Particle Collision near 1+1-D Horava-Lifshitz Black Holes

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- 1+1-D HL BLACK HOLES
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- CONCLUSION

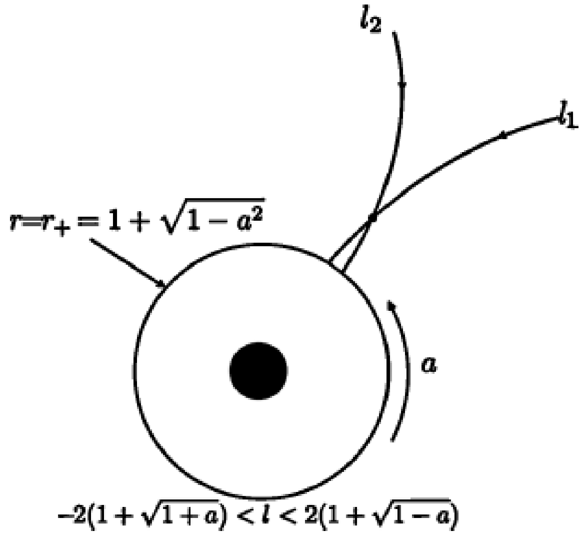
Motivation

Rotating BHs as particle accelerators

Bañados, Silk and West (2009)

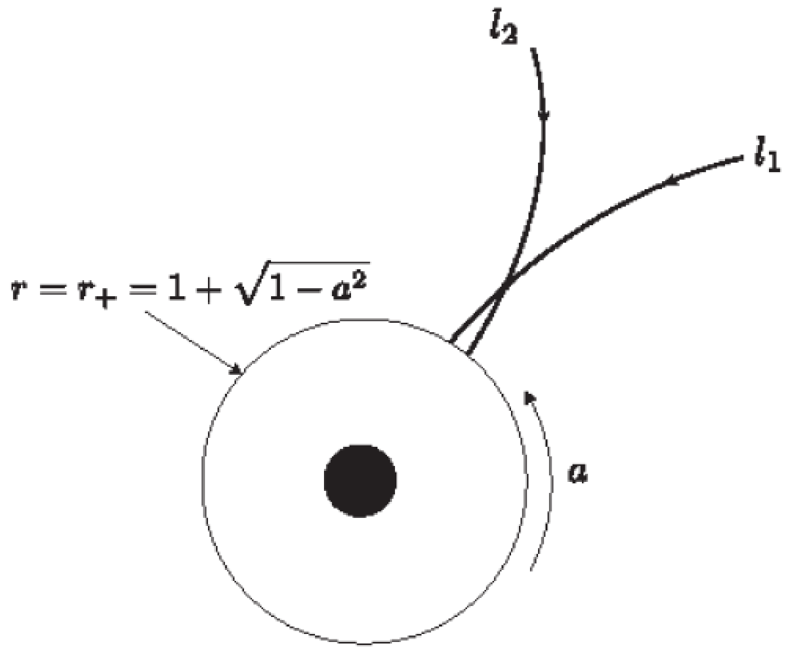
Banados, Silk, West (BSW Effect), Phys. Rev. Lett. **103**, 111102 (2009).

1. Drop two particles at rest at infinity on the equatorial plane.
2. Consider the collision of the two particles near the horizon.
3. Take the maximum rotation limit and fine-tune the angular momentum of either particle, then the centre-of-mass energy can be arbitrarily high!



- Robust?
- General condition?

Motivation



$$-2(1+\sqrt{1+a}) < l < 2(1+\sqrt{1-a})$$

FIG. 1. Schematic picture of two particles falling into a black hole with angular momentum a (per unit black hole mass) and colliding near the horizon. The allowed range of l for geodesics falling into the black hole is also given.

Banados, Silk, West (BSW Effect), Phys. Rev. Lett. **103**, 111102 (2009).

Is it also valid for near horizon of a black hole emerges even in 1+1- dimensional Hořava-Lifshitz gravity ?

1+1-D HL BLACK HOLE

D. Bazeia, F. A. Brito and F. G. Costa, Phys.Rev.D.91, (2015) 044026.

- The line element is $$ds^2 = -N(x)^2 dt^2 + \frac{dx^2}{N(x)^2}$$

where a general class of solutions is obtained as follows

$$N(x)^2 = 2C_2 + \frac{A}{\eta}x^2 - 2C_1 x + \frac{B}{\eta x} + \frac{C}{3\eta x^2}$$

- In the case of $C_2 = 1/2,\ B = -2M,\ \eta = 1$ and $A = C = C_1 = 0$, it gives a Schwarzschild-like solution;

$$N(x)^2 = 1 - \frac{2M}{x}$$

1+1-D HL BLACK HOLES

- On the other hand, the choice of the parameters, for ,

  $C_2 = 1/2, B = -2M,\ C = 3Q^2,\ \eta = 1$ and $A = C_1 = 0$

- gives a Reissner--Nordström-like solution.

$$N(x)^2 = 1 - \frac{2M}{x} + \frac{Q^2}{x^2}$$

- The new black hole solution which is derived by Bazeia et. al. is found by taking $C_1 \neq 0,\ C_2 \neq 0,\ B \neq 0$ $\quad A = C = 0$

-

$$N(x)^2 = 2C_2 - 2C_1 x + \frac{B}{\eta x}.$$

1+1-D HL BLACK HOLES

- This solution develops the following horizons

$$x_h^{\pm} = \frac{C_2}{2C_1} \pm \sqrt{\Delta}, \; \Delta = \frac{C_2^2}{4C_1^2} + \frac{B}{2\eta C_1}.$$

- As $\Delta = 0$ they degenerate, i.e. $x_h^+ = x_h^-$.
- The Hawking temperature is given in terms of the outer horizon as follows

$$T_H = \left. \frac{\left(N(x)^2\right)'}{4\pi} \right|_{x=x_h^+}.$$

1+1-D HL BLACK HOLES

- For the special case $C_2 = 0,\ C_1 = -M$ and $B = -2M$

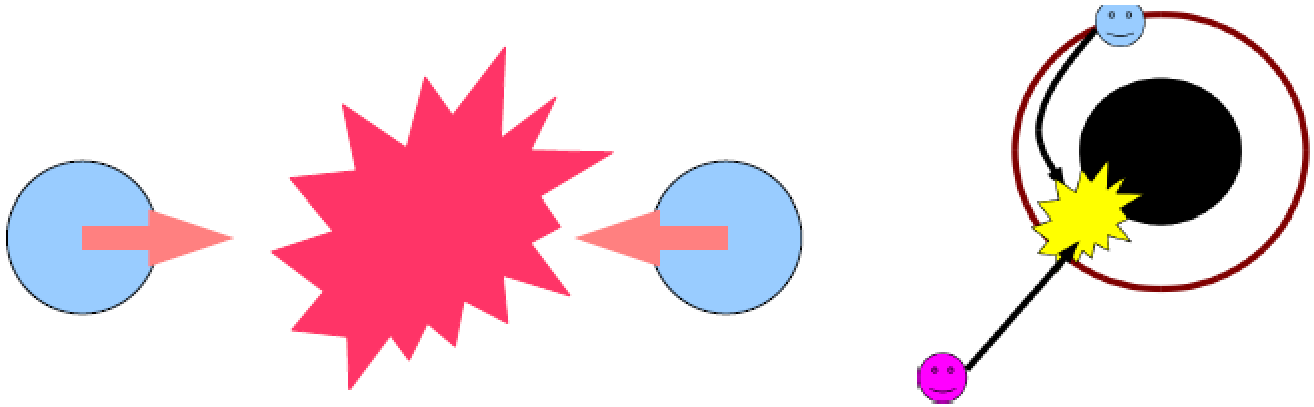
the horizons are independent of the mass :

$$x_h^{\pm} = \pm\frac{1}{\sqrt{\eta}} \ \ (\eta > 0)$$

The temperature is then given simply by $T_H = \frac{M}{\pi}.$

This is a typical relation between the Hawking temperature and the mass of black holes in dimensions

CM ENERGY OF PARTICLE COLLISION NEAR THE HORIZON OF THE 1+1 -D HL BLACK HOLE



- The CM energy of particles 1 and 2 at the same spacetime point is given by

$$E_{\rm cm}^2 = -(p_1 + p_2)^a (p_1 + p_2)_a = m_1^2 + m_2^2 - 2 g_{ab} p_1^a p_2^b.$$

- Coordinate-independent and in principle observable

CM ENERGY OF PARTICLE COLLISION NEAR THE HORIZON OF THE 1+1 -D HL BLACK HOLE

- Lagrangian equation $\mathcal{L} = -\frac{1}{2}[-N(x)^2\left(\frac{dt}{d\tau}\right)^2 + \frac{1}{N(x)^2}\left(\frac{dx}{d\tau}\right)^2]$
- The canonical momenta calculated as

$$p_t = \frac{d\mathcal{L}}{d\dot{t}} = N(x)^2\dot{t} = E$$

- Hence, $\dot{t} = \frac{E}{N(x)^2}$

- $p_x = \frac{d\mathcal{L}}{d\dot{x}} = \frac{-\dot{x}}{N(x)^2}$ and by using the normalization condition for time-like particles $g_{tt}(u^t)^2 + g_{xx}(u^x)^2 = -1$

CM ENERGY OF PARTICLE COLLISION NEAR THE HORIZON OF THE 1+1 -D HL BLACK HOLE

- The two-velocities can be written as,

$$u^t = \dot{t} = \frac{E}{N(x)^2}$$

$$u^x = \dot{x} = \sqrt{-V_{eff}} = \sqrt{N(x)^2 - E^2}.$$

- The CM energy is given by $E_{cm} = \sqrt{2}\sqrt{(1 - g_{\mu\nu} u_1^{\mu} u_2^{\nu})}$

$$\frac{E_{cm}^2}{2} = \left(1 + \frac{E_1 E_2}{N(x)^2} - \frac{\sqrt{E_1^2 - N(x)^2}\sqrt{E_2^2 - N(x)^2}}{N(x)^2}\right)$$

CM ENERGY OF PARTICLE COLLISION NEAR THE HORIZON OF THE 1+1 -D HL BLACK HOLE

- The lowest order term gives the CM energy of two particles as

$$\frac{E_{cm}^2}{2} = 1 + \frac{E_1 E_2 - |E_1 E_2|}{N(x)^2} + \frac{(E_1 E_2)^2}{2\,|E_1 E_2|}$$

- There are two cases for this CM energy, when $E_1 E_2 < 0$, the CM energy is reduced to $\frac{E_{cm}^2}{2} = 1 - \frac{2\,|E_1 E_2|}{N(x)^2}$

which is unbounded for $x \longrightarrow x_h.$

- when $E_1 E_2 > 0$, $E_{cm}^2 = \frac{(E_1 + E_2)^2}{|E_1 E_2|}$

SOME EXAMPLES

$E_1E_2 < 0$

- Schwarzchild-like Solution: $N(x)^2 = 1 - \frac{2M}{x}$

$x \longrightarrow x_h = 2M$

$$E_{cm}^2(x \longrightarrow x_h) = \infty$$

- Reissner-Nordstrom-like solution: $N(x)^2 = 1 - \frac{2M}{x} + \frac{Q^2}{x^2}$

$$E_{cm}^2(x \longrightarrow x_{h=M+\sqrt{(M^2-Q^2)}}) = \infty$$

When the location of particle 1 which has positive energy approachs the horizon , on the other hand the particle 2 escaping from the horizon with negative energy might give us the BSW effect

SOME EXAMPLES

$E_1 E_2 < 0$

- **The Non-Black Hole case:** $N(x)^2 = 2Mx - 1.$

$x \longrightarrow x_h = \frac{1}{2M}$

$$E_{cm}^2(x \longrightarrow x_h) = \infty$$

- **The Extremal case of the Reissner-Nordstrom like black hole:** $N(x)^2 = \left(1 - \frac{M}{x}\right)^2$ $E_{cm}^2(x \longrightarrow x_h) = \infty.$

- **Specific New Black Hole Case:** $N(x)^2 = 2Mx - \frac{2M}{\eta x}$

$$E_{cm}^2(x \longrightarrow x_h) = \infty$$

Hence the BSW effect arises here as well.

HAWKING PHOTON VERSUS AN INFALLING PARTICLE

- The massless photon of such an emission can naturally scatter an infalling particle or vice versa. This phenomenou is analogous to a Compton scattering taking place in 1+1-dimensions. Null-geodesics for a photon can be described simply by

$$\frac{dt}{d\lambda} = \frac{E_1}{N^2}$$
$$\frac{dx}{d\lambda} = \pm\sqrt{E_1^2 - N^2}$$

- The center-of-mass energy of a Hawking photon and the infalling particle can be taken now as

$$E_{cm}^2 = -(p^\mu + k^\mu)^2$$

$$E_{cm}^2 = m^2 - 2mg_{\mu\nu}u^\mu k^\nu,$$

HAWKING PHOTON VERSUS AN INFALLING PARTICLE

- The center-of-mass energy of a Hawking photon and the infalling particle can be taken now as

$$E_{cm}^2 = m^2 + \frac{2mE_1}{N^2}\left(E_2 + \sqrt{E_2^2 - N^2}\right).$$

- In the near horizon limit this reduces to
- $$E_{cm}^2 = m^2 + \frac{2mE_1}{N^2}\left(E_2 + |E_2| - \frac{N^2}{2|E_2|}\right).$$
- Note that for $E_2 < 0$ we have $$E_{cm}^2 = m^2\left(1 - \frac{E_1}{m|E_2|}\right)$$
- which is finite and therefore is not of interest. On the other hand for $E_2 > 0$, we obtain an unbounded one.

CONCLUSION

- Our aim is to show that the BSW effect which arises in higher dimensional black holes applies also in the 1+1- D.
- In other words the strong gravity near the event horizon effects the collision process with unlimeted source to turn it into a natural accelerator.
- Key property: presence of event horizon plus critical
- particle
- Finally, we must admit that absence of rotational effects in 1+1- D confines the problem to the level of a toy model.


THANK YOU