Hawking Radiation of Spin-1 Particles From Three Dimensional Rotating Hairy Black Hole

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Abstract

In the present article, we study the Hawking radiation (HR) of spin-1 particles – so-called vector particles – from a three dimensional ($3D$) rotating black hole with scalar hair (RBHWSH) using Hamilton-Jacobi (HJ) ansatz. Putting the Proca equation amalgamated with the WKB approximation in process, the tunneling spectrum of vector particles is obtained. We recover the standard Hawking temperature corresponding to the emission of these particles from RBHWSH.

I. INTRODUCTION

One of the most radical predictions of general relativity (GR) is the existence of black holes (BHs). According to the seminal works of Hawking [1–3], BHs are not entirely black. That was the surprising claim made by Hawking over forty years ago. Examining the behavior of quantum fluctuations around the event horizon of a BH, Hawking substantiated the theory that BHs radiate thermal radiation, with a constant temperature – so-called Hawking temperature – directly proportional to the surface gravity κ, which is the gravitational acceleration experienced at the BH's horizon:

$$T_H = \frac{\hbar\kappa}{2\pi}, \tag{1}$$

with the system of units $c = G = k_B = 1$. The works of Hawking and Bekenstein [4] – and of others [5–15], rederiving T_H in various ways – brings together the normally disparate areas of GR, quantum mechanics (QM), and thermodynamics. Enthusiasm for understanding of the underlying coordinations between these subjects of physics creates ample motivation for the study of HR (see for instance [16–27] and references therein).

Quantum fluctuations create a virtual particle pair near the BH horizon . While the particle with negative energy tunnels into the horizon (absorption), the other one having positive energy flies off into spatial infinity (emission) and produces the HR. Applying the WKB approximation, the emission and absorption probabilities of the tunneling particles give the tunneling rate Γ as [12, 28, 29]

$$\Gamma = \frac{P_{emission}}{P_{absorption}} = \exp(-2ImS) = \exp\left(-\frac{E_{net}}{T}\right), \tag{2}$$

where S is the action of the classically forbidden trajectory of the tunneling particle, which has a net energy E_{net} and temperature T. One of the methods for finding S is to use the HJ method. This method is generally performed by substituting a suitable ansatz, considering the symmetries of the spacetime, into the relativistic HJ equation. The resulting radial integral always possesses a pole located at the event horizon. However, using the residue theory the associated pole can be analytically evaded [30].

Recently, within the framework of the HJ method, the HR of spin-1 particles described by the Proca equation in $3D$ non-rotating static BHs has been studied by Kruglov [28]. These spin-1 particles are in fact the vector particles like Z and $W^{\pm}$ bosons, and they have significant role in the Standard Model [31]. Based on Kruglov's study [28], Chen et al. [32] have very recently investigated the HR of these bosons in the rotating BTZ geometry. Similar to the works of [28, 32], here we aim to study the HR of the vector particles in the $3D$ RBHWSH [33–36]. These BHs are solutions to the action in $3D$ Einstein gravity that is non-minimally coupled to scalar field ϕ. In the limit of $\phi = 0$, RBHWSH is nothing but the rotating BTZ BH [33, 37].

This paper is organized as follows. Sec. 2 introduces the geometrical and thermodynamical features of the $3D$ RBHWSH spacetime. In Sec. 3, we study the Proca equation for a massive boson in this geometry. Then, we employ the HJ method with the separation of variables technique to obtain the HR of the RBHWSH. Finally, in Sec.4, we present our remarks.

II. $3D$ RBHWSH SPACETIME

The action in a $3D$ Einstein gravity with a non-minimally coupled scalar field reads [33]

$$\mathcal{I} = \frac{1}{2}\int d^3x\sqrt{-g}\left[R - g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \xi R\phi^2 - 2V(\phi)\right], \tag{3}$$

where the coupling strength ξ between gravity and the scalar field is $1/8$. Furthermore, the scalar potential $V(\phi)$ is given by

$$V(\phi) = -\Lambda + \frac{1}{512}\left(\Lambda + \frac{\beta}{B^2}\right)\phi^6 + \frac{1}{512}(\frac{a^2}{B^4})\frac{(\phi^6 - 40\phi^4 + 640\phi^2 - 4608)\,\phi^{10}}{(\phi^2-8)^5}, \tag{4}$$

in which the parameters β, B, and a are integration constants, and Λ is the cosmological constant. The line-element of RBHWSH is given by

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{1}{f(r)}\mathrm{d}r^2 + r^2\left[\mathrm{d}\theta + \omega(r)\mathrm{d}t\right]^2, \tag{5}$$

with the metric functions:

$$f(r) = -M\left(1 + \frac{2B}{3r}\right) + r^2\Lambda + \frac{(3r+2B)^2 J^2}{36r^4}, \tag{6}$$

$$\omega(r) = -\frac{(3r+2B)\,J}{6r^3}, \tag{7}$$

where J is the angular momentum of the BH. The scalar field is represented by

$$\phi = \pm\sqrt{\frac{8B}{r+B}}. \tag{8}$$

It is worth noting that RBHWSH can be reduced to the rotating BTZ BH solution when $B = 0$ [33–36]. Following [35, 36], one can see that the mass, the Hawking temperature, the Bekenstein-Hawking entropy, and the angular velocity of the particle at the horizon of this BH are given by

$$M = \frac{J^2 l^2 \left(2B + 3r_+\right)^2 + 36r_+^6}{12l^2 r_+^3 \left(2B + 3r_+\right)}, \tag{9}$$

$$T_H = \frac{f'(r_+)}{4\pi} = \frac{\left(B + r_+\right)\left[36r_+^6 - J^2 l^2 (2B + 3r_+)^2\right]}{24\pi l^2 r_+^5 \left(2B + 3r_+\right)}, \tag{10}$$

$$S_{BH} = \frac{A_H}{4G}[1 - \xi\phi^2(r_+)] = \frac{4\pi r_+^2}{B + r_+}, \tag{11}$$

$$\Omega_H = -\omega(r_+) = \frac{\left(3r_+ + 2B\right) J}{6r_+^3}, \tag{12}$$

where $\Lambda = \frac{1}{l^2}$, and r_+ is referred to as the event horizon of the BH. In order to carry out analysis into finding r_+ values, we impose the condition of $f(r_+) = 0$, which yields a particular cubic equation. The solutions of that cubic equation are also given in detail by [35]. One can verify that the first law of thermodynamics:

$$dM = T_H dS_{BH} + \Omega_H dJ. \tag{13}$$

holds. On the other hand, calculation of the specific heat using $C_J = T_H \left(\frac{\partial S_{BH}}{\partial T_H}\right)_J$ proves that RBHWSH is locally stable when $r_+ > r_{ext}$. Here, r_{ext} represents the radius of an extremal RBHWSH that yields $T_H = 0$ [35].

III. HR OF SPIN-1 PARTICLES FROM RBHWSH

As described by Kruglov [28], the Proca equation for the massive vector particles having the wave function Φ is given by

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\Phi^{\nu\mu}) + \frac{m^2}{\hbar^2}\Phi^\nu = 0, \tag{14}$$

where

$$\Phi_{\nu\mu} = \partial_\nu \Phi_\mu - \partial_\mu \Phi_\nu. \tag{15}$$

Let us set the vector function as

$$\Phi_\nu = (c_0, c_1, c_2) \exp\left[\frac{i}{\hbar} S(t, r, \theta)\right], \tag{16}$$

and assume that the action is given by

$$S(t, r, \theta) = S_0(t, r, \theta) + \hbar S_1(t, r, \theta) + \hbar^2 S_2(t, r, \theta) + ... \tag{17}$$

According to the WKB approximation, we can further set

$$S_0(t, r, \theta) = -Et + L(r) + j\theta + \Bbbk, \tag{18}$$

where E and j are the energy and angular momentum of the spin-1 particles, respectively, and $\Bbbk$ is a (complex) constant. Substituting Eqs. (15), (16), (17), and (18) into Eq. (14) and considering the leading order in $\hbar$, we obtain a 3×3 matrix (let us say Ξ matrix) equation: $\Xi\,(c_1, c_2, c_3)^T = 0$ (the superscript T means the transition to the transposed vector). Thus, one can read the non-zero components of Ξ as follows

$$\begin{aligned}
\Xi_{11} &= A_1 - j^2, \\
\Xi_{12} &= \Xi_{21} = -A_2 \partial_r L(r), \\
\Xi_{13} &= \Xi_{31} = A_1 \omega(r) - jE, \\
\Xi_{22} &= \left(m^2 r^2 + j^2\right) f(r) - A_2, \\
\Xi_{23} &= \Xi_{32} = \partial_r L(r) \left[A_2 \omega(r)^2 - j f(r)^2\right], \\
\Xi_{33} &= \frac{A_1}{r^2} \left[f(r) - \omega(r)^2 r^2\right] - E^2,
\end{aligned} \tag{19}$$

where

$$A_1 = r^2 \left\{m^2 + f(r) \left[\partial_r L(r)\right]^2\right\}, \tag{20}$$

$$A_2 = r^2 f(r) \left[E + j\omega(r)\right]. \tag{21}$$

Upon the fact that any homogeneous system of linear equations (19) admits nontrivial solution if and only if $det\Xi = 0$, we obtain

$$det\Xi = \frac{m^2}{r^6}[A_1 + j^2 - \frac{A_2^2}{r^2 f(r)^3}]^2 = 0. \tag{22}$$

Solving for $L(r)$ yields

$$L_{\pm}(r) = \pm \int \sqrt{\frac{[E + \omega(r)j]^2 - f(r)(m^2 + \frac{j^2}{r^2})}{f(r)^2}} dr. \tag{23}$$

One can immediately observe from the above that when $\omega(r) = 0$, it reduces to the Kruglov's solution [28]. The L_+ corresponds to outgoing (moving away from the BH) spin-1 particles and L_- stands for the ingoing (moving towards the BH) spin-1 particles. The imaginary part of $L_{\pm}(r)$ can be calculated by using the pole deployed at the horizon. According to the complex path integration method via the Feynman's prescription [30] (see [12] for a similar process), we have

$$ImL_{\pm}(r) = \pm \frac{\pi}{f'(r_+)} E_{net}, \tag{24}$$

where

$$E_{net} = E + E_0 = E + \omega(r_+)j = E - j\Omega_H. \tag{25}$$

Therefore, the probabilities of the vector particles crossing the horizon out/in become

$$P_{emission} = \exp\left(-\frac{2}{\hbar} ImS\right) = \exp[-\frac{2}{\hbar}(ImL_+ + Im\mathbb{k})], \tag{26}$$

$$P_{absorption} = \exp\left(\frac{2}{\hbar} ImS\right) = \exp[-\frac{2}{\hbar}(ImL_- + Im\mathbb{k})]. \tag{27}$$

According to the classical definition of the BH, any outside particle certainly falls into the BH. So, we must have $P_{absorption} = 1$, which results in $Im\mathbb{k} = -ImL_-$. On the other hand, $L_+ = -L_-$ so that the total probability of radiating particles (as a consequence of QM) is

$$\Gamma = P_{emission} = \exp\left(-\frac{4}{\hbar} ImL_+\right) = \exp\left(-\frac{4\pi}{f'(r_+)} E_{net}\right). \tag{28}$$

Thus, comparing Eq. (28) with Eq. (2) we can recover the correct Hawking temperature (10) of RBHWSH:

$$T \equiv T_H = \frac{f'(r_+)}{4\pi} = \frac{(B + r_+)\left[36r_+^6 - J^2l^2(2B + 3r_+)^2\right]}{24\pi l^2 r_+^5 (2B + 3r_+)}. \tag{29}$$

IV. CONCLUSION

In this paper, we have used the Proca equation to compute the tunneling rate of outgoing vector particles across the event horizon of axially symmetric static rotating $3D$ RBHWSHs. For this purpose, we have ignored the back-reaction effects and substituted the HJ ansatz into the associated Proca equations. In the derivation of the tunneling rate within the framework of the WKB approximation, the calculation of the imaginary part of the action was the most important point. Using the complex path integration technique, we have shown that the tunneling rate is given by Eq. (26). The latter result allowed us to recover the standard Hawking temperature for RBHWSH.

Finally, a study about the vector particles in the higher dimensional BHs may reveal more information compared to the present case. This is going to be our next problem in the near future.

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