A Generalized Loading Algorithm for Adaptive Beamforming in Uniform Linear Array

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ABSTRACT

The aim of this work is focused on designing a new generalized loading approach for Uniform Linear Array (ULA). This method achieves robust adaptive beamforming against direction-of-arrival (DOA) mismatches by shaping the directional response of the adaptive Uniform Linear Array.

To achieve this purpose, we consider an ULA with N sensors, which are located at half wavelength spacing. The desired spatial signal impinges from a specific direction, such that the presumed direction has a mismatch with the true direction. Furthermore we assume that the desired signal components are present in the beamformer training data snapshots and that the data sample size is limited. Therefore, we desire to develop a robust adaptive beamformer to improve the performance against inaccuracies caused by limited sample size and the look direction mismatch by shaping the directional response of the array.

The ability of the directional response shaping (DRS) will follow the modified conventional loading methods for an adaptive ULA. However, it stresses on the range of specified direction (cut off angle) in the presence of undesired interferences. In this technique, a general loading matrix is considered which is derived from a weight function. This matrix is added to the estimated correlation matrix, such that the directional response of the beamformer is shaped. Also, the loaded matrix minimizes the output power of the beamformer. By using the loaded matrix, beamformer weight vector will approach to an optimal value regarding the output

SINR. The weight function can be chosen to further suppress the interferences by making the weight large in the vicinity of DOA's of interferences.

To demonstrate the capability of the proposed method, it is compared with some of the well-known methods such as Sample Matric Inversion (SMI), Loaded Diagonally Sample Matrix Inversion (LSMI), Robust Capon Beamformer (RCB), Iterative Minimum Variance Beamformer (IRMVB (Li's)) algorithms. The results clarify that, convergence of our method to optimal SINR in different conditions is superior.

Keywords: Adaptive Beamformer, Uniform Linear Array (ULA), Mismatch, Interferences, Generalized Loading matrix, Correlation Matrix

ÖZ

Bu çalışmanın amacı uyarlanır demet oluşturmada genel yükleme yaklaşımı geliştirmektir. Bu yaklaşım, uyarlanır doğrusal dizgenin yönsel tepkimesini şekillendirmek suretiyle, yönsel varma açısındaki uyumsuzluklara karşı dayanıklı hale gelir.

Bu amaca ulaşmak için duyarga sayısı N ve duyargalar arası uzaklığın yarı dalgaboyu olan bir tekdüze doğrusal dizge ele aldık. İstenen işaretin, gerçek yaklaşım yönü ile varsayılan yaklaşım yönü arasında bir uyumsuzluk olacak şekilde dizgeye ulaşmaktadır. Ayrıca, istenen işaretin demet-oluşturucunun alıştırma veri dizilerinde mevcut olduğu, ve örnek veri büyüklüğünün sınırlı olduğu varsayılmıştır.

Bu sorunlar karşısında uyarlanır demet-oluşturucunun başarımını artırmak için dizgenin yönsel tepkimesini şekillendirmeye çalıştık.

Bu yöntemde, bir ağırlık işlevinden elde edilen genel bir yükleme matrisi üzerinde durduk. Bu matris kestirilen ilinti matrisi ile toplanır, ve sonuçta demet-oluşturucunun yönsel tepkimesi şekillendirilmiş olur. Bu yolla demet-oluşturucunun ağırlık vektörü çıkış SINR'ı açısından en iyi durumu yaklaşır. Ağırlık işlevi, karışma işaretlerinin varış yön açılarının cıvarında büyük seçilerek bu işaretler daha etkili olarak bastırılır.

Önerilen yöntemin olumlu özelliklerini göstermek için, SMI, LSMI, RCB ve IRMVB gibi iyi bilinen diğer yöntemlerle karşılaştırıldı. Sonuçlar, önerilen yöntemin

farklı durumlar için en iyi SINR değerlerine yakınlığı açısından üstün olduğunu göstermiştir.

Anahtar Kelimeler: Uyarlanır Demet Oluşturmada, , Tekdüze Doğrusal Dizge, Uyumsuzluk, Karışım, Ilinti Matrisi, Genelliştirilmiş Yükleme Matrisi

Dedicated to

My beloved wife and dear son

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c	Rate of propagating
C(n,m)	General loading matrix
d	Interelement spacing
$E\{\cdot\}$	Expected value
E_s	Signal-plus-interference subspace
E_n	Noise subspace
F_c	Carrier frequency
$H(F_c)$	Directional response
$\mathrm{H}_d(\theta_d)$	Directional response of filter
I	Identity matrix
$\mathrm{i}(k)$	Interference
J_a	Cost function
N	Number of sensors (elements)
$\mathrm{n}(k)$	Thermal noise
P	Rank of interference
P_y	Output power of beamformer
Q	Noise covariance matrix
$Q_m(\phi)$	Eigen-beam
q_m	Eigen-vector
R	Correlation matrix
$\hat{\mathrm{R}}$	Sample covariance matrix

R_{i+n}	Interference plus noise correlation matrix
$s(k)$	Desired signal
$v(\phi)$	Array steering vector
w	Weight vector
$W_q(\phi)$	Quiescent response
$x(k)$	Array steering vector
$y(k)$	Output for narrowband beamformer
$\mathcal{P}\{.\}$	Operator to calculate Eigen-vectors
$\alpha \,\&\, \beta$	Shrinkage parameters
$\gamma(\theta_d)$	Weight function
Δ	Hermition error matrix
ε	Uncertainty level
ζ	Diagonal loading factor
η	Arbitrary constant
θ_c	Cut of angle
Λ	Diagonal matrix
σ_s	Wave length
σ_w	Power if noise
$\rho(\varphi)$	Normalized angular power density
τ	Delay time
ϕ	Direction of impinging signal
AU-IRCB	Adaptive Uncertainty Iterative Robust Capon Beamformer
CMT	Covariance Matrix Taper

DOA	Directional Of Arrival
DOF	Degree Of Freedom
DRS	Directional Response Shape
ESB	Eigen-Based Beamformer
FIR	Finite Impulse Response
FU-IRCB	Fixed Uncertainty Iterative Robust Capon Beamformer
GSC	Generalized Sidelobe Canceller
INR	Interference to Noise Ratio
IRMVB	Iterative Robust Minimum Variance Beamformer
LCMV	Linearly Constrained Minimum Variance
MVDR	Minimum Variance Distortionless Response
RCB	Robust Capon Beamformer
SCB	Standard Capon Beamformer
SMI	Sample Matrix Inversion
SOI	Signal Of Interest
ULA	Uniform Linear Array

Chapter 1

INTRODUCTION

1.1 Introduction

In many applications, the desired information which is extracted from an array of sensors is the content of a spatially propagating signal from a definite direction. The content may be a message contained in the signal, such as just the existence of the signal, as in radar and sonar, or in communications applications. Because of this, we want to linearly combine the signals from all the sensors in a way, that is, with a convenient weighting, so as to examine signals arriving from a particular angle.

Adaptive beamforming has various applications in sonar, radar, seismology, microphone array speech processing [1]–[5], and even, in wireless communications [6], [7]. When the practical problem for adaptive arrays is considered, the performance of adaptive beamforming methods might be inferior to the model case.

The performance of adaptive beamforming methods is known to reduce substantially if there are mismatches between the true and assumed array steering vector responses to the desired signal. Such mismatches may frequently occur in practical situations due to contravention of basic assumptions on the surroundings, look direction errors, sensor array or environment being non-stationary. This is particularly true when the desired signal components are present in the beamformer training data snapshots like passive locations, medical imaging, mobile communications and acoustics. In this case the beamformers produce main beam in the assumed direction of the desired signal and true signal is considered as the interference so the algorithm tries to suppress the true signal. For this occasion, the adaptive array performance is very sensitive to array and model imperfections [8], [9]. Similar performance degradation can be attributed to the inaccurate estimation of the covariance matrix even when the array steering vector to the desired signal is known, however the data samples size is limited.[9],[10], [11].

1.2 Background

In the last three decades, plenty of algorithms have been suggested to design and improve robust adaptive beamforming against slight mismatches.When precise data of direction-of-arrival (DOA) for the desired signal and interferences are presented, the main beam and nulls can be generated by array signal processing through shifted phase along the desired signal direction and direction of interferences. Several methods have been established for the special case of look direction mismatch.

The linearly constrained minimum variance (LCMV) beamformer [37], minimum variance distortionless response (MVDR) beamformer, signal blocking-based algorithms [12], sample matrix inversion (SMI) [11], Bayesian beamformer and generalized sidelobe canceller (GSC) are the widely held adaptive beamformers. Although these techniques are useful to reduce the signal look direction mismatch, they are not effective when the desired signal appears in the data snapshots and there is a mismatch between the assumed and true signal directions, in which case the beamformers encounter performance degradation problems.

Efforts are in progress to develop robust algorithms against mismatches, signal wave-front distortions and coherent and incoherent local scattering [13].

Other approaches aim to obtain robustness against common kinds of mismatches, for instance, diagonal loading of the sample covariance matrix [11],[14] which is a general technique that makes the beamformer robust against direction-of-arrival mismatch. Nevertheless, an important disadvantage of diagonal loading is that there is no trustworthy way to find the diagonal loading factor and if the chosen parameter is improper, the robustness of this technique will not be satisfactory. The method which has fast convergence is the sample matrix inversion (SMI) technique.

It uses the matrix inversion to have speedy convergence. Furthermore, in the SMI algorithm there is a block matrix which changes by different weight vectors. Another approach is covariance matrix taper (CMT) which is known to provide excellent robustness when the interference is non-stationary [15] Robustness against mismatches for the desired signal array response is acceptable. Another method is robust adaptive beamforming using worst case performance optimization [16]. The performance of this method is fairly close to the simple algorithm which is known as diagonal loading of the sample matrix inversion (LSMI) algorithm. Generalized sidelobe canceller (GSC) [20] is a technique that modifies its blocking matrix in order to extend the sharp nulls [21].

1.3 Organization

Chapter 2 provides details about beamforming in uniform linear array and explains how features are extracted in adaptive beamforming and used for MVDR and SMI. This is followed by an explanation for the methods of beamforming such as diagonally loaded, robust capon, Eigen-based and general rank signal beamformers take into the part in Chapter 3. Next in Chapter 4, the proposed method which is directional response shaped beamformer and the results that obtained via Monte Carlo simulation are presented and then discussed in varied situation in chapter 5. Finally, Chapter 6 makes some conclusion and purveys directions for our future work.

Chapter 2

BEAMFORMIN IN UNIFORM LINEAR ARRAYS

2.1 Introduction

Array signal processing is motivated with the retrieval of information from signals which are received using an array of sensors. These signals are broadcast spatially over a space, such as, air, and the samples are gathered from the wavefront by the sensor array. The desired information in the signal might be either the content of the signal which is considered in communications or the specific location of the source or reflection that produces the signal, like in radar and sonar applications. In every case, the sensor array data must be processed to draw out proper information. For linear arrays, the sensors are organized in patterns and located along a straight line. The most common applications of array signal processing, include radar, sonar, seismology, biomedicine, communications, astronomy, and imaging.

2.2 Uniform Linear Array

Uniform Linear array (ULA) is an antenna array configured of individual beam elements with equal spacing between the elements and can be employed to produce a directional radiation array. Every single element antenna has beam-patterns that are broad and they have low directivity that is not appropriate for long distance communications. A high directivity can still be achieved with single element antennas by increasing the electrical dimensions with respect to the wavelength and the physical size of the antenna. Antenna arrays come in different geometrical structures, the most common being linear arrays. Arrays commonly use identical antenna elements. The beam pattern of the array depends on the configuration, the distance between the elements, the amplitude and phase excitation of the elements, and also the radiation pattern of every sensor. Figure 2.1 shows the ULA, where interelement space is defined by d and single propagation signal impinges on the ULA from angle ϕ .

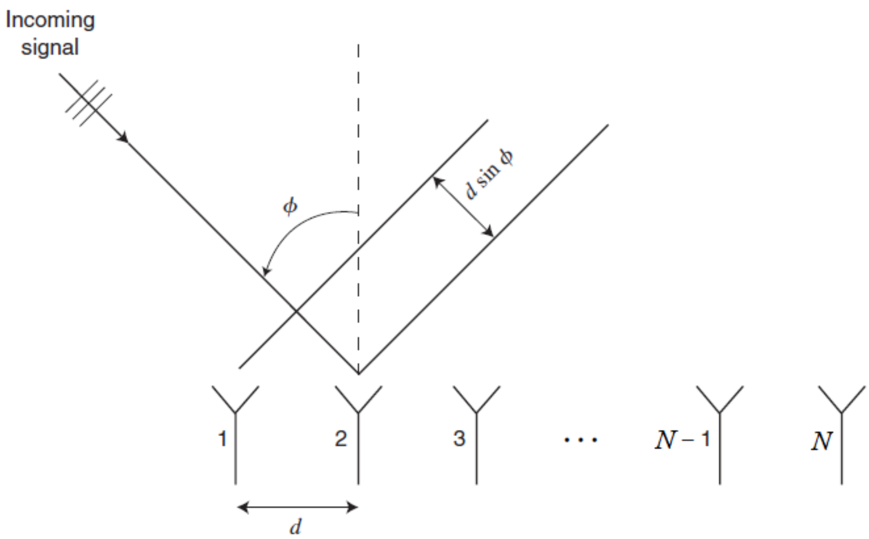

Figure 2.1: Impinging Signal on Uniform Linear Array [22]

For raising a model for a single spatial signal in interference and noise received by ULA, we assume a signal with angle ϕ which is discrete signal and contain the individual sensor signals

$$\mathrm{x}(n) = [x_1(\mathrm{n}) \quad x_2(\mathrm{n}) \quad \ldots \quad x_N(\mathrm{n})]^T \tag{2.1}$$

where N is the total number of sensors. A signal measurement of this vector is defined as an array snapshot. With respect to the (2.1) full array discrete time signal is constructed for every signal of interest (SOI) which is absorbed by individual sensors

$$\mathrm{x}(n) = v(\phi)s(n) + w(n) \tag{2.2}$$

where $v(\phi)$ is the array response vector and, $s(n) = H(F_c)s_0(n)$ is the impulse response of signal of interest (SOI) to n^{th} sensor, since $F_c = c/\lambda$. Often in spatial filtering is concerned to receive a signal arriving from a specified point ϕ , and assume the signal is narrowband, a usual selection for beamformer weight is the array response vector model as

$$v(\phi) = [1 \quad e^{-i2\pi[(d\sin\phi)/\lambda]} \quad \ldots \quad e^{-i2\pi[(d\sin\phi)/\lambda](N-1)}]^T \tag{2.3}$$

The spatial signal has a different propagation between two sensors because the space of elements is equal so the result of time delay can be:

$$\tau(\phi) = \frac{d\sin\phi}{c} \tag{2.4}$$

where c is the speed of propagation for signal. To end up the delay to the n^{th} element (sensor) will be

$$\tau_n(\phi) = (n-1)\frac{d\sin\phi}{c} \qquad , \quad 2 \le n \le N \tag{2.5}$$

It should be mentioned that full possible range for angle ϕ is $-90^o \le \phi \le 90^o$, and the space for sensor must be $d \le \frac{\lambda}{2}$, this conditions prevent to aliasing the signals and the signals will not be ambiguity.

2.3 Contractual Spatial Filtering (BEAMFORMING)

To extract the desired information from an array of sensors which includes a spatially propagating signal from a specific direction, it is needed to process weighting that emphasizes signals from a certain angle, and attenuates other signals; this procedure can be considered as forming a beam.

Beamforming is classified to be data-dependent or statistically optimum, depending on the approach to choose the weights. For data independent beamformer the weights are selected to obtain a desired response. However, in statistically optimum beamformer to obtain the optimized array response, the weights are chosen with respect to the statistics of array data. Commonly, the statistically optimum beamformer puts nulls at the angles of interference and tries to maximize the signal-to-noise ratio at beamformer output [19].

In general, a beamformer yields its output by developing a weighted combination of signals (Data vector) from the N elements of the sensor array

$$y(n) = w^H x(n) \tag{2.6}$$

where $$w = [w_1 \quad w_2 \quad \dots \quad w_N]^T \tag{2.7}$$

is the weight vector of beamformer. A standard tool for analyzing the performance of a beamformer is the response for a given weight vector w as a function of ϕ, which is known as the beam response. This direction of response is calculated as

$$W(\phi) = w^H v(\phi) \qquad -90^o \le \phi \le 90^o \tag{2.8}$$

The weight vector can be found by maximize the Signal to Interference plus Noise Ratio (SINR)

$$SINR = \frac{E\left\{| w^H s(n) v(\phi) |^2\right\}}{E\left\{| w^H \, x_{i+n}(n) |^2\right\}} = \frac{\sigma_s^2 \, | w^H v(\phi) |^2}{w^H R_{i+n} w} \tag{2.9}$$

The optimal solution to (2.9) is founded by minimizing the cost function ($w^H R_{i+n} w$) while the beam response is going to have unity gain ($w^H v(\phi) = 1$), so by using the Lagrange multiplier we can write

$$J = w^H R_{i+n} w + \lambda w^H v(\phi)$$

$$\frac{\delta J}{\delta w} = 2R_{i+n} + \lambda v(\phi) = 0 \Rightarrow w = -\frac{1}{2} \lambda R_{i+n}^{-1} v(\phi)$$

$$w^H v(\phi) = v^H(\phi) w = 1$$

$$\Rightarrow v^H(\phi) w = -\frac{1}{2}\lambda v^H(\phi) R_{i+n}^{-1} v(\phi) = 1$$

$$-\frac{1}{2}\lambda = \frac{1}{v(\phi)^H \mathrm{R}_{i+n}^{-1} v(\phi)}$$

$$\Rightarrow w_{opt} = \frac{\mathrm{R}_{i+n}^{-1} v(\phi)}{v(\phi)^H \mathrm{R}_{i+n}^{-1} v(\phi)} \tag{2.10}$$

where R_{i+n} is Interference-plus Noise correlation matrix.

2.4 Minimum Variance Distortionless Response Beamformer

2.4.1 Overview on MVDR

MVDR is a special item of Linearly Constrained Minimum Variance Beamforming (LCMV) in array processing that does not require having background information. In special cases, the only prior knowledge for MVDR is the desired signal of interest (SOI). MVDR rejects all other information. This is a major advantage in the case when a signal contains unknown and unidentified components such as noise and interferences. MVDR beamformer assumes that the direction of arrival (DOA) from the desired signal is identified in advance. Then, it operates as an adaptive filter to pass the desired signal with a particular gain, so that the output of the filter from unknown sources such as undesired signals (noise) and unwanted interferers can be minimized. It is noted that the specific gain to the SOI is taken to be unity [26].

Typically, MVDR use fairly Finite Impulse Response (FIR) method to design a filter that minimizes the output average power of linear filter to limit the desired response to the specific unit gain. Now, consider linear transversal filter for array processing with respect to the direction of arrival to desired signal by inputs $x_o(n), x_1(n), \ldots x_{M-1}(n)$ which gives the output

$$y(n) = \sum_{k=0}^{M-1} w_k^{\ *} x_k(n) \tag{2.10}$$

For special case of a sinusoidal excitation, $x(n)$ might be $e^{-i\phi k}$ where ϕ the angle that define the direction of arrival, then

$$y(n) = x_o(n) \sum_{k=0}^{M-1} w_k^{\ *} e^{-i\phi k} \tag{2.11}$$

The objective for constrained optimization problem is employing the approach to minimize the variance of the beamformer output subject to above constraint. The method of Lagrange multipliers can help us to solve the optimization problem. It compound two parts of optimization problem and gives

$$\mathrm{J} = \sum_{k=0}^{M-1} \sum_{l=0}^{M-1} w_k^{*} w_l x(n) - \mathrm{Re}[\lambda^{*} (\sum_{k=0}^{M-1} w_k^{*} d_l - g)] \tag{2.12}$$

$$g = \sum_{k=0}^{M-1} w_k^{\ *} e^{-i\phi k}$$

The solution to (2.12) is to obtain optimality criteria by minimizing the variance of the output subject to have unit gain for beamresponse by Minimum Variance Distortionless Response (MVDR) Beamforming. So,the weight vector w_o is computed by :

$$w_o = \frac{\mathrm{R}^{-1} v(\phi_o)}{v^H(\phi_o)\mathrm{R}^{-1} v(\phi_o)} \tag{2.13}$$

where R is the k×k correlation matrix, w_o is the k×1 optimum weight vector and $v(\phi_o)$ is the steering vector defined in (2.3).

2.4.2 Robust Adptive Beamformin by MVDR

2.4.2.1 Modeling the Signal

Assume a linear antenna array with N omni-directional antenna sensors where narrowband signal is received by the antenna array at the time *n* is represented as

$$\mathrm{x}(n) = \mathrm{s}(n) + \mathrm{i}(n) + \mathrm{w}(n) \tag{2.14}$$

where $\mathrm{s}(n)$, $\mathrm{i}(n)$ and $\mathrm{w}(n)$ represent the desired signal vector, interference vector, and noise vector. We assume that the desired signal is uncorrelated with the interferers and noise, while the received signal is supposed to be zero-mean. With respect to mentioned point source assumption, the desired signal $\mathrm{s}(n)$ is modeled as $\mathrm{s}(n) = s(n)v(\phi_s)$ where $s(n)$ is the signal waveform and $v(\phi_s)$ is the array steering vector accompanying the desired signal.

2.4.2.2 Minimum Variance Distortionless Rresponse Beamformer

The output for the beamformer at the time *n* is written as

$$y(n) = w^H x(n) \tag{2.15}$$

where w is the *N by* 1 weight beamforming vector of the array. If we assume that the direction-of-arrival (DOA) is known, then with the optimal weight vector *w* of the beamformer output SINR is maximized [24]

$$\mathrm{SINR} = \frac{\mathrm{E}[|\mathrm{w}^H \mathrm{s}|^2]}{\mathrm{E}[|\mathrm{w}^H(\mathrm{i}+\mathrm{n})|^2]} = \frac{\sigma_s^{\,2}\,|\mathrm{w}^H v(\phi_s)|^2}{\mathrm{w}^H \mathrm{R}_{\mathrm{i+n}} \mathrm{w}} \tag{2.16}$$

where $\sigma_s^{\,2}$ is the power for desired signal , $\mathrm{R}_{\mathrm{i+n}}$ is the *M* by *M* interference plus-noise covariance matrix, The MVDR beamformer is obtained by minimizing the variance (power) of interference and noise at the output of the adaptive beamformer. The optimization problem is:

$$\min_{\mathrm{w}} \mathrm{w}^H \mathrm{R}_{\mathrm{i+n}} \mathrm{w} \quad \text{s.t.} \quad \mathrm{w}^H v(\phi_s) = 1 \tag{2.17}$$

There is a solution to this optimization problem with respect to the MVDR beamformer by minimizing the cost function when the beam response is desired to unity gain and it gives result like this

$$\mathrm{w}_{MVDR} = \eta \mathrm{R}_{i+n}^{-1} v(\phi_s) \tag{2.18}$$

where η is defined by $\eta = \frac{1}{\mathrm{w}^H \mathrm{R}_{i+n}^{-1} \mathrm{w}}$

2.5 Sample Matrix Inversion (SMI) Adaptive Beamforming

Up to now, we have considered the optimum beamformer but have not discussed how the beamformer would be implemented practically. Optimality is accomplished by the assumption of having perfect knowledge of the interference-plus-noise correlation matrix (R_{i+n}), where it is utilized in (2.18). There is an adaptive method which is based on collected information from estimated covariance matrix. In fact, Sample Matrix Inversion (SMI) is a block adaptive method of the optimum beamformer that utilizes block of information to estimate the weight vector for adaptive beamforming

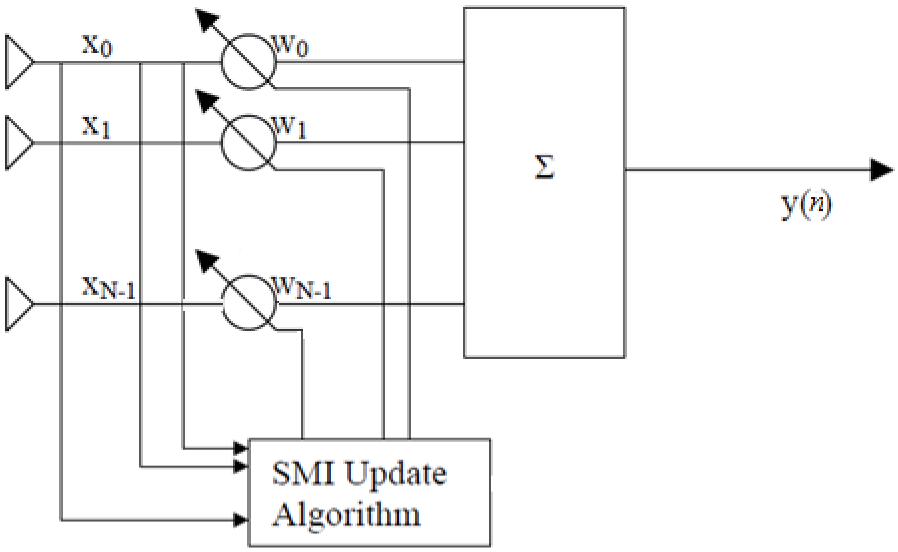

Figure 2.2: Sample Matrix Inversion Adaptive Beamformer [7]

Practically, the precise interference-plus-noise covariance matrix R_{i+n} is not available; hence what we can obtain in applications is just the number of training snapshots which is described by (2.14). Data array actually contains desired signal, interference and random noise. The sample estimated covariance matrix is utilized instead of interference-plus-noise covariance matrix in practice. The correlation is unknown and by using the maximum likelihood (ML) the algorithm estimates the correlation matrix from training snapshots. So the sample covariance matrix replaces the interference-plus-noise covariance matrix ($\mathrm{R}_{\mathrm{i+n}}$) [22]. The sample covariance matrix is

$$\hat{\mathrm{R}}_{i+n} = \frac{1}{K}\sum_{k=1}^{K} \mathrm{x}_{i+n}(n_k)\mathrm{x}_{i+n}^{H}(n_k) \tag{2.20}$$

where K is the number of training data samples which contain the desired signal. The maximum likelihood estimation for correlation matrix implies that if $K \to \infty$, then $\hat{\mathrm{R}}_{i+n} \to \mathrm{R}_{i+n}$. The total number of training data samples K is referred to the sample correlation support and the greater the number of samples the better the estimate of $\hat{\mathrm{R}}_{i+n}$ in the stationary model [22]. To compute the beamformer weight, $\hat{\mathrm{R}}_{i+n}$ is substituted into (2.13)

$$w_{SMI} = \frac{\hat{\mathrm{R}}_{i+n}^{-1} v(\phi_o)}{v^H(\phi_o)\hat{\mathrm{R}}_{i+n}^{-1} v(\phi_o)} \tag{2.21}$$

For implementing the SMI adaptive beamformer, we need the interference-plus-noise estimated correlation matrix, which does not include the desired signal **s***(n)*. But usually we do not have accurate knowledge of the locations and responses for array sensors. In addition, sometimes the angle-of-arrival of the desired signal is unknown exactly for the condition when we demand its true direction. The presence of the desired signal in the training data samples may result in the cancelation of the desired signal itself, and further loss in the performance of the adaptive beamformer.

One drawback of SMI method is that it may not be stable in the inversion of a covariance matrix when the number of snapshots is increased. The numerical stability is dependent on the mathematical accuracy of an array processor. Moreover, the SMI algorithm is not the proper approach to be executed as parallel array processors.

Another crucial deficiency of the SMI algorithm is that it does not provide appropriate robustness to mismatch between the assumed and true signal steering vectors. When the mismatch occurs, there is an unknown complex vector that defines the result of distortions for the steering vector. To sum up, the SMI beamformer tends to consider the signal components in array steering vector such as interference and attempts to suppress these signals by putting nulls instead of maintaining distortionless response for steering vector [23].

Chapter 3

ADAPTIVE BEAMFORMING METHODS

3.1 Introduction

An adaptive beamformer is a method in which adaptive spatial signal processing is performed with an array of sensors. The signals are collected in a way which increases the strength of a signal in specified direction. Moreover, the aim of this method (Beamforming) is to maximize the Signal Interference plus Noise Ratio (SINR) and attenuate the mismatches. Either there is desired signal in the beamforming training data or not. Hence we consider some methods such as diagonally Loaded Sample Matrix Inversion Beamformer (LSMI), Robust Capon Beamformer, Eigenspace–based beamformer and general rank signals, to emphasize the overview of some notable principles.

3.2 Diagonally Loaded Sample Matrix Inversion Beamformer (LSMI)

Clearly, by inadequate estimated sample size of covariance in the sample matrix inversion (SMI) algorithm, desired sidelobe level and distortionless mainlobe of adaptive arrays will not be achieved. Moreover, in most applications a limited number of training data samples are available, so to reach this goal, in numerous cases it can be advantageous to consider the optimum beamformer with respect to the eigenvalues (λ_m) and eigenvectors (q_m) for interference-plus-noise correlation matrix [22]

$$R_{i+n} = \sum_{m=1}^{N} \lambda_m q_m q_m^H \tag{3.1}$$

where the eigenvalues are $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_N$ and the rank of interference is P. we substitute (3.1) into the optimum beamformer weights

$$w_o = \alpha R_{i+n}^{-1} v(\phi_s) \tag{3.2}$$

where $R_{i+n}^{-1} = \sum_{m=1}^{N} \frac{1}{\lambda_m} q_m q_m^H$ and $\alpha = [v^H(\phi_s) R_{i+n}^{-1} v(\phi_s)]$

Then we have

$$W_o(\phi) = \frac{\alpha}{\sigma_w^2}\{W_q(\phi) - \sum_{m=1}^{N} \frac{\lambda_m - \sigma_w^2}{\lambda_m}[q_m^H v(\phi_s)] Q_m(\phi)\} \tag{3.3}$$

where $W_q(\phi) = v^H(\phi_s) v(\phi)$ is the quiescent response of the optimum beamformer and $Q_m(\phi) = q_m^H v(\phi)$ is the beam response of the mth eigenvector (eigenbeam). This equation is existed when the optimum conditions are described and rank of interference is less than the number of sensors and the smallest eigenvalues for R_{i+n} are eigenvalues which equal to the thermal noise power $\lambda_m = \sigma_w^2$. If we consider the (3.3) to the SMI adaptive beamormer, it will be

$$W_{smi}(\phi) = \frac{\alpha}{\hat{\lambda}_{\min}}\{W_q(\phi) - \sum_{m=1}^{N} \frac{\hat{\lambda}_m - \hat{\lambda}_{\min}}{\hat{\lambda}_m}[\hat{q}_m^H v(\phi_s)] \hat{Q}_m(\phi)\} \tag{3.4}$$

where $\hat{\lambda}_m$ is the eigenvalue and $\hat{q}_m$ is the eigenvector of $\hat{R}_{i+n}$, and respectively, $W_q(\phi)$ and $\hat{Q}_m(\phi)$ are the beampatterns of the quiescent weight vector and the *m*th eigenvector eigenbeam for SMI beamformer. The summation part is weighted eigenbeams which place nulls at angles of interferers. The weights for eigenbeams are characterized by the $\frac{(\hat{\lambda}_m - \hat{\lambda}_{\min})}{\hat{\lambda}_m}$ and the noise eigenvectors are selected to fill the residue of the interference-plus-noise space that is not spanned by the interference. In the ideal state, the noise eigenvectors should not affect the beam response since the eigenvalue for the true correlation matrix is $\lambda_m = \lambda_{\min} = \sigma_w^2$. Nevertheless, this relation does not hold for the SMI, because by increasing the number of samples the noise power eigenvalues change. So, the eigenbeams affect the response in a way by their deflection from the noise power. Therefore, as the eigenvalues are random variables that change according to the number of samples, response of beam suffers from the addition of casually weighted eigenbeams and consequently sidelobe level will be higher in the adaptive beampattern. So, to reduce the variation of the eigenvalues, a weighted identity matrix is added to the sample correlation matrix [27].

$$\hat{\mathrm{R}}_{dl} = \hat{\mathrm{R}}_{i+n} + \zeta \mathrm{I} \quad \text{and} \quad \zeta = \sigma_w^2 \tag{3.5}$$

This approach is known as Diagonal Loading. where the ζ is loading factor. This technique adds the loading level to all eigenvalues of correlation matrix which produce a bias in eigenvalues toward decrease their alteration. The diagonally loaded SMI adaptive beamformer is given by

$$w_{LSMI} = \frac{\hat{\mathrm{R}}_{dl}^{-1}\mathrm{v}(\phi_s)}{\mathrm{v}^H(\phi_s)\hat{\mathrm{R}}_{dl}^{-1}\mathrm{v}(\phi_s)} \tag{3.6}$$

It is clear that the diagonal loading method increase variance of the white noise by parameter ζ and it can improve the performance of the SMI adaptive beamformer with random signal array response mismatch [13]. Convergence for LSMI beamformer will be faster even while the number of snapshots is 2 times more than the number of sensors (2N)[21]. However, a main shortcoming of this method is that there is no trustable way to select an appropriate value for the loading factor, because the optimal choice depends on the unknown signal and interference factors [28].

To overcome the main disadvantage of diagonal loading method in [26] an approach has been proposed which attempts to solve the problem by developing the General Linear Combination based (GLC) beamformer.

Actually, when the number of sample size N is small, the sample covariance matrix $\hat{\mathrm{R}}$ is not a proper estimation of the true covariance matrix R. To attenuate this problem, in the GLC-based covariance matrix estimation, which is a shrinkage method [29], we consider a GLC of the sample covariance matrix $\hat{\mathrm{R}}$ and the identity matrix I to acquire a more precise estimate of R instead of $\hat{\mathrm{R}}$:

$$\tilde{\mathrm{R}} = \alpha \mathrm{I} + \beta \hat{\mathrm{R}} \quad \text{which} \qquad \tilde{\mathrm{R}} \geq 0 \tag{3.7}$$

where $\tilde{\mathrm{R}}$ is the improved estimation of R, α and β are the shrinkage parameters. To find the parameters $\tilde{\mathrm{R}}$ is minimized, as proposed in [32], with respect to MSE ($\tilde{\mathrm{R}}$) = $E\{\|\tilde{\mathrm{R}} - \mathrm{R}\|^2\}$. Note that $\alpha \geq 0$ and $\beta \geq 0$, because these guarantee that $\hat{\mathrm{R}} \geq 0$. By minimization of MSE for GLC the shrinkage parameters for M dimension (number of sensors) of array are calculated as:

$$MSE(\tilde{\mathrm{R}}) = \alpha^2 M - 2\alpha(1-\beta) tr(\mathrm{R}) + (1-\beta)^2 \|\mathrm{R}\|^2 + \beta^2 E\{\|\hat{\mathrm{R}} - \mathrm{R}\|^2\} \tag{3.8}$$

So, the optimal value for β and α can be found as

$$\beta_o = \frac{\gamma}{\rho + \gamma} \qquad \text{where} \qquad \gamma = \|\upsilon I - \mathrm{R}\|^2 \tag{3.9}$$

$$\alpha_o = \upsilon(1-\beta_o) = \upsilon \frac{\rho}{\rho + \gamma} \quad \text{where} \quad \rho \triangleq E\{\|\hat{\mathrm{R}} - \mathrm{R}\|^2\}, \upsilon = tr(\mathrm{R})/M \tag{3.10}$$

It should be considered that $\beta_o \in [0,1]$ and $\alpha_o \geq 0$. However α_o and β_o are directly dependent on the indefinite covariance matrix R. Therefore these parameters should be estimated by estimating ρ

$$\hat{\rho} = \frac{1}{N^2}\sum_{n=1}^{N} || \mathrm{x}(n) ||^4 - \frac{1}{N} || \hat{\mathrm{R}} ||^2 \tag{3.11}$$

Consequently, the estimated of α_o and β_o are achieved to guarantee the estimate of β_o is not negative [32]:

$$\hat{\alpha}_o = \min[\hat{\upsilon}\frac{\hat{\rho}}{|| \hat{\mathrm{R}} - \hat{\upsilon} I ||^2}, \hat{\upsilon}] \quad \text{where} \quad \hat{\upsilon} = tr(\hat{\mathrm{R}})/M \tag{3.12}$$

$$\hat{\beta}_o = 1 - \frac{\hat{\alpha}_o}{\upsilon} \tag{3.13}$$

Now, diagonally loaded estimate of covariance matrix can be written as

$$\tilde{\mathrm{R}}_{GLC} = \hat{\alpha}_o \mathrm{I} + \hat{\beta}_o \hat{\mathrm{R}} \tag{3.14}$$

Using the above relation instead of R in the Standard Capon Beamformer will give the GLC based robust adaptive beamformer

$$\mathrm{w}_{GLC} = \frac{\tilde{\mathrm{R}}_{GLC}^{-1}\mathrm{v}(\phi_s)}{\mathrm{v}^H(\phi_s)\tilde{\mathrm{R}}_{GLC}^{-1}\mathrm{v}(\phi_s)} \tag{3.15}$$

by rewriting the equation (3.15) for enhanced GLC based weight vector

$$\tilde{\mathrm{w}}_{GLC} = \frac{[\frac{\hat{\alpha}_o}{\hat{\beta}_o}\mathrm{I} + \hat{\mathrm{R}}]^{-1}\mathrm{v}(\phi_s)}{\mathrm{v}^H(\phi_s)[\frac{\hat{\alpha}_o}{\hat{\beta}_o}\mathrm{I} + \hat{\mathrm{R}}]^{-1}\mathrm{v}(\phi_s)} \tag{3.16}$$

It is clear that the GLC based robust adaptive beamformer is a kind of Diagonal Loading method with loading factor ($\frac{\hat{\alpha}_o}{\hat{\beta}_o}$) which is automatically determined from the data samples $\{\mathrm{x}(n)\}_{n=1}^{N}$.

3.3 Robust Capon Beamformer

3.3.1 Introduction

The standard Capon beamformer (SCB) [26] can be an optimal spatial filter if both the exact covariance matrix and the array steering vector are known. In this case, the array signal-to-interference-plus-noise ratio (SINR) output is maximized and interferences are rejected better. Nevertheless, usually the covariance matrix can be incorrectly estimated due to limited number of data samples, and the knowledge for array steering vector can be inaccurate because of look direction mismatch or differences between presumed signal arrival angle and the actual arrival angle [8]. Whenever these mismatches exist there is performance degradation of SCB. This degradation becomes more serious if the signal-of-interest (SOI) is present in the estimated covariance matrix. Therefore adaptive beamforming encounters small sample size complications and array steering vector errors. On the other hand, if the knowledge of signal-of-interest is imprecise, the performance for the Capon beamformer will be worse than the standard Capon beamformer.

3.3.2 Extension of Capon Beamformer

Now we survey extension of Capon Beamformer when the steering vectors are uncertain [29]. Assume an array including M sensors, and the covariance matrix of the array output vector is R. We consider R that has the following form:

$$\mathrm{R} = \sigma_s^2 v(\phi_s) v^H(\phi_s) + \sum_{p=1}^{P} \sigma_p^2 v(\phi_p) v^H(\phi_p) + Q \qquad (3.17)$$

where σ_s^2 and σ_p^2 are powers of signals impinging on the array; ϕ_s and ϕ_p are the parameters for the positions of sources which emit the signals. $v(.)$ is the array steering vector and Q is the noise covariance matrix given by $Q = \sigma^2 I$ (the covariance matrix has full rank despite of the rest of the terms each of which has rank one). With respect to this explanation, the first term of R is related to the SOI and the remaining term corresponds to the P interferences. To simplify notation, let $v(\phi_s) = v_s$.

The aim of this method is to extend the Capon Beamformer to determine the power of signal-of-interest even when just vague knowledge of its steering vector v_s is available. Specifically, consider that only knowledge about v_s is available which belongs to the uncertainty ellipsoid:

$$[v_s - \bar{v}]^H C^{-1} [v_s - \bar{v}] \leq 1 \tag{3.18}$$

where the $\bar{v}$ and C are given.

When the common formulation for beamforming is utilized to the SCB, it is going to determine the weight vector w_o $(\mathrm{M} \times 1)$ by linearly constrained quadratic problem:

$$\min_w w^H \mathrm{R} w \quad \text{subject to} \quad w^H v_s = 1 \tag{3.19}$$

It gives the solution like

$$w_o = \frac{\mathrm{R}^{-1} v_s}{v_s^H \mathrm{R}^{-1} v_s} \tag{3.20}$$

And to estimate of σ_s^2 by $w_o^H R w_o$ gives the

$$\tilde{\sigma}_s^2 = \frac{1}{v_s^H \mathrm{R}^{-1} v_s} \tag{3.21}$$

The latest RCB methods in [16] when there is uncertainty in v_s, the constraint on $w^H v_s$ in (3.19) is replaced by any vector v in the uncertainty set. Then the obtained w is utilized in $w^H R w$ to estimate the σ_s^2 of SCB. However, in the new approach, the Capon Beamformer problem in [29] is reformulated in a simple form when the uncertainty set is included. By proceeding in this manner, a robust estimate of σ_s^2 is obtained without any prior calculation for weight vector w [29].

In [29] is proved that $\tilde{\sigma}_s^2 = \hat{\sigma}_s^2$ with respect to the

$$\min_w w^H \hat{\mathrm{R}}^{-1} w \qquad \text{subject to} \qquad [v - \bar{v}]^H C^{-1} [v - \bar{v}] \le 1 \tag{3.22}$$

Now if the matrix C is decomposed ($C \succ 0$) and put it in (3.22), it will change to quadratic problem with a quadratic equality constraint [27]:

$$\min_w w^H \hat{\mathrm{R}}^{-1} w \qquad \text{subject to} \qquad || v - \bar{v} ||^2 = \varepsilon \tag{3.23}$$

where ε is defined as the uncertainty level. The solution to the RCB formulation in (3.23) can be obtained by the Lagrange multiplier method:

$$f = v^H \mathrm{R}^{-1} v + \lambda(|| v - \bar{v} ||^2 - \varepsilon) \tag{3.24}$$

By solving this optimization problem, $\hat{v}_s$ is obtained as

$$\hat{v}_s = \bar{v} - (\mathrm{I} + \lambda \mathrm{R})^{-1} \bar{v} \tag{3.25}$$

The Lagrange multiplier λ is obtained by solving the equation $g(\lambda) = || (I + \lambda \mathrm{R})^{-1} \bar{v} ||^2 = \varepsilon$ and then lower and upper bounds of λ are imposed.

To end up, $\hat{v}_s$ is specified by using (3.25), and $\hat{\sigma}_s^2$ is computed by using (3.21) where v_s is replaced with $\hat{v}_s$. Therefore, the main computational complexity of the RCB method arises from the eigendecomposition of the Hermitian matrix. So, the computational complexity of RCB is acceptable compared with the SCB [27]. Once there is estimation for signal-of-interest steering vector, the estimated weight vector can be obtained

$$\hat{w}_o = \frac{\hat{R}^{-1}\hat{v}_s}{\hat{v}_s^H \hat{R}^{-1}\hat{v}_s} = \frac{(R + \frac{1}{\lambda}I)^{-1}\overline{v}}{\overline{v}^H (R + \frac{1}{\lambda}I)^{-1} R (R + \frac{1}{\lambda}I)^{-1}\overline{v}} \quad (3.26)$$

Clearly, robust capon beamformer weight vector is in the form of diagonal loading. Robust Capon Beamformer will not support some problems where the uncertainty set of desired array steering vector utilized to achieve robustness against steering vector mismatches. Specifically, when large steering vector mismatches are present, the uncertainty set must expand to describe the increased error of the desired array steering vector. Hence, the output signal to interference-plus-noise ratio (SINR) for these beamformers is degraded due to their abilities to suppress the interference-plus-noise being weakened.

To overcome this problem, an approach has been proposed by [31] which utilize a small uncertainty sphere to search iteratively for the desired array steering vector. In this technique, the ability of interference-plus-noise suppression of the beamformer can be retained by maintaining its degrees of freedom (DOFs) also by using the modified desired array steering vector. The Iterative Robust Minimum Variance Beamformer (IRMVB) method yields greater output for SINR. By applying a stopping criterion the steering vector calculated by the IRMVB method is not permitted to converge to the steering vectors of the interferences [31].

The concept of the IRMVB (with spherical uncertainty set) is shown in Figure (3.1). When there is a mismatch in steering direction, the desired array steering vector s_o (corresponding to the desired signal direction θ_o) and the assumed array steering vector $\overline{s}_o$ (corresponding to the assumed desired signal direction $\overline{\theta}_o$) do not coincide.

If the errors are large then the size of the uncertainty sphere ε_1, used in (3.23) has to be larger [27]. Hence, the ability of the beamformer to suppress the interference will be weakened due to the increasing of the DOFs. To solve this problem, the IRMVB uses a small uncertainty sphere which is smaller than ε_1 ($\varepsilon_2 \leq \varepsilon_1$) to adjust the steering vector form $\overline{s}_o$ to approach s_o.

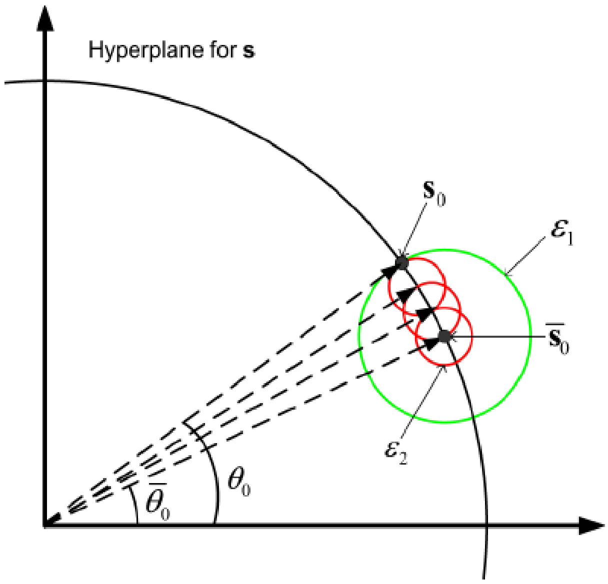

Figure 3.1: Concept of IRMVB Method [31]

This is done by using the constraint (3.23) (with ε_2 in place of ε_1) centered at the assumed desired array steering vector $\overline{s}_o$. At the first iteration $|| s_o - \overline{s}_o ||^2 = \varepsilon_2$ and the RCB is solved for the modified desired array steering vector. After every iteration, the calculated steering vector by IRMVB method is scaled. Again, the spherical constraint is imposed centered at the calculated steering vector of the prior iteration of IRMVB to solve for the following steering vector. This procedure is repeated until the desired array steering vector is reached. This can be achieved by using the stopping criteria. Then, IRMVB weight vector can be calculated by using the converged steering vector by (3.20).

Recently, in some reports [31] for the RCB, it has been mentioned that whenever large steering vector mismatch arises, degradation exists in signal-to-interference-plus-noise ratio (SINR). This is because the ability to suppress the interference is sacrificed whenever radius for the uncertainty sphere is increased (for instance in IRMVB) to have adequate uncertainty level. So, the degradation of SINR becomes substantial when the interferences are dominant. Therefore, having a Robust adaptive beamformer method that is able to maintain its interference suppression capability in the large mismatch case without increasing the radius of the uncertainty sphere is desirable.

The authors in [31] refer to the Iterative RCB with a small fixed uncertainty level as the Fixed Uncertainty Iterative Robust Capon Beamformer (FU-IRCB). Let ε_2 be the representative of the small fixed uncertainty. This technique calculates $\hat{v}$ by solving the RCB optimization iteratively in (3.23) when ε is replaced by ε_2 . The vector $\hat{v}$ is a function of λ with respect to the that ε_2 which is obtained by solving $g(\lambda)=\varepsilon_2$. At each iteration, $\bar{v}$ is updated from $\hat{v}$ of the pervious iteration. The iteration continues until λ reaches a proper small value. The convergence rate of the FU-IRCB depends on how fast λ converges to a small value. Since λ is dependent on the solution of $g(\lambda)=\varepsilon_2$, so it is directly related to the value of ε_2 . Therefore, a larger value of ε_2 will make its value to decrease at a faster rate. On the other hand, with large ε_2 , the interference suppression capability is sacrificed. The other shortcoming of the FU-IRCB is that it is needed a *severe* stopping criterion in order to avoid the convergence of the iteration to one of the strong interference steering vectors. This can be illustrated by the objective function of the RCB optimization in (3.23).

A method has been proposed by [31] which belong to the iterative RCB family with adaptive uncertainty. At every iteration, the uncertainty level is readjusted and then the estimated steering vector is updated for the new uncertainty level. When the uncertainty level approximately becomes zero the iteration converges. The new method is based on the geometric estimated vector for the mismatch. The estimation is according to the concept that the mismatch vector can be decomposed into two kinds of subspaces, which are the signal-plus-interference subspace and the subspace for noise. The signal component is calculated like a function of the projection of the assumed steering vector on signal subspace, while the noise component is calculated from its orthogonal projection.

3.4 Eigenspace Based Beamformer (ESB)

One of the methods of robust adaptive beamforming is the Eigenspace Based Beamformer. In this approach the weight vector is calculated by utilizing the subspace component for signal-plus-interference of the sample correlation matrix, which can degrade the disturbed noise subspace. One common property of this method (ESB) for adaptive beamforming usually is the eigen-decomposition of the steering vector space into subspaces associated with the signal and the noise components. In addition to the fast convergence advantage, the optimal weight vector with respect to the precise steering lies in the signal subspace. This beamformer needs to have prior information about signal subspace component and the number of sources [32] that can be estimated by the method proposed in [33].

If N samples are available, the covariance is calculated by using

$$\hat{R} = \frac{1}{N}\sum_{n=1}^{N} x(n)x^{H}(n)$$

The assumed desired signal steering vector is denoted as $\bar{v}$, v is defined as the true steering vector of the desired signal and the estimated steering vector of the desired signal is denoted as $\hat{v}$. In the eigenspace projection based robust adaptive beamforming, By using the assumed steering vector of desired signal, this method computes the projection of $\bar{v}$ onto the signal-plus- interference subspace, giving a modified estimation of the true desired signal steering vector. So eigendecomposition will be defined by

$$\hat{R} = E_s \Lambda_s E_s^H + E_n \Lambda_n E_n^H \qquad (3.27)$$

where the $N \times (P+1)$ matrix E_s contain the signal-plus-interference subspace eigenvectors and $N \times (N-P-1)$ matrix E_n comprises the noise subspace for $\hat{R}$. Also, the $(P+1) \times (P+1)$ matrix Λ_s includes the eigenvalues corresponding to E_s and Λ_n contains the eigenvalues for E_n respectively. P is the number of interfering signals.

The estimated true desired signal steering vector is calculated by

$$\hat{v} = E_s E_s^H \bar{v} \qquad (3.28)$$

where $E_s E_s^H$ is the projection matrix to the subspace of desired signal-plus-interference and the eigenspace based weight vector is given by

$$w_{ESB} = \alpha \hat{R}^{-1}\hat{v} = \alpha \hat{R}^{-1} E_s E_s^H \bar{v} = \alpha (E_s \Lambda_s^{-1} E_s^H \bar{v}) \qquad (3.29)$$

Recently, the application of eigen-subspace idea has been developed to deal with adaptive array beamforming where observation mismatch occurs [8], [35]. In the case of improper observation, the optimal weight vector established by the eigensubspace method comprises an undesired component existing on the noise subspace. For instance, degradation in performance of array is mostly produced by this undesired component. To overcome this problem, a robust method is offered in [8] by taking the projection of the assumed steering vector onto the steering signal subspace to remove the undesired noise component. The technique is proposed in [36] which adopts a linear combination of the eigenvectors of the signal subspace to eliminate the undesired noise component. Similarly, in the robust method of [35], it is established to find the orthogonal of the correct steering vector to the noise subspace. Another robust technique for Eigenspace based adaptive beamforming presented in [36] wants to remove the undesired component by minimizing the power of array output in the signal subspace.

Nevertheless, there are main disadvantages to utilize the ESB techniques for adaptive beamforming when steering errors are presented. It needs more complex computations to carry out the eigendecomposition for determining the signal subspace. The second one is that, this method is only applicable to the point signal source case, so, it is capable to eliminate the small pointing errors. The third one is that the performance of this approach is restricted when signal-to-noise ratio (SNR) is low.

3.5 General – rank Signals Beamformer

The output for narrowband beamformer is specified by $y(k) = w^H x(k)$ where $x(k) = [x_1(k) \quad x_2(k) \quad ... \quad x_N(k)]$ is the $N \times 1$ array steering vector and $w = [w \quad w \quad ... \quad w_N]^T$ is the $N \times 1$ weight vector of beamformer which has the N array sensors. Respectively the vector of training snapshot is defined by

$\mathrm{x}(k) = \mathrm{s}(k) + \mathrm{i}(k) + \mathrm{n}(k)$ where the $\mathrm{s}(k)$, $\mathrm{i}(k)$ and $\mathrm{n}(k)$ are the desired signal, interference and thermal noise. In the general rank signal case, the SINR is given by [37]

$$SINR = \frac{w(\phi)^H \mathrm{R}_s w(\phi)}{w(\phi)^H \mathrm{R}_{i+n} w(\phi)} \tag{3.30}$$

where

$$\mathrm{R}_s = E\{\mathrm{s}(k)\mathrm{s}^H(k)\} \tag{3.31}$$

$$\mathrm{R}_{i+n} = E\{(\mathrm{i}(k) + \mathrm{n}(k))(\mathrm{i}(k) + \mathrm{n}(k))^H\} \tag{3.32}$$

R_s is the covariance matrix for the general rank source, which is defined as a rank one matrix for a point signal source. However, in most practical conditions, the desired signal is modeled as an incoherently scattered (spatially distributed) source with randomly fluctuating wave-fronts such as in sonar and wireless communications, where rank $\{\mathrm{R}_s\} > 1$ .For instance, in the case of an incoherently scattered source R_s (source covariance matrix) is giving by [38], [39]

$$\mathrm{R}_s = \sigma_s^2 \int_{-\pi/2}^{\pi/2} \rho(\varphi) v(\varphi) v^H(\varphi) d\varphi \tag{3.33}$$

where $\rho(\varphi)$ is the normalized angular power density (i.e., $\int_{-\pi/2}^{\pi/2} \rho(\varphi) d\varphi = 1$) . In fact, the distributed source can be such that the desired signal covariance matrix can have any rank from 1 in to N.

It is important to note that in practical (real) situations, $\rho(\varphi)$ may be unclear [3], and there may be significant mismatches linked with the source location parameters, because their accuracy of estimation depends on how the model is related to $\rho(\varphi)$. Therefore, in the case of spread (incoherently scattered) sources, there might exist a significant mismatch between the assumed and real signal covariance matrices.

In this general case, the optimization problem takes the form

$$\min_{w} w^H \mathrm{R}_{\mathrm{i+n}} w \quad \text{subject to} \quad w^H \mathrm{R}_s w = 1 \tag{3.34}$$

Solution for (3.34) is found by minimization of the function

$$F(w,\lambda) = w^H \mathrm{R}_{\mathrm{i+n}} w + \lambda(1 - w^H \mathrm{R}_s w) \tag{3.35}$$

By solving the above equation by taking the gradient, where λ is the Lagrange multiplier, the following generalized eigenvalue problem is attained [13]

$$\mathrm{R}_{\mathrm{i+n}} w = \lambda \mathrm{R}_s w \tag{3.36}$$

where the λ can be considered as a generalized eigenvalue.The solution of optimization problem (3.34) can be considered in the case of generalized eigenvector corresponding to the smallest generalized eigenvalue of the matrix pencil ($\mathrm{R}_{\mathrm{i+n}}, \mathrm{R}_s$). Multiplying (3.36) by R_{i+n}^{-1} , the equation is written as

$$\mathrm{R}_{i+n}^{-1} \mathrm{R}_s w = \frac{1}{\lambda} w \tag{3.37}$$

which is determined as the characteristic equation to the matrix $\mathrm{R}_{i+n}^{-1} \mathrm{R}_s$. It is clear that the minimum generalized eigenvalue $\lambda_{\min}$ in (3.36) corresponds to the maximum value for eigenvalue $1/\lambda_{\min}$ in equation (3.37). Utilizing this fact, it is concluded that the optimum weight vector for the beamformer is

$$w_{opt} = \mathcal{P}\{\mathrm{R}_{i+n}^{-1} \mathrm{R}_s\} \tag{3.38}$$

where $\mathcal{P}\{.\}$ is the operator that calculates the principal eigenvector of a matrix. Although, in most practical applications the actual matrices $\mathrm{R}_{\mathrm{i+n}}$ and R_s are not available but if the estimate of these matrices is available, equation (3.38) purveys a proper solution to the adaptive beamformer of the general rank source case. In this case we will have a generalized version for the sample matrix inversion beamformer (SMI): $w_{smi} = \mathcal{P}\{\hat{\mathrm{R}}_{i+n}^{-1}\mathrm{R}_s\}$.

In practical cases, the signal covariance matrix is available with some errors. Hence, there is a definite mismatch between the actual signal covariance matrix and assumed signal covariance matrix (R_s). Consequently, in the mismatched situation, we have an unknown Hermitian error matrix (Δ), which describes the effect of the mentioned mismatches on the array response for the desired signal. We have $\bar{\mathrm{R}}_s = \mathrm{R}_s + \Delta \neq \mathrm{R}_s$. To overcome this kind of mismatches, some robust methods has been proposed in [40]

Chapter 4

DIRECTIONAL-RESPONSE-SHAPED BEAMFORMER (DRS)

4.1 Introduction

The goal of this chapter is to introduce our proposed method toward a generalized loading algorithm for adaptive beamforming in uniform linear Array (ULA). In this technique, we have worked on the possibility of applying the principle established in [41] to the adaptive beamforming problems.

The main obstacle to kinds of these filters is inaccuracies caused by limited sample size used in estimating the covariance matrix as well as look direction mismatch. The ability of the directional response shaping (DRS) will follow the modified conventional loading methods for an adaptive ULA. However, it describes the range of specified direction (cut off angle) in the presence of undesired interferences.

4.2 Derivation of the DRS Beamformer Algorithm

Consider the output of an adaptive beamformer, given by

$$y(n) = w^H \mathrm{x}(n) \tag{4.1}$$

where w^H is the adaptive weight vector (bemaformers gain)

$$w = [w_0 \quad w_1 \quad \ldots \quad w_{N-1}]^T \tag{4.2}$$

N is the number of elements (sensors), and $\mathrm{x}(k)$ is the input data vector, as

$$\mathrm{x}(n) = [x_o(n) \quad x_1(n) \quad \ldots \quad x_{N-1}(n)]^T \tag{4.3}$$

The beamformer output power is

$$P_y = E\{|\,\mathrm{y}(n)\,|^2\} = w^H \mathrm{R}_x w \tag{4.4}$$

where $$\mathrm{R}_x = E\{\mathrm{x}(n)\,\mathrm{x}^H(n)\} \tag{4.5}$$

is the correlation matrix of the input data vector $\mathrm{x}(k)$.

As aforementioned in the last chapters the MVDR is obtained by minimizing the output power of the beamformer (cost function) $J_x = w^H \mathrm{R}_x w$ subject to $w^H a(\theta_0) = 1$. However, in the practical case the precise correlation matrix of the input data vector is not available or in most applications a limited number of training data samples are available. Hence, in this technique the hermitian matrix is loaded to the estimated sample correlation matrix, and as a result to all eigenvalues of the correlation matrix, which produce a bias in eigenvalues toward decreasing their alteration to achieve the maximum SINR.So, our cost function will change to the

$$J = w^H (\mathrm{R}_x + \mathrm{R}_a) w = J_x + J_a \tag{4.6}$$

Now, the choice of R_a will be based on weighted noise gain of the array which can be expressed as

$$J_a = \int_{-\pi/2}^{\pi/2} \gamma(\theta_d)\,|\,\mathrm{H}_d(\theta_d)\,|^2 \, d\theta_d \tag{4.7}$$

where $$\mathrm{H}_d(\theta_d) = \sum_{k=0}^{N-1} w_k^* e^{-jk\theta_d} = w^H a(\theta_d) \tag{4.8}$$

is the directional response of the filter at the time k and $\gamma(\theta_d)$ is a weight function that is used to shape the directional response.

$$a(\theta_d) = [1 \quad e^{-j\theta_d} \quad e^{-j2\theta_d} \quad \ldots \quad e^{-j(N-1)\theta_d}\,] \tag{4.9}$$

is the array response vector where $\theta_d = \frac{2\pi d \sin\theta}{\lambda}$, $d = \frac{\lambda}{2}$ so θ_d is defined by $\theta_d = \pi \sin\theta$, where θ is the incidence angle of a plane wave.

Now (4.8) can be written as

$$R_a = \int_{-\pi/2}^{\pi/2} \gamma(\theta_d) a(\theta_d) a^H(\theta_d) d\theta_d \triangleq C \tag{4.10}$$

after substituting (4.11) in (4.7), J_a also can be written as

$$J_a = w^H C w \tag{4.11}$$

where C is a matrix with elements given by

$$C(n,m) = \int_{-\pi/2}^{\pi/2} \gamma(\theta_d) e^{j(m-n)\theta_d} d\theta_d \tag{4.12}$$

Now, if the direction range of the interferences is unknown or if it overlaps with the direction of the signal-of-interest, then in order to avoid suppression of the signal, it might be more plausible to utilize a weight function $\gamma(\theta_d)$ as shown in Fig.4.1.

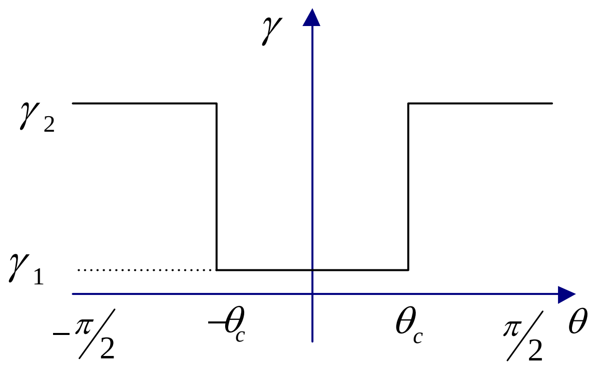

Figure 4.1: Weight Function Versus the θ [41]

The elements of matrix C for the weight function in Fig4.1 are given by

$$C(n,m)=\begin{cases}\frac{2(\gamma_1-\gamma_2)}{(m-n)}\sin((m-n)\theta_c)+\frac{2\gamma_2}{m-n}\sin((m-n)\frac{\pi}{2}) & m\neq n\\ 2\gamma_2(\frac{\pi}{2}-\theta_c)+2\gamma_1\theta_c & m=n\end{cases} \tag{4.13}$$

Hence the beamformer weight vector will be computed by

$$w_{DRS}=\frac{(\hat{\mathrm{R}}_x+\mathrm{C})^{-1}\bar{a}(\theta_d)}{\bar{a}^H(\theta_d)(\hat{\mathrm{R}}_x+\mathrm{C})^{-1}\bar{a}(\theta_d)} \tag{4.14}$$

where $\hat{\mathrm{R}}_x=\frac{1}{M}\sum_{k=1}^{M}x(k)x^H(k)$ and $\bar{a}(\theta_d)$ is the presumed steering vector. Where $\bar{a}(.)$ indicates presumed. The SINR of Directional-Response-Shaped will be given by

$$\mathrm{SINR}_{DRS}=\frac{\sigma_s^2\,|\,w_{DRS}^H a(\theta_d)\,|^2}{w_{DRS}^H\,\mathrm{R}_{i+n}\,w_{DRS}} \tag{4.15}$$

where R_{i+n} is the interference-plus-noise correlation matrix and given by $\mathrm{R}_{i+n}=\mathrm{R}_i+\mathrm{R}_n$, $\mathrm{R}_n=\sigma_n^2\mathrm{I}$, $\mathrm{R}_i=\sigma_i a(\varphi_d)a^H(\varphi_d)$ and $a(\varphi_d)$ is the array response vector for interferences defined by

$$a(\varphi_d)=[1\quad e^{-j\varphi_d}\quad e^{-j2\varphi_d}\quad \ldots\quad e^{-j(N-1)\varphi_d}] \tag{4.16}$$

4.3 Implementation Issues

In fact, for every method which is proposed, the computational complexity is one of the most important issues to consider. It becomes more crucial as the number of sensor is large and/or a large number of snapshots must be used.

The proposed method for beamformers has a computational complexity comparable with the traditional adaptive beamforming algorithms. The computational complexity of our techniques has been shown to be similar to the sample matrix inversion (SMI) with just N additions more. Also, it has a much lower complexity than the robust capon beamformer (RCB) and its iterative versions (IRMVB), since it does not require eigenvalue decomposition (EVD). Our beamformer's complexity is less than those of the Eigen-space based and general rank beamformers, because these techniques also involve the Eigen-decomposition of the correlation matrix.

Chapter 5

SIMULATIONS AND DISCUSSIONS

5.1 Introduction

In this chapter according to experimental methodology we assume four methods (SMI, LSMI, RCB, and IRMVB) for calculating output SINR versus the number of snapshots, output SINR versus the varied SNR, and normalized Beampattern versus the directional arrival. Then we compare our method with these approaches and results by using Tables and Figures.

5.2 Simulation Approach

In the simulation, we evaluate our approach by using Monte Carlo simulations. In all examples, we assume a uniform linear array (ULA) with $N = 10$ Omni-directional sensors spaced by half-wavelength. For each result, to obtain each simulated point, the average of 100 simulation runs is used. Through all scenarios, we assume that there is one desired and two interfering sources. The desired signal is always present in the training data samples, and the interference-to-noise ratio (INR) is considered to be 30 dB for all cases. The presumed steering vector is disturbed by white complex Gaussian noise which has zero mean and variance is supposed to be one in each sensor. The diagonal loading parameter is chosen to be $\lambda = 80$ for the LSMI algorithm in all examples, except the IRMVB (Li's method) which is selected to be $\lambda = 0$. Furthermore, in each example, we select the value of γ_2 that almost provides the optimal performance for our beamformer. The performance of the all techniques is compared with respect to the output SINR. Note that all of simulations have been done under the same conditions. The desired signal and interferers are plane waves which impinge on the array from 4^o, 20^o and 30^o, whereas the direction of assumed signal is 0^o. There is a 4^o look direction mismatch.

5.3 Simulations

The methods that have been evaluated in all simulations are 1) LSMI algorithm 2) benchmark SMI algorithm 3) RCB algorithm 4) IRMVB (Li's) method 5) proposed method, DRS algorithm.

First the performance of the methods is measured versus training data samples (snapshots) $M = 20:500$ for the fixed $SNR = 0(\text{dB})$.

Table 5.1: Performance of beamformers by various training data samples

SNR	0 (dB)												
Number of Snapshots	20	60	100	140	180	220	260	300	340	380	420	460	500
SMI	0.91	5.31	6.81	7.48	7.94	8.23	8.37	8.54	8.64	8.75	8.83	8.9	8.92
LSMI	5.21	6.44	6.78	6.87	6.97	7.03	7.07	7.08	7.09	7.12	7.16	7.15	7.16
RCB	5.29	6.43	6.7	6.82	6.87	6.97	7.02	7.04	7.03	7.05	7.07	7.08	7.09
IRMVB(Li's)	6.83	8.41	8.80	8.97	9.12	9.19	9.28	9.32	9.33	9.37	9.4	9.41	9.42
Proposed Method	6.90	8.46	8.87	9.08	9.19	9.28	9.34	9.37	9.39	9.42	9.46	9.48	9.49
Optimal	9.7	9.7	9.7	9.7	9.7	9.7	9.7	9.7	9.7	9.7	9.7	9.7	9.7

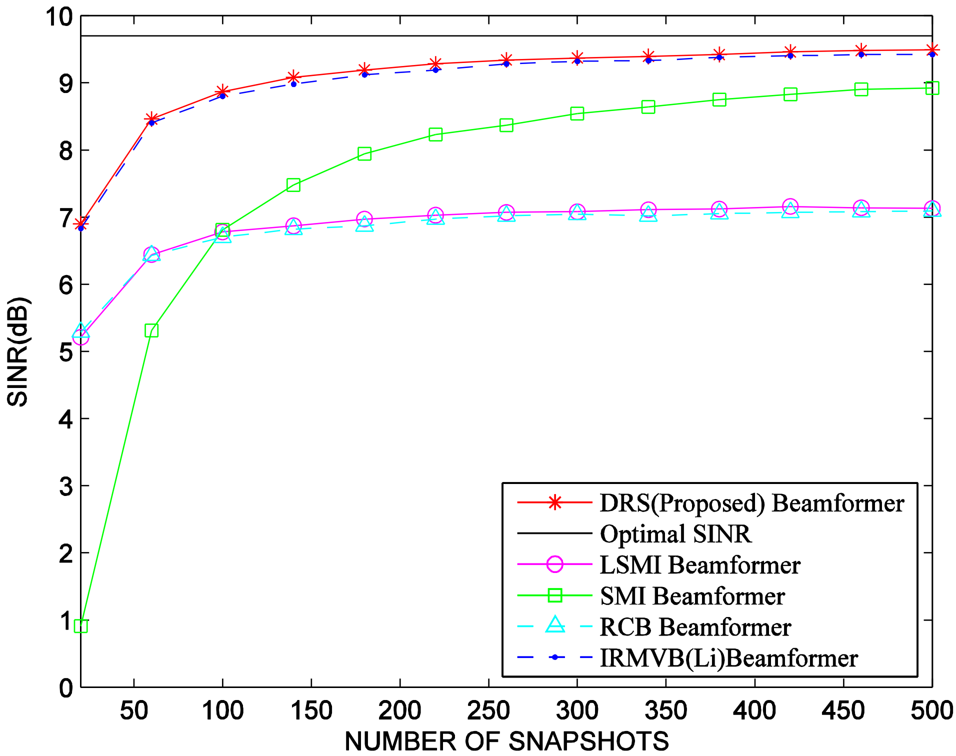

Figure 5.1: Output SINR Versus the Training Data Sample

The performance of the techniques versus the SNR for training data samples taking the values 100,200,300,400,500 are presented in Figure 5.2 up to 5.6.

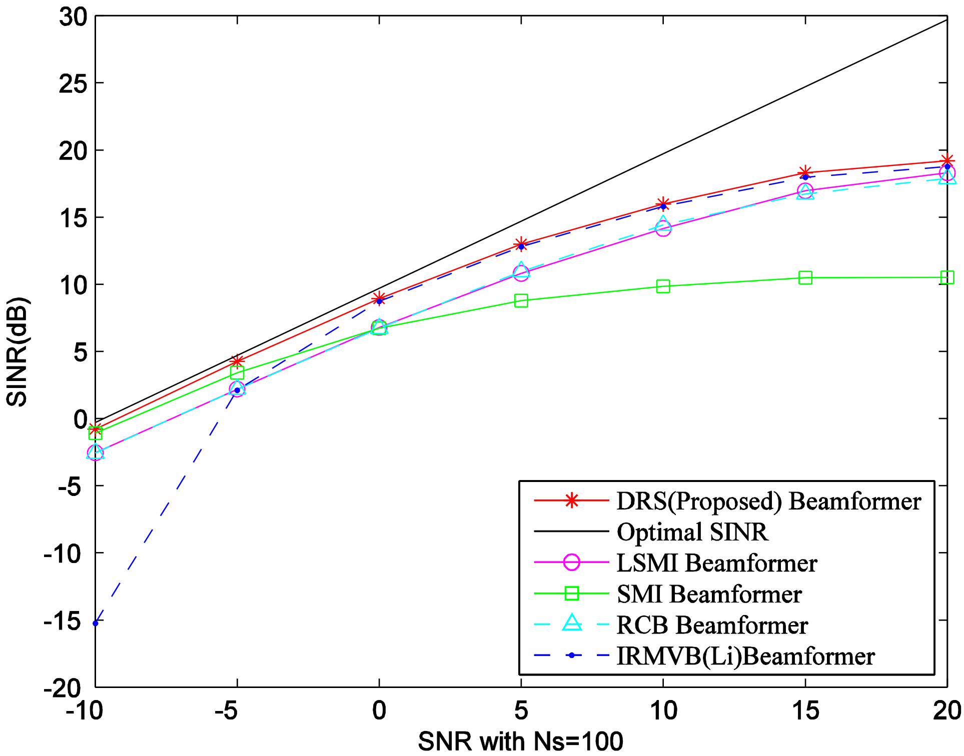

Figure 5.2: Output SINR Versus SNR with Training Data Sample=100

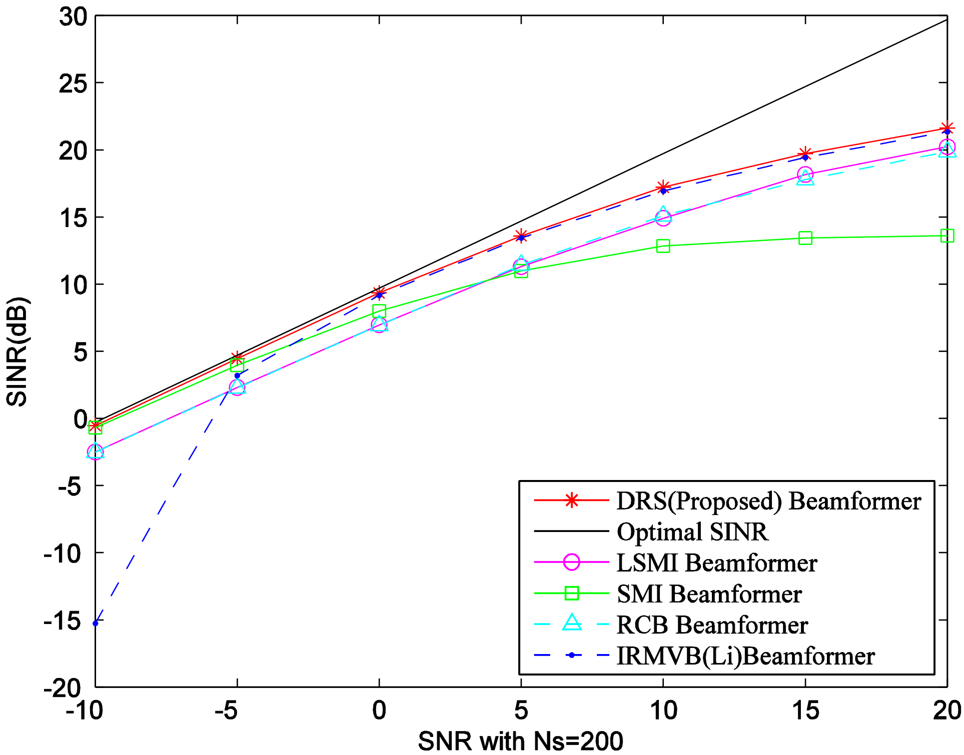

Figure 5.3: Output SINR Versus SNR with Training Data Sample=200

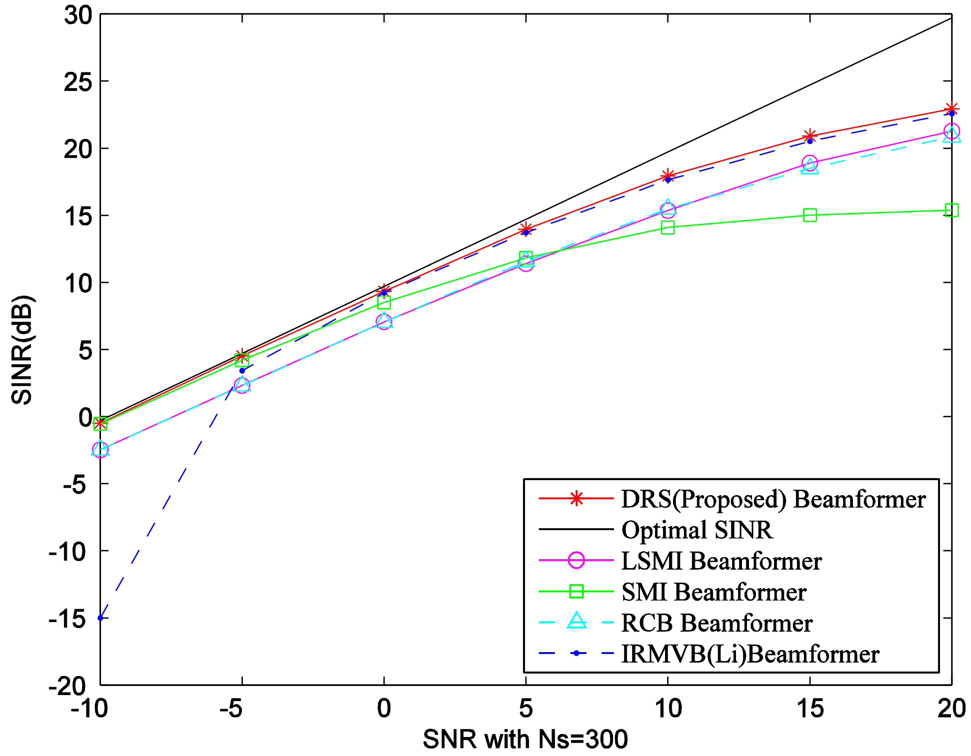

Figure 5.4: Output SINR Versus SNR with Training Data Sample=300

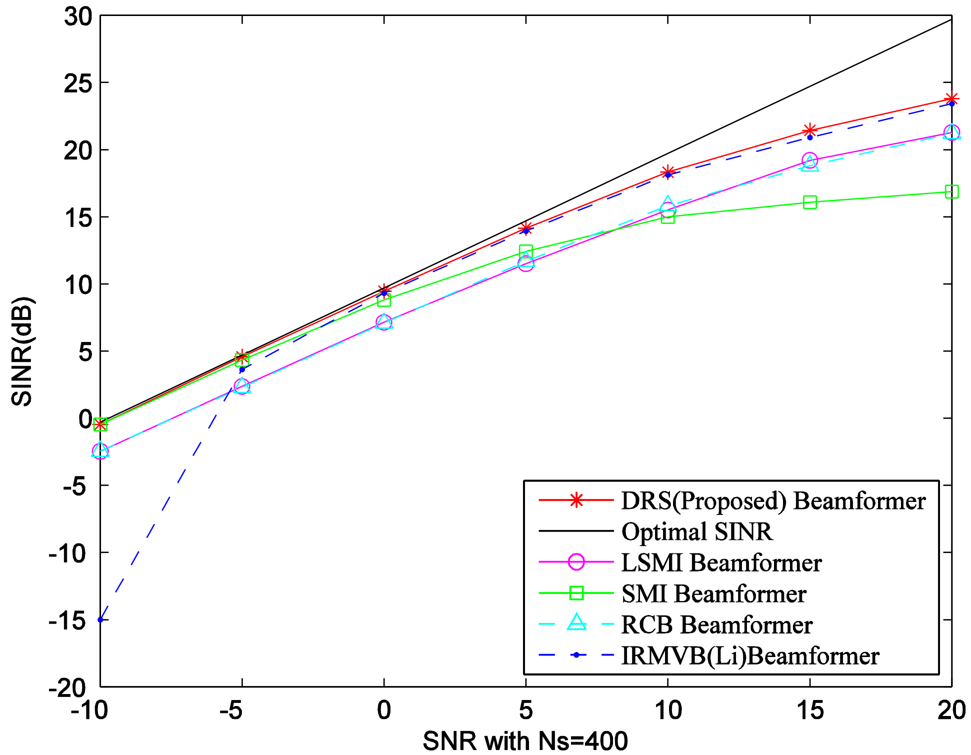

Figure 5.5: Output SINR Versus SNR with Training Data Sample=400

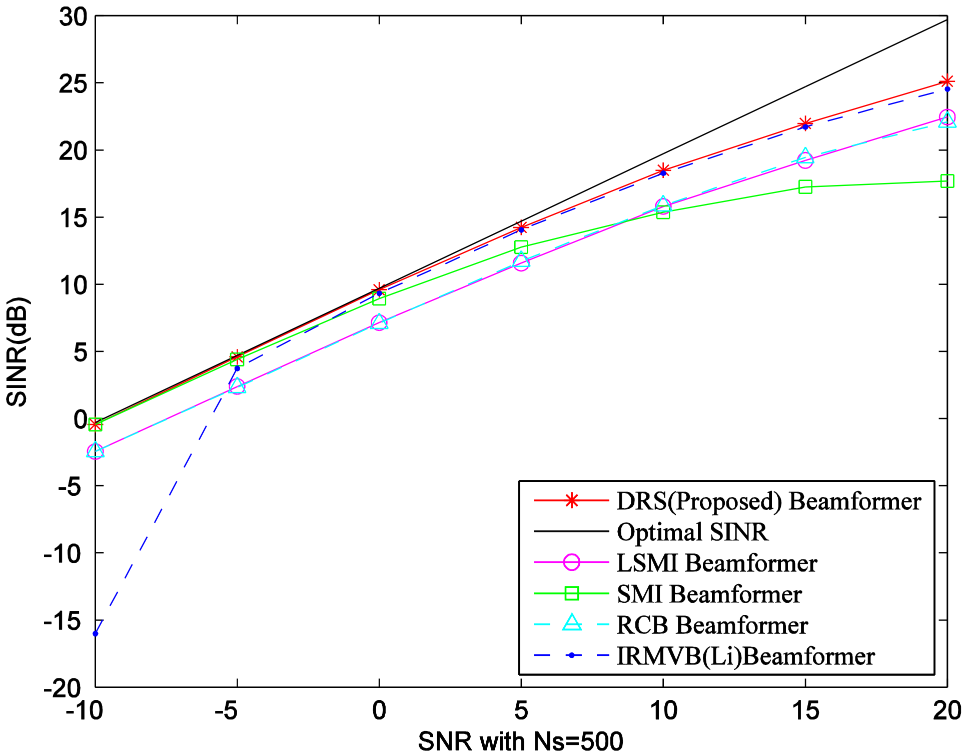

Figure 5.6: Output SINR Versus SNR with Training Data Sample=500

In order to show the ability of proposed method in handling the look direction mismatch, all methods are simulated for the same mismatch ranging from 1^o to 8^o as shown in Figure 5.7.

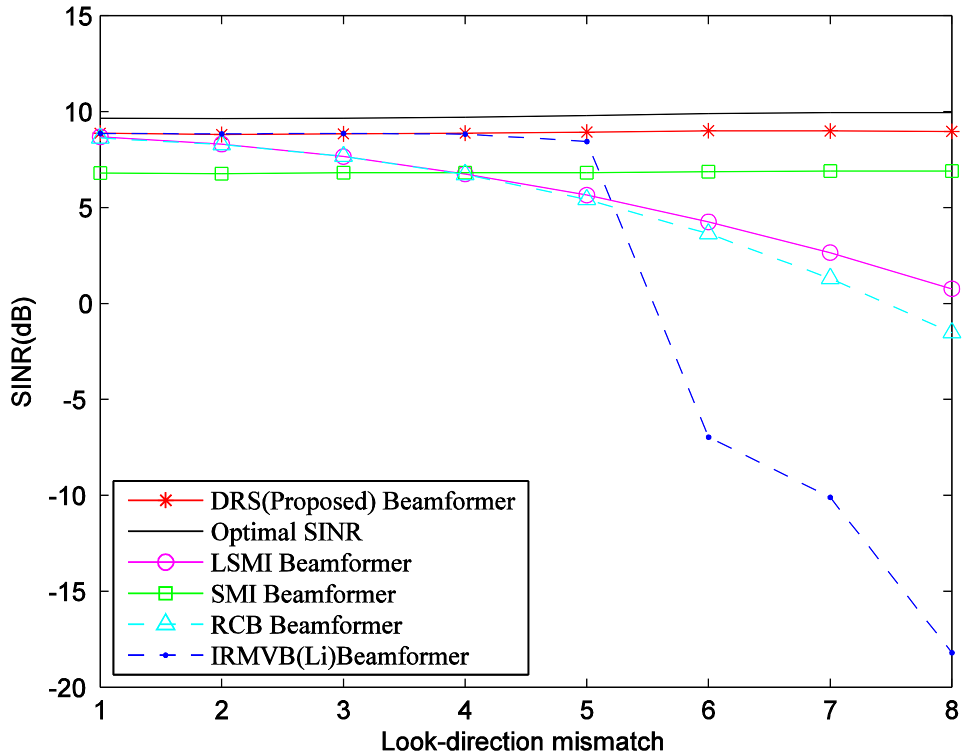

Figure 5.7: Outputs SINR Versus the Look-Direction Mismatch

If the direction range of the interferences is not known, then to avoid suppressing the desired signal, it may be more appropriate to utilize the weight function given in Fig. 4.1 by introducing the cut of angle (θ_c).

Figure 5.8 illustrates the ranges considered for cutoff angles from 7^o to 15^o in which the proposed DRS beamformer is simulated and compared with LSMI and SMI beamformers.

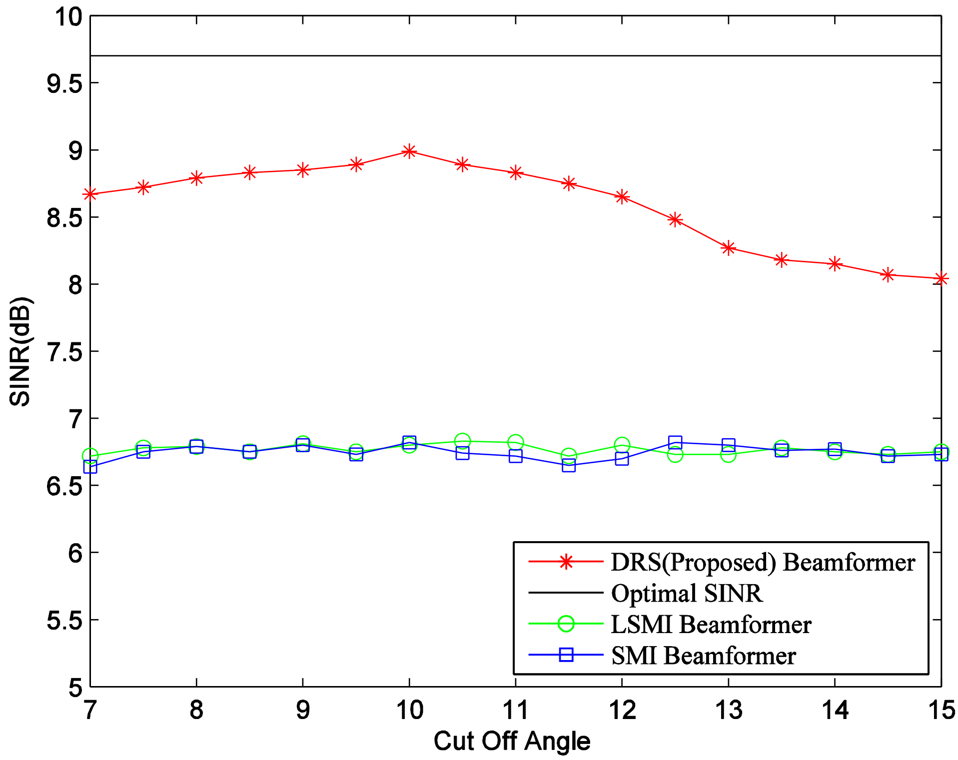

Figure 5.8: Outputs SINR Versus the Cut off Angle

To demonstrate capability of the interference suppression for our algorithm, we have plotted the normalized beampattern in comparison with the SMI benchmark, LSMI, RCB as well as IRMVB (Li's) methods.

We perform the simulation to show the effectiveness of the proposed technique in the presence of steering angle mismatch. A desired signal with SNR=0 dB is impinging on the array with direction angle $=4^o$ (except Figure 5.14 which is in no mismatch) under the 100 data snapshots.

The results for array output beam-patterns by using the DRS beamformer are compared with aforementioned methods in Figure 5.10 to Figure 5.16

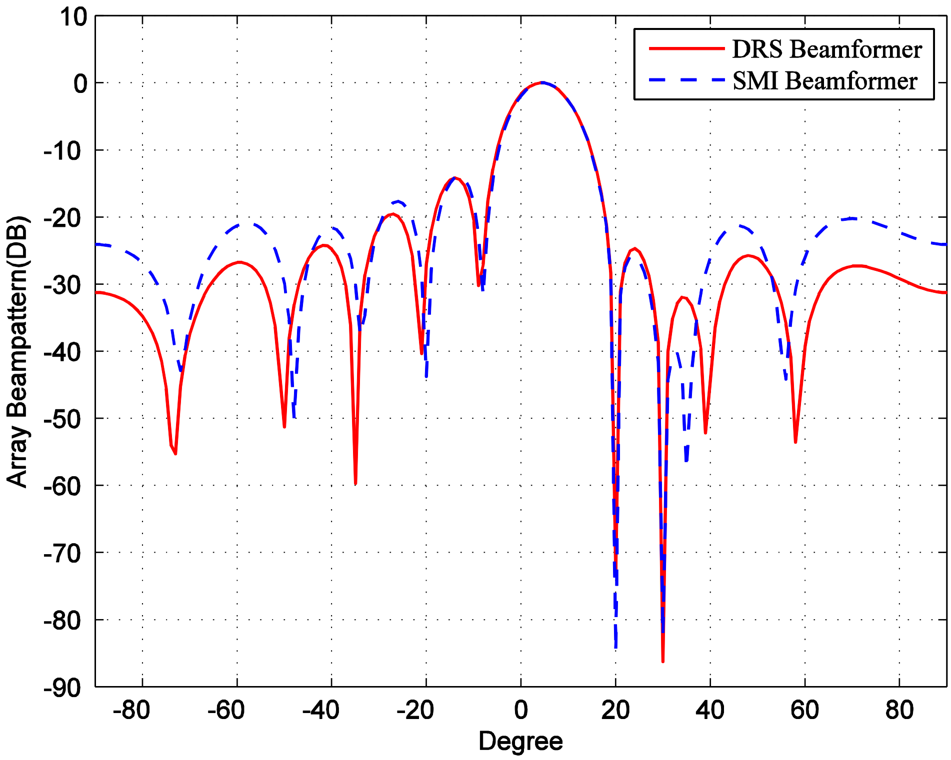

Figure 5.9: Output Beampatterns of Proposed (DRS) and SMI Beamformer

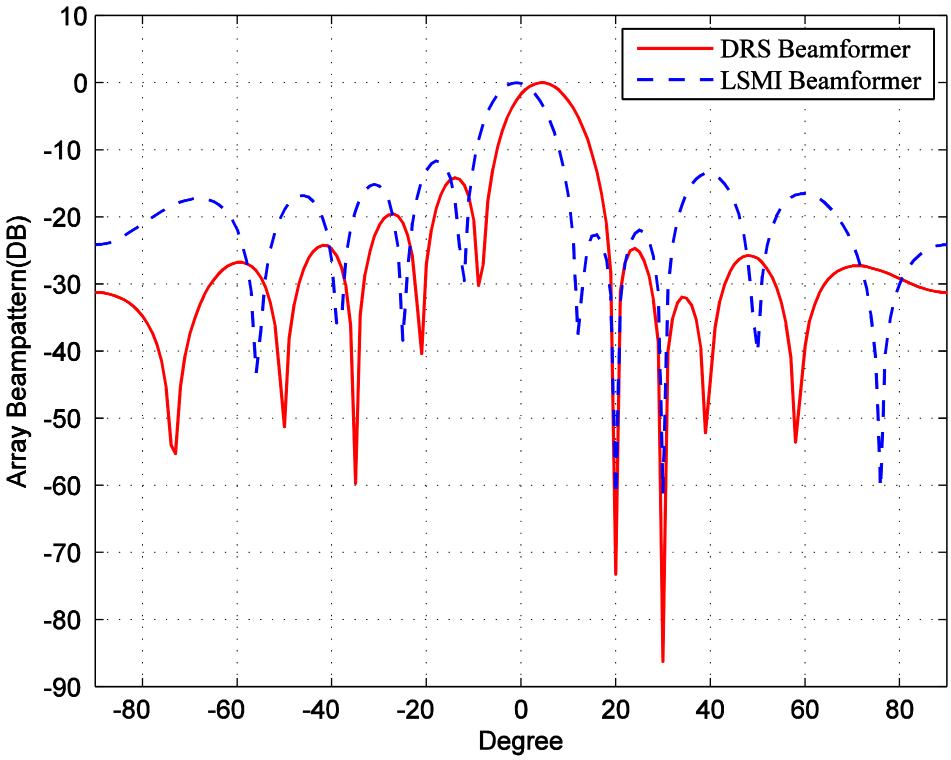

Figure 5.10: Output Beampatterns of Proposed (DRS) and LSMI Beamformer

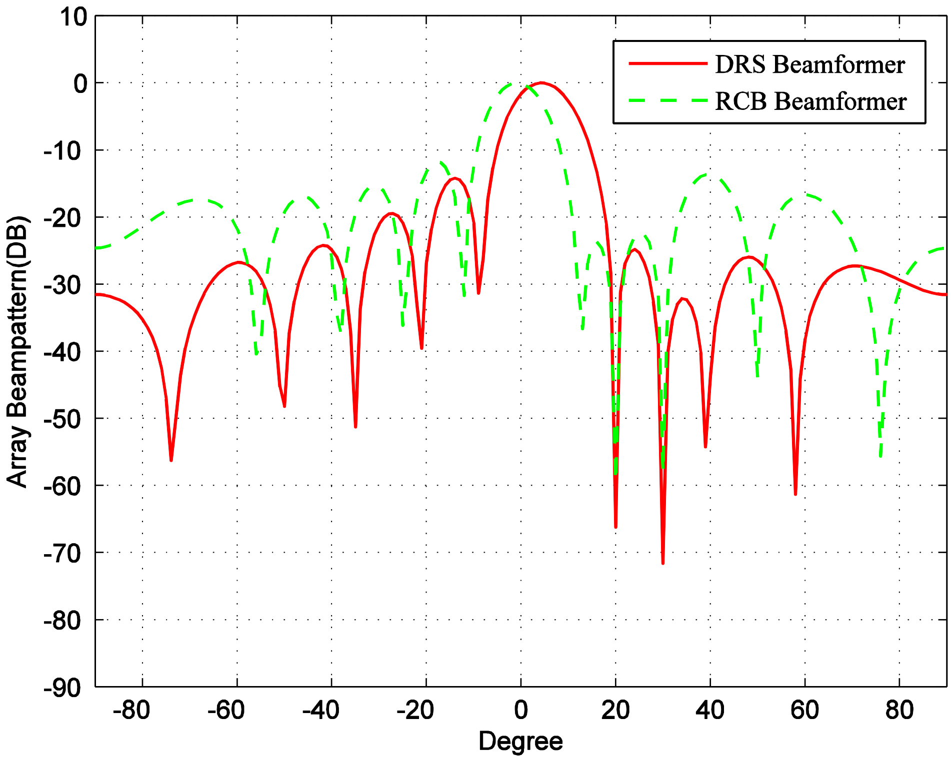

Figure 5.11: Output Beampatterns of Proposed (DRS) and RCB Beamformer

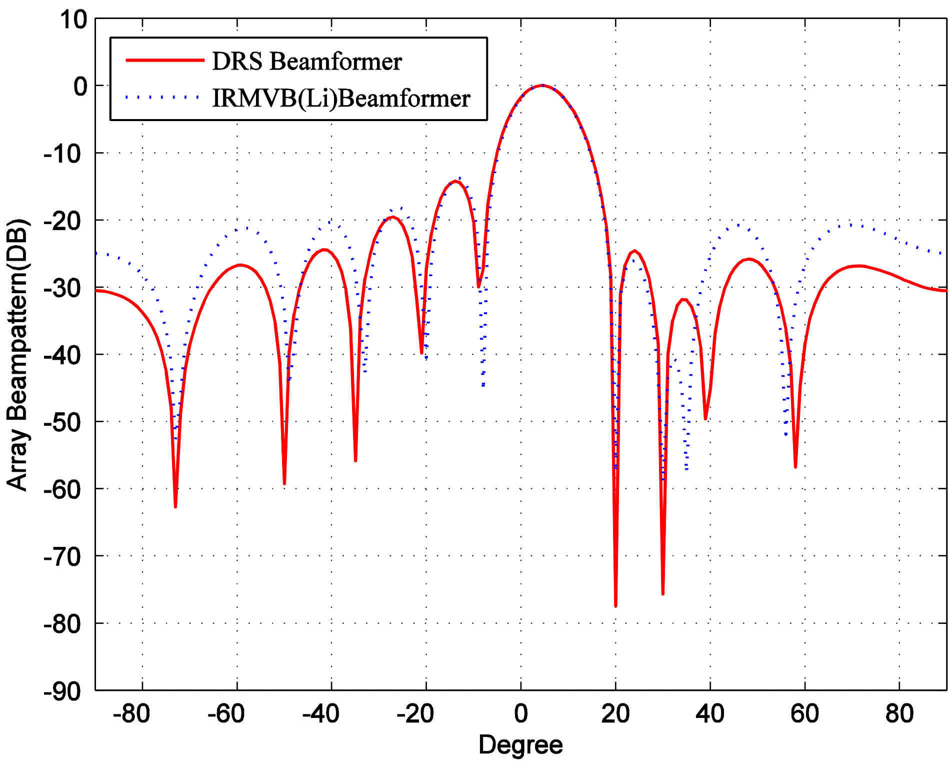

Figure 5.12: Output Beampatterns of proposed (DRS) and IRMVB Beamformer

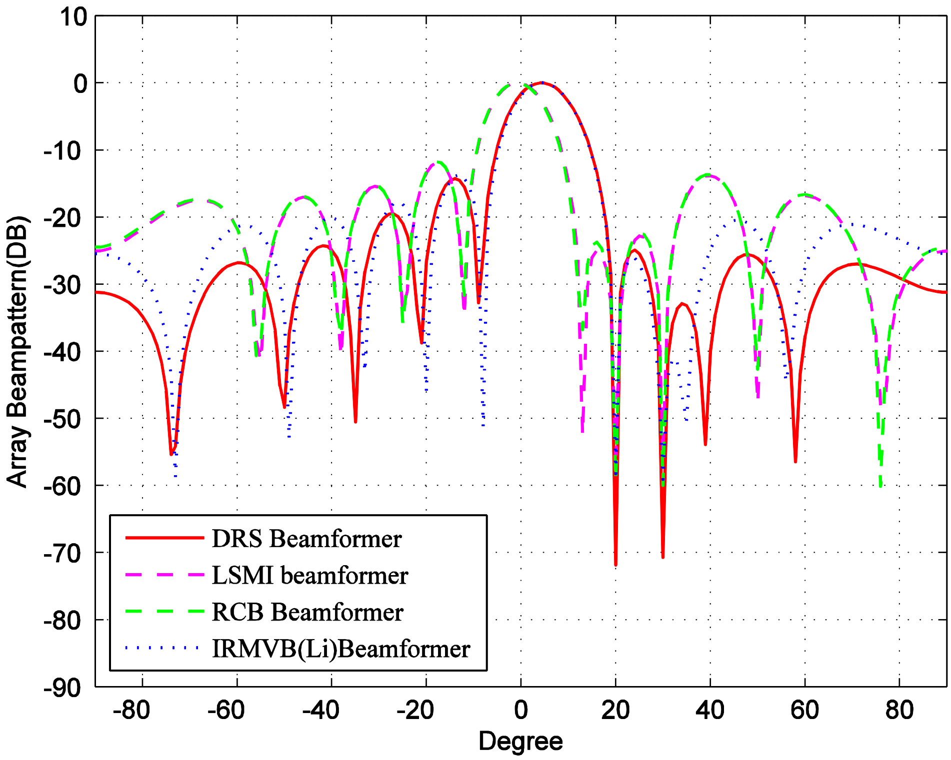

Figure 5.13: Comparing Existing Output Beampatterns with DRS Beamformer

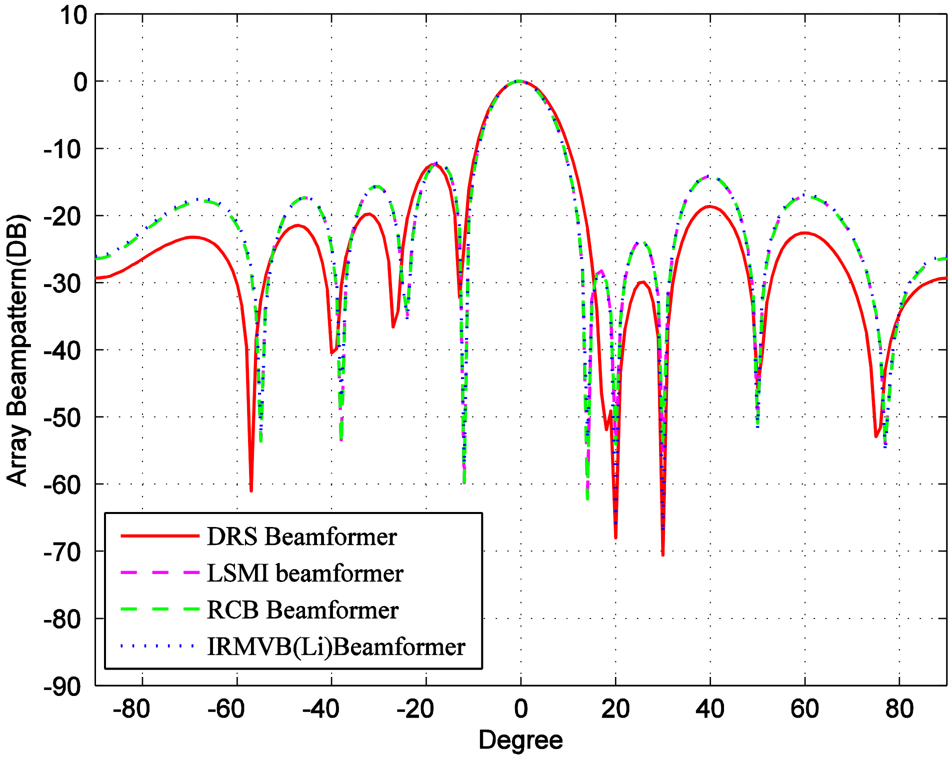

Figure 5.14: Beampatterns with Proposed (DRS) Beamformer without Mismatch

5.4 Discussion

Figures 5.1 to 5.15 clearly illustrate that in all simulations, the proposed beamformer by equation (4.15) gives better performance among the tested methods. The SINR values for the proposed beamformer are close to the optimal values in a wide range of N, SNR and mismatches for direction of arrival (DOA).

To evaluate the convergence of the proposed DRS method, power of SOI is fixed at 1 dB and simulation is iterated up to 500. The rate of convergence of DRS technique is demonstrated by computing the average output SINR at every iteration, Fig. 5.1 shows the output SINR versus the number of snapshots (data training samples) =500. Explicitly it is seen that the output SINR of the DRS method is significantly better than the diagonal loading methods whereas the SMI keeps its level to be improved. However, the IRMVB [31] (Li's) method approaches to the proposed method slightly for all N snapshots.

The aim of the simulations in (Fig 5.2 up to 5.6) is to compare the performance of output SINR against the input SNR varied from -10dB to 20dB. The resulting output SINR for each input SNR is averaged over 100 realizations. It can be considered that the DRS beamformer outperforms than the other tested beamformers at all SNRs. It can be seen that, although, by increasing the SNR, all existing algorithms approach to optimal value of SINR but, the proposed method‘s result is better than the others, whereas, the IRMVB method is acceptable only for SNR$>$0.

The performance of all algorithms is assessed in terms of output SINR (when SNR is fixed at 0 dB and number of snapshots is kept at 100) as shown in Fig 5.7, where the look direction mismatch is increased. The DRS method maintains its performance for all mismatch values. However, the IRMVB (Li's) algorithm drifts apart from the optimal SINR when the look direction mismatch is bigger than 5^o, whereas the rest of the methods yield slightly lower output SINR.

The interference suppression is considered by plotting the normalized beam-pattern with two interferences at 20^o and 30^o, $SNR = 0$ and 4^o mismatch under the 100 data training samples as given on Fig 5.10 − 5.14 to demonstrate the capability of DRS beamformer in comparison with the SMI, LSMI, RCB and IRMVB (Li's) beamformers. In all simulations there is superior interference rejection as a result of shaping the directional response to reach the desired steering vector. The beam-patterns of the proposed DRS method have deeper nulls formed at the interferences' directions of arrival (DOAs). Also, sidelobe levels are higher for all methods than those of the proposed (DRS) method.

One of the advantages of our method is noise suppression even in lower noise levels (SNRs). As a point of view, if Fig 5.10 up to Fig 5.15 are examined carefully, the robustness of the proposed beamformer versus the existing methods is clearly seen.

Fig 5.7 demonstrates that, by increasing the mismatch greater than 5^o, the output SINR for existing methods decrease, however, proposed method keeps it at the same level. This case is also shown by the normalized beampattern at mismatch= 7^o in Figure 5.15

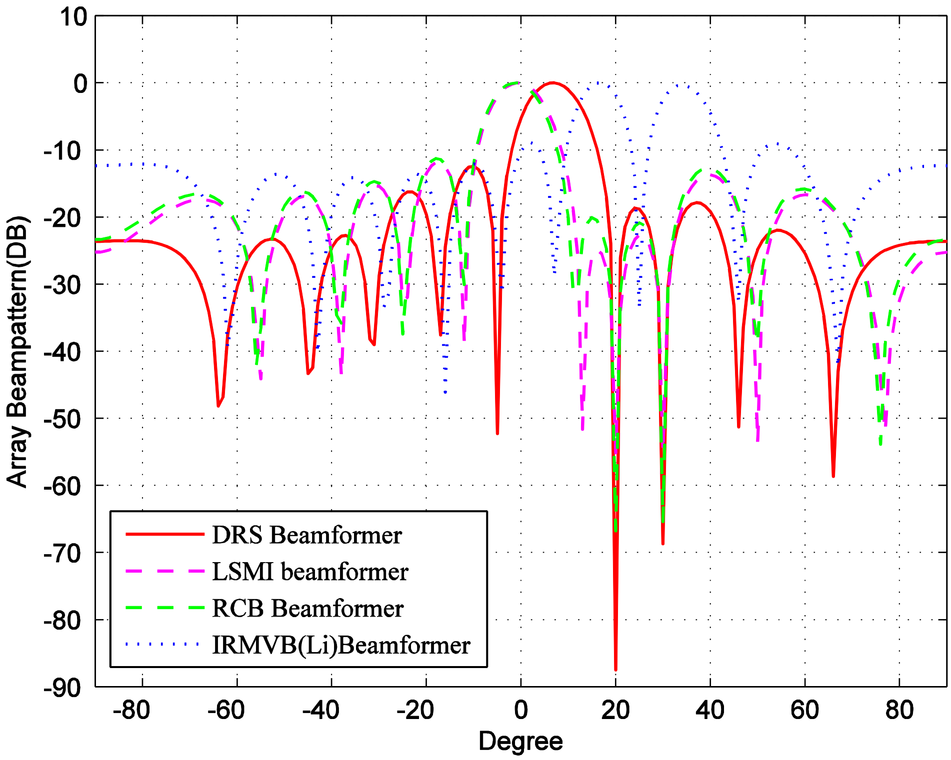

Figure 5.15: Output Beampatterns for all Methods with Mismatch= 7^o

By increasing the mismatch, proposed method follows the true steering vector. However, for IRMVB the main-lobe is disturbed and the side-lobe level gets higher, whereas the other methods can not follow the true steering vector.

Chapter 6

CONCLUSION AND FUTURE WORK

6.1 Conclusion

In this work, we proposed a new approach to robust adaptive beamforming based on the generalized loading of the covariance matrix. This method shows the capability of our algorithm in the presence of direction-of-arrival (DOA) mismatches of the desired signal by shaping the directional response of the adaptive ULA.

The performance of the proposed method is shown to reduce errors (look direction mismatch) between the true and assumed array steering vectors of the desired signal. To reach this aim, a general loading matrix is added to the estimated correlation matrix. This matrix is derived from a weight function which used to shape the directional response of the beamformer.

The computational cost of the proposed algorithm has been shown that is similar to the traditional adaptive beamforming methods. Furthermore, efficient implementations of our method for the processing state have been developed. In addition, numerical examples in terms of SINR and data training show that the proposed technique is robust to sample covariance matrix errors. Also, to evaluate the beamformers the computer simulations are considered in terms of SINR and normalized beam-pattern. It shows that by shaping the directional response, our approach can modify the mismatches in array steering vector for the desired signal.

The efficiency of this method is clarified while the desired signals are accompanied by noise and interferences. The algorithm suppresses the inferences with deeper nulls in the directional of interferences by shaping the directional response of the desired signal. Moreover, the simulation SINR versus the SNR indicates that the proposed method keeps its convergence with respect to the number of snapshots to optimality and is capable to suppress the noises with lower noise level (low SNR).

To end up, in situations with different types of desired signal errors, our beamformer is shown to consistently enjoy a significantly improved performance and faster convergence rate when compared with the existing adaptive beamforming methods.

6.2 Future Work

Extensive simulation studies have indicated that the proposed beamforming method performs better or as good as the existing methods in various scenarios. Theoretical analysis of the proposed method should be performed to verify the results obtained from simulation. Such an analysis will enable the optimal choice of the parameters used in the algorithm.

REFERENCES

[1] L. E. Brennan, J. D. Mallet and I. S. Reed, "Adaptive arrays in airborne MTI radar," *Antennas Propagation,* vol. 24, pp. 607-615, Sept. 1976.

[2] H. Cox, "Resolving power and sensitivity to mismatch of optimum array," *J. Acoust. Soc. Amer,* vol. 54, p. 771–758, 1973.

[3] A. B. Gershman, V. I. Turchin and V. A. Zverev, "Experimental results of localization of moving underwater signal by adaptive beamforming," *Signal Processing,* vol. 43, pp. 2249–2257 Oct. 1995.

[4] Y. Kameda and J. Ohga, "Adaptive microphone-array system for noise reduction," *Acoust., Speech, Signal Processing,* vol. ASSP-34, p. 1391–1400, Dec. 1986.

[5] J. Capon, R. J. Greenfield and R. J. Kolker, "Multidimensional maximum-likelihood processing for a large aperture seismic array," *Proc.IEEE,* vol. 55, p. 192–211, Feb. 1967.

[6] L. C. Godara, "Application of antenna arrays to mobile communications II. Beam-forming and direction-of-arrival considerations," *Proc. IEEE,* vol. 85, p. 1195–1245, Aug. 1997.

[7] T. S. Rapappport, *Smart Antennas: Adaptive Arrays, Algorithms, and Wireless*

Position Location, Piscataway; NJ: IEEE, 1998.

[8] D. D. Feldman and L. J. Griffiths, "A projection approach to robust adaptive beamforming," *Signal Processing,* vol. 42, p. 867–876, Apr. 1994.

[9] M. Wax and Y. Anu, "Performance analysis of the minimum variance beamformer in the presence of steering vector errors," *Signal Processing,* vol. 44, pp. 938–947, Apr. 1996.

[10] E. K. Hung and R. M. Turner, "A fast beamforming algorithm for large arrays," *Aerosp. Electron. Syst,* Vols. AES-19, pp. 598–607,, July 1983.

[11] B. D. Carlson, "Covariance matrix estimation errors and diagonal loading in adaptive arrays," *Aerosp. Electron. Syst,* vol. 24, p. 397–401, July 1988.

[12] L. C. Godara, "Error analysis of the optimal antenna array processors," *Aerosp. Electron. Syst,* vol. AES-22, p. 395–409, July 1986.

[13] A. B. Gershman, "Robust adaptive beamforming in sensor arrays," *Int.J. Electron. Commun,* vol. 53, p. 305–314, Dec. 1999.

[14] H. Cox, R. M. Zeskind and M. H. Owen, "Robust adaptive beamforming," *Acoust., Speech, Signal Processing,* Vols. ASSP-35, p. 1365–1376, Oct. 1987.

[15] J. Riba, J. Goldberg and A. G. Vazquez, "Robust beamforming for interference

rejection in mobile communications," *Signal Processing,* vol. 45, p. 271–275, Jan. 1997.

[16] S. Vorobyov, A. B. Gershman and a. Z. Luo, "Robust adaptive beamforming using worst-case performance optimization: A solution to the signal mismatch problem," *Signal Processing*, vol. 51, p. 313–324, Feb. 2003.

[17] L. Lel, X. Rongqing and L. Gaipeng, "Robust Adaptive beamforming based on Generalized Sidelobe Cancellation," in *CIE international Conference on Radar*, Shanghai , China, Oct. 16-19.

[18] M. H. ER, "Linear array Pattern Synthesis with Prescribed Broad Nulls," *Antennas and Propagation*, vol. 38, no. 9, pp. 1496-148, Dec. 1990.

[19] B. D. Van and K. M. Buckley, "Beamforming : A Versatile approach to Spatial Filtering," *ASSP Signal*, vol. 5, no.2, pp. 4-24 Apr. 1988.

[20] C. Wang and Z. Liu, "Minimum Variance Distortionless Response Beamforming for Tumor Segmentation in MRI," http://ir.lib.ncut.edu.tw/, 2000.

[21] S. A. Vorobyov, "Principles of minimum variance robust adaptive beamforming design," *Elsevier Signal Processing* , vol. 93, no.12, pp. 3264-3277, Aug. 2012.

[22] D. G. Manolakis, *Statistical and Adaptive Signal Processing*, London: Artech

house, 2005.

[23] J. Lee, K. P. Cheng and C. C. Wang, "Robust adaptive array beamforming under steering angle mismatch," *Signal Processing,* vol. 86, pp. 296-309, 2006.

[24] O. Ledoit and M. Wolf, "A well-conditioned estimator for large-dimensional covariance matrices," *Journal of Multivariate Analysis,* vol. 88, p. 365-411, 2004.

[25] J. Capon., "High resolution frequency-wave number spectrum analysis," *Proceedings of IEEE,* vol. 57, p. 1408-1418, Aug.1969.

[26] J. Li and L. Du, "Fully Automatic Computation of Diagonal Loading Levels for Robust Adaptive Beamforming," *Aerospace and Electronic Systems,* vol. 46, no. 1, pp. 449- 458 , 2010.

[27] J. Li and P. Stoica, "On Robust Capon Beamforming and Diagonal Loading," *Siganal Processing,* vol. 51, no. 7, pp. 1702-1715, Jul. 2003.

[28] T. L. Marzetta, "new interpretation for Capon's maximum likelihood method of frequency-wavenumber spectrum estimation," *Acoust., Speech, Signal Processing,* vol. ACSSP-31, p. 445–449, Apr. 1983.

[29] P. Stoica, Z. Wang and A. J. Li, "Robust Capon beamforming," *Signal Processing Lett,* vol. 10, no. 6, pp. 172-175, Jun. 2003.

[30] J. P. Lie, W. Ser and C. M. See, "Adaptive Uncertainty Based Iterative Robust Capon Beamformer Using Steering Vector Mismatch Estimation," *signal processing*, vol. 59, no. 9, pp. 4483-4488, Sep . 2011.

[31] S. E. Nai, W. S and Z. L. Yu, "Iterative Robust Minimum Variance Beamforming," *Sgnal Processing*, vol. 59, no. 4, pp. 1601-1611, Apr. 2011.

[32] C. L. Chang, "Performance of DMI and Eigenspace-Based Beamformers," *Antennas and Propagation*, vol. 40, no. 11, pp. 1336-1347, Nov. 1992.

[33] H. Akaike, "Information theory and an extension of the maximum likelihood principle, New York, *Spring New York*, 1998.

[34] W. S. Youn and C. K. Un, " Robust adaptive beamforming based on the eigenstructure method," *Signal Processing*, vol. 42, p. 1543–1547, june .1994.

[35] J. W. Kim and C. K. Un, "A robust adaptive array based on signal subspace approach," *Signal Processing*, vol. 41, p. 3166–3171, Nov. 1993.

[36] L. Chang and C. Yeh, "Effect of pointing errors on the performance of the projection beamformer," *IEEE Trans. on Antennas Propagat*, vol. 41, p. 1045–1056, Aug. 1993.

[37] R. A. Monzingo and T. W. Miller, "Introduction to Adaptive Arrays," in *Wiley*,

New York, 1980.

[38] Y. Meng, P. Stoica and K. M.Wong, "Estimation of the directions of arrival of spatially dispersed signals in array processing," *Proc. Inst. Elect Eng. Radar, Sonar Navigat*, vol. 143, p. 1-9, Feb. 1996.

[39] O. Besson and P. Stoica, "Decoupled estimation of DOA and angular spread for a spatially distributed source," *Signal Processing,* vol. 48, p. 1872–1882, July .2000.

[40] S. Shahbazpanahi, A. B. Gershman and Zhi.Quan. Luo, "Robust Adaptive Beamforming for General-Rank Signal Models," *Signal Processing*, vol. 51, no. 9, pp. 2257-2269, Sep.2003.

[41] O. Kokrer and A. Hocanin, "Frequency-response-shaped LMS adaptive filter," *Elsevier Digital Signal Processing,* pp. 855-869, Aug . 2006.