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RINDLER TYPE ACCELERATION IN $f(R)$ GRAVITY

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By choosing a fluid source in $f(R)$ gravity, defined by $f(R) = R - 12a\xi \ln|R|$, where a (Rindler acceleration) and ξ are both constants, the field equations correctly yield the Rindler acceleration term in the metric. We identify domains in which the weak energy conditions (WEC) and the strong energy conditions (SEC) are satisfied.

Keywords: $f(R)$ gravity; Rindler acceleration; exact solutions.

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Rindler acceleration is known to act on an observer accelerated in flat spacetime. Geometrically such a spacetime is represented by $ds^2 = -x^2\, dt^2 + dx^2 + dy^2 + dz^2$, where the acceleration in question acts in the x-direction.[1] The reason that this acceleration has become popular in recent years anew is due to an analogous effect detected in the Pioneer spacecrafts launched in 1972/1973. Observation of the spacecrafts over a long period revealed an attractive, mysterious acceleration toward Sun, an effect came to be known as the Pioneer anomaly.[2] Besides the MOdified Newton Dynamics (MOND)[3] to account for such an extraneous acceleration there has been attempts within general relativity for a satisfactory interpretation. From this token a field theoretical approach based on dilatonic source in general relativity was proposed by Grumiller to yield a Rindler type acceleration in the spacetime.[4,5] More recently, we attempted to interpret the Rindler acceleration term as a nonlinear electrodynamic effect with an unusual Lagrangian.[6] Therein the problematic energy conditions are satisfied but at the cost of extra structures such as global monopoles[7] which pop up naturally. In a different study global monopoles were proposed as source to create the acceleration term in the weak field approximation.[8]

In this paper, we show that a particular $f(R)$ gravity[9–14] with a fluid source accounts for the Rindler acceleration. The fluid satisfies the weak energy condition (WEC) and strong energy condition (SEC) in regions as depicted in Fig. 1.

The action for $f(R)$ gravity written as

$$S = \frac{1}{2\kappa} \int \sqrt{-g} f(R) d^4x + S_M \,, \tag{1}$$

in which $\kappa = 8\pi G = 1$, $f(R) = R - 12a\xi \ln|R|$, (a and ξ are constants) is a function of the Ricci scalar R and S_M is the physical source for a perfect fluid-type energy–momentum

$$T_\mu^\nu = \mathrm{diag}[-\rho, p, q, q] \tag{2}$$

with a state function $p = -\rho$. Note that for dimensional reasons the logarithmic argument should read $\left|\frac{R}{R_0}\right|$, where $\ln|R_0|$ accounts for the cosmological constant. In our analysis, however, we shall choose $|R_0| = 1$ to ignore the cosmological constant. The four-dimensional static spherically symmetric line element is given by

$$ds^2 = -A(r)dt^2 + \frac{1}{B(r)}\,dr^2 + r^2(d\theta^2 + \sin^2\theta\,d\varphi^2)\,, \tag{3}$$

where $A(r)$ and $B(r)$ are to be found. Let us add also that in the sequel, for convenience we shall make the choice $A(r) = B(r)$.

Variation of the action with respect to the metric yields the field equations

$$G_\mu^\nu = \frac{1}{F}T_\mu^\nu + \check{T}_\mu^\nu\,, \tag{4}$$

where G_μ^ν stands for the Einstein's tensor, with

$$\check{T}_\mu^\nu = \frac{1}{F}\left[\nabla^\nu\nabla_\mu F - \left(\Box F - \frac{1}{2}f + \frac{1}{2}RF\right)\delta_\mu^\nu\right]. \tag{5}$$

Our notation here is such that $\Box = \nabla^\mu\nabla_\mu = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\partial^\mu\right)$ and $\nabla^\nu\nabla_\mu h = g^{\lambda\nu}\nabla_\lambda h_{,\mu} = g^{\lambda\nu}(\partial_\lambda h_{,\mu} - \Gamma_{\lambda\mu}^\beta h_{,\beta})$ for a scalar function h. The field equations explicitly read as

$$FR_t^t - \frac{f}{2} + \Box F = \nabla^t\nabla_t F + T_t^t\,, \tag{6}$$

$$FR_r^r - \frac{f}{2} + \Box F = \nabla^r\nabla_r F + T_r^r\,, \tag{7}$$

$$FR_\theta^\theta - \frac{f}{2} + \Box F = \nabla^\theta\nabla_\theta F + T_\theta^\theta\,, \tag{8}$$

$$\left(F = \frac{df}{dR}\right), \tag{9}$$

which are independent. Note that the $\varphi\varphi$ equation is identical with $\theta\theta$ equation. By adding the four equations (i.e. tt, rr, $\theta\theta$ and $\varphi\varphi$) we find

$$FR - 2f + 3\Box F = T \,, \tag{10}$$

which is the trace of Eq. (4). As usual tt and rr components admit $\nabla^t \nabla_t F = \nabla^r \nabla_r F$ which in turn yields $F'' = 0$, with $' = \frac{d}{dr}$, and consequently

$$F = C_1 + C_2 r \,. \tag{11}$$

Here C_1 and C_2 are two integration constants, which for our purpose we set $C_1 = 1$ and $C_2 = \xi$. A detailed calculation gives the metric solution

$$A(r) = 1 - \frac{2m}{r} + 2ar \,, \tag{12}$$

which would lead to the following energy-momentum components

$$-\rho = p = \frac{(6a\xi - f)r^2 + 4(\xi - a)r - 6m\xi}{2r^2} \,, \tag{13}$$

$$q = -\frac{fr - 2\xi + 8a}{2r} \,, \tag{14}$$

with

$$f = f(R(r)) = -\left(\frac{12a}{r} + 12a\xi \ln\left(\frac{12a}{r} \right) \right). \tag{15}$$

It is observed that in the limit of R-gravity (i.e. $\xi \to 0$) one gets $f = R = -\frac{12a}{r}$ and therefore

$$-\rho = p = \frac{4a}{r} \,, \tag{16}$$

while

$$q = \frac{2a}{r} \,. \tag{17}$$

Naturally the integration constant m accounts for the constant mass while the Rindler acceleration constant, i.e. a, determines the properties of the fluid source. These are the results found in Refs. 4 and 5. In the other limit, once $a \to 0$ one can see from the vanishing Ricci scalar that F cannot be r dependent which means that ξ must be zero. This, in turn, reduces to the standard R-gravity.

Once more we note that the Rindler acceleration a is positive and $C_2 = \xi$ is positive too, to avoid any nonphysical solutions. Our final remark will be on the energy conditions.

For WEC one should have (i) $\rho \geq 0$, (ii) $\rho + p \geq 0$ and (iii) $\rho + q \geq 0$. One observes that the second condition is trivial while the third condition implies

$$3a\xi r^2 + (2a + \xi)r - 3m\xi \leq 0 \,, \tag{18}$$

which simply confines the range of r as

$$r \leq \frac{\sqrt{(2a + \xi)^2 + 36ma\xi^2} - (2a + \xi)}{6a\xi} \,. \tag{19}$$

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On the other hand, the first condition ($\rho \geq 0$) reads as

$$-6a\xi r^2 \ln\left(\frac{12a}{r}\right) - (\xi + 2a)r \leq 3a\xi r^2 + (2a + \xi)r - 3m\xi \leq 0 \,, \qquad (20)$$

which can be satisfied. Figure 1 displays the possible regions in which the WEC are satisfied. Clearly by a fixed value for the Rindler acceleration larger deviation from the standard R-gravity provides larger region of satisfaction of WEC. Once the value of ξ gets smaller and smaller, the region in which $\rho \geq 0$ and $\rho + q \geq 0$ gets narrower and narrower. In the limit $\xi \to 0$, this region disappears completely.
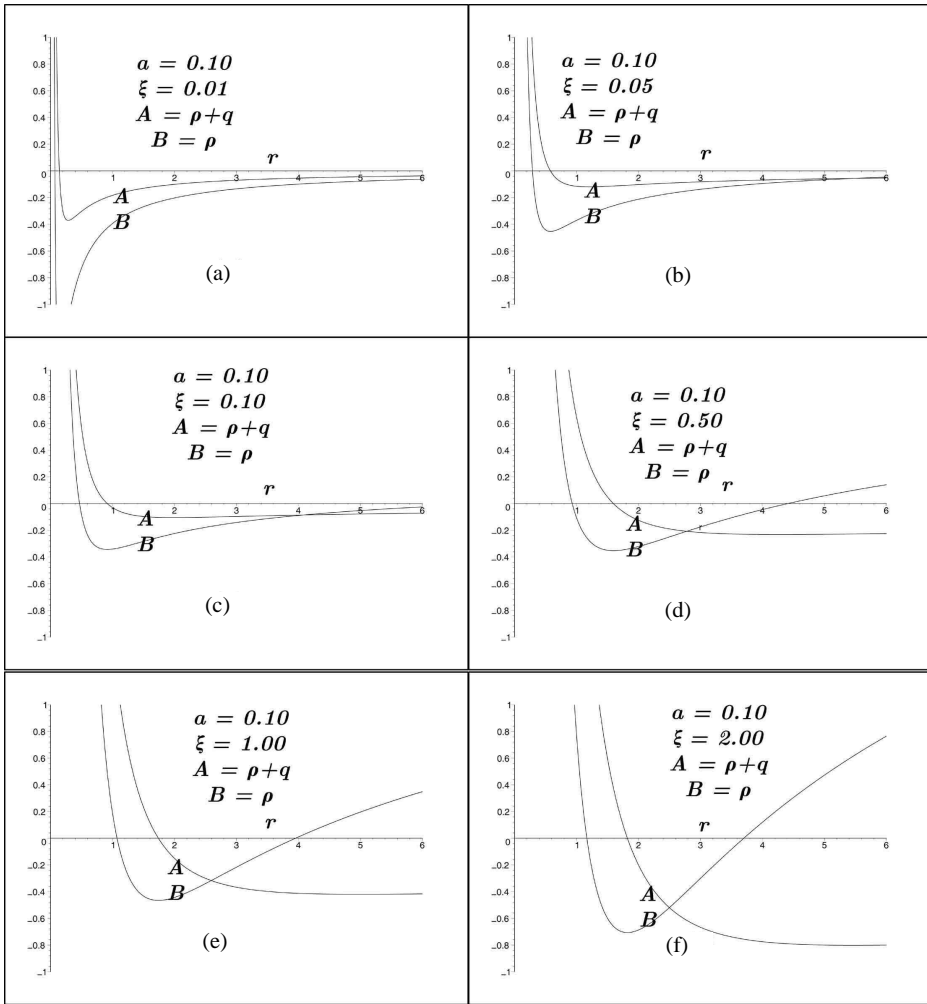
Fig. 1.   A plot of $\rho \, (= B)$ and $\rho + q \, (= A)$ for $m = 1$, $a = 0.1$ and various values of ξ. The value of ξ indicates the deviation of the theory from standard gravity. From the figure it is clear that by getting far from R-gravity the region in which WEC are satisfied (i.e. $\rho \geq 0$ and $\rho + q \geq 0$) is enlarged. We also note that having WEC satisfied, makes SEC satisfied as well.

In addition to the WEC one may also check the strong energy conditions (SEC) i.e. (i) $\rho \geq 0$, (ii) $\rho + q \geq 0$ and (iii) $\rho + p + 2q \geq 0$. The first two conditions have already been considered in WEC and the third condition becomes effectively equivalent with $q \geq 0$, i.e.

$$-6a\xi r \ln\left(\frac{12a}{r}\right) - (\xi + 2a) \leq 0 \,, \tag{21}$$

which upon (20) is satisfied trivially.

In conclusion, the Grumiller metric[4,5] would be physically acceptable (from energy point of view) if instead of R-gravity we adopt $f(R) = R - 12a\xi \ln|R|$ gravity. Herein a is just the Rindler acceleration and ξ is a parameter which shows the deviation of the new $f(R)$ gravity from the standard gravity. The external source consists of a fluid with an energy–momentum given by (2). The pressure of the fluid is negative by choice so that it plays the role of dark energy. The interesting feature of the Rindler modified Schwarzschild geometry[4,5] is that at large distances (i.e. outside the galaxy) there exists still an effective mass to yield nearly flat rotation curves. This result is irrelevant to whether the so-called Pioneer anomaly is a genuine case or not. Clearly our model excludes the flat space (i.e. $R = 0$) but becomes applicable to spacetimes with $0 < |R| < \infty$. Given the particular fluid source it is the unique $f(R)$ that yields the Grumiller metric[4,5] and satisfies WEC and SEC. It remains open, however, to explore new forms of $f(R)$ with alternative energy-momenta, other than the one found here, so that the same Rindler term will result with energy conditions satisfied. This will be part of our future project.

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