Gravitinos Tunneling From Traversable Lorentzian Wormholes

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Abstract Recent research shows that Hawking radiation (HR) is also possible around the trapping horizon of a wormhole. In this article, we show that the HR of gravitino (spin-3/2) particles from the traversable Lorentzian wormholes (TLWH) reveals a negative Hawking temperature (HT). We first introduce the TLWH in the past outer trapping horizon geometry (POTHG). Next, we derive the Rarita-Schwinger equations (RSEs) for that geometry. Then, using both the Hamilton-Jacobi (HJ) ansätz and the WKB approximation in the quantum tunneling method, we obtain the probabilities of the emission/absorption modes. Finally, we derive the tunneling rate of the emitted gravitino particles, and succeed to read the HT of the TLWH.

Keywords Hawking Radiation, Gravitino, Quantum Tunneling, Lorentzian Wormhole, Spin-3/2 Particles.

1 Introduction

An interesting phenomenon that corresponds to spontaneous emissions (as if a black body radiation) from a black hole (BH) is the HR. It is a semi-classical outcome of the quantum field theory (Hawking 1975, 1976). HR dramatically changed our way of looking to the BHs; they are not absolutely black and cold objects, rather they emit energy with a characteristic temperature: HT. Event horizon, where is an irreversible point (in classical manner) for any object including photons is the test-bed of the gedanken experiment for the HR. The studies concerning this phenomenon have been carrying on by using different methods. In particular, the quantum tunneling (Parikh & Wilczek 2000) of particles with different spins from the various BHs have gained momentum in the recent years (the reader may be referred to (Vanzo et al. 2011; Jing 2003; Kerner & Mann 2006, 2008a,b; Yale & Mann 2009; Yang et. al. 2014; Sharif & Javed 2013a; Kruglova 2014; Li & Ren 2008; Sharif & Javed 2013b; Ran 2014; Chen et. al. 2015a; Sakalli et. al. 2014; Sakalli & Ovgun 2015b,a; Gecim & Sucu 2015; Jan & Gohar 2014; Chen et. al. 2015b; Singh et. al. 2014; Dehghani 2015; Sakalli et. al. 2012) and references cited therein). Recently, it has been shown that HR of the bosons with spin-0 (scalar particles) and spin-1 (vector particles) from the TLWH (Morris & Thorne 1988), which is a bridge or tunnel between different regions of the spacetime is possible by using the POTHG (Gonzalez-Diaz 2010; Martin-Moruno & Gonzalez-Diaz 2009; Sakalli & Ovgun 2015c). Wormhole has been extensively studied in different areas (Garattini 2015; Kuffittig 2015; Rahaman et. al. 2014a,b, 2015; Halilsoy et. al. 2014). However, HT of the TLWH appears to be negative because of the phantom energy (exotic matter: the sum of the pressure and energy density is negative) that supports the broadness of the wormhole throat (Morris & Thorne 1988). In addition, it is a well-known fact that the virtual particle-antiparticle pairs are created near the horizon. In a BH spacetime the real particles with positive energy and temperature are emitted towards spatial infinity (Wald 1976). However, in the POTHG which is analog to the white hole geometry, the antiparticles come out from the horizon (Helou 2015a). In other words, our analysis predicts that the energy spectrum of the antiparticles leads to a negative temperature for the TLWH. For the subject of the white hole radiation, the reader may refer to (Peltola & Makela 2006).

As it is shown by Caldwell et al (Caldwell et. al. 2003), the dark matter (DM) (Hurst et. al. 2015)

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could have a phantom energy. In this regard, the phantom energy can keep apart every bound object until the Cosmos eventuates in the Big-Rip (Chimento & Lazkoz 2004). On the other hand, DM does not emit, reflect or absorb light, making it not just dark but entirely transparent. But if the DM particles strolling around a BH or a wormhole can produce gamma-rays would give a possibility to study the radiation of this mysterious matter (Liew 2013; Allahverdi et. al. 2015). DM has many candidates, and gravitino (spin-3/2) is one of them (Kawasaki & Moroi 1995; Davidson et. al. 2008). Gamma-ray decay of the gravitino DM has been very recently studied in (Allahverdi et. al. 2015). So, HR of the gravitinos from the BHs and/or wormholes could make an impact on the production of the DM. Behaviors of the gravitino's wave function are governed by the RSEs (Yale & Mann 2009; Corley 1999). So, our main motivation in this paper is to investigate the HR of the gravitino tunneling from the TLWH geometry. Using the RSEs and HJ method, we aim to regain the standard HT of the TLWH.

The structure of this paper is as follows. In Sec. II, we introduce the 3+1 dimensional TLWH (Martin-Moruno & Gonzalez-Diaz 2009) and analyzes the RSEs for the gravitino particles in the POTHG of the TLWH (Hayward 1994, 1998, 2009; Misner & Sharp 1964; Aminneborg 1998). We show that the RSEs are separable when a suitable HJ änsatz is employed. Then the radial equation can be reduced to a coefficient matrix equation that makes us possible to compute the probabilities of the emission/absorption of the gravitinos. Finally, we calculate the tunneling rate of the radiated gravitinos, and retrieve the standard HT of the TLWH. We summarize and discuss our results in Sec. III.

2 Quantum Tunneling of Gravitinos From 3+1 Dimensional TLWH

For the wave equation of the gravitino (spin-3/2) particles, we start with the massless (the mass has no remarkable effect in the computation of the quantum tunneling (Yale & Mann 2009)) RSEs (Corley 1999; Majhi & Samanta 2010; Chen & Huang 2015; Chen et. al. 2013):

$$i\gamma^{\nu}\left(D_{\nu}\right)\Psi_{\mu}=0, \tag{1}$$

$$\gamma^{\mu}\Psi_{\mu}=0, \tag{2}$$

where $\Psi_{\mu} \equiv \Psi_{\mu a}$ is a vector-valued spinor and the γ^{μ} matrices satisfy $\{\gamma^{\mu},\gamma^{\nu}\} = 2g^{\mu\nu}$. The first equation is the Dirac equation applied to every vector index of Ψ, while the second is a set of additional constraints to ensure that no ghost state propagates; that is, to ensure that Ψ represents only spin-3/2 fermions, with no spin-1/2 mixed states (Yale & Mann 2009; Majhi & Samanta 2010).

The covariant derivative obeys

$$D_{\mu}=\partial_{\mu}+\frac{i}{2}\Gamma_{\mu}^{\alpha\beta}J_{\alpha\beta}, \tag{3}$$

where

$$\begin{aligned}\Gamma_{\mu}^{\alpha\beta}&=g^{\beta\gamma}\Gamma_{\mu\gamma}^{\alpha},\\ J_{\alpha\beta}&=\frac{i}{4}\left[\gamma^{\alpha},\gamma^{\beta}\right],\\ \{\gamma^{\alpha},\gamma^{\beta}\}&=2g^{\alpha\beta}\times I.\end{aligned} \tag{4}$$

The metric of TLWH in the generalized retarded Eddington-Finkelstein coordinates (REFCs), which is the POTHG, is given by (Martin-Moruno & Gonzalez-Diaz 2009)

$$ds^2=-Fdu^2-2dudr+r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right), \tag{5}$$

where $F=1-2M/r$. Misner-Sharp energy is represented by $M=\frac{1}{2}r(1-\partial^{a}r\partial_{a}r)$ which becomes $M=\frac{1}{2}r_h$ on the trapping horizon (r_h) (Misner & Sharp 1964). Marginal surfaces having $F(r_h)=0$ are the past marginal surfaces in the REFCs (Gonzalez-Diaz 2010).

For solving the RSEs, we use the following Dirac γ-matrices:

$$\gamma^{u}=\frac{1}{\sqrt{F}}\begin{pmatrix}-i & -\sigma^3\\ -\sigma^3 & i\end{pmatrix},\quad \gamma^{r}=\sqrt{F}\begin{pmatrix}0 & \sigma^3\\ \sigma^3 & 0\end{pmatrix},$$
$$\gamma^{\theta}=\frac{1}{r}\begin{pmatrix}0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix},\quad \gamma^{\phi}=\frac{1}{r\sin\theta}\begin{pmatrix}0 & \sigma^2\\ \sigma^2 & 0\end{pmatrix}, \tag{6}$$

where the Pauli matrices are given by

$$\sigma^1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\sigma^2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},\sigma^3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}. \tag{7}$$

Gravitino wave function (ψ) has two spin states [spin up (i.e. positive $r-$direction) and spin down (i.e. negative $r-$direction)]:

$$\psi_{\nu\uparrow}=\left(a_{\nu},0,c_{\nu},0\right)e^{\frac{i}{\hbar}S_{\uparrow}(u,r,\theta,\phi)}, \tag{8}$$

$$\psi_{\nu\downarrow}=\left(0,b_{\nu},0,d_{\nu}\right)e^{\frac{i}{\hbar}S\downarrow(u,r,\theta,\phi)}, \tag{9}$$

where $S(u,r,\theta,\phi)$ denotes the gravitino action which is going to be expanded in powers of $\hbar$, and $a_\nu, b_\nu, c_\nu, d_\nu$ are the arbitrary constants. Here we shall only consider the spin up case, since the spin down case is fully analogous with it. The action for the spin-up states can be chosen as follows

$$S_\uparrow(u,r,\theta,\phi) = S_{\uparrow 0}(u,r,\theta,\phi) + \hbar S_{\uparrow 1}(u,r,\theta,\phi) + \hbar^2 S_{\uparrow 2}(u,r,\theta,\phi) + ... \tag{10}$$

Therefore, the corresponding RSEs become

$$\frac{1}{\sqrt{F}}\left[(ia_0 - c_0)\left(\partial_u S_{\uparrow 0}\right)\right] + \sqrt{F}(-c_0 \partial_r S_{\uparrow 0}) = 0, \tag{11}$$

$$\frac{1}{r\mathrm{sin}\theta}(-ic_0 \partial_\phi S_{\uparrow 0}) + \frac{1}{r}(-c_0 \partial_\theta S_{\uparrow 0}) = 0, \tag{12}$$

$$\frac{1}{\sqrt{F}}\left[(-a_0 - ic_0)\left(\partial_u S_{\uparrow 0}\right)\right] + \sqrt{F}(-a_0 \partial_r S_{\uparrow 0}) = 0, \tag{13}$$

$$\frac{1}{r\mathrm{sin}\theta}(-ia_0 \partial_\phi S_{\uparrow 0}) + \frac{1}{r}(-a_0 \partial_\theta S_{\uparrow 0}) = 0, \tag{14}$$

with the constraints equations:

$$\frac{-a_0 + c_0}{\sqrt{F}} + \sqrt{F}c_1 + \frac{d_2}{r} - \frac{id_3}{r\mathrm{sin}\theta} = 0 \tag{15}$$

$$\frac{-d_0}{\sqrt{F}} - \sqrt{F}d_1 + \frac{c_2}{r} + \frac{ic_3}{r\mathrm{sin}\theta} = 0 \tag{16}$$

$$\frac{a_0}{\sqrt{F}} + \sqrt{F}a_1 + \frac{b_2}{r} - \frac{ib_3}{r\mathrm{sin}\theta} = 0 \tag{17}$$

$$\frac{-b_0 + id_0}{\sqrt{F}} - \sqrt{F}b_1 + \frac{a_2}{r} + \frac{ia_3}{r\mathrm{sin}\theta} = 0 \tag{18}$$

Equations (15-18) are not important here. Because these equations give an independent wave solution, so that they have no effect on the action (Yale & Mann 2009).

Afterwards, the separation of variables method is applied to the action $S_{\uparrow 0}(u,r,\theta,\phi)$:

$$S_{\uparrow 0} = Eu - W(r) - j_\theta \theta - j_\phi \phi + K, \tag{19}$$

where E and (j_θ, j_ϕ) are energy and angular constants, respectively. However, K is an arbitrary complex constant. Thus, Eqs. (11-14) reduce to

$$-\frac{ia_0}{\sqrt{F}}E + \frac{c_0}{\sqrt{F}}E - c_0\sqrt{F}W' = 0, \tag{20}$$

$$\frac{-c_0}{r}\left(j_\theta + \frac{i}{\mathrm{sin}\theta}j_\phi\right) = 0, \tag{21}$$

$$\frac{a_0}{\sqrt{F}}E + \frac{ic_0}{\sqrt{F}}E - a_0\sqrt{F}W' = 0, \tag{22}$$

$$\frac{-a_0}{r}\left(j_\theta + \frac{i}{\mathrm{sin}\theta}j_\phi\right) = 0. \tag{23}$$

Equations (21) and (23) are about the solutions of (j_θ, j_ϕ), and they do not have contribution to the tunneling rate. For this reason, we simply ignore them. Namely, the master equations for the tunneling rate are Eqs. (20) and (22). To analyze them, we first consider the case of $a_0 = ic_0$ (Hui-Ling & Shu-Zheng 2009). Using Eqs. (20) and (22), we now have a solution for $W(r)$ as

$$W_1 = \int \frac{2E}{F}dr. \tag{24}$$

The integrand has a simple pole at $r = r_h$. Choosing the contour as a half loop going around this pole from left to right and integrating, one obtains

$$W_1 = \frac{i2\pi E}{F'(r_h)} = \frac{i\pi E}{\kappa|_H}. \tag{25}$$

where $\kappa|_H = \partial_r F/2|_{r=r_H}$ is the surface gravity at the horizon. On the other hand, if one sets $a_0 = -ic_0$, this time Eqs. (20) and (22) admit the following solution for $W(r)$:

$$W_2 = 0. \tag{26}$$

Hence, we can derive the ingoing/outgoing imaginary action solutions as

$$\mathrm{Im}\, S_1 = \mathrm{Im}\, W_1 + \mathrm{Im}\, K, \tag{27}$$

$$\mathrm{Im}\, S_2 = \mathrm{Im}\, W_2 + \mathrm{Im}\, K = \mathrm{Im}\, K. \tag{28}$$

We can now set S_1 for the action of absorbed (ingoing) gravitinos. We can tune their probability:

$$\Gamma_{in} = \exp(-2\,\mathrm{Im}\, S_1), \tag{29}$$

to %100 by letting

$$K = -\frac{i\pi E}{\kappa|_H}. \tag{30}$$

Consequently, the probability of the emitted (outgoing) gravitinos becomes

$$\Gamma_{out} = \exp(-2\,\mathrm{Im}\, S_2) = \exp(\frac{2\pi E}{\kappa|_H}). \tag{31}$$

Recalling the definition of the tunneling rate:

$$\Gamma = \frac{\Gamma_{out}}{\Gamma_{in}} = \exp(\frac{2\pi E}{\kappa|_H}), \tag{32}$$

which is also equivalent to the Boltzmann factor: $\Gamma = \exp(-E/T)$, we read the HT of the TLWH as follows

$$T_H = -\frac{\kappa|_H}{2\pi}, \tag{33}$$

which is a negative temperature. This result implies that if the trapping horizon remains in the past outer region, the wormhole throat would have a negative temperature (Martin-Moruno & Gonzalez-Diaz 2009). The phantom energy, which is the special case of the exotic matter could be the reason of that negative temperature (Gonzalez-Diaz 2010; Martin-Moruno & Gonzalez-Diaz 2009; Sakalli & Ovgun 2015c; Gonzalez-Diaz 2004; Saridakis et. al 2009; Velten et. al. 2013; Helou 2015b). On the other hand, when $K = 0$ in the action (19), it is possible to obtain the positive temperature: $T_H = +\frac{\kappa|_H}{2\pi}$. Although, the latter remark contradicts with the previous results (Martin-Moruno & Gonzalez-Diaz 2009; Sakalli & Ovgun 2015c) (and whence, one may easily get rid of the case of $K = 0$), however Hong and Kim (Hong & Kim 2006) showed that possibility of negative/positive temperature of the wormhole depends on the exotic matter distribution.

3 Conclusion

In this work, we have studied the HR of the gravitino particles from the TLWH in 3+1 dimensions. TLWH has been introduced in the POTHG. We have analyzed the RSEs in the background of the TLWH with the help of HJ method. The probabilities of the emitted/absorbed gravitino particles from the trapped horizon of the TLWH have been computed. After comparing the obtained tunneling rate with the Boltzmann factor, we have recovered the standard HT of the TLWH, which is a negative temperature. This is the special condition in which the high-energy states are more occupied than lower-energy states (Braun et. al. 2013). Another possibility of the negative temperature may originate from the exotic matter distribution of the wormhole (Hong & Kim 2006). Meanwhile, very recently it has been claimed by Helou (2015a) that HR does not occur in the POTHG. In fact, the latter debatable remark is based on the study of Firouzjaee & Ellis (2015) stating that cosmic matter flux may turn the HR off. On the other hand, Hayward show that switching off the radiation causes the wormhole to collapse to a Schwarzschild BH (Hayward 2002).

In summary, gravitinos can tunnel through wormhole [simply this can be thought as a wormhole with one entrance (BH) and one exit (white hole)]. In such a case, gravitinos tunnel from the BH with positive temperature, while they tunnel through the white hole with negative temperature. Thus our calculations are based on the exit of the wormhole, just as the white hole case. Besides, we have shown that positive temperature can be obtained by tuning the K-constant in the action (19) to zero. However, the latter result is a debatable issue, and it demands much deeper analysis. This will be our next venture in this line of study.

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