

# Double-bounce domain wall in Einstein–Yang–Mills–Scalar black holes

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**Abstract** We find Einstein–Yang–Mills (EYM) black hole solutions endowed with massless scalar hair in the presence of a potential  $V(\phi)$  as function of the scalar field  $\phi$ . Choosing  $V(\phi) = \text{constant}$  (or zero) sets the scalar field to vanish leaving us with the EYM black holes. Our class of black hole solutions is new so that they do not asymptotically go in general to any known limits. A particular case is given, however, which admits an asymptotically anti-de Sitter limit in 6-dimensional spacetime. The role of the potential  $V(\phi)$  in making double bounces (i.e. both a minimum and maximum radii) on a domain wall universe is highlighted.

## 1 Introduction

There has always been curiosity in obtaining exact solutions in Einstein's theory that contain new sources to encompass the underlying spacetime curvature. This covers sources such as scalar (charged, uncharged), cosmological constant, electromagnetism (linear, non-linear, electric, magnetic), Yang–Mills (YM) fields plus others as well as their combinations. The necessity of enrichment in sources can be traced back to gauge/gravity duality, but may find better justification from the more recent holographic superconductivity analogy. Once spherical symmetry and asymptotic flatness in the spacetime ansatz are assumed severe restrictions on possible solutions are inevitable. The uniqueness theorems and no-hair conjectures are a few of such restrictions that the solutions obtained must comply with.

In a recent study minimally coupled scalar field has been considered in Einstein–Maxwell theory with anti-de Sitter (AdS) asymptotics [1] (more recently, the same authors

studied also the static, planar solutions of Einstein–scalar gravity which results in an anti-de Sitter vacuum [2]).

In [1, 2] beside kinetic Lagrangian terms due to the scalar and Maxwell fields an additional potential  $V(\phi)$  constructed from scalar field has been supplemented. The constancy of such a potential can naturally be interpreted as a cosmological term. In some specific cases the scalar field is assumed to be of the form  $\phi \sim \ln r$  and upon this choice the potential  $V(\phi)$  can be determined from the dynamical equations. Among other things the model is shown to admit asymptotic Lifshitz black holes.

In this paper we add a Yang–Mills (YM) field instead of Maxwell to the Einstein–minimally coupled scalar field system and search for the resulting solutions. Our pure magnetic YM field is added through the Wu–Yang ansatz, which was introduced before [3, 4], and is known to extend easily to all higher dimensions. Our model of the Einstein–Scalar–YM system contains an indispensable potential function  $V(\phi)$  as a function of the scalar field. With the exception of specific dimensions and scalar field our solutions do not admit immediately recognizable asymptotes. In 6 dimensions, for instance we obtain an EYM–scalar solution which asymptotes to an anti-de Sitter spacetime. Although spherical symmetry allows more general ansatzes our choice in this paper will be such that  $-g_{tt} = g^{rr}$ , while the coefficient of the unit angular line element  $d\Omega_d^2$  is an arbitrary function  $R(r)^2$ . The asymptotic flatness condition requires that  $R(r) = r$ , which should be discarded in the present case since it annuls the scalar field through the field equations. We observe also that non-trivial scalar and YM fields must coexist only by virtue of a non-zero potential  $V(\phi)$ . The choice  $V(\phi) = \text{constant}$  (or zero) admits no scalar solution. Overall, our solutions can be interpreted as hairy black holes in an asymptotically non-flat spacetime. By setting the scalar field to zero we recover the EYM black holes obtained before [4]. Our model is exemplified with specific parameters of scalar hair. Next, in  $d + 2$ -dimensional bulk spacetime we define a  $d + 1$ -dimensional Domain Wall (DW)

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as a Friedmann–Robertson–Walker (FRW) universe by the proper boundary (junction) conditions given long ago by Darmois and Israel [5–8]. We establish such a DW universe and explore the conditions under which the FRW universe has double bounces. That is, the radius function of FRW universe will lie in between the minimum and maximum values. We have shown previously the existence of such double bounces in different theories [9, 10].

The paper is organized as follows. Our formalism of EYM system with a scalar field and potential is introduced in Sect. 2. Section 3 presents an exact solution to the system under certain ansatzes. Section 4 gives a solution with a different scalar field ansatz in 6-dimensional spacetime that asymptotically goes to anti-de Sitter spacetime. Domain-wall dynamics in our bulk space is introduced in Sect. 5. Our conclusion appears in Sect. 6.

## 2 The formalism

Following the formalism given in [1, 2] with the same unit convention (i.e.,  $c = 16\pi G = 1$ ) the  $d + 2$ -dimensional Einstein–Yang–Mills gravity coupled minimally to a scalar field  $\phi$  is given by

$$S = \int d^{d+2}x \sqrt{-g} [\mathcal{R} - 2(\partial\phi)^2 - L_{(YM)} - V(\phi)] \quad (1)$$

in which  $\mathcal{R}$  is the Ricci scalar,  $L_{(YM)} = \text{Tr}(F_{\mu\nu}^{(a)} F^{(a)\mu\nu})$  and  $V(\phi)$  is an arbitrary function of the scalar field  $\phi$ . The YM field 2-form components are given by

$$\mathbf{F}^{(a)} = \frac{1}{2} F_{\mu\nu}^{(a)} dx^\mu \wedge dx^\nu \quad (2)$$

with the internal index ( $a$ ) running over the degrees of freedom of the nonabelian YM gauge field. Variation of the action with respect to the metric  $g_{\mu\nu}$  gives the EYM field equations as

$$G_\mu^\nu = T_\mu^{\nu(YM)} + T_\mu^{\nu(\phi)} - \frac{1}{2} V(\phi) \delta_\mu^\nu \quad (3)$$

in which

$$T_\mu^{\nu(YM)} = 2 \text{Tr} \left( F_{\mu\alpha}^{(a)} F^{(a)\nu\alpha} - \frac{1}{4} F_{\gamma\sigma}^{(a)} F^{(a)\gamma\sigma} \delta_\mu^\nu \right), \quad (4)$$

$$T_\mu^{\nu(\phi)} = 2 \left( \partial_\mu \phi \partial^\nu \phi - \frac{1}{2} \partial_\rho \phi \partial^\rho \phi \delta_\mu^\nu \right).$$

Variation of the action with respect to the scalar field  $\phi$  yields

$$\nabla^2 \phi = \frac{1}{4} \frac{dV(\phi)}{d\phi}. \quad (5)$$

The  $SO(d + 1)$  gauge group YM potentials are given by [11]

$$\mathbf{A}^{(a)} = \frac{Q}{r^2} C_{(i)(j)}^{(a)} x^i dx^j,$$

$$Q = \text{YM magnetic charge}, \quad r^2 = \sum_{i=1}^{d+1} x_i^2,$$

$$2 \leq j + 1 \leq i \leq d + 1, \quad \text{and} \quad 1 \leq a \leq d(d + 1)/2,$$

$$x_1 = r \cos \theta_{d-1} \sin \theta_{d-2} \cdots \sin \theta_1, \quad (6)$$

$$x_2 = r \sin \theta_{d-1} \sin \theta_{d-2} \cdots \sin \theta_1,$$

$$x_3 = r \cos \theta_{d-2} \sin \theta_{d-3} \cdots \sin \theta_1,$$

$$x_4 = r \sin \theta_{d-2} \sin \theta_{d-3} \cdots \sin \theta_1,$$

...

$$x_d = r \cos \theta_1,$$

in which  $C_{(b)(c)}^{(a)}$  are the non-zero structure constants of the  $\frac{d(d+1)}{2}$ -parameter Lie group  $\mathcal{G}$  [3, 4]. The metric ansatz is spherically symmetric which reads

$$ds^2 = -U(r) dt^2 + \frac{dr^2}{U(r)} + R(r)^2 d\Omega_d^2, \quad (7)$$

with the only unknown functions  $U(r)$  and  $R(r)$  and the solid angle element

$$d\Omega_d^2 = d\theta_1^2 + \sum_{i=2}^d \prod_{j=1}^{i-1} \sin^2 \theta_j d\theta_i^2, \quad (8)$$

with

$$0 \leq \theta_d \leq 2\pi, \quad 0 \leq \theta_i \leq \pi, \quad 1 \leq i \leq d - 1.$$

Variation of the action with respect to  $\mathbf{A}^{(a)}$  implies the YM equations

$$\mathbf{d} \star \mathbf{F}^{(a)} + \frac{1}{\sigma} C_{(b)(c)}^{(a)} \mathbf{A}^{(b)} \wedge \star \mathbf{F}^{(c)} = 0, \quad (9)$$

in which  $\sigma$  is a coupling constant and “ $\star$ ” means duality. The YM invariant is given by

$$\text{Tr}(F_{\mu\nu}^{(a)} F^{(a)\mu\nu}) = \frac{d(d-1)Q^2}{r^4} \quad (10)$$

and

$$\text{Tr}(F_{t\alpha}^{(a)} F^{(a)t\alpha}) = \text{Tr}(F_{r\alpha}^{(a)} F^{(a)r\alpha}) = 0, \quad (11)$$

while

$$\text{Tr}(F_{\theta_i\alpha}^{(a)} F^{(a)\theta_i\alpha}) = \frac{(d-1)Q^2}{r^4}, \quad (12)$$

which leads us to the closed form of the energy-momentum tensor

$$T_{\mu}^{\nu(YM)} = -\frac{d(d-1)Q^2}{2R^4} \times \text{diag}\left[1, 1, \frac{(d-4)}{d}, \frac{(d-4)}{d}, \dots, \frac{(d-4)}{d}\right]. \tag{13}$$

The above field equations can be rearranged as

$$\frac{R''}{R} = -\frac{2}{d}(\phi')^2, \quad (UR^d\phi')' = \frac{1}{4}R^d\frac{dV}{d\phi}, \tag{14}$$

$$(UR^d)'' = d(d-1)R^{d-2} + 2R^d\left(T_{\theta_i}^{\theta_i(YM)} + \frac{2}{d}T_r^r(YM)\right) - \frac{d+2}{d}R^dV, \tag{15}$$

$$(R^{d-1}UR')' = (d-1)R^{d-2} + \frac{2}{d}R^d T_r^r(YM) - \frac{1}{d}R^dV, \tag{16}$$

in which a ‘prime’ denotes derivative with respect to  $r$ . As introduced in Refs. [1, 2] we define new variables

$$F(r) = -\frac{2}{d}(\phi')^2, \quad R = e^{\int Y dr}, \quad u = UR^d, \tag{17}$$

which reduce the field equations to

$$Y' + Y^2 = F(r), \quad (u\phi')' = \frac{1}{4}e^{d\int Y dr}\frac{dV}{d\phi}, \tag{18}$$

$$u'' = d(d-1)e^{(d-2)\int Y dr} + 2e^{d\int Y dr}\left(T_{\theta_i}^{\theta_i(YM)} + \frac{2}{d}T_r^r(YM)\right) - \frac{d+2}{d}e^{d\int Y dr}V, \tag{19}$$

$$(uY)' = (d-1)e^{(d-2)\int Y dr} + \frac{2}{d}e^{d\int Y dr}T_r^r(YM) - \frac{1}{d}e^{d\int Y dr}V. \tag{20}$$

A combination of the latter two equations yields

$$u'' - (d+2)(uY)' = -2(d-1)e^{(d-2)\int Y dr} + 2e^{d\int Y dr}(T_{\theta_i}^{\theta_i(YM)} - T_r^r(YM)) \tag{21}$$

which is a  $V$ -independent equation and can be integrated once to

$$u' - (d+2)uY = \int \{-2(d-1)e^{(d-2)\int Y dr} + 2e^{d\int Y dr}(T_{\theta_i}^{\theta_i(YM)} - T_r^r(YM))\} dr + C_1. \tag{22}$$

This is further integrated to obtain

$$u = R^{d+2}\left[\int\left(\frac{1}{R^{d+2}}\left[\int\{-2(d-1)R^{d-2} + 2R^d(T_{\theta_i}^{\theta_i(YM)} - T_r^r(YM))\} dr + C_1\right] dr + C_2\right)\right] \tag{23}$$

and consequently the potential reads from (19)

$$V = \frac{d^2(d-1)}{(d+2)R^2} + \frac{2d}{(d+2)}\left(T_{\theta_i}^{\theta_i(YM)} + \frac{2}{d}T_r^r(YM)\right) - \frac{d}{(d+2)R^d}u''. \tag{24}$$

### 3 Exact solutions

We start with an ansatz for the scalar field  $\phi = \alpha \ln(\frac{r}{r_0})$  in which  $r_0$  and  $\alpha$  are two real constants. The  $Y$ -equation then takes the Riccati form

$$Y' + Y^2 = -\frac{2\alpha^2}{d r^2}, \tag{25}$$

which admits a solution for  $Y$  given by

$$Y = \frac{A}{r} \tag{26}$$

with

$$A^2 - A + \frac{2\alpha^2}{d} = 0, \quad 0 < A < 1. \tag{27}$$

Note that  $A = 0, 1$  make the scalar field vanish, so we exclude them. Knowing  $Y$  we find

$$R = \left(\frac{r}{r_1}\right)^A, \quad (r_1 = \text{cons.}) \tag{28}$$

and therefore

$$u = r^{A(d+2)} \left[ \int \left( \frac{\int [-2(d-1)r^{A(d-2)}r_1^{-A(d-2)} + 4(d-1)Q^2r^{A(d-4)}r_1^{-A(d-4)}] dr + C_1}{r^{A(d+2)}} \right) dr + \frac{C_2}{r_1^{A(d+2)}} \right] \tag{29}$$

which becomes

$$u = \begin{cases} \frac{(d-1)r^{2+A(d-2)}r_1^{-A(d-2)}}{(A(d-2)+1)(2A-1)} - \frac{2(d-1)Q^2r^{A(d-4)+2}r_1^{-A(d-4)}}{(A(d-4)+1)(3A-1)} - \frac{C_1}{A(d+2)-1}r + C_2\left(\frac{r}{r_1}\right)^{A(d+2)}, & A \neq \frac{1}{2}, \frac{1}{3}, \frac{1}{d+2}, \\ -\frac{4(d-1)\ln(r)r_1^{-\frac{d-2}{2}}r^{\frac{d+2}{2}}}{d} - \frac{8(d-1)Q^2r^{\frac{d}{2}}r_1^{-\frac{d-4}{2}}}{d-2} - \frac{2C_1}{d}r + C_2\left(\frac{r}{r_1}\right)^{\frac{(d+2)}{2}}, & A = \frac{1}{2}, \\ -\frac{9(d-1)r_1^{-\frac{d+2}{3}}r^{\frac{d+4}{3}}}{(d+1)} + 12Q^2r_1^{-\frac{d+4}{3}}r^{\frac{d+2}{3}}\ln r - \frac{3C_1}{d-1}r + C_2\left(\frac{r}{r_1}\right)^{\frac{(d+2)}{3}}, & A = \frac{1}{3}, \\ -\frac{(d-1)(d+2)^2}{d^2r_1^{\frac{d-2}{d+2}}}r^{\frac{3d+2}{d+2}} + \frac{Q^2(d+2)^2}{d-1}r^{\frac{3d}{d+2}} + C_1r\ln r + C_2\frac{r}{r_1}, & A = \frac{1}{d+2}. \end{cases} \tag{30}$$

The potential accordingly takes the form from (24)

$$V = \begin{cases} \frac{r_1^{2A}}{r^{2A}}\frac{d(d-1)(A-1)}{(d+2)(2A-1)} + \frac{r_1^{4A}}{r^{4A}}\frac{(1-A)d(d-1)Q^2}{(3A-1)} - dA[A(d+2)-1]C_2\frac{r^{2(A-1)}}{r_1^{2A}}, & A \neq \frac{1}{2}, \frac{1}{3}, \frac{1}{d+2}, \\ \frac{d(d-1)Q^2r_1^2}{r^2} - \frac{d^2C_2}{4r_1r} + \frac{(d-1)r_1}{r}(d+2+d\ln r), & A = \frac{1}{2}, \\ 2(d-1)d\left(\frac{r_1}{r}\right)^{\frac{2}{3}} - \frac{1}{9}\frac{d(d-1)C_2}{r^{\frac{4}{3}}r_1^{\frac{2}{3}}} - \left(\frac{r_1}{r}\right)^{\frac{4}{3}}\frac{dQ^2(4(d-1)\ln r+3d+9)}{3}, & A = \frac{1}{3}, \\ (d^2-1)\left(\frac{r_1}{r}\right)^{\frac{2}{d+2}} - \left(\frac{r_1}{r}\right)^{\frac{4}{d+2}}d(d+1)Q^2 - \frac{r_1^{\frac{d}{d+2}}dC_1}{d+2}r^{-2\frac{d+1}{d+2}}, & A = \frac{1}{d+2}. \end{cases} \tag{31}$$

Let us note that  $r_1$  is a constant introduced for dimensional reasons and without loss of generality we set  $r_1 = 1$  in the sequel.

### 3.1 Asymptotic functions

In this subsection we give the asymptotic behaviors of the general solution found above. To do so we first rewrite the solution (30) in terms of  $R$  (with  $r_1 = 1$ ),

$$u(R) = \begin{cases} \frac{(d-1)R^{\frac{2+A(d-2)}{A}}}{(A(d-2)+1)(2A-1)} - \frac{2(d-1)Q^2R^{\frac{A(d-4)+2}{A}}}{(A(d-4)+1)(3A-1)} - \frac{C_1}{A(d+2)-1}R^{\frac{1}{A}} + C_2R^{d+2}, & A \neq \frac{1}{2}, \frac{1}{3}, \frac{1}{d+2}, \\ -\frac{8(d-1)\ln(R)R^{d+2}}{d} - \frac{8(d-1)Q^2R^d}{d-2} - \frac{2C_1}{d}R^2 + C_2R^{d+2}, & A = \frac{1}{2}, \\ -\frac{9(d-1)R^{d+4}}{(d+1)} + 36Q^2R^{d+2}\ln R - \frac{3C_1}{d-1}R^3 + C_2R^{d+2}, & A = \frac{1}{3}, \\ -\frac{(d-1)(d+2)^2}{d^2}R^{3d+2} + \frac{Q^2(d+2)^2}{d-1}R^{3d} + C_1(d+2)R^{d+2}\ln R + C_2R^{d+2}, & A = \frac{1}{d+2} \end{cases} \tag{32}$$

and

$$V(R) = \begin{cases} \frac{1}{R^2}\frac{d(d-1)(A-1)}{(d+2)(2A-1)} + \frac{1}{R^4}\frac{(1-A)d(d-1)Q^2}{(3A-1)} - A(A(d+2)-1)dC_2\frac{R^{\frac{2(A-1)}{A}}}{R^A}, & A \neq \frac{1}{2}, \frac{1}{3}, \frac{1}{d+2}, \\ \frac{d(d-1)Q^2}{R^4} - \frac{d^2C_2}{4R^2} + \frac{(d-1)}{R^2}, & A = \frac{1}{2}, \\ \frac{2(d-1)d}{R^2} - \frac{1}{9}\frac{d(d-1)C_2}{R^4} - \frac{dQ^2(d+3+\frac{4(d-1)}{9}\ln R)}{R^4}, & A = \frac{1}{3}, \\ \frac{(d^2-1)}{R^2} - \frac{d(d+1)Q^2}{R^4} - \frac{dC_1}{(d+2)R^{2(d+1)}}, & A = \frac{1}{d+2}. \end{cases} \tag{33}$$

We also note that in terms of  $R$  the line element (7) reads

$$ds^2 = -U(R) dt^2 + \frac{R^{\frac{2(1-A)}{A}} dR^2}{A^2 U(R)} + R^2 d\Omega_d^2, \tag{34}$$

in which  $U(R) = \frac{u}{R^d}$ . For  $0 < A < \frac{1}{2}$  while  $R \rightarrow \infty$  the line element becomes

$$ds^2 \simeq \xi^2 R^{\frac{2-2A}{A}} dt^2 - \frac{dR^2}{A^2 \xi^2} + R^2 d\Omega_d^2 \tag{35}$$

in which  $\xi^2 = -\frac{(d-1)}{(A(d-2)+1)(2A-1)}$  so that  $R$  turns time-like. For  $\frac{1}{2} < A < 1$ , we obtain asymptotically

$$ds^2 \simeq -C_2 R^2 dt^2 + \frac{R^{\frac{2(1-2A)}{A}} dR^2}{A^2 C_2} + R^2 d\Omega_d^2. \tag{36}$$

It is observed that since  $A \neq 1$  asymptotic form ( $R \rightarrow \infty$ ) excludes the case of AdS. For the case when  $A = \frac{1}{2}$  one easily finds

$$ds^2 \simeq \lambda^2 \ln(R) R^2 dt^2 - \frac{4dR^2}{\lambda^2 \ln(R)} + R^2 d\Omega_d^2 \tag{37}$$

where  $\lambda^2 = \frac{8(d-1)}{d}$ . It is needless to restate that the roles of  $t$  and  $R$  change, i.e.  $R$  becomes a time-like coordinate. The asymptotic metrics (35) and (37) suggest also that we have no Lifshitz [11] asymptotes.

#### 4 Asymptotically anti/de Sitter solutions for $d = 4$

In this section we consider a different ansatz for the scalar field which reads

$$\phi = \xi \ln\left(1 + \frac{r_0}{r}\right) \tag{38}$$

in which  $\xi$  and  $r_0 > 0$  are some real constants. We plug  $\phi$  into (14) to find

$$R = \chi r \left(1 + \frac{r_0}{r}\right)^{\beta + \frac{1}{2}} \tag{39}$$

in which  $\chi$  is an integration constant and  $\beta^2 = \frac{1}{4} - \frac{2}{d}\xi^2$ . We note that due to this condition  $\beta$  remains bounded as

$$-\frac{1}{2} \leq \beta \leq \frac{1}{2} \tag{40}$$

and consequently

$$-\frac{1}{2\sqrt{2}} \leq \xi \leq \frac{1}{2\sqrt{2}}. \tag{41}$$

The general solution given for  $u$ , unfortunately is not integrable in closed form for a general dimension  $d + 2$  and for

a general  $\beta$ . But one can see that in  $d + 2$  dimension the specific choice, i.e.,  $\beta = -1/2$  yields

$$U = \frac{1}{\chi^2} - \frac{(d-1)Q^2}{\chi^4 r^2 (d-3)} - \frac{C_1}{r^{d-1} \chi^d (d+1)} + \chi^2 C_2 r^2 \tag{42}$$

which in turn must give the general EYM solution for  $d > 3$ . This, of course, implies  $\chi = 1$ ,  $C_1 = m(d + 1)$  and  $C_2 = -\frac{1}{3}\Lambda$  which casts the solution into

$$U = 1 - \frac{(d-1)Q^2}{(d-3)r^2} - \frac{m}{r^{d-1}} - \frac{1}{3}\Lambda r^2. \tag{43}$$

Also the corresponding potential  $V$  reads

$$V = \frac{1}{3}d(d+1)\Lambda \tag{44}$$

which means that  $\Lambda$  represents the cosmological constant.

Another case that we can integrate (23) is  $d = 4$  with  $\beta = 0$  (consequently  $\xi^2 = \frac{1}{2}$ ) giving

$$U = \frac{r_0^2(1+x)}{x^2} \Lambda - \frac{3(2m - 12Q^2 r_0 - r_0^3)(1+x) \ln(1+x)}{x^2 r_0^3} - \frac{m(x+2)(x^2 - 6x - 6)}{2x r_0^3 (1+x)} - \frac{6Q^2(6+9x+2x^2)}{x r_0^2 (1+x)} - \frac{7x+6}{2x(1+x)}, \tag{45}$$

for  $x = \frac{r_0}{r}$ . We add also that to get the correct flat limit we set  $\chi = 1$  and  $C_2 = \Lambda$ . Our other integration constant  $C_1$  is related to the mass of the black hole solution so we set it as  $m$ . Here it is worth to see  $\lim_{r \rightarrow 0/x \rightarrow \infty} U = -\infty$  which clearly implies that the solution admits an event horizon and therefore the solution is a black hole.

Next, we find the form of the potential which is given by

$$V = 2 \left[ \frac{6\Pi_1 + (m - 6Q^2 r_0)\Pi_2}{r_0^5} \exp(-\sqrt{2}\phi) + 2\Lambda\Pi_3 \exp\left(-\frac{\phi}{\sqrt{2}}\right) + \frac{\Pi_4}{r_0^2} \exp(-\sqrt{2}\phi) \right], \tag{46}$$

where

$$\Pi_1 = \left( \exp\left(\frac{\phi}{\sqrt{2}}\right) \left( \exp\left(\frac{\phi}{\sqrt{2}}\right) + 3 \right) + 1 \right) \times (-2m + 12Q^2 r_0 + r_0^3) \left( \frac{\phi}{\sqrt{2}} \right),$$

$$\Pi_2 = \left[ \exp(\sqrt{2}\phi) - 1 \right] \left[ \exp\left(\frac{\phi}{\sqrt{2}}\right) \left( \exp\left(\frac{\phi}{\sqrt{2}}\right) + 28 \right) + 1 \right],$$

$$\begin{aligned} \Pi_3 &= \left[ \exp\left(\frac{\phi}{\sqrt{2}}\right) \left( \exp\left(\frac{\phi}{\sqrt{2}}\right) + 3 \right) + 1 \right], \\ \Pi_4 &= \left[ \exp\left(\frac{\phi}{\sqrt{2}}\right) - 1 \right] \left[ \exp\left(\frac{\phi}{\sqrt{2}}\right) \left( 16 \exp\left(\frac{\phi}{\sqrt{2}}\right) \right. \right. \\ &\quad \left. \left. + 13 \right) + 1 \right]. \end{aligned}$$

It is easy to observe that

$$\lim_{\substack{r \rightarrow \infty \\ (x \rightarrow 0)}} V = 20\Lambda, \tag{47}$$

in which for zero cosmological constant it vanishes.

### 5 Dynamics of domain-walls

In this section we consider a  $d + 2$ -dimensional bulk action supplemented by surface terms [12]

$$\begin{aligned} S &= \int_{\mathcal{M}} d^{d+2}x \sqrt{-g} [\mathcal{R} - 2(\partial\phi)^2 - L_{(YM)} - V(\phi)] \\ &\quad + \int_{\Sigma} d^{d+1}x \sqrt{-h} \{K\} + \int_{\Sigma} d^{d+1}x \sqrt{-h} L_{(DW)}, \end{aligned} \tag{48}$$

where the DW Lagrangian will be given by  $L_{(DW)} = -\hat{V}(\phi)$  as the induced potential on the DW.  $\{K\}$  stands for the trace of the extrinsic curvature tensor  $K_{ij}$  of DW with the induced metric  $h_{ij}$  ( $h = |g_{ij}|$ ).

Herein  $\Sigma$  is the  $(d + 1)$ -dimensional DW in a  $(d + 2)$ -dimensional bulk  $\mathcal{M}$  which splits the background bulk into two  $(d + 2)$ -dimensional spacetimes  $\mathcal{M}_{\pm}$ . Note that  $\pm$  refer to the normal directions on the DW. The metric we shall work on is given by (7) whose parameters are chosen such that  $\lim_{r \rightarrow \infty} U(r) = \infty$  and for the case of BH  $r_h < r = a$ . For the non-BH case we make the choice  $0 < r = a$ . Let us impose now the following condition:

$$-U(a) \left( \frac{dt}{d\tau} \right)^2 + \frac{1}{U(a)} \left( \frac{da}{d\tau} \right)^2 = -1 \tag{49}$$

in which the location of the DW is given by  $r = a(\tau)$ , with the proper time  $\tau$ . Therefore the line element on the DW becomes

$$ds_{dw}^2 = -d\tau^2 + a(\tau)^2 d\Omega_d^2, \tag{50}$$

which is a FRW metric in  $(d + 1)$  dimensions with the radius function  $a(\tau)$ . Imposing the boundary conditions, i.e., the Darmois–Israel conditions, on the wall [5–8] leads to

$$-\langle K_i^j \rangle - \langle K \rangle \delta_i^j = S_i^j, \tag{51}$$

where

$$S_{ij} = \frac{1}{\sqrt{-h}} \frac{2\delta}{\delta g^{ij}} \int d^{d+1}x \sqrt{-h} (-\hat{V}(\phi)) \tag{52}$$

is the surface energy-momentum tensor on the DW. Note that the bracket  $\langle \cdot \rangle$  denotes a jump across  $\Sigma$ . The latter yields

$$S_i^j = -\hat{V}(\phi) \delta_i^j, \tag{53}$$

which, after (50) and (52), gives the energy density  $\sigma$  and the surface pressures  $p_{\theta_i}$  for generic metric functions  $f(r)$  and  $R(r)$  with  $r = a(\tau)$ . The overall results are given by

$$\sigma = -S_{\tau}^{\tau} = -\frac{d}{4\pi} \left( \sqrt{U(a) + \dot{a}^2} \frac{R'}{R} \right), \tag{54}$$

$$S_{\theta_i}^{\theta_i} = p_{\theta_i} = \frac{1}{8\pi} \left( \frac{U' + 2\ddot{a}}{\sqrt{U(a) + \dot{a}^2}} + 2(d - 1) \sqrt{U(a) + \dot{a}^2} \frac{R'}{R} \right) \tag{55}$$

in which a dot “.” and prime “'” means  $\frac{d}{d\tau}$  and  $\frac{d}{da}$ , respectively. The junction conditions, therefore, read

$$\frac{d}{4\pi} \left( \sqrt{U(a) + \dot{a}^2} \frac{R'}{R} \right) = -\hat{V}(\phi), \tag{56}$$

$$\begin{aligned} \frac{1}{8\pi} \left( \frac{U' + 2\ddot{a}}{\sqrt{U(a) + \dot{a}^2}} + 2(d - 1) \sqrt{U(a) + \dot{a}^2} \frac{R'}{R} \right) \\ = -\hat{V}(\phi), \end{aligned} \tag{57}$$

which upon using

$$\frac{U' + 2\ddot{a}}{\sqrt{U(a) + \dot{a}^2}} = \frac{2}{\dot{a}} \frac{d}{d\tau} (\sqrt{U(a) + \dot{a}^2}) \tag{58}$$

and Eq. (57) can be cast into

$$\begin{aligned} \frac{U' + 2\ddot{a}}{\sqrt{U(a) + \dot{a}^2}} = \frac{2}{\dot{a}} \frac{d}{d\tau} \left( -\frac{4\pi}{d} \hat{V}(\phi) \frac{R}{R'} \right) \\ = \frac{d}{da} \left( -\frac{8\pi}{d} \hat{V}(\phi) \frac{R}{R'} \right). \end{aligned} \tag{59}$$

Finally we find

$$\frac{d}{da} \left( \hat{V}(\phi) \frac{R}{R'} \right) = \hat{V}(\phi). \tag{60}$$

This equation admits the simple relation between  $R(r)$  and  $\hat{V}(\phi)$  given by

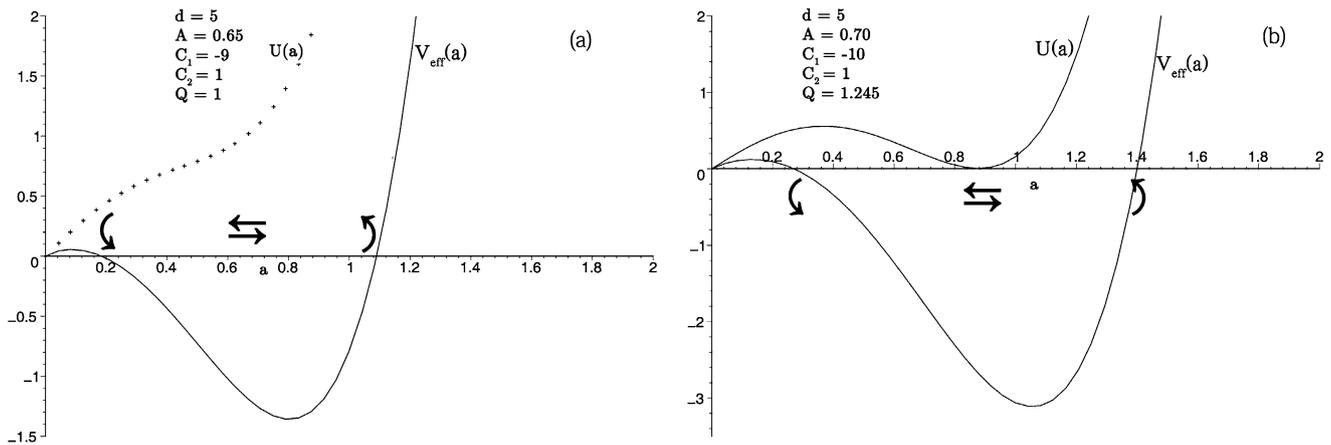
$$R'(r) = \xi \hat{V}(\phi) \tag{61}$$

with  $\xi = \text{constant}$ . Using above with some manipulation we obtain the 1-dimensional equation

$$\dot{a}^2 + V_{\text{eff}}(a) = 0, \tag{62}$$

where the effective potential is defined by

$$V_{\text{eff}}(a) = U(a) - \left( \frac{4\pi R}{d\xi} \right)^2. \tag{63}$$



**Fig. 1** Examples of Einstein–Yang–Mills–Scalar (EYMS) solutions with double bounces on the domain wall. For technical reasons we restrict ourselves to  $d = 5$  alone. (a) is the case of non-black hole, while (b) refers to the case of an extremal black hole. The effective potential

$V_{\text{eff}}(a)$  is given in Eq. (63). From Eq. (62) the universe admits only the possibility of  $V_{\text{eff}}(a) < 0$ . This gives rise to an oscillatory FRW universe on the DW for such an EYMS system when supplemented by a scalar potential in the Lagrangian

In order that (62) admits a solution as the radius on the DW universe we must have  $V_{\text{eff}}(a) < 0$ , which naturally restricts the probable forms of the potential function  $V(\phi)$ . Further, (61) yields

$$\hat{V}(\phi) = \frac{Ar^{A-1}}{\xi}. \tag{64}$$

The following boundary condition for the scalar field (see Eq. (38) in [12]) holds automatically:

$$\frac{\partial \phi}{\partial R} = -\frac{d}{R} \frac{1}{\hat{V}(\phi)} \frac{\partial \hat{V}(\phi)}{\partial \phi} \tag{65}$$

(after fixing  $\xi = 2A$  for convenience). Based on Eq. (63) we plot  $V_{\text{eff}}(a)$  for the cases of a non-black hole (Fig. 1a) and an extremal black hole (Fig. 1b) as examples to verify double bounces.

### 6 Conclusion

Einstein–Yang–Mills (EYM) fields, minimally coupled with a massless scalar field  $\phi$  supplemented by a scalar potential  $V(\phi)$  are considered. The EYMS system admits exact, both black hole and non-black hole solutions. Depending on the scalar field the potential  $V(\phi)$  has a large spectrum of possible values with the YM field. The novelty with the inclusion of the potential  $V(\phi)$  has significant contributions. Firstly, if the potential  $V(\phi)$  is set to zero the scalar field becomes trivial. That is, the YM field within Einstein’s general relativity at least in the assumed ansatz spacetime cannot coexist with a minimally coupled scalar field. The non-asymptotically flat and non-asymptotically AdS black hole solutions are obtained by our formalism with scalar hair.

Depending on the dimensionality of the spacetime and the scalar field ansatz the picture may change entirely. This we show with an explicit example in 6-dimensional spacetime which admits an anti-de Sitter asymptote. Secondly, the richness brought in by the additional potential  $V(\phi)$  (i.e. Sect. 3) in the bulk spacetime of dimension  $(d + 2)$  is employed in the construction of a Domain Wall (DW) universe as a brane in  $(d + 1)$ -dimensions. Specifically, we are interested to construct a DW brane as a FRW universe which admits both a minimum and a maximum bounce. We have shown that this is possible, so that once our world is such a brane it will oscillate between these limits ad infinitum (see Fig. 1). Finally, since a constant potential amounts to a cosmological term a space-filling uniform scalar field may be the cause of the cosmological constant.

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