8    9
4    10    11    12
dof
{0, 0, 0, 1, 2, 0, 3, 4, 5, 0, 0, 0}
S = Table[0., {sr, 1, 3 * noden - rest}, {sc, 1, m}];
mi = 0;
```

Figure 24. Writing of Elemental Transformation Matrix and Equilibrium Equation

Third part of this section includes computer codes to store and assemble all of the elemental equilibrium equations to one matrix which is termed global equilibrium equation matrix as shown in Figure 25. Also the final equilibrium equation matrix is shown in Figure 26.

```
(*STORE [bg] INTO [S] *)
Do[
 node1 = inc[[i, 2]];
 node2 = inc[[i, 3]];
 k1 = 3 * node1 - 2;
 k2 = 3 * node1 - 1;
 k3 = 3 * node1;
 mi = mi + 1;
 c1 = 3 * mi - 2;
 c2 = 3 * mi - 1;
 c3 = 3 * mi;
 Print[i, "---", k1, ":", k2, ":", k3];
 k4 = 3 * node2 - 2;
 k5 = 3 * node2 - 1;
 k6 = 3 * node2;
 Print[i, "---", k4, ":", k5, ":", k6];
 kc1 = dof[[k1]];
 kc2 = dof[[k2]];
 kc3 = dof[[k3]];
 Print[i, "...", kc1, ":", kc2, ":", kc3];
 kc4 = dof[[k4]];
 kc5 = dof[[k5]];
 kc6 = dof[[k6]];
 Print["member  =", mi, c1, ":", c2, ":", c3];
 Print[i, "...", kc4, ":", kc5, ":", kc6];
 Print[i, "^", bgmem[[i]][[1, 1]], ":",
  bgmem[[i]][[2, 1]], ":", bgmem[[i]][[3, 1]]];
 If[kc1 ≠ 0, S[[kc1, i]] = S[[kc1, i]] + bgmem[[i]][[1, 1]]];
 If[kc2 ≠ 0, S[[kc2, i]] = S[[kc2, i]] + bgmem[[i]][[2, 1]]];
 If[kc3 ≠ 0, S[[kc3, i]] = S[[kc3, i]] + bgmem[[i]][[3, 1]]];
 If[kc4 ≠ 0, S[[kc4, i]] = S[[kc4, i]] + bgmem[[i]][[4, 1]]];
 If[kc5 ≠ 0, S[[kc5, i]] = S[[kc5, i]] + bgmem[[i]][[5, 1]]];
 If[kc6 ≠ 0,
  S[[kc6, i]] = S[[kc6, i]] + bgmem[[i]][[6, 1]]],

 {i, 1, m}]
```

Figure 25. Assembling of Elemental Equilibrium Equation into Global Matrix

THE EQUILIBRIUM MATRIX

```
MatrixForm[S]
```

0.	-0.227921	0.	0.	-1.	0.
1.	0.341882	0.	0.	0.	0.
0.	0.227921	0.218218	-0.683486	0.	0.
0.	-0.341882	0.436436	-0.256307	0.	0.
0.	0.911685	0.872872	0.683486	0.	0.

```
Dimensions[S]
{5, 6}

MatrixRank[S]
5
```

Figure 26. Assembled Global Equilibrium Equation

Also scatter plot of global equilibrium equation is shown in Figure 27.

THE SCATTER PLOT OF EQUILIBRIUM MATRIX

```
MatrixPlot[S]
```

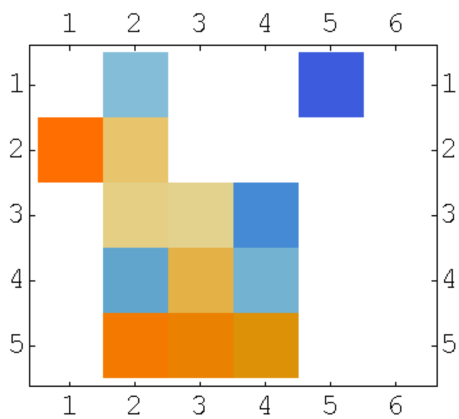
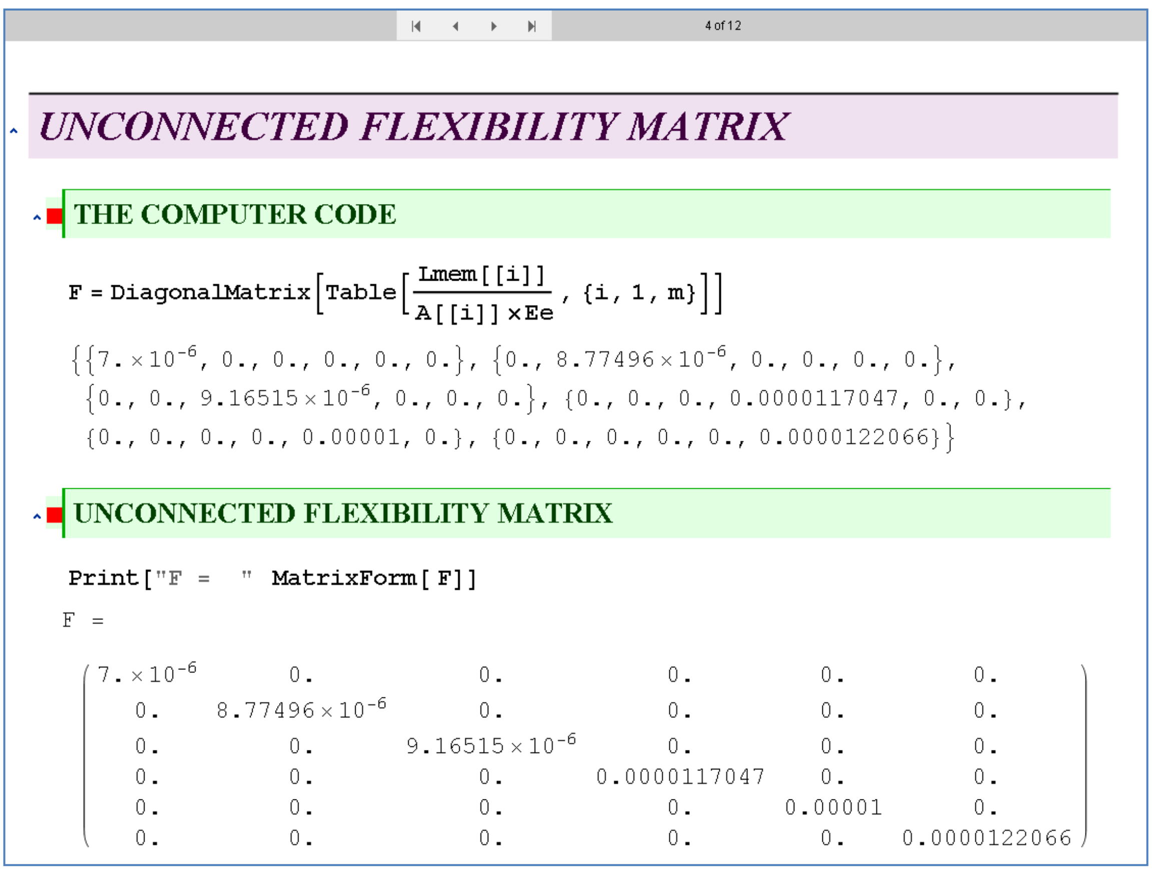


Figure 27. Scatter Plot of Equilibrium Equation

5.3.2 Computer Codes to Solve Generated Equilibrium Equations (EE) with Null Space Method

In this section after generation of global equilibrium equation according to the algorithm of Figure 4 the analysis procedure continues with null space property of integrated force method. For this purpose, first the flexibility condition is calculated as shown Figure 28 by using of equation 3.24 and 3.25:

$$[G] = \begin{bmatrix} f_1 & & & 0 \\ & f_2 & & \\ & & \ddots & \\ 0 & & & f_s \end{bmatrix} \qquad f_s = \frac{L_i}{E_i A_i}$$



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UNCONNECTED FLEXIBILITY MATRIX

THE COMPUTER CODE

```
F = DiagonalMatrix[Table[Lmem[[i]] / (A[[i]] × Ee), {i, 1, m}]]
```

{{7. × 10^-6, 0., 0., 0., 0., 0.}, {0., 8.77496 × 10^-6, 0., 0., 0., 0.}, {0., 0., 9.16515 × 10^-6, 0., 0., 0.}, {0., 0., 0., 0.0000117047, 0., 0.}, {0., 0., 0., 0., 0.00001, 0.}, {0., 0., 0., 0., 0., 0.0000122066}}

UNCONNECTED FLEXIBILITY MATRIX

```
Print["F =  " MatrixForm[F]]
```

F =

$$\begin{pmatrix} 7.\times10^{-6} & 0. & 0. & 0. & 0. & 0. \\ 0. & 8.77496\times10^{-6} & 0. & 0. & 0. & 0. \\ 0. & 0. & 9.16515\times10^{-6} & 0. & 0. & 0. \\ 0. & 0. & 0. & 0.0000117047 & 0. & 0. \\ 0. & 0. & 0. & 0. & 0.00001 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0.0000122066 \end{pmatrix}$$

Figure 28. Flexibility Matrix

The scatter plot of flexibility matrix is shown in Figure 29.

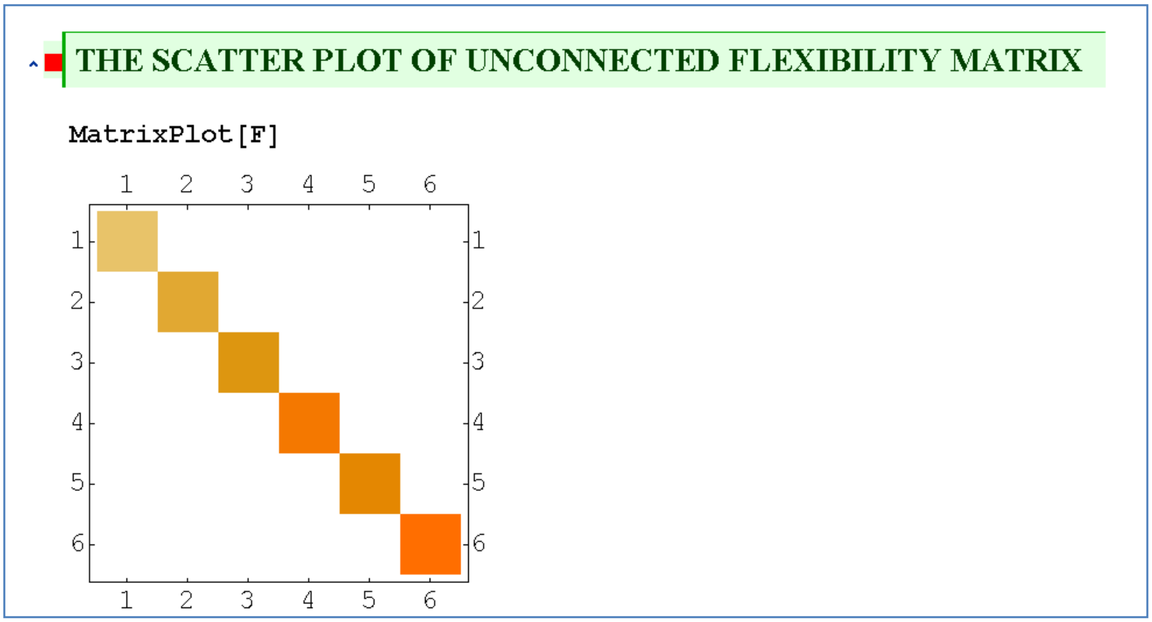

Figure 29. Scatter Plot of Flexibility Matrix

Then the compatibility condition is written by using of null space property of equilibrium equation as shown in Figure 30.

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COMPATIBILITY CONDITIONS (CC)

```
nlsp = NullSpace[S];
MatrixForm[nlsp]
```

(0. 0. 0. 0. 0. 1.)

```
Dimensions[nlsp]
```

{1, 6}

```
cc = nlsp.F;
MatrixForm[cc]
```

(0. 0. 0. 0. 0. 0.0000122066)

```
Dimensions[S]
```

{5, 6}

```
MatrixPlot[cc]
```

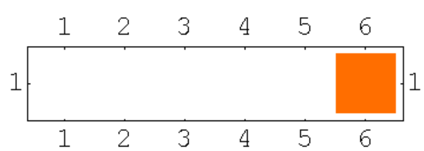

Figure 30. Compatibility Condition Matrix

Now, the generated equilibrium equation and obtained compatibility condition are coupled (Figure 31). Then the S matrix of space truss structure shown in Figure 16 is obtained.

In this step by using of equation 3.3 internal forces and member end forces can be calculated as shown in Figures 32 and 33, respectively:

[S]{F}= {P*}

COPULE COMPATIBILITY CONDITIONS WITH EQUILIBRIUM EQUATIONS (IFM MATRIX)

THE IFM MATRIX

```
ifm = Join[S, cc];

Dimensions[ifm]
```

{6, 6}

```
MatrixQ[ifm]
```

True

```
MatrixForm[ifm]
```

$$\begin{pmatrix} 0. & -0.227921 & 0. & 0. & -1. & 0. \\ 1. & 0.341882 & 0. & 0. & 0. & 0. \\ 0. & 0.227921 & 0.218218 & -0.683486 & 0. & 0. \\ 0. & -0.341882 & 0.436436 & -0.256307 & 0. & 0. \\ 0. & 0.911685 & 0.872872 & 0.683486 & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0.0000122066 \end{pmatrix}$$

```
Det[ifm]
```

-7.26166×10^{-6}

THE SCATTER PLOT OF THE IFM MATRIX

```
MatrixPlot[ifm]
```

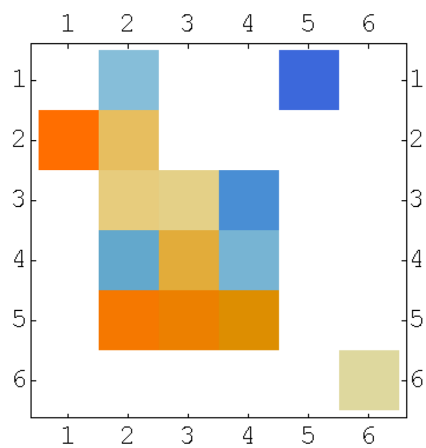

Figure 31. Coupling of EE Matrix and CC Matrix and Its Scatter Plot

INDEPENDENT FORCES

```
P = Table[0., {sr, 1, 3 * noden - rest}, {sc, 1, 1}];

(*FORM THE JOINT LOAD VECTOR...P...*)

Do[k1 = 3 * i - 2;
 k2 = 3 * i - 1;
 k3 = 3 * i;
 kc1 = dof[[k1]];
 kc2 = dof[[k2]];
 kc3 = dof[[k3]];
 If[kc1 ≠ 0, P[[kc1, 1]] = P[[kc1, 1]] + applfrcs[[i, 2]]];
 If[kc2 ≠ 0, P[[kc2, 1]] = P[[kc2, 1]] + applfrcs[[i, 3]]];
 If[kc3 ≠ 0, P[[kc3, 1]] = P[[kc3, 1]] + applfrcs[[i, 4]]];
 Print[i, "   ", k1, "   ", k2, "   ", k3, "   ", kc1, "   ",
  kc2, "   ", kc3],

 {i, 1, noden}]
(*................................*)
```

```
1   1   2   3   0   0   0
2   4   5   6   1   2   0
3   7   8   9   3   4   5
4   10   11   12   0   0   0
```

```
MatrixForm[P]
```

$$\begin{pmatrix} 0. \\ 0. \\ 0. \\ 100. \\ 0. \end{pmatrix}$$

```
initial = Table[0., {sr, 1, di}, {sc, 1, 1}]
```

```
{{0.}}
```

```
MatrixQ[P]
```

```
True
```

```
Pact = Join[P, initial]
```

```
{{0.}, {0.}, {0.}, {100.}, {0.}, {0.}}
```

```
Pfinal = Pact
{{0.}, {0.}, {0.}, {100.}, {0.}, {0.}}

indFrcs = LinearSolve[ifm, Pfinal]
{{42.8571}, {-125.357}, {130.931}, {0.}, {28.5714}, {0.}}

Print["independent forces  ", MatrixForm[indFrcs]]
```

$$\text{independent forces} \quad \begin{pmatrix} 42.8571 \\ -125.357 \\ 130.931 \\ 0. \\ 28.5714 \\ 0. \end{pmatrix}$$

Figure 32. Independent Forces of Members

MEMBER END FORCES

```
mend = 0;
endfrcs = {};

Do[
 mend = mend + 1;
 endfrc = (indFrcs[[mend, 1]] × bmem[[i]]);
 Print["member   ", i, "    ", MatrixForm[endfrc]];
 AppendTo[endfrcs, endfrc],
 {i, 1, m}]
```

member 1 $\begin{pmatrix} -42.8571 \\ 42.8571 \end{pmatrix}$

member 2 $\begin{pmatrix} 125.357 \\ -125.357 \end{pmatrix}$

member 3 $\begin{pmatrix} -130.931 \\ 130.931 \end{pmatrix}$

member 4 $\begin{pmatrix} 0. \\ 0. \end{pmatrix}$

member 5 $\begin{pmatrix} -28.5714 \\ 28.5714 \end{pmatrix}$

member 6 $\begin{pmatrix} 0. \\ 0. \end{pmatrix}$

```
endfrcs;
```

Figure 33. Member End Forces

In this package the deformation of elements is calculated by relation (Figure 34):

$$\beta = [G]\{F\}$$

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DEFORMATIONS

```
β = F.indFrcs;
Print[" β=  " MatrixForm[β]]
```

$$\beta = \begin{pmatrix} 0.0003 \\ -0.0011 \\ 0.0012 \\ 0. \\ 0.000285714 \\ 0. \end{pmatrix}$$

Figure 34. Deformation of Elements

Here, the computer codes are written to calculate the nodal displacements by using of equation 3.4 and 3.5 as shown in Figure 35.

$$\{X\}=[J][G][F]$$

$$J = \left[[S]^{-1} \right]^{T}$$

DISPLACEMENTS

THE COMPUTER CODE

```
invIFM = Inverse[ifm];

tinvIFM = Transpose[invIFM];

jd = Take[tinvIFM, 3*noden - rest];

Dimensions[jd]

{5, 6}

Disp = jd.F.indFrcs

{{-0.000285714}, {0.0003},
 {-0.00111239}, {0.0031603}, {0.000072722}}
```

THE NODAL DISPLACEMENTS MATRIX

```
Print[" displacements =   " Chop[MatrixForm[Disp]]];
```

$$\text{displacements} = \begin{pmatrix} -0.000285714 \\ 0.0003 \\ -0.00111239 \\ 0.0031603 \\ 0.000072722 \end{pmatrix}$$

THE SCATTER PLOT OF THE DISPLACEMENTS

```
MatrixPlot[Disp]
```

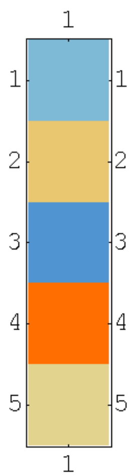

Figure 35. Nodal Displacements Matrix and Its Scatter Plot

In this section which is the last step of IFM via null space, the reactions of supports are calculated by related written computer codes as shown in Figure 36.

THE REACTIONS MATRIX

```
MatrixForm[reactions]
```

$$\begin{pmatrix} -28.5714 \\ -100. \\ -114.286 \\ 114.286 \\ 28.5714 \\ 0. \\ 0. \end{pmatrix}$$

THE SCATTER PLOT OF REACTIONS MATRIX

```
MatrixPlot[reactions]
```

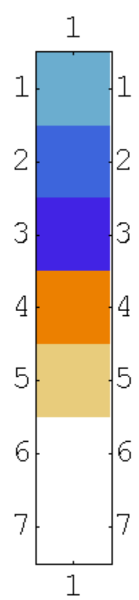

Figure 36. Support Reactions Matrix and Its Scatter Plot

5.3.3 Computer Codes to Solve Generated Equilibrium Equations (EE) with Singular Value Decomposition Method

Almost all of the writing of computer codes for IFM via singular value decomposition are similar to those in IFM via null space. The main difference between the null pace and singular value decomposition is related to generation of compatibility condition. The computer codes for this method of IFM are shown in Figures 37 and 38 in which the equations 3.16, 3.21 are used:

$$[M]=\left\lfloor [I]-[A]^T\left([A]^T\right)^{pinv} \right\rfloor$$

$$\left([A]^T\right)^{pinv}=\left([A][A]^T\right)^{-1}[A]$$

$$[M]=[M_u][M_\delta][M_v]^T$$

$$[M]=[M_u]\begin{bmatrix}[NS]\\ [0]\end{bmatrix}$$

$$[C]=[NS][G]$$

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COMPATIBILITY CONDITIONS (CC)

SINGULAR VALUE DECOMPOSITION (SVD)

```
Spinv = (Inverse[(S.Transpose[S])]).S;
mm = IdentityMatrix[Length[inc]] - (Transpose[S].Spinv);
{u, w, v} = SingularValueDecomposition[mm];
Print[" u = " MatrixForm[u]]
Print[" w = " MatrixForm[w]]
Print[" v = " MatrixForm[v]]
```

u =

$$\begin{pmatrix} 0. & -0.197487 & 0.0261359 & -0.0549663 & 0.675368 & 0.707936 \\ 0. & 0.440396 & 0.780504 & 0.257782 & 0.311346 & -0.182969 \\ 0. & -0.104723 & 0.415747 & -0.891825 & -0.142621 & 0.0222535 \\ 0. & -0.867854 & 0.358482 & 0.266675 & -0.0190799 & -0.216425 \\ 0. & -0.0539668 & -0.297952 & -0.253101 & 0.652869 & -0.64654 \\ 1. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$w = \begin{pmatrix} 1. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

v =

$$\begin{pmatrix} 0. & 0.210067 & -0.06137 & -0.072503 & 0.689827 & 0.686285 \\ 0. & -0.596046 & 0.583357 & 0.42835 & -0.062006 & 0.342191 \\ 0. & -0.064345 & 0.562908 & -0.822877 & 0.0208556 & -0.0378638 \\ 0. & 0.767707 & 0.54061 & 0.305575 & -0.158018 & 0.00446907 \\ 0. & 0.0841745 & -0.216341 & -0.201885 & -0.703483 & 0.640675 \\ 1. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

Figure 37. Computer Codes for Find Singular Value Decomposition (SVD) of EE

COMPATIBILITY CONDITIONS MATRIX

```
c2 = Chop[Inverse[u].mm];
{row, col} = Dimensions[S];
c1 = Take[c2, col - row, col];
cc = c1.F;
Print["cc=" MatrixForm[cc]]
cc= ( 0.  0.  0.  0.  0.  0.0000122066 )

MatrixForm[cc]
( 0.  0.  0.  0.  0.  0.0000122066 )

Dimensions[S]
{5, 6}
```

SCATTER PLOT OF COMPATIBILITY CONDITION MATRIX

```
MatrixPlot[cc]
```

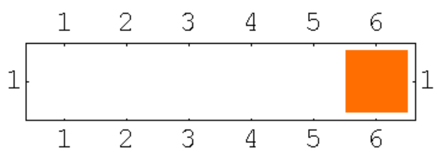

Figure 38. Compatibility Condition Matrix via SVD and Its Scatter Plot

5.3.4 Computer Codes to Solve Generated Equilibrium Equations (EE) with Dual Integrated Force Method

In this section the computer code for dual integrated force method are written which are used to obtain displacements after generation of global equilibrium equation matrix. According to the chapter three and algorithm of Figure 15, since the primary unknowns are displacements the internal force will be back calculated. To start analysis of space truss by this method flexibility matrix should be generated, but since the flexibility matrix is same for IFMD and IFM, then computer codes is similar to that in IFM (Figure 39).

UNCONNECTED FLEXIBILITY MATRIX

THE COMPUTER CODE

```
F = DiagonalMatrix[Table[Lmem[[i]] / (A[[i]] × Ee), {i, 1, m}]]
```

{{7. × 10^-6, 0., 0., 0., 0., 0.}, {0., 8.77496 × 10^-6, 0., 0., 0., 0.},
{0., 0., 9.16515 × 10^-6, 0., 0., 0.}, {0., 0., 0., 0.0000117047, 0., 0.},
{0., 0., 0., 0., 0.00001, 0.}, {0., 0., 0., 0., 0., 0.0000122066}}

UNCONNECTED FLEXIBILITY MATRIX

```
Print["F =   " MatrixForm[F]]
```

F =

7. × 10^-6	0.	0.	0.	0.	0.
0.	8.77496 × 10^-6	0.	0.	0.	0.
0.	0.	9.16515 × 10^-6	0.	0.	0.
0.	0.	0.	0.0000117047	0.	0.
0.	0.	0.	0.	0.00001	0.
0.	0.	0.	0.	0.	0.0000122066

THE SCATTER PLOT OF UNCONNECTED FLEXIBILITY MATRIX

```
MatrixPlot[F]
```

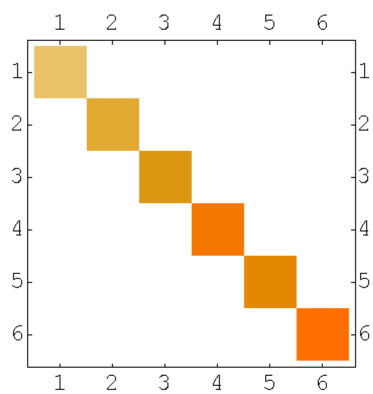

Figure 39. Generation of Flexibility Matrix and Scatter Plot

After writing of flexibility matrix the Pseudostiffness Matrix $[K]_{ifmd}$ should be generated by equation 3.23. The computer codes and $[K]_{ifmd}$ is shown in Figure 40:

$$[K]_{ifmd} = [A][G]^{-1}[A]^T$$

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PSEUDOSTIFFNESS MATRIX

THE GLOBAL PSEUDOSTIFFNESS MATRIX

```
kifmd = S.Inverse[F].Transpose[S];
Print["[k]ifmd =   " MatrixForm[kifmd]]
Dimensions[kifmd]
```

$$[k]_{ifmd} = \begin{pmatrix} 105920. & -8880.04 & -5920.03 & 8880.04 & -23680.1 \\ -8880.04 & 156177. & 8880.04 & -13320.1 & 35520.2 \\ -5920.03 & 8880.04 & 51027.3 & 16478.1 & 4551.18 \\ 8880.04 & -13320.1 & 16478.1 & 39715.3 & -8921.72 \\ -23680.1 & 35520.2 & 4551.18 & -8921.72 & 217763. \end{pmatrix}$$

{5, 5}

THE SCATTER PLOT OF THE PSEUDOSTIFFNESS MATRIX

```
MatrixPlot[kifmd]
```

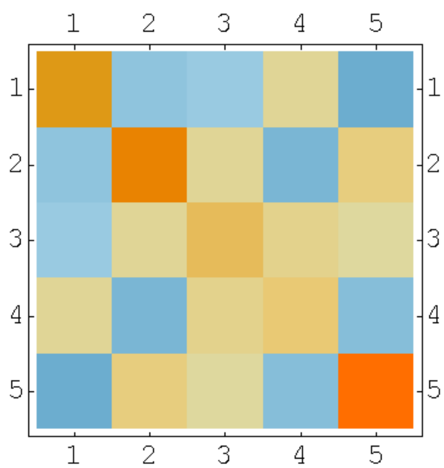

Figure 40. $[K]_{ifmd}$ Matrix Computer Codes and Scatter Plot

Then the related computer codes are written to obtain the displacements by using of equation 3.22 (Figure 41).

$$[K]_{ifmd}\{X\} = \{P\}_{ifmd}$$

DISPLACEMENTS

THE COMPUTER CODE

```
P = Table[0., {sr, 1, 3 * noden - rest}, {sc, 1, 1}];

Do[k1 = 3 * i - 2;
 k2 = 3 * i - 1;
 k3 = 3 * i;
 kc1 = dof[[k1]];
 kc2 = dof[[k2]];
 kc3 = dof[[k3]];
 If[kc1 ≠ 0, P[[kc1, 1]] = P[[kc1, 1]] + applfrcs[[i, 2]]];
 If[kc2 ≠ 0, P[[kc2, 1]] = P[[kc2, 1]] + applfrcs[[i, 3]]];
 If[kc3 ≠ 0, P[[kc3, 1]] = P[[kc3, 1]] + applfrcs[[i, 4]]];
 Print[i, "  ", k1, "  ", k2, "  ", k3, "  ", kc1, "  ",
  kc2, "  ", kc3],

 {i, 1, noden}]
1  1  2  3  0  0  0
2  4  5  6  1  2  0
3  7  8  9  3  4  5
4  10  11  12  0  0  0

MatrixForm[P]
( 0.
  0.
  0.
  100.
  0. )

Dimensions[P]
{5, 1}

xdisp = LinearSolve[kifmd, P];
```

Figure 41. Computer Codes for Calculation of Displacements

The matrix of nodal displacements and its scatter plot are shown in Figure 42.

THE NODAL DISPLACEMENTS MATRIX

```
Print["displacements =  " MatrixForm[ xdisp]]
Dimensions[xdisp]
MatrixQ[xdisp]
```

$$\text{displacements} = \begin{pmatrix} -0.000285714 \\ 0.0003 \\ -0.00111239 \\ 0.0031603 \\ 0.000072722 \end{pmatrix}$$

```
{5, 1}

True
```

SCATTER PLOT OF DISPLACEMENT MATRIX

```
MatrixPlot[xdisp]
```

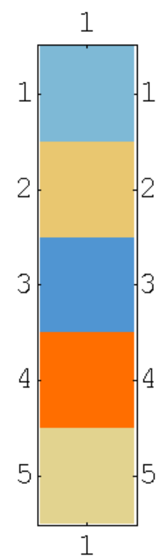

Figure 42. Nodal Displacements Matrix and Its Scatter Plot

In this section deformation of members are obtained by equation 3.11 and related computer codes are presented in Figure 43.

$$\{\beta\} = [A]^T \{X\}$$

7 of 11

DEFORMATIONS

```
deform = Transpose[S].xdisp;
Print["deformations =  " MatrixForm[Chop[ deform]]]
Dimensions[deform]
MatrixQ[deform]
```

$$\text{deformations} = \begin{pmatrix} 0.0003 \\ -0.0011 \\ 0.0012 \\ 0 \\ 0.000285714 \\ 0 \end{pmatrix}$$

```
{6, 1}

True
```

Figure 43. Deformation Matrix of Members

Then internal forces are calculated by equation 3.11 and 3.28 (Figure 44).

$$\{F\} = [G]^{-1}[A]^{T}\{X\}$$

$$\{\beta\} = [A]^{T}\{X\}$$

INTERNAL FORCES

```
frcsIND = Inverse[F].deform;
Print["Independent forces= "
  MatrixForm[Chop[frcsIND]]]
Dimensions[frcsIND]
MatrixQ[frcsIND]
```

$$\text{Independent forces=} \begin{pmatrix} 42.8571 \\ -125.357 \\ 130.931 \\ 0 \\ 28.5714 \\ 0 \end{pmatrix}$$

```
{6, 1}

True
```

Figure 44. Matrix of Internal Forces

Then support reactions can be evaluated (Figure 45).

SUPPORT REACTIONS MATRIX

```
Print["Reactions = " MatrixForm[Chop[reactions]]]
```

$$\text{Reactions} = \begin{pmatrix} -28.5714 \\ -100. \\ -114.286 \\ 114.286 \\ 28.5714 \\ 0 \\ 0 \end{pmatrix}$$

SCATTER PLOT OF REACTIONS MATRIX

```
MatrixPlot[reactions]
```

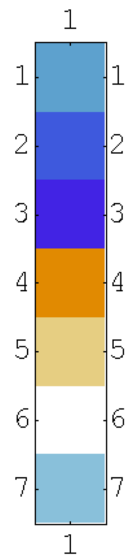

Figure 45. Support Reactions Matrix

In the reporting phase following results can be seen:

- Member independent forces and end forces
- Deformations of elements
- Displacements of nodes
- Support reactions
- Degree of indeterminacy

Chapter 6

ILLUSTRATIVE EXAMPLES

6.1 Introduction

In this chapter 6 examples are presented that first two examples are solved with IFM via null space, second pair of examples are solved by IFM via singular value decomposition and last two examples are solved by dual integrated force method. Also to prove and compare the results the software of Mastan is used. The obtained results by Mastan are presented at the end of each example.

Solving Example 1 and 2 by IFM via null space

Reporting phase for IFM via null space consists of:

- Member independent forces
- Member end forces
- Deformations of elements
- Displacements
- Support reactions

6.2 Example 1

This truss consists of 12 member and 6 nodes. The nodal properties and elemental connectivity information are shown in Tables 6 and 7. The area for all members is A=0.003m² and the modulus of elasticity is $E = 2\times10^8$.

Table 6. Nodal Data of Example 1

Node Number	Coordinate(m)			Applied Load(kN)			Support Restraint		
	x	y	z	x	y	z	x	y	z
1	0	0	0	0	0	0	1	1	1
2	0	2	0	0	0	0	1	0	1
3	0	1	2	0	0	0	1	1	1
4	3	0	0	0	0	-45	0	0	0
5	3	2	0	0	0	-30	0	0	0
6	3	1	2	0	0	0	0	0	0

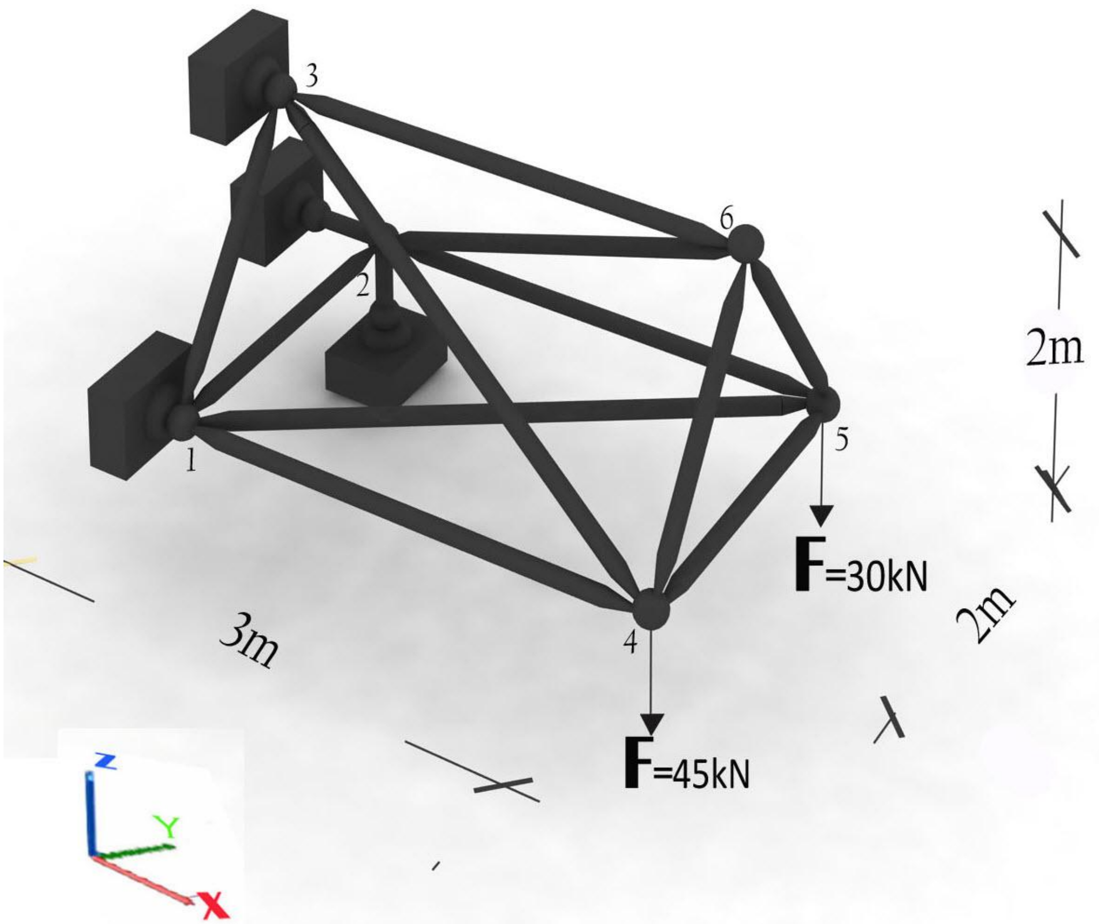

Figure 46. Space Truss of Example 1

Table 7. Elemental Data of Example 1

Element Number	Connectivity		Element Number	Connectivity	
	Start Node	End Node		Start Node	End Node
1	1	2	7	2	6
2	1	3	8	3	6
3	2	3	9	3	4
4	1	4	10	5	6
5	1	5	11	4	6
6	2	5	12	4	5

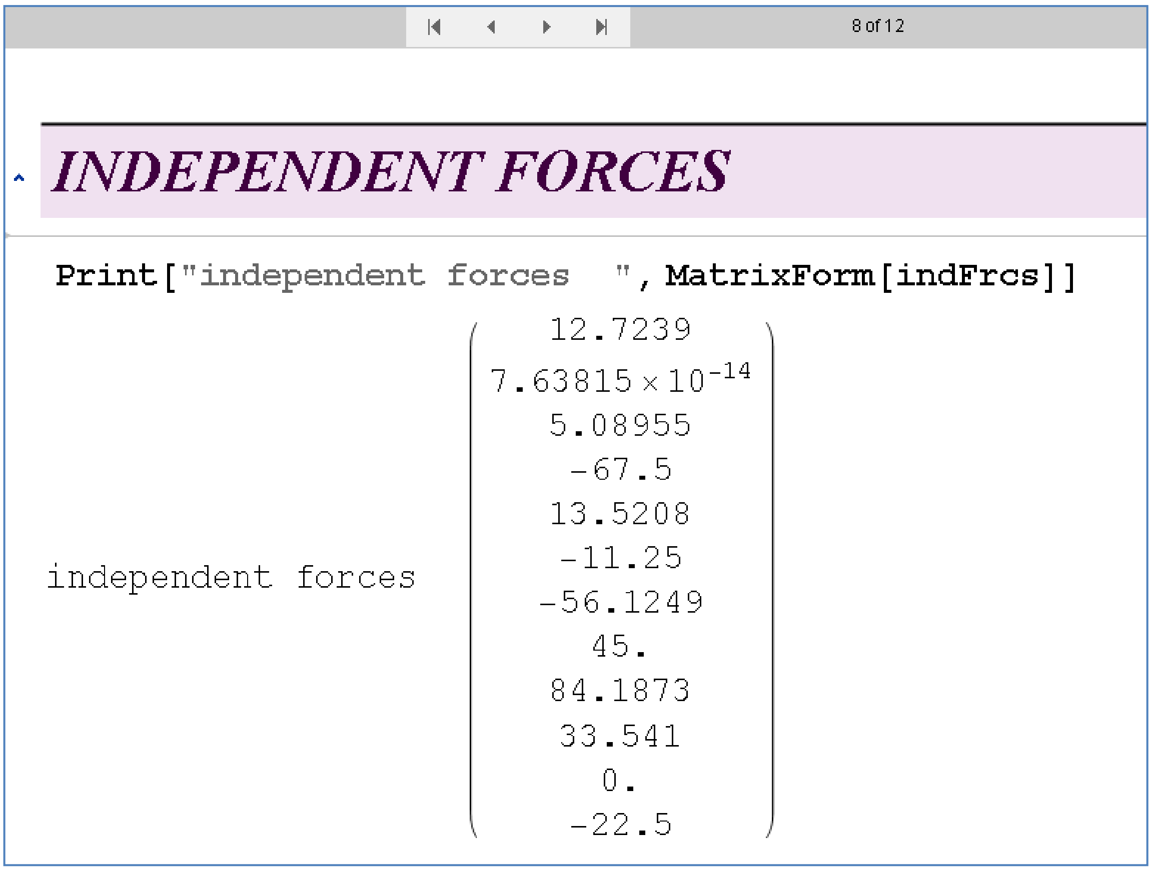

Figure 47. Reporting Phase; Member Forces for Example 1.

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INDEPENDENT FORCES

MEMBER END FORCES

member	1	$\begin{pmatrix} -12.7239 \\ 12.7239 \end{pmatrix}$
member	2	$\begin{pmatrix} -7.63815 \times 10^{-14} \\ 7.63815 \times 10^{-14} \end{pmatrix}$
member	3	$\begin{pmatrix} -5.08955 \\ 5.08955 \end{pmatrix}$
member	4	$\begin{pmatrix} 67.5 \\ -67.5 \end{pmatrix}$
member	5	$\begin{pmatrix} -13.5208 \\ 13.5208 \end{pmatrix}$
member	6	$\begin{pmatrix} 11.25 \\ -11.25 \end{pmatrix}$
member	7	$\begin{pmatrix} 56.1249 \\ -56.1249 \end{pmatrix}$
member	8	$\begin{pmatrix} -45. \\ 45. \end{pmatrix}$
member	9	$\begin{pmatrix} -84.1873 \\ 84.1873 \end{pmatrix}$
member	10	$\begin{pmatrix} -33.541 \\ 33.541 \end{pmatrix}$
member	11	$\begin{pmatrix} 0. \\ 0. \end{pmatrix}$
member	12	$\begin{pmatrix} 22.5 \\ -22.5 \end{pmatrix}$

Figure 48. Reporting Phase; Member End Forces for Example 1

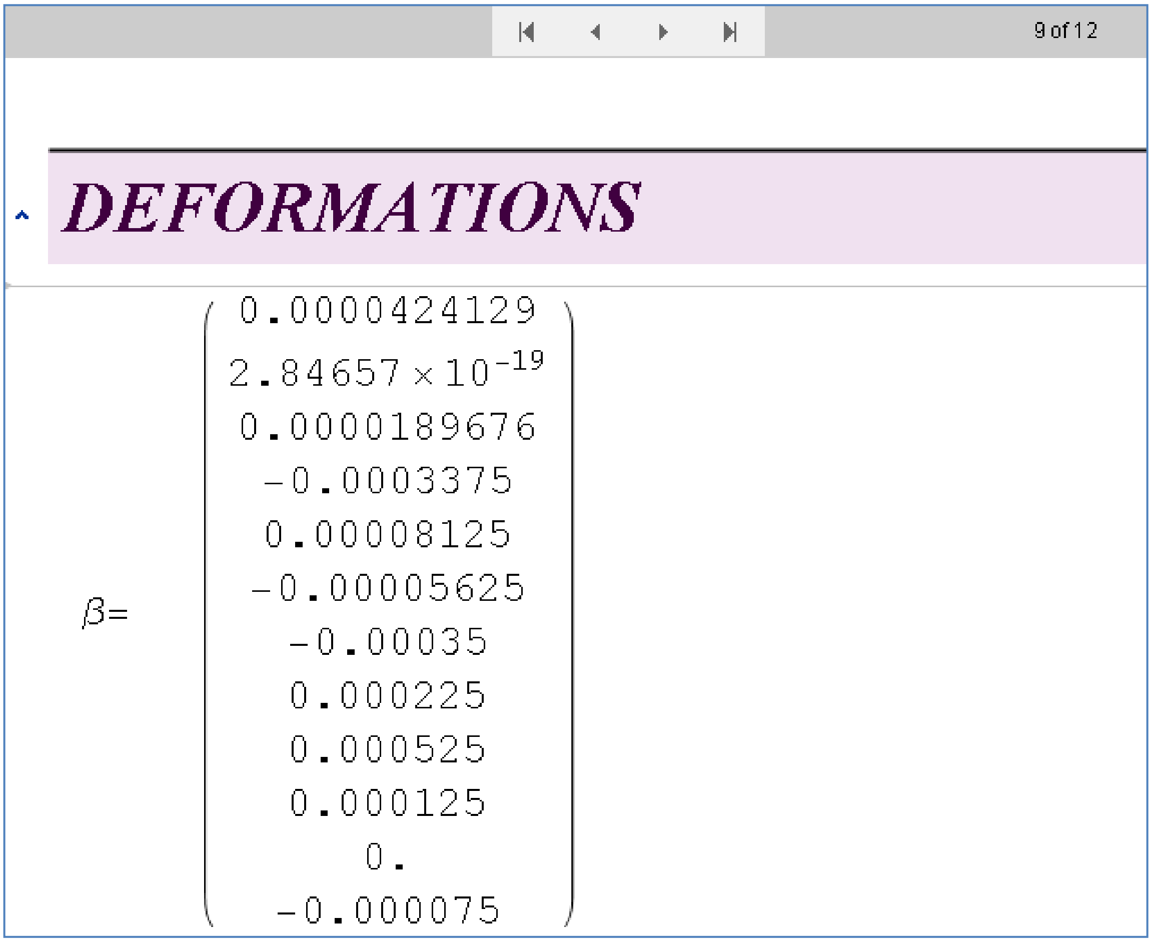

Figure 49. Reporting Phase; Deformations of Elements for Example 1.

DISPLACEMENTS

THE COMPUTER CODE

THE NODAL DISPLACEMENTS MATRIX

```
Print[" displacements =   " Chop[MatrixForm[Disp]]];
```

$$\text{displacements} = \begin{pmatrix} 0.0000424129 \\ -0.0003375 \\ 0.000305851 \\ -0.00164136 \\ -0.00005625 \\ 0.000230851 \\ -0.00103783 \\ 0.000225 \\ -0.000474939 \\ -0.00125097 \end{pmatrix}$$

THE SCATTER PLOT OF THE DISPLACEMENTS

```
MatrixPlot[Disp]
```

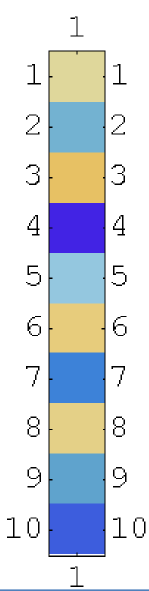

Figure 50. Reporting Phase; Nodal Displacements for Example 1.

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REACTIONS OF SUPPORTS

THE COMPUTER CODE

THE REACTIONS MATRIX

```
MatrixForm[reactions]
```

$$\begin{pmatrix} 56.25 \\ -20.2239 \\ -6.83177\times 10^{-14} \\ 56.25 \\ 25.4478 \\ -112.5 \\ 20.2239 \\ 49.5522 \end{pmatrix}$$

THE SCATTER PLOT OF REACTIONS MATRIX

```
MatrixPlot[reactions]
```

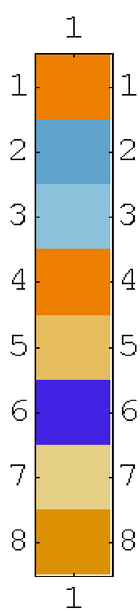

Figure 51. Reporting Phase; Support Reactions for Example 1.

6.3 Mastan Results for Example 1

To compare and prove obtained results by written computer codes, the Mastan software is used and the results are presented below.

```
***********   MASTAN2 v3.3.1   ***********

Time:  08:57:42      Date:  05/25/2012

Problem Title:   not provided
**************


##############################
Results of Structural Analysis
##############################

General Information:
      Structure Analyzed as:  Space Truss
      Analysis Type:  First-Order Elastic

Analytical Results:

(i)   Displacements at Step # 1, Applied Load Ratio = 1.0000

  Deflections
     Node       X-disp          Y-disp          Z-disp
       1     0.0000e+000     0.0000e+000     0.0000e+000
       2     0.0000e+000     4.2413e-005     0.0000e+000
       3     0.0000e+000     0.0000e+000     0.0000e+000
       4    -3.3750e-004     3.0585e-004    -1.6414e-003
       5    -5.6250e-005     2.3085e-004    -1.0378e-003
       6     2.2500e-004    -4.7494e-004    -1.2510e-003

(ii)  Element Results at Step # 1, Applied Load Ratio = 1.0000

  Internal End Forces (Note: Refers to local coordinates)
    Element  Node         Fx             Fy             Fz
       1       1    -1.2724e+001    0.0000e+000    0.0000e+000
               2     1.2724e+001    0.0000e+000    0.0000e+000
       2       1     0.0000e+000    0.0000e+000    0.0000e+000
               3     0.0000e+000    0.0000e+000    0.0000e+000
       3       2    -5.0896e+000    0.0000e+000    0.0000e+000
               3     5.0896e+000    0.0000e+000    0.0000e+000
       4       1     6.7500e+001    0.0000e+000    0.0000e+000
               4    -6.7500e+001    0.0000e+000    0.0000e+000
       5       1    -1.3521e+001    0.0000e+000    0.0000e+000
               5     1.3521e+001    0.0000e+000    0.0000e+000
       6       2     1.1250e+001    0.0000e+000    0.0000e+000
               5    -1.1250e+001    0.0000e+000    0.0000e+000
```

```
     7        2    5.6125e+001    0.0000e+000    0.0000e+000
              6   -5.6125e+001    0.0000e+000    0.0000e+000
     8        3   -4.5000e+001    0.0000e+000    0.0000e+000
              6    4.5000e+001    0.0000e+000    0.0000e+000
     9        3   -8.4187e+001    0.0000e+000    0.0000e+000
              4    8.4187e+001    0.0000e+000    0.0000e+000
    10        5   -3.3541e+001    0.0000e+000    0.0000e+000
              6    3.3541e+001    0.0000e+000    0.0000e+000
    11        4    0.0000e+000    0.0000e+000    0.0000e+000
              6    0.0000e+000    0.0000e+000    0.0000e+000
    12        4    2.2500e+001    0.0000e+000    0.0000e+000
              5   -2.2500e+001    0.0000e+000    0.0000e+000

(iii) Reactions at Step # 1, Applied Load Ratio = 1.0000

  Forces
     Node          Rx              Ry              Rz
      1       5.6250e+001    -2.0224e+001     0.0000e+000
      2       5.6250e+001        FREE         2.5448e+001
      3      -1.1250e+002     2.0224e+001     4.9552e+001

  Moments
     Node          Mx              My              Mz
            *** No Reaction Moments Exist ***

#####################################
End of Results of Structural Analysis
#####################################
```

6.4 Example 2

The truss of example 2 as shown in Figure 62 consists of 18 member with 8 nodes and the properties of nodes and members are presented in Tables 8 and 9, respectively. The section area for members is $A = 0.002\,m^2$ and modulus of elasticity is $E = 2\times10^8 kN/m^2$.

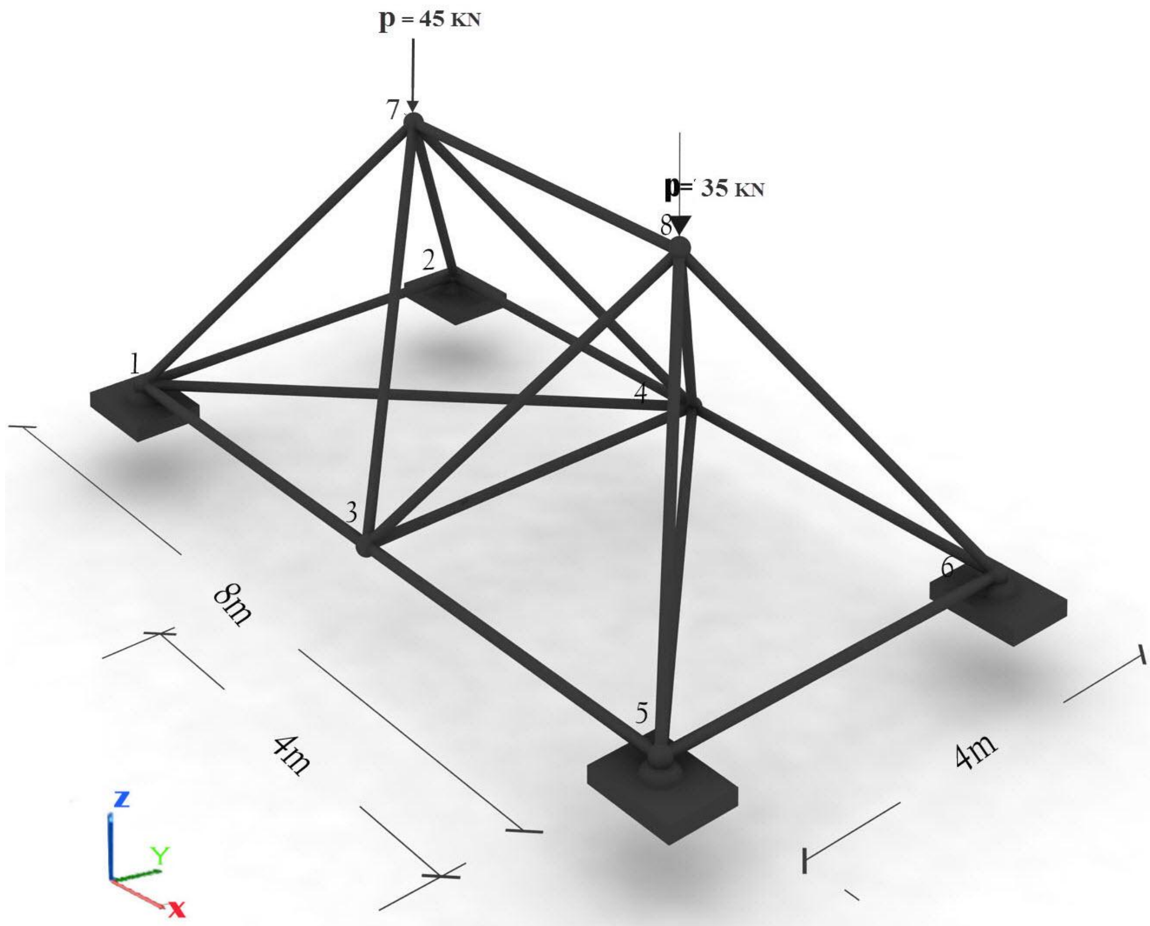

Figure 52. Space Truss of Example 2

Table 8. Nodal Data for Space Truss of Example 2

Node Number	Coordinate			Applied Load			Restraint		
	x	y	z	x	y	z	x	y	z
1	0	0	0	0	0	0	1	1	1
2	0	4	0	0	0	0	1	1	1
3	4	0	0	0	0	0	0	0	0
4	4	4	0	0	0	0	0	0	0
5	8	0	0	0	0	0	1	1	1
6	8	4	0	0	0	0	1	1	1
7	2	2	3.5	0	0	-45	0	0	0
8	6	2	3.5	0	0	-35	0	0	0

Table 9. Member Data for Space Truss of Example 2

Element Number	Connectivity		Element Number	Connectivity	
	Start Node	End Node		Start Node	Start Node
1	1	2	10	1	7
2	2	4	11	2	7
3	3	4	12	3	7
4	1	3	13	4	7
5	1	4	14	3	8
6	4	6	15	4	8
7	5	6	16	6	8
8	3	5	17	5	8
9	4	5	18	7	8

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INDEPENDENT FORCES

```
Print["independent forces  ", MatrixForm[indFrcs]]
```

independent forces $\begin{pmatrix} 4.64884\times10^{-15} \\ 0.533333 \\ -1.22125\times10^{-15} \\ 0.706677 \\ 0.266667 \\ -0.533333 \\ -1.04749\times10^{-14} \\ -0.706677 \\ -0.266667 \\ -27.3385 \\ -27.3043 \\ -1.59002 \\ -1.62426 \\ 1.59002 \\ 1.62426 \\ -24.1243 \\ -24.09 \\ -22.8571 \end{pmatrix}$

Figure 53. Member Forces for Space Truss of Example 2.

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INDEPENDENT FORCES

MEMBER END FORCES

member	1	$\begin{pmatrix} -4.64884 \times 10^{-15} \\ 4.64884 \times 10^{-15} \end{pmatrix}$
member	2	$\begin{pmatrix} -0.533333 \\ 0.533333 \end{pmatrix}$
member	3	$\begin{pmatrix} 1.22125 \times 10^{-15} \\ -1.22125 \times 10^{-15} \end{pmatrix}$
member	4	$\begin{pmatrix} -0.706677 \\ 0.706677 \end{pmatrix}$
member	5	$\begin{pmatrix} -0.266667 \\ 0.266667 \end{pmatrix}$
member	6	$\begin{pmatrix} 0.533333 \\ -0.533333 \end{pmatrix}$
member	7	$\begin{pmatrix} 1.04749 \times 10^{-14} \\ -1.04749 \times 10^{-14} \end{pmatrix}$
member	8	$\begin{pmatrix} 0.706677 \\ -0.706677 \end{pmatrix}$
member	9	$\begin{pmatrix} 0.266667 \\ -0.266667 \end{pmatrix}$
member	10	$\begin{pmatrix} 27.3385 \\ -27.3385 \end{pmatrix}$
member	11	$\begin{pmatrix} 27.3043 \\ -27.3043 \end{pmatrix}$
member	12	$\begin{pmatrix} 1.59002 \\ -1.59002 \end{pmatrix}$
member	13	$\begin{pmatrix} 1.62426 \\ -1.62426 \end{pmatrix}$
member	14	$\begin{pmatrix} -1.59002 \\ 1.59002 \end{pmatrix}$
member	15	$\begin{pmatrix} -1.62426 \\ 1.62426 \end{pmatrix}$
member	16	$\begin{pmatrix} 24.1243 \\ -24.1243 \end{pmatrix}$
member	17	$\begin{pmatrix} 24.09 \\ -24.09 \end{pmatrix}$
member	18	$\begin{pmatrix} 22.8571 \\ -22.8571 \end{pmatrix}$

Figure 54. Member End Forces of Example 2.

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DEFORMATIONS

$$\beta = \begin{pmatrix} 4.64884 \times 10^{-20} \\ 5.33333 \times 10^{-6} \\ -1.22125 \times 10^{-20} \\ 7.06677 \times 10^{-6} \\ 3.77123 \times 10^{-6} \\ -5.33333 \times 10^{-6} \\ -1.04749 \times 10^{-19} \\ -7.06677 \times 10^{-6} \\ -3.77123 \times 10^{-6} \\ -0.000307559 \\ -0.000307173 \\ -0.0000178878 \\ -0.000018273 \\ 0.0000178878 \\ 0.000018273 \\ -0.000271398 \\ -0.000271013 \\ -0.000228571 \end{pmatrix}$$

Figure 55. Element Deformations of Example 2.

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DISPLACEMENTS

THE COMPUTER CODE

THE NODAL DISPLACEMENTS MATRIX

$$\text{displacements} = \begin{pmatrix} 7.06677 \times 10^{-6} \\ 0 \\ -0.000502551 \\ 5.33333 \times 10^{-6} \\ 0 \\ -0.000502551 \\ 0.000117386 \\ -4.33359 \times 10^{-7} \\ -0.000462263 \\ -0.000111186 \\ 4.33359 \times 10^{-7} \\ -0.000412227 \end{pmatrix}$$

THE SCATTER PLOT OF THE DISPLACEMENTS

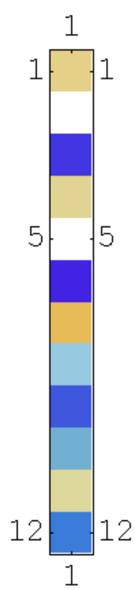

Figure 56. Nodal Displacements and Its Scatter Plot for Example 2.

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REACTIONS OF SUPPORTS

THE COMPUTER CODE

THE REACTIONS MATRIX

```
MatrixForm[reactions]
```

$$\begin{pmatrix} 11.2552 \\ 11.9619 \\ 21.2633 \\ 11.6019 \\ -12.1352 \\ 21.2367 \\ -11.6019 \\ 10.8952 \\ 18.7367 \\ -11.2552 \\ -10.7219 \\ 18.7633 \end{pmatrix}$$

THE SCATTER PLOT OF REACTIONS MATRIX

```
MatrixPlot[reactions]
```

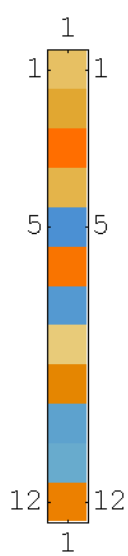

Figure 57. Support Reactions and Scatter Plot of Example 2.

6.5 Mastan Results for Example 2

```
***********   MASTAN2 v3.3.1   ***********

Time:  09:14:36      Date:  05/25/2012

Problem Title:   not provided
**************
##############################
Results of Structural Analysis
##############################

General Information:
      Structure Analyzed as:  Space Truss
      Analysis Type:  First-Order Elastic

Analytical Results:

(i)   Displacements at Step # 1, Applied Load Ratio = 1.0000

  Deflections
     Node       X-disp          Y-disp          Z-disp
       1     0.0000e+000     0.0000e+000     0.0000e+000
       2     0.0000e+000     0.0000e+000     0.0000e+000
       3     7.0668e-006    -3.7754e-020    -5.0255e-004
       4     5.3333e-006     0.0000e+000    -5.0255e-004
       5     0.0000e+000     0.0000e+000     0.0000e+000
       6     0.0000e+000     0.0000e+000     0.0000e+000
       7     1.1739e-004    -4.3336e-007    -4.6226e-004
       8    -1.1119e-004     4.3336e-007    -4.1223e-004

(ii)  Element Results at Step # 1, Applied Load Ratio = 1.0000

  Internal End Forces (Note: Refers to local coordinates)
    Element  Node         Fx             Fy             Fz
       1       1    0.0000e+000    0.0000e+000    0.0000e+000
               2    0.0000e+000    0.0000e+000    0.0000e+000
       2       2   -5.3333e-001    0.0000e+000    0.0000e+000
               4    5.3333e-001    0.0000e+000    0.0000e+000
       3       3   -3.7754e-015    0.0000e+000    0.0000e+000
               4    3.7754e-015    0.0000e+000    0.0000e+000
       4       1   -7.0668e-001    0.0000e+000    0.0000e+000
               3    7.0668e-001    0.0000e+000    0.0000e+000
       5       1   -2.6667e-001    0.0000e+000    0.0000e+000
               4    2.6667e-001    0.0000e+000    0.0000e+000
       6       4    5.3333e-001    0.0000e+000    0.0000e+000
               6   -5.3333e-001    0.0000e+000    0.0000e+000
       7       5    0.0000e+000    0.0000e+000    0.0000e+000
               6    0.0000e+000    0.0000e+000    0.0000e+000
       8       3    7.0668e-001    0.0000e+000    0.0000e+000
               5   -7.0668e-001    0.0000e+000    0.0000e+000
       9       4    2.6667e-001    0.0000e+000    0.0000e+000
               5   -2.6667e-001    0.0000e+000    0.0000e+000
      10       1    2.7339e+001    0.0000e+000    0.0000e+000
```

```
           7   -2.7339e+001    0.0000e+000    0.0000e+000
     11    2    2.7304e+001    0.0000e+000    0.0000e+000
           7   -2.7304e+001    0.0000e+000    0.0000e+000
     12    3    1.5900e+000    0.0000e+000    0.0000e+000
           7   -1.5900e+000    0.0000e+000    0.0000e+000
     13    4    1.6243e+000    0.0000e+000    0.0000e+000
           7   -1.6243e+000    0.0000e+000    0.0000e+000
     14    3   -1.5900e+000    0.0000e+000    0.0000e+000
           8    1.5900e+000    0.0000e+000    0.0000e+000
     15    4   -1.6243e+000    0.0000e+000    0.0000e+000
           8    1.6243e+000    0.0000e+000    0.0000e+000
     16    6    2.4124e+001    0.0000e+000    0.0000e+000
           8   -2.4124e+001    0.0000e+000    0.0000e+000
     17    5    2.4090e+001    0.0000e+000    0.0000e+000
           8   -2.4090e+001    0.0000e+000    0.0000e+000
     18    7    2.2857e+001    0.0000e+000    0.0000e+000
           8   -2.2857e+001    0.0000e+000    0.0000e+000

(iii) Reactions at Step # 1, Applied Load Ratio = 1.0000

  Forces
     Node          Rx              Ry              Rz
       1       1.1255e+001     1.1962e+001     2.1263e+001
       2       1.1602e+001    -1.2135e+001     2.1237e+001
       5      -1.1602e+001     1.0895e+001     1.8737e+001
       6      -1.1255e+001    -1.0722e+001     1.8763e+001

  Moments
     Node          Mx              My              Mz
             *** No Reaction Moments Exist ***

#####################################
End of Results of Structural Analysis
#####################################
```

Solving Examples 3 and 4 by Integrated Force Method via Null Space

In this section examples 3 and 4 are solved by integrated force method via singular value decomposition. The reporting phase for IFM via singular value decomposition is similar to IFM via null space as:

- Member independent forces
- Member end forces
- Deformations of elements
- Displacements
- Support reactions

6.6 Example 3

In this example the space truss has 10 joints and 25 member that properties of nodes and elements are presented in Tables 10 and 11, respectively. The section area for all the members is $A = 0.0025\,m^2$ and modulus of elasticity is $E = 2 \times 10^8\,kN/m^2$.

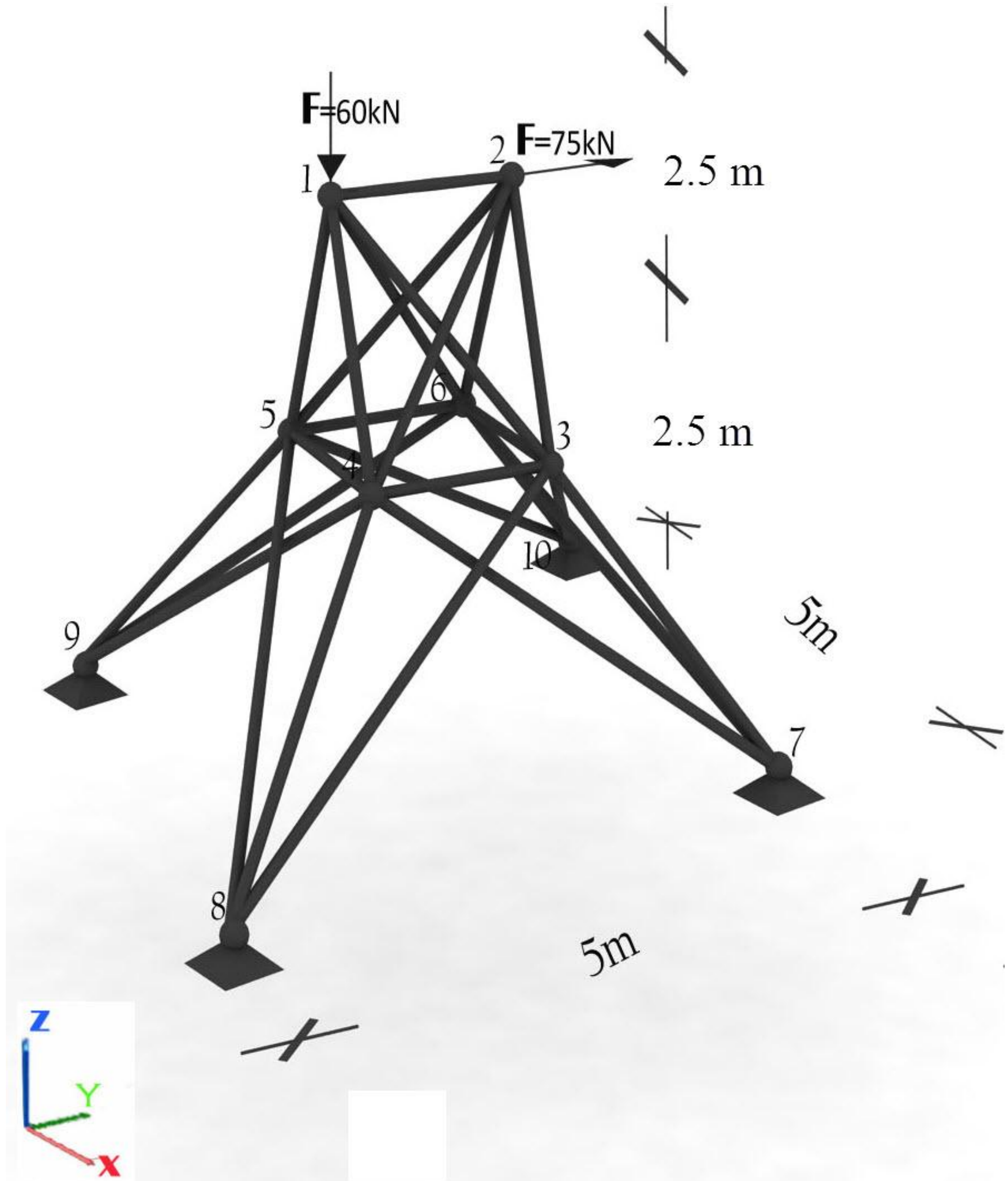

Figure 58. Space Truss of Example 3

Table 10. Nodal Data of Space Truss for Example 3.

Node Number	Coordinate			Applied Load			Restraint		
	x	y	z	x	y	z	x	y	z
1	1.5	2.5	5	0	0	-60	0	0	0
2	3.5	2.5	5	0	75	0	0	0	0
3	3.5	1.5	2.5	0	0	0	0	0	0
4	1.5	1.5	2.5	0	0	0	0	0	0
5	1.5	3.5	2.5	0	0	0	0	0	0
6	3.5	3.5	2.5	0	0	0	0	0	0
7	5	0	0	0	0	0	1	1	1
8	0	0	0	0	0	0	1	1	1
9	0	5	0	0	0	0	1	1	1
10	5	5	0	0	0	0	1	1	1

Table 11. Elemental Connectivity of Space Truss for Example 3

Element Number	Connectivity		Element Number	Connectivity	
	Start Node	End Node		Start Node	End Node
1	1	2	14	5	8
2	1	5	15	4	8
3	1	4	16	3	8
4	1	3	17	4	7
5	1	6	18	6	7
6	2	5	19	3	7
7	2	4	20	5	9
8	2	6	21	6	9
9	2	3	22	4	9
10	4	5	23	5	10
11	5	6	24	6	10
12	3	6	25	3	10
13	3	4			

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MEMBER FORCES

INDEPENDENT FORCES

$$\text{independent forces} \quad \begin{pmatrix} 7.06769 \\ -30.741 \\ -24.3658 \\ -9.89715 \\ -1.95573 \\ -51.3753 \\ 39.5224 \\ -59.7291 \\ 69.2443 \\ 5.13092 \\ 6.9158 \\ 0.596746 \\ -5.08876 \\ -16.8524 \\ 20.8599 \\ 41.1237 \\ -13.0978 \\ -10.914 \\ 37.9703 \\ -64.7808 \\ -36.3493 \\ -3.42906 \\ -14.9253 \\ -40.6242 \\ 9.82671 \end{pmatrix}$$

Figure 59. Member Forces of Space Truss for Example 3.

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MEMBER FORCES

INDEPENDENT FORCES

MEMBER END FORCES

member	1	-7.06769	7.06769
member	2	30.741	-30.741
member	3	24.3658	-24.3658
member	4	9.89715	-9.89715
member	5	1.95573	-1.95573
member	6	51.3753	-51.3753
member	7	-39.5224	39.5224
member	8	59.7291	-59.7291
member	9	-69.2443	69.2443
member	10	-5.13092	5.13092
member	11	-6.9158	6.9158
member	12	-0.596746	0.596746

Figure 60. Member End Forces for Example 3.

member 13	5.08876	-5.08876
member 14	16.8524	-16.8524
member 15	-20.8599	20.8599
member 16	-41.1237	41.1237
member 17	13.0978	-13.0978
member 18	10.914	-10.914
member 19	-37.9703	37.9703
member 20	64.7808	-64.7808
member 21	36.3493	-36.3493
member 22	3.42906	-3.42906
member 23	14.9253	-14.9253
member 24	40.6242	-40.6242
member 25	-9.82671	9.82671

Figure 61. Member End Forces for Example 3 (continued).

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DEFORMATIONS

$$\beta = \begin{pmatrix} 0.0000282708 \\ -0.000165545 \\ -0.000131214 \\ -0.0000663921 \\ -0.0000131194 \\ -0.000344636 \\ 0.000265125 \\ -0.000321651 \\ 0.000372892 \\ 0.0000205237 \\ 0.0000276632 \\ 2.38699 \times 10^{-6} \\ -0.0000203551 \\ -0.000153533 \\ 0.000136787 \\ 0.000374655 \\ -0.000119327 \\ -0.0000994316 \\ 0.000248988 \\ -0.000424796 \\ -0.000331158 \\ -0.0000312402 \\ -0.000135976 \\ -0.000266391 \\ 0.0000895256 \end{pmatrix}$$

Figure 62. Element Deformations for Example 3.

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DISPLACEMENTS

THE COMPUTER CODE

THE NODAL DISPLACEMENTS

$$\text{displacements} = \begin{pmatrix} -0.000362652 \\ 0.000637831 \\ -0.000359737 \\ -0.000334382 \\ 0.0019673 \\ 0.0000317329 \\ 0.000178055 \\ 0.0000817108 \\ 0.000384351 \\ 0.00019841 \\ 0.000118159 \\ -0.000010546 \\ -0.000154678 \\ 0.000138682 \\ -0.000381099 \\ -0.000127015 \\ 0.0000840978 \\ -0.000375119 \end{pmatrix}$$

Figure 63. Nodal Displacements for Example 3

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SUPPORT REACTIONS

THE COMPUTER CODE

REACTIONS MATRIX

$$\text{reactions} = \begin{pmatrix} 3.71363 \\ -4.67245 \\ -15.7739 \\ -35.5913 \\ -10.1365 \\ -29.2261 \\ 58.6951 \\ -44.2412 \\ 71.2261 \\ -26.8174 \\ -15.9498 \\ 33.7739 \end{pmatrix}$$

SCATTER PLOT OF REACTIONS MATRIX

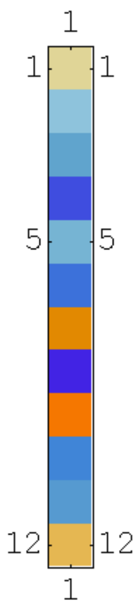

Figure 64. Support Reactions and Its Scatter Plot for Example 3.

6.7 Mastan Result for Example 3

```
***********   MASTAN2 v3.3.1   ***********

Time:  09:33:47      Date:  05/25/2012

Problem Title:   not provided
**************


##############################
Results of Structural Analysis
##############################

General Information:
      Structure Analyzed as:  Space Truss
      Analysis Type:  First-Order Elastic

Analytical Results:

(i)   Displacements at Step # 1, Applied Load Ratio = 1.0000

  Deflections
     Node       X-disp           Y-disp           Z-disp
       1    -3.6265e-004     6.3783e-004    -3.5974e-004
       2    -3.3438e-004     1.9673e-003     3.1733e-005
       3     1.7805e-004     8.1711e-005     3.8435e-004
       4     1.9841e-004     1.1816e-004    -1.0546e-005
       5    -1.5468e-004     1.3868e-004    -3.8110e-004
       6    -1.2701e-004     8.4098e-005    -3.7512e-004
       7     0.0000e+000     0.0000e+000     0.0000e+000
       8     0.0000e+000     0.0000e+000     0.0000e+000
       9     0.0000e+000     0.0000e+000     0.0000e+000
      10     0.0000e+000     0.0000e+000     0.0000e+000


(ii)  Element Results at Step # 1, Applied Load Ratio = 1.0000

  Internal End Forces (Note: Refers to local coordinates)
    Element  Node        Fx              Fy              Fz
       1       1    -7.0677e+000    0.0000e+000    0.0000e+000
               2     7.0677e+000    0.0000e+000    0.0000e+000
       2       1     3.0741e+001    0.0000e+000    0.0000e+000
               5    -3.0741e+001    0.0000e+000    0.0000e+000
       3       1     2.4366e+001    0.0000e+000    0.0000e+000
               4    -2.4366e+001    0.0000e+000    0.0000e+000
       4       1     9.8971e+000    0.0000e+000    0.0000e+000
               3    -9.8971e+000    0.0000e+000    0.0000e+000
       5       1     1.9557e+000    0.0000e+000    0.0000e+000
               6    -1.9557e+000    0.0000e+000    0.0000e+000
       6       2     5.1375e+001    0.0000e+000    0.0000e+000
               5    -5.1375e+001    0.0000e+000    0.0000e+000
       7       2    -3.9522e+001    0.0000e+000    0.0000e+000
               4     3.9522e+001    0.0000e+000    0.0000e+000
```

```
      8       2    5.9729e+001    0.0000e+000    0.0000e+000
              6   -5.9729e+001    0.0000e+000    0.0000e+000
      9       2   -6.9244e+001    0.0000e+000    0.0000e+000
              3    6.9244e+001    0.0000e+000    0.0000e+000
     10       4   -5.1309e+000    0.0000e+000    0.0000e+000
              5    5.1309e+000    0.0000e+000    0.0000e+000
     11       5   -6.9158e+000    0.0000e+000    0.0000e+000
              6    6.9158e+000    0.0000e+000    0.0000e+000
     12       3   -5.9675e-001    0.0000e+000    0.0000e+000
              6    5.9675e-001    0.0000e+000    0.0000e+000
     13       3    5.0888e+000    0.0000e+000    0.0000e+000
              4   -5.0888e+000    0.0000e+000    0.0000e+000
     14       5    1.6852e+001    0.0000e+000    0.0000e+000
              8   -1.6852e+001    0.0000e+000    0.0000e+000
     15       4   -2.0860e+001    0.0000e+000    0.0000e+000
              8    2.0860e+001    0.0000e+000    0.0000e+000
     16       3   -4.1124e+001    0.0000e+000    0.0000e+000
              8    4.1124e+001    0.0000e+000    0.0000e+000
     17       4    1.3098e+001    0.0000e+000    0.0000e+000
              7   -1.3098e+001    0.0000e+000    0.0000e+000
     18       6    1.0914e+001    0.0000e+000    0.0000e+000
              7   -1.0914e+001    0.0000e+000    0.0000e+000
     19       3   -3.7970e+001    0.0000e+000    0.0000e+000
              7    3.7970e+001    0.0000e+000    0.0000e+000
     20       5    6.4781e+001    0.0000e+000    0.0000e+000
              9   -6.4781e+001    0.0000e+000    0.0000e+000
     21       6    3.6349e+001    0.0000e+000    0.0000e+000
              9   -3.6349e+001    0.0000e+000    0.0000e+000
     22       4    3.4291e+000    0.0000e+000    0.0000e+000
              9   -3.4291e+000    0.0000e+000    0.0000e+000
     23       5    1.4925e+001    0.0000e+000    0.0000e+000
             10   -1.4925e+001    0.0000e+000    0.0000e+000
     24       6    4.0624e+001    0.0000e+000    0.0000e+000
             10   -4.0624e+001    0.0000e+000    0.0000e+000
     25       3   -9.8267e+000    0.0000e+000    0.0000e+000
             10    9.8267e+000    0.0000e+000    0.0000e+000

(iii) Reactions at Step # 1, Applied Load Ratio = 1.0000

  Forces
     Node          Rx               Ry               Rz
      7       3.7136e+000    -4.6724e+000    -1.5774e+001
      8      -3.5591e+001    -1.0136e+001    -2.9226e+001
      9       5.8695e+001    -4.4241e+001     7.1226e+001
     10      -2.6817e+001    -1.5950e+001     3.3774e+001

  Moments
     Node          Mx               My               Mz
             *** No Reaction Moments Exist ***

#####################################
End of Results of Structural Analysis
#####################################
```

6.8 Example 4

This example includes a space truss which has 12 joints and 30 members as shown in Figure 83. The properties of nodes and elements are presented in Tables 12 and 13. The member area section is $A = 0.002\,m^2$ and modulus of elasticity is $E = 2\times10^8\,kN/m^2$.

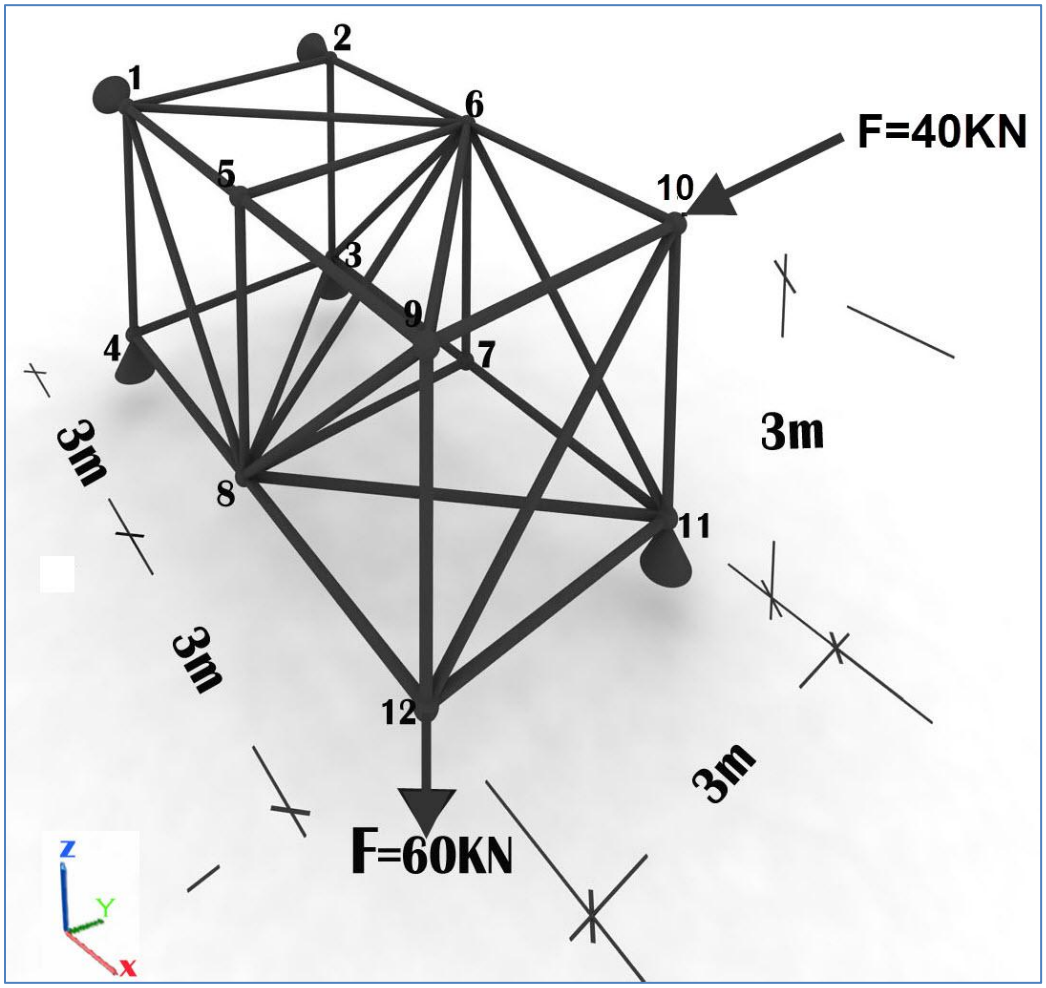

Figure 65. Space Truss for Example 4.

Table 12. Nodal Data of Space Truss of Example 4.

Node Number	Coordinate			Applied Load			Restraint		
	x	y	z	x	y	z	x	y	z
1	0	0	3	0	0	0	1	1	1
2	0	3	3	0	0	0	1	1	1
3	0	3	0	0	0	0	1	1	1
4	0	0	0	0	0	0	1	1	1
5	3	0	3	0	0	0	0	0	0
6	3	3	3	0	0	0	0	0	0
7	3	3	0	0	0	0	0	0	0
8	3	0	0	0	0	0	0	0	0
9	6	0	3	0	0	0	0	0	0
10	6	3	3	0	-40	0	0	0	0
11	6	3	0	0	0	0	1	1	1
12	6	0	0	0	0	-60	0	0	0

Table 13. Member Connectivity of Space Truss Example 4.

Element Number	Connectivity		Element Number	Connectivity	
	Start Node	End Node		Start Node	End Node
1	1	4	16	6	8
2	1	2	17	7	8
3	2	3	18	8	12
4	3	4	19	8	11
5	3	7	20	8	9
6	3	6	21	5	9
7	2	6	22	6	9
8	1	6	23	6	11
9	1	5	24	6	10
10	1	8	25	7	11
11	4	8	26	9	12
12	3	8	27	10	12
13	5	8	28	11	12
14	5	6	29	10	11
15	6	7	30	9	10

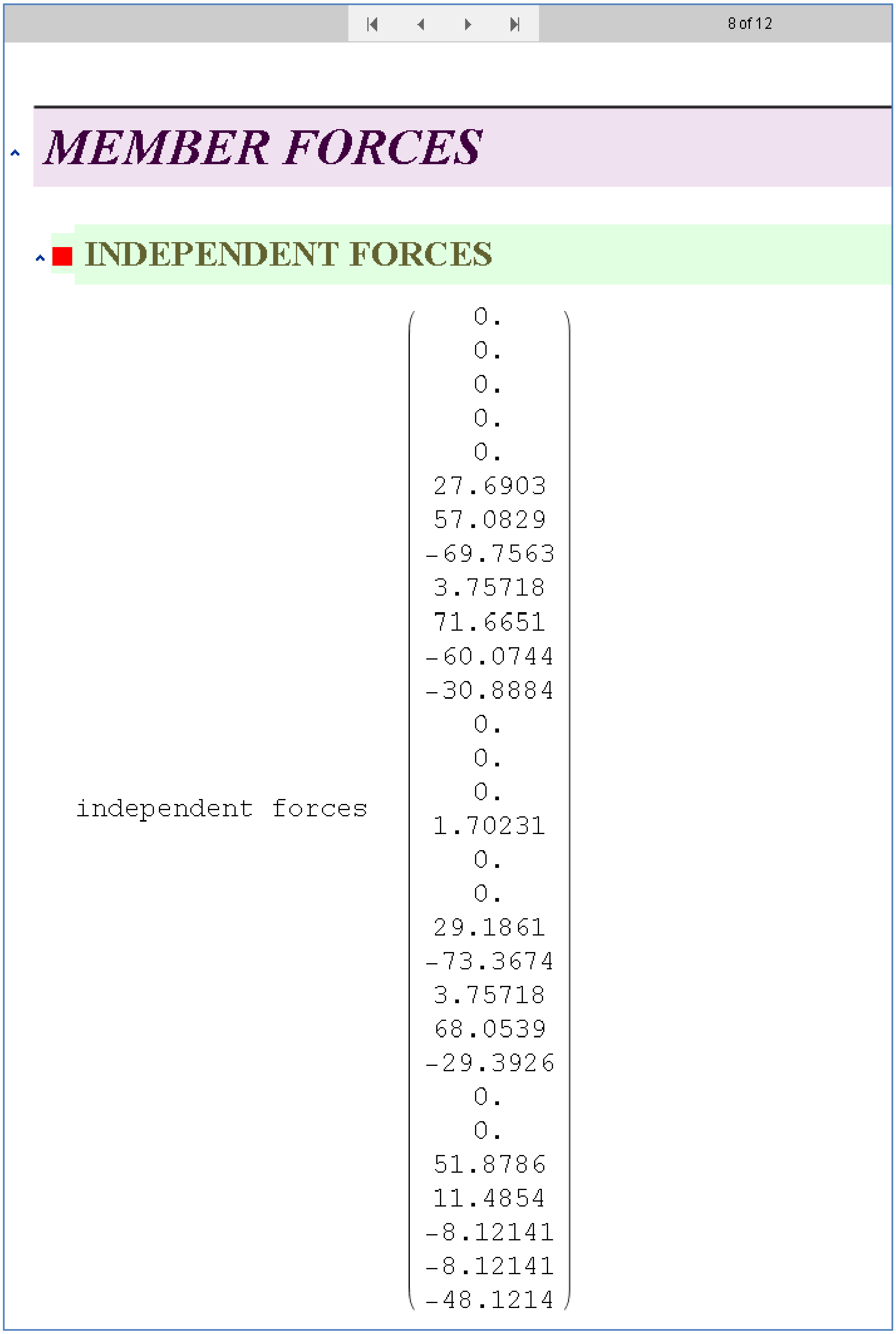

Figure 66. Member Forces of Space Truss for Example 4.

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MEMBER FORCES

INDEPENDENT FORCES

MEMBER END FORCES

member	1	$\begin{pmatrix} 0. \\ 0. \end{pmatrix}$
member	2	$\begin{pmatrix} 0. \\ 0. \end{pmatrix}$
member	3	$\begin{pmatrix} 0. \\ 0. \end{pmatrix}$
member	4	$\begin{pmatrix} 0. \\ 0. \end{pmatrix}$
member	5	$\begin{pmatrix} 0. \\ 0. \end{pmatrix}$
member	6	$\begin{pmatrix} -27.6903 \\ 27.6903 \end{pmatrix}$
member	7	$\begin{pmatrix} -57.0829 \\ 57.0829 \end{pmatrix}$
member	8	$\begin{pmatrix} 69.7563 \\ -69.7563 \end{pmatrix}$
member	9	$\begin{pmatrix} -3.75718 \\ 3.75718 \end{pmatrix}$
member	10	$\begin{pmatrix} -71.6651 \\ 71.6651 \end{pmatrix}$
member	11	$\begin{pmatrix} 60.0744 \\ -60.0744 \end{pmatrix}$
member	12	$\begin{pmatrix} 30.8884 \\ -30.8884 \end{pmatrix}$
member	13	$\begin{pmatrix} 0. \\ 0. \end{pmatrix}$

Figure 67. Member End Forces for Example 4.

member	13	$\begin{pmatrix} 0. \\ 0. \end{pmatrix}$
member	14	$\begin{pmatrix} 0. \\ 0. \end{pmatrix}$
member	15	$\begin{pmatrix} 0. \\ 0. \end{pmatrix}$
member	16	$\begin{pmatrix} -1.70231 \\ 1.70231 \end{pmatrix}$
member	17	$\begin{pmatrix} 0. \\ 0. \end{pmatrix}$
member	18	$\begin{pmatrix} 0. \\ 0. \end{pmatrix}$
member	19	$\begin{pmatrix} -29.1861 \\ 29.1861 \end{pmatrix}$
member	20	$\begin{pmatrix} 73.3674 \\ -73.3674 \end{pmatrix}$
member	21	$\begin{pmatrix} -3.75718 \\ 3.75718 \end{pmatrix}$
member	22	$\begin{pmatrix} -68.0539 \\ 68.0539 \end{pmatrix}$
member	23	$\begin{pmatrix} 29.3926 \\ -29.3926 \end{pmatrix}$
member	24	$\begin{pmatrix} 0. \\ 0. \end{pmatrix}$
member	25	$\begin{pmatrix} 0. \\ 0. \end{pmatrix}$
member	26	$\begin{pmatrix} -51.8786 \\ 51.8786 \end{pmatrix}$
member	27	$\begin{pmatrix} -11.4854 \\ 11.4854 \end{pmatrix}$
member	28	$\begin{pmatrix} 8.12141 \\ -8.12141 \end{pmatrix}$
member	29	$\begin{pmatrix} 8.12141 \\ -8.12141 \end{pmatrix}$
member	30	$\begin{pmatrix} 48.1214 \\ -48.1214 \end{pmatrix}$

Figure 68. Member End Forces for Example 4 (continued).

DEFORMATIONS

$$\beta = \begin{pmatrix} 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0.0002937 \\ 0.000428121 \\ -0.000739877 \\ 0.0000281789 \\ 0.000760123 \\ -0.000450558 \\ -0.000327621 \\ 0. \\ 0. \\ 0. \\ 0.0000180557 \\ 0. \\ 0. \\ 0.000309565 \\ -0.000778179 \\ 0.0000281789 \\ 0.000721821 \\ -0.000311755 \\ 0. \\ 0. \\ 0.000389089 \\ 0.000121821 \\ -0.0000609106 \\ -0.0000609106 \\ -0.000360911 \end{pmatrix}$$

Figure 69. Deformations of Elements for Example 4.

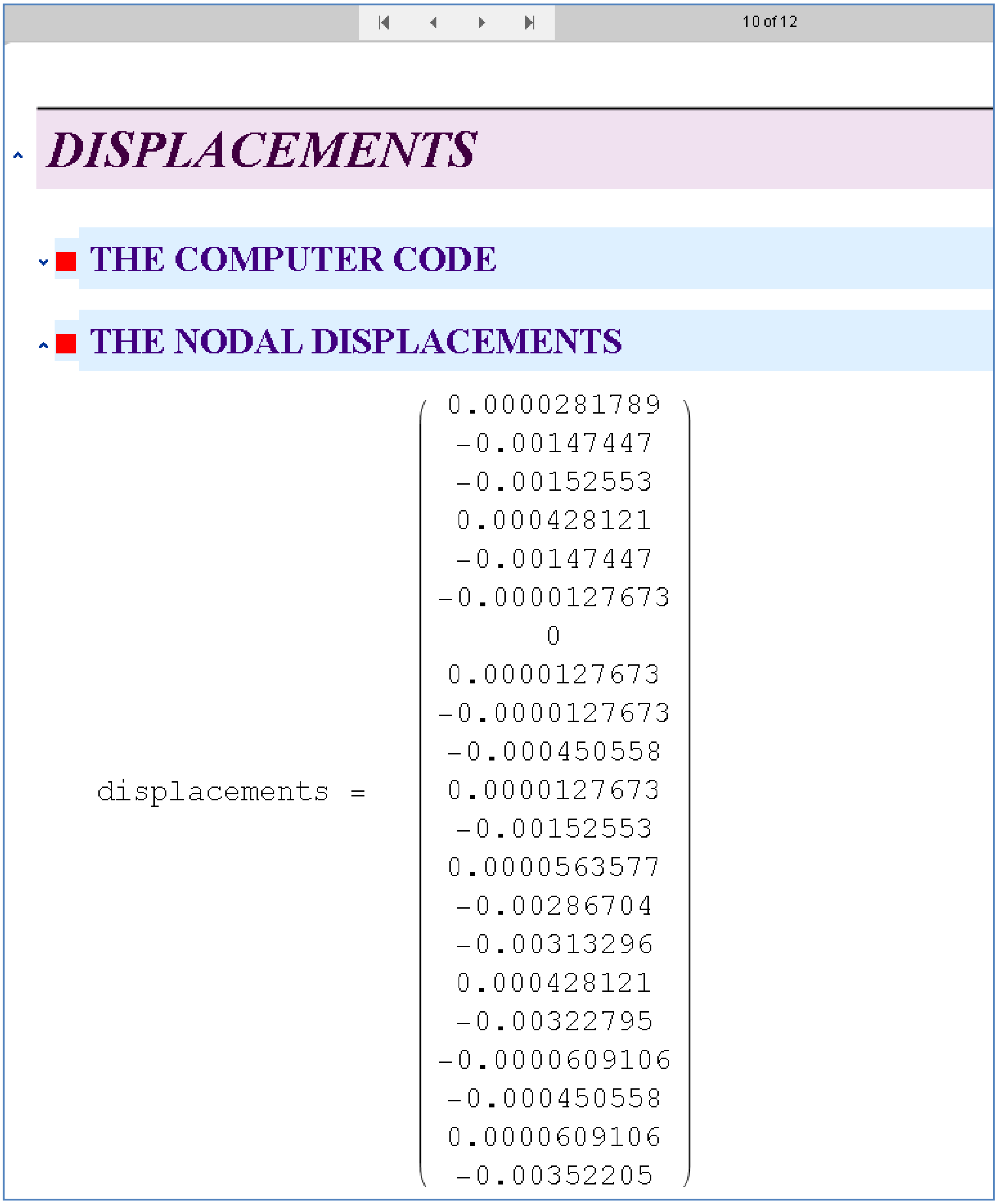

Figure 70. Nodal Displacements for Example 4.

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SUPPORT REACTIONS

THE COMPUTER CODE

REACTIONS MATRIX

$$
reactions = \begin{pmatrix} -5.10693 \\ 49.3251 \\ 50.6749 \\ -57.0829 \\ 0 \\ 0 \\ 2.2614 \\ -21.8414 \\ -19.58 \\ 60.0744 \\ 0 \\ 0 \\ -0.146035 \\ 12.5163 \\ 28.9051 \end{pmatrix}
$$

SCATTER PLOT OF REACTIONS MATRIX

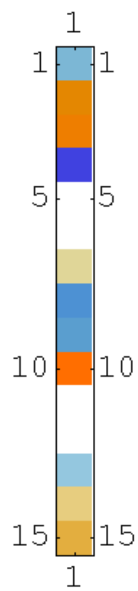

Figure 71. Support Reactions for Example 4.

6.9 Mastan Results for Example 4

```
***********   MASTAN2 v3.3.1   ***********

Time:  09:28:26      Date:  05/25/2012

Problem Title:   not provided
**************


##############################
Results of Structural Analysis
##############################

General Information:
      Structure Analyzed as:  Space Truss
      Analysis Type:  First-Order Elastic

Analytical Results:

(i)   Displacements at Step # 1, Applied Load Ratio = 1.0000

  Deflections
     Node       X-disp          Y-disp          Z-disp
       1     0.0000e+000     0.0000e+000     0.0000e+000
       2     0.0000e+000     0.0000e+000     0.0000e+000
       3     0.0000e+000     0.0000e+000     0.0000e+000
       4     0.0000e+000     0.0000e+000     0.0000e+000
       5     2.8179e-005    -1.4745e-003    -1.5255e-003
       6     4.2812e-004    -1.4745e-003    -1.2767e-005
       7     0.0000e+000     1.2767e-005    -1.2767e-005
       8    -4.5056e-004     1.2767e-005    -1.5255e-003
       9     5.6358e-005    -2.8670e-003    -3.1330e-003
      10     4.2812e-004    -3.2279e-003    -6.0911e-005
      11     0.0000e+000     0.0000e+000     0.0000e+000
      12    -4.5056e-004     6.0911e-005    -3.5221e-003

 (ii)  Element Results at Step # 1, Applied Load Ratio = 1.0000

  Internal End Forces (Note: Refers to local coordinates)
    Element  Node         Fx             Fy             Fz
       1       1    0.0000e+000    0.0000e+000    0.0000e+000
               4    0.0000e+000    0.0000e+000    0.0000e+000
       2       1    0.0000e+000    0.0000e+000    0.0000e+000
               2    0.0000e+000    0.0000e+000    0.0000e+000
       3       2    0.0000e+000    0.0000e+000    0.0000e+000
               3    0.0000e+000    0.0000e+000    0.0000e+000
       4       3    0.0000e+000    0.0000e+000    0.0000e+000
               4    0.0000e+000    0.0000e+000    0.0000e+000
       5       3    0.0000e+000    0.0000e+000    0.0000e+000
               7    0.0000e+000    0.0000e+000    0.0000e+000
       6       3   -2.7690e+001    0.0000e+000    0.0000e+000
               6    2.7690e+001    0.0000e+000    0.0000e+000
```

7	2	-5.7083e+001	0.0000e+000	0.0000e+000
	6	5.7083e+001	0.0000e+000	0.0000e+000
8	1	6.9756e+001	0.0000e+000	0.0000e+000
	6	-6.9756e+001	0.0000e+000	0.0000e+000
9	1	-3.7572e+000	0.0000e+000	0.0000e+000
	5	3.7572e+000	0.0000e+000	0.0000e+000
10	1	-7.1665e+001	0.0000e+000	0.0000e+000
	8	7.1665e+001	0.0000e+000	0.0000e+000
11	4	6.0074e+001	0.0000e+000	0.0000e+000
	8	-6.0074e+001	0.0000e+000	0.0000e+000
12	3	3.0888e+001	0.0000e+000	0.0000e+000
	8	-3.0888e+001	0.0000e+000	0.0000e+000
13	5	0.0000e+000	0.0000e+000	0.0000e+000
	8	0.0000e+000	0.0000e+000	0.0000e+000
14	5	-2.8422e-014	0.0000e+000	0.0000e+000
	6	2.8422e-014	0.0000e+000	0.0000e+000
15	6	-8.8818e-016	0.0000e+000	0.0000e+000
	7	8.8818e-016	0.0000e+000	0.0000e+000
16	6	-1.7023e+000	0.0000e+000	0.0000e+000
	8	1.7023e+000	0.0000e+000	0.0000e+000
17	7	0.0000e+000	0.0000e+000	0.0000e+000
	8	0.0000e+000	0.0000e+000	0.0000e+000
18	8	0.0000e+000	0.0000e+000	0.0000e+000
	12	0.0000e+000	0.0000e+000	0.0000e+000
19	8	-2.9186e+001	0.0000e+000	0.0000e+000
	11	2.9186e+001	0.0000e+000	0.0000e+000
20	8	7.3367e+001	0.0000e+000	0.0000e+000
	9	-7.3367e+001	0.0000e+000	0.0000e+000
21	5	-3.7572e+000	0.0000e+000	0.0000e+000
	9	3.7572e+000	0.0000e+000	0.0000e+000
22	6	-6.8054e+001	0.0000e+000	0.0000e+000
	9	6.8054e+001	0.0000e+000	0.0000e+000
23	6	2.9393e+001	0.0000e+000	0.0000e+000
	11	-2.9393e+001	0.0000e+000	0.0000e+000
24	6	-2.1316e-014	0.0000e+000	0.0000e+000
	10	2.1316e-014	0.0000e+000	0.0000e+000
25	7	0.0000e+000	0.0000e+000	0.0000e+000
	11	0.0000e+000	0.0000e+000	0.0000e+000
26	9	-5.1879e+001	0.0000e+000	0.0000e+000
	12	5.1879e+001	0.0000e+000	0.0000e+000
27	10	-1.1485e+001	0.0000e+000	0.0000e+000
	12	1.1485e+001	0.0000e+000	0.0000e+000
28	11	8.1214e+000	0.0000e+000	0.0000e+000
	12	-8.1214e+000	0.0000e+000	0.0000e+000
29	10	8.1214e+000	0.0000e+000	0.0000e+000
	11	-8.1214e+000	0.0000e+000	0.0000e+000
30	9	4.8121e+001	0.0000e+000	0.0000e+000
	10	-4.8121e+001	0.0000e+000	0.0000e+000

(iii) Reactions at Step # 1, Applied Load Ratio = 1.0000

Forces

Node	Rx	Ry	Rz
1	-5.1069e+000	4.9325e+001	5.0675e+001

```
       2      -5.7083e+001     0.0000e+000     0.0000e+000
       3       2.2614e+000    -2.1841e+001    -1.9580e+001
       4       6.0074e+001     0.0000e+000     0.0000e+000
      11      -1.4604e-001     1.2516e+001     2.8905e+001

  Moments
    Node           Mx               My               Mz
             *** No Reaction Moments Exist ***

#####################################
End of Results of Structural Analysis
#####################################
```

Solving Examples 5 and 6 with Dual Integrated Force Method

In this section the examples 5 and 6 are solved by dual integrated force method and the reporting phase in this method consists of:

- Nodal displacements
- Element deformations
- Member independent forces
- Member end forces
- Support reactions

6.10 Example 5

In this example the space truss consists of 16 joints and 39 elements. The properties of truss are presented in Tables 14 and 15. The section area of members is $A = 0.0015 m^2$ and the modulus of elasticity is $E = 2\times10^8 \, kN/m^2$.

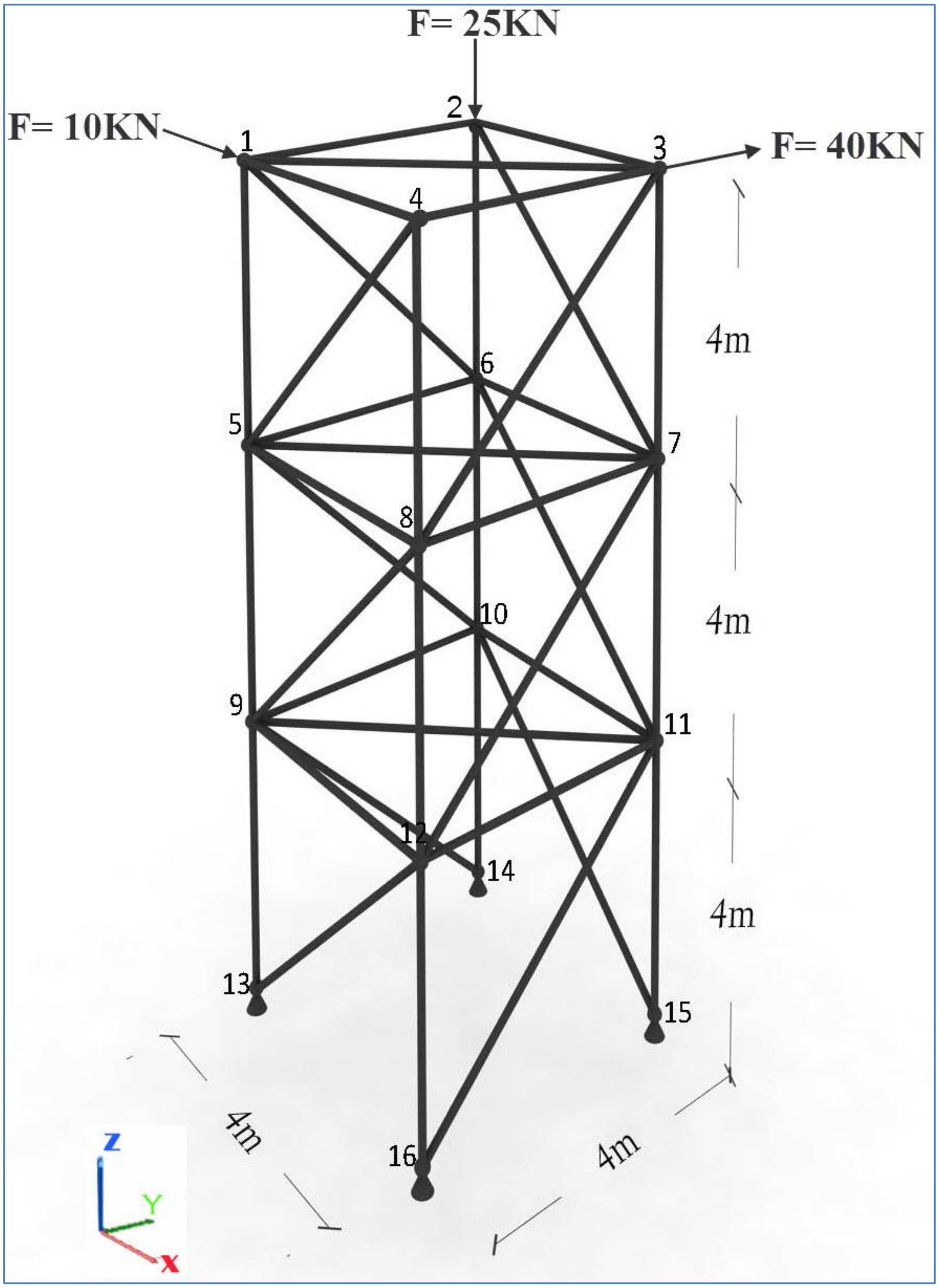

Figure 72. Space Truss for Example 5.

Table 14. Nodal Data of Space Truss of Example 5.

Node Number	Coordinate			Applied Load			Restraint		
	x	y	z	x	y	z	x	y	z
1	0	0	12	10	0	0	0	0	0
2	0	4	12	0	0	-25	0	0	0
3	4	4	12	0	40	0	0	0	0
4	4	0	12	0	0	0	0	0	0
5	0	0	8	0	0	0	0	0	0
6	0	4	8	0	0	0	0	0	0
7	4	4	8	0	0	0	0	0	0
8	4	0	8	0	0	0	0	0	0
9	0	0	4	0	0	0	0	0	0
10	0	4	4	0	0	0	0	0	0
11	4	4	4	0	0	0	0	0	0
12	4	0	4	0	0	0	0	0	0
13	0	0	0	0	0	0	1	1	1
14	0	4	0	0	0	0	1	1	1
15	4	4	0	0	0	0	1	1	1
16	4	0	0	0	0	0	1	1	1

Table 15. Element Data of Space Truss of Example 5.

Element Number	Coordinate		Element Number	Coordinate	
	Start Node	End Node		Start Node	End Node
1	1	2	21	7	12
2	2	3	22	8	9
3	3	4	23	5	9
4	1	4	24	6	10
5	1	3	25	7	11
6	1	6	26	8	12
7	2	7	27	9	10
8	4	5	28	10	11
9	3	8	29	11	12
10	1	5	30	9	12
11	2	6	31	9	11
12	3	7	32	9	14
13	4	8	33	10	15
14	5	6	34	11	16
15	6	7	35	12	13
16	7	8	36	9	13
17	5	8	37	10	14
18	5	7	38	11	15
19	5	10	39	12	16
20	6	11			

DISPLACEMENTS

THE COMPUTER CODE

THE NODAL DISPLACEMENTS MATRIX

$$\text{displacements} = \begin{pmatrix} 0.00355298 \\ 0.00574146 \\ 0.00111499 \\ -0.00141711 \\ 0.00574146 \\ -0.00171499 \\ -0.00148828 \\ 0.010984 \\ -0.00248501 \\ 0.00334848 \\ 0.010984 \\ 0.0000850094 \\ 0.00181124 \\ 0.00304351 \\ 0.00104382 \\ -0.00143067 \\ 0.00311468 \\ -0.00131049 \\ -0.00152366 \\ 0.00644014 \\ -0.00202285 \\ 0.00158491 \\ 0.00690231 \\ 0.000289514 \\ 0.000559124 \\ 0.00090185 \\ 0.00067515 \\ -0.000968517 \\ 0.000994845 \\ -0.000741816 \\ -0.00104867 \\ 0.00247331 \\ -0.00119152 \\ 0.00034564 \\ 0.00291365 \\ 0.000258184 \end{pmatrix}$$

Figure 73. Nodal Displacements of Example 5

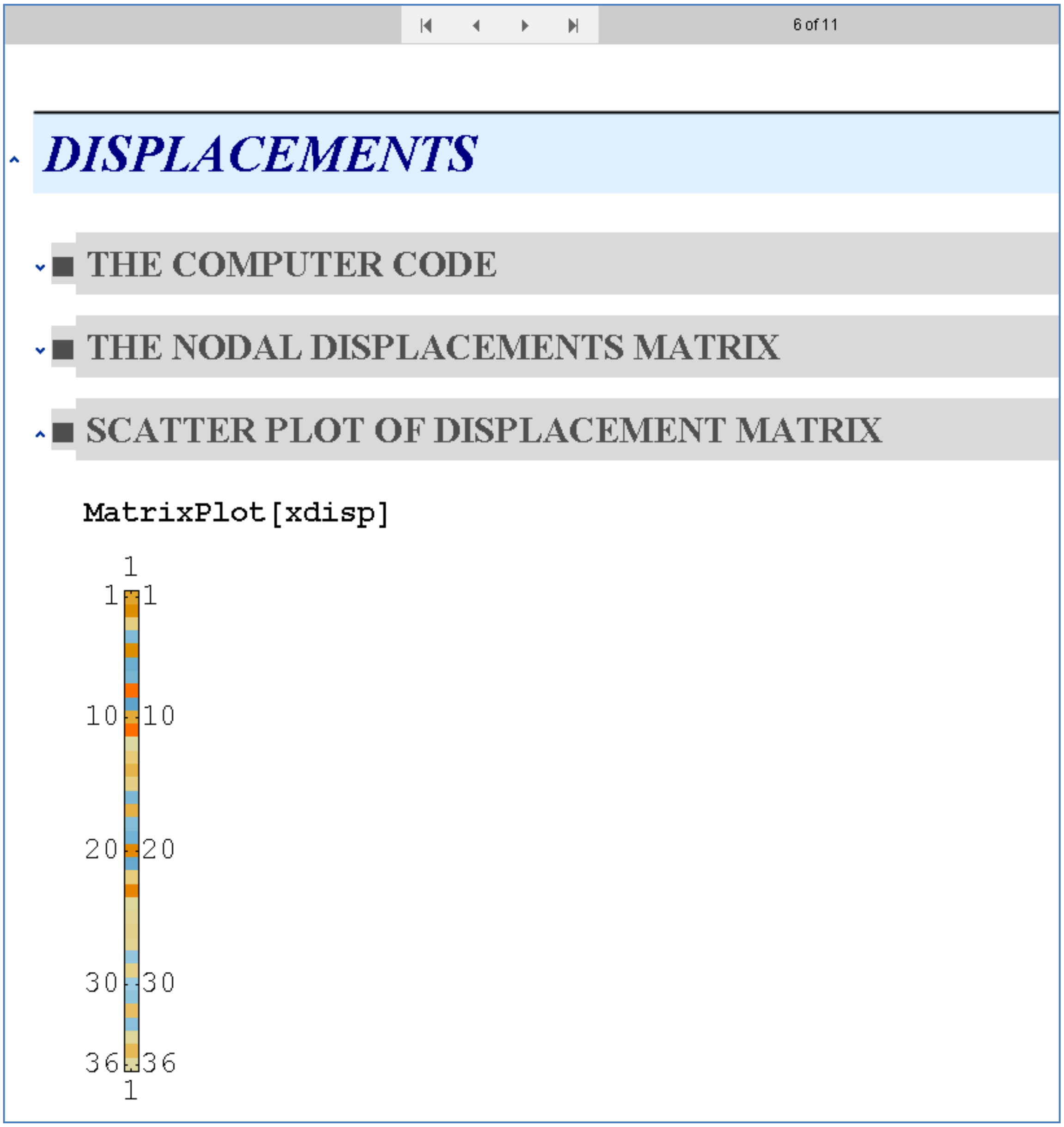

Figure 74. Scatter Plot of Nodal Displacements for Example 5.

DEFORMATIONS

$$
\text{deformations} = \begin{pmatrix}
0 \\
-0.0000711709 \\
0 \\
-0.000204504 \\
0.000142342 \\
-0.000142342 \\
0.000142342 \\
0.000409008 \\
0.000924325 \\
0.0000711709 \\
-0.000404504 \\
-0.000462162 \\
-0.000204504 \\
0.0000711709 \\
-0.0000929952 \\
-0.000462162 \\
-0.000226329 \\
0.0000436488 \\
-0.00018599 \\
0.00018599 \\
0.000880676 \\
0.000452657 \\
0.00036867 \\
-0.00056867 \\
-0.00083133 \\
0.0000313297 \\
0.0000929952 \\
-0.0000801507 \\
-0.000440338 \\
-0.000213484 \\
-0.0000256891 \\
-0.000160301 \\
0.000160301 \\
0.000906365 \\
0.000426968 \\
0.00067515 \\
-0.000741816 \\
-0.00119152 \\
0.000258184
\end{pmatrix}
$$

Figure 75. Member Deformation for Example 5.

INTERNAL FORCES

Independent forces=

0
-5.33781
0
-15.3378
7.54881
-7.54881
7.54881
21.6909
49.0197
5.33781
-30.3378
-34.6622
-15.3378
5.33781
-6.97464
-34.6622
-16.9746
2.31482
-9.86363
9.86363
46.7049
24.0058
27.6503
-42.6503
-62.3497
2.34973
6.97464
-6.0113
-33.0254
-16.0113
-1.36237
-8.50126
8.50126
48.0673
22.6434
50.6362
-55.6362
-89.3638
19.3638

Figure 76. Member Forces for Example 5.

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INTERNAL FORCES

MEMBER END FORCES

member 1 $\begin{pmatrix} 0. \\ 0. \end{pmatrix}$

member 2 $\begin{pmatrix} 5.33781 \\ -5.33781 \end{pmatrix}$

member 3 $\begin{pmatrix} 0. \\ 0. \end{pmatrix}$

member 4 $\begin{pmatrix} 15.3378 \\ -15.3378 \end{pmatrix}$

member 5 $\begin{pmatrix} -7.54881 \\ 7.54881 \end{pmatrix}$

member 6 $\begin{pmatrix} 7.54881 \\ -7.54881 \end{pmatrix}$

member 7 $\begin{pmatrix} -7.54881 \\ 7.54881 \end{pmatrix}$

member 8 $\begin{pmatrix} -21.6909 \\ 21.6909 \end{pmatrix}$

member 9 $\begin{pmatrix} -49.0197 \\ 49.0197 \end{pmatrix}$

member 10 $\begin{pmatrix} -5.33781 \\ 5.33781 \end{pmatrix}$

member 11 $\begin{pmatrix} 30.3378 \\ -30.3378 \end{pmatrix}$

member 12 $\begin{pmatrix} 34.6622 \\ -34.6622 \end{pmatrix}$

member 13 $\begin{pmatrix} 15.3378 \\ -15.3378 \end{pmatrix}$

member 14 $\begin{pmatrix} -5.33781 \\ 5.33781 \end{pmatrix}$

member 15 $\begin{pmatrix} 6.97464 \\ -6.97464 \end{pmatrix}$

member 16 $\begin{pmatrix} 34.6622 \\ -34.6622 \end{pmatrix}$

member 17 $\begin{pmatrix} 16.9746 \\ -16.9746 \end{pmatrix}$

Figure 77. Member End Forces of Example 5.

member 18 $\begin{pmatrix} -2.31482 \\ 2.31482 \end{pmatrix}$

member 19 $\begin{pmatrix} 9.86363 \\ -9.86363 \end{pmatrix}$

member 20 $\begin{pmatrix} -9.86363 \\ 9.86363 \end{pmatrix}$

member 21 $\begin{pmatrix} -46.7049 \\ 46.7049 \end{pmatrix}$

member 22 $\begin{pmatrix} -24.0058 \\ 24.0058 \end{pmatrix}$

member 23 $\begin{pmatrix} -27.6503 \\ 27.6503 \end{pmatrix}$

member 24 $\begin{pmatrix} 42.6503 \\ -42.6503 \end{pmatrix}$

member 25 $\begin{pmatrix} 62.3497 \\ -62.3497 \end{pmatrix}$

member 26 $\begin{pmatrix} -2.34973 \\ 2.34973 \end{pmatrix}$

member 27 $\begin{pmatrix} -6.97464 \\ 6.97464 \end{pmatrix}$

member 28 $\begin{pmatrix} 6.0113 \\ -6.0113 \end{pmatrix}$

member 29 $\begin{pmatrix} 33.0254 \\ -33.0254 \end{pmatrix}$

member 30 $\begin{pmatrix} 16.0113 \\ -16.0113 \end{pmatrix}$

member 31 $\begin{pmatrix} 1.36237 \\ -1.36237 \end{pmatrix}$

member 32 $\begin{pmatrix} 8.50126 \\ -8.50126 \end{pmatrix}$

member 33 $\begin{pmatrix} -8.50126 \\ 8.50126 \end{pmatrix}$

member 34 $\begin{pmatrix} -48.0673 \\ 48.0673 \end{pmatrix}$

member 35 $\begin{pmatrix} -22.6434 \\ 22.6434 \end{pmatrix}$

member 36 $\begin{pmatrix} -50.6362 \\ 50.6362 \end{pmatrix}$

member 37 $\begin{pmatrix} 55.6362 \\ -55.6362 \end{pmatrix}$

member 38 $\begin{pmatrix} 89.3638 \\ -89.3638 \end{pmatrix}$

member 39 $\begin{pmatrix} -19.3638 \\ 19.3638 \end{pmatrix}$

Figure 78. Member End Forces of Example 5.

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SUPPORT REACTIONS

THE COMPUTER CODE

SUPPORT REACTIONS MATRIX

$$\text{Reactions} = \begin{pmatrix} -16.0113 \\ 0 \\ -66.6475 \\ 0 \\ -6.0113 \\ 61.6475 \\ 6.0113 \\ 0 \\ 83.3525 \\ 0 \\ -33.9887 \\ -53.3525 \end{pmatrix}$$

SCATTER PLOT OF REACTIONS MATRIX

Out[88]=

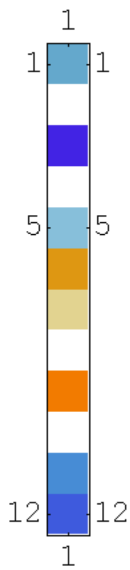

Figure 79. Support Reaction for Example 5.

6.11 Mastan Results for Example 5

```
***********   MASTAN2 v3.3.1   ***********

Time:  10:14:48      Date:  05/25/2012

Problem Title:   not provided
**************


##############################
Results of Structural Analysis
##############################

General Information:
      Structure Analyzed as:  Space Truss
      Analysis Type:  First-Order Elastic

Analytical Results:

(i)   Displacements at Step # 1, Applied Load Ratio = 1.0000

  Deflections
     Node       X-disp          Y-disp          Z-disp
       1     3.5530e-003     5.7415e-003     1.1150e-003
       2    -1.4171e-003     5.7415e-003    -1.7150e-003
       3    -1.4883e-003     1.0984e-002    -2.4850e-003
       4     3.3485e-003     1.0984e-002     8.5009e-005
       5     1.8112e-003     3.0435e-003     1.0438e-003
       6    -1.4307e-003     3.1147e-003    -1.3105e-003
       7    -1.5237e-003     6.4401e-003    -2.0228e-003
       8     1.5849e-003     6.9023e-003     2.8951e-004
       9     5.5912e-004     9.0185e-004     6.7515e-004
      10    -9.6852e-004     9.9485e-004    -7.4182e-004
      11    -1.0487e-003     2.4733e-003    -1.1915e-003
      12     3.4564e-004     2.9136e-003     2.5818e-004
      13     0.0000e+000     0.0000e+000     0.0000e+000
      14     0.0000e+000     0.0000e+000     0.0000e+000
      15     0.0000e+000     0.0000e+000     0.0000e+000
      16     0.0000e+000     0.0000e+000     0.0000e+000

(ii)  Element Results at Step # 1, Applied Load Ratio = 1.0000

  Internal End Forces (Note: Refers to local coordinates)
    Element  Node        Fx              Fy              Fz
       1       1    5.6843e-014    0.0000e+000    0.0000e+000
               2   -5.6843e-014    0.0000e+000    0.0000e+000
       2       2    5.3378e+000    0.0000e+000    0.0000e+000
               3   -5.3378e+000    0.0000e+000    0.0000e+000
       3       3    0.0000e+000    0.0000e+000    0.0000e+000
               4    0.0000e+000    0.0000e+000    0.0000e+000
       4       1    1.5338e+001    0.0000e+000    0.0000e+000
               4   -1.5338e+001    0.0000e+000    0.0000e+000
       5       1   -7.5488e+000    0.0000e+000    0.0000e+000
```

	3	7.5488e+000	0.0000e+000	0.0000e+000
6	1	7.5488e+000	0.0000e+000	0.0000e+000
	6	-7.5488e+000	0.0000e+000	0.0000e+000
7	2	-7.5488e+000	0.0000e+000	0.0000e+000
	7	7.5488e+000	0.0000e+000	0.0000e+000
8	4	-2.1691e+001	0.0000e+000	0.0000e+000
	5	2.1691e+001	0.0000e+000	0.0000e+000
9	3	-4.9020e+001	0.0000e+000	0.0000e+000
	8	4.9020e+001	0.0000e+000	0.0000e+000
10	1	-5.3378e+000	0.0000e+000	0.0000e+000
	5	5.3378e+000	0.0000e+000	0.0000e+000
11	2	3.0338e+001	0.0000e+000	0.0000e+000
	6	-3.0338e+001	0.0000e+000	0.0000e+000
12	3	3.4662e+001	0.0000e+000	0.0000e+000
	7	-3.4662e+001	0.0000e+000	0.0000e+000
13	4	1.5338e+001	0.0000e+000	0.0000e+000
	8	-1.5338e+001	0.0000e+000	0.0000e+000
14	5	-5.3378e+000	0.0000e+000	0.0000e+000
	6	5.3378e+000	0.0000e+000	0.0000e+000
15	6	6.9746e+000	0.0000e+000	0.0000e+000
	7	-6.9746e+000	0.0000e+000	0.0000e+000
16	7	3.4662e+001	0.0000e+000	0.0000e+000
	8	-3.4662e+001	0.0000e+000	0.0000e+000
17	5	1.6975e+001	0.0000e+000	0.0000e+000
	8	-1.6975e+001	0.0000e+000	0.0000e+000
18	5	-2.3148e+000	0.0000e+000	0.0000e+000
	7	2.3148e+000	0.0000e+000	0.0000e+000
19	5	9.8636e+000	0.0000e+000	0.0000e+000
	10	-9.8636e+000	0.0000e+000	0.0000e+000
20	6	-9.8636e+000	0.0000e+000	0.0000e+000
	11	9.8636e+000	0.0000e+000	0.0000e+000
21	7	-4.6705e+001	0.0000e+000	0.0000e+000
	12	4.6705e+001	0.0000e+000	0.0000e+000
22	8	-2.4006e+001	0.0000e+000	0.0000e+000
	9	2.4006e+001	0.0000e+000	0.0000e+000
23	5	-2.7650e+001	0.0000e+000	0.0000e+000
	9	2.7650e+001	0.0000e+000	0.0000e+000
24	6	4.2650e+001	0.0000e+000	0.0000e+000
	10	-4.2650e+001	0.0000e+000	0.0000e+000
25	7	6.2350e+001	0.0000e+000	0.0000e+000
	11	-6.2350e+001	0.0000e+000	0.0000e+000
26	8	-2.3497e+000	0.0000e+000	0.0000e+000
	12	2.3497e+000	0.0000e+000	0.0000e+000
27	9	-6.9746e+000	0.0000e+000	0.0000e+000
	10	6.9746e+000	0.0000e+000	0.0000e+000
28	10	6.0113e+000	0.0000e+000	0.0000e+000
	11	-6.0113e+000	0.0000e+000	0.0000e+000
29	11	3.3025e+001	0.0000e+000	0.0000e+000
	12	-3.3025e+001	0.0000e+000	0.0000e+000
30	9	1.6011e+001	0.0000e+000	0.0000e+000
	12	-1.6011e+001	0.0000e+000	0.0000e+000
31	9	1.3624e+000	0.0000e+000	0.0000e+000
	11	-1.3624e+000	0.0000e+000	0.0000e+000
32	9	8.5013e+000	0.0000e+000	0.0000e+000

	14	-8.5013e+000	0.0000e+000	0.0000e+000
33	10	-8.5013e+000	0.0000e+000	0.0000e+000
	15	8.5013e+000	0.0000e+000	0.0000e+000
34	11	-4.8067e+001	0.0000e+000	0.0000e+000
	16	4.8067e+001	0.0000e+000	0.0000e+000
35	12	-2.2643e+001	0.0000e+000	0.0000e+000
	13	2.2643e+001	0.0000e+000	0.0000e+000
36	9	-5.0636e+001	0.0000e+000	0.0000e+000
	13	5.0636e+001	0.0000e+000	0.0000e+000
37	10	5.5636e+001	0.0000e+000	0.0000e+000
	14	-5.5636e+001	0.0000e+000	0.0000e+000
38	11	8.9364e+001	0.0000e+000	0.0000e+000
	15	-8.9364e+001	0.0000e+000	0.0000e+000
39	12	-1.9364e+001	0.0000e+000	0.0000e+000
	16	1.9364e+001	0.0000e+000	0.0000e+000

(iii) Reactions at Step # 1, Applied Load Ratio = 1.0000

Forces

Node	Rx	Ry	Rz
13	-1.6011e+001	0.0000e+000	-6.6648e+001
14	0.0000e+000	-6.0113e+000	6.1648e+001
15	6.0113e+000	0.0000e+000	8.3352e+001
16	0.0000e+000	-3.3989e+001	-5.3352e+001

Moments

Node	Mx	My	Mz

*** No Reaction Moments Exist ***

#####################################
End of Results of Structural Analysis
#####################################

6.12 Example 6

This example of space truss consists of 32 nodes and 96 members. The properties of joints and members are presented in Tables 16 and 17. And also, the section area for members is $A = 0.002\, m^2$ and the modulus of elasticity is $E = 2 \times 10^8\, kN / m^2$.

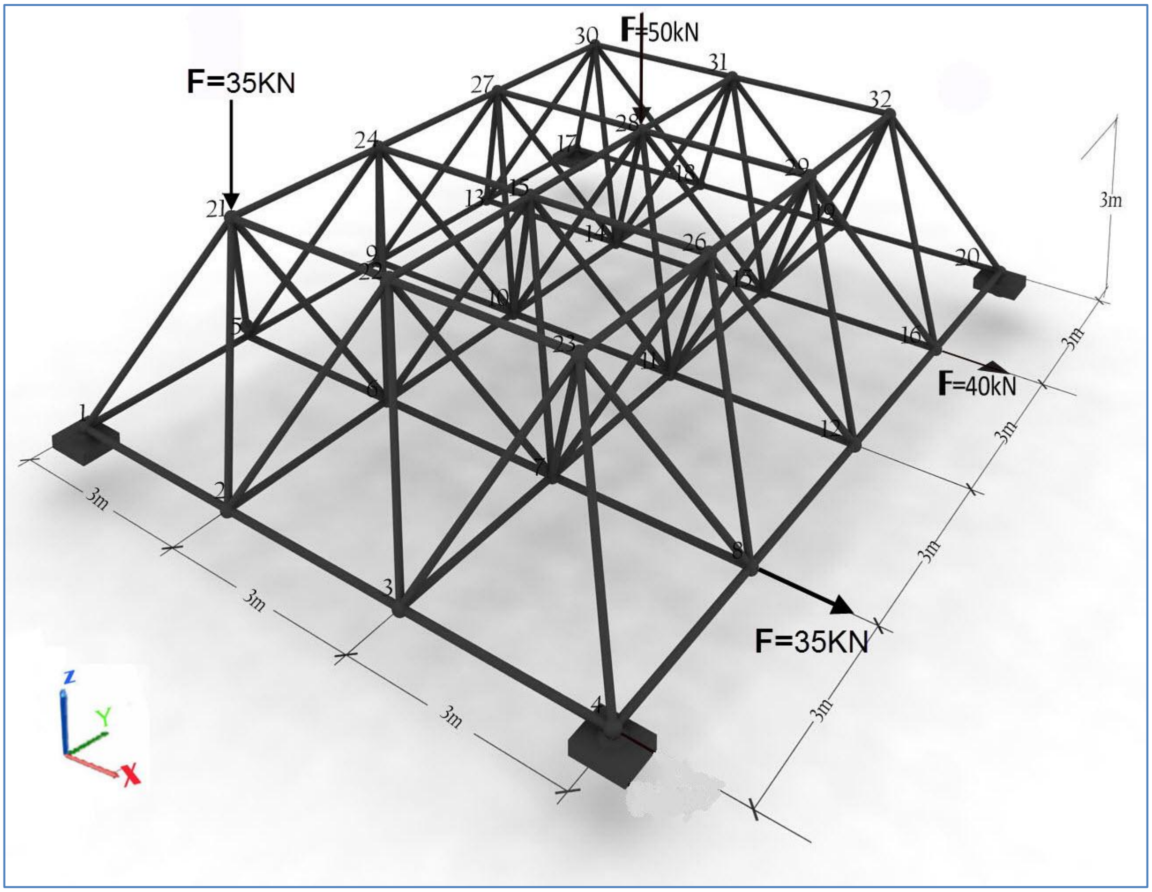

Figure 80. Space Truss of Example 6.

Table 16. Nodal Data of Space Truss of Example 6.

Node Number	Coordinate			Applied Loads			Restraint		
	x	y	z	x	y	z	x	y	z
1	0	0	0	0	0	0	1	1	1
2	3	0	0	0	0	0	0	0	0
3	6	0	0	0	0	0	0	0	0
4	9	0	0	0	0	0	1	1	1
5	0	3	0	0	0	0	0	0	0
6	3	3	0	0	0	0	0	0	0
7	6	3	0	0	0	0	0	0	0
8	9	3	0	35	0	0	0	0	0
9	0	6	0	0	0	0	0	0	0
10	3	6	0	0	0	0	0	0	0
11	6	6	0	0	0	0	0	0	0
12	9	6	0	0	0	0	0	0	0
13	0	9	0	0	0	0	0	0	0
14	3	9	0	0	0	0	0	0	0
15	6	9	0	0	0	0	0	0	0
16	9	9	0	40	0	0	0	0	0
17	0	12	0	0	0	0	1	1	1
18	3	12	0	0	0	0	0	0	0
19	6	12	0	0	0	0	0	0	0
20	9	12	0	0	0	0	1	1	1
21	1.5	1.5	3	0	0	-35	0	0	0
22	4.5	1.5	3	0	0	0	0	0	0
23	7.5	1.5	3	0	0	0	0	0	0
24	1.5	4.5	3	0	0	0	0	0	0
25	4.5	4.5	3	0	0	0	0	0	0
26	7.5	4.5	3	0	0	0	0	0	0
27	1.5	7.5	3	0	0	0	0	0	0
28	4.5	7.5	3	0	0	-50	0	0	0
29	7.5	7.5	3	0	0	0	0	0	0
30	1.5	10.5	3	0	0	0	0	0	0
31	4.5	10.5	3	0	0	0	0	0	0
32	7.5	10.5	3	0	0	0	0	0	0

Table 17. Elemental Data for Example 6

Element Number	Connectivity		Element Number	Connectivity	
	Start Node	End Node		Start Node	End Node
1	1	2	39	6	25
2	2	6	40	7	25
3	5	6	41	11	25
4	1	5	42	10	25
5	1	21	43	11	15
6	2	21	44	14	15
7	6	21	45	10	28
8	5	21	46	11	28
9	6	10	47	15	28
10	9	10	48	14	28
11	5	9	49	15	19
12	5	24	50	18	19
13	6	24	51	14	31
14	10	24	52	15	31
15	9	24	53	19	31
16	10	14	54	18	31
17	13	14	55	3	4
18	9	13	56	4	8
19	9	27	57	7	8
20	13	27	58	3	23
21	14	27	59	4	23
22	10	27	60	8	23
23	14	18	61	7	23
24	17	18	62	8	12
25	13	17	63	11	12
26	13	30	64	7	26
27	14	30	65	8	26
28	18	30	66	12	26
29	17	30	67	11	26
30	2	3	68	12	16
31	3	7	69	15	16
32	6	7	70	11	29
33	2	22	71	12	29
34	3	22	72	16	29
35	7	22	73	15	29
36	6	22	74	16	20

37	7	11	75	19	20
38	10	11	76	15	32
77	16	32	87	22	25
78	20	32	88	25	28
79	19	32	89	28	31
80	21	22	90	22	23
81	24	25	91	25	26
82	27	28	92	28	29
83	30	31	93	31	32
84	21	24	94	23	26
85	24	27	95	26	29
86	27	30	96	29	32

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DISPLACEMENTS

THE COMPUTER CODE

THE NODAL DISPLACEMENTS MATRIX

$$\text{displacements} = \begin{pmatrix} 0.00016496 \\ -5.8138 \times 10^{-6} \\ -0.000669505 \\ 0.000180408 \\ -0.0000137148 \\ -0.000281531 \\ 0.00147637 \\ 0.0000122176 \\ -0.00167606 \\ 0.00147637 \\ -5.8138 \times 10^{-6} \\ -0.000996862 \\ 0.00160374 \\ -0.0000137148 \\ -0.000305443 \\ 0.00186624 \\ -0.0000233667 \\ 0.000748146 \\ 0.00156004 \\ 0.000033646 \\ -0.00184938 \\ 0.00156004 \\ 0.0000436846 \\ -0.00118964 \\ 0.00163202 \\ -6.83407 \times 10^{-6} \\ -0.000398237 \\ 0.00163202 \\ -4.5108 \times 10^{-6} \\ 0.000539577 \\ 0.00140255 \\ 0.0000417727 \\ -0.00147433 \\ 0.00140255 \\ 0.000152077 \\ -0.000963766 \\ 0.00160878 \\ 0.0000556728 \\ -0.000371367 \\ 0.00190878 \\ 0.0000209389 \\ 0.000728973 \\ 0.000138984 \end{pmatrix}$$

Figure 81. Nodal Displacements of Example 6

0.000152077
-0.000461945
0.000165647
0.0000556728
-0.000256213
0.000545936
0.000443441
-0.000978315
0.000423144
0.000108712
-0.000581188
0.000358307
-0.000100958
0.0000329523
0.000814392
0.000161925
-0.00143685
0.000791879
0.0000623683
-0.000754512
0.000756251
-0.0000121613
0.000109981
0.000853973
-0.000142482
-0.00138384
0.000817376
-0.0000225682
-0.000959121
0.000764393
0.000030307
0.0000762418
0.000551122
-0.0003978
-0.000737163
0.000458025
-0.000102556
-0.000542434
0.000380223
0.0000984261
0.0000111715

Figure 82. Nodal Displacements of Example 6 (continued)

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DISPLACEMENTS

THE COMPUTER CODE

THE NODAL DISPLACEMENTS MATRIX

SCATTER PLOT OF DISPLACEMENT MATRIX

```
MatrixPlot[xdisp]
```

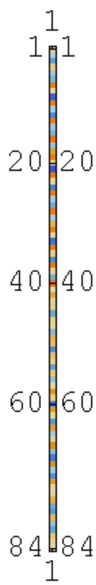

Figure 83. Scatter Plot of Nodal Displacements for Example 6.

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DEFORMATIONS

$$
\begin{pmatrix}
0.00016496 \\
0 \\
0 \\
0.0000122176 \\
-0.00039488 \\
-0.000224268 \\
0.000211582 \\
0.0000138161 \\
0.0000494984 \\
0 \\
0.0000214284 \\
-0.0000138161 \\
-0.0000205208 \\
0.0000542895 \\
-0.0000199525 \\
0.000108393 \\
0 \\
8.12669 \times 10^{-6} \\
0.0000199525 \\
-0.0000748491 \\
1.21659 \times 10^{-6} \\
0.00005368 \\
0 \\
0.000138984 \\
-0.0000417727 \\
0.0000748491 \\
0.000308129 \\
-0.000168482 \\
-0.000214495 \\
0.000015448 \\
0 \\
0.000127374 \\
0.000224268 \\
-0.000293785 \\
0.000206851 \\
-0.000137334 \\
6.88069 \times 10^{-6} \\
0.0000719796 \\
-0.0000537267 \\
-4.1619 \times 10^{-6} \\
0.0000238366
\end{pmatrix}
$$

Figure 84. Deformations of Elements for Example 6.

deformations =	0.000034052 0.0000625068 0.00020623 -0.000142021 -0.000131806 -0.000124867 -0.000163806 0 0.0000266627 -0.00014554 0.000265522 -0.000288465 0.000168482 -0.000180408 -0.0000233667 0.0002625 0.000293785 -0.000160589 0.000063334 -0.00019653 0.000018856 0 -6.15915×10^{-6} -0.000063334 9.89063×10^{-6} 0.0000596026 0.0000254497 0.0003 0.0000483669 -9.89063×10^{-6} -0.0000695829 0.0000311067 -0.0000209389 -0.000165647 -0.000171762 0.0000695829 -0.000186286 0.000288465 -0.000122792	

Figure 85. Deformations of Elements for Example 6 (continued).

-0.0000225124
-0.0000365977
-0.0000930974
-0.000281515
-0.000304407
-0.000255318
-0.0000463441
-0.0000849365
-0.0000799882
-0.0000648366
-0.0000356289
-0.0000529824
-0.0000778021
0.0000887971
0.0000424683
0.0000681191

Figure 86. . Deformations of Elements for Example 6 (continued).

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CALCULATE THE DEGREE OF INDETERMINANCY

di = m + rest − 3 × noden

12

Figure 87. Calculation of Degree of Indeterminacy

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INTERNAL FORCES

$$
\begin{pmatrix}
21.9947 \\
0 \\
0 \\
1.62902 \\
-42.9891 \\
-24.4152 \\
23.0341 \\
1.50411 \\
6.59978 \\
0 \\
2.85712 \\
-1.50411 \\
-2.23402 \\
5.91029 \\
-2.17216 \\
14.4524 \\
0 \\
1.08356 \\
2.17216 \\
-8.14854 \\
0.132446 \\
5.84393 \\
0 \\
18.5312 \\
-5.56969 \\
8.14854 \\
33.5448 \\
-18.342 \\
-23.3513 \\
2.05974 \\
0 \\
16.9832 \\
24.4152 \\
-31.9832 \\
22.5191 \\
-14.9511 \\
0.917426 \\
9.59728 \\
-5.84902 \\
-0.45309 \\
2.595 \\
3.70711 \\
8.33424 \\
27.4973 \\
-15.4613 \\
-14.3492 \\
-13.5938
\end{pmatrix}
$$

Figure 88. Member Forces for Example 6.

Independent forces=	-17.8329 0 3.55503 -15.8443 28.9064 -31.4041 18.342 -24.0544 -3.11557 35. 31.9832 -17.4827 6.89494 -21.3954 2.51413 0 -0.670523 -6.89494 1.07676 6.48871 3.39329 40. 5.26552 -1.07676 -7.57523 3.38647 -2.79185 -22.0863 -18.699 7.57523 -20.2803 31.4041 -16.3723 -3.00166 -4.87969 -12.413 -37.5354 -40.5876 -34.0425 -6.17922 -11.3249 -10.6651 -8.64488 -4.75053 -7.06432 -10.3736 11.8396 5.66244 9.08255

Figure 89. Member Forces for Example 6 (continued).

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INTERNAL FORCES

MEMBER END FORCES

member	1	$\begin{pmatrix} -21.9947 \\ 21.9947 \end{pmatrix}$
member	2	$\begin{pmatrix} -9.93852 \times 10^{-15} \\ 9.93852 \times 10^{-15} \end{pmatrix}$
member	3	$\begin{pmatrix} 0. \\ 0. \end{pmatrix}$
member	4	$\begin{pmatrix} -1.62902 \\ 1.62902 \end{pmatrix}$
member	5	$\begin{pmatrix} 42.9891 \\ -42.9891 \end{pmatrix}$
member	6	$\begin{pmatrix} 24.4152 \\ -24.4152 \end{pmatrix}$
member	7	$\begin{pmatrix} -23.0341 \\ 23.0341 \end{pmatrix}$
member	8	$\begin{pmatrix} -1.50411 \\ 1.50411 \end{pmatrix}$
member	9	$\begin{pmatrix} -6.59978 \\ 6.59978 \end{pmatrix}$
member	10	$\begin{pmatrix} 0. \\ 0. \end{pmatrix}$
member	11	$\begin{pmatrix} -2.85712 \\ 2.85712 \end{pmatrix}$
member	12	$\begin{pmatrix} 1.50411 \\ -1.50411 \end{pmatrix}$
member	13	$\begin{pmatrix} 2.23402 \\ -2.23402 \end{pmatrix}$
member	14	$\begin{pmatrix} -5.91029 \\ 5.91029 \end{pmatrix}$
member	15	$\begin{pmatrix} 2.17216 \\ -2.17216 \end{pmatrix}$
member	16	$\begin{pmatrix} -14.4524 \\ 14.4524 \end{pmatrix}$
member	17	$\begin{pmatrix} -2.89121 \times 10^{-14} \\ 2.89121 \times 10^{-14} \end{pmatrix}$
member	18	$\begin{pmatrix} -1.08356 \\ 1.08356 \end{pmatrix}$
member	19	$\begin{pmatrix} -2.17216 \\ 2.17216 \end{pmatrix}$
member	20	$\begin{pmatrix} 8.14854 \\ -8.14854 \end{pmatrix}$
member	21	$\begin{pmatrix} -0.132446 \\ 0.132446 \end{pmatrix}$
member	22	$\begin{pmatrix} -5.84393 \\ 5.84393 \end{pmatrix}$
member	23	$\begin{pmatrix} -1.0842 \times 10^{-14} \\ 1.0842 \times 10^{-14} \end{pmatrix}$

Figure 90. Member End Forces for Example 6.

member 24 $\begin{pmatrix} -18.5312 \\ 18.5312 \end{pmatrix}$

member 25 $\begin{pmatrix} 5.56969 \\ -5.56969 \end{pmatrix}$

member 26 $\begin{pmatrix} -8.14854 \\ 8.14854 \end{pmatrix}$

member 27 $\begin{pmatrix} -33.5448 \\ 33.5448 \end{pmatrix}$

member 28 $\begin{pmatrix} 18.342 \\ -18.342 \end{pmatrix}$

member 29 $\begin{pmatrix} 23.3513 \\ -23.3513 \end{pmatrix}$

member 30 $\begin{pmatrix} -2.05974 \\ 2.05974 \end{pmatrix}$

member 31 $\begin{pmatrix} 4.06576 \times 10^{-15} \\ -4.06576 \times 10^{-15} \end{pmatrix}$

member 32 $\begin{pmatrix} -16.9832 \\ 16.9832 \end{pmatrix}$

member 33 $\begin{pmatrix} -24.4152 \\ 24.4152 \end{pmatrix}$

member 34 $\begin{pmatrix} 31.9832 \\ -31.9832 \end{pmatrix}$

member 35 $\begin{pmatrix} -22.5191 \\ 22.5191 \end{pmatrix}$

member 36 $\begin{pmatrix} 14.9511 \\ -14.9511 \end{pmatrix}$

member 37 $\begin{pmatrix} -0.917426 \\ 0.917426 \end{pmatrix}$

member 38 $\begin{pmatrix} -9.59728 \\ 9.59728 \end{pmatrix}$

member 39 $\begin{pmatrix} 5.84902 \\ -5.84902 \end{pmatrix}$

member 40 $\begin{pmatrix} 0.45309 \\ -0.45309 \end{pmatrix}$

member 41 $\begin{pmatrix} -2.595 \\ 2.595 \end{pmatrix}$

member 42 $\begin{pmatrix} -3.70711 \\ 3.70711 \end{pmatrix}$

member 43 $\begin{pmatrix} -8.33424 \\ 8.33424 \end{pmatrix}$

member 44 $\begin{pmatrix} -27.4973 \\ 27.4973 \end{pmatrix}$

member 45 $\begin{pmatrix} 15.4613 \\ -15.4613 \end{pmatrix}$

member 46 $\begin{pmatrix} 14.3492 \\ -14.3492 \end{pmatrix}$

member 47 $\begin{pmatrix} 13.5938 \\ -13.5938 \end{pmatrix}$

member 48 $\begin{pmatrix} 17.8329 \\ -17.8329 \end{pmatrix}$

member 49 $\begin{pmatrix} -4.51751 \times 10^{-15} \\ 4.51751 \times 10^{-15} \end{pmatrix}$

member 50 $\begin{pmatrix} -3.55503 \\ 3.55503 \end{pmatrix}$

Figure 91. Member End Forces for Example 6 (continued).

member	51	$\begin{pmatrix} 15.8443 \\ -15.8443 \end{pmatrix}$
member	52	$\begin{pmatrix} -28.9064 \\ 28.9064 \end{pmatrix}$
member	53	$\begin{pmatrix} 31.4041 \\ -31.4041 \end{pmatrix}$
member	54	$\begin{pmatrix} -18.342 \\ 18.342 \end{pmatrix}$
member	55	$\begin{pmatrix} 24.0544 \\ -24.0544 \end{pmatrix}$
member	56	$\begin{pmatrix} 3.11557 \\ -3.11557 \end{pmatrix}$
member	57	$\begin{pmatrix} -35. \\ 35. \end{pmatrix}$
member	58	$\begin{pmatrix} -31.9832 \\ 31.9832 \end{pmatrix}$
member	59	$\begin{pmatrix} 17.4827 \\ -17.4827 \end{pmatrix}$
member	60	$\begin{pmatrix} -6.89494 \\ 6.89494 \end{pmatrix}$
member	61	$\begin{pmatrix} 21.3954 \\ -21.3954 \end{pmatrix}$
member	62	$\begin{pmatrix} -2.51413 \\ 2.51413 \end{pmatrix}$
member	63	$\begin{pmatrix} 2.89121 \times 10^{-14} \\ -2.89121 \times 10^{-14} \end{pmatrix}$
member	64	$\begin{pmatrix} 0.670523 \\ -0.670523 \end{pmatrix}$
member	65	$\begin{pmatrix} 6.89494 \\ -6.89494 \end{pmatrix}$
member	66	$\begin{pmatrix} -1.07676 \\ 1.07676 \end{pmatrix}$
member	67	$\begin{pmatrix} -6.48871 \\ 6.48871 \end{pmatrix}$
member	68	$\begin{pmatrix} -3.39329 \\ 3.39329 \end{pmatrix}$
member	69	$\begin{pmatrix} -40. \\ 40. \end{pmatrix}$
member	70	$\begin{pmatrix} -5.26552 \\ 5.26552 \end{pmatrix}$
member	71	$\begin{pmatrix} 1.07676 \\ -1.07676 \end{pmatrix}$
member	72	$\begin{pmatrix} 7.57523 \\ -7.57523 \end{pmatrix}$
member	73	$\begin{pmatrix} -3.38647 \\ 3.38647 \end{pmatrix}$

Figure 92. Member End Forces for Example 6 (continued).

member	74	$\begin{pmatrix} 2.79185 \\ -2.79185 \end{pmatrix}$
member	75	$\begin{pmatrix} 22.0863 \\ -22.0863 \end{pmatrix}$
member	76	$\begin{pmatrix} 18.699 \\ -18.699 \end{pmatrix}$
member	77	$\begin{pmatrix} -7.57523 \\ 7.57523 \end{pmatrix}$
member	78	$\begin{pmatrix} 20.2803 \\ -20.2803 \end{pmatrix}$
member	79	$\begin{pmatrix} -31.4041 \\ 31.4041 \end{pmatrix}$
member	80	$\begin{pmatrix} 16.3723 \\ -16.3723 \end{pmatrix}$
member	81	$\begin{pmatrix} 3.00166 \\ -3.00166 \end{pmatrix}$
member	82	$\begin{pmatrix} 4.87969 \\ -4.87969 \end{pmatrix}$
member	83	$\begin{pmatrix} 12.413 \\ -12.413 \end{pmatrix}$
member	84	$\begin{pmatrix} 37.5354 \\ -37.5354 \end{pmatrix}$
member	85	$\begin{pmatrix} 40.5876 \\ -40.5876 \end{pmatrix}$
member	86	$\begin{pmatrix} 34.0425 \\ -34.0425 \end{pmatrix}$
member	87	$\begin{pmatrix} 6.17922 \\ -6.17922 \end{pmatrix}$
member	88	$\begin{pmatrix} 11.3249 \\ -11.3249 \end{pmatrix}$
member	89	$\begin{pmatrix} 10.6651 \\ -10.6651 \end{pmatrix}$
member	90	$\begin{pmatrix} 8.64488 \\ -8.64488 \end{pmatrix}$
member	91	$\begin{pmatrix} 4.75053 \\ -4.75053 \end{pmatrix}$
member	92	$\begin{pmatrix} 7.06432 \\ -7.06432 \end{pmatrix}$
member	93	$\begin{pmatrix} 10.3736 \\ -10.3736 \end{pmatrix}$
member	94	$\begin{pmatrix} -11.8396 \\ 11.8396 \end{pmatrix}$
member	95	$\begin{pmatrix} -5.66244 \\ 5.66244 \end{pmatrix}$
member	96	$\begin{pmatrix} -9.08255 \\ 9.08255 \end{pmatrix}$

Figure 93. Member End Forces for Example 6 (continued).

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SUPPORT REACTIONS

THE COMPUTER CODE

SUPPORT REACTIONS MATRIX

$$\text{Reactions} = \begin{pmatrix} -4.44449 \\ 15.9212 \\ 35.1004 \\ -31.1917 \\ 10.2529 \\ 14.2746 \\ -8.99813 \\ -15.1028 \\ 19.0662 \\ -30.3657 \\ -11.0712 \\ 16.5588 \end{pmatrix}$$

SCATTER PLOT OF REACTIONS MATRIX

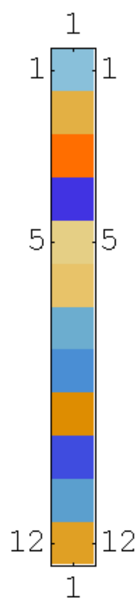

Figure 94. Support Reactions of Example 6 and Its Scatter Plot.

6.13 Mastan Results for Example 6

```
***********   MASTAN2 v3.3.1   ***********

Time:  10:37:17      Date:  05/25/2012

Problem Title:   not provided
**************

##############################
Results of Structural Analysis
##############################

General Information:
      Structure Analyzed as:  Space Truss
      Analysis Type:  First-Order Elastic

Analytical Results:

(i)   Displacements at Step # 1, Applied Load Ratio = 1.0000

  Deflections
     Node       X-disp          Y-disp          Z-disp
       1     0.0000e+000     0.0000e+000     0.0000e+000
       2     1.6496e-004    -5.8138e-006    -6.6950e-004
       3     1.8041e-004    -1.3715e-005    -2.8153e-004
       4     0.0000e+000     0.0000e+000     0.0000e+000
       5     1.4764e-003     1.2218e-005    -1.6761e-003
       6     1.4764e-003    -5.8138e-006    -9.9686e-004
       7     1.6037e-003    -1.3715e-005    -3.0544e-004
       8     1.8662e-003    -2.3367e-005     7.4815e-004
       9     1.5600e-003     3.3646e-005    -1.8494e-003
      10     1.5600e-003     4.3685e-005    -1.1896e-003
      11     1.6320e-003    -6.8341e-006    -3.9824e-004
      12     1.6320e-003    -4.5108e-006     5.3958e-004
      13     1.4026e-003     4.1773e-005    -1.4743e-003
      14     1.4026e-003     1.5208e-004    -9.6377e-004
      15     1.6088e-003     5.5673e-005    -3.7137e-004
      16     1.9088e-003     2.0939e-005     7.2897e-004
      17     0.0000e+000     0.0000e+000     0.0000e+000
      18     1.3898e-004     1.5208e-004    -4.6195e-004
      19     1.6565e-004     5.5673e-005    -2.5621e-004
      20     0.0000e+000     0.0000e+000     0.0000e+000
      21     5.4594e-004     4.4344e-004    -9.7832e-004
      22     4.2314e-004     1.0871e-004    -5.8119e-004
      23     3.5831e-004    -1.0096e-004     3.2952e-005
      24     8.1439e-004     1.6193e-004    -1.4369e-003
      25     7.9188e-004     6.2368e-005    -7.5451e-004
      26     7.5625e-004    -1.2161e-005     1.0998e-004
      27     8.5397e-004    -1.4248e-004    -1.3838e-003
      28     8.1738e-004    -2.2568e-005    -9.5912e-004
      29     7.6439e-004     3.0307e-005     7.6242e-005
      30     5.5112e-004    -3.9780e-004    -7.3716e-004
      31     4.5802e-004    -1.0256e-004    -5.4243e-004
```

32	3.8022e-004	9.8426e-005	1.1171e-005

(ii) Element Results at Step # 1, Applied Load Ratio = 1.0000

Internal End Forces (Note: Refers to local coordinates)

Element	Node	Fx	Fy	Fz
1	1	-2.1995e+001	0.0000e+000	0.0000e+000
	2	2.1995e+001	0.0000e+000	0.0000e+000
2	2	-6.6613e-016	0.0000e+000	0.0000e+000
	6	6.6613e-016	0.0000e+000	0.0000e+000
3	5	2.8422e-014	0.0000e+000	0.0000e+000
	6	-2.8422e-014	0.0000e+000	0.0000e+000
4	1	-1.6290e+000	0.0000e+000	0.0000e+000
	5	1.6290e+000	0.0000e+000	0.0000e+000
5	1	4.2989e+001	0.0000e+000	0.0000e+000
	21	-4.2989e+001	0.0000e+000	0.0000e+000
6	2	2.4415e+001	0.0000e+000	0.0000e+000
	21	-2.4415e+001	0.0000e+000	0.0000e+000
7	6	-2.3034e+001	0.0000e+000	0.0000e+000
	21	2.3034e+001	0.0000e+000	0.0000e+000
8	5	-1.5041e+000	0.0000e+000	0.0000e+000
	21	1.5041e+000	0.0000e+000	0.0000e+000
9	6	-6.5998e+000	0.0000e+000	0.0000e+000
	10	6.5998e+000	0.0000e+000	0.0000e+000
10	9	2.8422e-014	0.0000e+000	0.0000e+000
	10	-2.8422e-014	0.0000e+000	0.0000e+000
11	5	-2.8571e+000	0.0000e+000	0.0000e+000
	9	2.8571e+000	0.0000e+000	0.0000e+000
12	5	1.5041e+000	0.0000e+000	0.0000e+000
	24	-1.5041e+000	0.0000e+000	0.0000e+000
13	6	2.2340e+000	0.0000e+000	0.0000e+000
	24	-2.2340e+000	0.0000e+000	0.0000e+000
14	10	-5.9103e+000	0.0000e+000	0.0000e+000
	24	5.9103e+000	0.0000e+000	0.0000e+000
15	9	2.1722e+000	0.0000e+000	0.0000e+000
	24	-2.1722e+000	0.0000e+000	0.0000e+000
16	10	-1.4452e+001	0.0000e+000	0.0000e+000
	14	1.4452e+001	0.0000e+000	0.0000e+000
17	13	0.0000e+000	0.0000e+000	0.0000e+000
	14	0.0000e+000	0.0000e+000	0.0000e+000
18	9	-1.0836e+000	0.0000e+000	0.0000e+000
	13	1.0836e+000	0.0000e+000	0.0000e+000
19	9	-2.1722e+000	0.0000e+000	0.0000e+000
	27	2.1722e+000	0.0000e+000	0.0000e+000
20	13	8.1485e+000	0.0000e+000	0.0000e+000
	27	-8.1485e+000	0.0000e+000	0.0000e+000
21	14	-1.3245e-001	0.0000e+000	0.0000e+000
	27	1.3245e-001	0.0000e+000	0.0000e+000
22	10	-5.8439e+000	0.0000e+000	0.0000e+000
	27	5.8439e+000	0.0000e+000	0.0000e+000
23	14	3.5527e-015	0.0000e+000	0.0000e+000
	18	-3.5527e-015	0.0000e+000	0.0000e+000
24	17	-1.8531e+001	0.0000e+000	0.0000e+000
	18	1.8531e+001	0.0000e+000	0.0000e+000

25	13	5.5697e+000	0.0000e+000	0.0000e+000
	17	-5.5697e+000	0.0000e+000	0.0000e+000
26	13	-8.1485e+000	0.0000e+000	0.0000e+000
	30	8.1485e+000	0.0000e+000	0.0000e+000
27	14	-3.3545e+001	0.0000e+000	0.0000e+000
	30	3.3545e+001	0.0000e+000	0.0000e+000
28	18	1.8342e+001	0.0000e+000	0.0000e+000
	30	-1.8342e+001	0.0000e+000	0.0000e+000
29	17	2.3351e+001	0.0000e+000	0.0000e+000
	30	-2.3351e+001	0.0000e+000	0.0000e+000
30	2	-2.0597e+000	0.0000e+000	0.0000e+000
	3	2.0597e+000	0.0000e+000	0.0000e+000
31	3	4.6629e-015	0.0000e+000	0.0000e+000
	7	-4.6629e-015	0.0000e+000	0.0000e+000
32	6	-1.6983e+001	0.0000e+000	0.0000e+000
	7	1.6983e+001	0.0000e+000	0.0000e+000
33	2	-2.4415e+001	0.0000e+000	0.0000e+000
	22	2.4415e+001	0.0000e+000	0.0000e+000
34	3	3.1983e+001	0.0000e+000	0.0000e+000
	22	-3.1983e+001	0.0000e+000	0.0000e+000
35	7	-2.2519e+001	0.0000e+000	0.0000e+000
	22	2.2519e+001	0.0000e+000	0.0000e+000
36	6	1.4951e+001	0.0000e+000	0.0000e+000
	22	-1.4951e+001	0.0000e+000	0.0000e+000
37	7	-9.1743e-001	0.0000e+000	0.0000e+000
	11	9.1743e-001	0.0000e+000	0.0000e+000
38	10	-9.5973e+000	0.0000e+000	0.0000e+000
	11	9.5973e+000	0.0000e+000	0.0000e+000
39	6	5.8490e+000	0.0000e+000	0.0000e+000
	25	-5.8490e+000	0.0000e+000	0.0000e+000
40	7	4.5309e-001	0.0000e+000	0.0000e+000
	25	-4.5309e-001	0.0000e+000	0.0000e+000
41	11	-2.5950e+000	0.0000e+000	0.0000e+000
	25	2.5950e+000	0.0000e+000	0.0000e+000
42	10	-3.7071e+000	0.0000e+000	0.0000e+000
	25	3.7071e+000	0.0000e+000	0.0000e+000
43	11	-8.3342e+000	0.0000e+000	0.0000e+000
	15	8.3342e+000	0.0000e+000	0.0000e+000
44	14	-2.7497e+001	0.0000e+000	0.0000e+000
	15	2.7497e+001	0.0000e+000	0.0000e+000
45	10	1.5461e+001	0.0000e+000	0.0000e+000
	28	-1.5461e+001	0.0000e+000	0.0000e+000
46	11	1.4349e+001	0.0000e+000	0.0000e+000
	28	-1.4349e+001	0.0000e+000	0.0000e+000
47	15	1.3594e+001	0.0000e+000	0.0000e+000
	28	-1.3594e+001	0.0000e+000	0.0000e+000
48	14	1.7833e+001	0.0000e+000	0.0000e+000
	28	-1.7833e+001	0.0000e+000	0.0000e+000
49	15	7.1054e-015	0.0000e+000	0.0000e+000
	19	-7.1054e-015	0.0000e+000	0.0000e+000
50	18	-3.5550e+000	0.0000e+000	0.0000e+000
	19	3.5550e+000	0.0000e+000	0.0000e+000
51	14	1.5844e+001	0.0000e+000	0.0000e+000
	31	-1.5844e+001	0.0000e+000	0.0000e+000

52	15	-2.8906e+001	0.0000e+000	0.0000e+000
	31	2.8906e+001	0.0000e+000	0.0000e+000
53	19	3.1404e+001	0.0000e+000	0.0000e+000
	31	-3.1404e+001	0.0000e+000	0.0000e+000
54	18	-1.8342e+001	0.0000e+000	0.0000e+000
	31	1.8342e+001	0.0000e+000	0.0000e+000
55	3	2.4054e+001	0.0000e+000	0.0000e+000
	4	-2.4054e+001	0.0000e+000	0.0000e+000
56	4	3.1156e+000	0.0000e+000	0.0000e+000
	8	-3.1156e+000	0.0000e+000	0.0000e+000
57	7	-3.5000e+001	0.0000e+000	0.0000e+000
	8	3.5000e+001	0.0000e+000	0.0000e+000
58	3	-3.1983e+001	0.0000e+000	0.0000e+000
	23	3.1983e+001	0.0000e+000	0.0000e+000
59	4	1.7483e+001	0.0000e+000	0.0000e+000
	23	-1.7483e+001	0.0000e+000	0.0000e+000
60	8	-6.8949e+000	0.0000e+000	0.0000e+000
	23	6.8949e+000	0.0000e+000	0.0000e+000
61	7	2.1395e+001	0.0000e+000	0.0000e+000
	23	-2.1395e+001	0.0000e+000	0.0000e+000
62	8	-2.5141e+000	0.0000e+000	0.0000e+000
	12	2.5141e+000	0.0000e+000	0.0000e+000
63	11	2.8422e-014	0.0000e+000	0.0000e+000
	12	-2.8422e-014	0.0000e+000	0.0000e+000
64	7	6.7052e-001	0.0000e+000	0.0000e+000
	26	-6.7052e-001	0.0000e+000	0.0000e+000
65	8	6.8949e+000	0.0000e+000	0.0000e+000
	26	-6.8949e+000	0.0000e+000	0.0000e+000
66	12	-1.0768e+000	0.0000e+000	0.0000e+000
	26	1.0768e+000	0.0000e+000	0.0000e+000
67	11	-6.4887e+000	0.0000e+000	0.0000e+000
	26	6.4887e+000	0.0000e+000	0.0000e+000
68	12	-3.3933e+000	0.0000e+000	0.0000e+000
	16	3.3933e+000	0.0000e+000	0.0000e+000
69	15	-4.0000e+001	0.0000e+000	0.0000e+000
	16	4.0000e+001	0.0000e+000	0.0000e+000
70	11	-5.2655e+000	0.0000e+000	0.0000e+000
	29	5.2655e+000	0.0000e+000	0.0000e+000
71	12	1.0768e+000	0.0000e+000	0.0000e+000
	29	-1.0768e+000	0.0000e+000	0.0000e+000
72	16	7.5752e+000	0.0000e+000	0.0000e+000
	29	-7.5752e+000	0.0000e+000	0.0000e+000
73	15	-3.3865e+000	0.0000e+000	0.0000e+000
	29	3.3865e+000	0.0000e+000	0.0000e+000
74	16	2.7919e+000	0.0000e+000	0.0000e+000
	20	-2.7919e+000	0.0000e+000	0.0000e+000
75	19	2.2086e+001	0.0000e+000	0.0000e+000
	20	-2.2086e+001	0.0000e+000	0.0000e+000
76	15	1.8699e+001	0.0000e+000	0.0000e+000
	32	-1.8699e+001	0.0000e+000	0.0000e+000
77	16	-7.5752e+000	0.0000e+000	0.0000e+000
	32	7.5752e+000	0.0000e+000	0.0000e+000
78	20	2.0280e+001	0.0000e+000	0.0000e+000
	32	-2.0280e+001	0.0000e+000	0.0000e+000

```
    79      19   -3.1404e+001    0.0000e+000    0.0000e+000
            32    3.1404e+001    0.0000e+000    0.0000e+000
    80      21    1.6372e+001    0.0000e+000    0.0000e+000
            22   -1.6372e+001    0.0000e+000    0.0000e+000
    81      24    3.0017e+000    0.0000e+000    0.0000e+000
            25   -3.0017e+000    0.0000e+000    0.0000e+000
    82      27    4.8797e+000    0.0000e+000    0.0000e+000
            28   -4.8797e+000    0.0000e+000    0.0000e+000
    83      30    1.2413e+001    0.0000e+000    0.0000e+000
            31   -1.2413e+001    0.0000e+000    0.0000e+000
    84      21    3.7535e+001    0.0000e+000    0.0000e+000
            24   -3.7535e+001    0.0000e+000    0.0000e+000
    85      24    4.0588e+001    0.0000e+000    0.0000e+000
            27   -4.0588e+001    0.0000e+000    0.0000e+000
    86      27    3.4042e+001    0.0000e+000    0.0000e+000
            30   -3.4042e+001    0.0000e+000    0.0000e+000
    87      22    6.1792e+000    0.0000e+000    0.0000e+000
            25   -6.1792e+000    0.0000e+000    0.0000e+000
    88      25    1.1325e+001    0.0000e+000    0.0000e+000
            28   -1.1325e+001    0.0000e+000    0.0000e+000
    89      28    1.0665e+001    0.0000e+000    0.0000e+000
            31   -1.0665e+001    0.0000e+000    0.0000e+000
    90      22    8.6449e+000    0.0000e+000    0.0000e+000
            23   -8.6449e+000    0.0000e+000    0.0000e+000
    91      25    4.7505e+000    0.0000e+000    0.0000e+000
            26   -4.7505e+000    0.0000e+000    0.0000e+000
    92      28    7.0643e+000    0.0000e+000    0.0000e+000
            29   -7.0643e+000    0.0000e+000    0.0000e+000
    93      31    1.0374e+001    0.0000e+000    0.0000e+000
            32   -1.0374e+001    0.0000e+000    0.0000e+000
    94      23   -1.1840e+001    0.0000e+000    0.0000e+000
            26    1.1840e+001    0.0000e+000    0.0000e+000
    95      26   -5.6624e+000    0.0000e+000    0.0000e+000
            29    5.6624e+000    0.0000e+000    0.0000e+000
    96      29   -9.0825e+000    0.0000e+000    0.0000e+000
            32    9.0825e+000    0.0000e+000    0.0000e+000

(iii) Reactions at Step # 1, Applied Load Ratio = 1.0000

  Forces
     Node          Rx               Ry               Rz
       1      -4.4445e+000     1.5921e+001     3.5100e+001
       4      -3.1192e+001     1.0253e+001     1.4275e+001
      17      -8.9981e+000    -1.5103e+001     1.9066e+001
      20      -3.0366e+001    -1.1071e+001     1.6559e+001

  Moments
     Node          Mx               My               Mz
             *** No Reaction Moments Exist ***

#####################################
End of Results of Structural Analysis
#####################################
```

Chapter 7

CONCLUSION

7.1 Summary of the Work

The main aim of this study is to show usage of the integrated force method and dual integrated force method by generation of equilibrium equation in space truss analysis. Therefore, a computer code is written to generate the equilibrium equations automatically as explained in Chapters 2 and 4. This assembled equilibrium equation is used in both integrated force method and dual integrated force method to analyze indeterminate space truss structures.

Dual integrated force method, subset of displacement method, is used to calculate nodal displacement as primary unknowns, and then the global stiffness matrix is used. Integrated force method is subset of force method in which two approaches are used:

- Null Space of Equilibrium Equation
- Singular Value Decomposition of Equilibrium Equation.

Therefore three computer codes are written to analyze the indeterminate space truss structures as presented in chapter 5. The written programs have further usages for:

- Educational aims: all of the steps in the analysis process are illustrated by outputs and results at each section of solution

- Practical aims: since the written programs are for space truss structures, they can be used in practical analysis.

7.2 Summary of Contributions

Written computer codes are used to:

- Write and assemble of equilibrium equations
- Obtain flexibility matrix and compatibility condition
- Assemble [S] matrix by coupling EE matrix and CC matrix
- Generate global stiffness matrix
- Analyze the space truss with three different methods

The written computer codes have following characteristics and advantages:

Characteristics:

- Flexibility of data input procedure.
- Flexibility of output procedure.
- Analysis packages are easy to run and there is no need to read any manual.
- Subroutines operation for scatter plot.
- The used relations and formulas are expressed at each section of analysis.

Advantages:

- Any indeterminate space truss with any number of nodes and elements can be analyzed by the written computer codes
- The written computer codes are perceptible step by step and easy to use
- The analysis procedure of space truss structures are very hard and time consuming, then these computer codes make the analysis time shorter and faster.

7.3 Recommendations for Future Researches

- By addition of related computer codes, the nodal settlements and thermal effects can be considered.
- These computer codes can be developed to analyze the space frame structures.
- Incorporate analysis programs for frame and beam can be added to obtain overall structural analysis package

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APPENDICES

Appendix A: Data Input and Calculation Phase for Example 1

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INPUT DATA

NUMBER OF NODES

```
noden = 6;
```

X-Y-Z COORDINATE

cord =

1	0.	0.	0.
2	0.	2.	0.
3	0.	1.	2.
4	3.	0.	0.
5	3.	2.	0.
6	3.	1.	2.

;

ELEMENT CONECTIVITY

```
m = 12;
```

inc =

1	1	2
2	1	3
3	2	3
4	1	4
5	1	5
6	2	5
7	2	6
8	3	6
9	3	4
10	5	6
11	4	6
12	4	5

;

Figure 95. Input Phase; Data for Example 1

RESTRAINT TABLE

0=free and 1=restrained

freet =

1	1	1	1
2	1	0	1
3	1	1	1
4	0	0	0
5	0	0	0
6	0	0	0

;

APPLIED FORCES

applfrcs =

1	0.	0.	0.
2	0.	0.	0.
3	0.	0.	0.
4	0.	0.	-45.
5	0.	0.	-30.
6	0.	0.	0.

;

MODULUS OF ELASTICITY AND CROSS-SECTION AREA

Ee = $2.\times 10^{8}$;

```
A = {0.003, 0.003, 0.003, 0.003, 0.003, 0.003, 0.003, 0.003,
   0.003, 0.003, 0.003, 0.003};
```

Figure 96. Input Phase; Data for Example 1(continued)

3 of 12

EQUILIBRIUM EQUATIONS

THE COMPUTER CODE

THE EQUILIBRIUM MATRIX

```
MatrixForm[S]
```

1.	0.	0.447214	0.	0.	0.	0.267261	0.	0.	0.	0.	0.
0.	0.	0.	1.	0.	0.	0.	0.	0.801784	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	-0.267261	0.	-0.447214	-1.
0.	0.	0.	0.	0.	0.	0.	0.	-0.534522	0.	-0.894427	0.
0.	0.	0.	0.	0.83205	1.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.5547	0.	0.	0.	0.	0.447214	0.	1.
0.	0.	0.	0.	0.	0.	0.	0.	0.	-0.894427	0.	0.
0.	0.	0.	0.	0.	0.	0.801784	1.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	-0.267261	0.	0.	-0.447214	0.447214	0.
0.	0.	0.	0.	0.	0.	0.534522	0.	0.	0.894427	0.894427	0.

Figure 97. Calculation Phase; Equilibrium Equations Matrix for Example 1

THE SCATTER PLOT OF EQUILIBRIUM MATRIX

```
MatrixPlot[S]
```

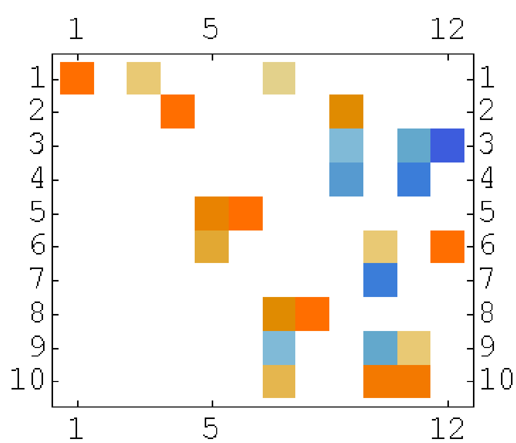

Figure 98. Scatter Plot of Equilibrium Equations Matrix for Example 1

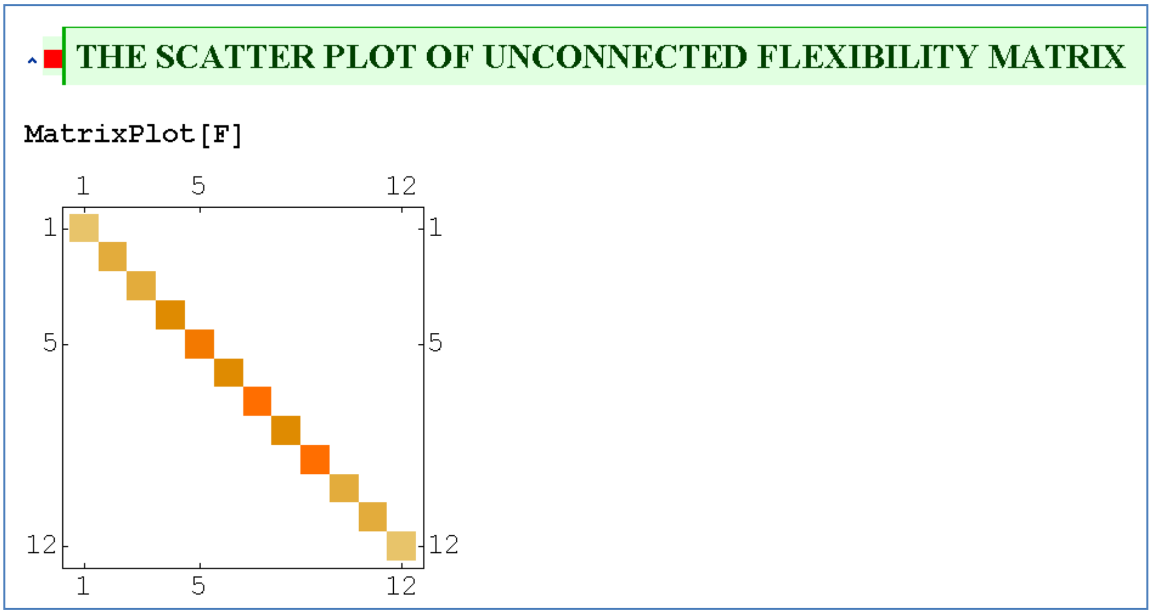

Figure 99. Scatter Plot of Flexibility Matrix

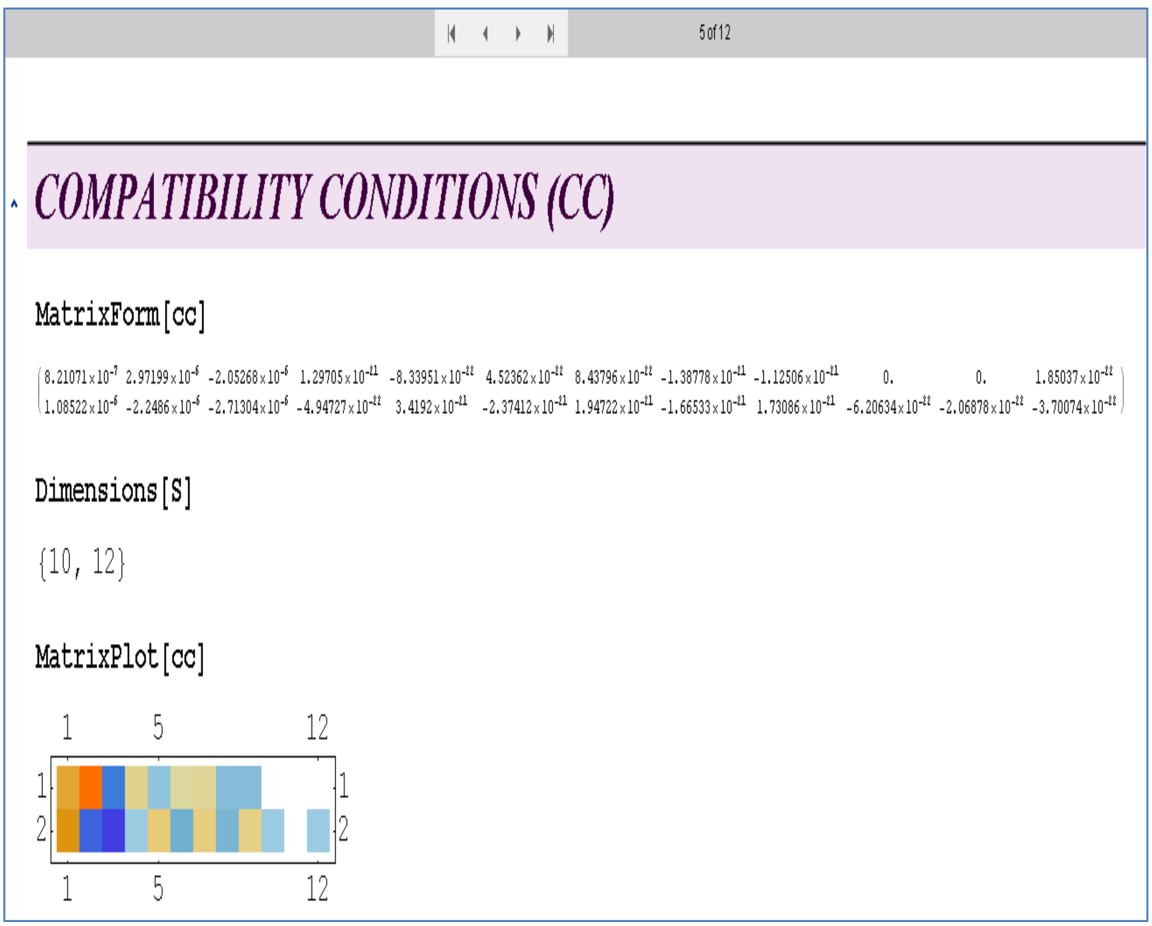

Figure 100. Compatibility Conditions for Example 1

THE SCATTER PLOT OF THE IFM MATRIX

```
MatrixPlot[ifm]
```

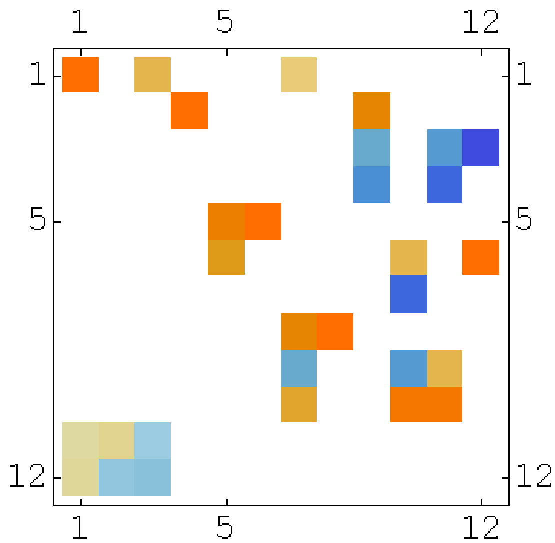

Figure 101. Scatter Plot of IFM Matrix for Example 1

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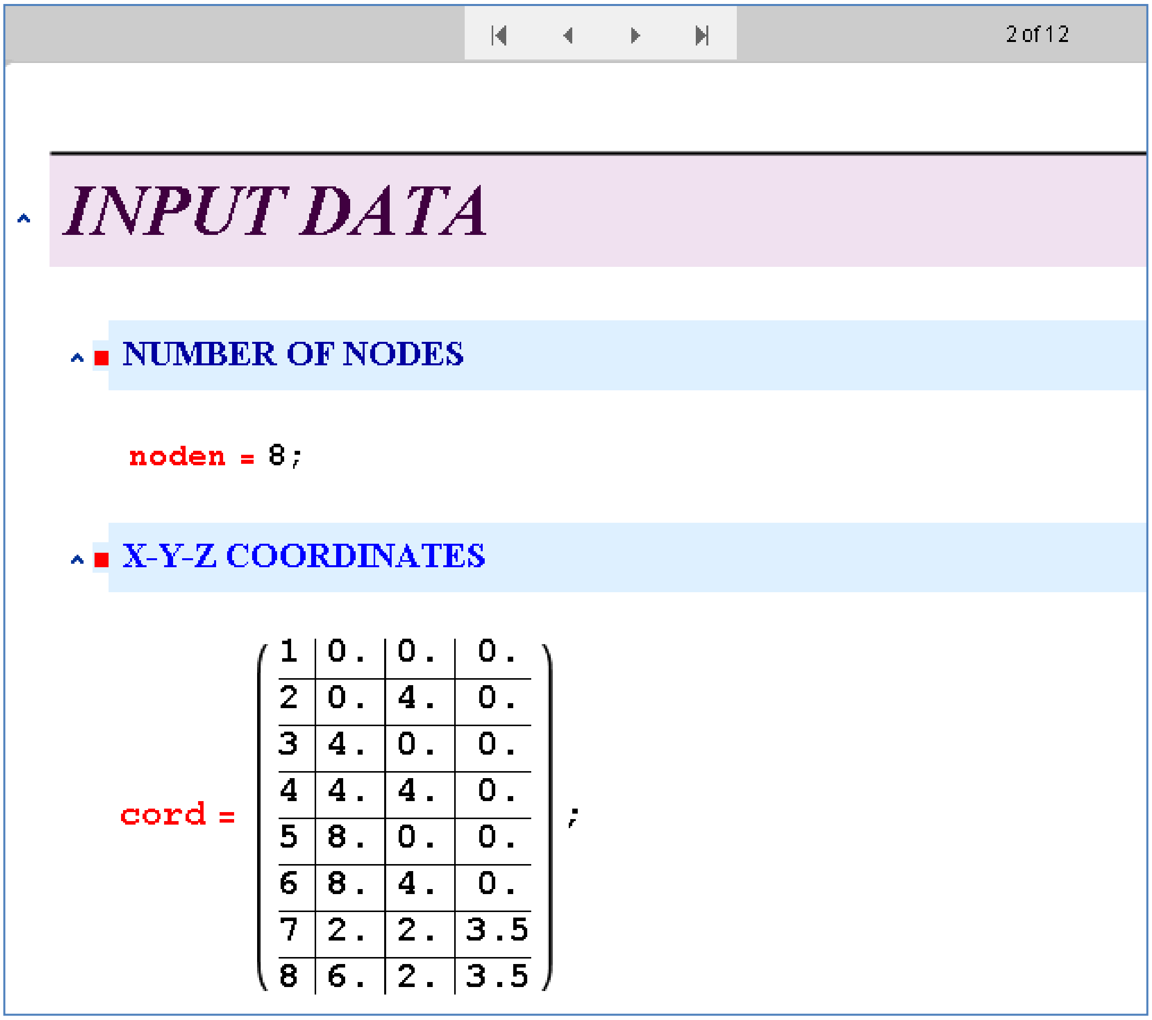
CACULATE DEGREE OF INDETERMINANCY

```
di = m + rest - 3 × noden
2
```

Figure 102. Calculation Phase; Degree of Indeterminacy for Example 1

Appendix B: Data Input and Calculation Phase for Example 2


2 of 12

INPUT DATA

NUMBER OF NODES

```
noden = 8;
```

X-Y-Z COORDINATES

cord =

1	0.	0.	0.
2	0.	4.	0.
3	4.	0.	0.
4	4.	4.	0.
5	8.	0.	0.
6	8.	4.	0.
7	2.	2.	3.5
8	6.	2.	3.5

;

Figure 103. Input Phase; Data for Example 2

INPUT DATA

NUMBER OF NODES

X-Y-Z COORDINATES

ELEMENT CONNECTIVITY

```
m = 18;
```

inc =

1	1	2
2	2	4
3	3	4
4	1	3
5	1	4
6	4	6
7	5	6
8	3	5
9	4	5
10	1	7
11	2	7
12	3	7
13	4	7
14	3	8
15	4	8
16	6	8
17	5	8
18	7	8

;

RESTRAINT TABLE

0=free and 1=restrained

freet =

1	1	1	1
2	1	1	1
3	0	0	0
4	0	0	0
5	1	1	1
6	1	1	1
7	0	0	0
8	0	0	0

;

Figure 104. Input Phase; Data for Example 2(continued)

INPUT DATA

NUMBER OF NODES

X-Y-Z COORDINATES

ELEMENT CONNECTIVITY

RESTRAINT TABLE

APPLIED FORCES

applfrcs =

1	0.	0.	0.
2	0.	0.	0.
3	0.	0.	0.
4	0.	0.	0.
5	0.	0.	0.
6	0.	0.	0.
7	0.	0.	-45.
8	0.	0.	-35.

;

MODULLUS OF ELASTICITY AND CROSS-SECTION AREA

$Ee = 2. \times 10^{8};$

```
A = {0.002, 0.002, 0.002, 0.002, 0.002, 0.002,
    0.002, 0.002, 0.002, 0.002, 0.002, 0.002, 0.002,
    0.002, 0.002, 0.002, 0.002, 0.002};
```

Figure 105. Input Phase; Data for Example 2(continued)

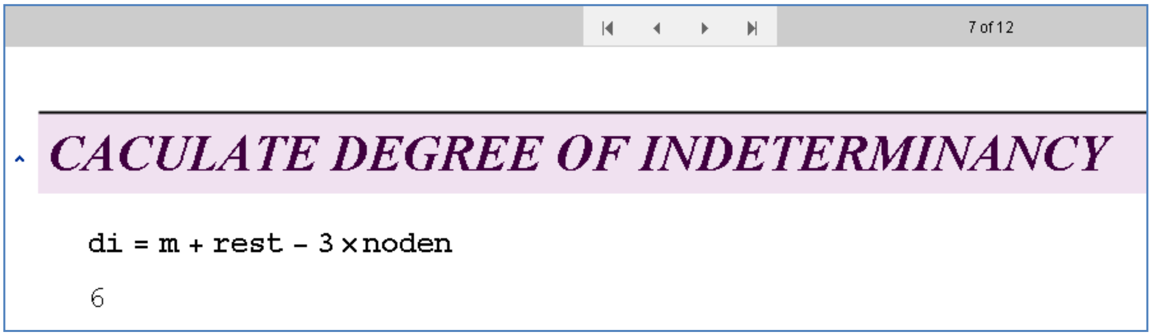


Figure 106. Calculation Phase; Degree of Indeterminacy

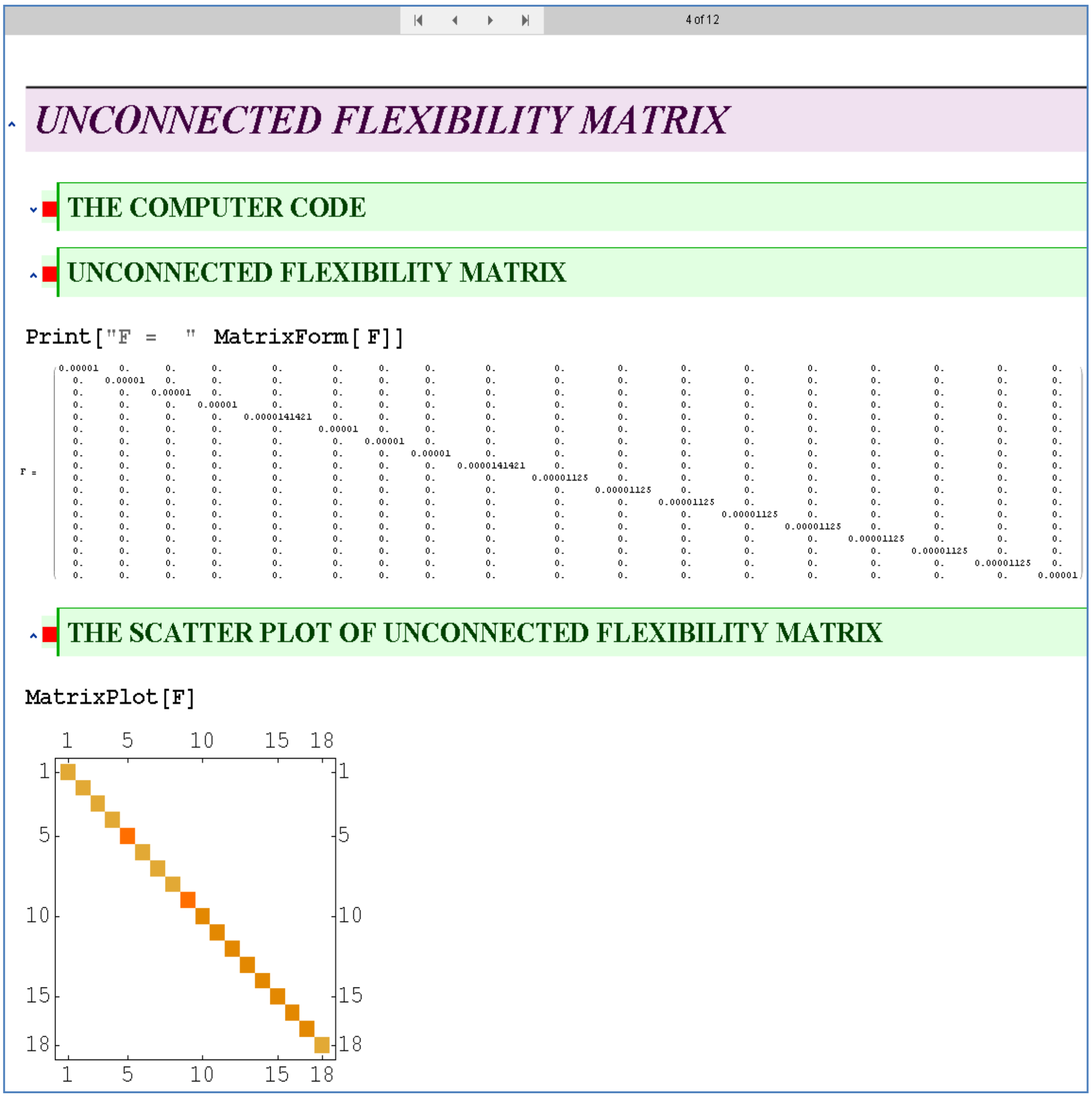


Figure 107. Calculation Phase; Flexibility Matrix and Scatter Plot for Example 2

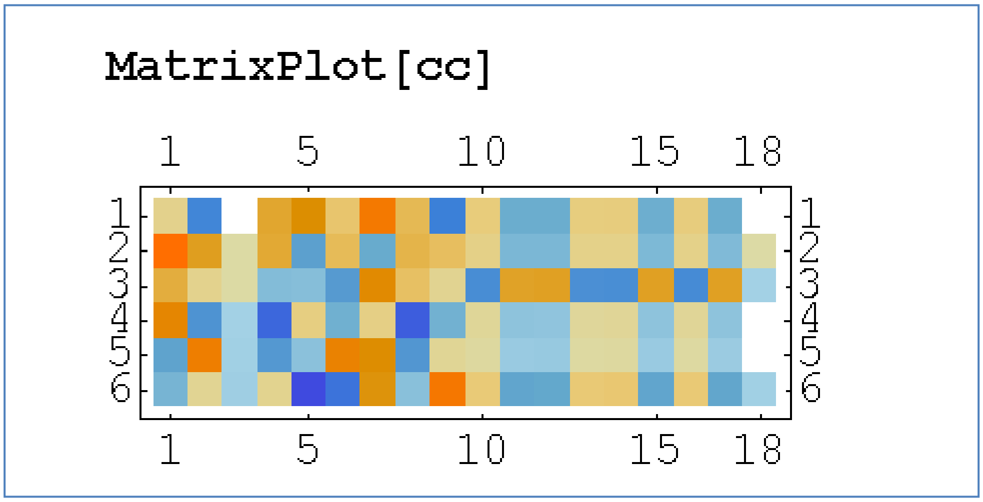


Figure 108. Calculation Phase; Scatter Plot of Compatibility Condition for Example 2

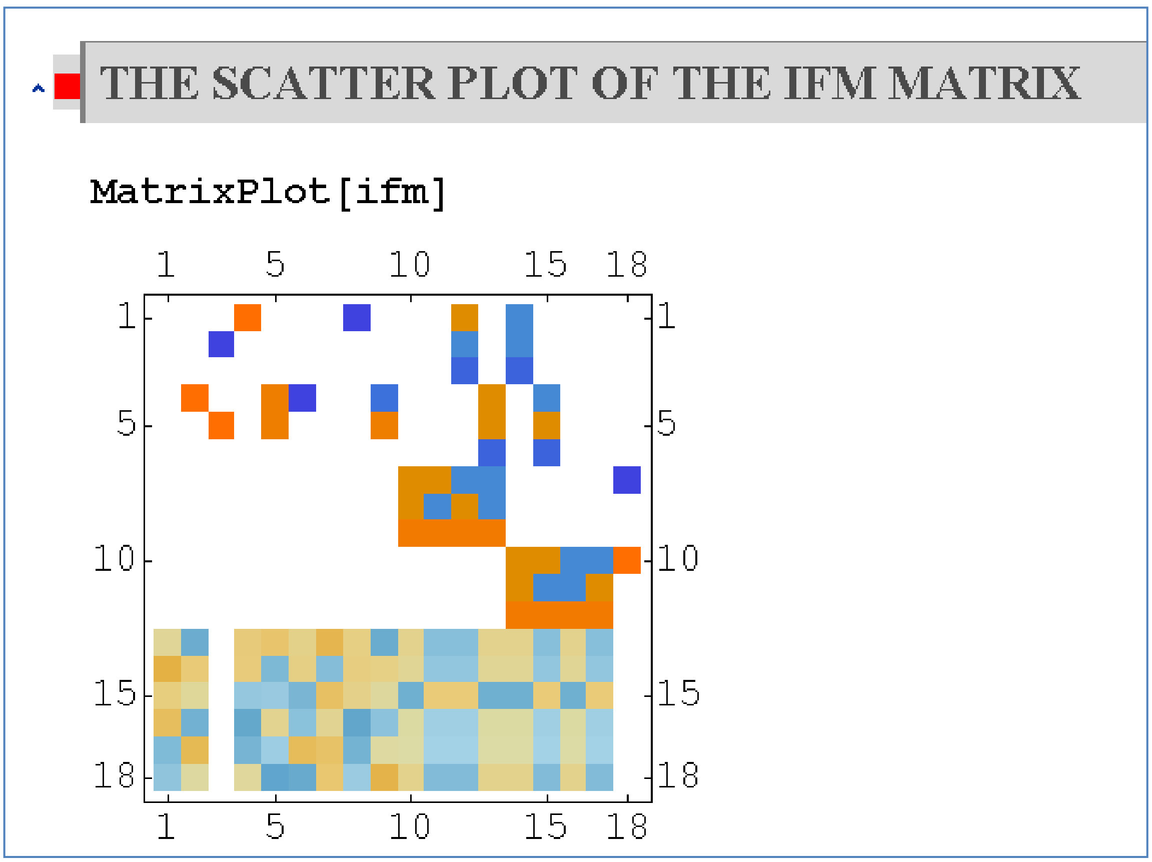
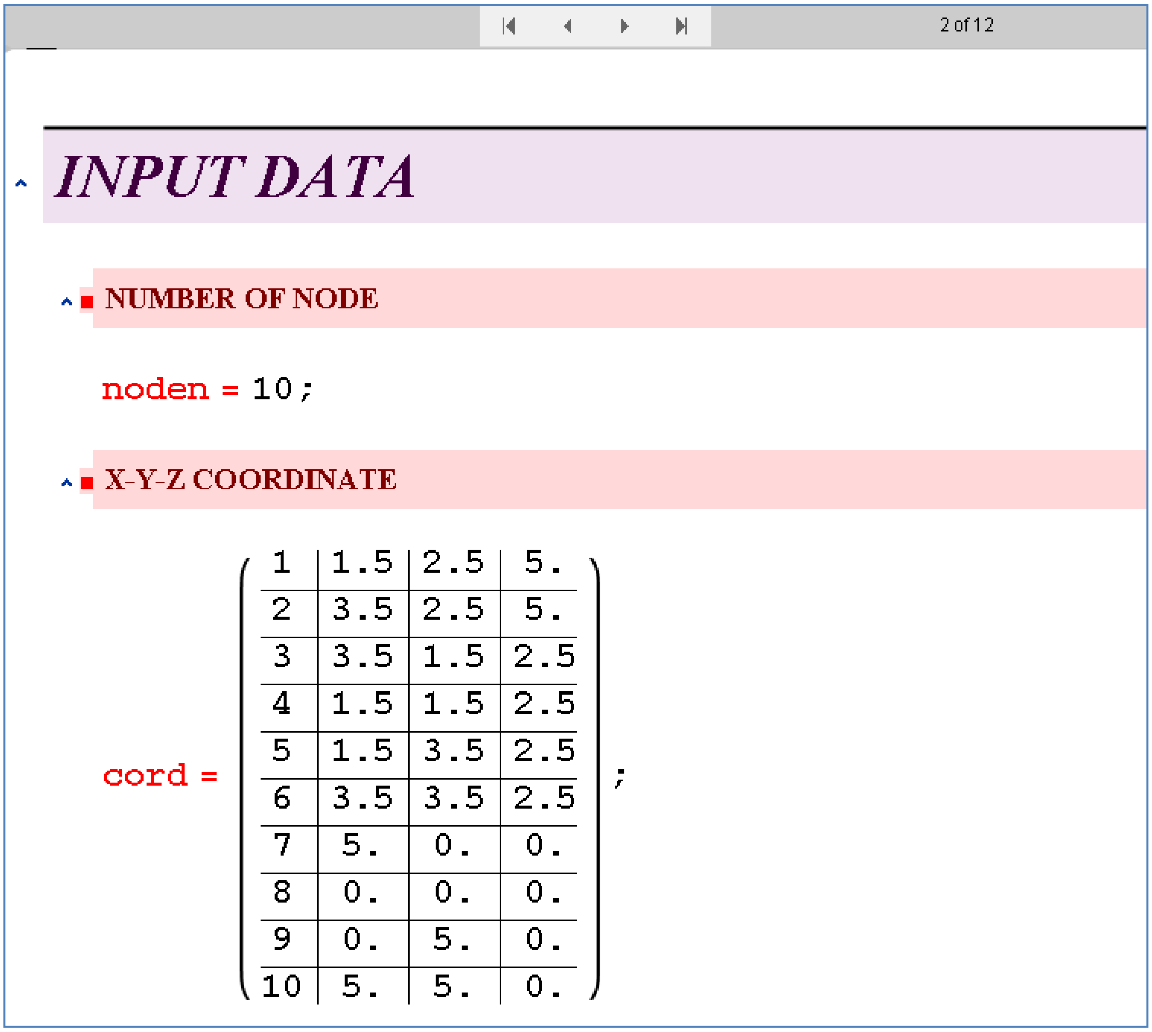


Figure 109. Calculation Phase; Scatter Plot of IFM Matrix for Example 2

Appendix C: Data Input and Calculation Phase for Example 3



1	1.5	2.5	5.
2	3.5	2.5	5.
3	3.5	1.5	2.5
4	1.5	1.5	2.5
5	1.5	3.5	2.5
6	3.5	3.5	2.5
7	5.	0.	0.
8	0.	0.	0.
9	0.	5.	0.
10	5.	5.	0.

Figure 110. Input Phase; Data for Example 3

2 of 12

INPUT DATA

NUMBER OF NODE

X-Y-Z COORDINATE

ELEMENT CONNECTIVITY

```
m = 25;
```

inc =

1	1	2
2	1	5
3	1	4
4	1	3
5	1	6
6	2	5
7	2	4
8	2	6
9	2	3
10	4	5
11	5	6
12	3	6
13	3	4
14	5	8
15	4	8
16	3	8
17	4	7
18	6	7
19	3	7
20	5	9
21	6	9
22	4	9
23	5	10
24	6	10
25	3	10

;

Figure 111. Input Phase; Data for Example 3(continued)

INPUT DATA

NUMBER OF NODE

X-Y-Z COORDINATE

ELEMENT CONNECTIVITY

RESTRAINT TABLE

0=free and 1=restrained

freet =

1	0	0	0
2	0	0	0
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0
7	1	1	1
8	1	1	1
9	1	1	1
10	1	1	1

;

APPLIED FORCES

applfrcs =

1	0.	0.	-60.
2	0.	75.	0.
3	0.	0.	0.
4	0.	0.	0.
5	0.	0.	0.
6	0.	0.	0.
7	0.	0.	0.
8	0.	0.	0.
9	0.	0.	0.
10	0.	0.	0.

;

Figure 112. Input Phase; Data for Example 3

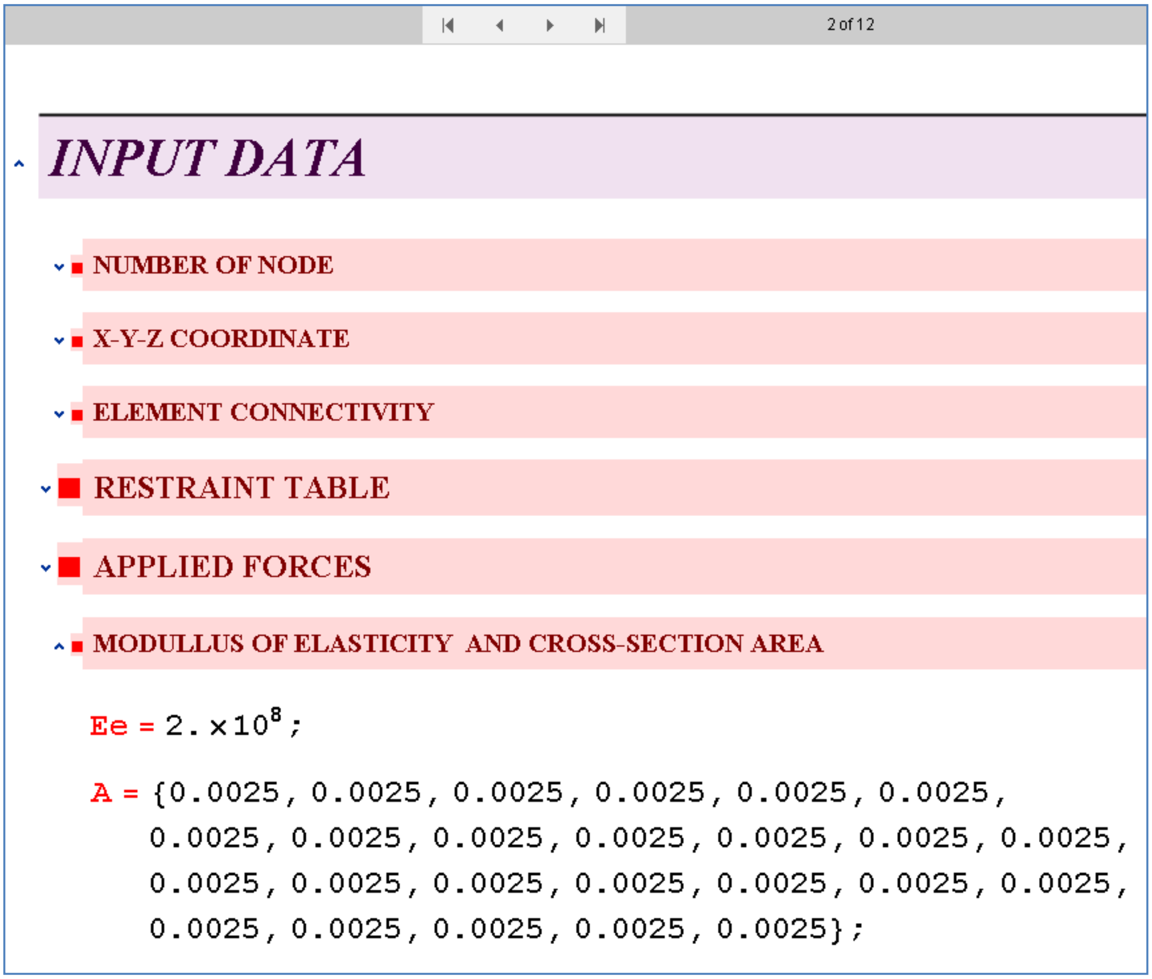


Figure 113. Input Phase; Data for Example 3

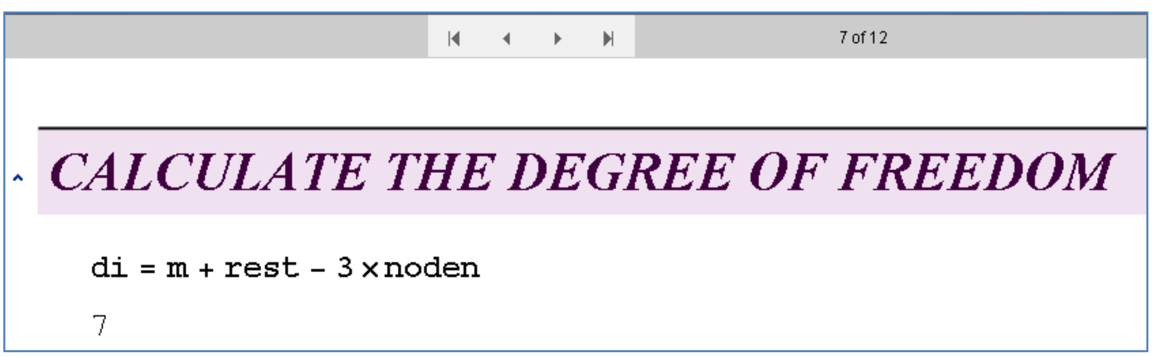


Figure 114. Calculation Phase; Degree of Indeterminacy

THE SCATTER PLOT OF UNCONNECTED FLEXIBILITY MATRIX

```
MatrixPlot[F]
```

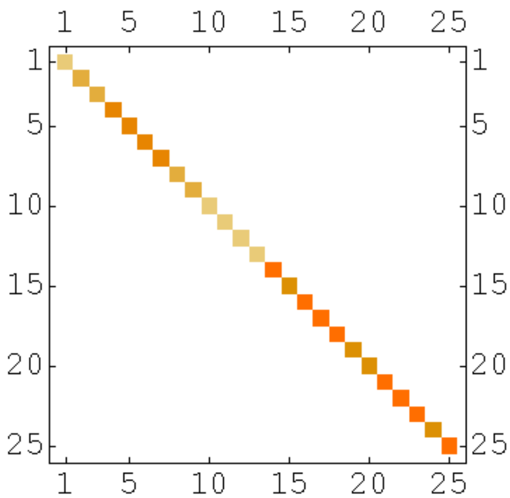


Figure 115. Calculation Phase; Scatter Plot of Unconnected Flexibility Matrix

SCATTER PLOT OF COMPATIBILITY CONDITION MATRIX

```
MatrixPlot[cc]
```

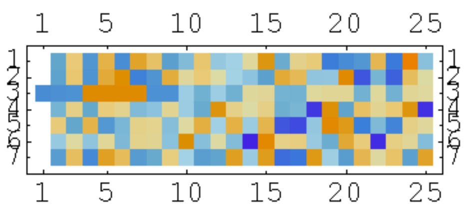


Figure 116. Calculation Phase; Scatter Plot of Compatibility Conditions

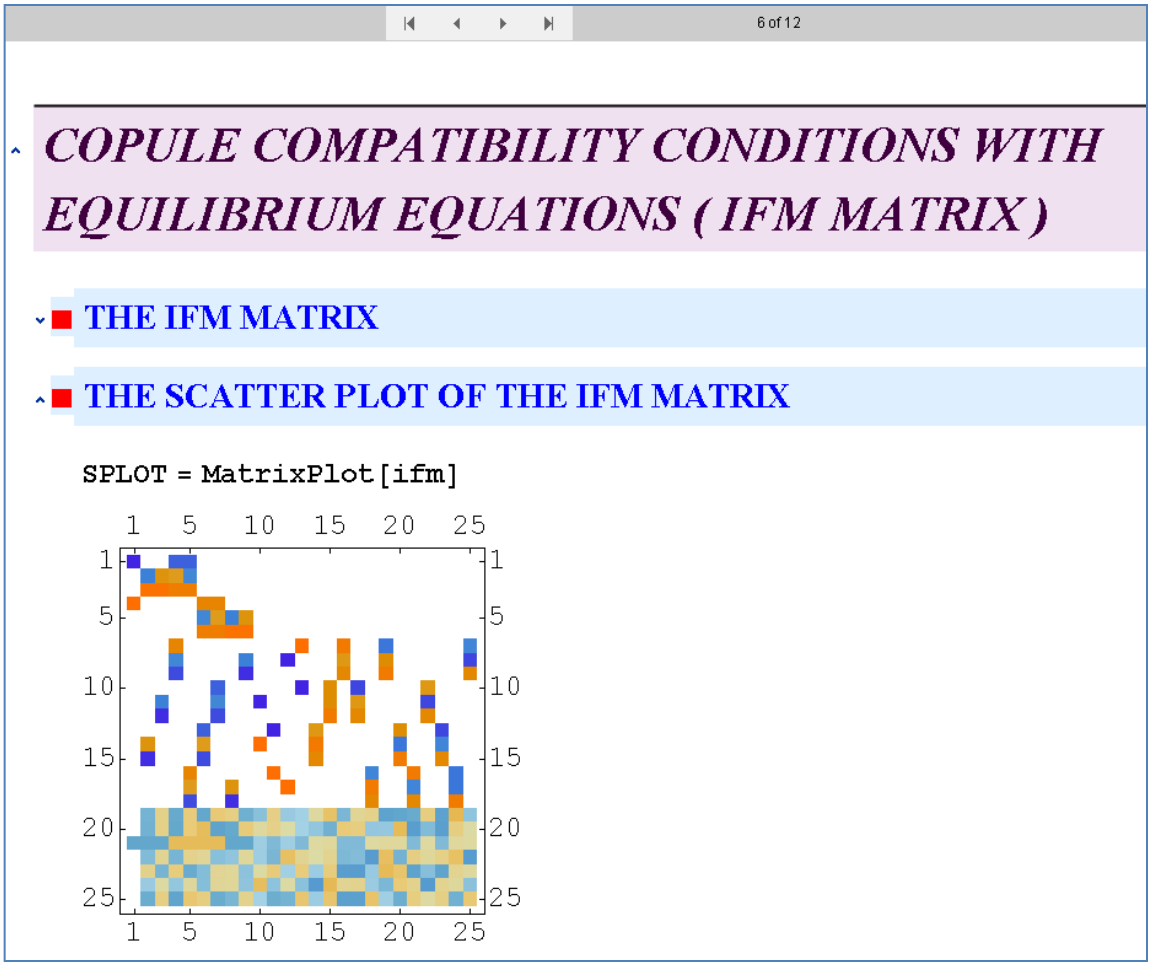


Figure 117. Calculation Phase; Scatter Plot of IFM Matrix

Appendix D: Data Input and Calculation Phase for Example 4

2 of 12

INPUT DATA

NUMBER OF NODE

```
noden = 12;
```

X-Y-Z COORDINATE

cord =

1	0.	0.	3.
2	0.	3.	3.
3	0.	3.	0.
4	0.	0.	0.
5	3.	0.	3.
6	3.	3.	3.
7	3.	3.	0.
8	3.	0.	0.
9	6.	0.	3.
10	6.	3.	3.
11	6.	3.	0.
12	6.	0.	0.

;

Figure 118. Input Phase; Data for Example 4

INPUT DATA

NUMBER OF NODE

X-Y-Z COORDINATE

ELEMENT CONECTIVITY

```
m = 30;
```

inc =

1	1	4
2	1	2
3	2	3
4	3	4
5	3	7
6	3	6
7	2	6
8	1	6
9	1	5
10	1	8
11	4	8
12	3	8
13	5	8
14	5	6
15	6	7
16	6	8
17	7	8
18	8	12
19	8	11
20	8	9
21	5	9
22	6	9
23	6	11
24	6	10
25	7	11
26	9	12
27	10	12
28	11	12
29	10	11
30	9	10

;

Figure 119. Input Phase; Data for Example 4 (continued)

INPUT DATA

NUMBER OF NODE

X-Y-Z COORDINATE

ELEMENT CONECTIVITY

RESTRAINT TABLE

0=free and 1=restrained

freet =

1	1	1	1
2	1	1	1
3	1	1	1
4	1	1	1
5	0	0	0
6	0	0	0
7	0	0	0
8	0	0	0
9	0	0	0
10	0	0	0
11	1	1	1
12	0	0	0

;

Figure 120. Input Phase; Data for Example 4 (continued)

INPUT DATA

NUMBER OF NODE

X-Y-Z COORDINATE

ELEMENT CONECTIVITY

RESTRAINT TABLE

APPLIED FORCES

applfrcs =

1	0.	0.	0.
2	0.	0.	0.
3	0.	0.	0.
4	0.	0.	0.
5	0.	0.	0.
6	0.	0.	0.
7	0.	0.	0.
8	0.	0.	0.
9	0.	0.	0.
10	0.	-40.	0.
11	0.	0.	0.
12	0.	0.	-60.

;

MODULLUS OF ELASTICITY AND CROSS-SECTION AREA

Ee = $2.\times 10^{8}$;

A = {0.002, 0.002};

Figure 121. Input Phase; Data for Example 4 (continued)

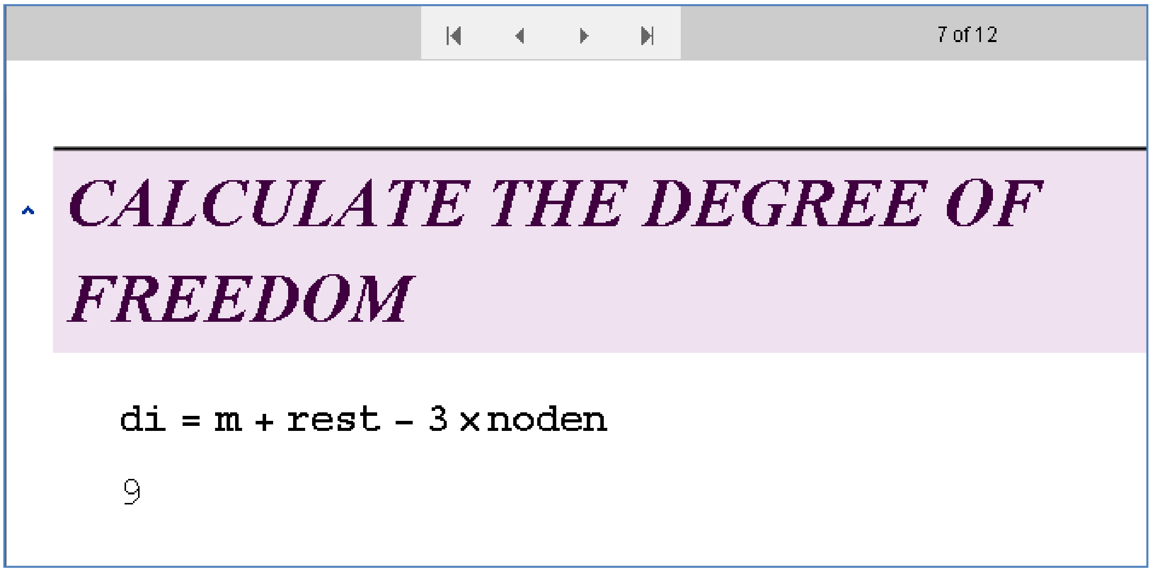


Figure 122. Calculation Phase; Degree of Indeterminacy

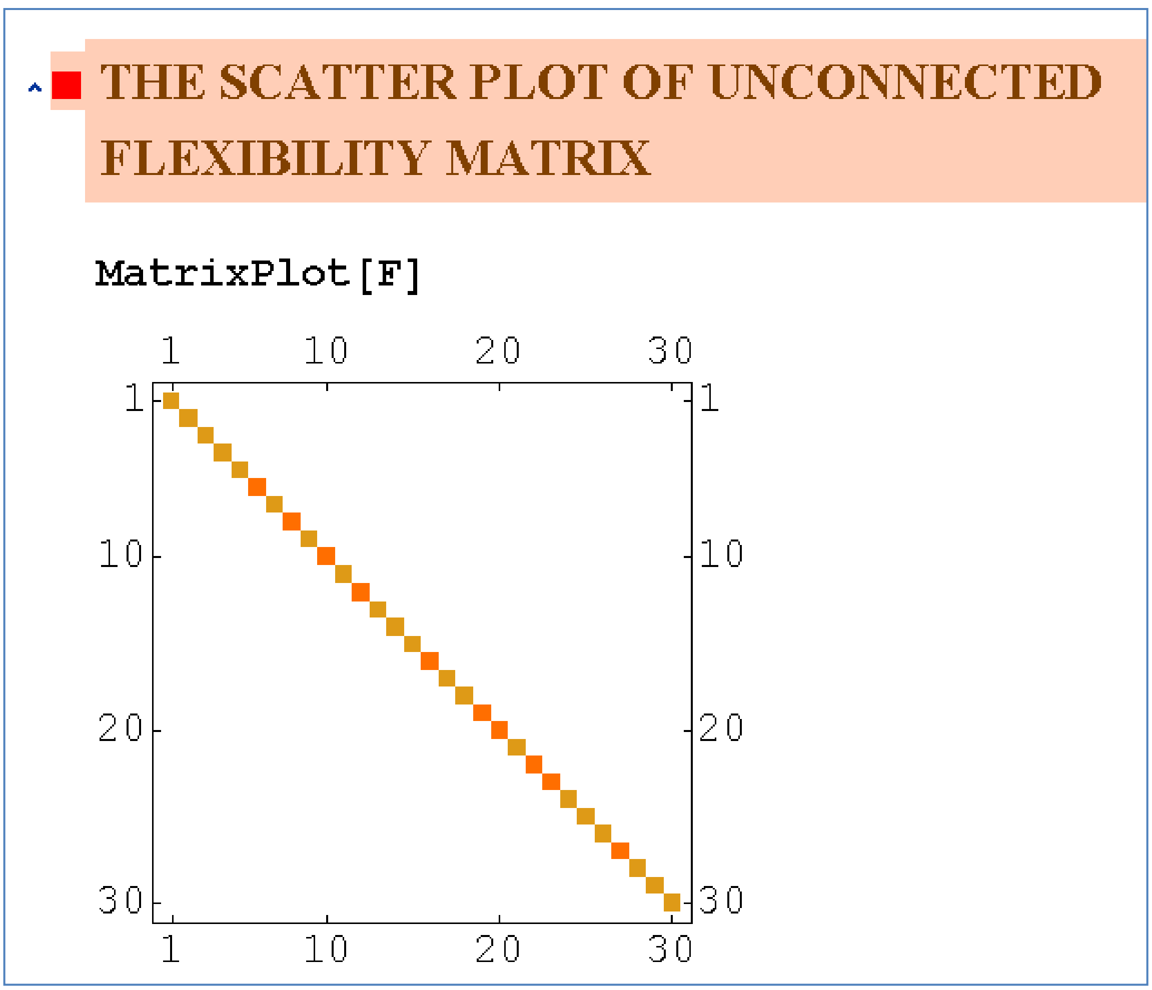


Figure 123. Calculation Phase; Scatter Plot of Unconnected Flexibility Matrix

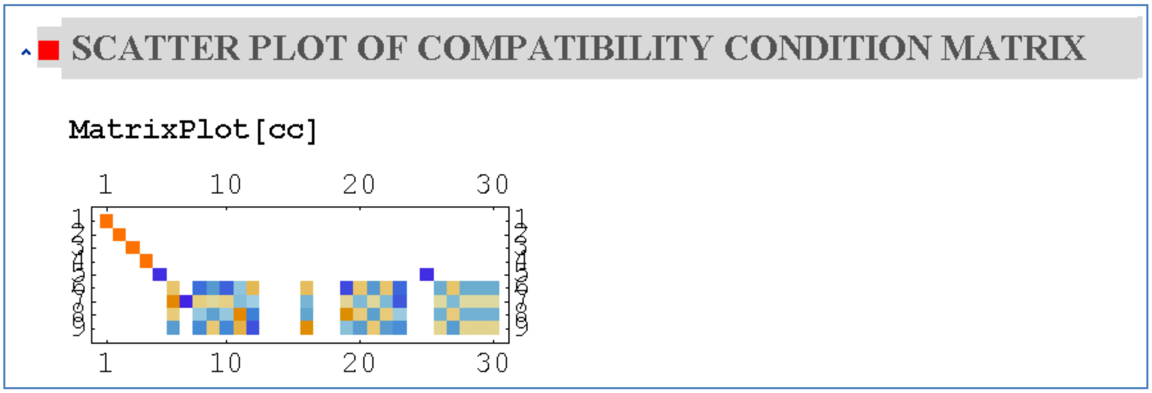


Figure 124. Calculation Phase; Scatter Plot of Compatibility Conditions

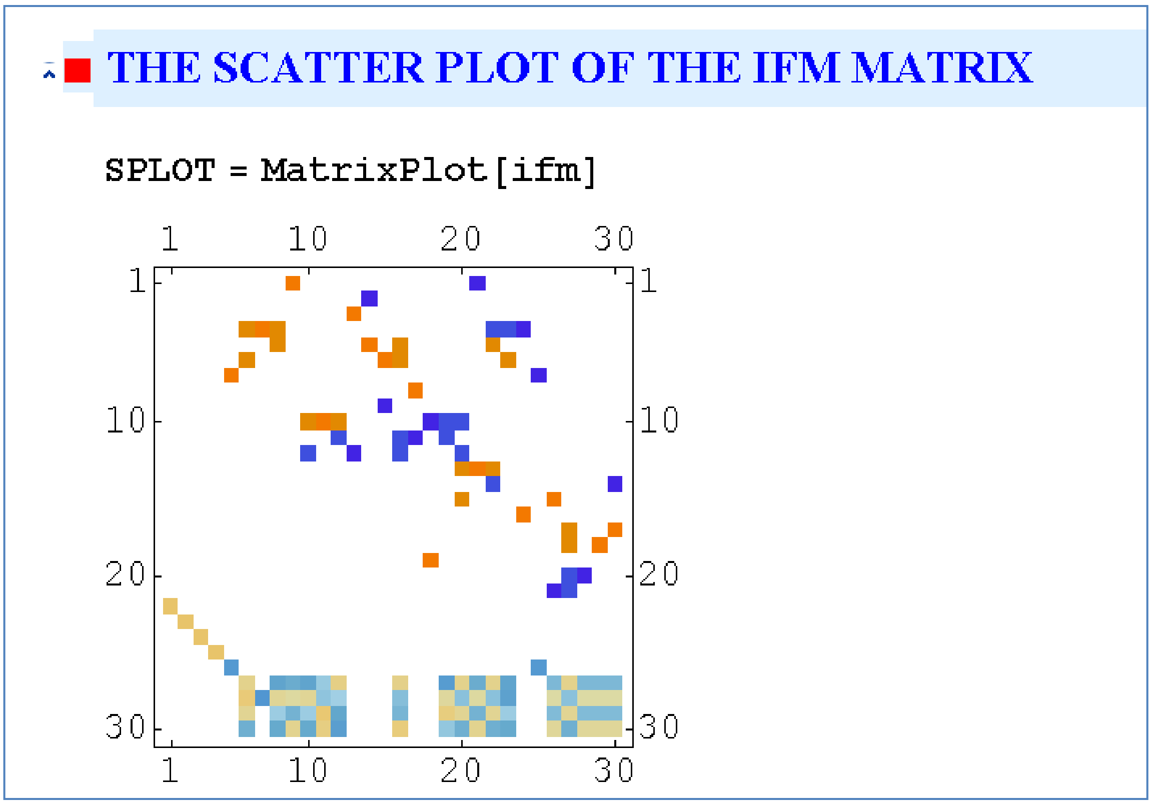


Figure 125. Calculation Phase; Scatter Plot of IFM Matrix